Production of Kaluza-Klein States at LHC and Implication for Dark Matter"

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MOTIVATION

- The Universe has several interesting and intriguing features. Big Bang hypothesis is universally accepted. It is supported by cosmological observations. Our Universe is very cold, very old and it is spatially flat. The standard model of cosmology is based on GR and Cosmological Principle. However, there are reasons to believe cosmological model is not complete.

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- The microscopic world is described by $SU(3) \otimes SU(2) \otimes U(1)$, the standard model (SM). SM is tested with great accuracy. It is also argued that SM is incomplete. GUT was proposed to unify three forces (except gravity). There is no experimental evidence for GUT.

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- Superstring theory (SST) is expected to unify four forces of Nature. It is consistently defined in $D = 10$. We live and do experiments in $D = 4$. A way out is to appeal to Kaluza-Klein compactification: extra 6 spatial dimensions are compact.So far we have no experimental evidence in favour of string theory.

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  Visible Matter: $\sim 4\%$, Dark Matter (DM) $\sim 24\%$, Dark Energy $\sim 71\%$. A simplest way to underrstand Dark Energy is to introduce cosmological constant, Λ, in Einstein-Hilbert action. *Cosmological Constant* problem has not been resolved to every ones satisfaction.

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- $\Omega_{total} = \frac{\rho}{\rho_{crit}}$ where $\rho_{crit} = \frac{3H^2}{8\pi G_N}$; with $H^{-1} = 2998 h^{-1} Mpc$. Here $h = 0.69 \pm 0.06$ is the Hubble constant; *it is the normalized expansion rate*.

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- The consensus is $\Omega_{total} = 1$. The cosmological experiments lead to inference that $\Omega_{DM} = 0.3 \pm .03$. DM is baptized as cold dark matter (CDM) due to its nonrelativistic nature.

- We should keep in mind (a) If DM particles couple to SM multiplets and DM is stable, its decay to SM particles must be prevented by a symmetry. (b) Alternatively, DM might interact very weakly and its lifetime is very long. It behaves effectively like a stable particle if its lifetime is large in cosmological time scale. (c) Or it does not experience forces of SM and interacts only gravitationally.

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- Hope: Cosmological data and SM will explain origin of DM. The limit of CMD mass is in the range 10 GeV to 1000 GeV. Axion, introduced to resolve strong CP problem, is also DM candiate and its mass estimation lies in the range $m_a = 10^{-5} \text{ to } 10^{-2}$ eV; however, there are no experimental evidences of them.

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- If SUSY were discovered in LHC we would have a strong candidate for DM in LSP (neutralino) since R-parity conservation would stabilize LSP. A SUSYSM will resolve two important issues: (i) gauge hierarchy in microscopic physics and (ii) DM in cosmology. Alas! SUSY is yet to be discovered at LHC

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- The Large Radius Compactification (LRC) (ADD and AADD) scenario offers the prospect of observing stringy states and KK states - if radius is order of TeV. ATLAS and CMS put the lower bound on masses to be 2 TeV to 6 TeV - with some inputs from models.

Present Work: AKN+JM

- Strategy and Summary of present work:
  We incorporate following ingredients: (a) LRC paradigm. (b) Universal Extra Dimension (UED) hypothesis. The fields of $D = 4$ theory of SM are promoted to $\hat{D} = 4 + n$ dimensions. $\hat{D}$-dimensional theory is compactification to $D = 4$. The SM spectrum is zero modes of $\hat{D}$-dimensional theory. This proposal is not free from problems. Which theory is problem free!!

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- Rather than proposing one more model, we adopt general principles of local quantum field theories (e.g. LSZ). (a) Existence of Hilbert space. (b) Lorentz invariance and existence unitary representation of Poincaré group. (c) A Poincaré invariant unique vacuum and (d) micro-causality: $[O(x), O(x')] = 0 \textit{ for } (x - x')^2 < 0$. VEV of operators are tempered distributions i.e. their Fourier Transform is polynomially bounded.

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- The analyticity properties of scattering amplitude proved in this framework are nonperturbative.

- We derive bounds on total inelastic scattering cross section for *proton* $+$ *proton* $\rightarrow KK$ states. The ingredients are:
  (a) Analyticity of scattering amplitude, $F(s, t)$, i.e. proof of fixed-t dispersion relations for $F(s, t)$.
  (b) Crossing symmetry.
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  Results (a) - (c) are proved from general axioms of local QFT. In addition (d) Convergence of partial expansion in Lehmann ellipse.
- The bounds for inelastic reaction $p + p \rightarrow KK$:

$$\sigma^{ab\rightarrow cd} \leq \frac{4\pi}{T_0}(ln\frac{s}{s_0})^2$$

Remarks: (i) T_0 is threshold for t-channel reaction (crude statement) in the TeV^2 range. $T_0 \sim \frac{1}{R^2}$; R, radius of S^1. (ii) s_0 is introduced to make *log* simensionless - it is assumed to be universal constant and taken to be order of 16 GeV^2 as PDG fits of all σ_t data.

- Bound on nonforward diffrential cross section; $\theta \neq 0, \pi$

$$\frac{d\sigma^{ab\to cd}}{d\Omega} \leq (\frac{1}{4\pi T_0^{3/2}})\frac{\sqrt{s}}{sin\theta}[log\frac{s}{s_0}]^3$$

It is proved from boundedness of inelastic partial wave amplitudes via unitarity relation. For small angles $sin\theta \approx \theta$. item¡2 -¿ Remarks: What goes in proof of these bounds?
I. Proof of fixed-t dispersion relation for an inelastic reaction: $a + b \to c + d$. For us a, b are colliding protons and c, d are KK states. Reaction respects all conservation laws. II. The partial wave expansion converges inside a complex domain; Lehmann Ellipse. Usually Legendre Polynomials converge for $cos\theta$ between $(-1, +1)$. Proof of existence of Lehmann ellipse is necessary to derive dispersion relation. III. Unitarity constraint on partial wave amplitude $f_l^{ab\to cd}$. For elastic case a positivity condition holds:
$0 \leq |f_l^{el}(s)|^2 \leq Im\ f_l^{el}(s) \leq 1$. For inelastic case no such constraint exists. The inequality is:
$|f_l^{ab\to cd}|(s) \leq \sqrt{Im\ f_l^{el}(s)}$

- TECHNICAL DETAILS: for $p + p \to KK$ we consider a $D = 5$ theory of scalars in flat space. We compactify x^4 on a CIRCLE: $R^{4,1} \to R^{3,1} \otimes S^1$.
  Compctification from $\hat{D}$ to $4 = D$ is accomplished by adopting Scherk-Schwarz (SS) procedure.
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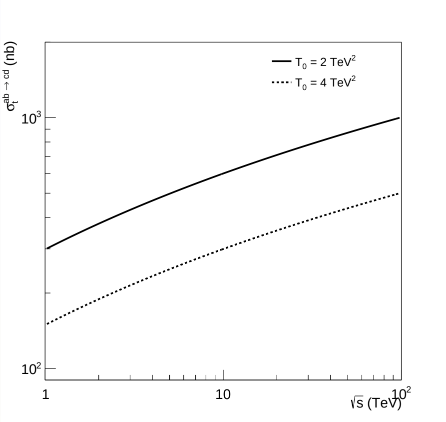
- Weaknesses: We cannot fix T_0 as a *number*, unlike in hadronic collisions.
  Recall Froissart (1961): All σ_t were constant. He had an undetermined CONSTANT prefactor, C'. Martin fixed C'. Only ISR showed rising σ_t^{pp}. Our bounds, we hope, will be useful to experimenalists.



$\sigma_t^{ab\to cd}$ (nb)
$T_0 = 2$ TeV^2
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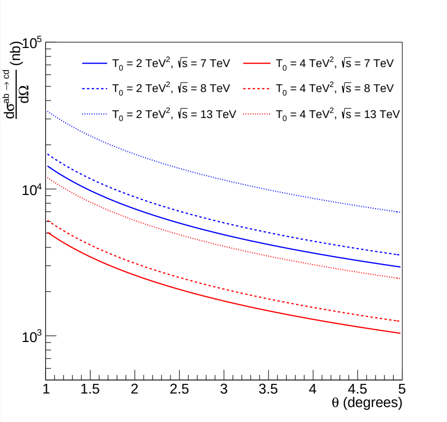
10^3
10^2
1
10
10^2
$\sqrt{s}$ (TeV)



$\frac{d\sigma^{ab\to cd}}{d\Omega}$ (nb)
$T_0 = 2$ TeV^2, $\sqrt{s} = 7$ TeV
$T_0 = 4$ TeV^2, $\sqrt{s} = 7$ TeV
$T_0 = 2$ TeV^2, $\sqrt{s} = 8$ TeV
$T_0 = 4$ TeV^2, $\sqrt{s} = 8$ TeV
$T_0 = 2$ TeV^2, $\sqrt{s} = 13$ TeV
$T_0 = 4$ TeV^2, $\sqrt{s} = 13$ TeV
10^5
10^4
10^3
1
1.5
2
2.5
3
3.5
4
4.5
5
θ (degrees)

SUMMARY AND CONCLUSIONS

- We considered production of Kalulza-Klein states from S^1 compactification of a $D = 5$ theory. In the LRC scenario where . masses of excited KK states lie in TeV scale. They be might discovered at CERN-LHC.

SUMMARY AND CONCLUSIONS

- We considered production of Kalulza-Klein states from S^1 compactification of a $D = 5$ theory. In the LRC scenario where . masses of excited KK states lie in TeV scale. They be might discovered at CERN-LHC.
- Our bounds are rigorously derived from Lorentz invariance, microcausality and other axioms of LSZ. They are nonperturbative results. In a detailed investigation we have shown that the Hilbert space of $D = 4$ theory is union of Hilbert spaces designated by KK charge n. n is quantized and is conserved. Thus lowest KK state is stable (modulo anomalies).
- The differential cross section exhibits forward peak.We derived bounds on cross section starting from general properties of S-matrix. QFT. These bounds might be useful for experimentalists.
  For Mathematical details see: arXIv:2303.09896

THANK YOU