Femtoscopy week, Gyöngyös, 2023

On structures in diffractive dissociation

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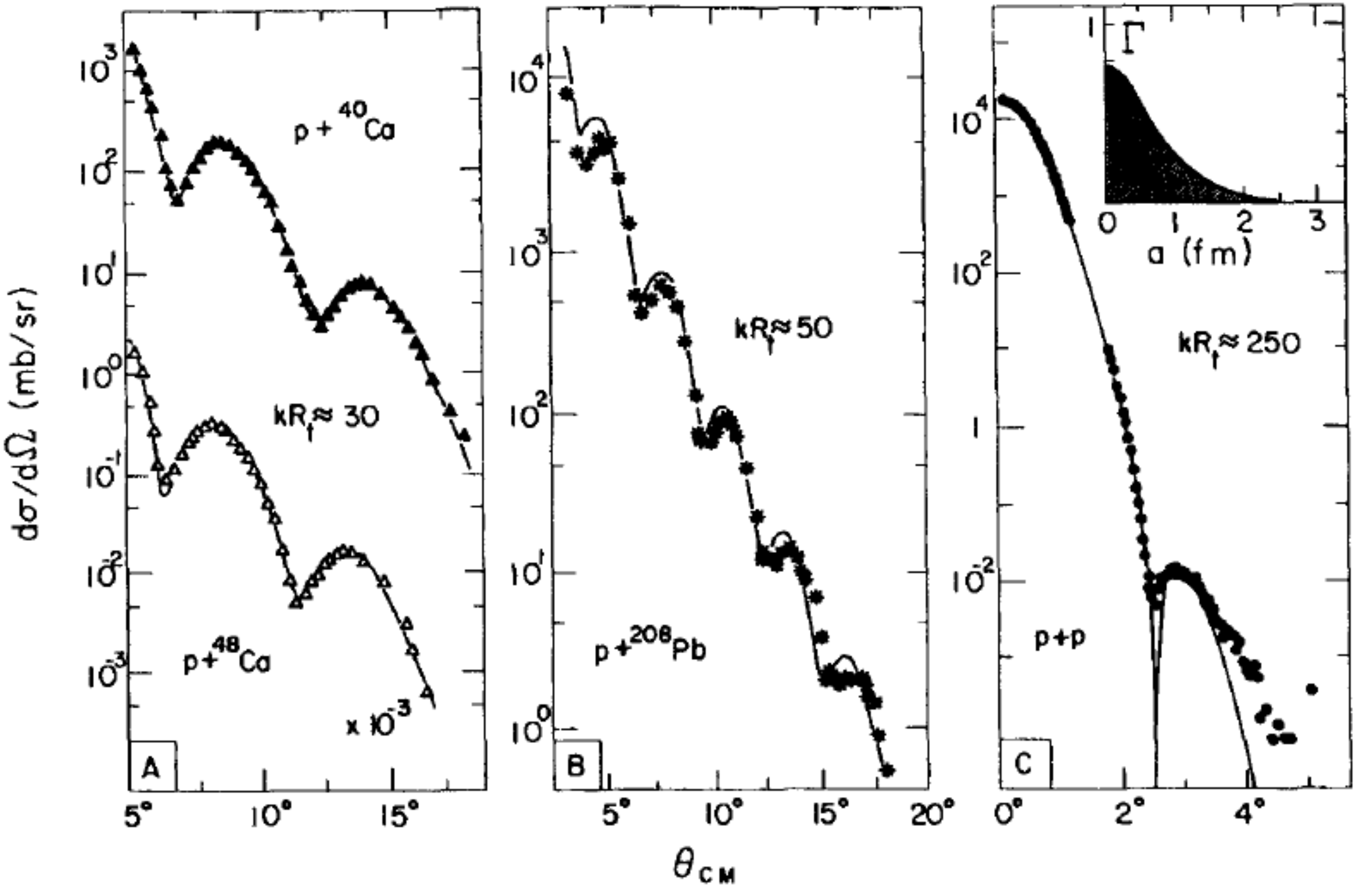
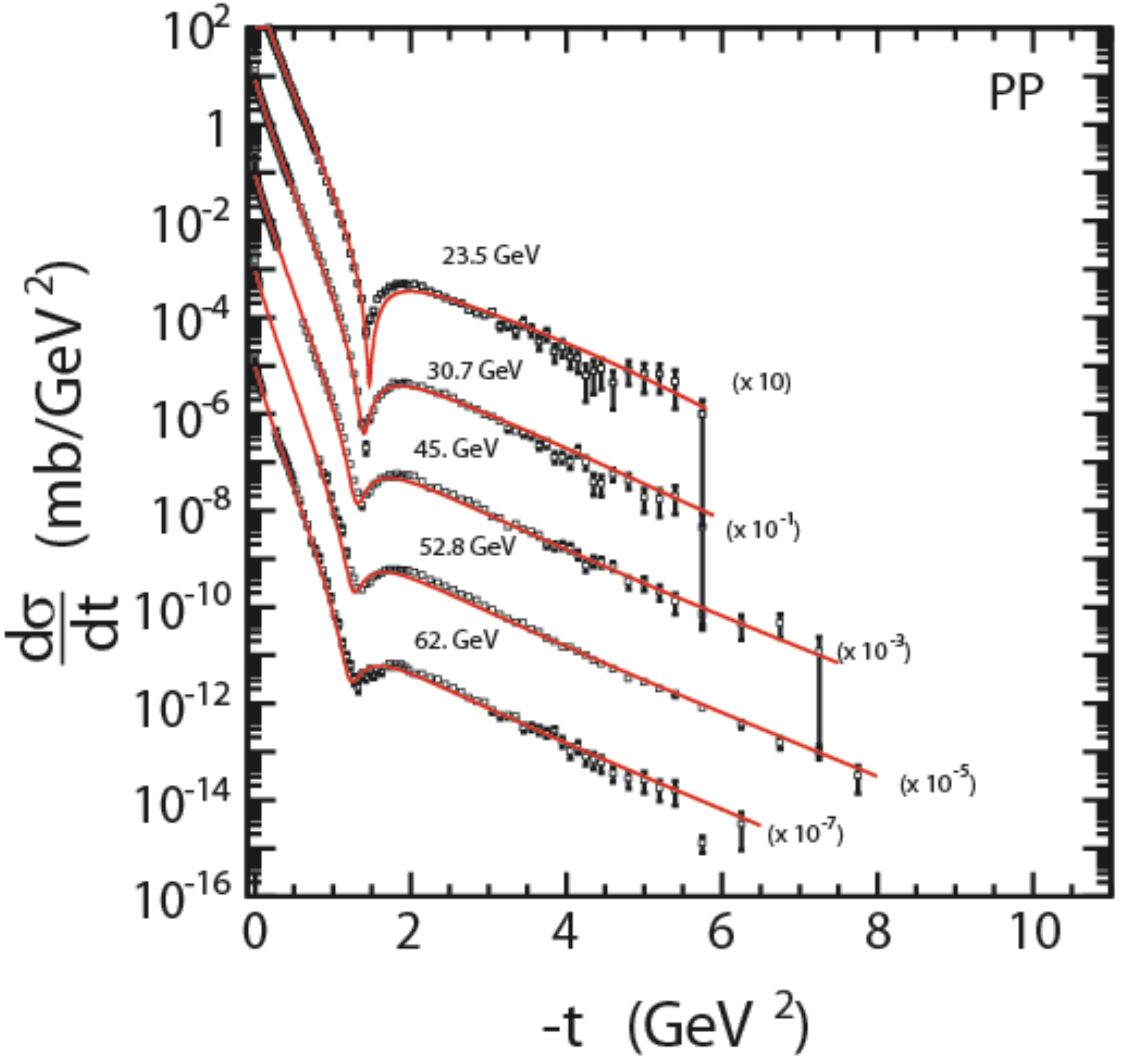
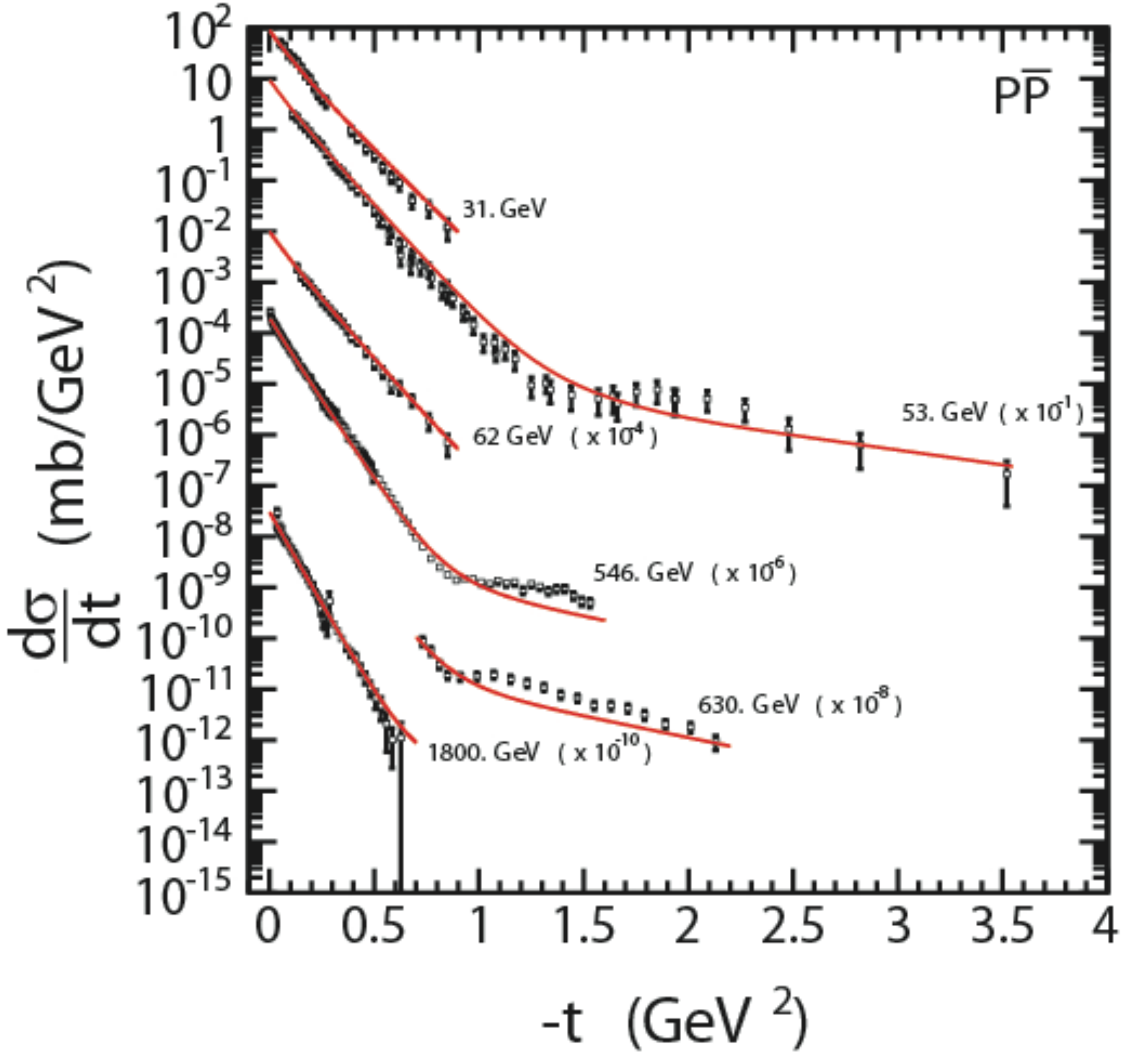
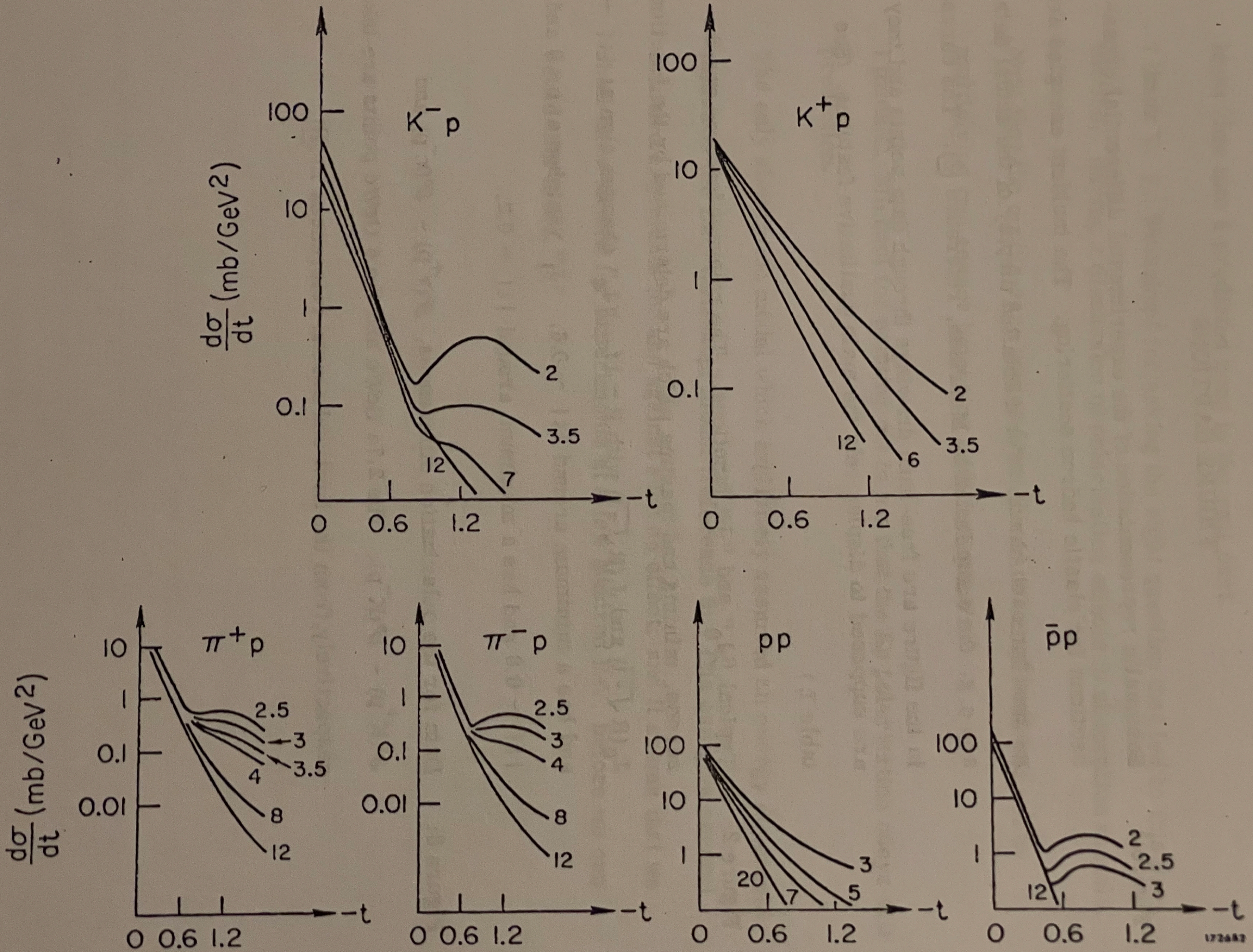


$d\sigma/d\Omega$ (mb/sr)
p + ^{40}Ca
kR$_f$ ≈ 30
p + ^{48}Ca
x 10^{-3}
A
kR$_f$ ≈ 50
p + ^{208}Pb
B
Γ
a (fm)
kR$_f$ ≈ 250
p + p
C
θ_{CM}

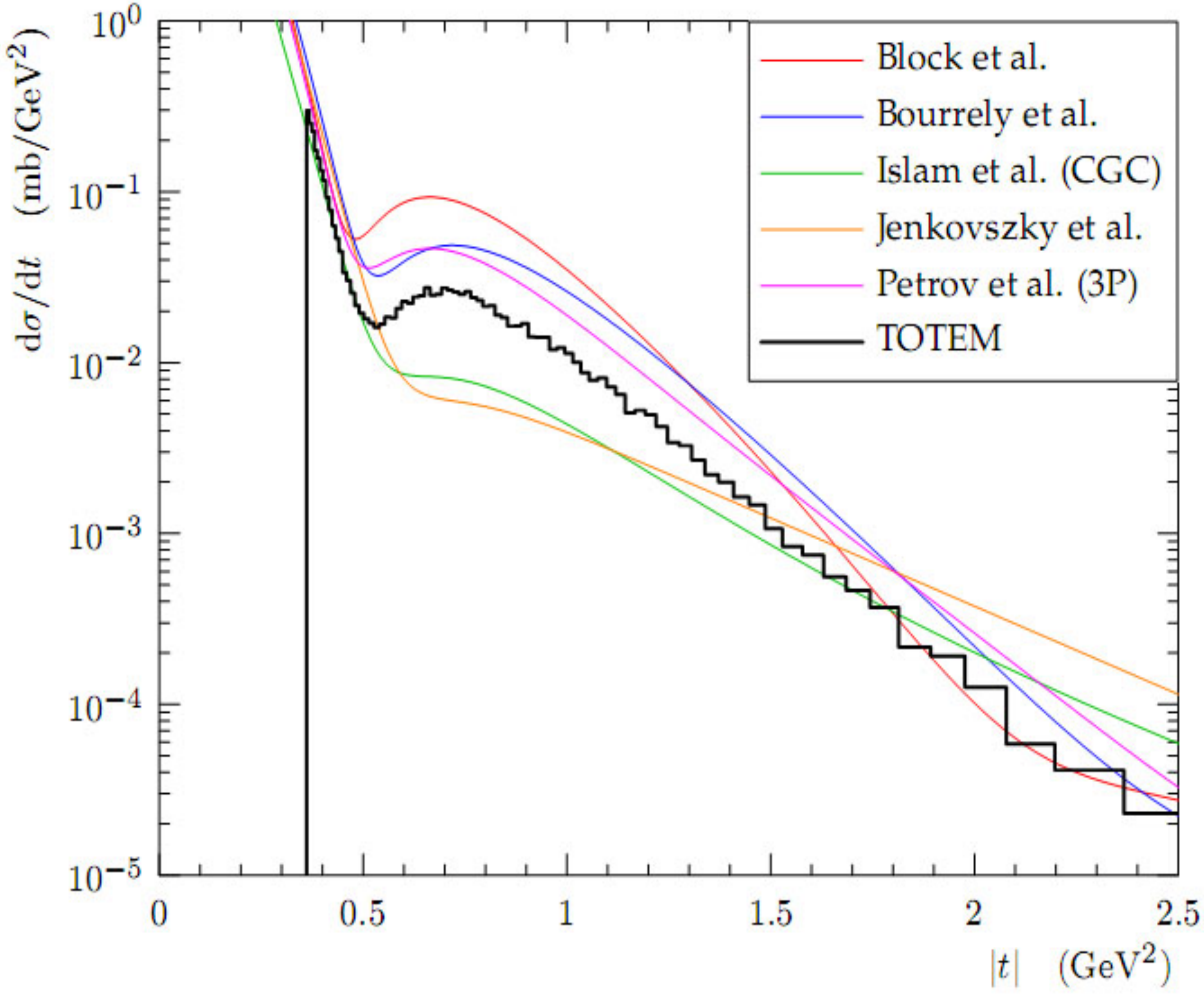

PP
23.5 GeV
30.7 GeV
45. GeV
52.8 GeV
62. GeV
(x 10)
(x 10^-1)
(x 10^-3)
(x 10^-5)
(x 10^-7)
dσ/dt (mb/GeV^2)
-t (GeV^2)
10^2
1
10^-2
10^-4
10^-6
10^-8
10^-10
10^-12
10^-14
10^-16
0
2
4
6
8
10

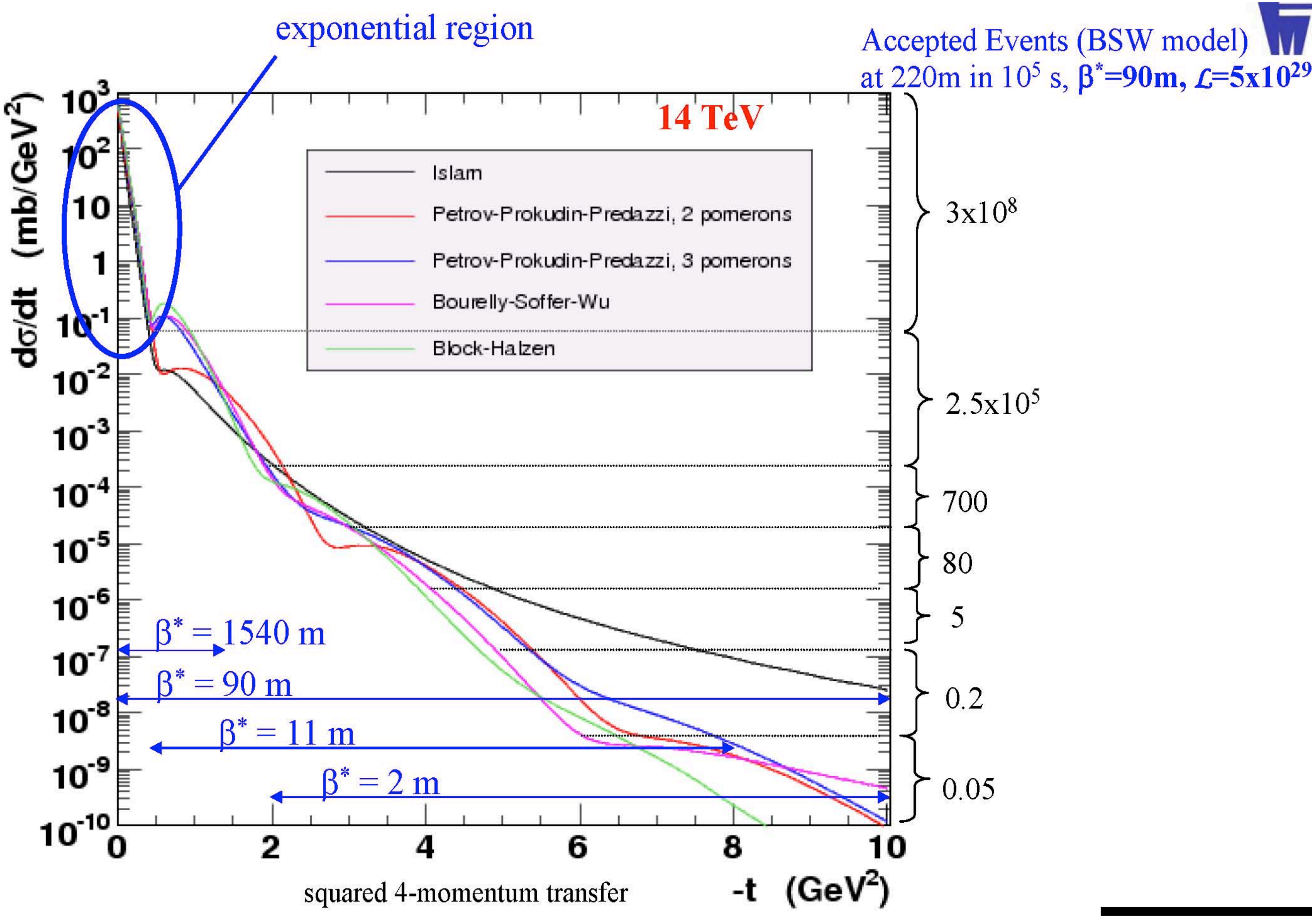


PP̄
31. GeV
53. GeV (x 10-1)
62 GeV (x 10-4)
546. GeV (x 10-6)
630. GeV (x 10-8)
1800. GeV (x 10-10)
dσ/dt (mb/GeV 2)
-t (GeV 2)


K⁻p
K⁺p
π⁺p
π⁻p
pp
p̄p
dσ/dt (mb/GeV²)
-t
100
10
1
0.1
0.01
0
0.6
1.2
2
2.5
3
3.5
4
5
6
7
8
12
20

CERN LHC, TOTEM Collab., June 26, 2011:





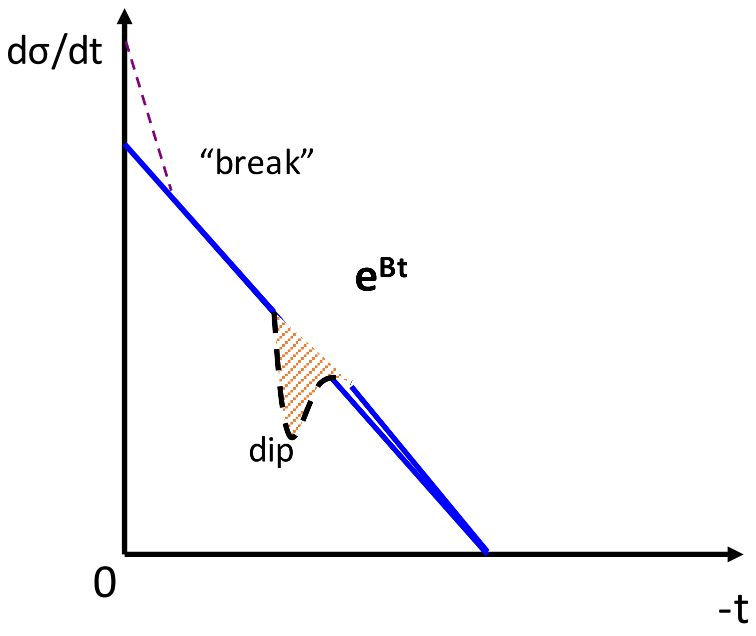
exponential region
Accepted Events (BSW model)
at 220m in 10^5 s, β*=90m, L=5x10^29
14 TeV
dσ/dt (mb/GeV²)
Islam
Petrov-Prokudin-Predazzi, 2 pomerons
Petrov-Prokudin-Predazzi, 3 pomerons
Bourelly-Soffer-Wu
Block-Halzen
3x10^8
2.5x10^5
700
80
5
0.2
0.05
β* = 1540 m
β* = 90 m
β* = 11 m
β* = 2 m
squared 4-momentum transfer
-t (GeV²)

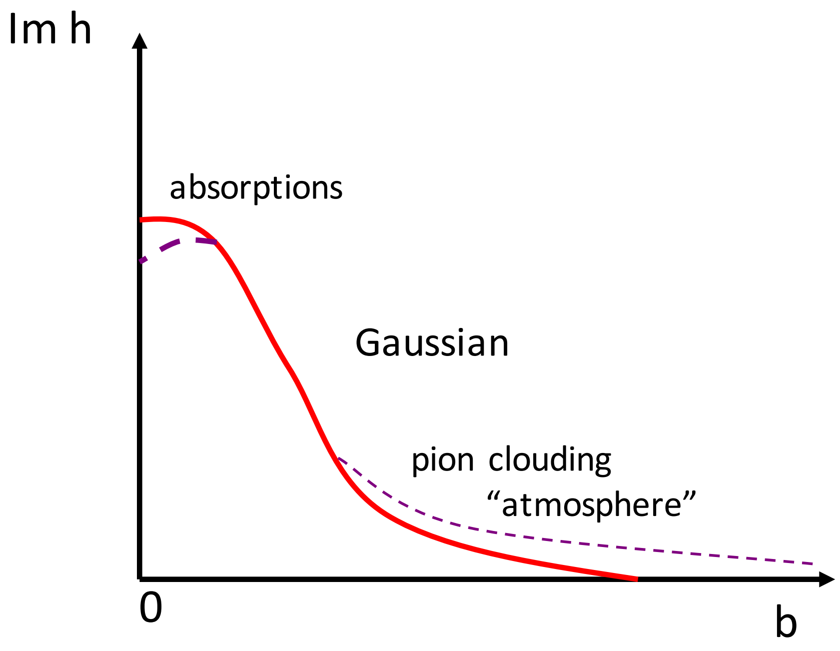
Geometrical scaling (GS), saturation and unitarity

1. On-shell (hadronic) reactions (s,t, Q^2=m^2);

t ←→ b **transformation:** $h(s,b) = \int_0^\infty d\sqrt{-t}\sqrt{-t}A(s,t)$

and dictionary:

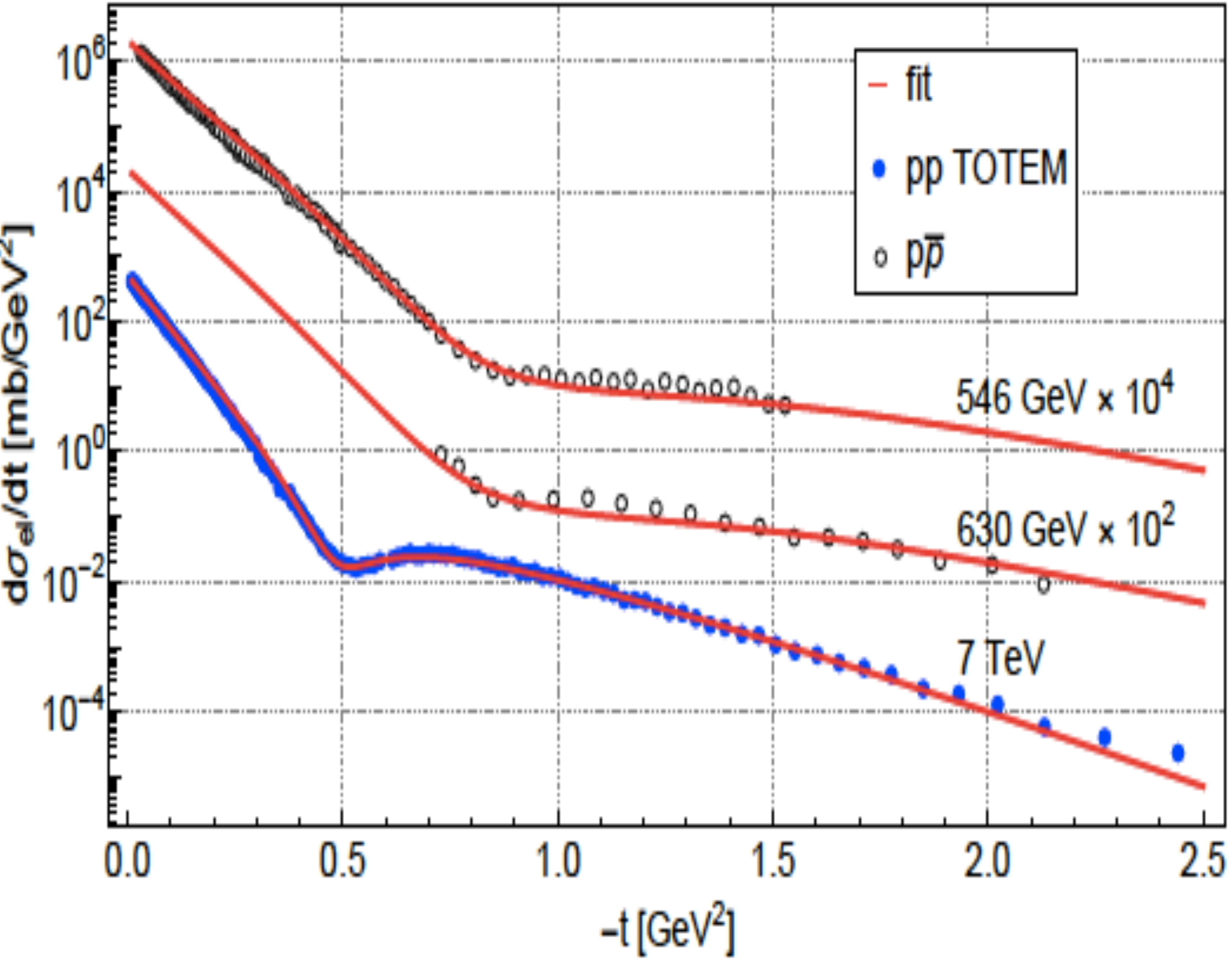




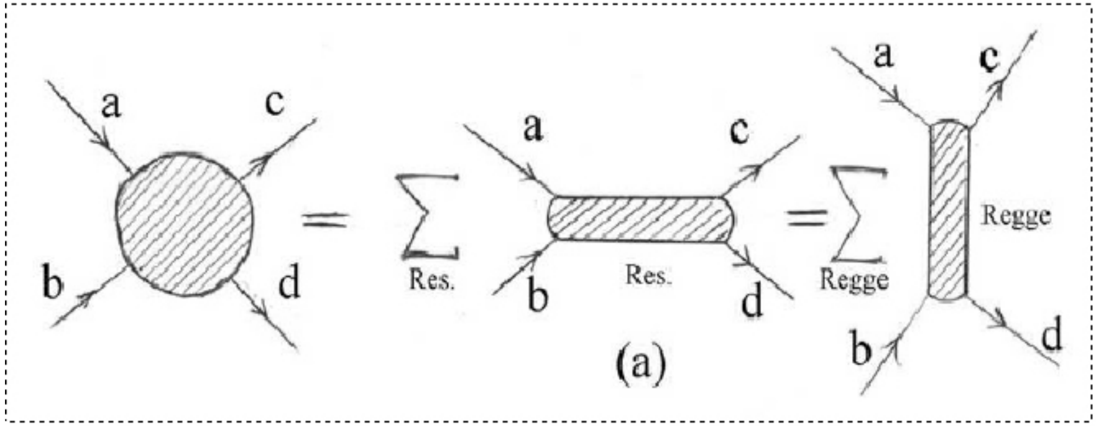
Invert the problem: Glauber – Wakaizume,

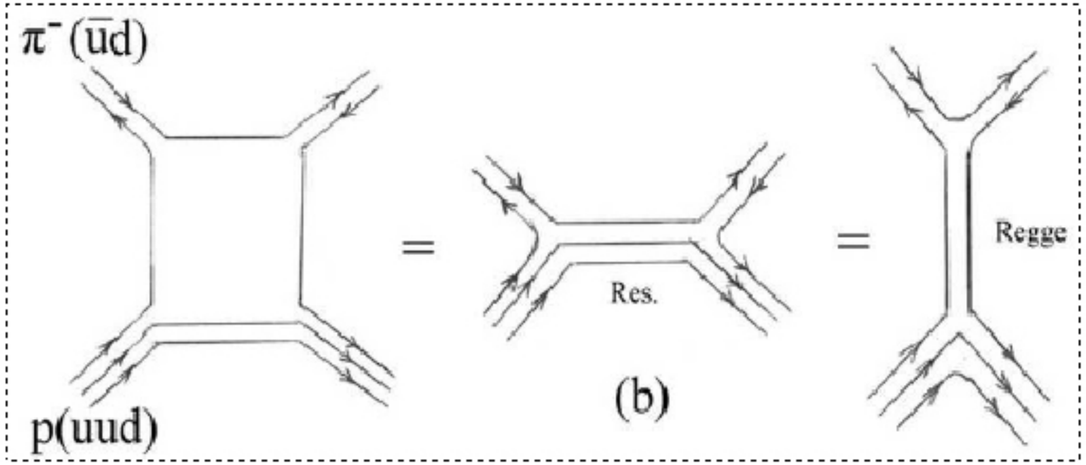
S. Wakaizume, Progress Theor. Phys. 60 (1978) 1050:

By using the Glauber multiple scattering theory, restore the internal structure of the nucleon from the fits to the pp data .



fit
pp TOTEM
$p\bar{p}$
546 GeV × 10^4
630 GeV × 10^2
7 TeV
$d\sigma_{el}/dt$ [mb/GeV^2]
$-t$ [GeV^2]


a
b
c
d
Res.
Regge
(a)


$\pi^-(\bar{u}d)$
p(uud)
Res.
Regge
(b)

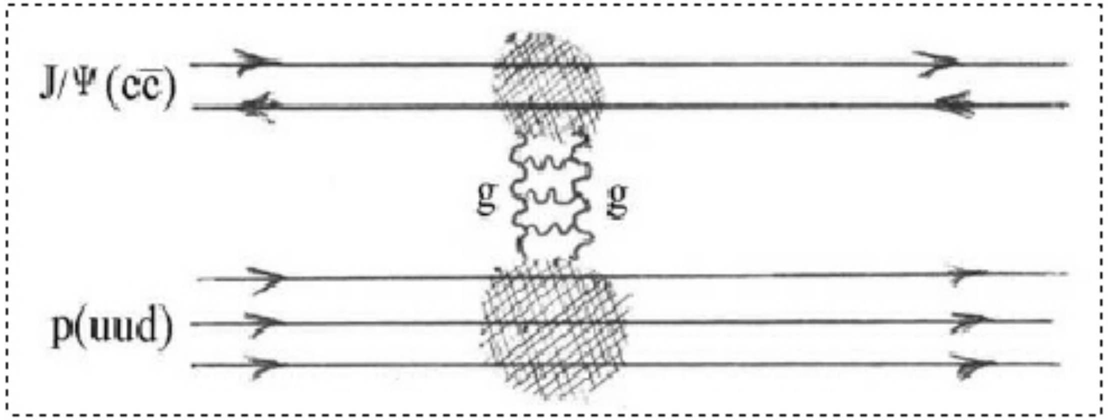

$J/\Psi\,(c\bar{c})$
g
g
p(uud)

TABLE I: Two-component duality

$\mathcal{I}mA(a+b\to c+d) =$	R	Pomeron
$s-$channel	$\sum A_{Res}$	Non-resonant background
$t-$channel	$\sum A_{Regge}$	Pomeron $(I = S = B = 0;\ C = +1)$
Duality quark diagram	Fig. 1b	Fig. 2
High energy dependence	$s^{\alpha-1},\ \alpha < 1$	$s^{\alpha-1},\ \alpha \geq 1$

$$\sigma_t(s) = \frac{4\pi}{s} Im A(s,t=0); \quad \frac{d\sigma}{dt} = \frac{\pi}{s^2}|A(s,t)|^2; \quad n(s);$$

$$\sigma_{el} = \int_{t_{min}\approx -s/2\approx\infty}^{t_{thr.}\approx 0} \frac{d\sigma}{dt} dt; \quad \sigma_{in} = \sigma_t - \sigma_{el}; \quad B(s,t) = \frac{d}{dt} \ln\left(\frac{d\sigma}{dt}\right);$$

$$A_{pp}^{p\bar{p}}(s,t) = P(s,t) \pm O(s,t) + f(s,t) \pm \omega(s,t) \rightarrow_{LHC} \approx P(s,t) \pm O(s,t),$$

where P, O, f. ω are the Pomeron, odderon and non-leading Reggeon contributions.

α(0)\C	+	-
1	P	O
1/2	f	ω

NB: The S-matrix theory (including Regge pole) is applicable to asymptotically free states only (not to quarks and gluons)!

The Pomeron is a dipole in the $j-$plane

$$A_P(s,t) = \frac{d}{d\alpha_P}\Big[e^{-i\pi\alpha_P/2}G(\alpha_P)\Big(s/s_0\Big)^{\alpha_P}\Big] = \tag{1}$$

$$e^{-i\pi\alpha_P(t)/2}\Big(s/s_0\Big)^{\alpha_P(t)}\Big[G'(\alpha_P) + \Big(L - i\pi/2\Big)G(\alpha_P)\Big].$$

Since the first term in squared brackets determines the shape of the cone, one fixes

$$G'(\alpha_P) = -a_P e^{b_P[\alpha_P - 1]}, \tag{2}$$

where $G(\alpha_P)$ is recovered by integration, and, as a consequence, the Pomeron amplitude can be rewritten in the following "geometrical" form

$$A_P(s,t) = i\frac{a_P\ s}{b_P\ s_0}[r_1^2(s)e^{r\ \ (s)[\alpha_P-1]} - \varepsilon_P r_2^2(s)e^{r\ \ (s)[\alpha_P-1]}], \tag{3}$$

where $r_1^2(s) = b_P + L - i\pi/2,\ \ r_2^2(s) = L - i\pi/2,\ \ L \equiv \ln(s/s_0)$.

The differential cross section of elastic (EL) proton-proton scattering is:

$$\frac{d\sigma_{EL}}{dt} = A_{EL}\beta^2(t)|\eta(t)|^2 \left(\frac{s}{s_0}\right)^{2\alpha_P(t)-2},$$

where A_{EL} is a free parameter including normalization. The proton-pomeron coupling is: $\beta^2(t) = e^{bt}$, where b is a free parameter, $b \approx 1.97$ GeV^{-2}. The pomeron trajectory is $\alpha_P(t) = 1 + \epsilon + \alpha' t$, where $\epsilon \approx 0.08$ and $\alpha' \approx 0.3$ GeV^{-2}. The signature factor is $\eta(t) = e^{-i\frac{\pi}{2}\alpha(t)}$; its contribution to the differential cross section is $|\eta(t)|^2 = 1$, therefore we ignore it in what follows.

$$A_{pp}^{p\bar{p}}(s,t) = P(s,t) \pm O(s,t) + f(s,t) \pm \omega(s,t) \to_{LHC} P(s,t) \pm O(s,t),$$

where P is the Pomeron contribution and O is that of the Odderon.

$$P(s,t) = i\frac{as}{bs_0}(r_1^2(s)e^{r_1^2(s)[\alpha_P(t)-1]} - \epsilon r_2^2(s)e^{r_2^2(s)[\alpha_P(t)-1]}),$$

where $r_1^2(s) = b + L - \frac{i\pi}{2}, \quad r_2^2(s) = L - \frac{i\pi}{2}$ with $L \equiv \ln\frac{s}{s_0}$; $\alpha_P(t)$ is the Pomeron trajectory and a, b, s_0 and ϵ are free parameters.

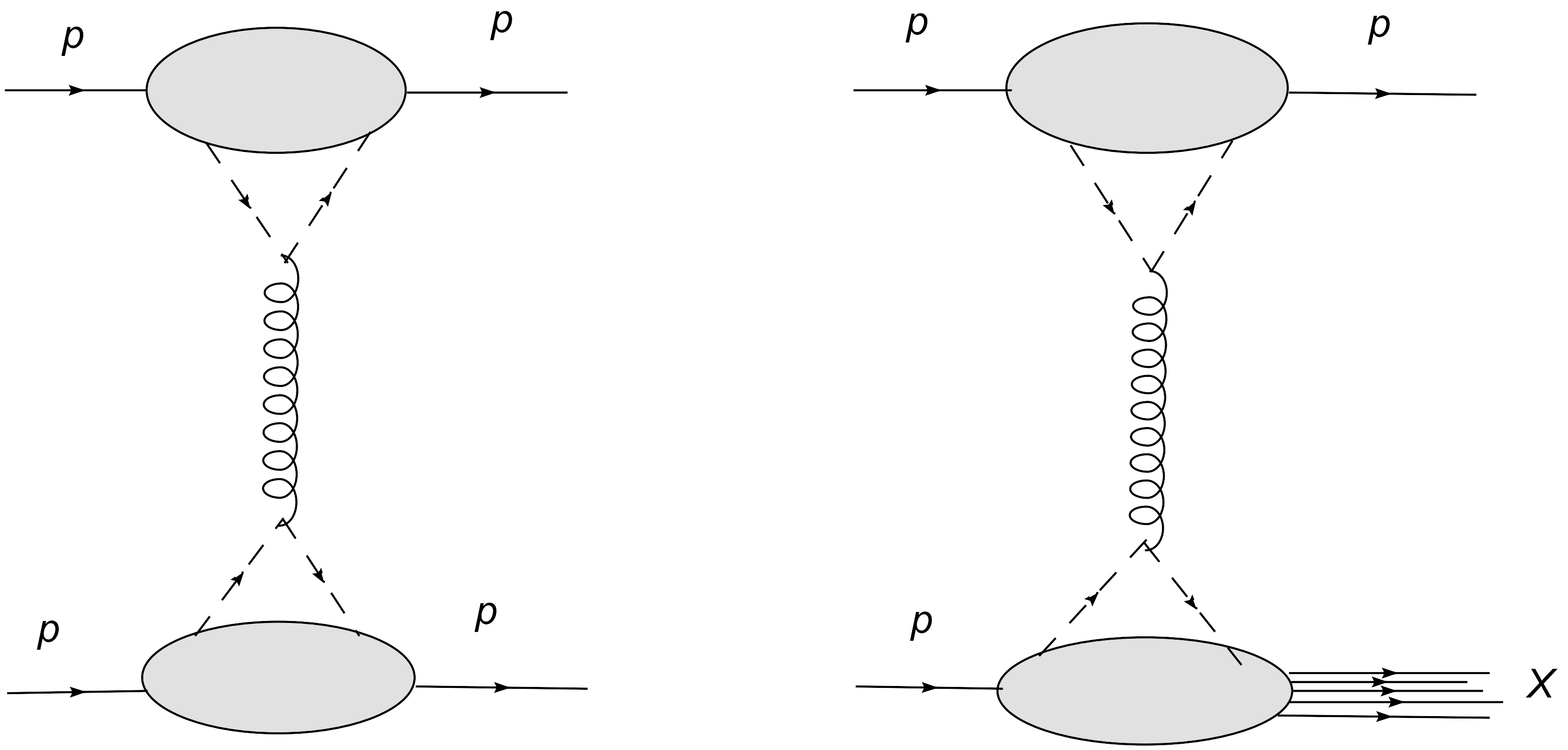
P and f (second column) have positive C-parity, thus entering in the scattering amplitude with the same sign in pp and $\bar{p}p$ scattering, while the Odderon and ω (third column) have negative C-parity, thus entering pp and $\bar{p}p$ scattering with opposite signs, as shown below:

$$A\left(s,t\right)_{pp}^{\bar{p}p} = A_P\left(s,t\right) + A_f\left(s,t\right) \pm \left[A_\omega\left(s,t\right) + A_O\left(s,t\right)\right], \tag{1}$$

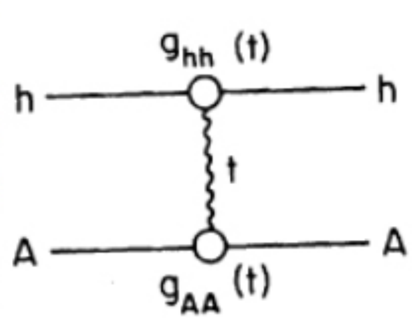
where the symbols P, f, O, ω stand for the relevant Regge-pole amplitudes and the super(sub)script, evidently, indicate $\bar{p}p(pp)$ scattering with the relevant choice of the signs in the sum.

$$A_P(s,t) = \frac{d}{d\alpha_P}\left[e^{-i\pi\alpha_P/2}G(\alpha_P)\Big(s/s_0\Big)^{\alpha_P}\right] =$$

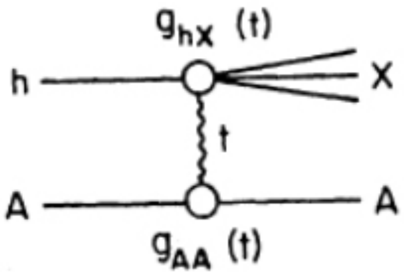
$$e^{-i\pi\alpha_P(t)/2}\Big(s/s_0\Big)^{\alpha_P(t)}\Big[G'(\alpha_P) + \Big(L - i\pi/2\Big)G(\alpha_P)\Big].$$



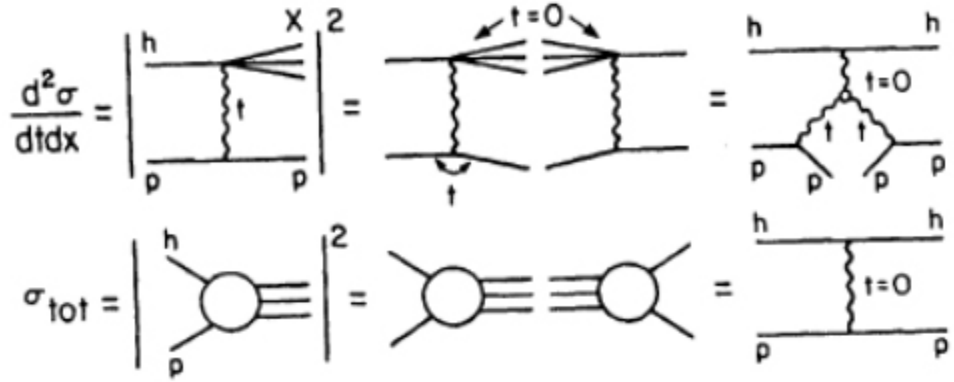
p
p
p
p
p
p
p
X



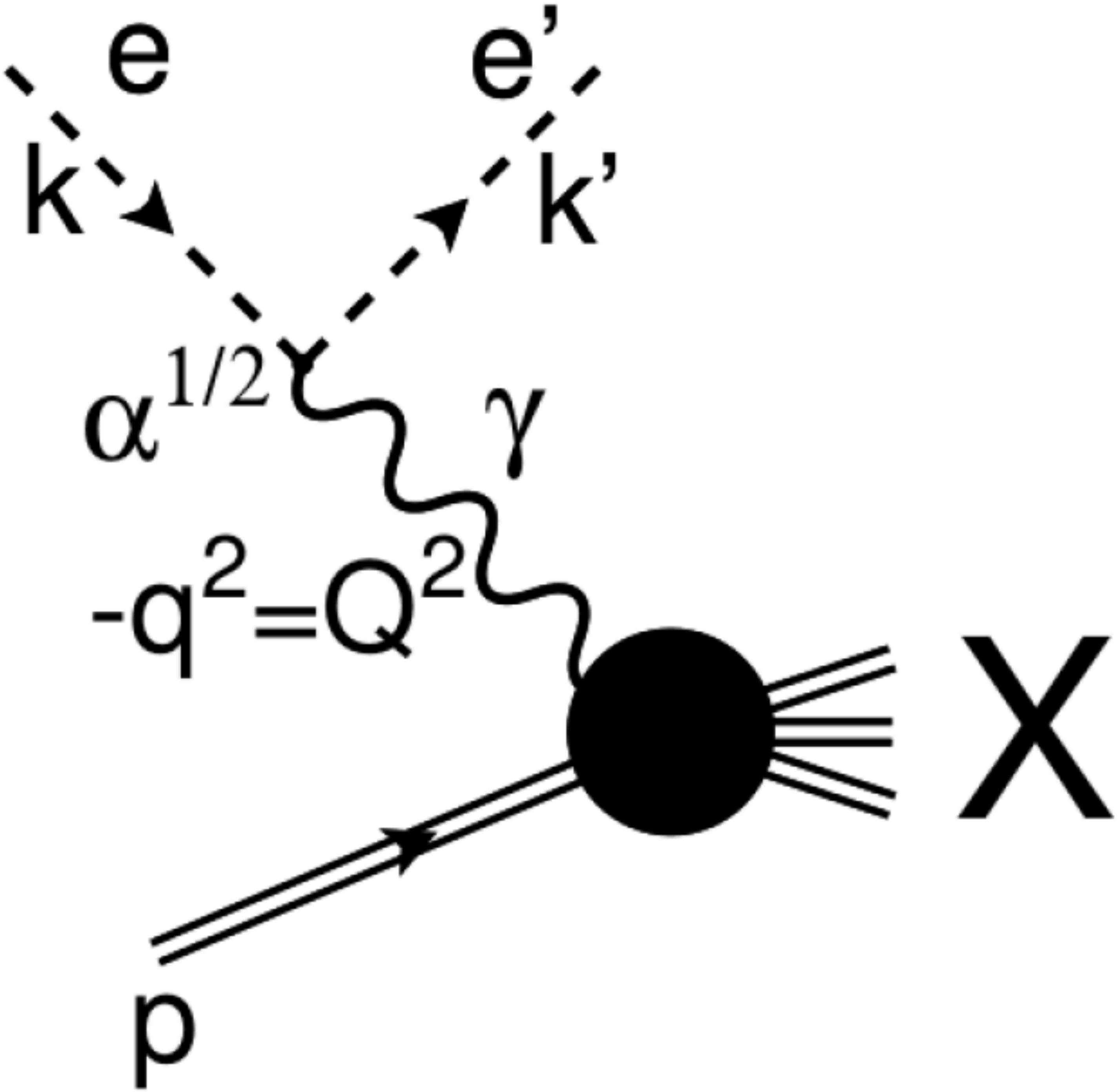
g_hh (t)
h
h
t
A
A
g_AA (t)



g_hX (t)
h
X
t
A
A
g_AA (t)



d²σ/dtdx =
h
X
2
t
p
p
=
t=0
t
=
h
h
t=0
t
t
p
p
p
p
σ_tot =
h
p
2
=
=
h
h
t=0
p
p


e
e’
k
k’
$\alpha^{1/2}$
γ
$-q^2=Q^2$
X
p

The differential cross section of proton-proton single diffraction (SD) is:

$$2 \cdot \frac{d^2\sigma_{SD}}{dt dM_X^2} = A_{SD}\beta^2(t)\tilde{W}_2^{Pp}(M_X^2, t)\left(\frac{s}{M_X^2}\right)^{2\alpha_P(t)-2},$$

where $\tilde{W}_2^{Pp}(M_X^2, t) \sim F_2^p(M_X^2, t)$.
Similarly, the differential cross section of proton-proton double diffraction (DD) process is:

$$\frac{d^3\sigma_{DD}}{dt dM_X^2 dM_Y^2} = A_{DD}\tilde{W}_2^{Pp}(M_X^2, t)\tilde{W}_2^{Pp}(M_Y^2, t)\left(\frac{s s_0}{M_X^2 M_Y^2}\right)^{2\alpha_P(t)-2}.$$

Similar to the case of elastic scattering, the double differential cross section for the SDD reaction, by Regge factorization, can be written as

$$\frac{d^2\sigma}{dtdM_X^2} = \frac{9\beta^4[F^p(t)]^2}{4\pi\sin^2[\pi\alpha_P(t)/2]}(s/M_X^2)^{2\alpha_P(t)-2}\times \left[\frac{W_2}{2m}\left(1-M_X^2/s\right) - mW_1(t+2m^2)/s^2\right], \tag{1}$$

where $W_i, \quad i=1,2$ are related to the structure functions of the nucleon and $W_2 \gg W_1$. For high M_X^2, the $W_{1,2}$ are Regge-behaved, while for small M_X^2 their behavior is dominated by nucleon resonances. The knowledge of the inelastic form factors (or transition amplitudes) is crucial for the calculation of low-mass diffraction dissociation.

Similar to the case of elastic scattering, the Dipole SD amplitude is recovered by differentiation (for simplicity (we set $s_0 = 1$ GeV^2)):

$$T_{DP} = \frac{d}{d\alpha} T(s,t,M^2) = e^{-i\pi\alpha/2} s^{\alpha} [G' F_2 + F_2' G + (L - i\pi/2) G F_2],$$

where $L = \ln(s/(1 GeV^2))$ and the primes imply differentiation in $\alpha(t)$.

The extrema (dip(s) and bump(s)) are calculated by a standard procedure, i.e. by equating to zero the derivative of the cross section:

$$\frac{d|T_{SD}|^2}{d\alpha} = \frac{1}{2}\Big(\frac{s^2}{s_0^2}\Big)^{\alpha}\Big[GF' + F(LG + G')\Big]\Big[8F'G' + 4G\Big(2LF' + F''\Big)$$

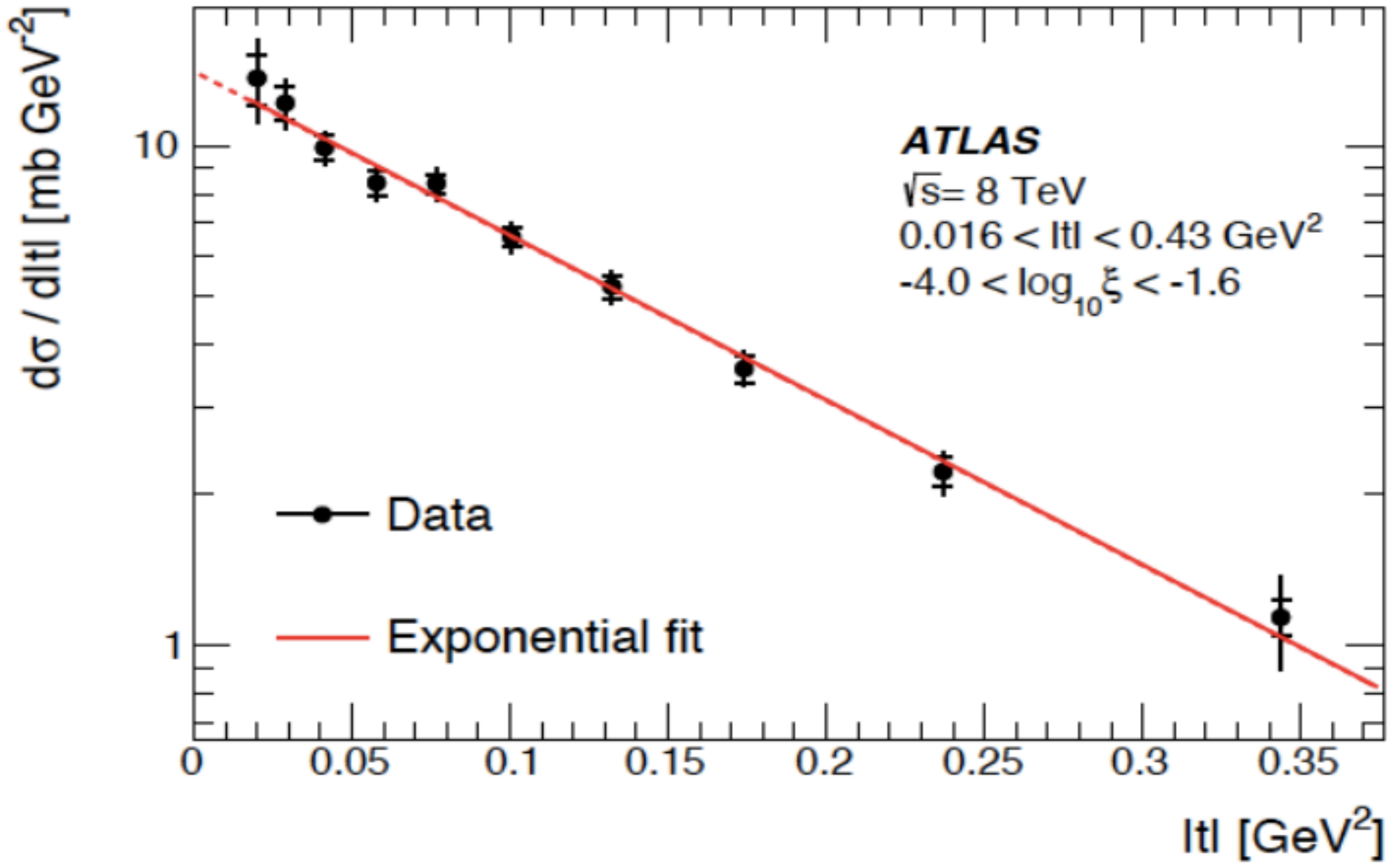
$$+F(4L^2 + \pi^2)G + 4\Big(2LG' + G''\Big)\Big],$$

where $L = \ln(s/s_0))$ and the primes imply differentiation in $\alpha(t)$.

Similar to the case of elastic scattering, the Dipole SD amplitude is recovered by differentiation (for simplicity (we set $s_0 = 1$ GeV^2)):

$$T_{DP} = \frac{d}{d\alpha} T(s,t,M^2) = e^{-i\pi\alpha/2} s^{\alpha}[G' F_2 + F_2' G + (L - i\pi/2) G F_2], \quad (19)$$

where $L = \ln(s/(1GeV^2))$ and the primes imply differentiation in $\alpha(t)$.



ATLAS
√s= 8 TeV
0.016 < |t| < 0.43 GeV²
-4.0 < log₁₀ξ < -1.6
Data
Exponential fit
dσ / d|t| [mb GeV⁻²]
|t| [GeV²]
10
1
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35

Experimentally known fact [1] that the triple-pomeron coupling is nearly independent of t, so that $g_{PPP}(t) = g_{PPP}(0)$. Then for the t-dependent part of the amplitude of the SD process we have:

$$A_{SD}^{SP}(s, M^2, \alpha) \sim \eta(\alpha) G(\alpha) \left(s/M^2\right)^{\alpha}, \tag{12}$$

where the t-dependence resulting from $g_{Ppp}(t)$ is accounted by $G(\alpha)$. Then the t-dependent part of the dipole pomeron amplitude is obtained as:

$$A_{SD}^{DP}(s, M^2, \alpha) = \frac{d}{d\alpha} A_{SD}^{SP}(s, M^2, \alpha) \sim e^{-i\pi\alpha/2} \left(s/M^2\right)^{\alpha} \left[G'(\alpha) + \left(L_{SD} - i\pi/2\right) G(\alpha)\right] \tag{13}$$

where

$$L_{SD} \equiv \ln\left(s/M^2\right). \tag{14}$$

Then double differential cross section for the SD process resulting from the dipole pomeron amplitude is:

$$\frac{d^2\sigma_{SD}}{dt\,dM^2} = \frac{1}{M^2} \left(G'^2(\alpha) + 2L_{SD} G(\alpha) G'(\alpha) + G^2(\alpha) \left(L_{SD}^2 + \frac{\pi^2}{4} \right) \right) \left(s/M^2\right)^{2\alpha(t)-2} \sigma^{Pp}(M^2) \tag{15}$$

where

$$\sigma^{Pp}(M^2) = \sigma_{ns}^{Pp}(M^2) + \sigma_0^{Pp}(M^2). \tag{16}$$

The resonanceless part is given as:

$$\sigma_0^{Pp}(M^2) = g_{PPP}g_{Ppp}(0)\left(M^2\right)^{\alpha(0)-1} = \sigma_0\tau^8(M_X^2)\left(M_X^2\right)^{\alpha(0)-1}. \tag{17}$$

where $\sigma_0 = 2.82$ mb or 7.249 GeV^{-2} and

$$\tau(M_X^2) = \frac{e^{-M_X^2/m_0^2} - 1}{e^{-M_X^2/m_0^2} + 1}, \quad m_0^2 = 1\ \text{GeV}^2.$$

Here $\tau^8(M_X^2)$ is included [1] in $\sigma_{t,0}^{Pp}(M_X^2)$ to suppress it in the region $M_X^2 < (m_p + m_{\pi^0})^2$ where no dissociation occurs . A simple t-independent form for the low-mass Pp total cross section containing resonance contributions can be written as:

$$\sigma_{t,\text{res}}^{Pp}(M^2) = \frac{8\pi}{M^2}\Im\text{m}\, A_{\text{res}}^{Pp}(M^2, \tilde{t}=0)\ , \tag{18}$$

with

$$\Im\text{m}\, A_{\text{res}}^{Pp}(M_X^2, \tilde{t}) = \sum_J \frac{[f(\tilde{t})]^{J+3/2}\Im\text{m}\alpha_{N^*}(M_X^2)}{(J - \Re\text{e}\alpha_{N^*}(M_X^2))^2 + (\Im\text{m}\,\alpha_{N^*}(M_X^2))^2}. \tag{19}$$

where α_{N^*} is the nucleon trajectory,

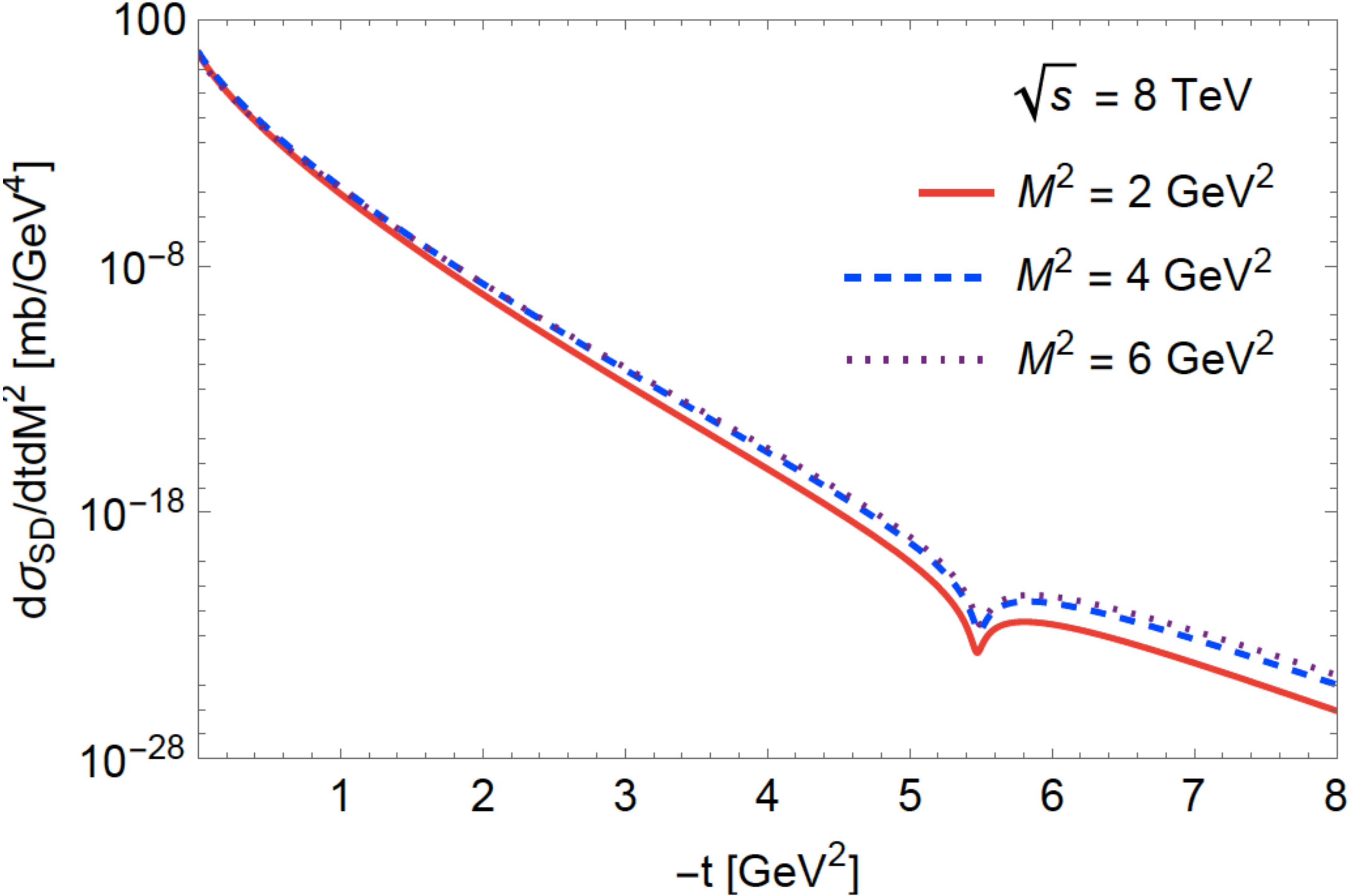
$$f(\tilde{t}) = (1 - \tilde{t}/t_0)^{-2}, \tag{20}$$

and $t_0 = 0.71$ GeV^2. The explicit form of the nucleon trajectory is given in Refs. [3, 4]. Resonances on this trajectory appear with total spins $J = 5/2, 9/2, 13/2, \ldots$.

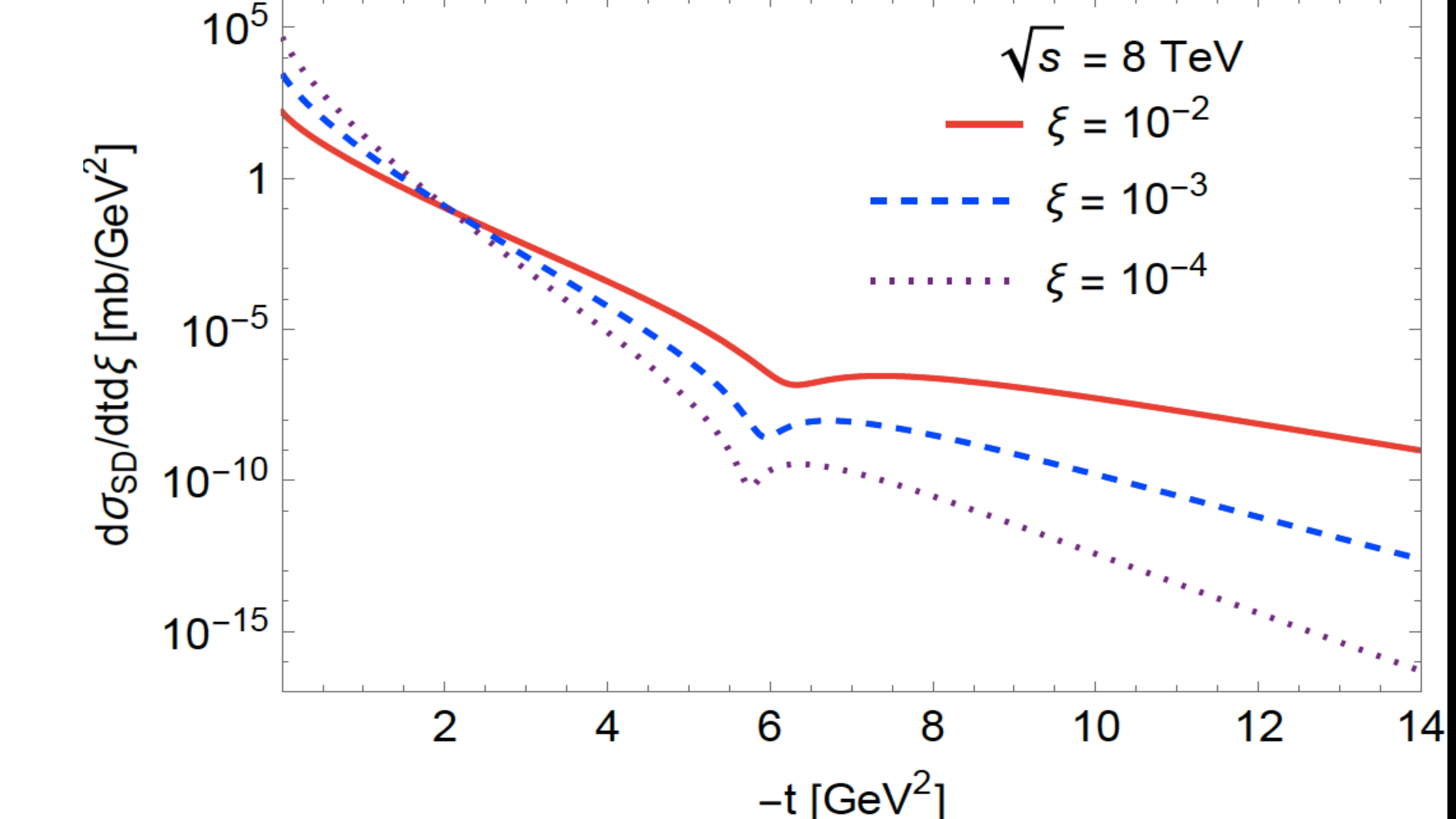
$$t_{dip}^{SD} = \frac{1}{\alpha' b} \ln \frac{\gamma b L_{SD}}{b + L},$$

$$t_{bump}^{SD} = \frac{1}{\alpha' b} \ln \frac{\gamma b (4 L_{SD}^2 + \pi^2)}{4(b + L_{SD})^2 + \pi^2},$$

$$L_{SD} \equiv \ln\left(s / M^2\right) = -\ln \xi.$$



100
10^{-8}
10^{-18}
10^{-28}
1
2
3
4
5
6
7
8
$d\sigma_{SD}/dtdM^2$ [mb/GeV^4]
$-t$ [GeV^2]
$\sqrt{s}$ = 8 TeV
M^2 = 2 GeV^2
M^2 = 4 GeV^2
M^2 = 6 GeV^2



$\sqrt{s}$ = 8 TeV
$\xi = 10^{-2}$
$\xi = 10^{-3}$
$\xi = 10^{-4}$
$d\sigma_{SD}/dtd\xi$ [mb/GeV^2]
10^5
1
10^{-5}
10^{-10}
10^{-15}
2
4
6
8
10
12
14
$-t$ [GeV^2]

NEW: Odderon and three-gluon exchange at large |t|:

Donnachie and Landshoff (Z. Phys. C2 (1972)55; arXiv 1112.2485; 1309.1292):

Quark counting ruresbeyond the dip=bump : $G(t)\sim s/t^4$ (PL **B38** (1996)6317);

Transition from non-perturbative "soft" (exponential cone) to hard (power-like behaviour, perturbative QCD, or quark counting rules) can be realized in dual amplitudes with Mandelstam analyticity (DAMA) by using logarithmic (here, the Odderon) trajectories.

$$D(s,t) = \int_0^1 dx \left(\frac{x}{g}\right)^{-\alpha(s')} \left(\frac{1-x}{g}\right)^{-\alpha(t')} . \tag{7}$$

Here $s' = s(1-x)$, $\quad t' = tx$ and g is a dimensionless parameter, $g > 1$. Only one, leading trajectory was included and it was chosen in a simple, but representative form:

$$\alpha(s) = \alpha_0 - \gamma \ln \left(\frac{1 + \beta\sqrt{s_0 - s}}{1 + \beta\sqrt{s_0}} \right) , \tag{8}$$

Conclusion

The origin and nature of the prominent structures in elastic proton scattering is still not understood. Diffractive dissociation may clarify it.

Thanks you the invitation to this *Femtoscopy Week*