Theory Perspective on FCC Physics

Zurich Phenomenology Workshop
Jan 9th 2024

Matthew McCullough
A Theorist’s Perspective on FCC Physics

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Where do we stand in 2024?

Coupling Strength

Calculability

Scale

?
We should be honest with ourselves about ignorance.
Personal **theorist perspective**: The broadest exploration possible is the best hope we have for **progress**. Leaning too heavily on theorist priors makes me very nervous...
FCC-ee

60000000000000000000000 Clean Z-Bosons
FCC-ee

20000000 Clean Higgs Bosons
Energy frontier is about direct exploration of new states. Across photon, gluon, \((W&Z)\) and five-flavour scheme for quarks, FCC-hh collides different initial states. **Broad exploration.** Plenty of partons colliding at very high energy.
FCC-hh

Energy frontier is about direct exploration of new states. Across photon, gluon, (W&Z) and five-flavour scheme for quarks, FCC-hh collides

\[ N = 144, 196 \]

different initial states. **Broad exploration.** Writing resonance cross section as

\[ \sigma = r \frac{C_{yy}}{s} \]

where

\[ C_{gg} = \frac{\pi^2}{8} \int_{\tau}^{1} dx \frac{f_g(x)f_g(\tau x)}{x}, \quad C_{qq} = \frac{4\pi^2}{9} \int_{\tau}^{1} dx \frac{f_q(x)f_q(\tau x) + f_q(x)f_q(\tau x)}{x} \]

and

\[ r = (2S + 1)B_{yy}B_{xx} \frac{\Gamma_R}{M_R} \]
Then, recalling, \[ r = (2S + 1)B_{yy}B_{xx} \frac{\Gamma_R}{M_R} \]
...the number of direct resonance production events you get above 10 TeV at FCC-hh is:
Then, recalling, the number of events you get above 10 TeV at FCC-hh is: 

\[ \begin{array}{cc}
Q^* \rightarrow jj & 2.5 \text{ ab}^{-1} \\
Z'_{TC2} \rightarrow t\bar{t} & 30 \text{ ab}^{-1} \\
Z'_{SSM} \rightarrow t\bar{t} & 100 \text{ ab}^{-1} \\
G_{RS} \rightarrow W^+W^- & 5 \text{ fb}^{-1} \\
Z'_{SSM} \rightarrow t\bar{t} & \text{Discovery} \\
Z'_{SSM} \rightarrow \tau^+\tau^- & \text{Discovery}
\end{array} \]
What are the questions we want answered?
What’s going on with flavour?
Where do matter mass patterns come from?

Clearly something going on here...

Present state of affairs à la Periodic Table, if we’re being honest with ourselves...

Figures borrowed from A. Greljo.
6000000000000 Clean Z-Bosons

<table>
<thead>
<tr>
<th>Particle production ($10^9$)</th>
<th>$B^0 / \bar{B}^0$</th>
<th>$B^+ / B^-$</th>
<th>$B_s^0 / \bar{B}_s^0$</th>
<th>$\Lambda_b / \bar{\Lambda}_b$</th>
<th>$c\bar{c}$</th>
<th>$\tau^- / \tau^+$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Belle II</td>
<td>27.5</td>
<td>27.5</td>
<td>n/a</td>
<td>n/a</td>
<td>65</td>
<td>45</td>
</tr>
<tr>
<td>FCC-ee</td>
<td>300</td>
<td>300</td>
<td>80</td>
<td>80</td>
<td>600</td>
<td>150</td>
</tr>
</tbody>
</table>

Incredible flavour factory!

\[ M_{12} = (M_{12})_{SM} \times (1 + h_{d,s} \sigma_{d,s}) \]
\[ \frac{C_{ij}^2}{\Lambda^2} (q_i, L \gamma_\mu q_j, L)^2 \]
\[ \lambda_{ij}^t = V_{ti}^* V_{tj} \]
\[ h \approx 1.5 \frac{|C_{ij}|^2}{|\lambda_{ij}^t|^2} \frac{(4\pi)^2}{G_F \Lambda^2} \approx \frac{|C_{ij}|^2}{|\lambda_{ij}^t|^2} \left( \frac{4.5 \text{ TeV}}{\Lambda} \right)^2 \]
\[ \sigma = \arg(C_{ij} \lambda_{ij}^{t*}) \]
A number of flavour-violating interactions generated by new heavy states would be most strongly constrained by their running into unflavoured EW precision observables!

Tera-Z is not another LEP, but a literal quantum leap towards the smallest distance scales...

Taken from 2311.00020
What’s up with the Higgs Boson?
What’s up with the Higgs Boson?

Every scalar we encountered until now has properties (mass, vev, etc) that are calculable within some more fundamental theory:
What’s up with the Higgs Boson?

What about the Higgs?

The Standard Model, our best description of nature, breaks down at short distances: It is an effective field theory, to be replaced by something more fundamental at shorter distance scales.
What about the Higgs? We know what happens with pions...

Backdrop

H

Calculable

LHC

$\Lambda \gg m_H$

No symmetry at EW scale to permit this.

Theory

No symmetry at EW scale to permit this.
What about the Higgs? We know what happens with pions…

Backdrop

Calculable

H
LHC
Theory

No symmetry at EW scale to permit this.

Could the Higgs be a composite “pseudo-Namb-Goldstone boson” (pNGB)?

Question that’s been asked many times… Kaplan, Georgi, Dimopoulos 1984 etc.

\[ \Lambda \gg m_H \]
Naturalness – pNGB Higgs

Vanilla pNGB Higgs scenarios have a potential which looks like

\[ V(h) = \epsilon f^2 \Lambda^2 F(h/f) \]

Where F is a generic function. Not so difficult to have a light Higgs

\[ m_h^2 \sim \epsilon \Lambda^2 \]

If one has \( \epsilon \ll 1 \). This is not fully possible in concrete models, since this is controlled by a symmetry which is already broken in SM.

However...
Vanilla pNGB Higgs scenarios have a potential which looks like

\[ V(h) = \epsilon f^2 \Lambda^2 F(h/f) \]

Where \( F \) is a generic function. The position of the minimum of the potential doesn’t care about the overall coefficient:

\[ V'(h) = 0 \iff F'(h/f) = 0 \]

So, if this is to occur at \( h = \nu \ll f \) then one has to fine-tune the contributions to the potential from the UV physics.
Naturalness – pNGB Higgs

Vanilla pNGB Higgs scenarios have a potential which looks like

\[ V(h) = \epsilon f^2 \Lambda^2 F \left( \frac{h}{f} \right) \]

Where F is a generic function. However, it is generic that the operator

\[ \mathcal{O}_H \sim \frac{1}{f^2} \left( \partial^\mu |H|^2 \right)^2 \]

is generated. This modifies all Higgs couplings by an amount

\[ \delta_\kappa \sim \frac{v^2}{f^2} \]
Vanilla pNGB Higgs scenarios have a potential which looks like
\[ V \sim \frac{k}{f} \]
However, it is generic, like for pions, that the operator is generated. This modifies all Higgs couplings by an amount

\[ \delta \kappa \sim \frac{v^2}{f^2} \]

So, in vanilla scenarios, Higgs coupling measurements suggest that if the Higgs is a pNGB then there must be some fine-tuning of parameters at least at the 10% or so level!
Vanilla NGB Higgs scenarios have a potential which looks like

\[ F \text{ is a generic function.} \]

However, it is generic, like for pions, that the operator is generated. This modifies all Higgs couplings by an amount

\[ \delta \kappa \sim \frac{\nu^2}{f^2} \]

Compositeness Scale

Naturalness – pNGB Higgs

The composite Nambu–Goldstone Higgs

Giuliano Panico, Andrea Wulzer

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The composite Higgs scenario, in which the Higgs emerges as a composite pseudo-Nambu–Goldstone boson, is extensively reviewed in these Notes. The material is presented in a pedagogical fashion, with great emphasis on the conceptual and technical foundations of the construction. A comprehensive summary of the flavor, collider and electroweak precision phenomenology is also presented.
Let’s scrutinize the assumptions...

\[ V(h) = \epsilon f^2 \Lambda^2 F(h/f) \]

Assumption until now has been that the symmetry is broken in the most minimal ways.

Technically: Breaking “spurion” is in a low-index irrep of the global symmetry.
Now assume some small explicit breaking "spurion" in a symmetric irrep with "n" indices:

\[ V_\epsilon = \frac{\lambda}{f^{n-4}} \epsilon a_1, a_2, \ldots, a_n \phi^{a_1} \phi^{a_2} \ldots \phi^{a_n} \]

For the pNGB fields this generates a potential:

\[ V = \epsilon m_\rho^2 f^2 G_n^{(N-1)/2} (\cos \Pi / f) \]

Gegenbauer function!
Now assume some small explicit breaking "spurion" in a symmetric irrep with \( n \) indices:

For the pNGB fields this generates a potential:

\[
V_\epsilon = \frac{1}{f^4} \phi a_n \phi^*(a_n^* \phi) \]

Gegenbauer function:
Getting to know Gegenbauer

The Gegenbauer potential looks like:

\[
\langle \Pi \rangle \approx \frac{j_{\lambda+1/2,1}}{n + \lambda} \approx \frac{5.1}{n}
\]

Global minimum at naturally small field values:
Gegenbauer’s Twin

Gegenbauer contribution allows to naturally realise $v \ll f$. On the other hand, for a standard pNGB Higgs model the top sector doesn’t allow $\epsilon$ to be arbitrarily small...

Twin Higgs models, however, address that particular aspect. Could “Gegenbauer’s Twin” allow both $\epsilon \ll 1$ and $v \ll f$?
Consider some standard pNGB Higgs construction and, inspired by pions, allow for an additional source of explicit symmetry breaking, in n-index irrep of global symmetry.

\[ \mathcal{L} = \mathcal{L}_{\text{Old}} + \epsilon \mathcal{L}_{S_n \neq 0} \]

What happens?
Gegenbauer’s Twin

Predictions, in absolute terms:

Example point. Low tuning (low cutoff), 3% single-coupling correction, 70% self-coupling correction.
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Gegenbauer’s Twin

HL-LHC Expectations

Example point. Low tuning (low cutoff), 3% single-coupling correction, 70% self-coupling correction.
Gegenbauer’s Twin

HL-LHC Expectations & FCC-ee

Example point. Low tuning (low cutoff), 3% single-coupling correction, 70% self-coupling correction.

Only 2σ at HL-LHC but 15σ at FCC-ee!
What is the Higgs Potential?

Important because it determines how the Universe froze in the EW sector, giving mass to gauge bosons, fermions, the Higgs...
What is the Higgs Potential?

...because it determines how the Universe will end...

\[ V(H) = \frac{1}{2} m_h^2 h^2 + \frac{1}{6} \lambda h^3 + \ldots \]
What is the Higgs Potential?

But, do we really need to measure Higgs self-interaction? Surely new physics, if it affects the potential, would also affect additional, better-measured observables?

...because it determines how the Universe will end...
Custodial Quadruplet

Could such a theory actually exist? Yes: The custodial quadruplet scalar. Projecting the \((4, 4)\) of \(SU(2)_L \times SU(2)_R\) onto EW group we have

\[
(4, 4) \rightarrow 4_{1/2} + 4_{3/2}
\]

and including all couplings to the Higgs we have for scalar quadruplet

\[
\mathcal{L}_{SO(4)} = -\lambda \left( H^* H^*(\epsilon H)\Phi + \frac{1}{\sqrt{3}} H^* H^* H^* \tilde{\Phi} \right) + \text{h.c.}
\]

which has exactly the pattern described.
Custodial Quadruplet

Higgs self-coupling is modified at dim-6 at tree-level, all other couplings modified at dim-6 one-loop, or dim-8. All calculable, giving

\[-\frac{\delta_{VV}}{\delta_{h^3}} = 3 \left( \frac{m_h}{4\pi v} \right)^2 + \left( \frac{m_h}{M} \right)^2 \approx \frac{1}{200} + \frac{1}{580} \left( \frac{3 \text{ TeV}}{M} \right)^2\]
Custodial Quadruplet

Higgs self-coupling is modified at dim-6 at tree-level, all other couplings modified at dim-6 one-loop, or dim-8. All calculable, giving

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Custodial Quadruplet

Higgs self-coupling is modified at dim-6 at tree-level, all other couplings modified at dim-6 one-loop, or dim-8. All calculable, giving

\[ \delta_{h^3} + \left( \frac{m_h}{M} \right)^2 \approx \frac{1}{200} + \frac{1}{580} \left( \frac{3 \text{ TeV}}{M} \right)^2 \]

Beware overly-assertive theorist priors...

Durieux, MM, Salvioni. 2022
Dark Sectors

Evidence for dark matter is now overwhelming

- Rotation curves
- CMB
- Large scale structure
- Velocity dispersions
- Gravitational lensing (Bullet Cluster)
- ....

Yet we have no clue what it is at the particle level!
Is the Higgs a portal to new dark sector states?

After all, $|H|^2$ is the most relevant interaction involving SM fields! Even if generated at microscopic scales $|H|^2 \chi^2$ stays relevant all the way down to the Higgs scale...
Is the Higgs a portal to new dark sector states?

Orders of magnitude improvement in coverage of exotic Higgs decays.
A final word on SM theory...

Numerous exciting challenges for SM theory calculations. EW and QCD precision calculations pushed to new frontiers.

- Mixed 3 and partial 4-loop calculations required for EWPOs.
- Also increased precision in simulation, resummation, hadronization, Monte Carlo, non-perturbative understanding required...
- See talks by Frixione, Weinzierl, Monni, Skands, Rojo, Ferrario Ravasio.

Generally: Factor 500 reduction in stat uncertainty compared to LEP, so big theory targets...
Any “perspective” is subjective. I have presented snippets of my own, subjective, perspective.

However, to create the most exciting future for particle physics we require a strategy that is the 
**as robust as possible** against subjective theory perspectives!
Conclusions

**Electroweak. QCD. Flavour. Higgs.**

The FCC Physics Programme uniquely covers each area in paradigm-shifting depth.
Status of Higgs Couplings

What are experimental limits on modifications of couplings relative to Standard Model prediction?

ATLAS, Nature, 2022
Status of Higgs Couplings

What are experimental limits on modifications of couplings relative to Standard Model prediction?

Only $\kappa_2$ and $\kappa_t$ free

ATLAS
$\sqrt{s} = 13$ TeV, 126–139 fb$^{-1}$

All other $\kappa$ fixed to SM

Observed

Lidija Živković, Yesterday
Status of Higgs Couplings

What are experimental limits on modifications of couplings relative to Standard Model prediction?

There is a chasm between how well we know the Higgs self-coupling, as compared to the other couplings, yet it is the most important of them all!

Lidija Živković, Yesterday
Future of Higgs Self-Coupling

Future facilities can give us valuable new insights into the nature of the Higgs potential.

Rich interplay between direct/indirect, HL-LHC, Higgs factory, future High energy machines.

\[ V(H) = \frac{1}{2} m_h^2 h^2 + \frac{1}{6} \lambda h^3 + \ldots \]

1905.03764
Gegenbauer’s Twin

If tuning calculations interest you (I understand if not...), we followed conservative approach

\[ \delta = \begin{pmatrix} \frac{\partial \log v^2}{\partial \log \epsilon} & \frac{\partial \log v^2}{\partial \log m_h^2} \\ \frac{\partial \log m_h^2}{\partial \log \epsilon} & \frac{\partial \log m_h^2}{\partial \log m_h^2} \end{pmatrix} \quad \Delta = \left( \sum \text{eigenvalues} (\delta^T \delta) \right)^{-1/2} \]

As Gegenbauer gives vev, tuning dominated by

\[ \left( \frac{\partial \log v^2}{\partial \log a} \right)^{-1} = \frac{8\pi^2 m_h^2}{3y_t^4 f^2 (1 - \frac{3v^2}{f^2} + \frac{2v^4}{f^4})} \]

So, compared to standard Twin expect improvement of

\[ \frac{\Delta}{2v^2/f^2} \approx \frac{4\pi^2 m_h^2}{3y_t^4 v^2} \approx 4 \]
Gegenbauer’s Twin

Quantitatively:

Estimate of Craig & Howe seems robust.

Twin model of Barbieri, Greco, Rattazzi, Wulzer.
Gegenbauer’s Twin

Generalising Gegenbauer story to pNGB Twin Higgs for $\text{SO}(8) \to \text{SO}(7)$ and going to Unitary gauge the top-sector contributions to the Higgs potential are

$$V_t \approx \frac{3y_t^4 f^4}{64\pi^2} \left[ \sin^4 \frac{h}{f} \log \frac{a}{\sin^2 h/f} + \cos^4 \frac{h}{f} \log \frac{a}{\cos^2 h/f} \right]$$

Whereas the symmetric n-index irrep gives

$$V_G^{(n)} = \epsilon m_\rho^2 f^2 G_n^{3/2} (\cos 2h/f)$$

Note: This is radiatively stable at all scales.
Consider a simple scenario that could apply to the Higgs boson.

Example SO(N+1):

\[
\mathcal{L} = \frac{1}{2} \partial_\mu \phi \cdot \partial^\mu \phi - \frac{\lambda}{4} \left( \phi \cdot \phi - \frac{f^2}{2} \right)^2
\]

We get N massless pNGBs with decay constant “f” and unbroken SO(N).
Consider a simple scenario that could apply to the Higgs boson.

Example SO(N+1):

We get N massless pNGBs with decay constant "f" and unbroken SO(N).

Beyond Minimality

\[ \mathcal{L} = -\frac{1}{2} \left( \partial \phi_{\text{RE}} \right)^2 - \frac{1}{2} \left( \partial \phi_{\text{IM}} \right)^2 - \frac{1}{2} \left( \partial \phi_{\text{IM}} \right)^2 \]

We get...