MODEL INDEPENDENT FEMTOSCOPIC METHODS

with possible applications

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Statistical analysis of large amount of data

Laguerre expansions
Gaussian expansions
Levy expansions
Predictions from trends
Possible areas of applications

Motivation: details

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Model independent shape analysis of correlations in 1, 2 or 3 dimensions

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Basic problem: first approximations usually Gaussian or exponential.
When data improve, they frequently become unsatisfactory.
Parameter estimates may become unreliable, need for precision.

Examples: Laguerre expansions

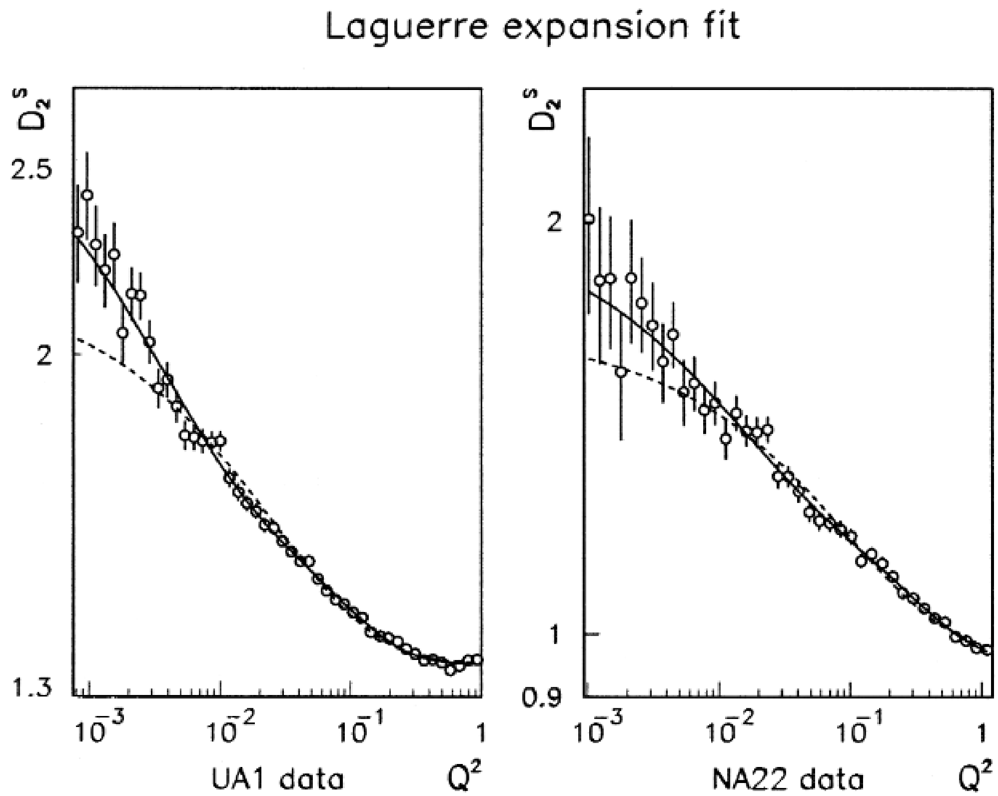

Fig. 1. The figures show D_2^s which is proportional to the two-particle Bose–Einstein correlation function, as measured by the UA1 and the NA22 Collaborations. The dashed lines stand for the exponential fit, which clearly underestimates the measured points at low value of the squared invariant momentum difference Q_I^2 (note the logarithmic horizontal scale). The solid lines stand for the fits with the Laguerre expansion method, which is able to reproduce the data with a statistically acceptable χ^2/NDF. The fit results are summarized in Table 1.

For describing nearly exponential data

Motivation: details

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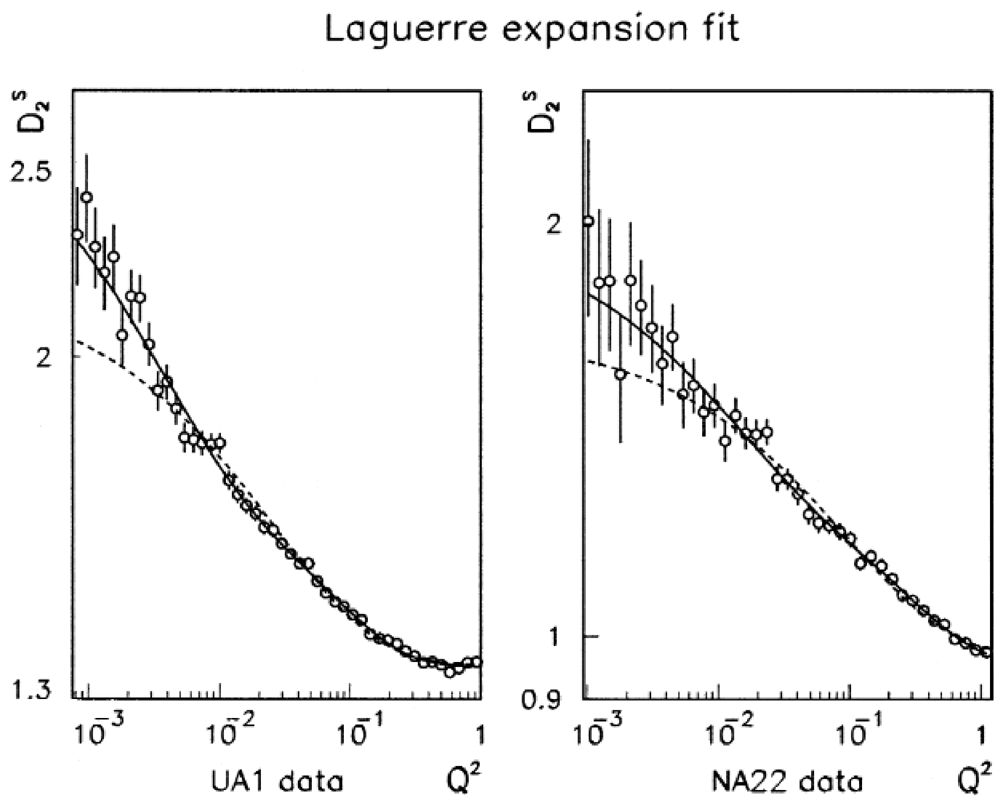

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Example 2: strong non-exponential behavior

universe MDPI

Article

Lévy α-Stable Model for the Non-Exponential Low-$|t|$ Proton–Proton Differential Cross-Section

Tamás Csörgő [1,2], Sándor Hegyi [2] and István Szanyi [1,2,3,*]

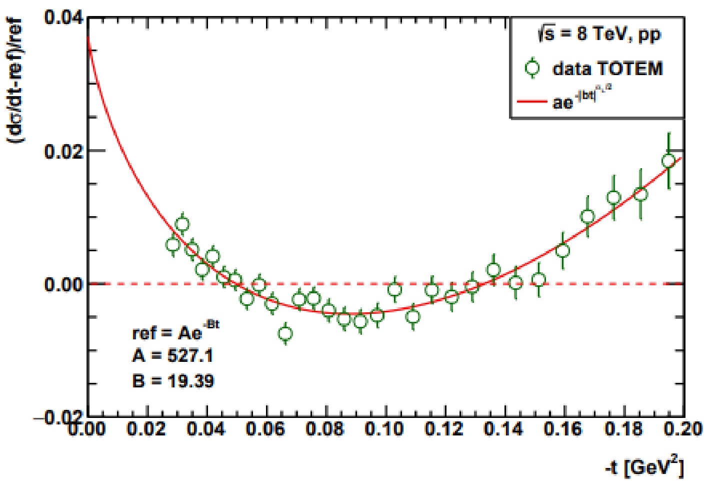

Figure 2. The ratio, $(d\sigma/dt - ref)/ref$, evaluated from the TOTEM pp elastic differential cross-section data at $\sqrt{s} = 8$ TeV [8]. The curve corresponds to the fitted model defined by Equation (44).

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Deviation from a reference exponential indicates a strong non-exponential behaviour of data. How to characterize this well?

Model-independent Levy expansions

$$t = QR, \quad (1)$$

$$C_2(t) = N\left\{1 + \lambda \exp(-t^\alpha) \sum_{n=0}^{\infty} c_n L_n(t|\alpha)\right\}, \quad (2)$$

$$t = QR_L,$$

$$w(t) = \exp(-t),$$

$$\int_0^\infty dt \exp(-t) L_n(t) L_m(t) \propto \delta_{n,m},$$

For $\alpha = 1$, these reduce to Laguerre polynomials and the Lévy expansion reduces to the Laguerre expansion of Ref. [4]

$$L_0(t \,|\, \alpha = 1) = 1, \quad (7)$$

$$L_1(t \,|\, \alpha = 1) = t - 1, \quad (8)$$

$$L_2(t \,|\, \alpha = 1) = t^2 - 4t + 2. \quad (9)$$

For describing nearly exponential data

Model-indep. Gauss/Edgeworth expansions

The $\alpha = 2$ case provides a new expansion around a Gaussian shape that is defined for non-negative values of t only

$$L_0(t\,|\,\alpha = 2) = 1\,, \tag{10}$$

$$L_1(t\,|\,\alpha = 2) = \frac{1}{2}\left\{\sqrt{\pi}t - 1\right\}\,,$$

$$L_2(t\,|\,\alpha = 2) = \frac{1}{16}\left\{2(\pi-2)t^2 - 2\sqrt{\pi}t + (4-\pi)\right\}\,.$$

$$t = QR,$$

$$C_2(t) = N\left\{1 + \lambda\exp(-t^\alpha)\sum_{n=0}^{\infty} c_n L_n(t|\alpha)\right\},$$

$$t = \sqrt{2}\,QR_E, \tag{23}$$

$$w(t) = \exp(-t^2/2), \tag{24}$$

$$\int_{-\infty}^{\infty} dt\,\exp(-t^2/2)H_n(t)H_m(t) \propto \delta_{n,m}, \tag{25}$$

where the order-n Hermite polynomial is defined as

$$H_n(t) = \exp(t^2/2)\left(-\frac{d}{dt}\right)^n \exp(-t^2/2). \tag{26}$$

The general form of Eq. (10) takes the particular form of the Edgeworth expansion [5,6,15] as:

$$C_2(Q) = \mathcal{N}\left\{1 + \lambda_E\exp(-Q^2R_E^2) \times\left[1 + \frac{\kappa_3}{3!}H_3(\sqrt{2}\,QR_E) + \frac{\kappa_4}{4!}H_4(\sqrt{2}\,QR_E) + \ldots\right]\right\}. \tag{27}$$

Two different Gaussian expansion (t>0, or not)

Model-independent Levy expansions

The Lévy expansion of short-range correlation functions results in the following formula which can be easily fitted to a given data set as

$$
\begin{aligned}
t &= QR\,, && (1)\\
C_2(t) &= N\left\{1+\lambda\exp(-t^\alpha)\sum_{n=0}^{\infty} c_n L_n(t|\alpha)\right\}\,, && (2)
\end{aligned}
$$

These Lévy polynomials were introduced in Ref. [6]; the first three are:

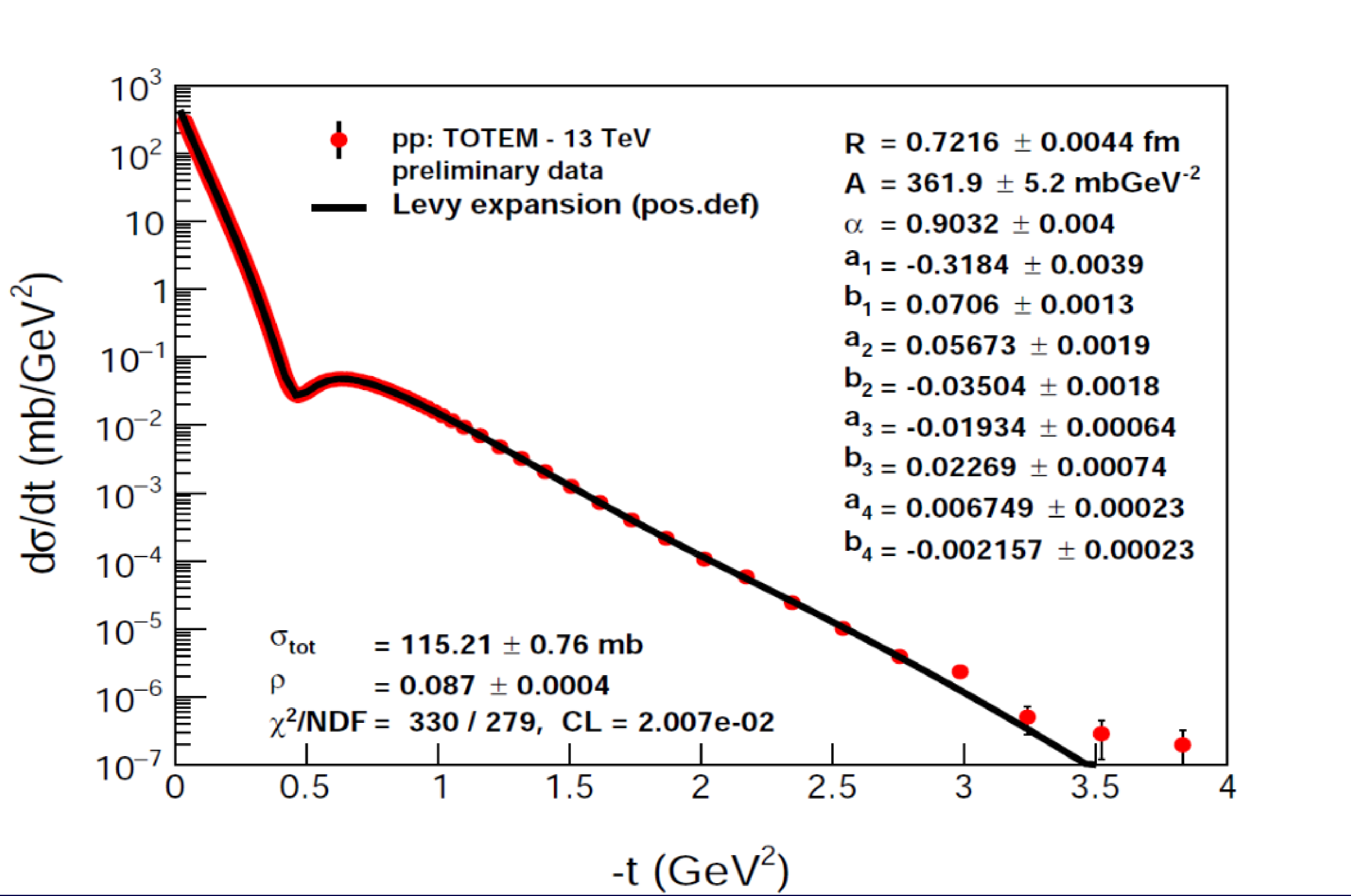
$$
\begin{aligned}
L_0(t\,|\,\alpha) &= 1\,, && (3)\\
L_1(t\,|\,\alpha) &= \det\begin{pmatrix}\mu_{0,\alpha} & \mu_{1,\alpha}\\ 1 & t\end{pmatrix}\,,\\
L_2(t\,|\,\alpha) &= \det\begin{pmatrix}\mu_{0,\alpha} & \mu_{1,\alpha} & \mu_{2,\alpha}\\ \mu_{1,\alpha} & \mu_{2,\alpha} & \mu_{3,\alpha}\\ 1 & t & t^2\end{pmatrix}\,, \qquad \textit{etc.}
\end{aligned}
$$

where

$$\mu_{n,\alpha} = \int_0^\infty \mathrm{d}t\, t^n \exp(-t^\alpha) = \frac{1}{\alpha}\Gamma\left(\frac{n+1}{\alpha}\right)$$

For describing nearly Levy data

Examples: Levy expansions



For describing data over 10 orders of magnitude