

# GLOBAL FITS OF PROTON PDFs WITH NON-LINEAR CORRECTIONS

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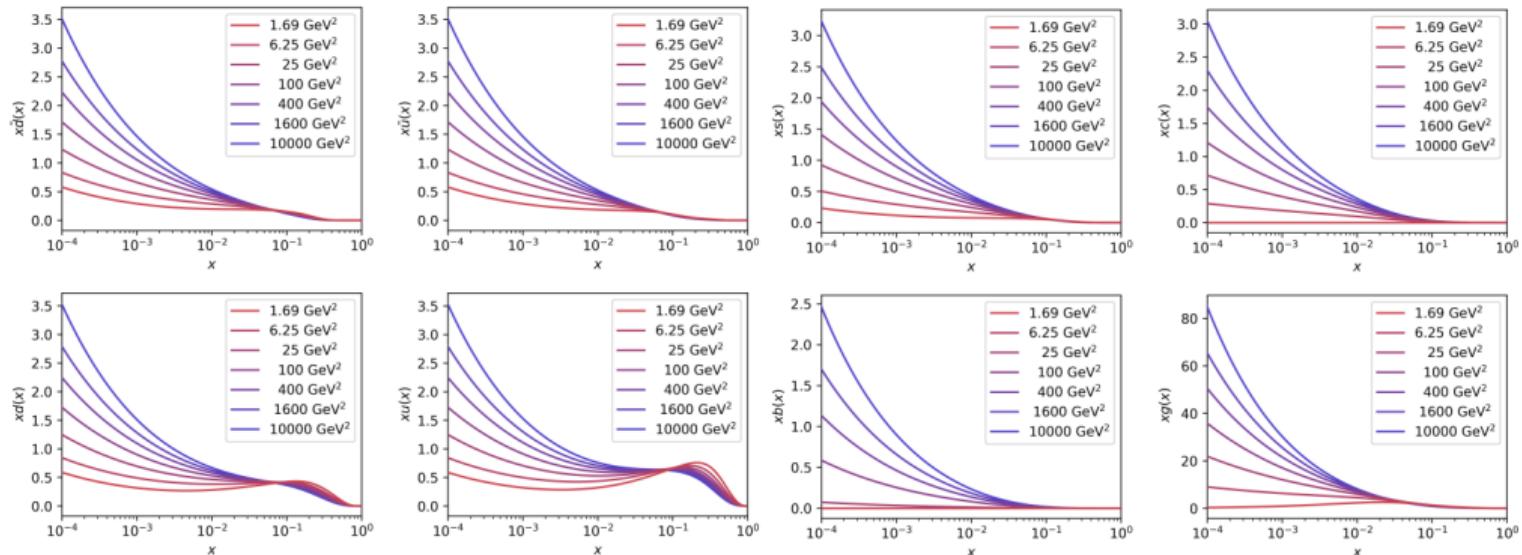


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# REMINDER - DGLAP EVOLUTION

$$Q^2 \frac{d}{dQ^2} \left( \frac{f_i(x, Q^2)}{f_g(x, Q^2)} \right) = \sum_j \frac{\alpha_s}{2\pi} \int_x^1 \frac{d\xi}{\xi} \begin{pmatrix} P_{q_i q_j} \left( \frac{x}{\xi} \right) & P_{q_i g} \left( \frac{x}{\xi} \right) \\ P_{g q_j} \left( \frac{x}{\xi} \right) & P_{gg} \left( \frac{x}{\xi} \right) \end{pmatrix} \left( f_j(\xi, Q^2) \right) \left( f_g(\xi, Q^2) \right)$$



## PROBLEM

Rapidly rising gluon at small  $x$  and large  $Q^2$  violates unitarity.

# GLUON RECOMBINATION — GLR-MQ EQUATION

[Phys. Rep. 100 (1983) 1, Nucl. Phys. B268 (1986) 427]

Based on double-leading-logarithmic approximation in  $Q^2$  and  $\frac{1}{x}$

$$\begin{aligned}\frac{dx_B G(x_B, Q^2)}{d \ln Q^2} &= \text{linear terms} - 5.05 \left( \frac{\alpha_s}{RQ} \right)^2 \int_{x_B}^{x_0} \frac{dx_1}{x_1} [x_1 G^2(x_1, Q^2)]^2 \\ \frac{dx_B S(x_B, Q^2)}{d \ln Q^2} &= \text{linear terms} - 0.0010625 \left( \frac{\alpha_s}{RQ} \right)^2 [x_1 G^2(x_1, Q^2)]^2 \\ &\quad - 0.32 \frac{\alpha_s}{Q^2} \int_{x_B}^{x_0} \frac{dx_1}{x_1} \frac{x_B}{x_1} P_{MQ}^{GG \rightarrow q\bar{q}} x_1 H(x_1, Q^2)\end{aligned}$$

with

$$\frac{dx_1 H(x_1, Q^2)}{d \ln Q^2} = - 5.05 \left( \frac{\alpha_s}{RQ} \right)^2 \int_{x_B}^{x_0} \frac{dz}{z} [z G^2(z, Q^2)]^2$$

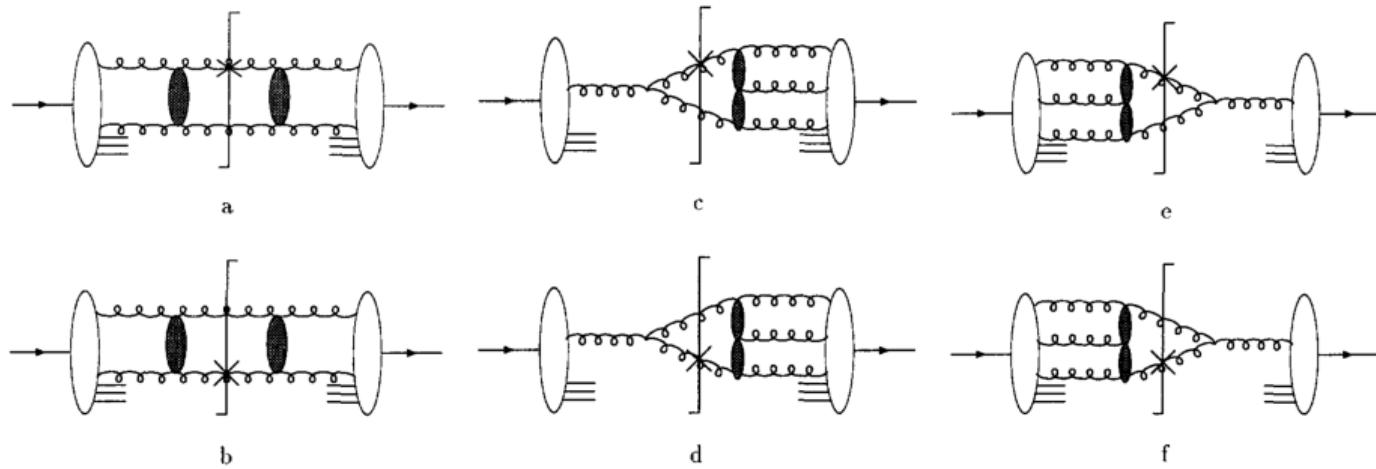
- ▶  $\frac{dx_B S(x_B, Q^2)}{d \ln Q^2}$  does not appear naturally; requires special treatments, i.e. mixing in NLL contributions
- ▶ **Violates momentum sum rules**

## ZHU + RUAN APPROACH

[Nucl. Phys. B 559 (1999), 378-392]

Based on leading logarithmic approximation in  $Q^2$

- ▶ Valid over the entire  $x$  range
- ▶ Includes transitions to quarks (and can be extended to  $q\bar{q} \rightarrow G$ , etc.)



- ▶  $2 \rightarrow 2$  diagrams lead to antiscreening (a,b)
- ▶  $2 \rightarrow 3$  diagrams lead to screening (c,d,e,f)
- ▶ Same recombination functions, but different kinematic regimes

## ZHU + RUAN APPROACH

[Nucl. Phys. B 559 (1999), 378-392]

$$\frac{dx_B G(x_B, Q^2)}{d \ln Q^2} = \text{linear terms} + \frac{9}{32\pi^2} \left( \frac{1}{RQ} \right)^2 \int_{x_B/2}^{1/2} dx_1 x_B x_1 G^2(x_1, Q^2) \sum_i P_i^{GG \rightarrow G(x_1, x_B)}$$

$$- \frac{9}{16\pi^2} \left( \frac{1}{RQ} \right)^2 \int_{x_B}^{1/2} dx_1 x_B x_1 G^2(x_1, Q^2) \sum_i P_i^{GG \rightarrow G(x_1, x_B)}$$

$$\frac{dx_B S(x_B, Q^2)}{d \ln Q^2} = \text{linear terms} + \frac{9}{32\pi^2} \left( \frac{1}{RQ} \right)^2 \int_{x_B/2}^{1/2} dx_1 x_B x_1 G^2(x_1, Q^2) \sum_i P_i^{GG \rightarrow q\bar{q}(x_1, x_B)}$$

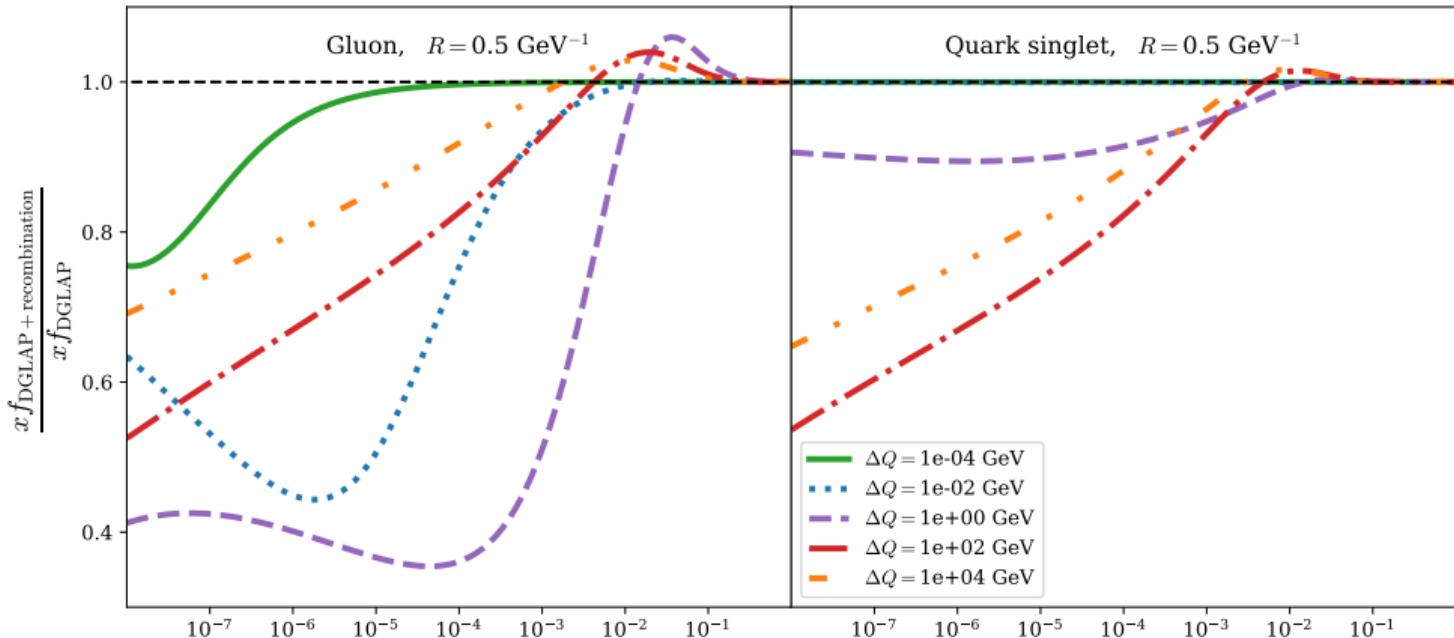
$$- \frac{9}{16\pi^2} \left( \frac{1}{RQ} \right)^2 \int_{x_B}^{1/2} dx_1 x_B x_1 G^2(x_1, Q^2) \sum_i P_i^{GG \rightarrow q\bar{q}(x_1, x_B)}$$

- The parameter  $R$  can be interpreted as the size of the transverse area where gluon overlap leads to non-linear corrections.

$$\left. \begin{aligned} \frac{d \int_0^1 dx_B x_B G(x_B, Q^2)}{d \ln Q^2} &= 0 \\ \frac{d \int_0^1 dx_B x_B q(x_B, Q^2)}{d \ln Q^2} &= 0 \end{aligned} \right\} \Rightarrow \text{Momentum is conserved.}$$

## RESULTS - $Q$ -DEPENDENCE OF NON-LINEAR CORRECTIONS

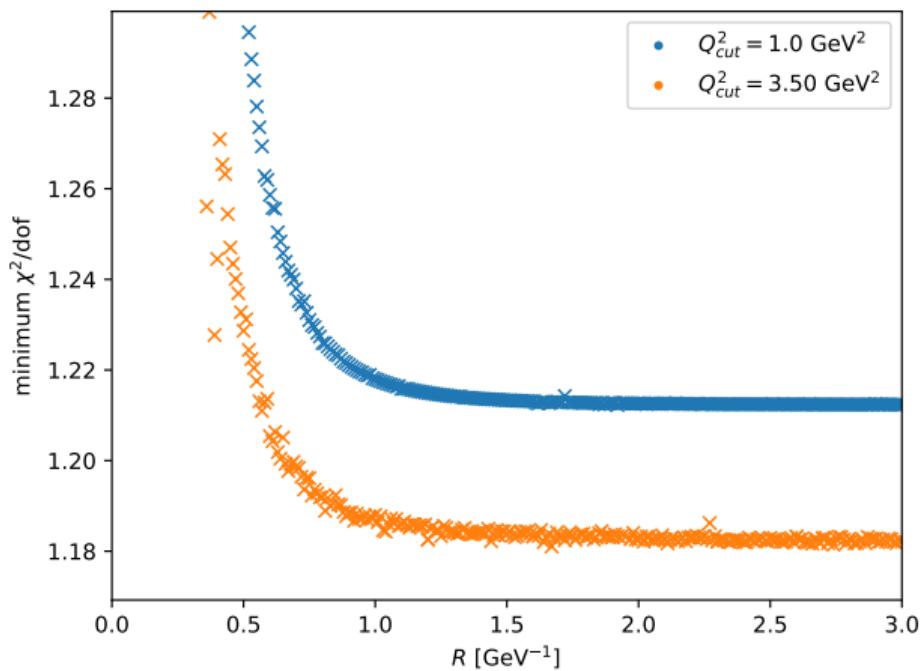
- Non-linear corrections applied to the evolution of a fixed set of PDFs (CJ15)



- Largest correction at  $Q = \mathcal{O}(1) \text{ GeV}$  for gluons and  $Q = \mathcal{O}(100) \text{ GeV}$  for quarks

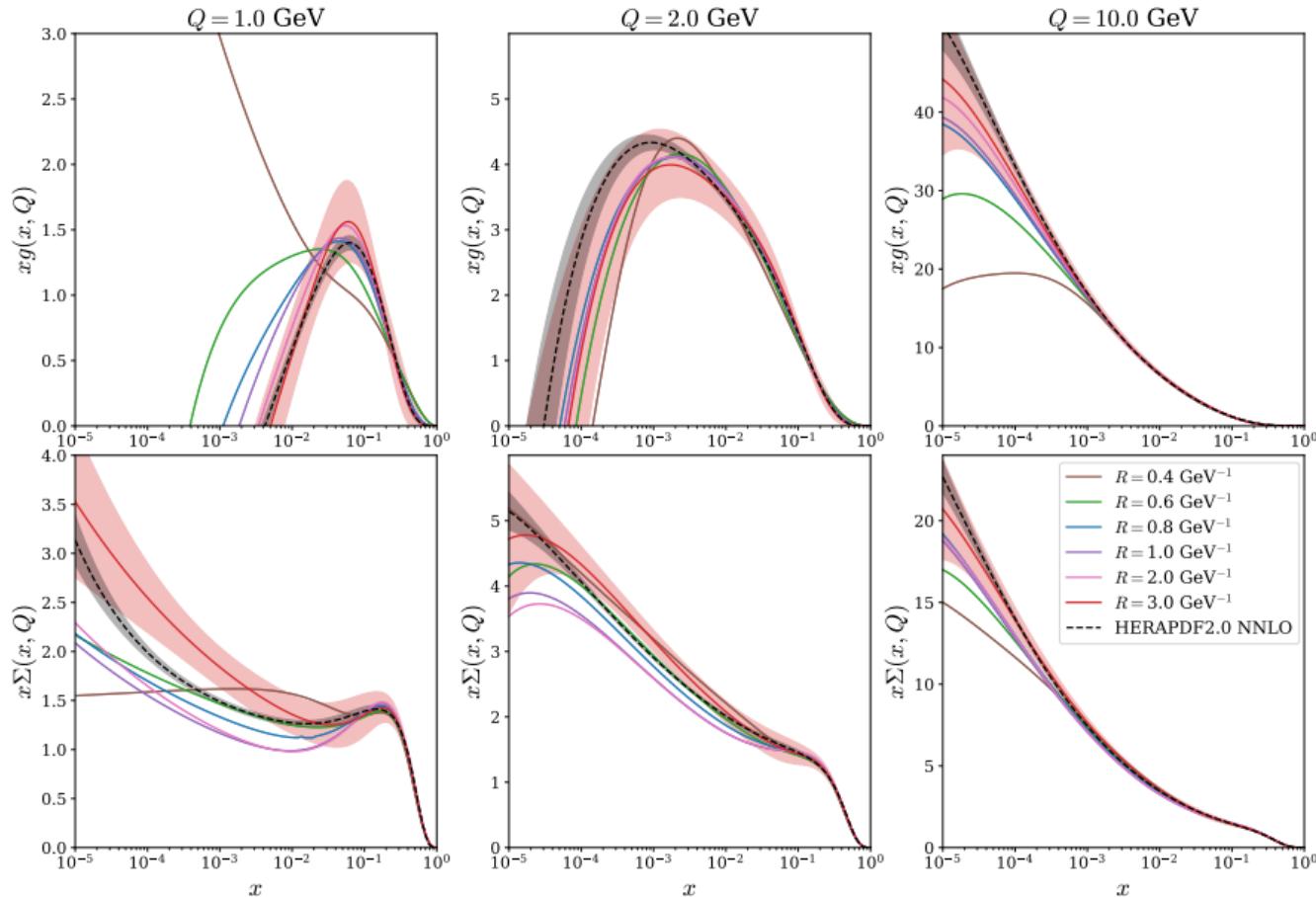
## *R*-DEPENDENT PDF FITS

- ▶ HERAPDF2.0 parameterization and methodology, NNLO
- ▶ 1568 / 1636 data points (BCDMS, HERA and NMC DIS)

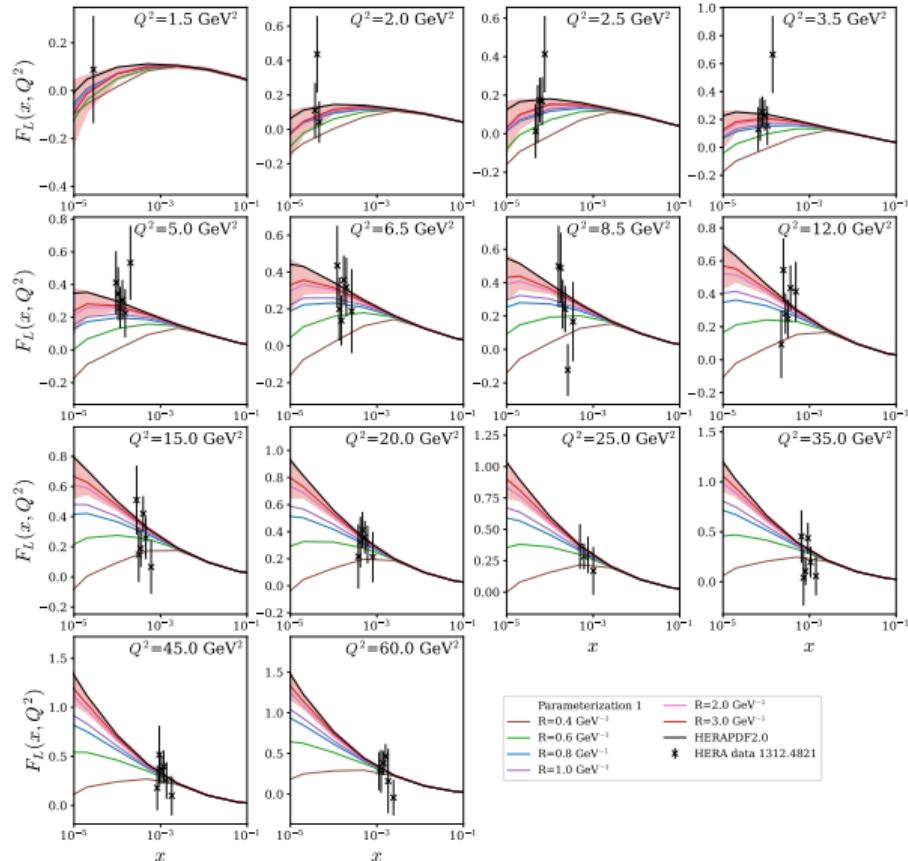


The entire procedure has also been repeated with a different parameterization for the gluon PDF, but no significant differences were observed (see paper for details)

# $R$ -DEPENDENT PDF FITS

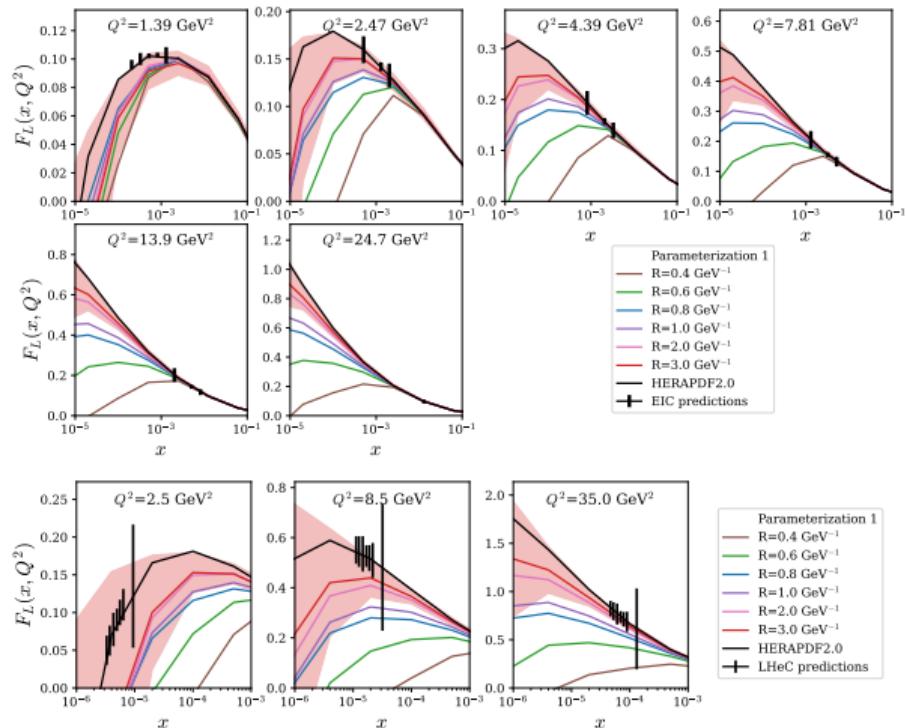


# COMPARING TO HERA $F_L$ DATA



- ▶ Data is well described for  $R > 0.6 \text{ GeV}^{-1}$
- ▶ Cannot distinguish between higher  $R$  values due to either  $x$  being too high or the uncertainties too large

# COMPARING TO $F_L$ PREDICTIONS FOR FUTURE EXPERIMENTS



[E. Aschenauer et al., <https://www.phenix.bnl.gov/WWW/publish/elke/EIC/EIC-R&D-Tracking/Meetings/fl.pdf>]

[P. Agostini et al., J.Phys.G 48 (2021) 11, 110501]

- ▶ Future data will be significantly more sensitive to recombination effects

# CONCLUSIONS AND OUTLOOK

## Conclusions

- ▶ Gluon recombination offers a possible explanation for saturation
- ▶ GLR(-MQ) equations violate momentum conservation → calculation by Zhu+Ruan avoids this problem
- ▶ Implemented in HOPPET + xFitter to produce new global proton PDF fits
- ▶ Current DIS data shows no signs of gluon recombination → Lower bound  $R > 0.5 \text{ GeV}^{-1}$

## Outlook

- ▶ Tools (modified HOPPET) and results (LHAPDFs) will be made available
- ▶ Future EIC / LHeC data will put tighter constraints on strength of non-linear effects