Next-to-leading Order Photon+Jet Cross Section

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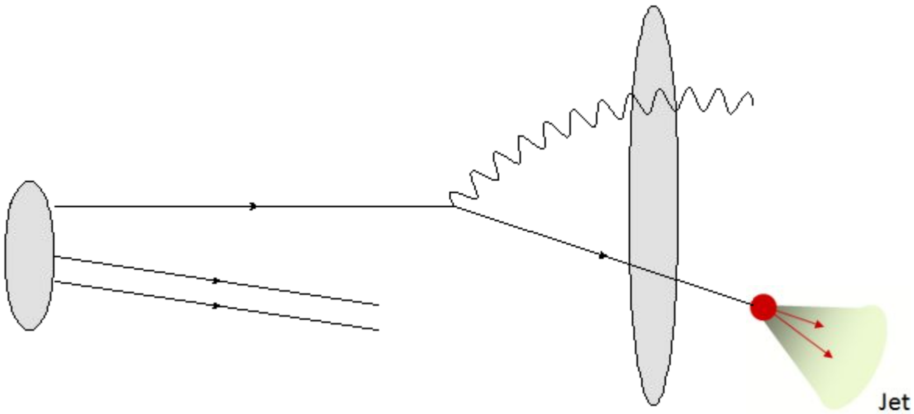
Forward Jet Production

The basic setup: a large-x parton from the proton scatters off the small-x gluon distribution in the target nucleus. The large-x parton is most likely a quark.
We adopt the formalism of the LC outgoing state, using the CGC effective theory together with the hybrid factorization.



Quark emitting a photon in the presence of a shockwave.

The Evolved Incoming State

The time evolution of the initial (bare) quark state is given by:

$$\left|q_\lambda^\alpha(q^+,\,\boldsymbol{q})\right\rangle_{in} \equiv U(0,\,-\infty)\,\left|q_\lambda^\alpha(q^+,\,\boldsymbol{q})\right\rangle$$

Where $U(t,\,t_0)$ denotes the evolution operator, defined as

$$U(t,\,t_0) = T\,\exp\left\{-i\int_{t_0}^{t} dt_1\,V(t_1)\right\}$$

Where V denotes the interaction part of the QCD Hamiltonian.
The expansion for U corresponds to Dyson's series:

$$U(t,t_0) = 1 - i\int_{t_0}^{t} dt_1 V(t_1) + (-i)^2\int_{t_0}^{t} dt_1 \int_{t_0}^{t_1} dt_2 V(t_1)V(t_2) + \cdots$$
$$+ (-i)^n \int_{t_0}^{t} dt_1 \int_{t_0}^{t_1} dt_2 \cdots \int_{t_0}^{t_{n-1}} dt_n V(t_1)V(t_2)\cdots V(t_n) + \cdots.$$

The state which encodes the information both on the ***time evolution*** and ***interaction with the target nucleus*** is:

$$\left|q_\lambda^\alpha(q^+,\,\boldsymbol{q})\right\rangle_{out} \equiv U(\infty,\,0)\,\hat{S}\,U(0,\,-\infty)\,\left|q_\lambda^\alpha(q^+,\,\boldsymbol{q})\right\rangle$$

hep-ph/0106240,
A. Kovner and U.
Wiedemann

The LO Forward Photon+Jet Cross Section

From the production state we can pass easily to the quark-gluon dijet cross section:

$$\frac{d\sigma_{\rm LO}^{qA\to q\gamma+X}}{d^3k\,d^3p}(2\pi)\delta(k^+ + p^+ - q^+) \equiv \frac{1}{2N_c}\,{}_{q\gamma}^{out}\langle q_\lambda^\alpha(q^+,\,\boldsymbol{q})|\,\hat{\mathcal{N}}_q(p)\,\hat{\mathcal{N}}_\gamma(k)\,|q_\lambda^\alpha(q^+,\,\boldsymbol{q})\rangle_{q\gamma}^{out}$$

The following number density operators were introduced:

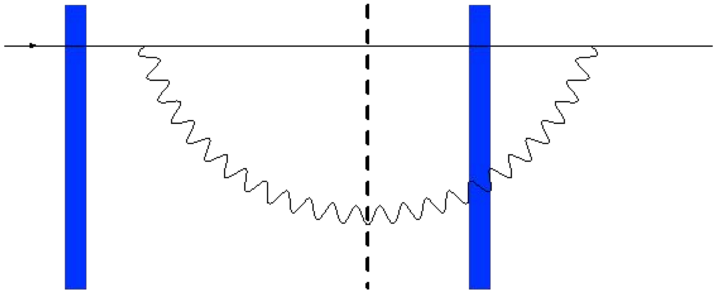
$$\hat{\mathcal{N}}_q(p) \equiv \frac{1}{(2\pi)^3}\,b_\lambda^{\alpha\dagger}(p)\,b_\lambda^\alpha(p) \qquad \hat{\mathcal{N}}_\gamma(k) = \frac{1}{(2\pi)^3}\,\alpha_i^\dagger(k)\,\alpha_i(k)$$

Then the result for the leading-order dijet cross section is given by:

$$\frac{d\sigma_{\rm LO}^{qA\to q\gamma+X}}{dk^+\,d^2\boldsymbol{k}\,dp^+\,d^2\boldsymbol{p}} = \frac{2\alpha_{e.m.}}{(2\pi)^6 q^+}\boxed{\frac{1+(1-\vartheta)^2}{\vartheta}}\boxed{\delta(q^+ - k^+ - p^+)}$$
$$\times\int_{\boldsymbol{x},\,\overline{\boldsymbol{x}},\,\boldsymbol{y},\,\overline{\boldsymbol{y}}}\boxed{\frac{\boldsymbol{R}\cdot\overline{\boldsymbol{R}}}{\boldsymbol{R}^2\,\overline{\boldsymbol{R}}^2}}\,{\rm e}^{-i\boldsymbol{p}\cdot(\boldsymbol{x}-\overline{\boldsymbol{x}})-i\boldsymbol{k}\cdot(\boldsymbol{y}-\overline{\boldsymbol{y}})}$$
$$\times\boxed{\left[\mathcal{S}\left(\boldsymbol{w},\,\overline{\boldsymbol{w}}\right) - \mathcal{S}\left(\boldsymbol{x},\,\overline{\boldsymbol{w}}\right) - \mathcal{S}\left(\boldsymbol{w},\,\overline{\boldsymbol{x}}\right) + \mathcal{S}\left(\boldsymbol{x},\,\overline{\boldsymbol{x}}\right)\right]}$$

hep-ph/0205037v1
By F. Gelis and J. J. Marian

An example for contribution which is included in the leading order result:



The dashed line, “the cut”, is the final state (the detector). The dipole is defined by:

$$\mathcal{S}(\overline{\boldsymbol{w}}, \boldsymbol{w}) \equiv \frac{1}{N_c} \operatorname{tr}\left[V^{\dagger}(\overline{\boldsymbol{w}})\, V(\boldsymbol{w})\right]$$

From the partonic cross section we can find the quark channel contribution by convolution with the PDF:

$$\left.\frac{d\sigma_{\mathrm{LO}}^{pA\to q\gamma+X}}{d^3p\, d^3k}\right|_{q-channel} = \int dx_p\, q_f(x_p, \mu^2)\, \frac{d\sigma_{\mathrm{LO}}^{qA\to q\gamma+X}}{d^3p\, d^3k}$$

For measuring a photon+jet one has to convolute the result above with fragmentation/jet function:

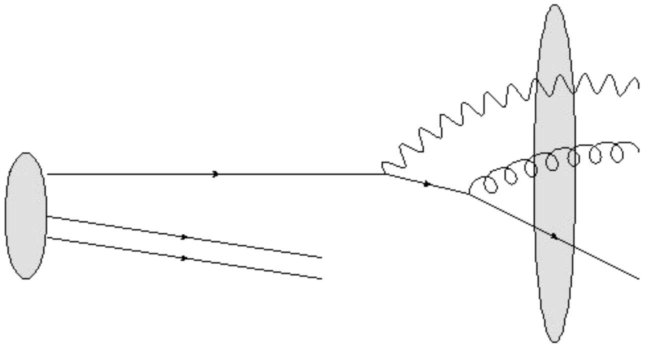
$$\frac{d\sigma_{LO}^{pA\to jet+\gamma+X}}{d^3p\, d^3k} = \int \frac{dz_1}{z_1^3} \int \frac{dz_2}{z_2^3} \int dx_p\, q_f(x_p, \mu^2)\, \frac{d\sigma_{LO}^{pA\to q\gamma+X}}{d^3p\, d^3k} D_{jet/q}(z_1)\, J_\gamma(z_2)$$

The Dijet+Photon Setup

Two possible configurations of 3 particles in the final state which are relevant for the cross section at order $\alpha_s \, \alpha_{e.m.}$:

a) ***Photon, quark and a gluon (quark channel)***,
b) ***Quark, anti-quark and a photon (gluon channel)***.

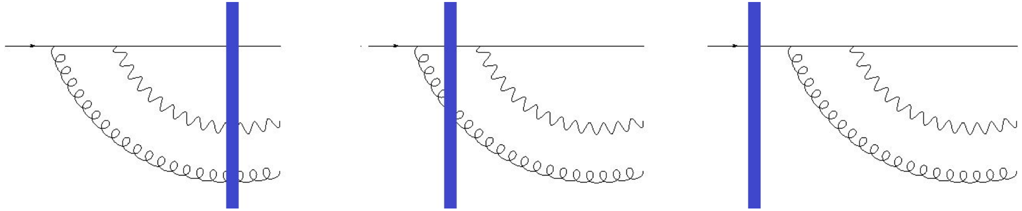
The production of these configurations happen via two successive parton splittings (in the light-cone formalism, there are also 1->3 instantaneous vertices).



An example for a contribution with 3 particles in the final state

From Photon+Dijet to “real” NLO Photon+Jet

Direct and interference (regular and instantaneous emissions):



hep-ph/1802.01398v1,
hep-ph/1810.11273.

The leading-order contribution to the photon+dijet cross section is:

$$\frac{d\sigma_{\mathrm{LO}}^{qA\to qg\gamma+X}}{d^3q_1\, d^3q_2\, d^3q_3}(2\pi)\delta(q_1+q_2+q_3-q^+) \equiv \frac{1}{2N_c}\ {}_{qg\gamma}\langle q_\lambda^\alpha(q^+,\boldsymbol{q})|\ \hat{\mathcal{N}}_q(q_1)\hat{\mathcal{N}}_g(q_2)\hat{\mathcal{N}}_\gamma(q_3)\ |q_\lambda^\alpha(q^+,\boldsymbol{q})\rangle_{qg\gamma}$$

The hadronic cross section is given by:

$$\frac{d\sigma^{pA\to q\gamma g+X}}{d^3q_\gamma\, d^3q_q\, d^3q_g} = \int dx_p\, q_f(x_p,\, \mu^2)\, \frac{d\sigma^{qA\to q\gamma g+X}}{d^3q_\gamma\, d^3q_q\, d^3q_g}$$

The real contribution for the NLO photon+jet cross section is related to photon+jet cross section by the integration over the unmeasured gluon:

$$\frac{d\sigma_{Rnlo}^{qA\to q\gamma+X}}{d^3q_\gamma\, d^3q_q} = \int d^3q_g\, \frac{d\sigma^{qA\to q\gamma g+X}}{d^3q_\gamma\, d^3q_q\, d^3q_g}$$

hep-ph/2009.11930

The Real Cross Section

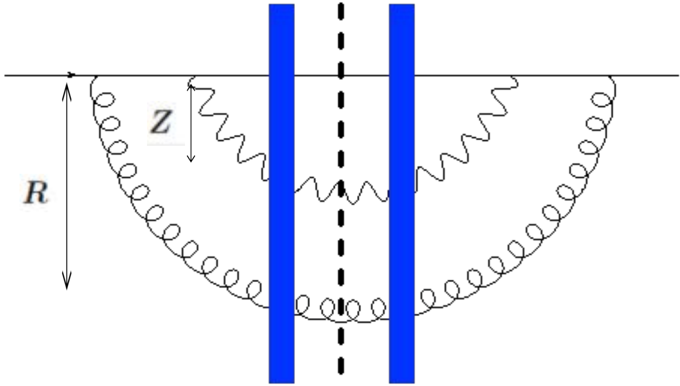
The quark quark anti-quark contribution reads:

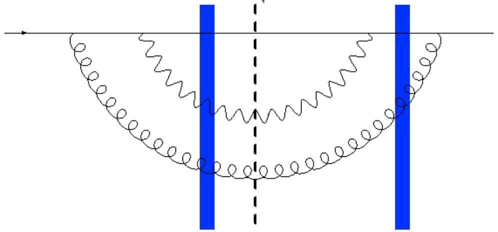
$$\frac{d\sigma^{qA\to q\gamma g+X}}{dk_1^+\, d^2\boldsymbol{k}_1\, dk_2^+\, d^2\boldsymbol{k}_2\, dk_3^+\, d^2\boldsymbol{k}_3} = \frac{\alpha_s\, \alpha_{e.m.}\, C_F}{2(2\pi)^{10}(q^+)^2}\,\delta(q^+ - k_1^+ - k_2^+ - k_3^+)$$

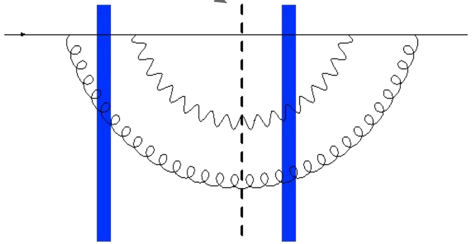
$$\times \int_{\overline{\boldsymbol{x}},\, \overline{\boldsymbol{z}},\, \overline{\boldsymbol{z}}',\, \boldsymbol{x},\, \boldsymbol{z},\, \boldsymbol{z}'} e^{-i\boldsymbol{k}_1\cdot(\boldsymbol{x}-\overline{\boldsymbol{x}}) - i\boldsymbol{k}_2\cdot(\boldsymbol{z}-\overline{\boldsymbol{z}}) - i\boldsymbol{k}_3\cdot(\boldsymbol{z}'-\overline{\boldsymbol{z}}')} \boxed{\frac{\boldsymbol{R}^i\, \boldsymbol{Z}^j\, \overline{\boldsymbol{R}}^m\, \overline{\boldsymbol{Z}}^n}{\boldsymbol{Z}^2\, \overline{\boldsymbol{Z}}^2}}$$

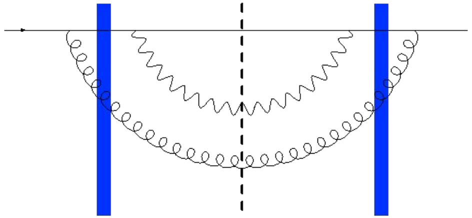
$$\times \Big[\boxed{\mathcal{K}_0^{ijmn}(\boldsymbol{x},\, \boldsymbol{z},\, \boldsymbol{z}',\, \overline{\boldsymbol{x}},\, \overline{\boldsymbol{z}},\, \overline{\boldsymbol{z}}',\, \vartheta,\, \xi)}\, \boxed{\mathcal{W}_0\left(\boldsymbol{z},\, \boldsymbol{z}',\, \overline{\boldsymbol{z}},\, \overline{\boldsymbol{z}}'\right)}$$

$$\boxed{-\, (\boldsymbol{z},\, \boldsymbol{z}' \to \boldsymbol{y}) \,-\, (\overline{\boldsymbol{z}},\, \overline{\boldsymbol{z}}' \to \overline{\boldsymbol{y}}) \,+\, (\boldsymbol{z},\, \boldsymbol{z}' \to \boldsymbol{y}\ \&\ \overline{\boldsymbol{z}},\, \overline{\boldsymbol{z}}' \to \overline{\boldsymbol{y}})}\ \Big]$$





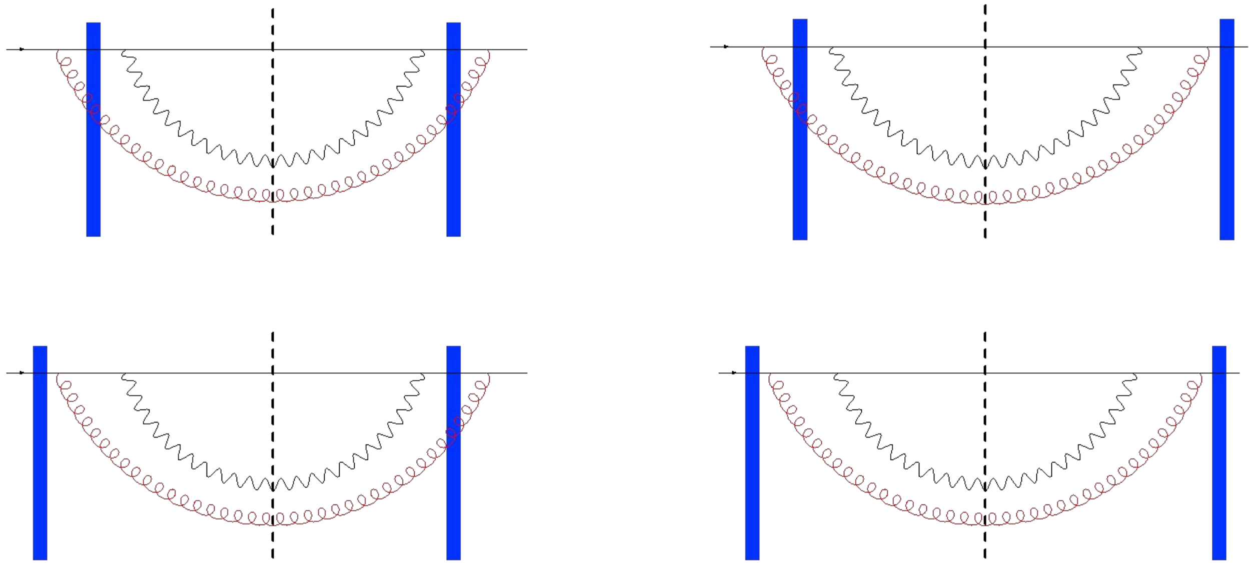




Similar structure to hep-ph/2009.11930.

Recovering the JIMWLK Evolution

In the limit where the gluon become soft (eikonal emission vertex = no recoil of the emitter), the general NLO result has to reduce to one step in the real part of the production JIMWLK evolution of the LO photon+jet production cross section. We managed to show that this is indeed the case in our result.



The four diagrams precisely reproduce the the BK evolution of the dipole $\mathcal{S}\left(\boldsymbol{w}, \overline{\boldsymbol{w}}\right)$ of the leading order cross-section.

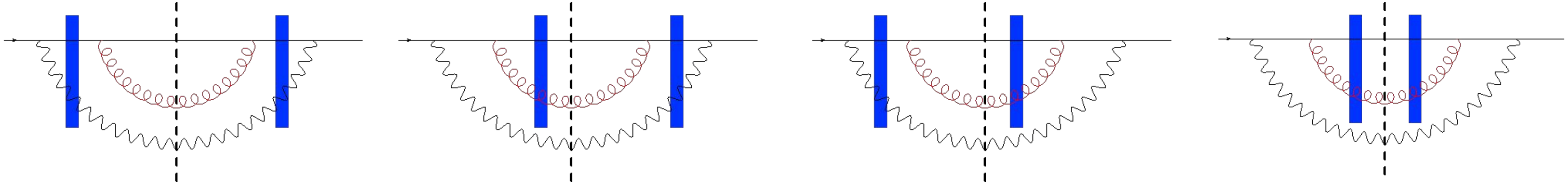
$$\frac{d\sigma^{qA\to q\gamma+X}_{\mathrm{NLO},1}}{dk_1^+\, d^2\boldsymbol{k}_1\, dk_3^+\, d^2\boldsymbol{k}_3} \simeq \boxed{\frac{\alpha_{e.m.}}{(2\pi)^5}\, \frac{1+(1-\vartheta)^2}{2\vartheta q^+}\, \delta(q^+ - k_1^+ - k_3^+) \times \int_{\overline{\boldsymbol{x}}, \overline{\boldsymbol{z}}', \boldsymbol{x}, \boldsymbol{z}'} e^{-i\boldsymbol{k}_1\cdot(\boldsymbol{x}-\overline{\boldsymbol{x}}) - i\boldsymbol{k}_3\cdot(\boldsymbol{z}'-\overline{\boldsymbol{z}}')}\, \frac{(\boldsymbol{x}-\boldsymbol{z}')\cdot(\overline{\boldsymbol{x}}-\overline{\boldsymbol{z}}')}{(\boldsymbol{x}-\boldsymbol{z}')^2(\overline{\boldsymbol{x}}-\overline{\boldsymbol{z}}')^2}}$$

$$\times \boxed{\frac{\bar{\alpha}_s}{2\pi} \int_0^1 \frac{d\xi}{\xi} \int_{\boldsymbol{z}} \frac{2(\boldsymbol{w}-\boldsymbol{z})\cdot(\overline{\boldsymbol{w}}-\boldsymbol{z})}{(\boldsymbol{w}-\boldsymbol{z})^2(\overline{\boldsymbol{w}}-\boldsymbol{z})^2} \left[\mathcal{S}\left(\boldsymbol{w}, \overline{\boldsymbol{w}}\right) - \mathcal{S}(\boldsymbol{w}, \boldsymbol{z})\,\mathcal{S}(\boldsymbol{z}, \overline{\boldsymbol{w}})\right]}$$

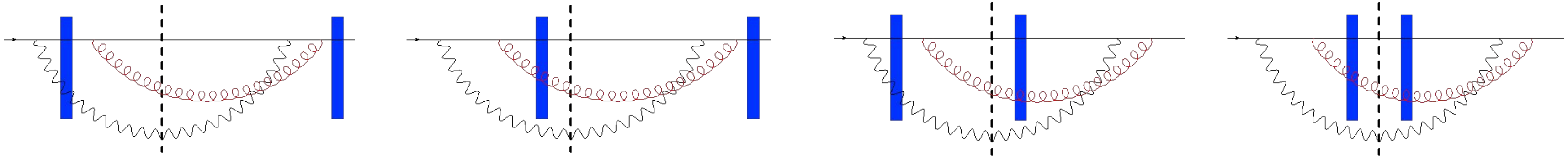
LO result

BK evolution of $\mathcal{S}\left(\boldsymbol{w}, \overline{\boldsymbol{w}}\right)$

Similarly, the following diagrams generate the evolutions of the dipole $\mathcal{S}\left(\boldsymbol{x}, \overline{\boldsymbol{x}}\right)$:

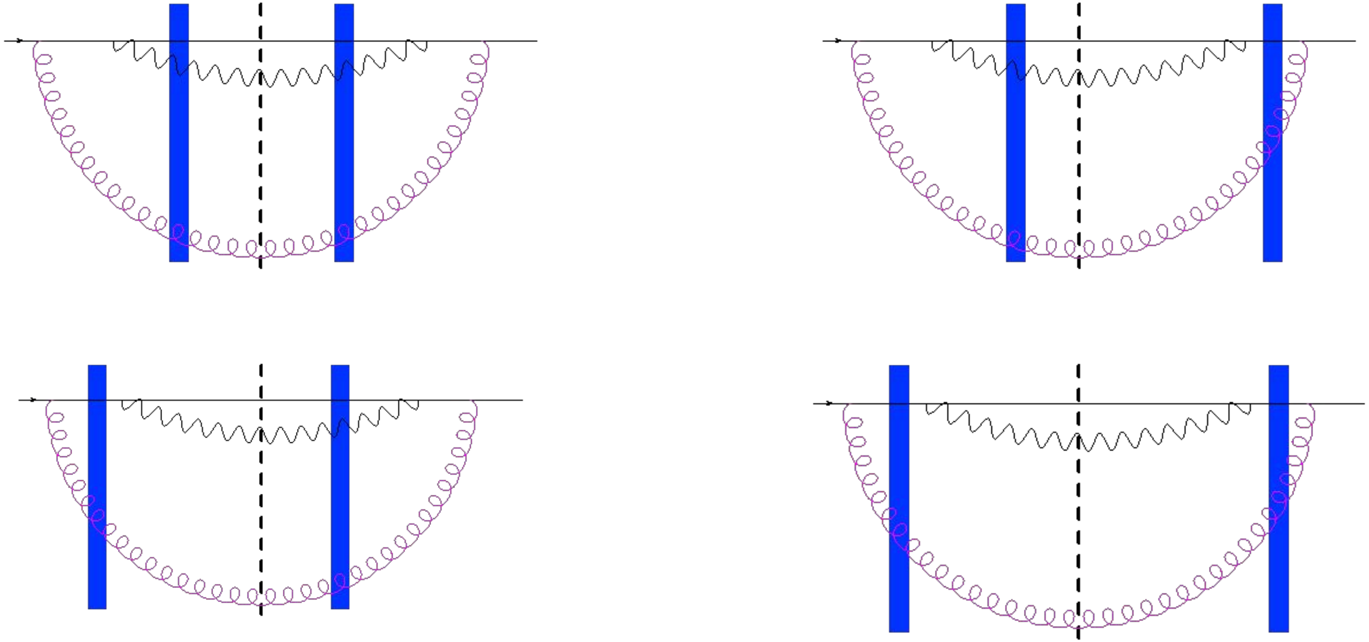


For the dipole $\mathcal{S}\left(\boldsymbol{w}, \overline{\boldsymbol{x}}\right)$:

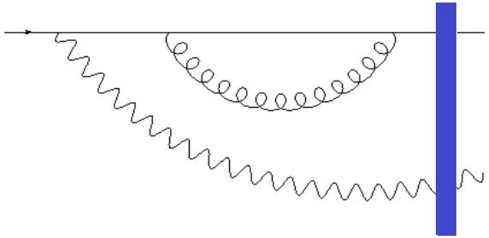


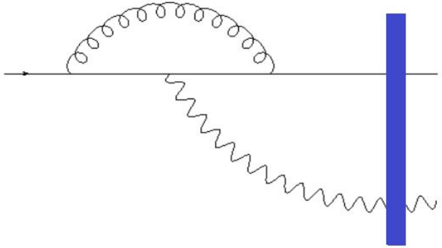
Recovering the Real DGLAP Evolution

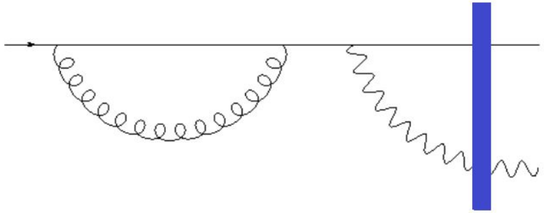
In the collinear limit, when the separation between partons become arbitrarily large, we recover the DGLAP evolution of the initial quark pdf and final quark jet function.

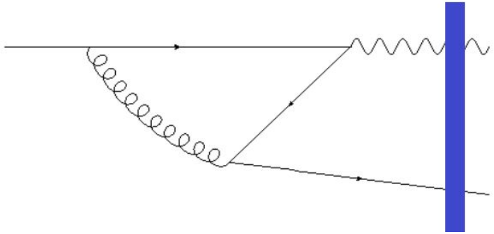


The Virtual Contributions









Both UV and IR divergences are involved. Two IR logs appear:

$$\ln\left(\frac{\Lambda}{q^+}\right)\ln\left(\frac{\tilde{k}^2}{\mu^2_{\overline{MS}}}\right) \qquad\qquad \ln^2\left(\frac{\Lambda}{\vartheta(1-\vartheta)q^+}\right)$$

These are canceled when combining all the various contributions.

Summary

1) The results for the NLO cross section for photon+jet production of an incoming quark are on the way.

2) Short-distance poles has been shown to cancel between pairs of diagrams.
The IR logs are canceled when combining together the virtual diagrams.

3) Partial match has been established between the eikonal (collinear) limit of the result and the JIMWLK (DGLAP) evolution of the LO cross section for photon+jet production.
(complete match with JIMWLK emerges once converting to the new formalism.)