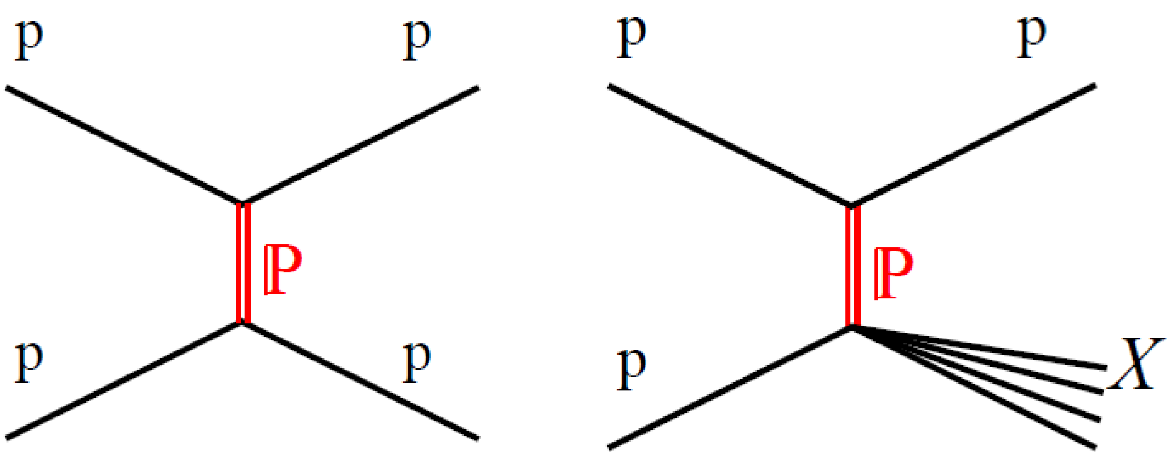
Dip-bump structures in pp elastic scattering and single diffractive dissociation

István Szanyi

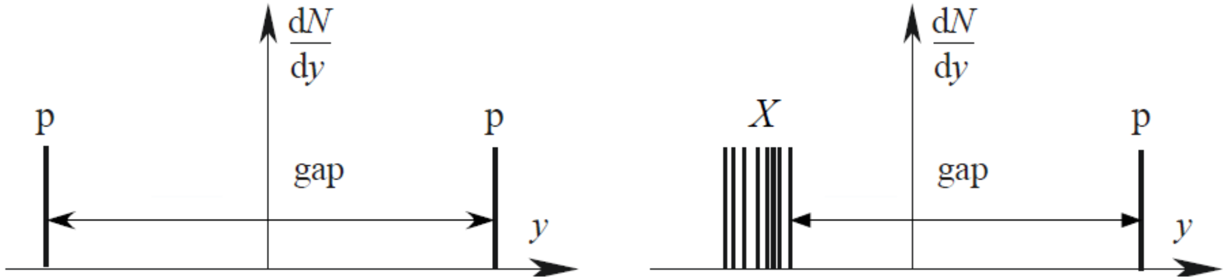
in collaboration with László Jenkovszky

9th Day of Femtoscopy
30-31 October 2023, Gyöngyös, Hungary

Elastic pp scattering and single diffractive dissociation



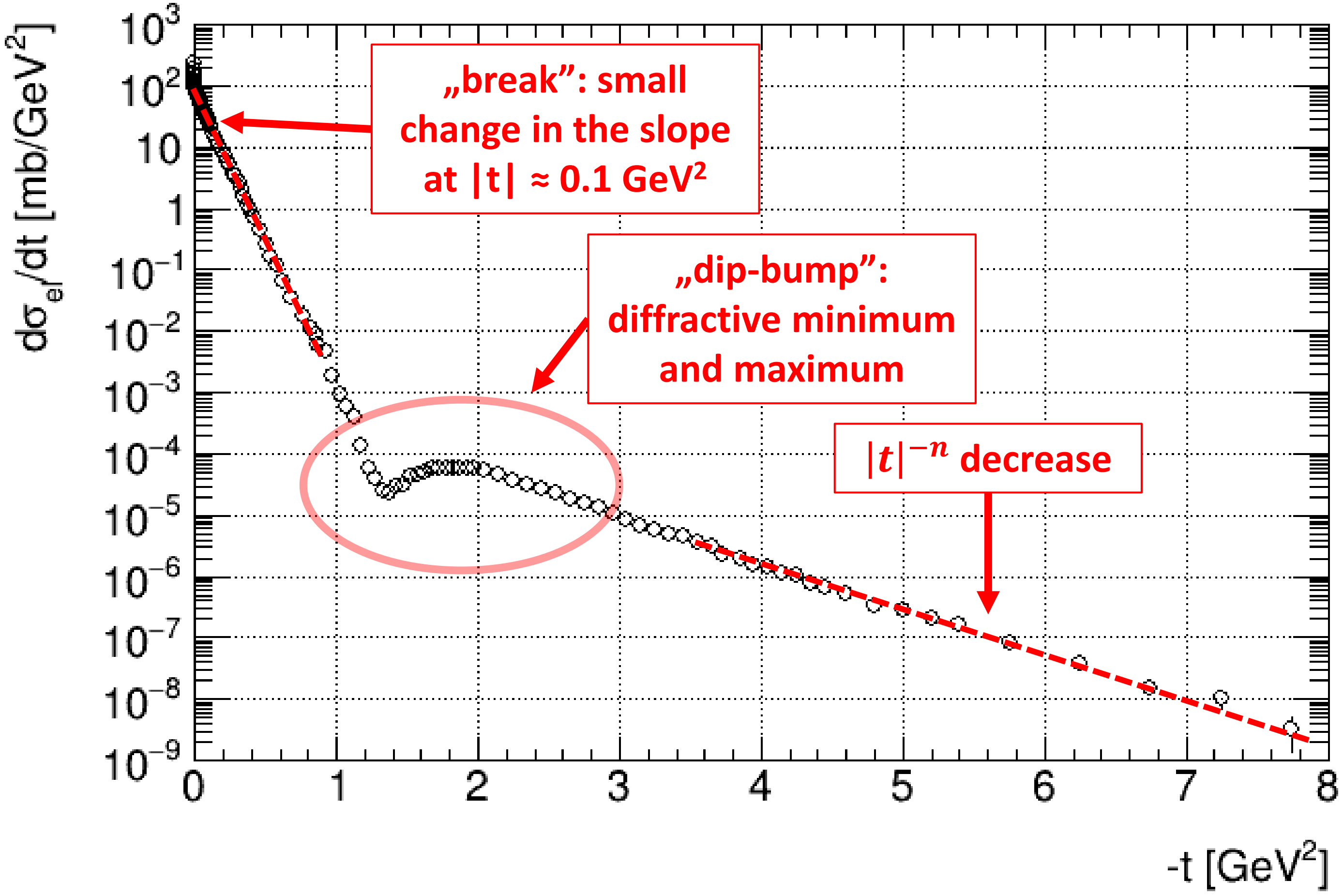
Leading order Pomeron exchange graph contributing to pp elastic scattering and to pp single diffractive dissociation

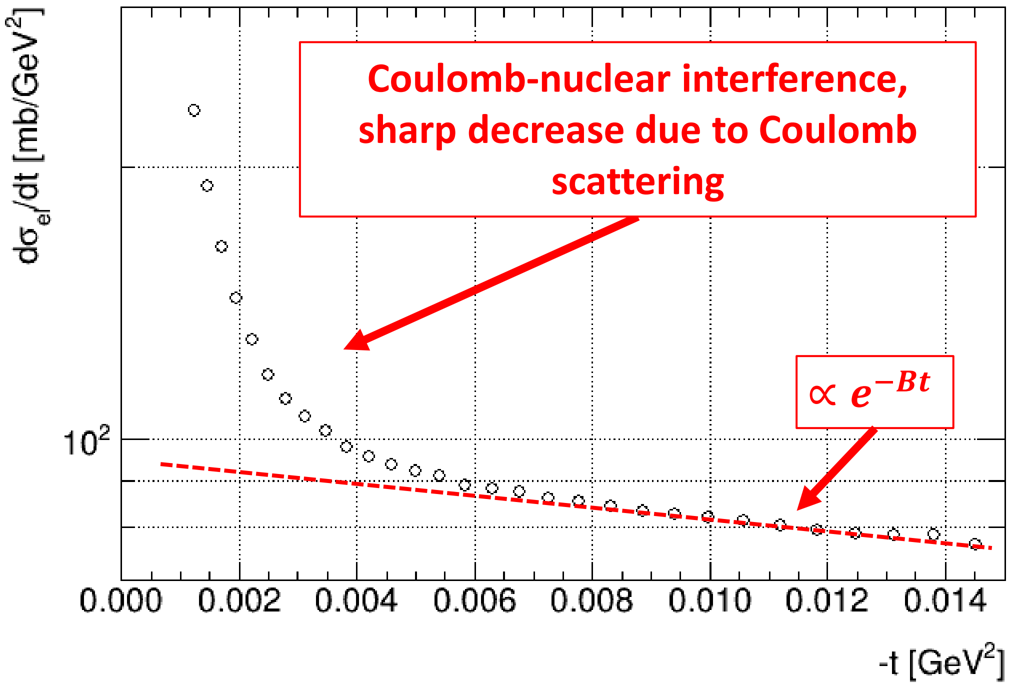


Schematic rapidity distribution of outgoing particles in pp elastic scattering and in pp single diffractive dissociation

Structures in elastic pp differential cross section

- measurements at CERN ISR in the 1970s revealed the characteristic structures of the high energy elastic pp differential cross section

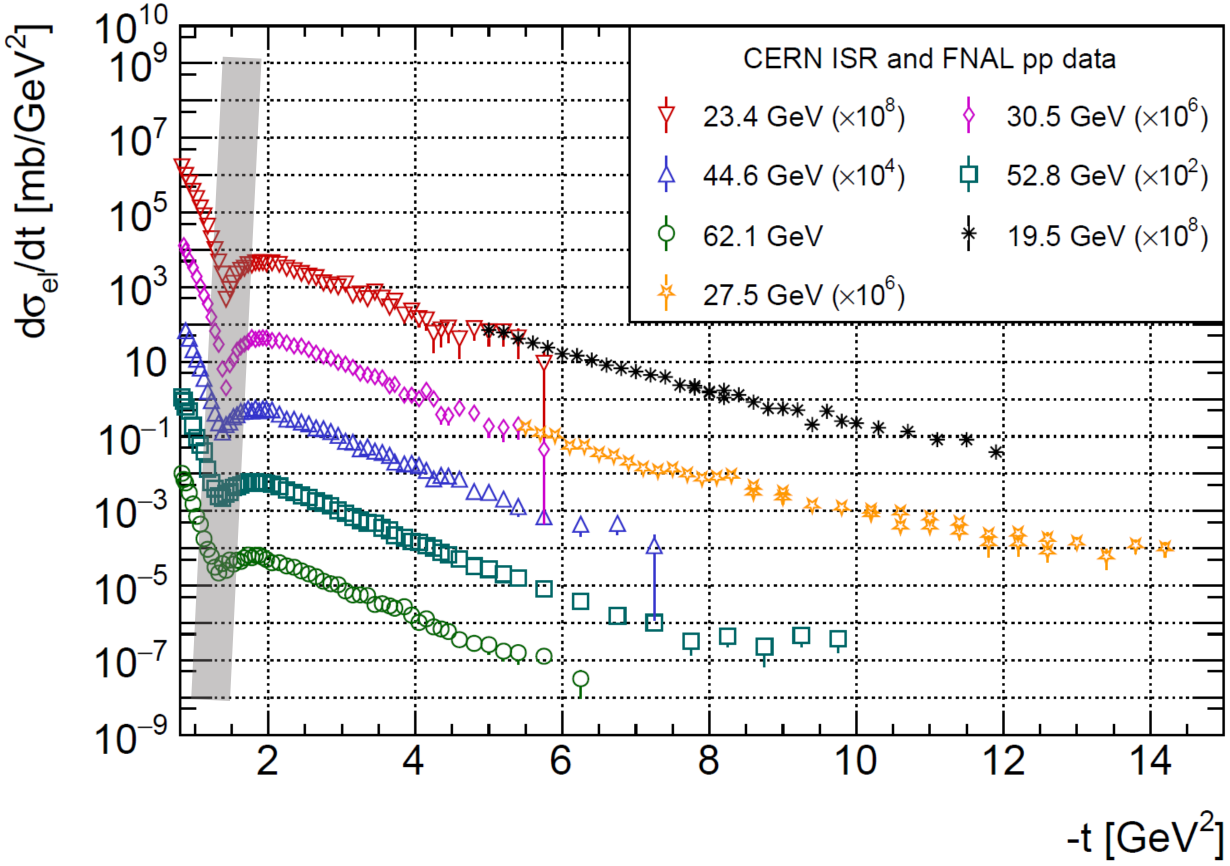




Elastic pp differential cross section measured at CERN ISR at $\sqrt{s} = 53.8$ GeV

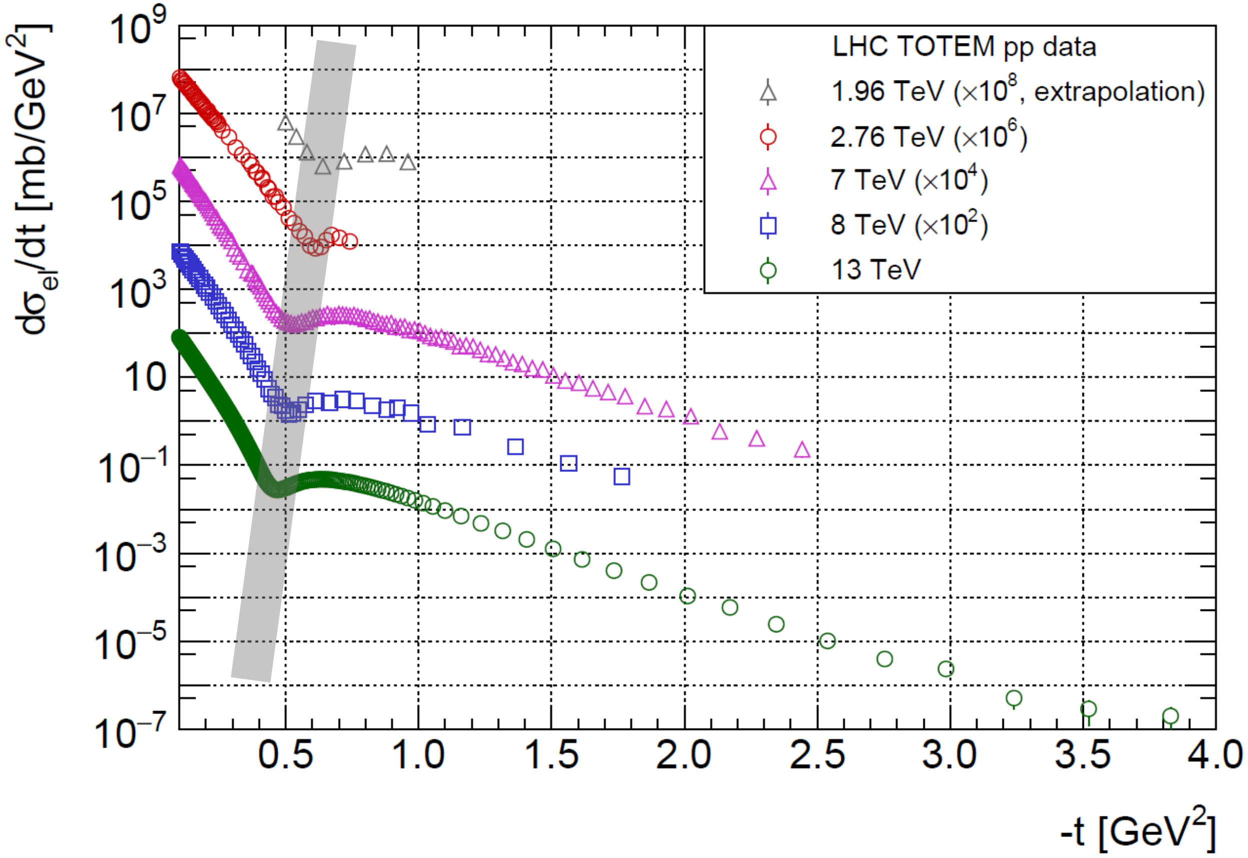
Elastic pp $d\sigma_{el}/dt$ measurements at medium and high |t|

E. Nagy et al., Nucl. Phys. B 150, 221 (1979)
W. Faissler et al., Phys. Rev. D 23, 33 (1981)



the position of the dip and the bump moves to lower |t| values as the CM energy increases

TOTEM Collab., EPL 95:4, 41001 (2011)
TOTEM Collab., Eur. Phys. J. C 79:10, 861 (2019)
TOTEM Collab., Eur. Phys. J. C 80:2, 91 (2020)
TOTEM Collab., Eur. Phys. J. C 82:3, 263 (2022)
TOTEM & D0 Collabs., Phys. Rev. Lett. 127:6, 062003



no secondary dip-bump structures are observed in the |t|range measured up to now

Dipole Regge model

L. L. Jenkovszky and A. N. Wall, Czech. J. Phys. B26, 447 (1976)
L. L. Jenkovszky, Fortsch. Phys.34, 791 (1986)

- basic assumptions:
  - the asymptotic behaviour of the scattering amplitude $\mathrm{A}(\mathrm{s}, \mathrm{t})$ is determined by an isolated j-plane pole of the second order (dipole)
  - the residue at the pole is independent of t, t-dependence enters only through the trajectory
- the partial wave amplitude is obtained as a derivative of a simple pole:

$$\boxed{a_j(t) \equiv a(j,t) = \frac{d}{d\alpha(t)}\left[\frac{\beta(j)}{j-\alpha(t)}\right] = \frac{\beta(j)}{[j-\alpha(t)]^2}}$$

- **the dipole scattering amplitude is obtained as a derivative of a simple pole scattering amplitude:**

$$\boxed{\mathrm{A}^{\mathrm{DP}}(s,\alpha) = \frac{d}{d\alpha}\mathrm{A}^{\mathrm{SP}}(s,\alpha) = e^{-\frac{i\pi\alpha}{2}}\left(\frac{s}{s_0}\right)^{\alpha}\left[G'(\alpha) + \left(L - \frac{i\pi}{2}\right)G(\alpha)\right]}$$

$$\boxed{\mathrm{A}^{\mathrm{SP}}(s,\alpha) = e^{-\frac{i\pi\alpha}{2}}G(\alpha)\left(\frac{s}{s_0}\right)^{\alpha}} \qquad \boxed{\alpha = \alpha(t)} \qquad \boxed{L \equiv ln\frac{s}{s_0}}$$

Dipole Regge model

L. L. Jenkovszky and A. N. Wall, Czech. J. Phys. B26, 447 (1976)
L. L. Jenkovszky, Fortsch. Phys.34, 791 (1986)

$$\mathrm{A}^{\mathrm{DP}}(s,\alpha) = e^{-\frac{i\pi\alpha}{2}}\left(\frac{s}{s_0}\right)^{\alpha}\left[G'(\alpha) + \left(L - \frac{i\pi}{2}\right)G(\alpha)\right]$$

- motivated by the shape of the diffraction cone (exponential decrease), the paramterization of $G'(\alpha)$ is:

$$G'(\alpha) = a e^{b[\alpha-\alpha_0]}$$

(α_0 is the intercept of the trajectory)

- $G(\alpha)$ is obtained by by integrating $G'(\alpha)$:

$$G(\alpha) = \int G'(\alpha)d\alpha = a\left(\frac{e^{b[\alpha-\alpha_0]}}{b} - \gamma\right)$$

- introducing that $\varepsilon = \gamma b$ the amplitude can be rewritten as:

$$\mathrm{A}^{\mathrm{DP}}(\mathrm{s},\mathrm{t}) = \mathrm{i}\frac{a}{b}\left(\frac{\mathrm{s}}{\mathrm{s}_0}\right)^{\alpha_0} e^{-\frac{i\pi}{2}(\alpha_0-1)}\left[\mathrm{r}_1^2(\mathrm{s})e^{\mathrm{r}_1^2(\mathrm{s})[\alpha(\mathrm{t})-\alpha_0]} - \varepsilon\, \mathrm{r}_2^2(\mathrm{s})e^{\mathrm{r}_2^2(\mathrm{s})[\alpha(\mathrm{t})-\alpha_0]}\right]$$

$$\mathrm{r}_1^2(\mathrm{s}) = \mathrm{b} + \mathrm{L}(\mathrm{s}) - \mathrm{i}\pi/2 \qquad \mathrm{r}_2^2(\mathrm{s}) = \mathrm{L}(\mathrm{s}) - \mathrm{i}\pi/2$$

Model for elastic pp and $\bar{p}p$ scattering amplitude

$$A(s,t)^{\bar{p}p}_{pp} = A_P^{DP}(s,t) + A_f^{SP}(s,t) \pm [A_O^{DP}(s,t) + A_\omega^{SP}(s,t)]$$

- **the dipole pomeron and odderon amplitudes are:**

$$A_P^{DP}(s,t) = e^{-\frac{i\pi\alpha_P(t)}{2}} \left(\frac{s}{s_{0P}}\right)^{\alpha_P(t)} \left[G'_P(t) + \left(L_P(s) - \frac{i\pi}{2}\right) G_P(t)\right]$$

$$A_O^{DP}(s,t) = -iA_{P\to O}^{DP}(s,t)$$

$$G'_P(t) = a_P e^{b_P[\alpha_P(t)-\alpha_P(0)]}$$

$$G_P(t) = a_P\left(e^{b_P[\alpha_P(t)-\alpha_P(0)]}/b_P - \gamma_P\right)$$

(with free parameters labeled by "O")

$$L_P(s) = \ln\frac{s}{s_{0P}}$$

$$\alpha_P(t) = 1 + \delta_P + \alpha'_P t$$

- **the simple pole f and ω reggeon amplitudes are:**

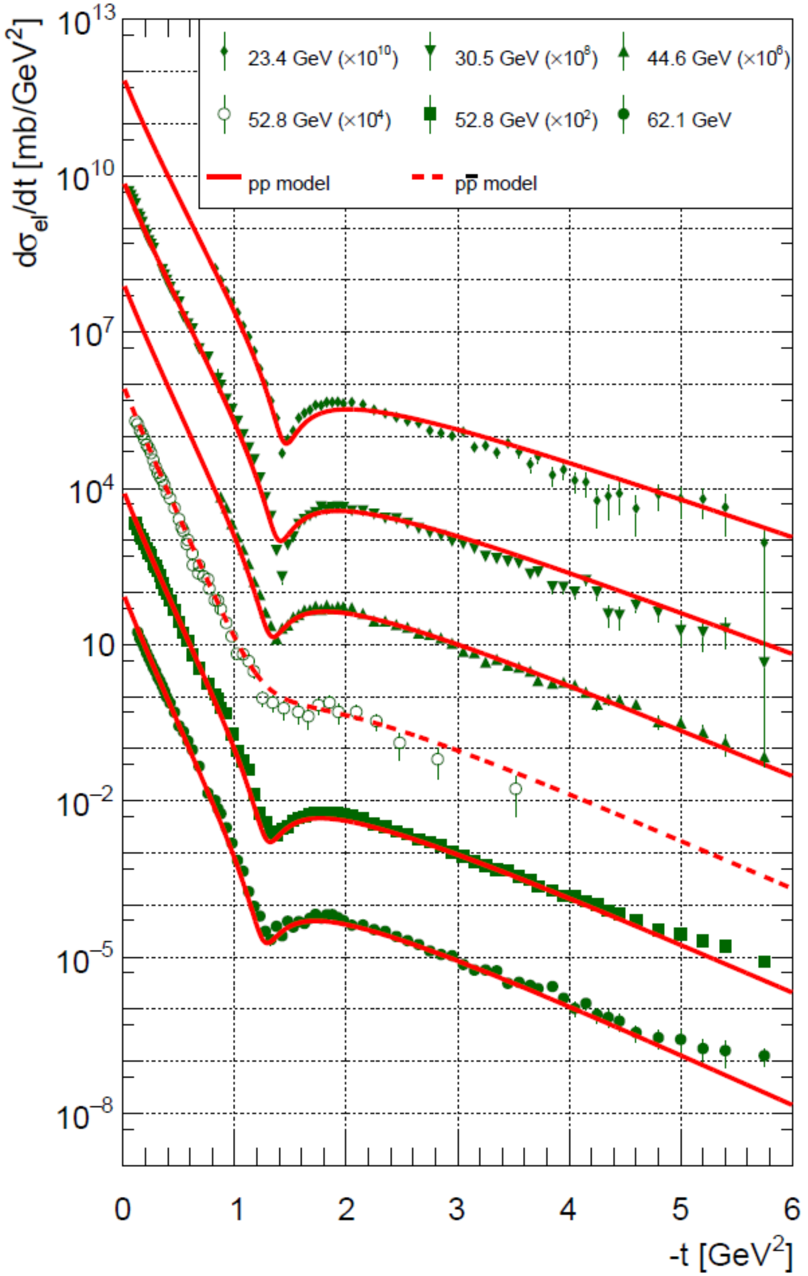
$$\mathbf{A_f}(s,t) = -a_f e^{-\frac{i\pi\alpha_f(t)}{2}} (s/s_{0f})^{\alpha_f(t)} e^{b_f t}$$

$$\mathbf{A_\omega}(s,t) = -i\mathbf{A_{f\to\omega}}(s,t)$$

$$\alpha_f(t) = \alpha_f^0 + \alpha'_f t$$

(with free parameters labeled by "ω")

ISR $d\sigma_{el}/dt$ data and the model

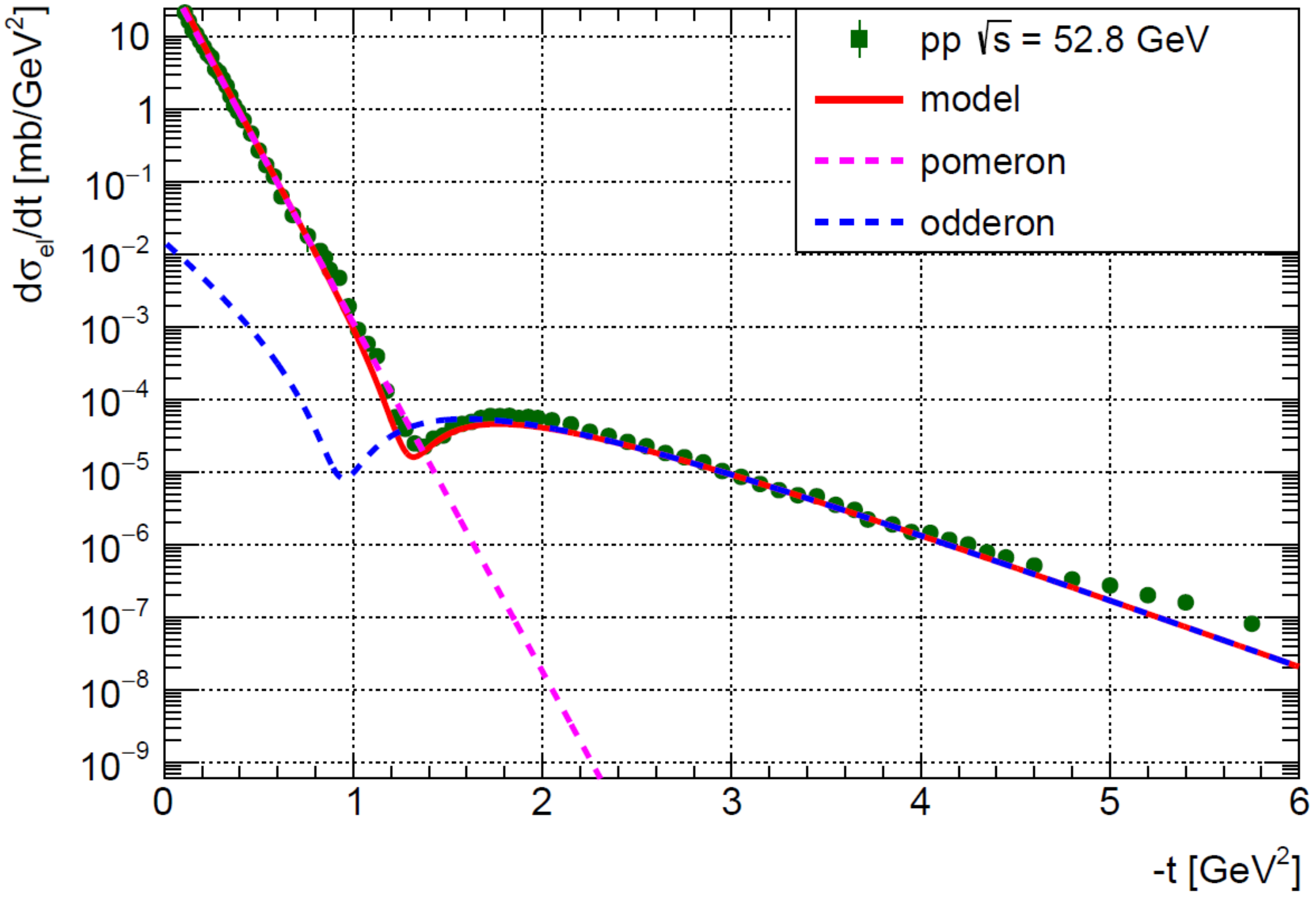


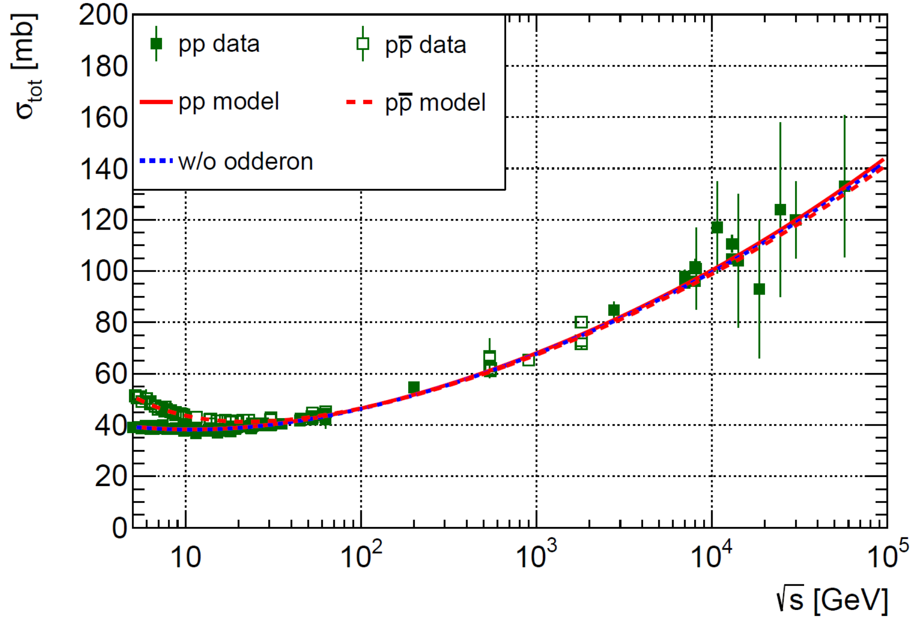
pomeron	odderon	f-reggeon	ω-reggeon
$\delta_P = 0.043$	$\delta_O = 0.14$	$\alpha_f^0 = 0.69$	$\alpha_\omega^0 = 0.44$
$\alpha'_P = 0.36$	$\alpha'_O = 0.13$	$\alpha'_f = 0.84$	$\alpha'_\omega = 0.93$
$a_P = 9.10$	$a_O = 0.029$	$a_f = 15.4$	$a_\omega = 9.69$
$b_P = 8.47$	$b_O = 6.96$	$b_f = 4.78$	$b_\omega = 3.5$
$\gamma_P = 0$	$\gamma_O = 0.11$	-	-
$s_{0P} = 2.88$	$s_{0O} = 1$	$s_{0f} = 1$	$s_{0\omega} = 1$

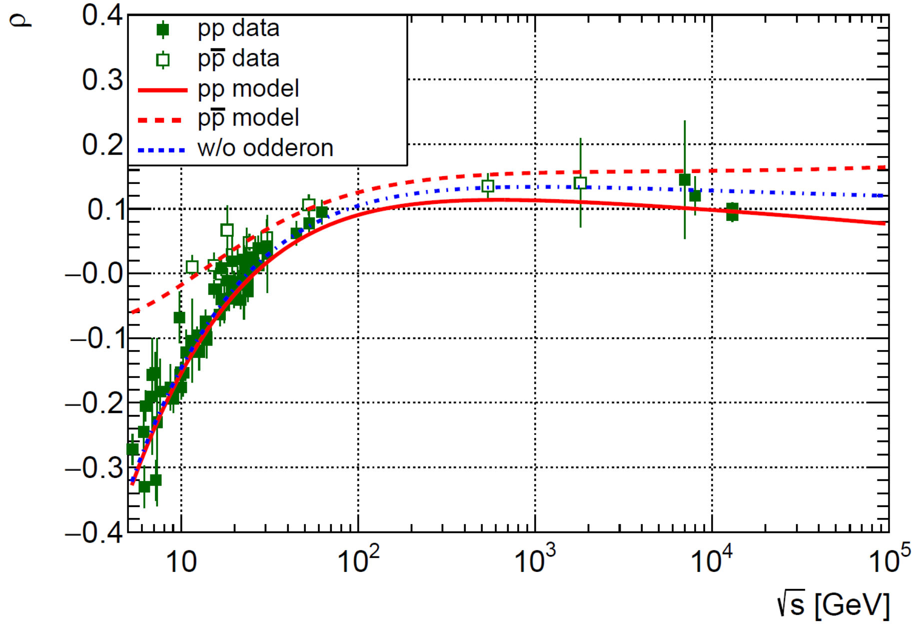
Fit to proton-proton and proton-antiproton differential cross section data at ISR energy region, and to ρ and total cross section data from 5 GeV up to the highest energies

$d\sigma_{el}/dt$ with P and O contribution, ρ and σ_{tot} w/o O

the odderon contribution takes over completely after the bump but at low-|t| the odderon contribution is small



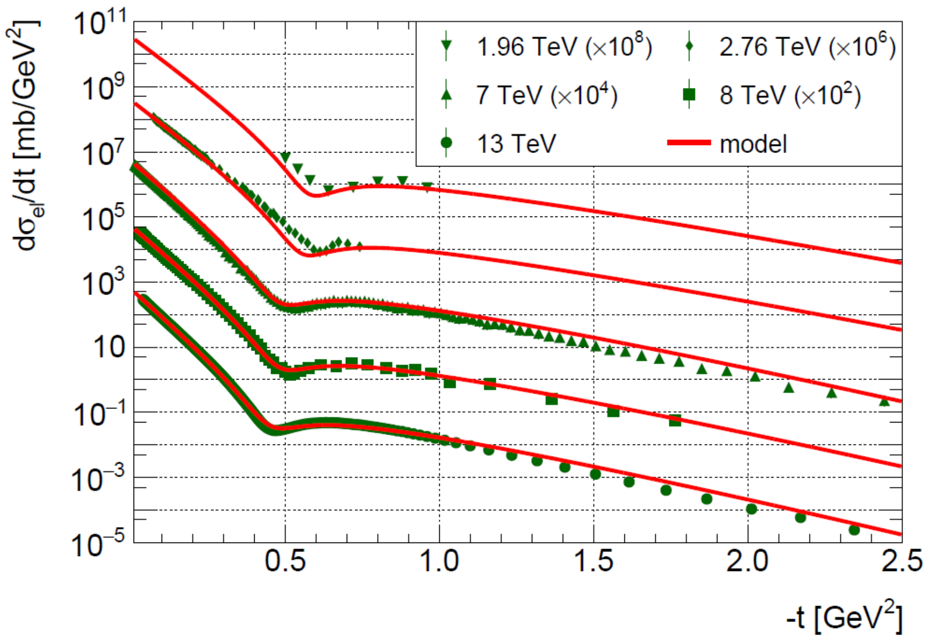


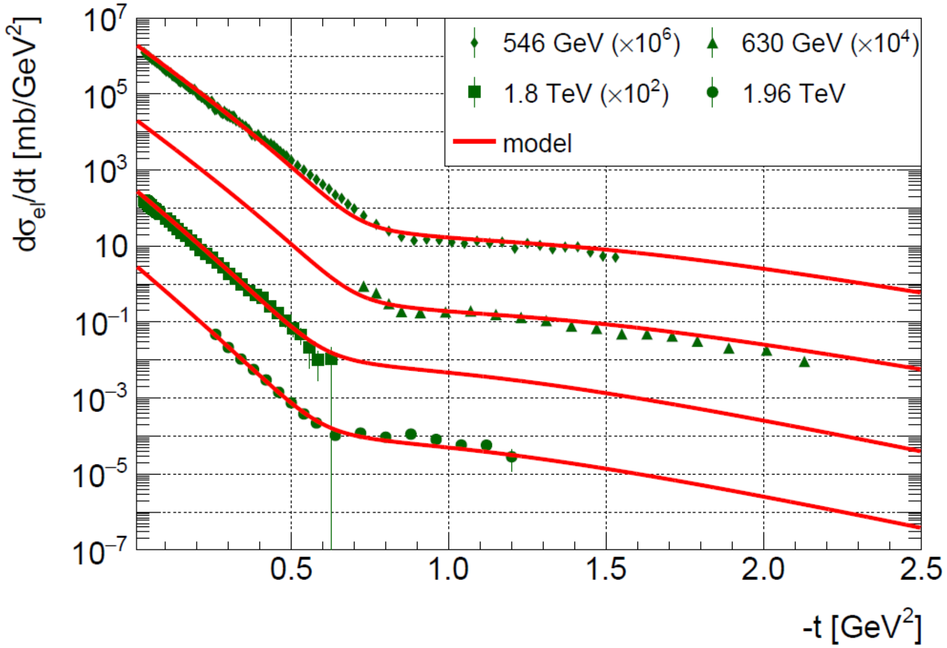


SPS + TEVATRON + LHC $d\sigma_{el}/dt$ data and the model

fit to proton-proton and proton-antiproton differential cross section data, and to ρ and total cross section data from 0.5 TeV up to the highest energies

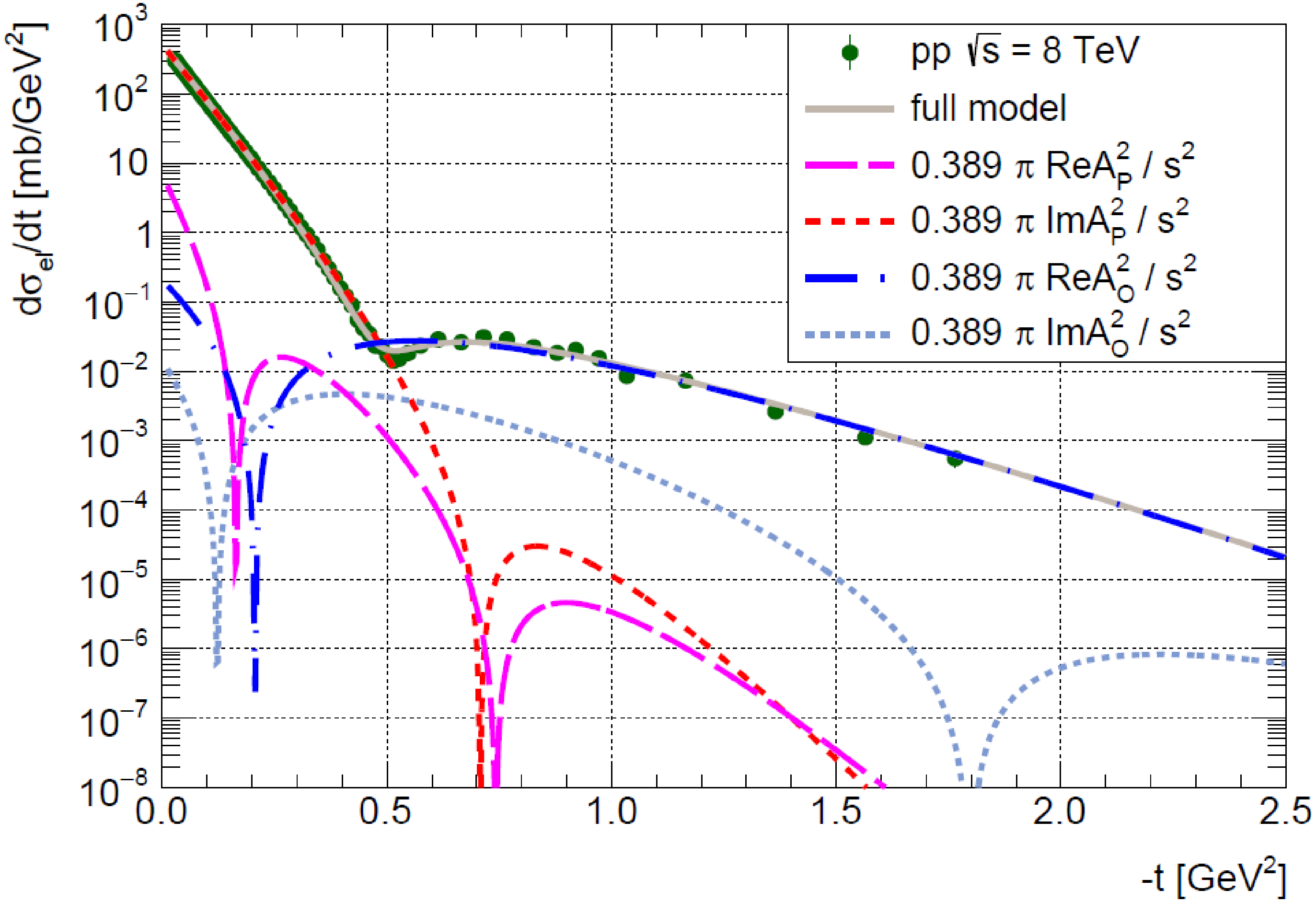
Pomeron	Odderon
$\delta_P = 0.02865$	$\delta_O = 0.2042$
$\alpha'_P = 0.4284$	$\alpha'_O = 0.1494$
$a_P = 45.63$	$a_O = 0.01934$
$b_P = 4.873$	$b_O = 2.160$
$\gamma_P = 0.06085$	$\gamma_O = 0.4866$
$s_{0P} = 11.26$	$s_{0O} = 1.03$

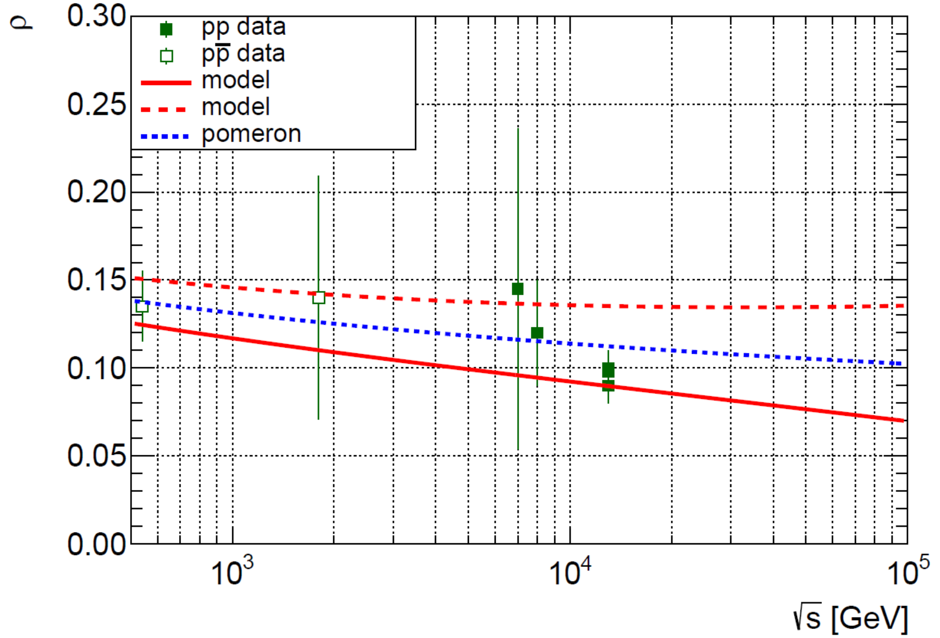


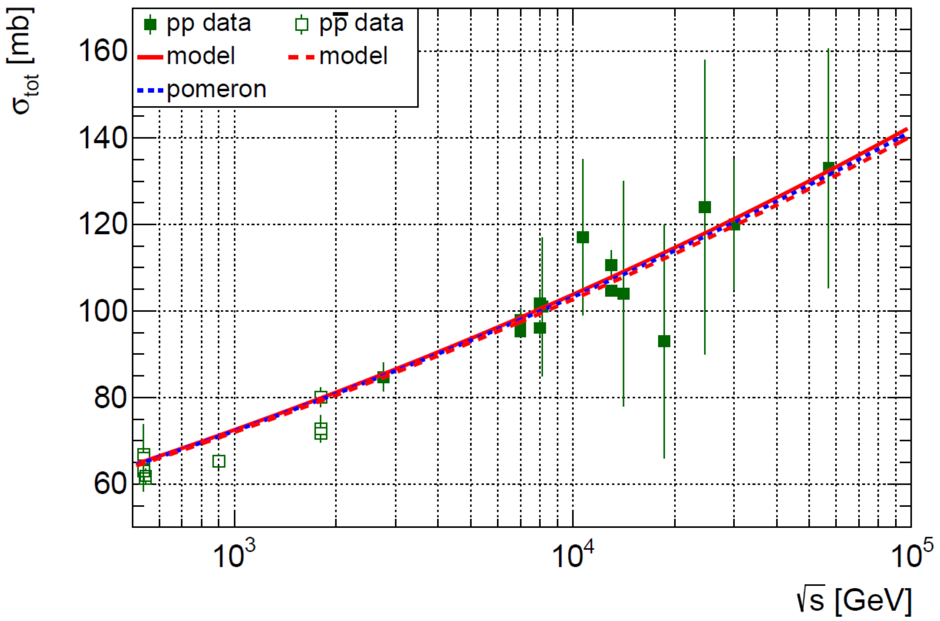


$d\sigma_{el}/dt$ with P and O contribution, ρ and σ_{tot} w/o O

the odderon contribution takes over completely after the bump but at low-|t| the odderon contribution is small







Dip and bump position in $d\sigma_{el}/dt$ in a dipole Regge model

$$\mathrm{A}^{\mathrm{DP}}(s,\alpha) = e^{-\frac{i\pi\alpha}{2}}\left(\frac{s}{s_0}\right)^{\alpha}\left[G'(\alpha) + \left(L - \frac{i\pi}{2}\right)G(\alpha)\right]$$

$$G'(\alpha) = a e^{b[\alpha-1-\delta]}$$

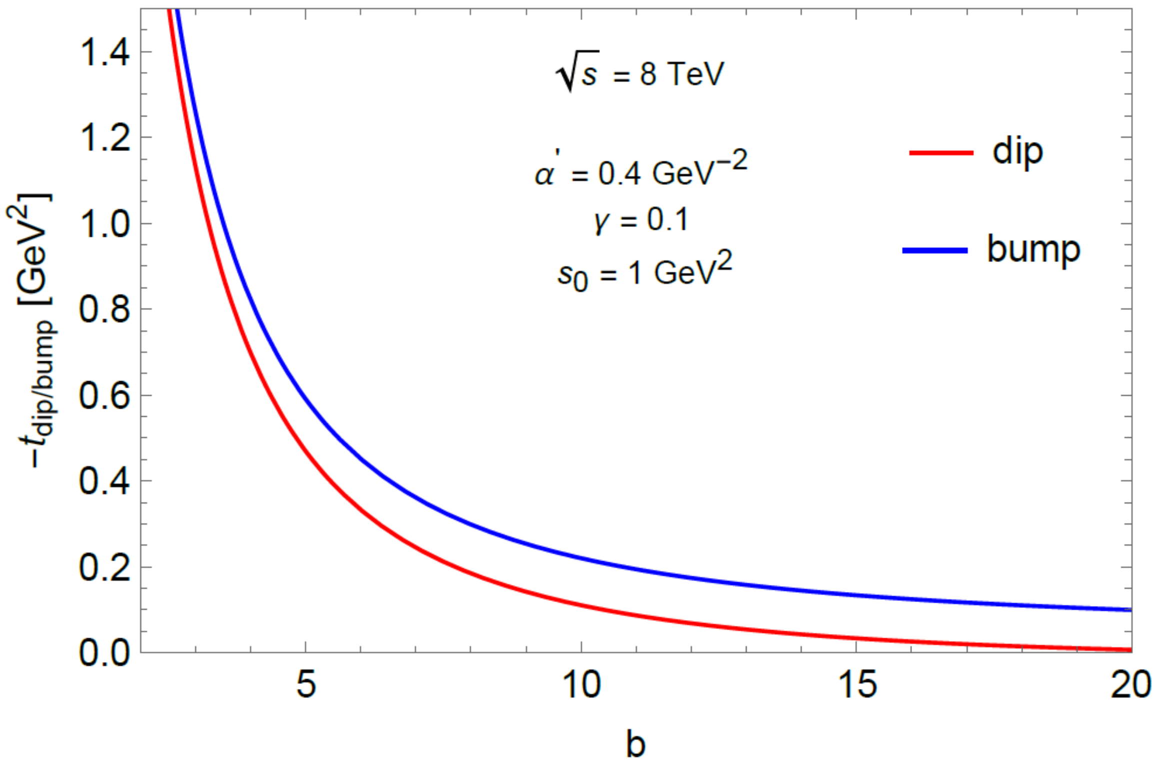
$$G(\alpha) = a\left(\frac{e^{b[\alpha-1-\delta]}}{b} - \gamma\right)$$

$$\alpha = 1 + \delta + \alpha' \mathrm{t}$$

$$L = ln\frac{s}{s_0}$$

$$-t_{min} = \frac{1}{\alpha' b}\ln\frac{b+L}{\gamma b L}$$

$$-t_{max} = \frac{1}{\alpha' b}\ln\frac{4(b+L)^2+\pi^2}{\gamma b(4L^2+\pi^2)}$$



the position of the dip and of the bump goes to smaller –t values as slope parameter rises

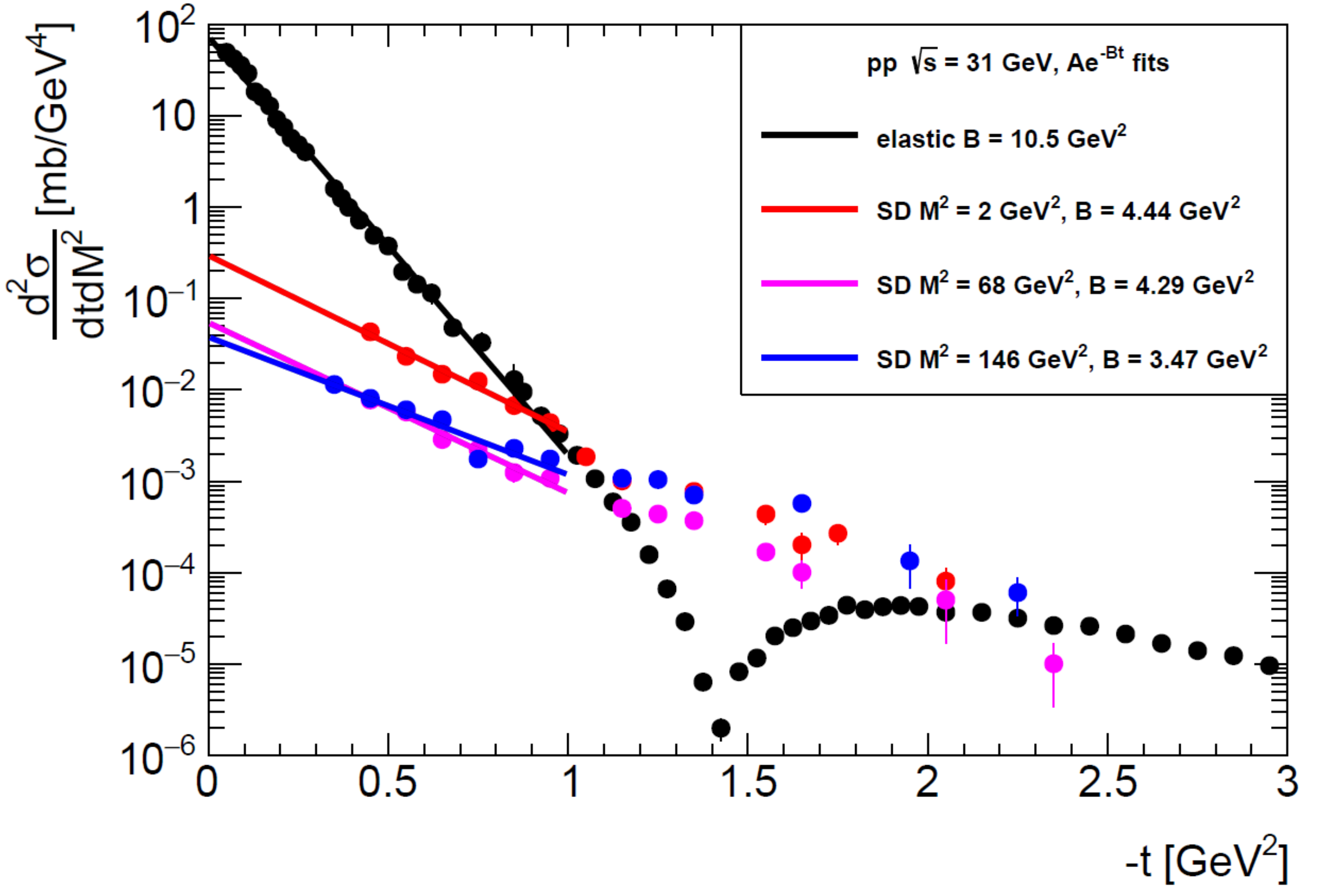
Dip-bump structures in single diffractive dissociation?

- measurements of pp single diffractive dissociation at ISR do not show a dip-bump structure at |t| values where such a structure is observed in elastic pp scattering

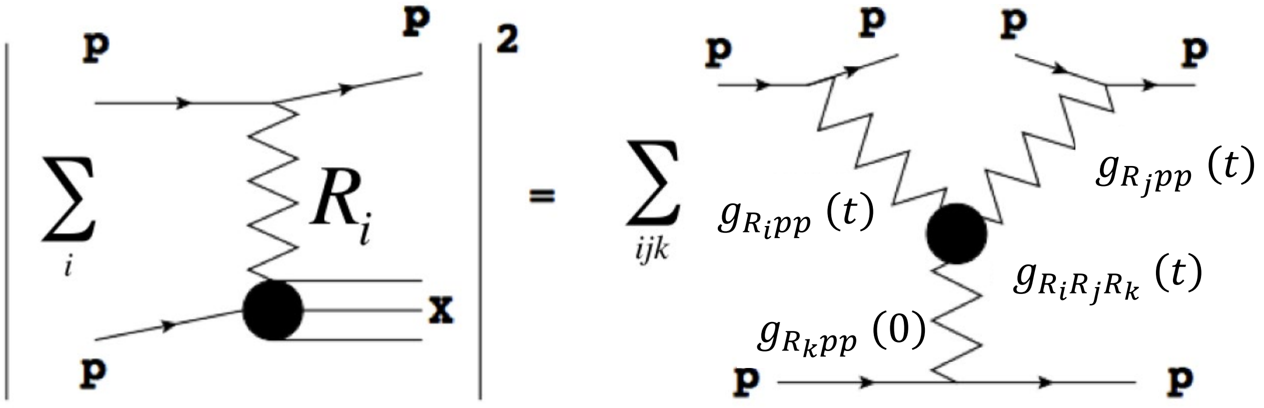
  M.G. Albrow et al., Nucl. Phys. B72, 376 (1974)

- it can be explained in a framework of a dipole Regge model in which the dip-bump structure moves to higher |t| values as the value of the slope parameter decreases

- **a dipole odderon+pomeron Regge approach can be used to predict dip-bump structures in pp single diffractive dissociation at LHC energies**



Triple Regge approach for single diffraction (SD)



when $s \gg M^2 \gg t$, the differential cross section is given by a sum of triple-Reggeon contributions

$\mathbb{P}\mathbb{P}$	$\mathbb{P}\mathbb{P}$	$f_2 f_2$	$f_2 f_2$	$f_2\mathbb{P}$	$\mathbb{P} f_2$	$f_2\mathbb{P}$	$\omega\mathbb{P}$	
$\mathbb{P}$	f_2	$\mathbb{P}$	f_2	$\mathbb{P}$	$\mathbb{P}$	f_2	ω	$\cdots$

$$\frac{d^2\sigma_{SD}}{dt\,dM^2} = \sum_{ijk} \frac{1}{16\pi^2}\frac{s_0}{s^2}\, g_{R_ipp}\,(t)\, g_{R_jpp}\,(t) \left(\frac{s}{M^2}\right)^{\alpha_i(t)+\alpha_j(t)} g_{R_iR_jR_k}\,(t)\, g_{R_kpp}\,(0) \left(\frac{M^2}{s_0}\right)^{\alpha_{R_k}(0)} \cos\left(\frac{\pi}{2}\left(\alpha_i(t)-\alpha_j(t)\right)\right)$$

Dipole Regge approach for single diffraction (SD)

- in the triple Regge approach the triple pomeron vertex results the following contribution for the double differential SD cross section:

$$\boxed{\frac{d^2\sigma_{SD}^{PPP}}{dtdM^2} = \frac{1}{16\pi^2}\frac{1}{M^2}g_{Ppp}^2(t)\left(\frac{s}{M^2}\right)^{2\alpha_P(t)-2} g_{PPP}(t)g_{Ppp}(0)(M^2)^{\alpha_{0P}-1}}$$

- g_{PPP} is found to be t-independent
- **assumption: the t-dependent part of the amplitude of the SD process has the form in case the pomeron a simple pole:**

$$\boxed{A_{SD}^{SP}(s, M^2, \alpha(t)) \sim \mathrm{e}^{-\frac{\mathrm{i}\pi\alpha}{2}}G(\alpha)(s/M^2)^{\alpha}}$$

- $G(\alpha)$ incorporates the t-dependece coming from $\mathrm{g}_{\mathrm{Ppp}}(t)$
- a dipole pomeron amplitude is obtained as:

$$\boxed{A_{SD}^{DP}(s, M^2, \alpha) = \frac{d}{d\alpha}A_{SD}^{SP}(s, M^2, \alpha) \sim e^{-\frac{i\pi\alpha}{2}}\left(\frac{s}{M^2}\right)^{\alpha}\left[G'(\alpha) + \left(L_{SD} - \frac{i\pi}{2}\right)G(\alpha)\right]} \qquad \boxed{L_{SD} \equiv \ln(s/M^2)}$$

Dipole Regge approach for single diffraction (SD)

- the double differential cross section for the SD process resulting from the dipole pomeron amplitude is:

$$\frac{d^2\sigma_{SD}^{PPP}}{dt dM^2} = \frac{1}{M^2}\left({G_P}'^2(\alpha_P) + 2L_{SD}G_P(\alpha_P){G_P}'(\alpha_P) + G_P^2(\alpha_P)\left(L_{SD}^2 + \frac{\pi^2}{4}\right)\right)\left(\frac{s}{M^2}\right)^{2\alpha_P(t)-2} \sigma^{Pp}(M^2)$$

$$G_P'(\alpha_P) = a_P e^{b_P[\alpha_P - 1 - \delta_P]}$$

$$\alpha_P = 1 + \delta_P + \alpha_P' t$$

$$G_P(\alpha_P) = \int G'(\alpha_P) d\alpha_P = a_P\left(\frac{e^{b_P[\alpha_P - 1 - \delta_P]}}{b_P} - \gamma_P\right)$$

$$\sigma^{Pp}(M^2) = g_{PPP}\, g_{Ppp}(0)(M^2)^{\delta_P}$$

- Using the $\xi = M^2/s$ proton's relative momentum loss variable, we have:

$$\frac{d^2\sigma_{SD}^{PPP}}{dt d\xi} = \left({G_P}'^2(\alpha_P) + 2L_{SD}G_P(\alpha_P){G_P}'(\alpha_P) + G_P^2(\alpha_P)\left(L_{SD}^2 + \frac{\pi^2}{4}\right)\right)\xi^{1-2\alpha_P(t)}\sigma^{Pp}(s\xi)$$

$$L_{SD} \equiv \ln(s/M^2) = -\ln\xi$$

- **dip and bump appear at:**

$$-t_{dip}^{SD} = \frac{1}{\alpha' b}\ln\frac{b + L_{SD}}{\gamma b L_{SD}}$$

$$-t_{bump}^{SD} = \frac{1}{\alpha' b}\ln\frac{4(b + L_{SD})^2 + \pi^2}{\gamma b(4L_{SD}^2 + \pi^2)}$$

Odderon contribution in SD in form of an OOP vertex

- the odderon-odderon-pomeron vertex results the following contribution for the double differential SD cross section:

$$\boxed{\frac{d^2\sigma_{SD}^{OOP}}{dtdM^2} = \frac{1}{16\pi^2}\frac{1}{M^2}g_{Opp}^2(t)(s/M^2)^{2\alpha_O(t)-2}g_{OOP}(t)g_{Ppp}(0)(M^2)^{\delta_P}}$$

- **assumption: $g_{OOP}(t)$ is t-independent and the t-dependent part of the odderon amplitude of the SD process has the form:**

$$\boxed{A_{SD}^{SP}(s,M^2,\alpha_O) \sim \mathrm{e}^{-\frac{\mathrm{i}\pi\alpha}{2}}G_O(\alpha_O)(s/M^2)^{\alpha_O}}$$

$$\boxed{G_O'(\alpha_O) = ae^{b[\alpha_O-1]}}$$

$$\boxed{G_O(\alpha_O) = \int G_O'(\alpha_O)d\alpha_O}$$

- $G_O(\alpha_O)$ incorporates the t-dependece coming from $\mathrm{g}_{\mathrm{Opp}}(t)$

- a dipole odderon contribution to the cross section is obtained as:

$$\boxed{\frac{d^2\sigma_{SD}^{OOP}}{dtdM^2} = \frac{1}{M^2}\left({G_O}'^2(\alpha_O) + 2L_{SD}G_O(\alpha_O)G'(\alpha_O) + G_O^2(\alpha_O)\left(L_{SD}^2 + \frac{\pi^2}{4}\right)\right)(s/M^2)^{2\alpha_O(t)-2}\sigma^{Pp}(M^2)}$$

(the a parameter of $G_O(\alpha_O)$ accounts also in the defference between g_{OOP} and g_{PPP})

RRP and pion contribution in SD

- RRP contribution:

$$\boxed{\frac{d^2\sigma_{SD}^{RRP}}{dtdM^2} = \frac{1}{M^2} a_R e^{b_R \alpha_R(t)} (s/M^2)^{2\alpha_R(t)-2} \sigma^{Pp}(M^2)}$$

$$\boxed{\alpha_R(t) = 1 + \delta_R + \alpha'_R\, t}$$

- the pion exchange contribution:

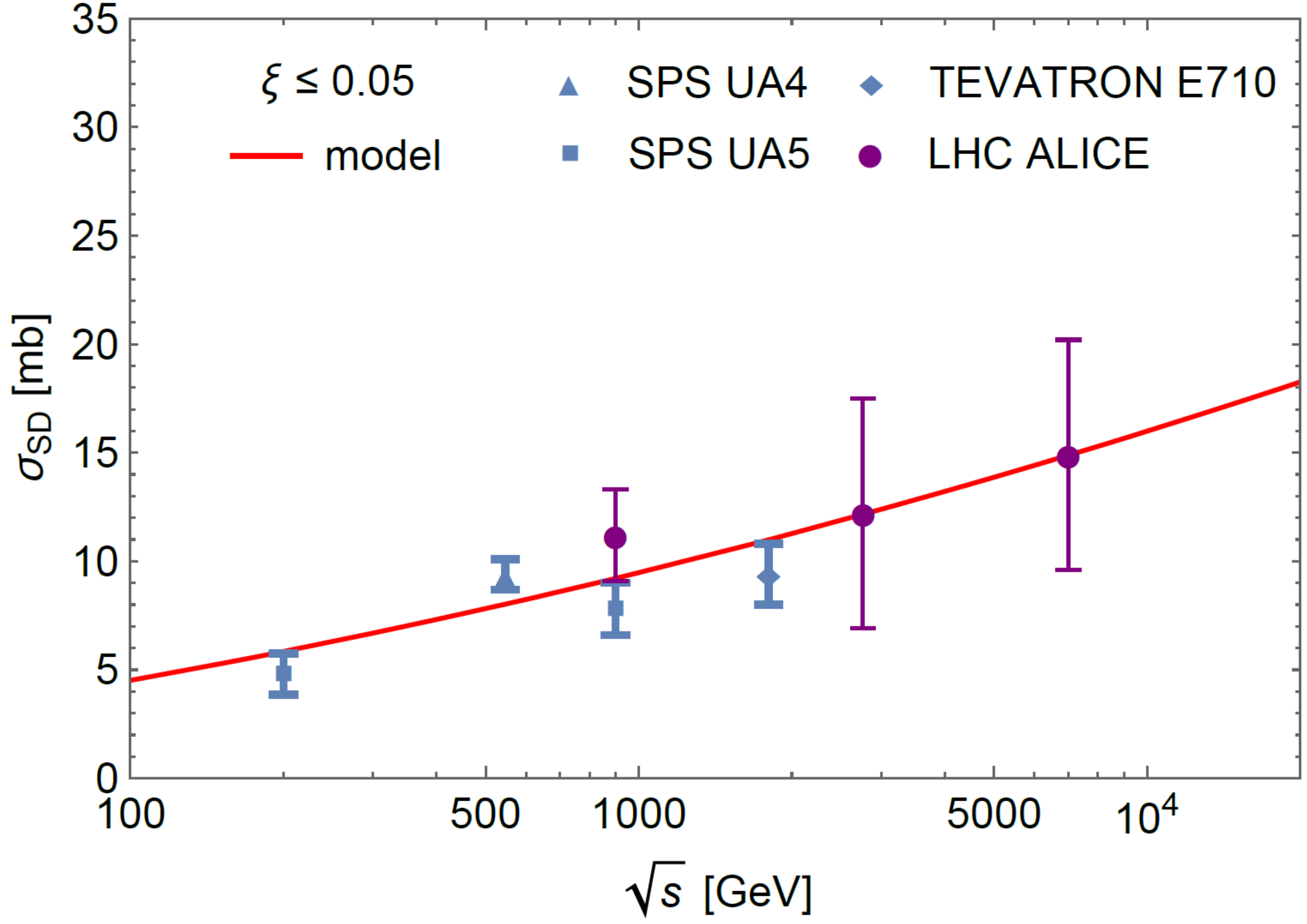
$$\frac{d^2\sigma}{d\xi dt} = f_{\pi/p}(\xi,t)\sigma^{\pi p}(s\xi)$$

$$f_{\pi/p}(\xi,t) = \frac{1}{4\pi}\frac{g^2_{\pi pp}}{4\pi}\frac{|t|}{(t-m_\pi^2)^2} G_1^2(t)\xi^{1-2\alpha_\pi(t)} \qquad G_1(t) = \frac{2.3-m_\pi^2}{2.3-t} \qquad \frac{g^2_{\pi p}}{4\pi} = 13.3 \qquad \alpha_\pi(t) = \alpha'_\pi\,(t-m_\pi^2)$$

- The full double SD differential cross section is written as:

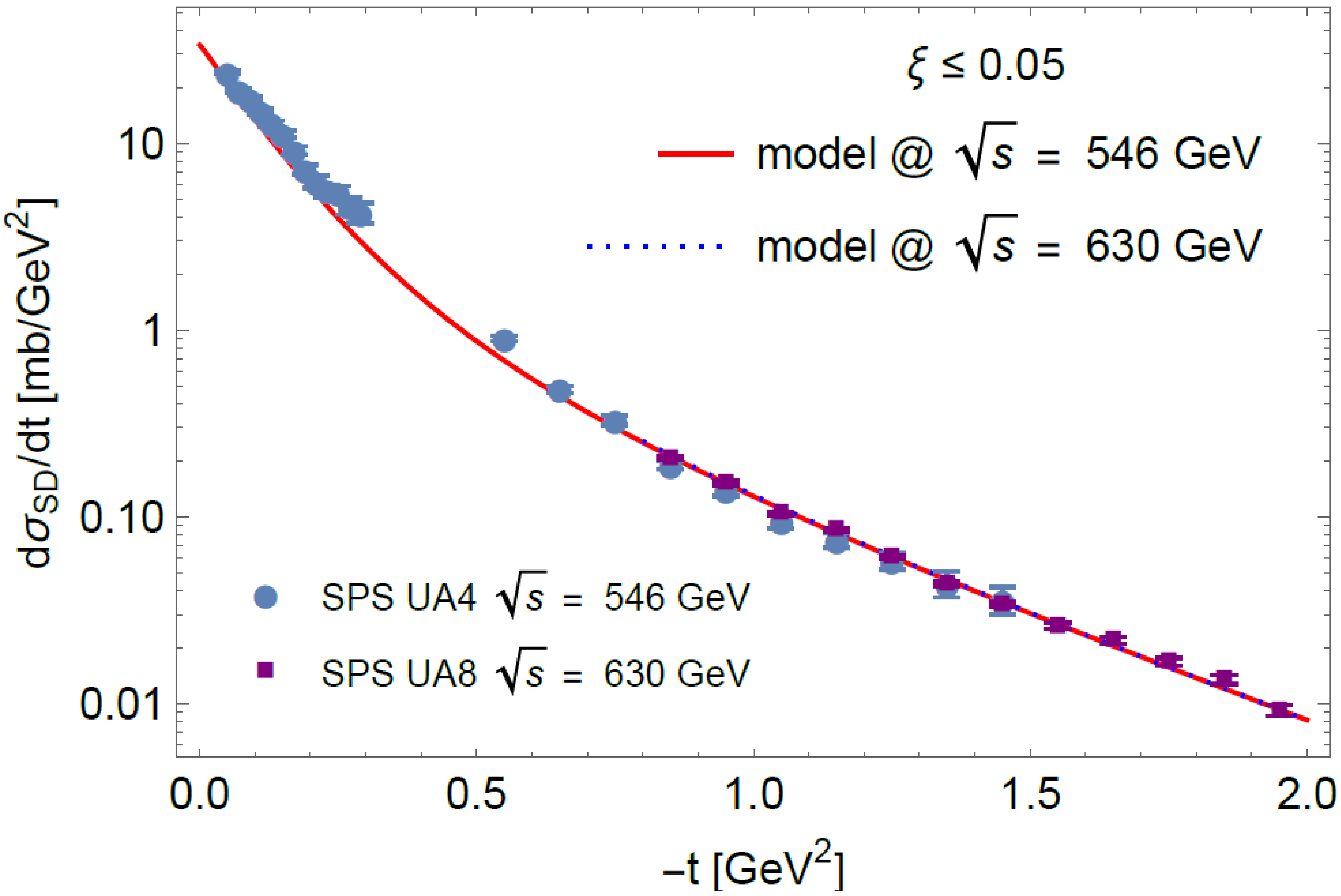
$$\boxed{\frac{d^2\sigma_{SD}}{dtdM^2} = \frac{d^2\sigma_{SD}^{PPP}}{dtdM^2} + \frac{d^2\sigma_{SD}^{OOP}}{dtdM^2} + \frac{d^2\sigma_{SD}^{RRP}}{dtdM^2} + \frac{d^2\sigma_{SD}^{\pi}}{dtdM^2}}$$

Total SD cross section

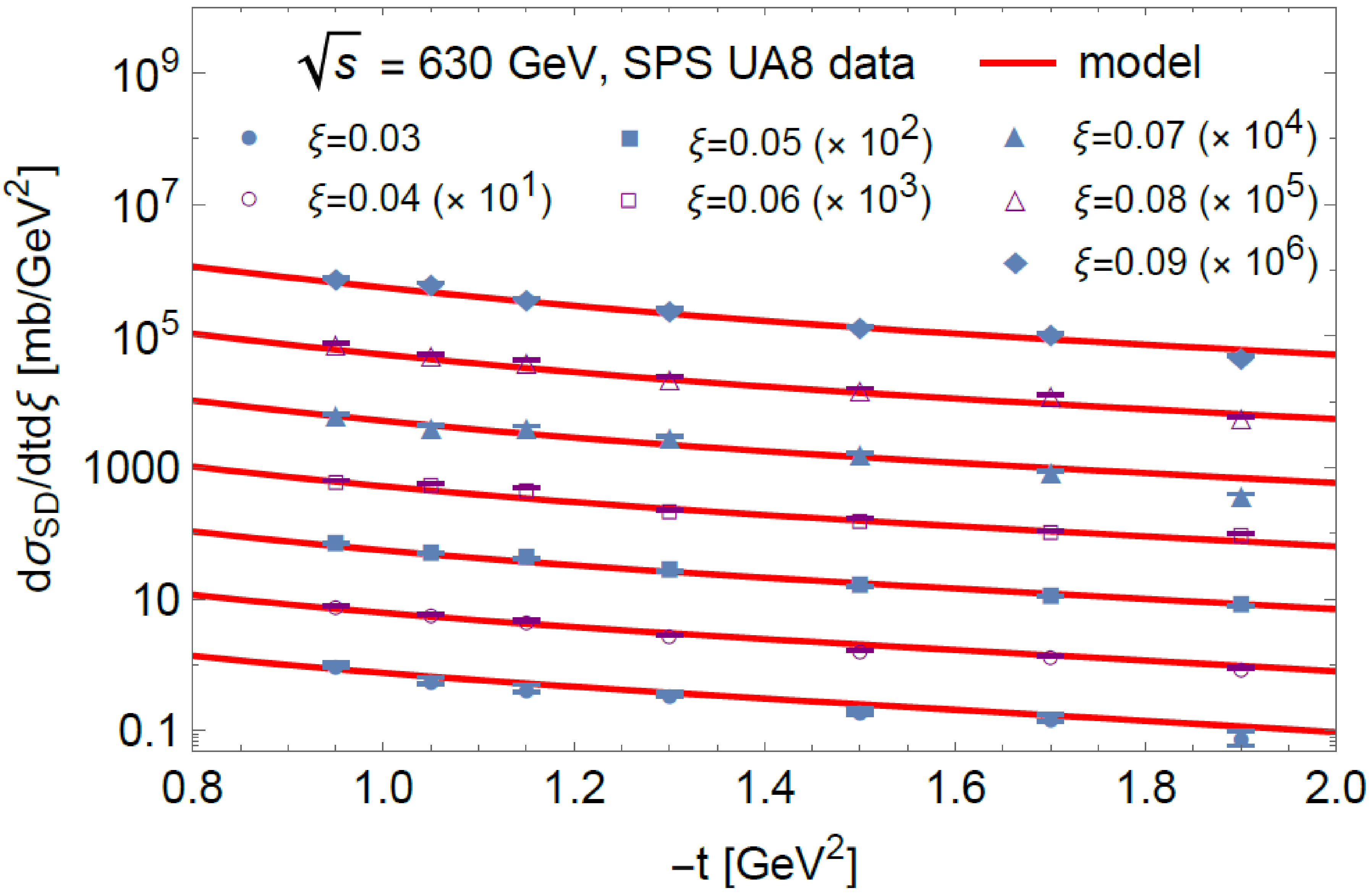


Dipole Pomeron	Dipole Odderon	Simple pole Reggeon
$\delta_P = 0$	$\delta_O = 0$	$\delta_R = -0.45$
$\alpha'_P = 0.43$	$\alpha'_O = 0.15$	$\alpha'_R = 0.93$
$a_P = 0.32$	$a_O = 0.084$	$a_O = 2.5$
$b_P = 2.86$	$b_O = 1.18$	$b_O = 0.0$
$\gamma_P = 0.061$	$\gamma_O = 0.49$	-

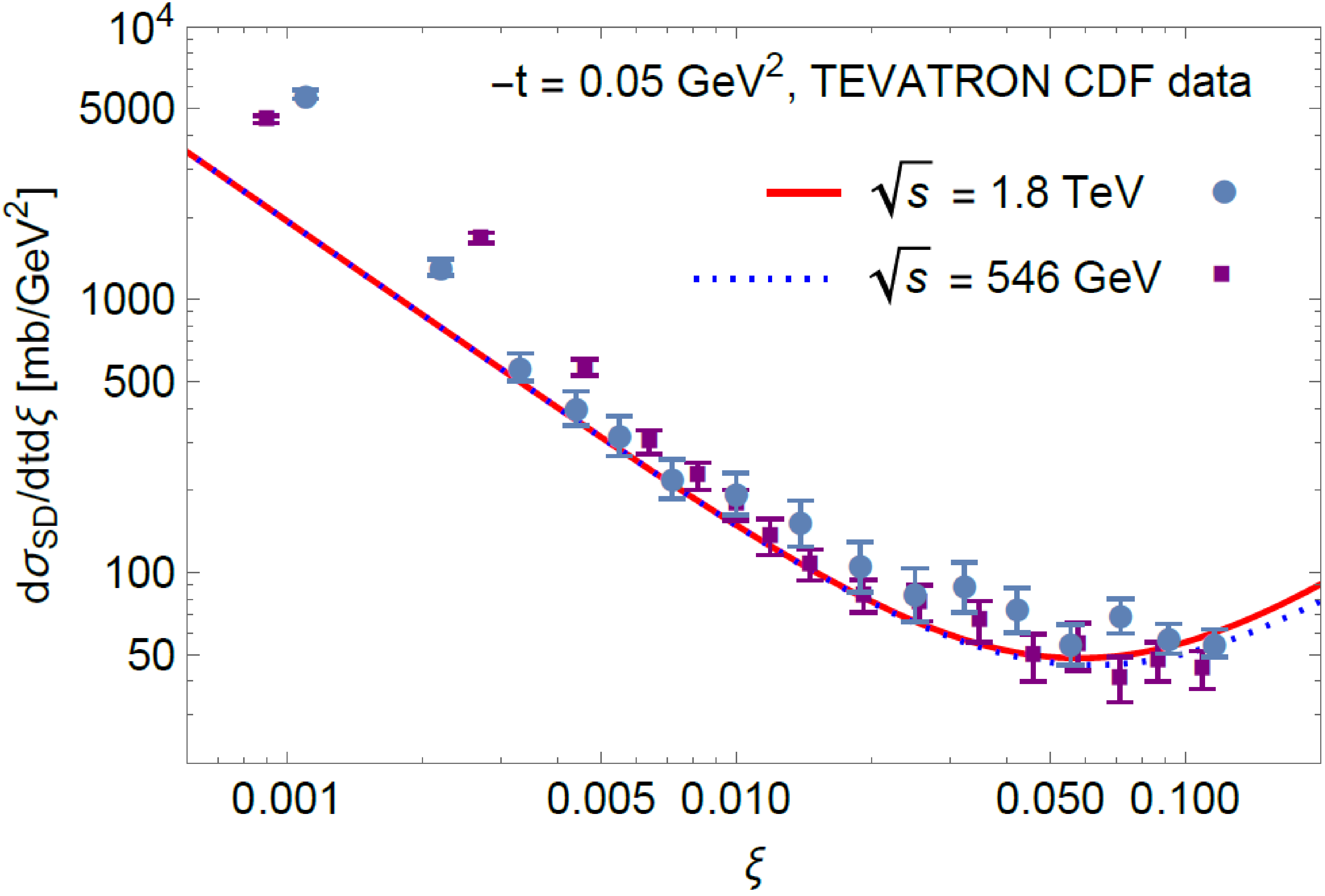
t dependence of the SD process at SPS energies



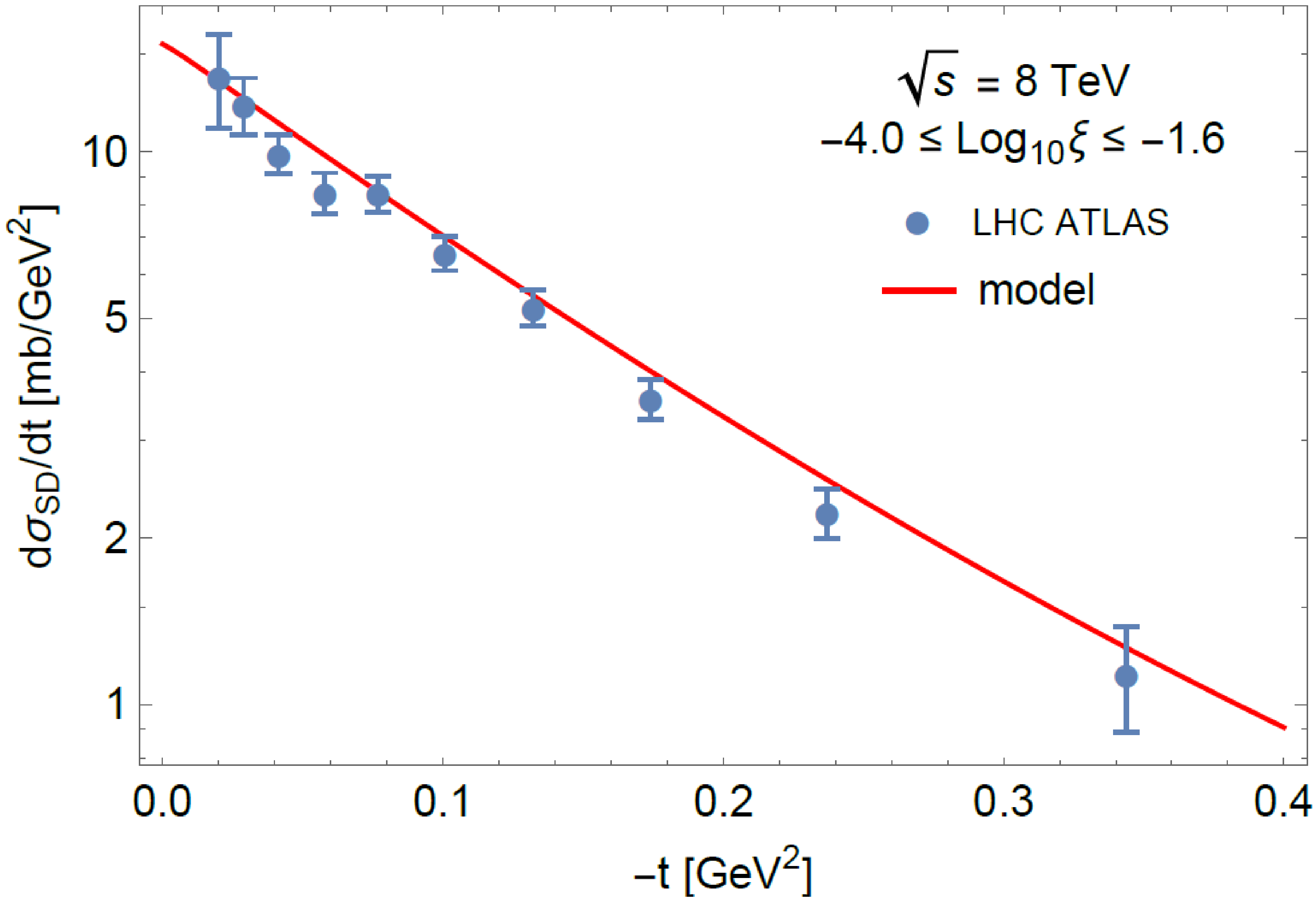
t dependence of the SD process at 630 GeV



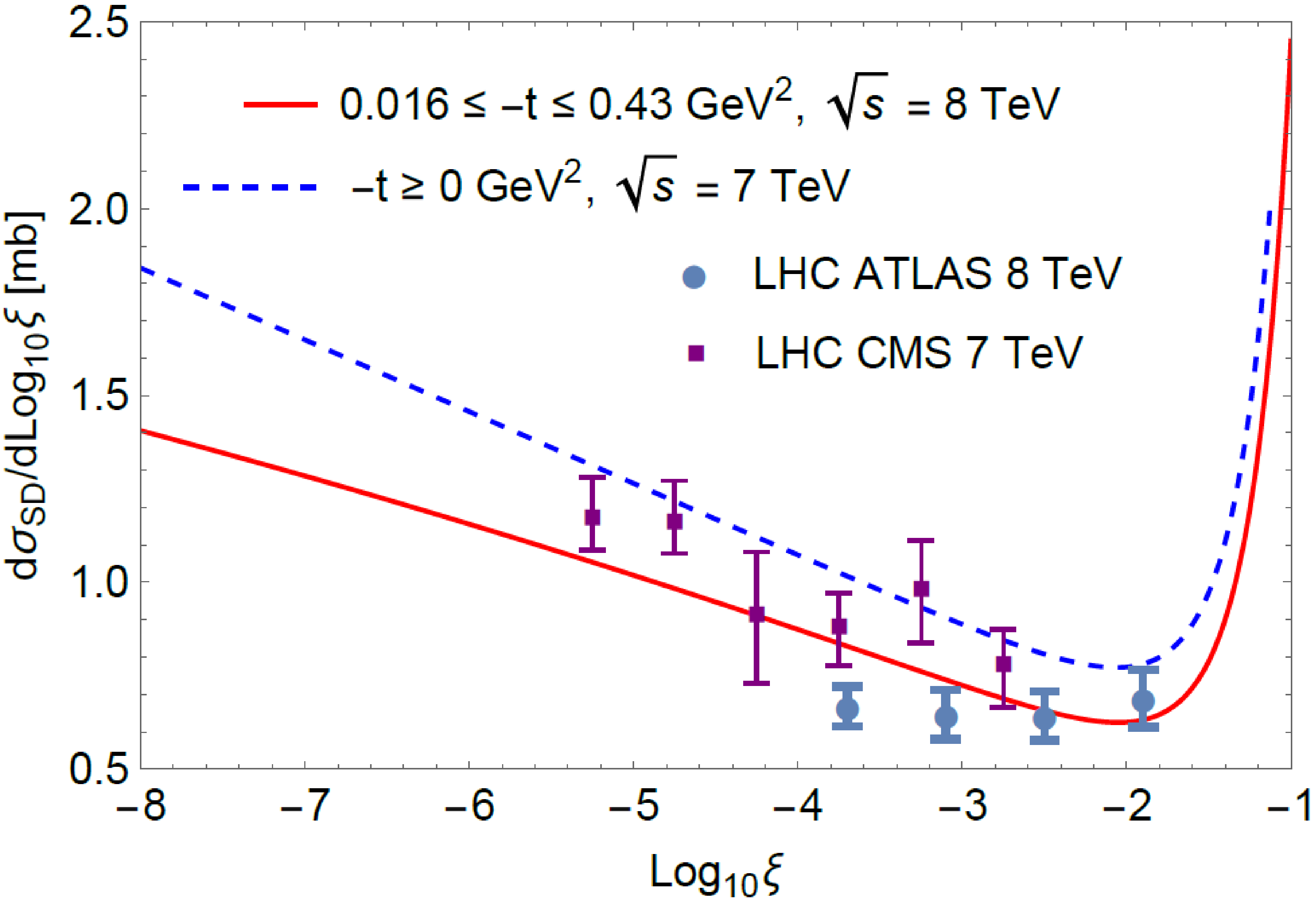
ξ dependence of the SD process at 546 GeV and 1.8 TeV



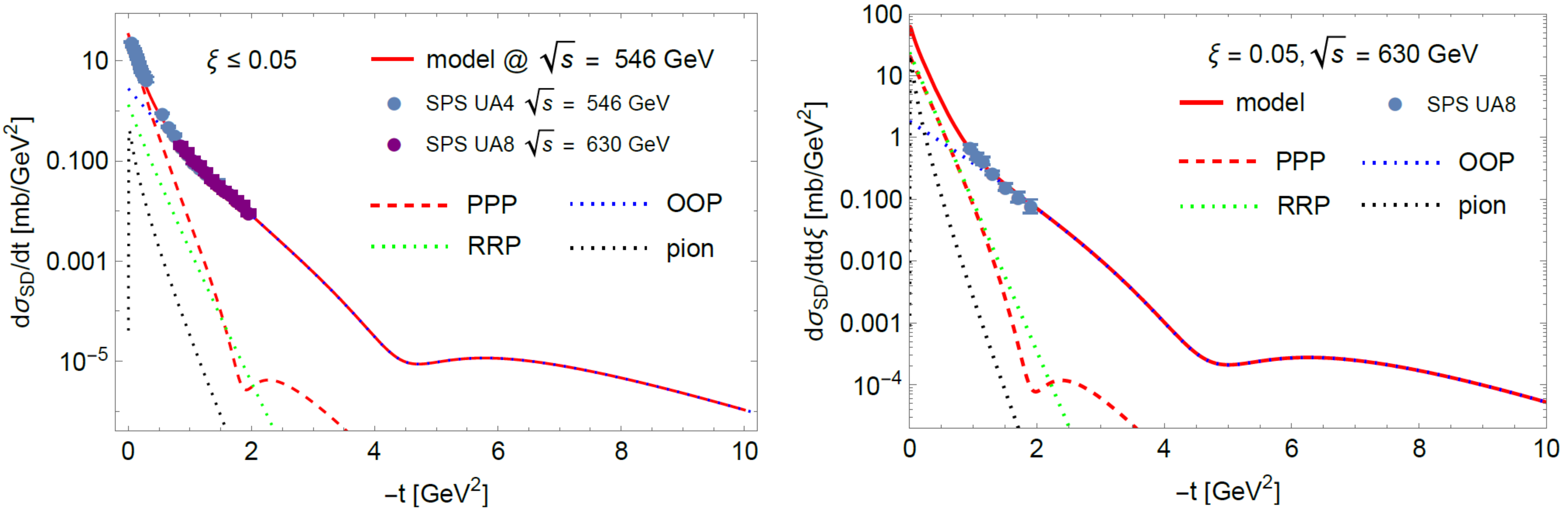
t dependence of the SD process at 8 TeV



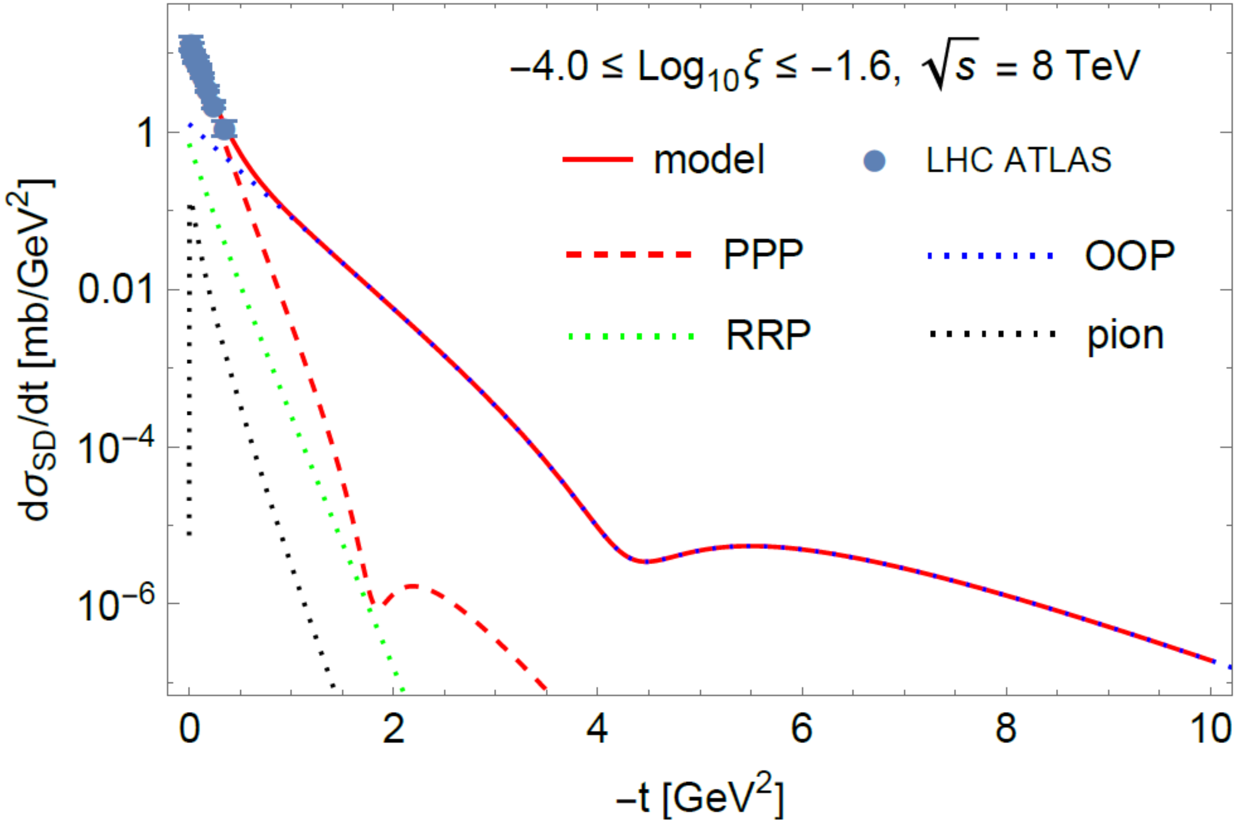
ξ dependence of the SD process at 7 and 8 TeV

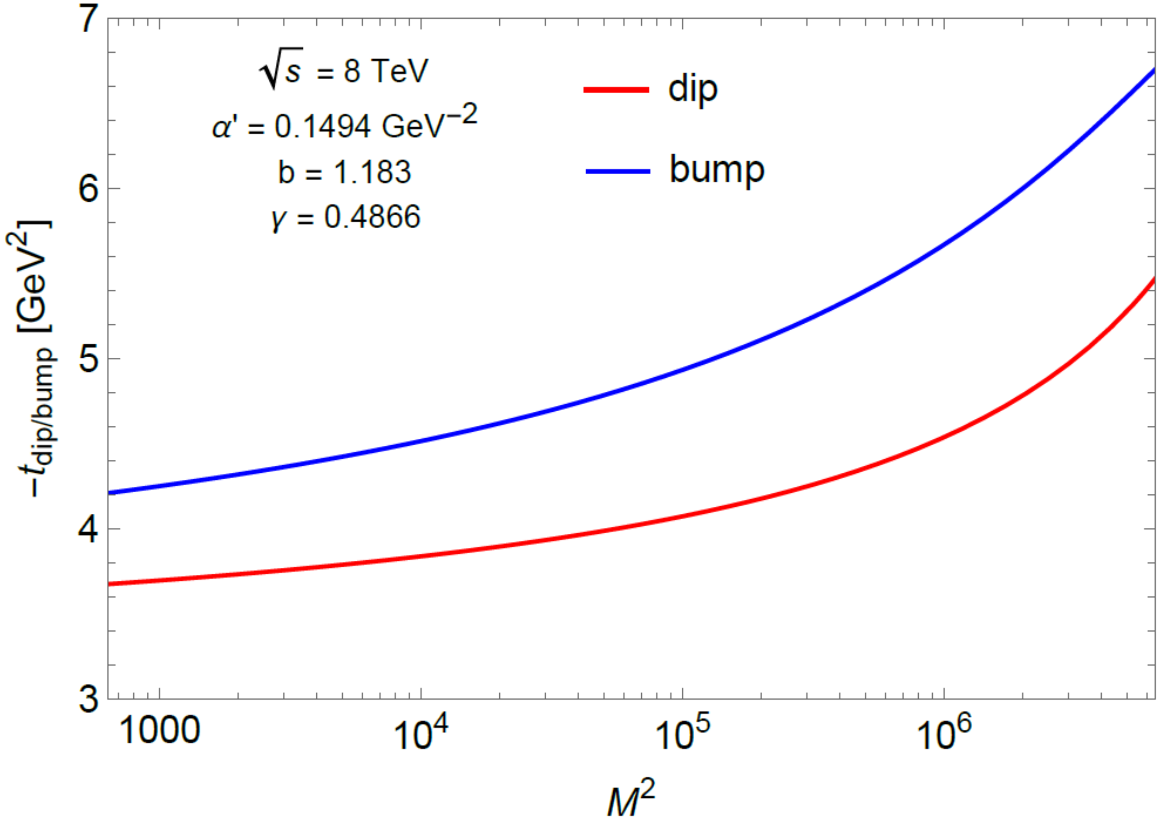


dip-bump in –t at SPS energies



dip-bump in –t at LHC



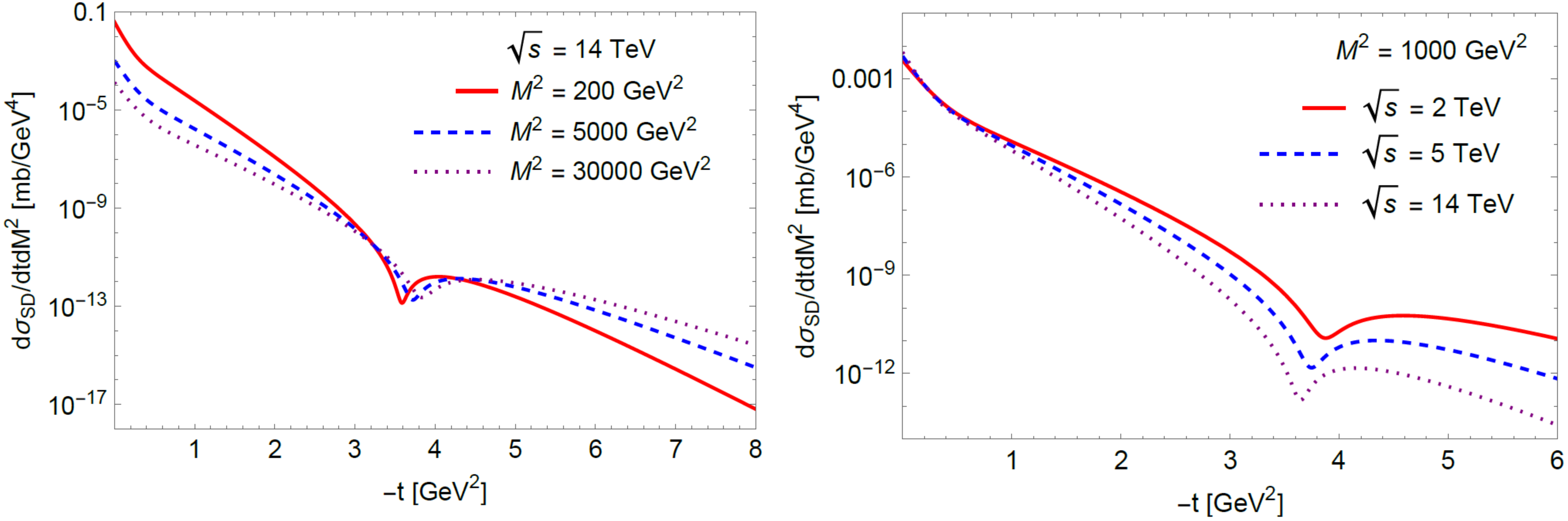


$$-t_{dip}^{SD} = \frac{1}{\alpha' b} \ln \frac{b + L_{SD}}{\gamma b L_{SD}}$$

$$-t_{bump}^{SD} = \frac{1}{\alpha' b} \ln \frac{4(b + L_{SD})^2 + \pi^2}{\gamma b (4L_{SD}^2 + \pi^2)}$$

$$L_{SD} = \ln(s/M^2)$$

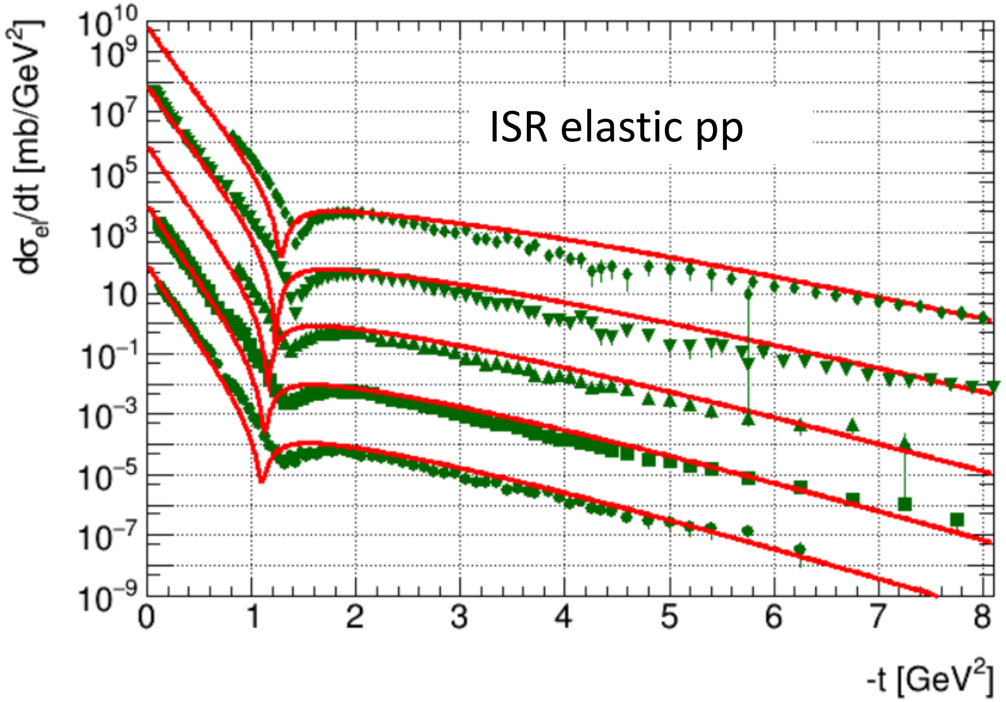
t, M^2 and s dependence of the SD process at LHC energies

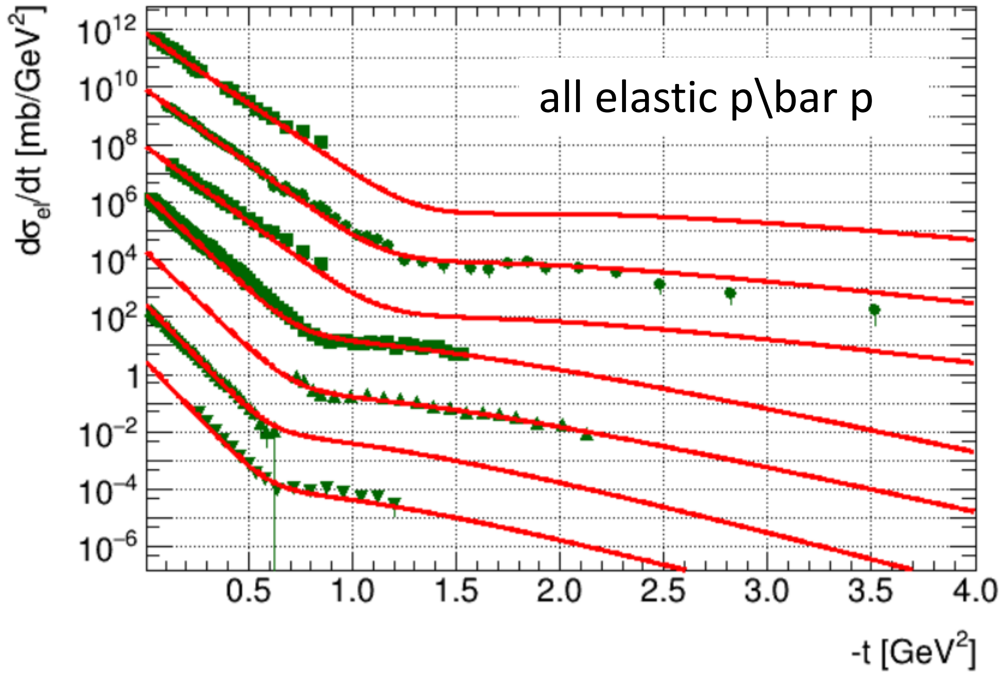


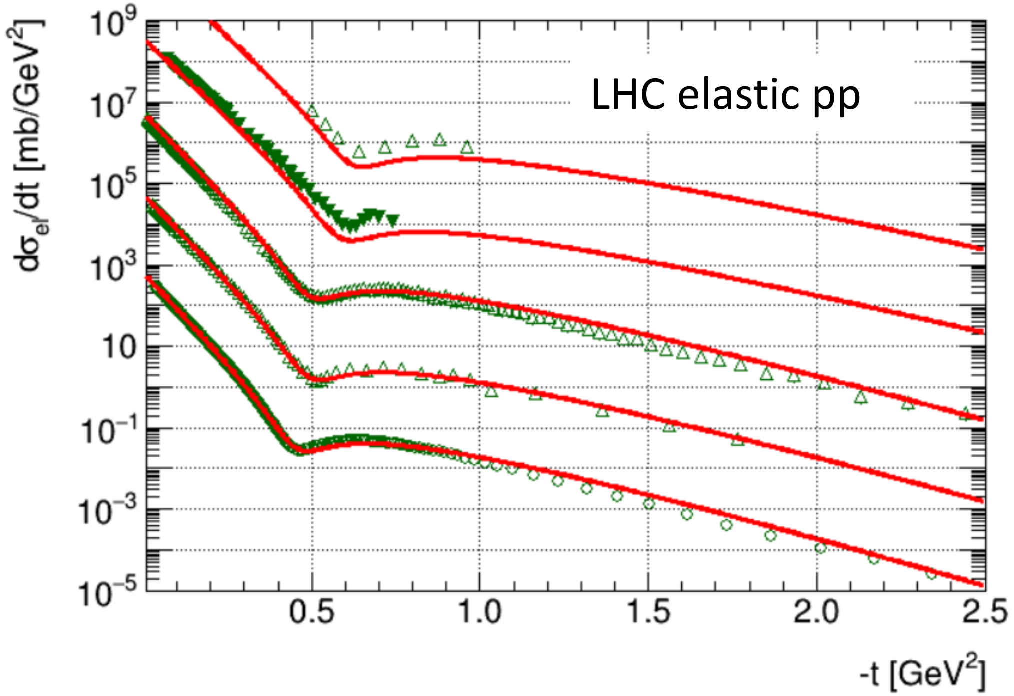
Summary

- **dip-bump structure is predicted in SD process** and it results from a dipole odderon contribution
- as the M^2 rises the slope of the t distribution decreases and the position of the dip-bump structure goes to higher -t values
- the calculations predict a dip-bump structure in -t at LHC around 4 GeV^2
- it would be interesting to check experimentally if such a dip-bump structure is present in the SD process

Thank you for your attention!

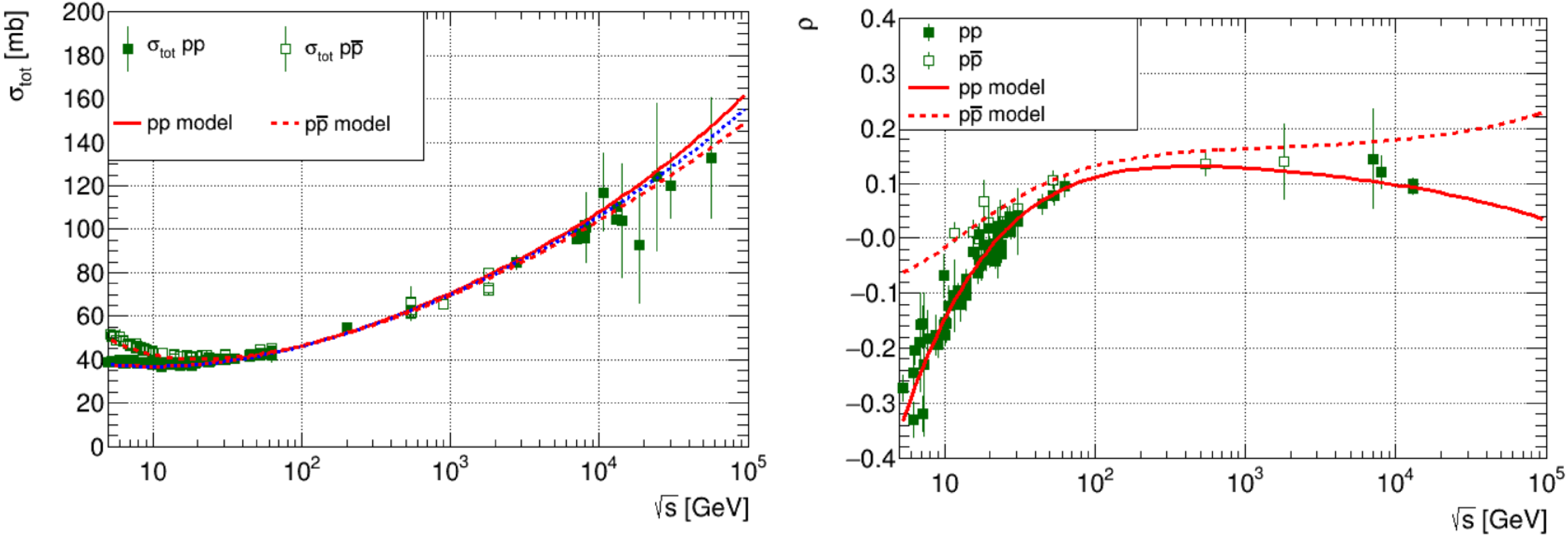






Pomeron	Odderon
$\delta_P = 0.04677 \pm 0.00004$	$\delta_O = 0.28606 \pm 0.00013$
$\alpha_{1P} = 0.4144 \pm 0.0002$	$\alpha_{1O} = 0.1745 \pm 0.0001$
$\alpha_{2P} = 0.0000$ (fixed)	$\alpha_{2O} = 0.0000$ (fixed)
$a_P = 2.2488 \pm 0.0036$	$a_O = 0.0202 \pm 0.0001$
$b_P = 5.5835 \pm 0.0139$	$b_O = 0.4015 \pm 0.0013$
$\epsilon_P = 0.0604 \pm 0.0005$	$\epsilon_O = 1.0036 \pm 0.0000$
$s_{0P} = 1.0001 \pm 0.0003$	$s_{0O} = 2.7211 \pm 0.0038$

f-reggeon	ω-reggeon
$\alpha_{0f} = 0.6869$ (fixed)	$\alpha_{0\omega} = 0.4380$ (fixed)
$\alpha_{1f} = 0.8400$ (fixed)	$\alpha_{1\omega} = 0.9300$ (fixed)
$a_f = -15.4042$ (fixed)	$a_\omega = 9.6945$ (fixed)
$b_f = 4.7842$ (fixed)	$b_\omega = 3.5101$ (fixed)
$s_{0f} = 1.0000$ (fixed)	$s_{0\omega} = 1.0000$ (fixed)



σ_{tot} [mb]
σ_{tot} pp
σ_{tot} $p\bar{p}$
pp model
$p\bar{p}$ model
$\sqrt{s}$ [GeV]
ρ
pp
$p\bar{p}$
pp model
$p\bar{p}$ model
$\sqrt{s}$ [GeV]

t and ξ dependence of the SD process at LHC energies