

Recent Developments in Vincia & Pythia

Peter Skands — U of Oxford & Monash U.



1. Perturbative Uncertainties (in Showers)
2. Sector Showers & NNLO Matching
3. EW Showers and Resonance Decays
4. From Showers to Jets: Colour Confusion

... including some questions for discussion ...

Note: see talk by Silvia (Monday) for $N^{(n)}$ LL showers (PanScales, Alaric, etc)



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1 Perturbative Uncertainties in Showers



Standard for Shower Uncertainties: Renormalization-scale variations

Example: PYTHIA's DGLAP-based shower

$$|M_{n+1}|^2 \sim \sum_{i \in \text{partons}} \underbrace{\frac{\alpha_s^{\text{MC}}(\mu_i^2)}{4\pi}}_{\mu_i^2 \propto p_{\perp i}^2} \underbrace{\mathcal{C}_i}_{\substack{2C_F \text{ for quark,} \\ C_A \text{ for gluon}}} \underbrace{\left(\frac{P_i(z)}{Q_i^2} \right)}_{\substack{\text{DGLAP Splitting Kernel} \\ \text{(Or dipole/antenna/...)}}}|M_n|^2 \underbrace{\Delta_n(t_n, t_{n+1})}_{\substack{\text{Sudakov factor} \\ t \text{ is the shower evolution/} \\ \text{ordering variable}}}$$



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Varying μ_i only induces terms **proportional to the shower splitting kernels**

Actual higher-order MEs **also have:**

Non-singular terms (dominate far from singular limits),

Non-trivial colour factors outside collinear limits,

Higher-order log terms not captured exactly by $\Delta_n(t_n, t_{n+1})$



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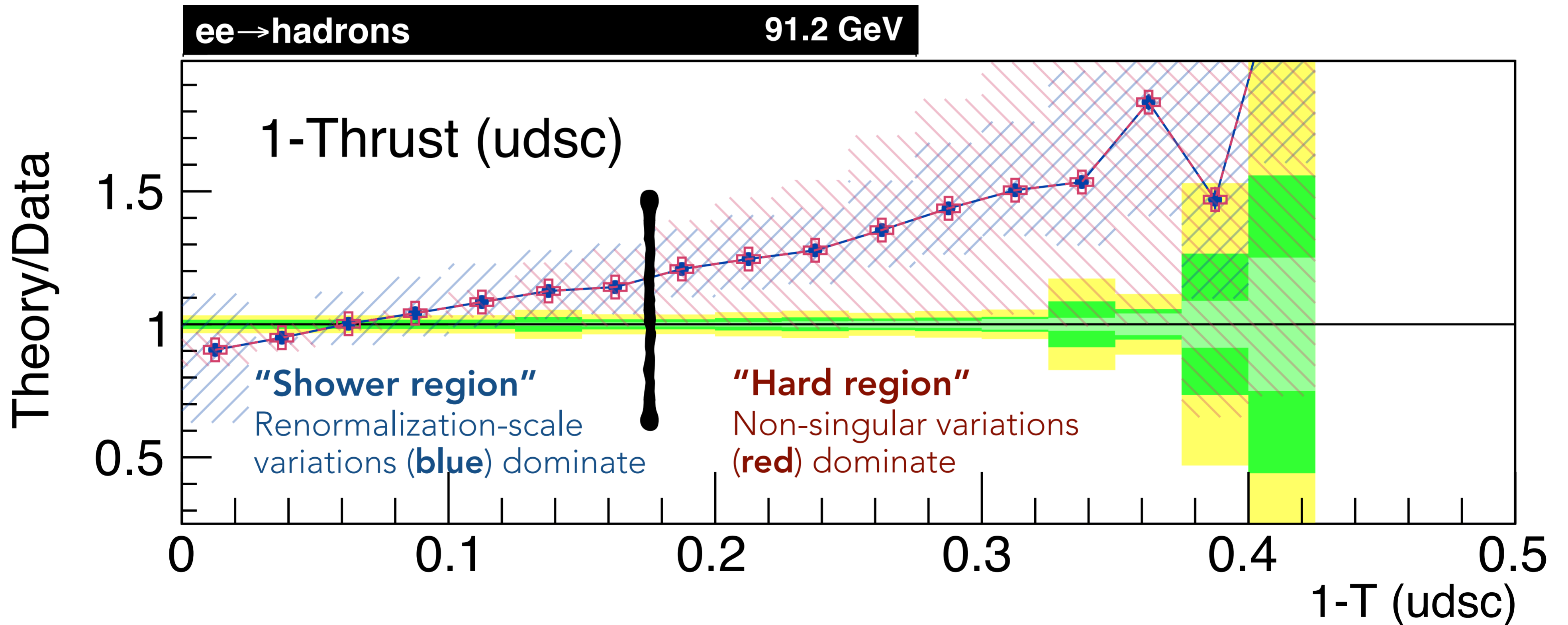
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- Non-trivial colour factors** outside collinear limits,
- Higher-order log terms** not captured exactly by $\Delta_n(t_n, t_{n+1})$

Vary μ_R and these
 [Hartgring, Laenen, PS
 JHEP 10 (2013) 127]

Non-Singular Variations: Example

Example from Mrenna & PS, "Automated Parton-Shower Variations in Pythia 8", *PRD* 94 (2016) 7

Can vary **renormalisation-scale** and **non-singular terms** independently

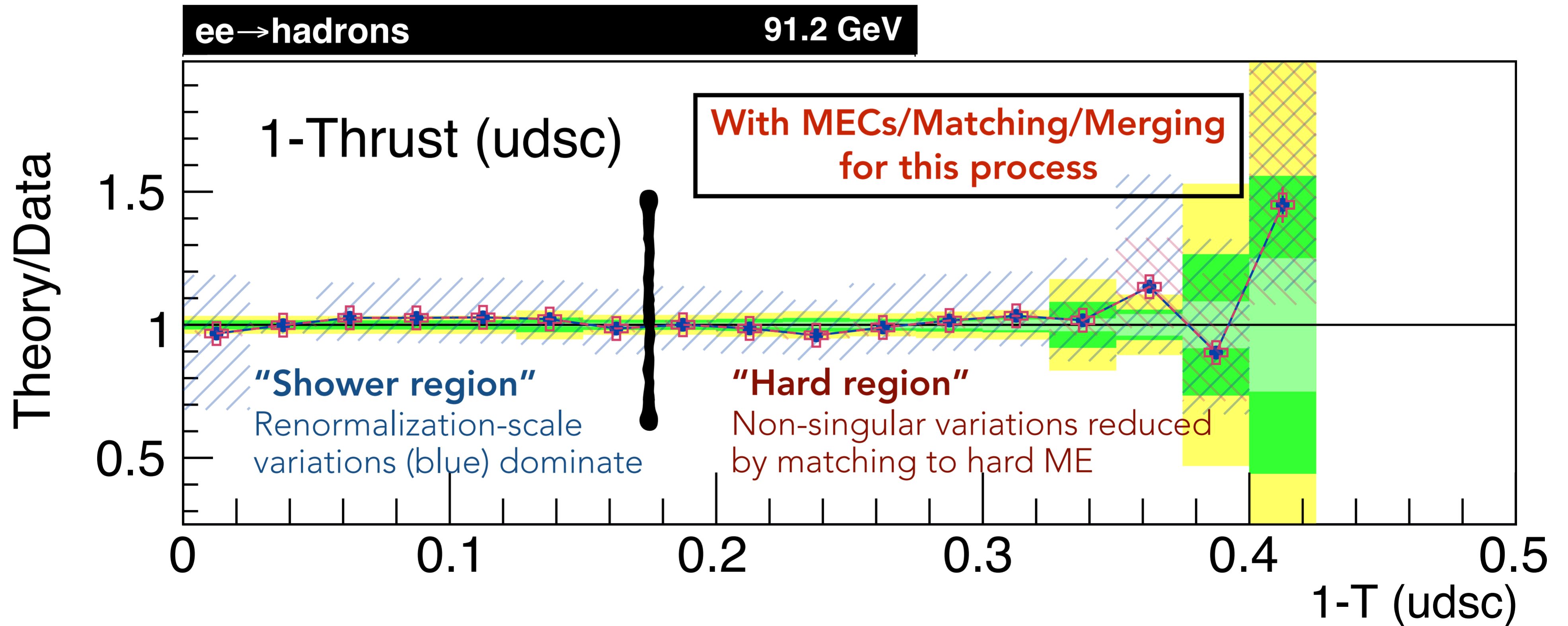


Note: ME corrections were switched off for illustration here. Would reduce **red** band, but not **blue**.

(Non-Singular Variations: Effect of Matching to Matrix Elements)

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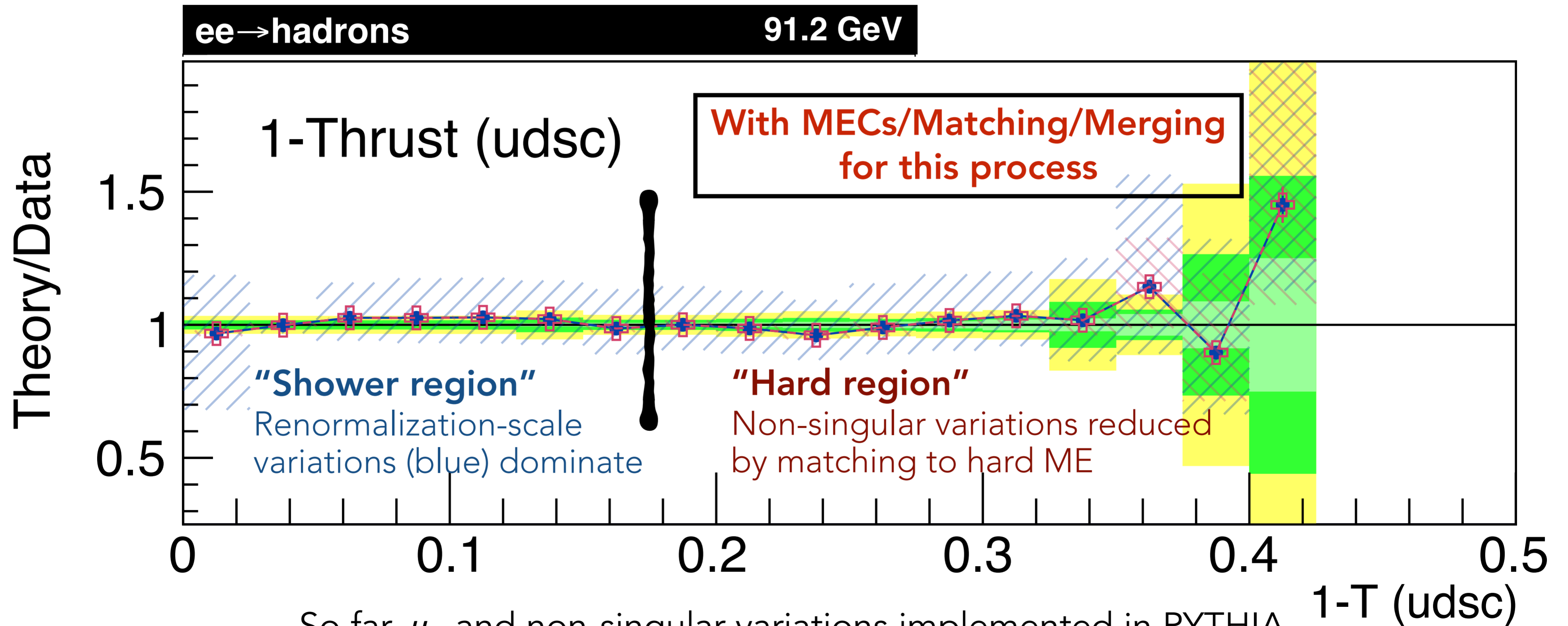
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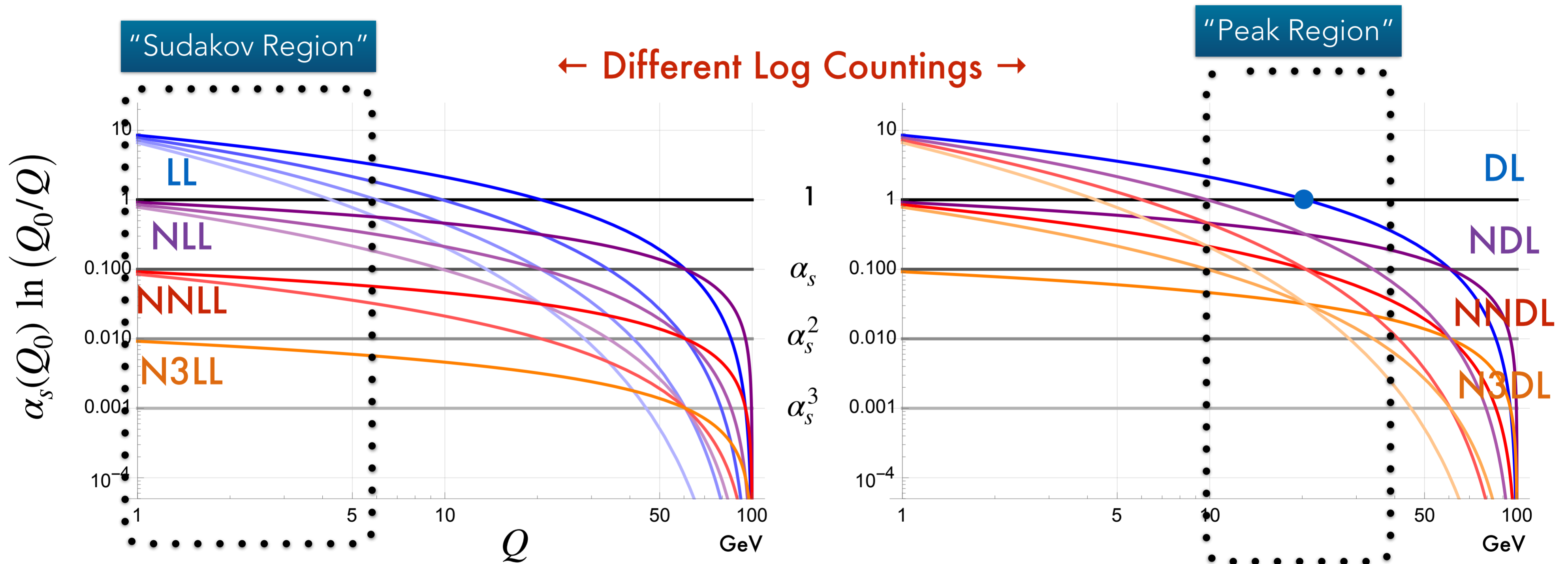


So far, μ_R and non-singular variations implemented in PYTHIA

Being re-implemented in VINCIA. Plan to add colour and Sudakov variations as well.

(Uncertainties: note on the size of uncontrolled log terms)

Schematic Example: starting scale $Q_0 = 100$ GeV



Conventional ("Caesar-style") log counting

Based on $\alpha_s L \sim 1$

Exponentiated "double-log" counting

Based on $\alpha_s L^2 \sim 1$

② Sector Showers in VINCIA

[PS & Villarejo JHEP 11 \(2011\) 150](#)

[Brooks, Preuss, PS JHEP 07 \(2020\) 032](#)

VINCIA's shower is unique in being a "Sector Shower"

Partition N-gluon Phase Space into N "sectors" (using step functions).

Each sector corresponds to one specific gluon being the "softest" in the event — the one you would cluster if you were running a jet algorithm (ARCLUS)

Inside each sector, **only a single kernel is allowed to contribute** (the most singular one)!

Sector Kernel = the eikonal for the soft gluon and its collinear DGLAP limits for $z > 0.5$.

→ Unique properties: shower operator becomes **bijective** and is a true **Markov chain**

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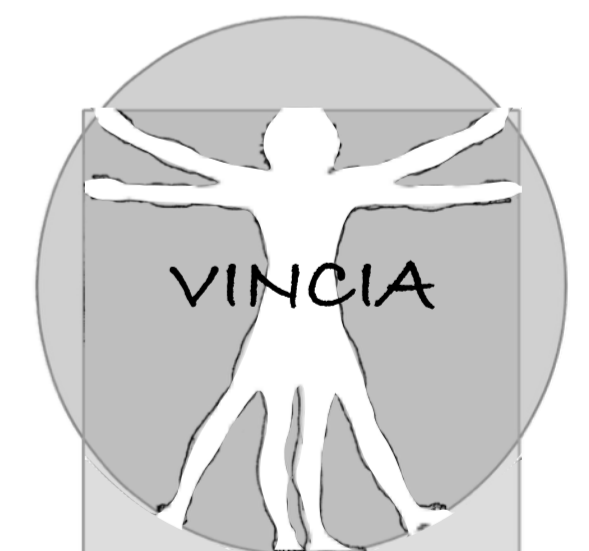
The crucial aspect:

Only a single history contributes to each phase-space point !

⇒ **Factorial growth of number of histories reduced to constant!**

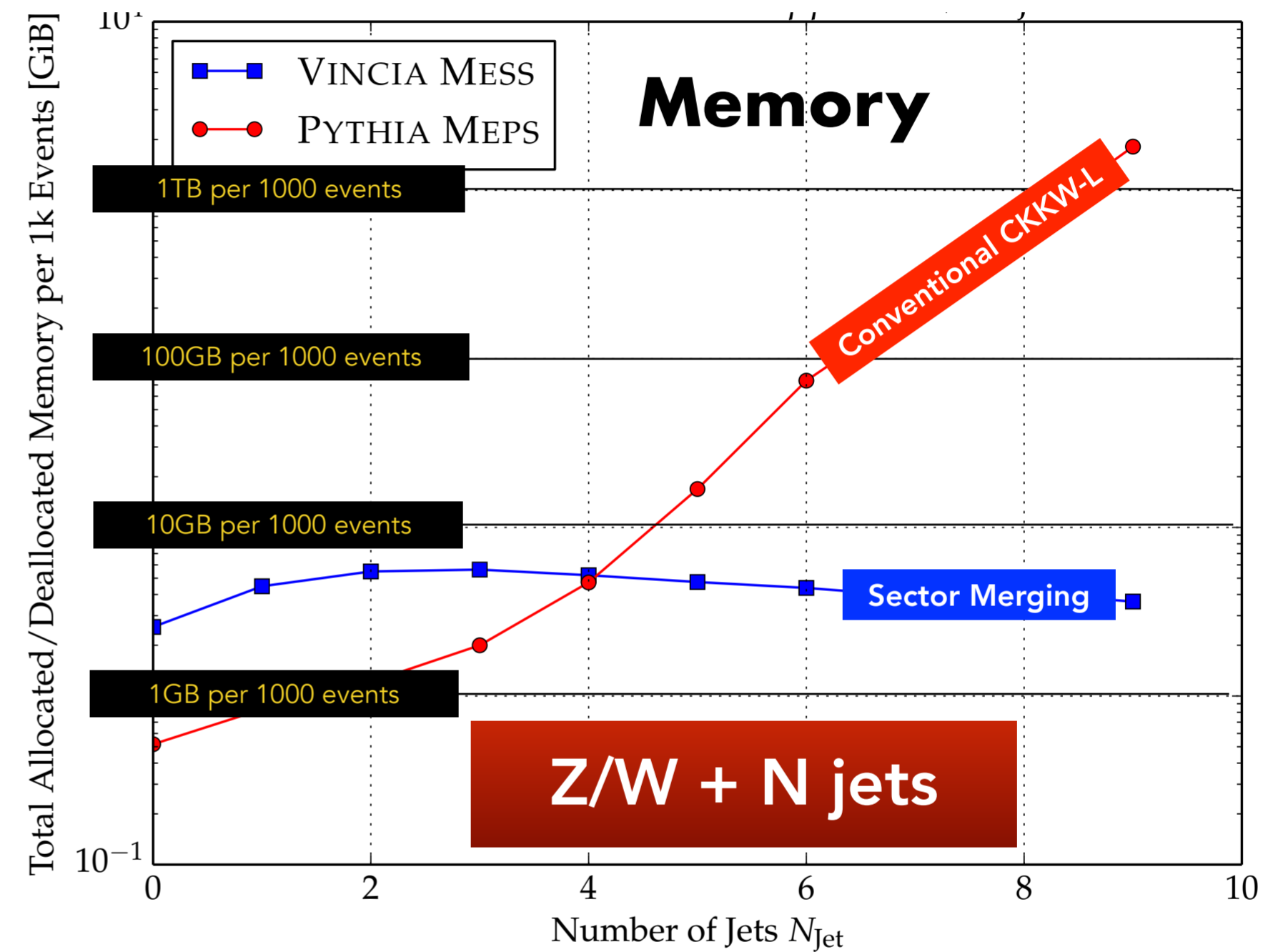
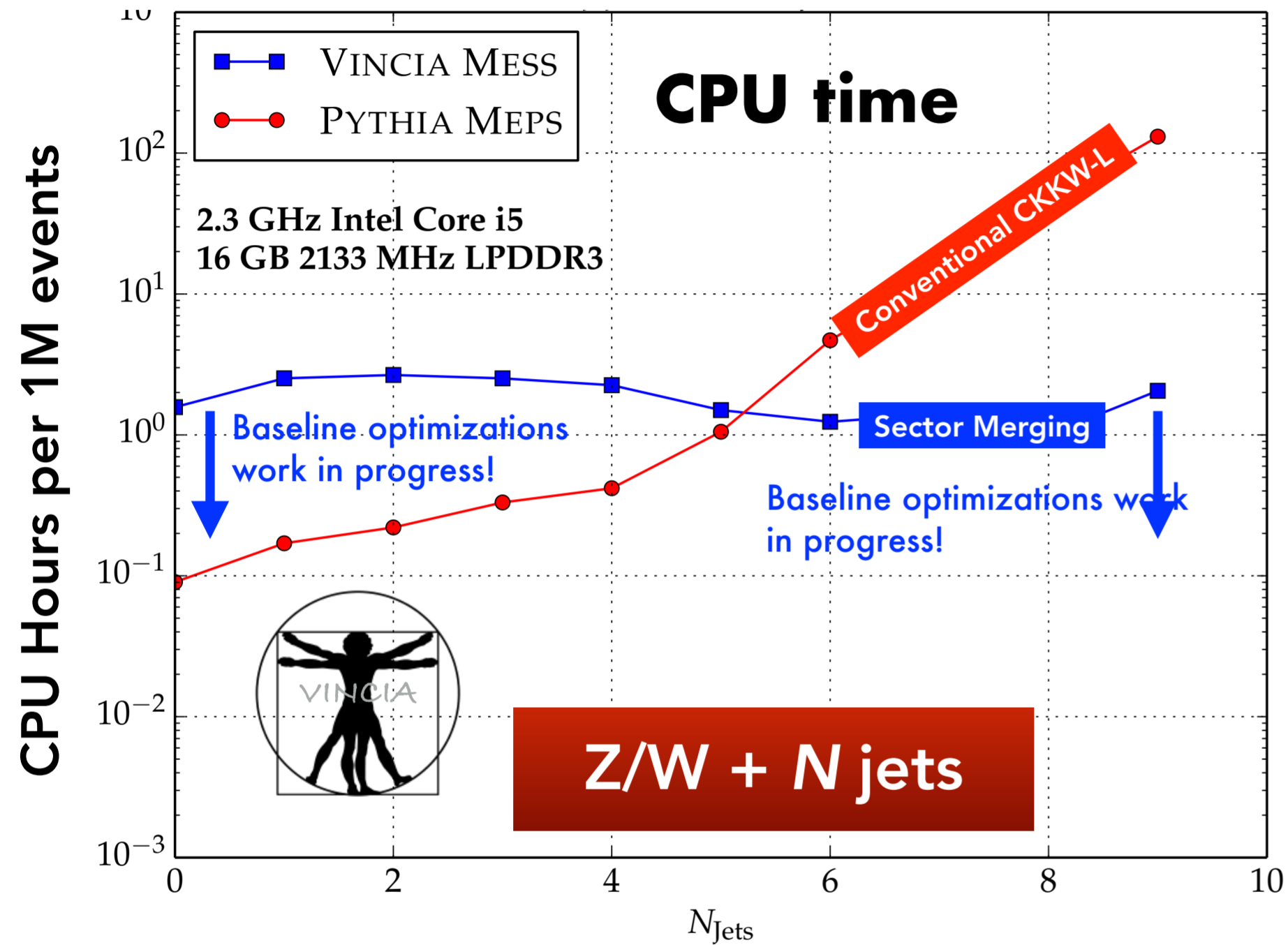
(And the number of sectors only grows linearly with the number of gluons)

($g \rightarrow q\bar{q} \rightarrow$ leftover factorial in number of *same-flavour* quarks; not a big problem)



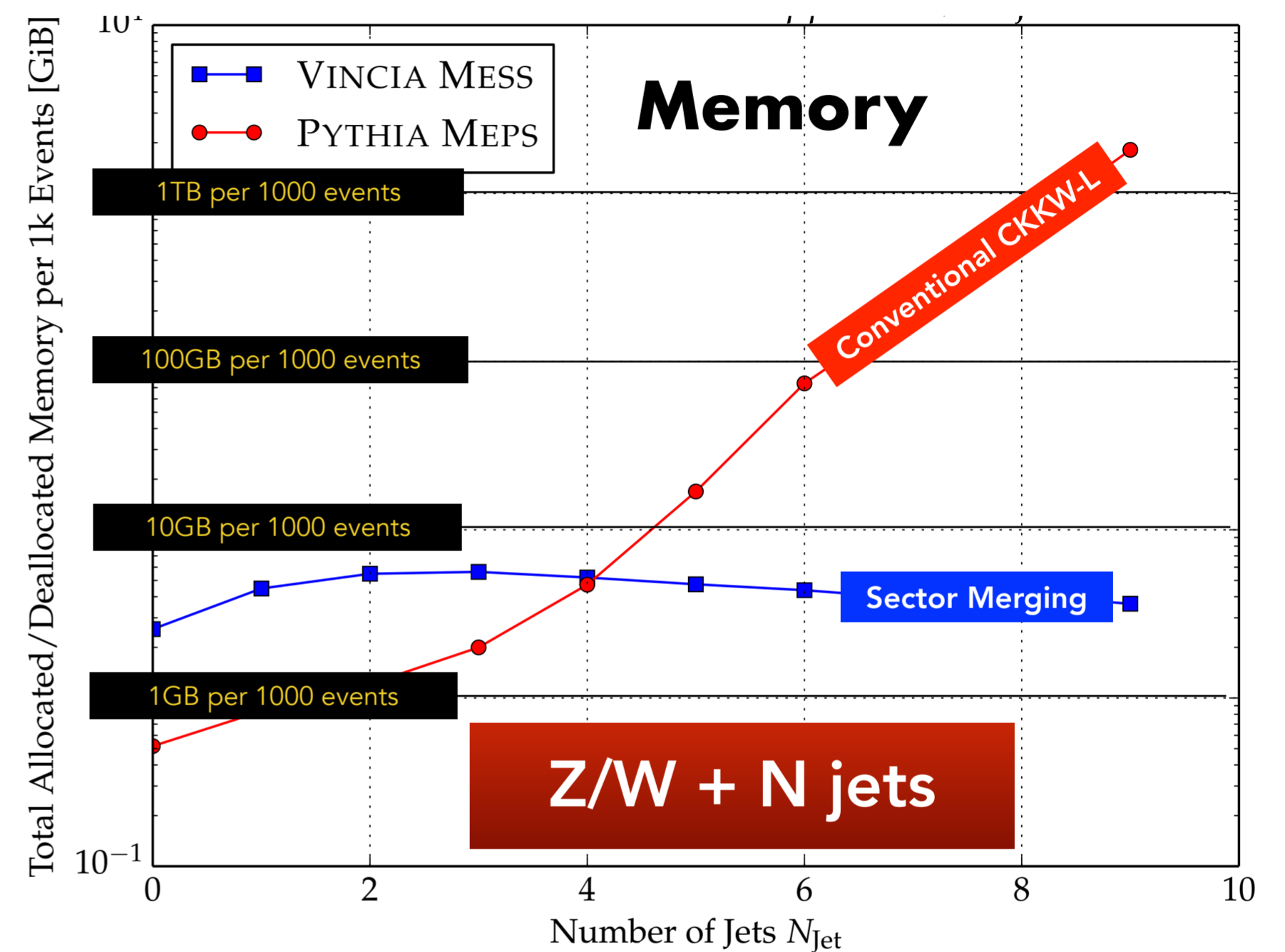
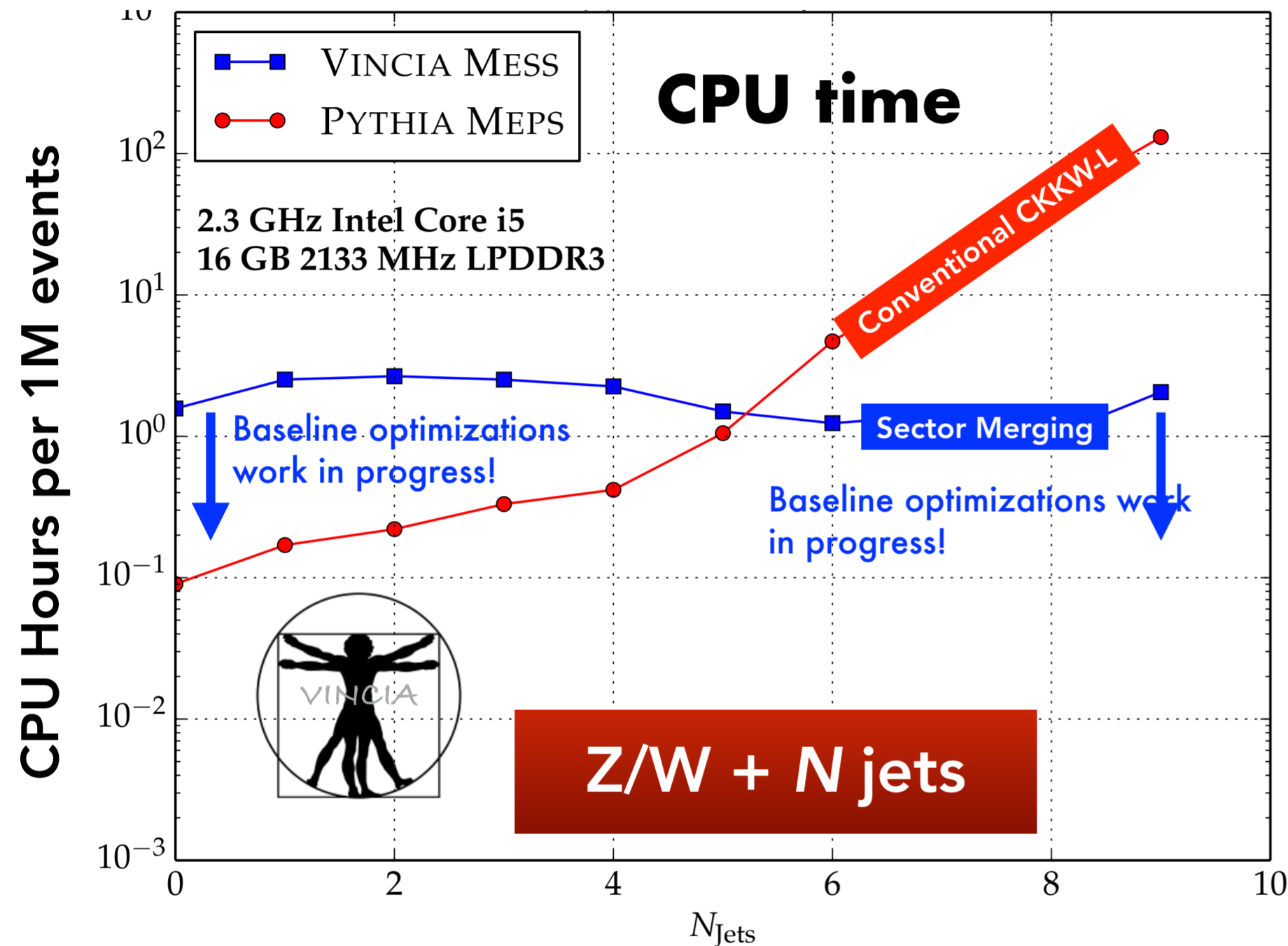
Sectorized CKKW-L Merging publicly available from Pythia 8.306

[Brooks & Preuss \(2021\) "Efficient multi-jet merging with the VINCIA sector shower"](#)



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Extensions now pursued:

Sectorized **matching at NNLO** (proof of concepts in [arXiv:2108.07133](https://arxiv.org/abs/2108.07133) & [arXiv:2310.18671](https://arxiv.org/abs/2310.18671))

Sectorized **iterated tree-level ME corrections** (demonstrated in PS & Villarejo [arXiv:1109.3608](https://arxiv.org/abs/1109.3608))

Sectorized **multi-leg merging at NLO** (active research grants, with **C. Preuss, Wuppertal**)

Sectorized Matching at NNLO (in VINCIA)

Idea: harness the power of showers as efficient phase-space generators

a.k.a. **"ME Corrections"** Sjöstrand et al. (1986, 2001); Giele, Kosower, PS (2011); Lopez-Villarejo, PS (2011)

a.k.a. **"Forward-Branching"** PS generation Weinzierl, Kosower (1999); Draggiotis, v. Hameren, Kleiss (2000);
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Conventional Fixed-Order phase-space generation (eg VEGAS)



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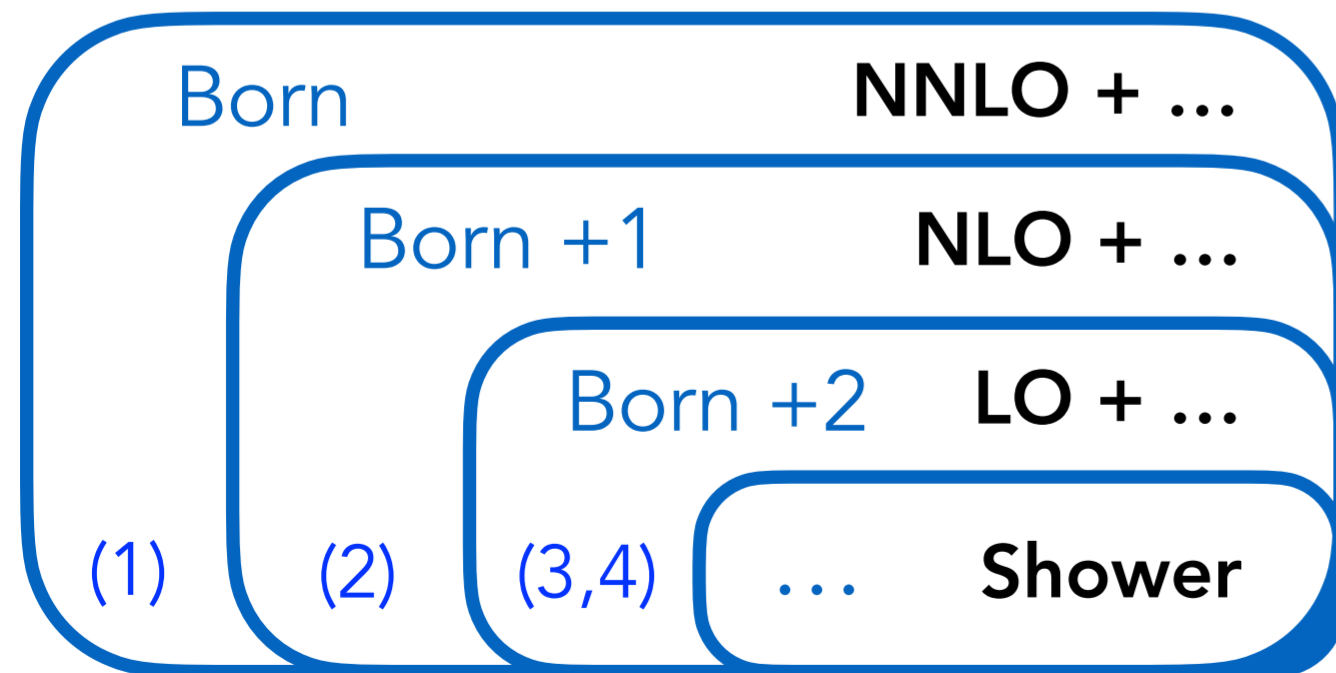
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Conventional **Fixed-Order** phase-space generation (eg VEGAS)



Nested phase-space generation in a **Shower Markov Chain**



Need:

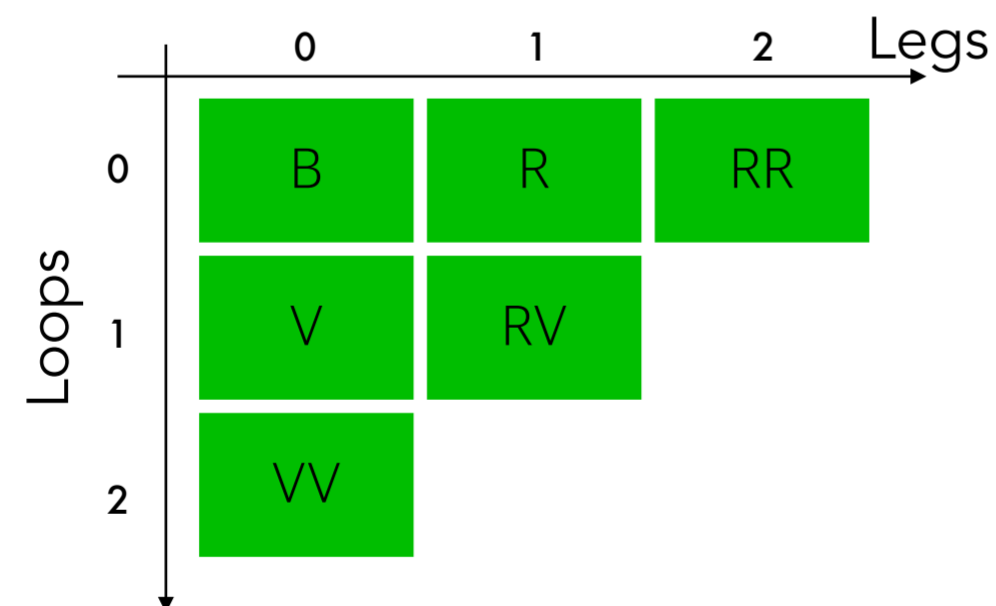
- (1) Born-local NNLO K -factors: $k_{\text{NNLO}}(\Phi_2)$
- (2) NLO MECs in the first $2 \mapsto 3$ shower branching: $w_{2 \mapsto 3}^{\text{NLO}}(\Phi_3)$
- (3) LO MECs for second (iterated) $2 \mapsto 3$ shower branching: $w_{3 \mapsto 4}^{\text{LO}}(\Phi_4)$
- (4) Direct $2 \mapsto 4$ branchings for unordered sector with LO MECs: $w_{2 \mapsto 4}^{\text{LO}}(\Phi_4)$

① Weight each Born-level event by local K -factor

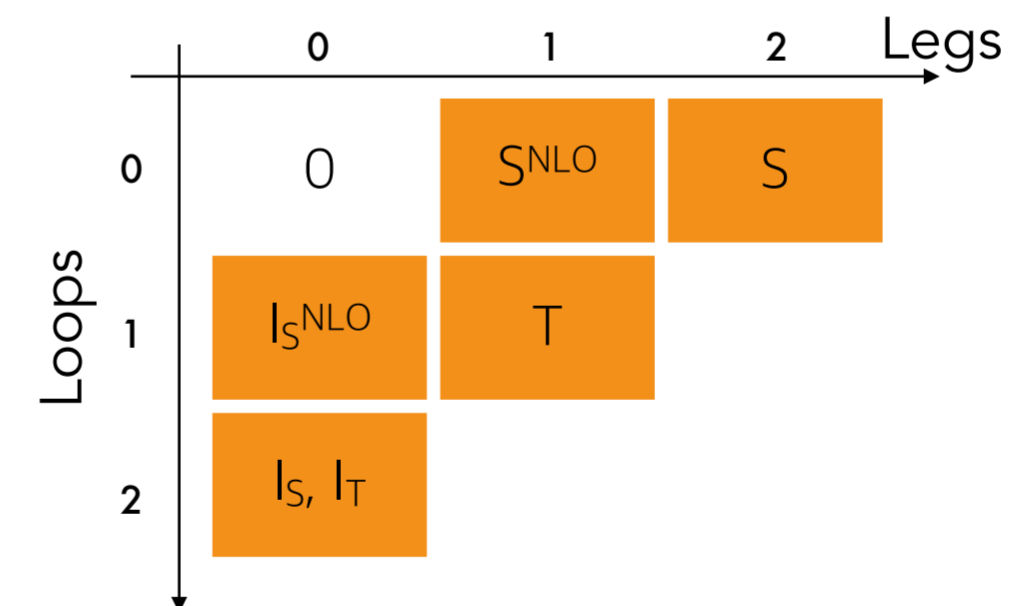
Campbell, Hoeche, Li, Preuss, PS (2023)

$$\begin{aligned}
 k_{\text{NNLO}}(\Phi_2) = & 1 + \frac{V(\Phi_2)}{B(\Phi_2)} + \frac{I_S^{\text{NLO}}(\Phi_2)}{B(\Phi_2)} + \frac{VV(\Phi_2)}{B(\Phi_2)} + \frac{I_T(\Phi_2)}{B(\Phi_2)} + \frac{I_S(\Phi_2)}{B(\Phi_2)} \\
 & + \int d\Phi_{+1} \left[\frac{R(\Phi_2, \Phi_{+1})}{B(\Phi_2)} - \frac{S^{\text{NLO}}(\Phi_2, \Phi_{+1})}{B(\Phi_2)} + \frac{RV(\Phi_2, \Phi_{+1})}{B(\Phi_2)} - \frac{T(\Phi_2, \Phi_{+1})}{B(\Phi_2)} \right] \\
 & + \int d\Phi_{+2} \left[\frac{RR(\Phi_2, \Phi_{+2})}{B(\Phi_2)} - \frac{S(\Phi_2, \Phi_{+2})}{B(\Phi_2)} \right]
 \end{aligned}$$

Fixed-Order Coefficients:



Subtraction Terms (not tied to shower formalism):



Note: **requires** "Born-local" NNLO subtraction terms. Currently only for simplest cases.

Interested in discussing & exploring connections with local subtraction schemes

②, ③, ④ Shower Markov chain with Second-Order Corrections

Key aspect

up to matched order, include **process-specific NLO corrections** into shower evolution:

② correct first branching to exclusive ($< t'$) NLO rate: [Hartgring, Laenen, PS (2013)]

Born \rightarrow Born + 1
Sudakov Factor

$$\Delta_{2 \rightarrow 3}^{\text{NLO}}(t_0, t') = \exp \left\{ - \int_{t'}^{t_0} d\Phi_{+1} \underline{A_{2 \rightarrow 3}(\Phi_{+1})} w_{2 \rightarrow 3}^{\text{NLO}}(\Phi_2, \Phi_{+1}) \right\}$$

③ correct second branching to LO ME: [Giele, Kosower, PS (2011); Lopez-Villarejo, PS (2011)]

Born + 1 \rightarrow Born + 2
Sudakov Factor

$$\Delta_{3 \rightarrow 4}^{\text{LO}}(t', t) = \exp \left\{ - \int_t^{t'} d\Phi'_{+1} \underline{A_{3 \rightarrow 4}(\Phi'_{+1})} w_{3 \rightarrow 4}^{\text{LO}}(\Phi_3, \Phi'_{+1}) \right\}$$

④ add direct 2 \rightarrow 4 branching and correct it to LO ME: [Li, PS (2017)]

Born \rightarrow Born + 2
Sudakov Factor

$$\Delta_{2 \rightarrow 4}^{\text{LO}}(t_0, t) = \exp \left\{ - \int_t^{t_0} d\Phi_{+2} \underline{A_{2 \rightarrow 4}(\Phi_{+2})} w_{2 \rightarrow 4}^{\text{LO}}(\Phi_2, \Phi_{+2}) \right\}$$

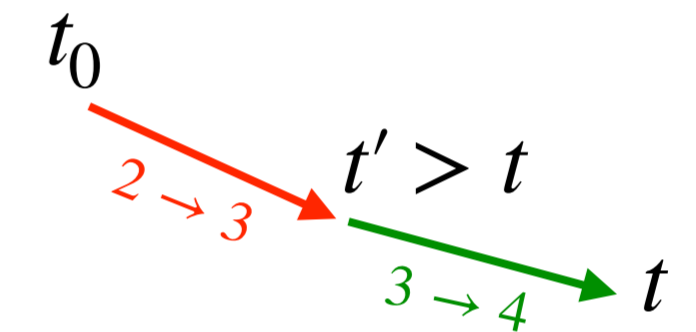
\Rightarrow entirely based on **MECs** and **sectorisation**

\Rightarrow **by construction**, expansion of extended shower **matches NNLO singularity structure**

But shower kernels **do not define NNLO subtraction terms**¹ (!)

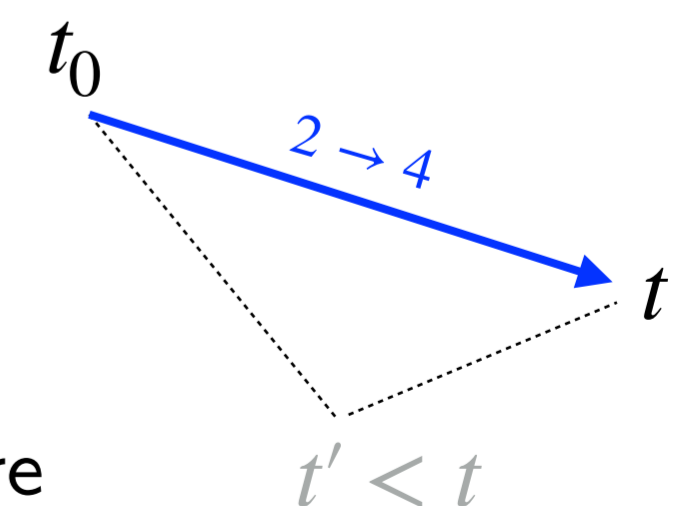


Iterated:
(Ordered)



Direct:

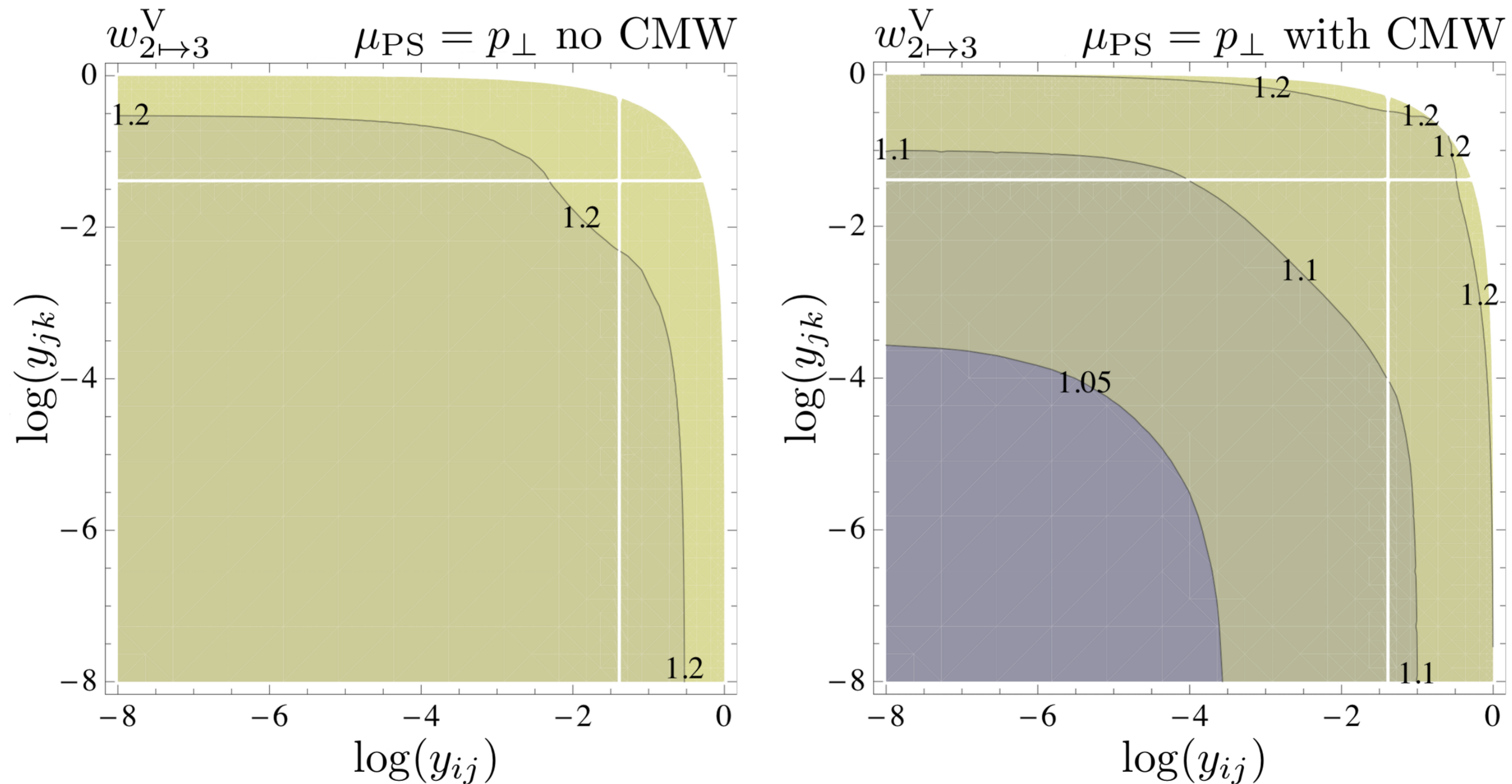
(Unordered)



Size of the Real-Virtual Correction Factor (2)

$$\underline{w_{2\rightarrow 3}^{\text{NLO}}} = w_{2\rightarrow 3}^{\text{LO}} (1 + w_{2\rightarrow 3}^{\text{V}})$$

studied **analytically** in detail for $Z \rightarrow q\bar{q}$ in [Hartgring, Laenen, PS JHEP 10 \(2013\) 127](#)

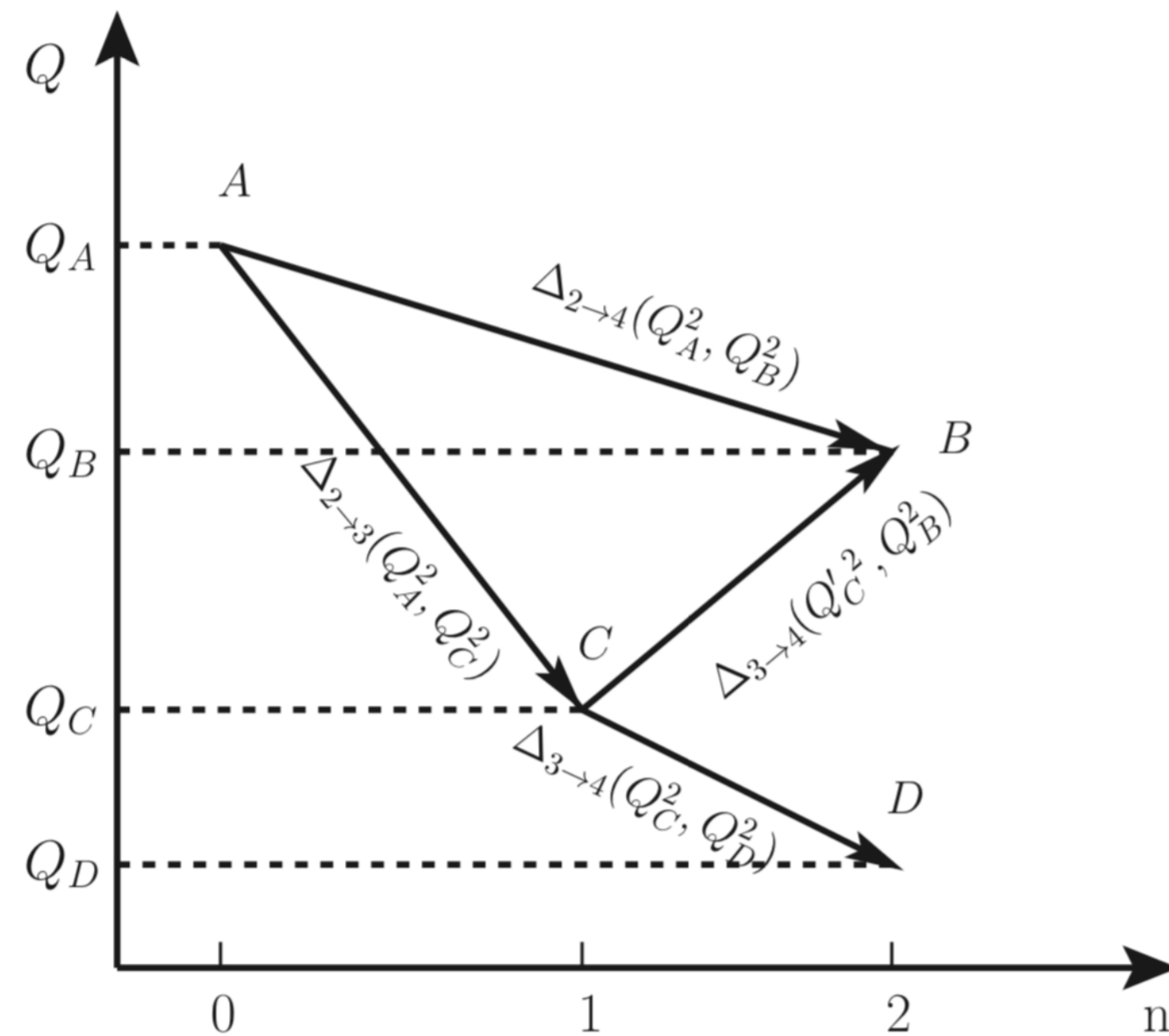


\Rightarrow now: **generalisation & (semi-)automation** in VINCIA in form of NLO MECs

(Combining iterated $n \rightarrow n + 1$ and direct $n \rightarrow n + 2$ branchings)

A priori, direct $2 \mapsto 4$ and iterated $2 \mapsto 3$ branchings **overlap in ordered** region.

In **sector showers**, iterated $2 \mapsto 3$ branchings are **always strictly ordered**.



Divide double-emission phase space into **strongly-ordered** and **unordered** region:

[Li, Skands 1611.00013]

$$d\Phi_{+2} = \underbrace{d\Phi_{+2}^>}_{\text{u.o.}} + \underbrace{d\Phi_{+2}^<}_{\text{s.o.}}$$

$d\Phi_{+2}^<$: **single-unresolved** limits \Rightarrow iterated $2 \mapsto 3$

$d\Phi_{+2}^>$: **double-unresolved** limits \Rightarrow direct $2 \mapsto 4$

Restriction on double-branching phase space enforced by additional veto:

$$d\Phi_{+2}^> = \sum_j \theta \left(p_{\perp,+2}^2 - \hat{p}_{\perp,+1}^2 \right) \Theta_{ijk}^{\text{sct}} d\Phi_{+2}$$

Preview: VINCIA NNLO+PS for $H \rightarrow b\bar{b}$

[Coloretti, Gehrmann-de Ridder, Preuss, JHEP 06 \(2022\) 009](#)

Fixed-Order Reference = EERAD3 NLO $H \rightarrow b\bar{b}g$: already quite optimised

Uses analytical MEs, “folds” phase space to cancel azimuthally antipodal points, and uses antenna subtraction (\rightarrow smaller # of NLO subtraction terms than Catani-Seymour or FKS).

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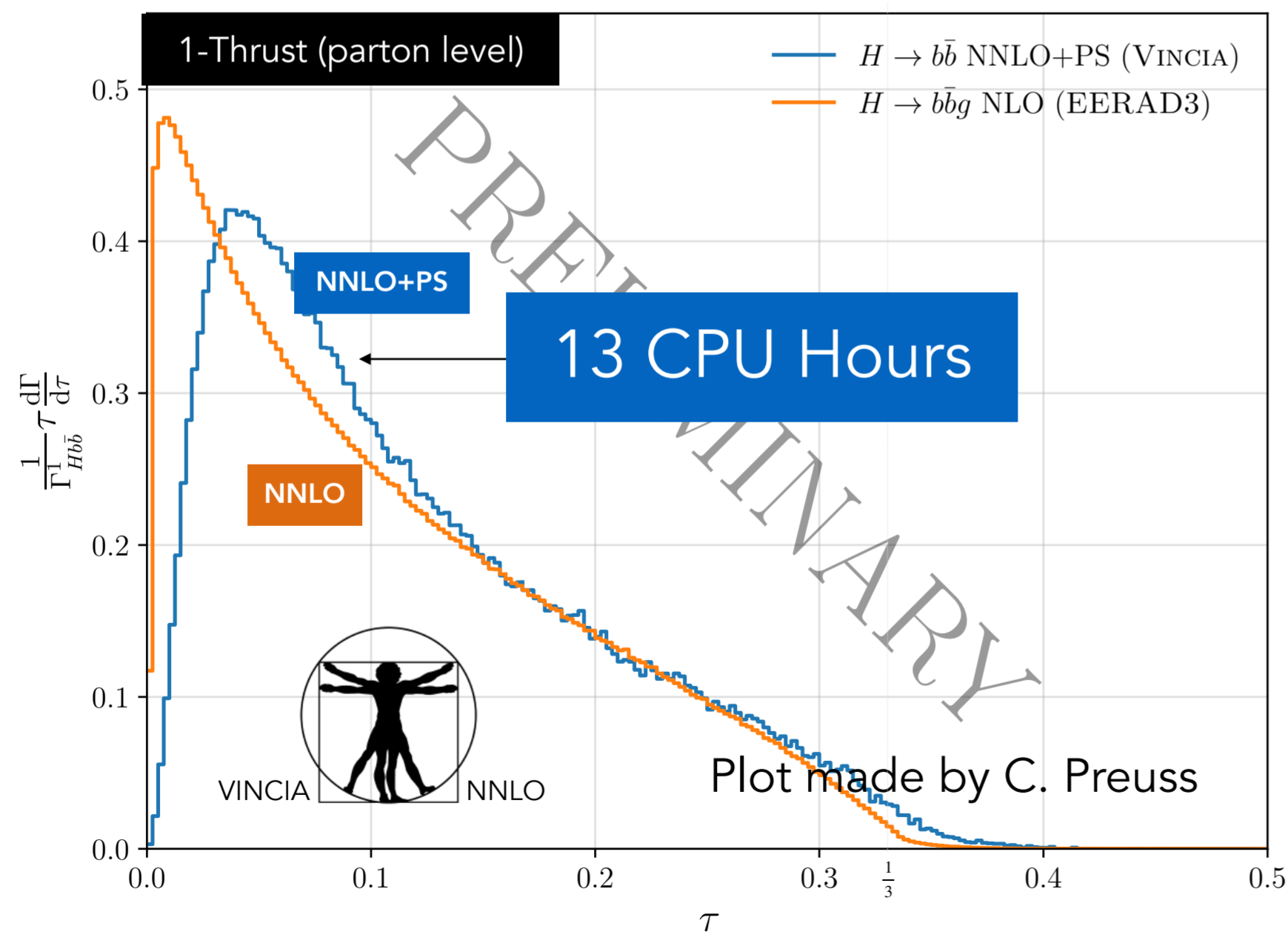
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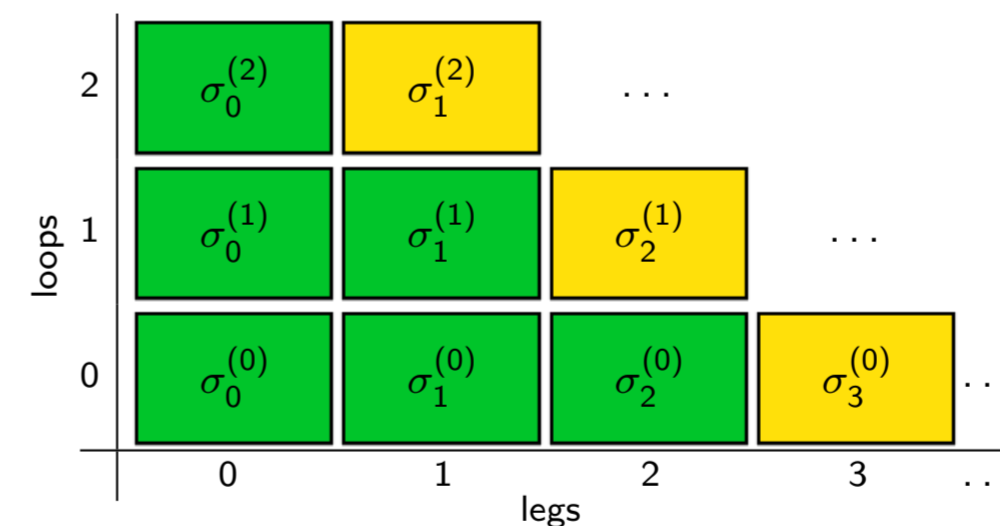
VINCIA NNLO+PS: shower as phase-space generator: efficient & no negative weights

► Looks ~ 5 x faster than EERAD3 (for similar unweighted stats) + is matched to shower \implies includes resummation; can calculate any IR safe observable; can be hadronised \rightarrow IR sensitive observables, etc.



Note:

NNLO accuracy in $H \rightarrow 2j$ implies NLO correction in first emission and LO correction in second emission.



So for Thrust,
NNLO $H \rightarrow b\bar{b}$
is effectively
NLO for $\tau < 1/3$
LO for $\tau > 1/3$

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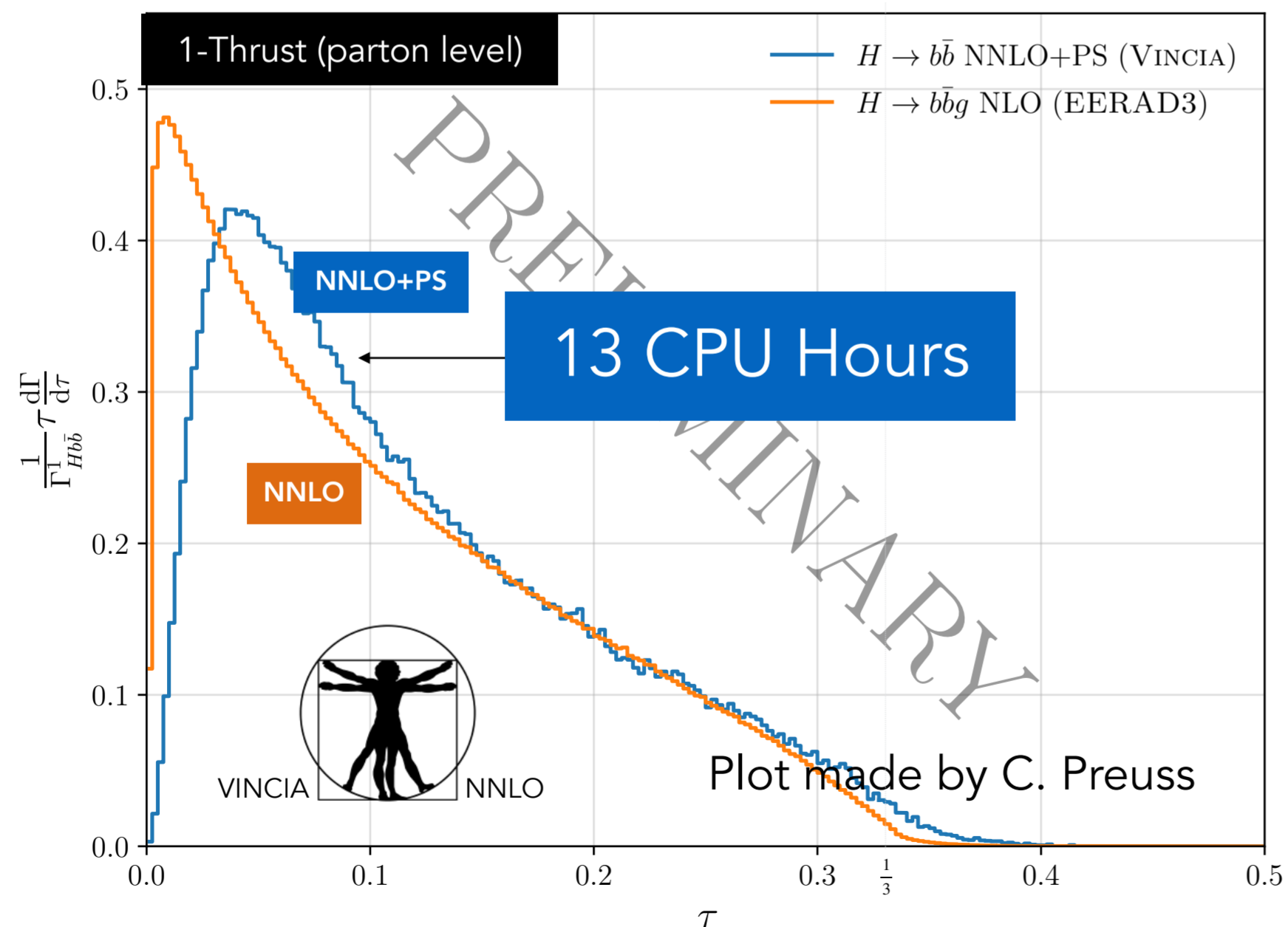
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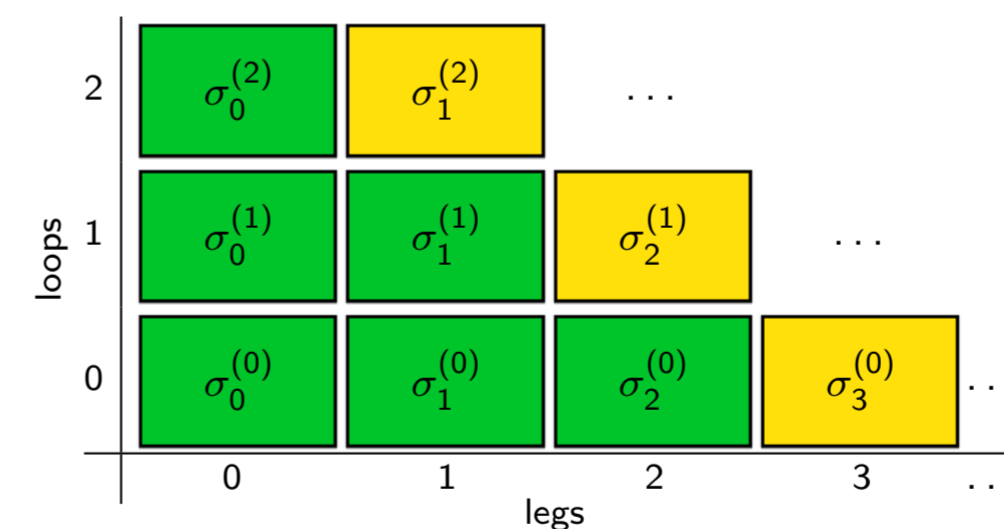
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Proof of concepts done for H & Z \rightarrow 2

Work remains to extend to pp, ep, and ee $\rightarrow n \geq 3$
(& on marrying this formalism with NⁿLL accuracy)

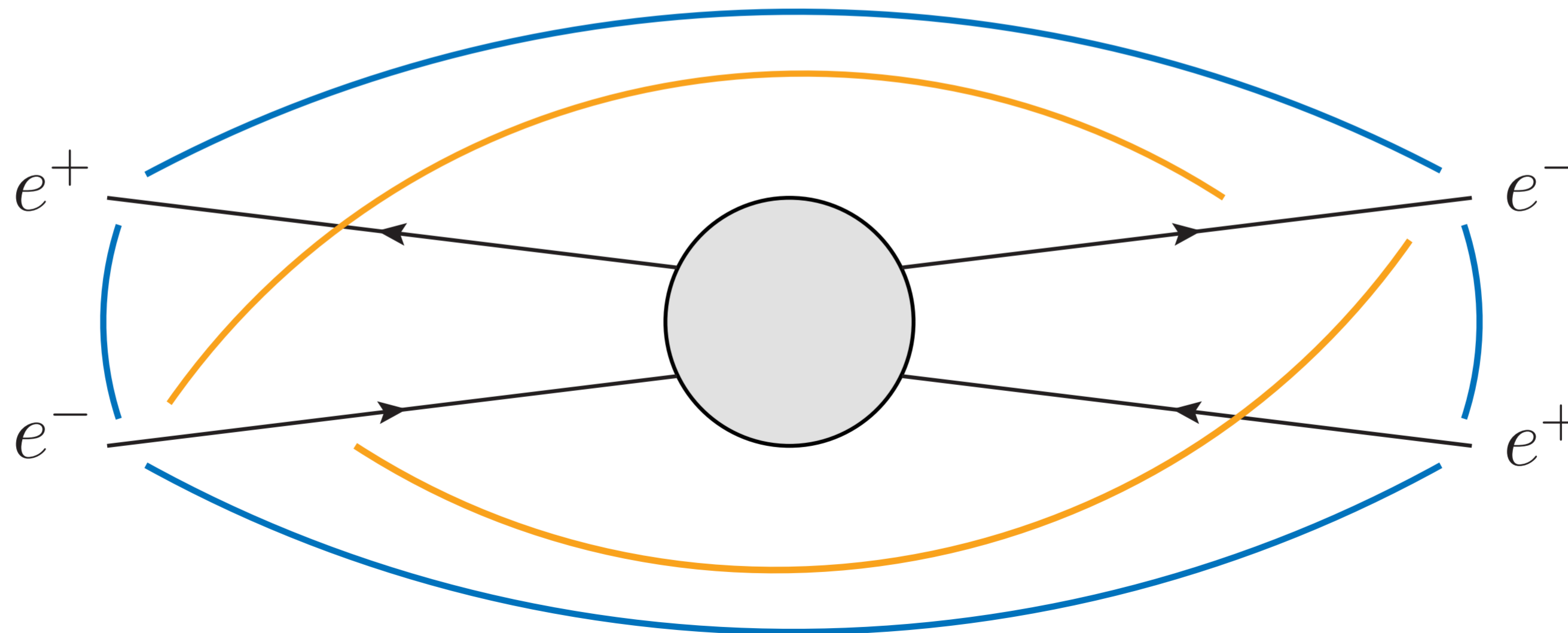
③ Electroweak Radiation in VINCIA

Main component: **soft photon emission**

[Dittmaier, 2000]
$$|M_{n+1}(\{p\}, p_j)|^2 = -8\pi\alpha \sum_{x,y} \sigma_x Q_x \sigma_y Q_y \frac{s_{xy}}{s_{xj} s_{yj}} |M_n(\{p\})|^2$$

Example: **Quadrupole final state (4-fermion: $e^+e^+e^-e^-$)**

- Opposite-charge pairs \blacktriangleright positive terms
- Same-charge pairs \blacktriangleright negative terms



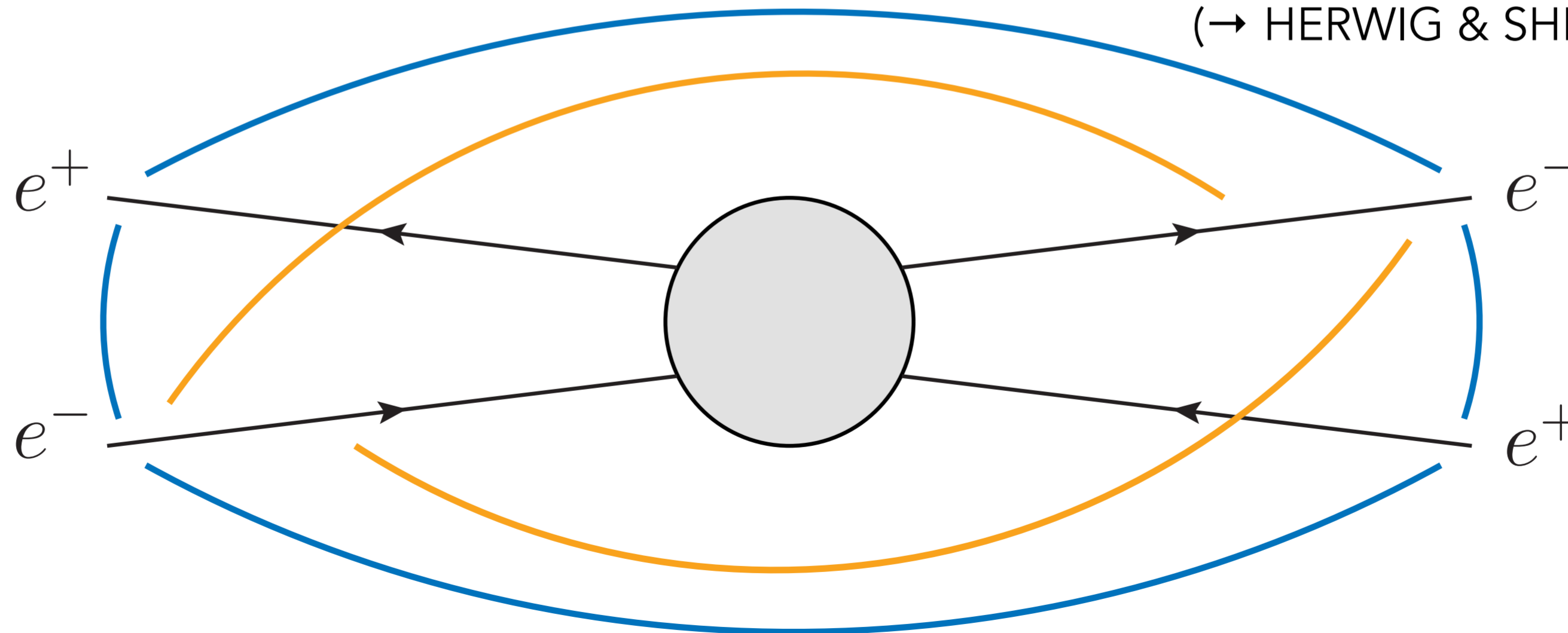
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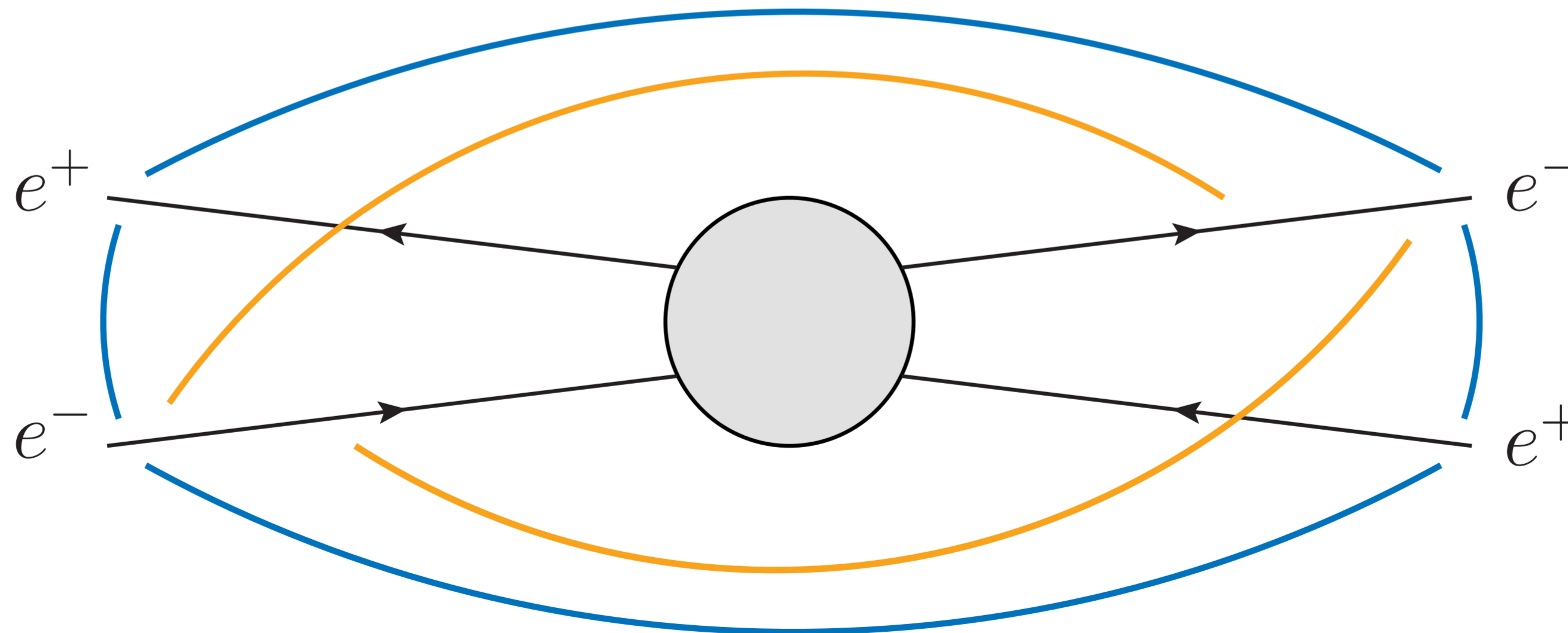
- Opposite-charge pairs \blacktriangleright positive terms
- Same-charge pairs \blacktriangleright negative terms \longleftarrow **Not well suited for showers**
(\rightarrow HERWIG & SHERPA use YFS)



QED Multipole Showers in VINCIA

Sectorize QED phase space: for each possible photon emission kinematics p_γ , find the 2 charged particles with respect to which that photon is softest ➤ “Dipole Sector”

Use dipole-antenna *kinematics* for that sector, but sum **all** the positive and negative antenna terms (w spin dependence) to find **coherent emission *probability* > 0**

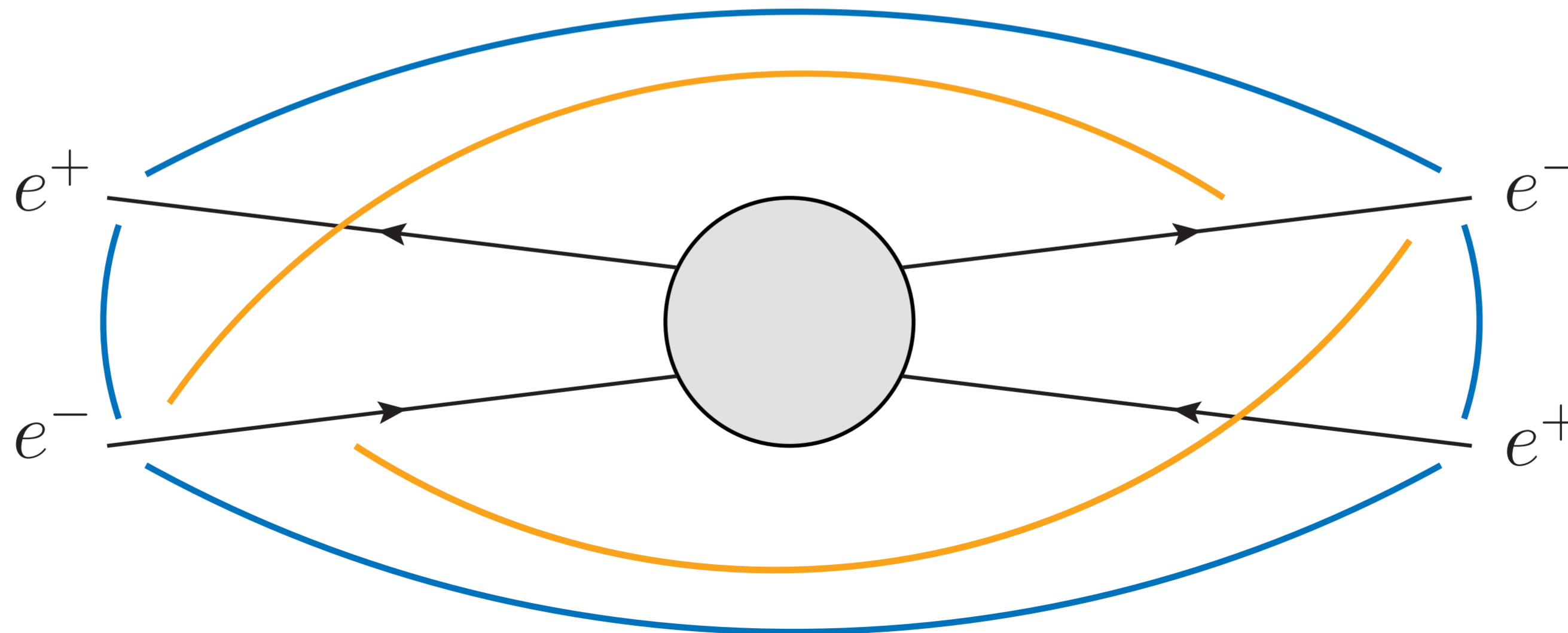


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⇒ QED shower with **full soft multipole coherence** **and** **DGLAP collinear limits**
and no negative weights [[Kleiss & Verheyen \(2017\)](#); [PS & Verheyen \(2020\)](#)]

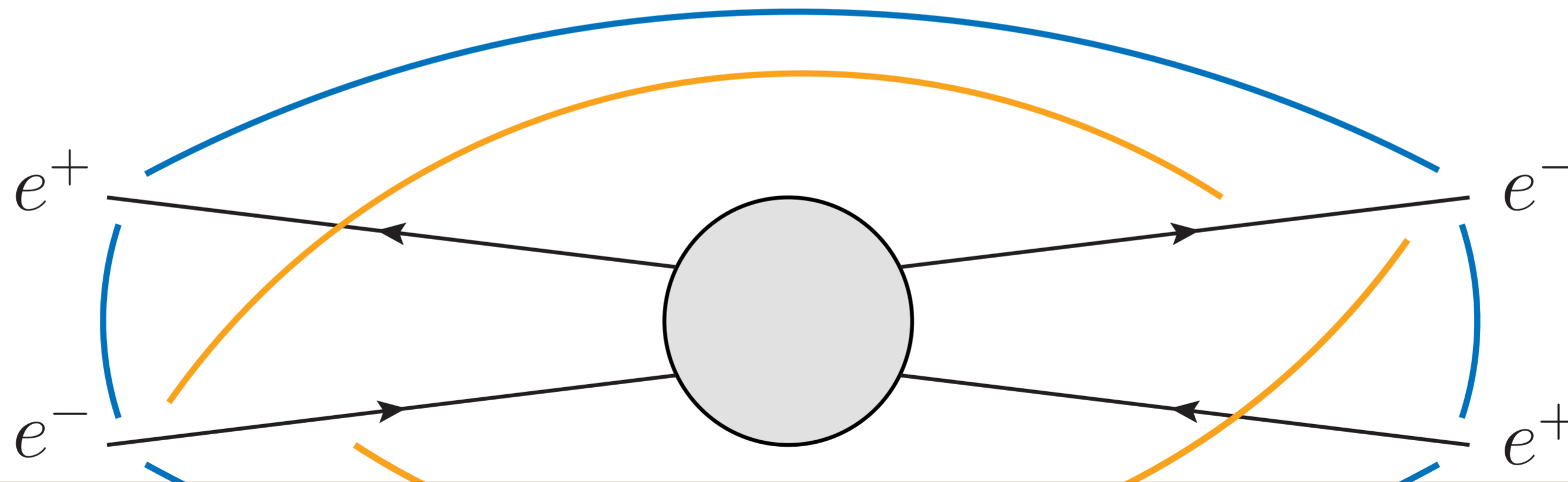


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Available in PYTHIA 8; directly applicable also to $e^+e^- \rightarrow Z/\gamma^* \rightarrow f\bar{f}$ and $e^+e^- \rightarrow W^+W^- \rightarrow 4f$
Also accounts for **initial-final interference** via **interleaved resonance decays**; discussed later

Example of QED multipole interferences

High-mass Drell-Yan

$$u\bar{u} \rightarrow Z/\gamma^* \rightarrow e^+e^-$$

$$m_{ee}^2 > 1 \text{ TeV}, p_{\perp,e} > 25 \text{ GeV and } |\eta_e| < 3.5$$

$$p_{\perp,\gamma} > 0.5 \text{ GeV and } |\eta_\gamma| < 3.5$$

PYTHIA

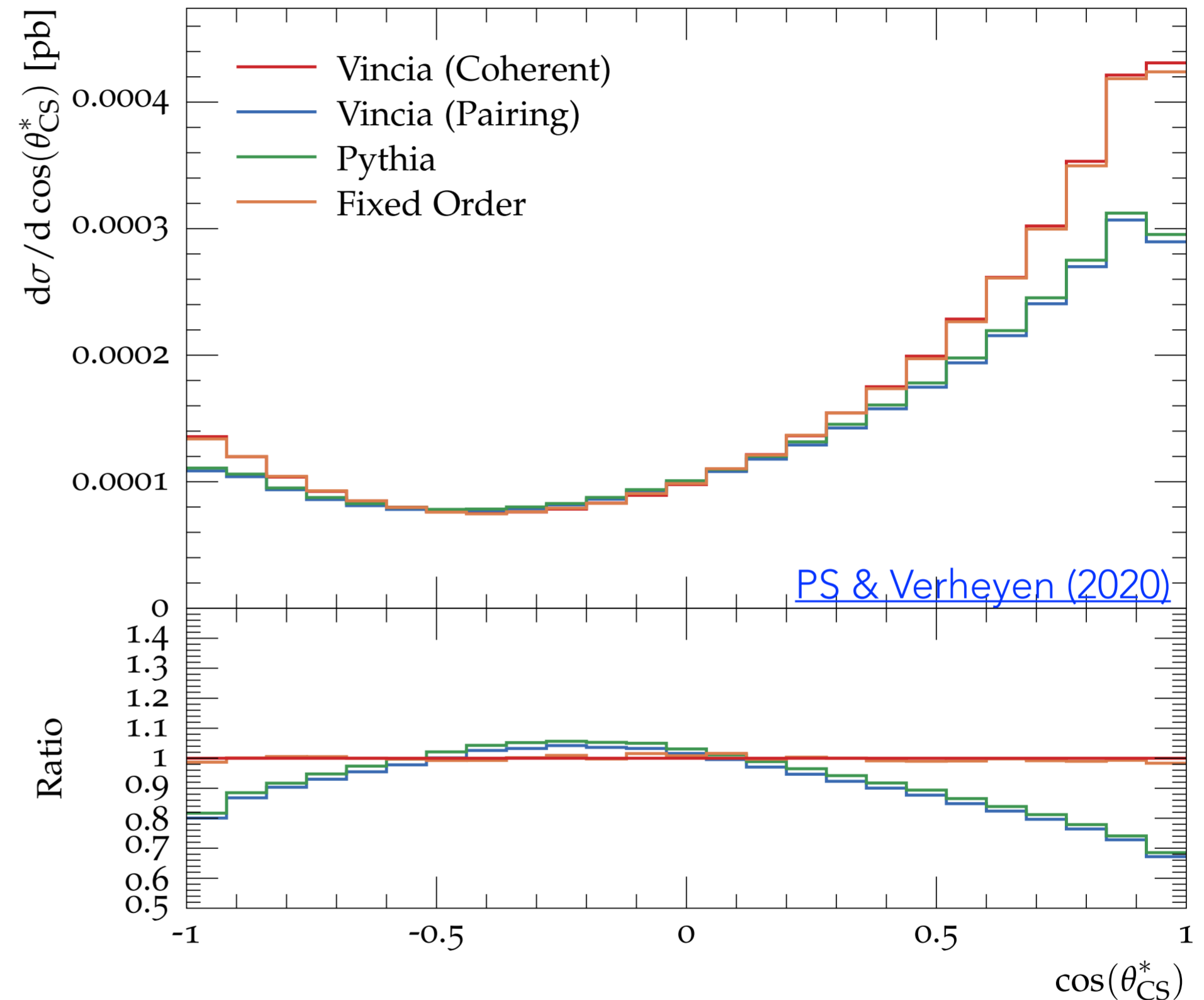
Factorizes $u\bar{u}$ and e^+e^- radiation

VINCIA

1) **Coherent** = full multipole treatment

2) **Pairing** ~ PYTHIA: only consider "maximally screening" charge pairs; no genuine multipole effects

$u\bar{u} \rightarrow Z/\gamma^* \rightarrow e^+e^- \gamma$ (Dressed, no QCD, $p_{\perp,\gamma} < 5 \text{ GeV}$)



$$\cos \theta_{CS}^* = 2 \frac{p_{ee}^z}{|p_{ee}^z|} \frac{p_{e^+}^+ p_{e^-}^- - p_{e^+}^- p_{e^-}^+}{m_{ee} \sqrt{m_{ee}^2 + p_{\perp,ee}^2}},$$

Angle between the incoming quark and the outgoing electron in the Collins-Soper frame, using longitudinal boost of ee pair as stand-in for ambiguous quark direction

Example of QED multipole interferences

High-mass Drell-Yan

$$u\bar{u} \rightarrow Z/\gamma^* \rightarrow e^+e^-$$

$$m_{ee}^2 > 1 \text{ TeV}, p_{\perp,e} > 25 \text{ GeV and } |\eta_e| < 3.5$$

$$p_{\perp,\gamma} > 0.5 \text{ GeV and } |\eta_\gamma| < 3.5$$

PYTHIA

Factorizes $u\bar{u}$ and e^+e^- radiation

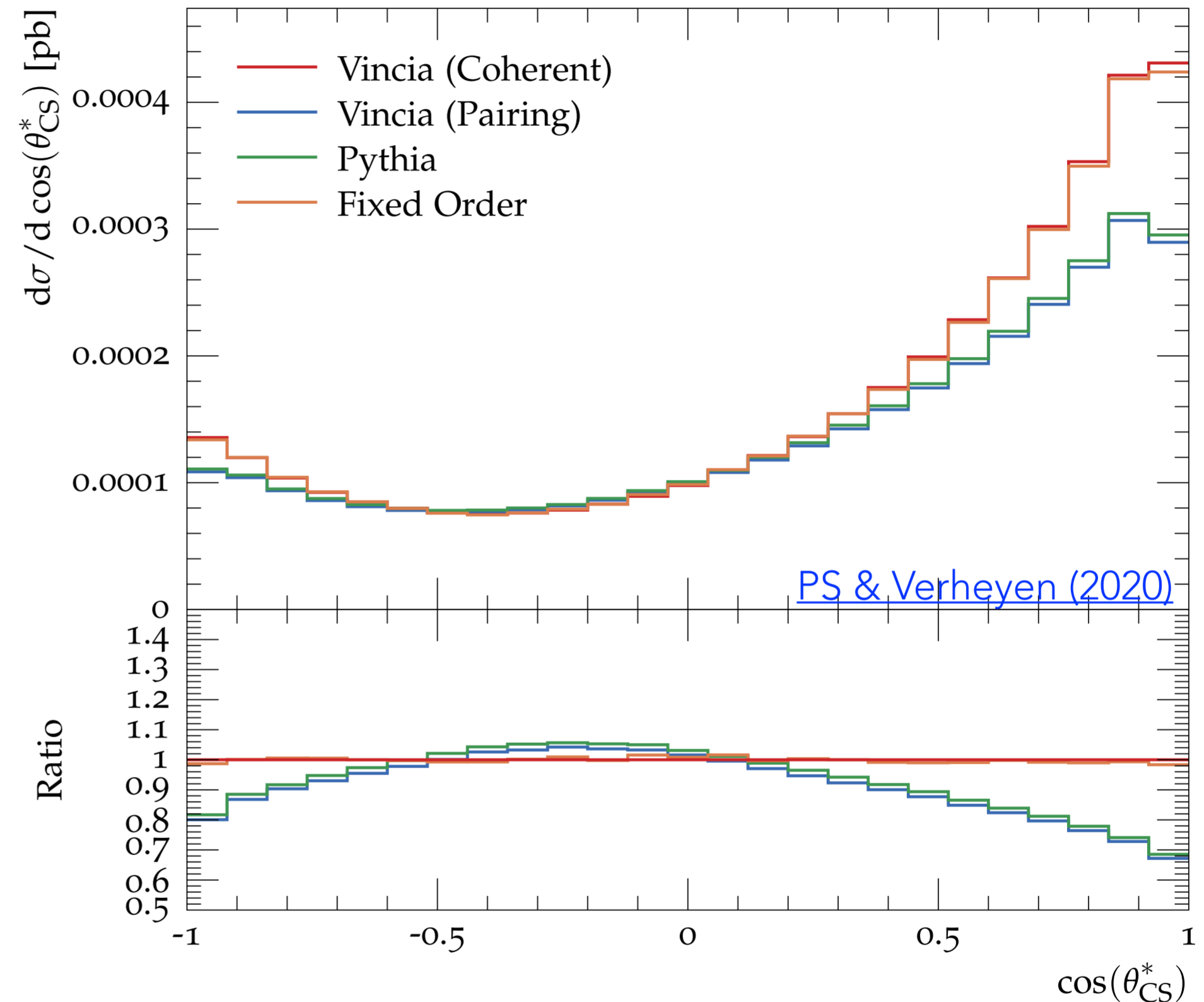
VINCIA

1) **Coherent** = full multipole treatment

2) **Pairing** ~ PYTHIA: only consider "maximally screening" charge pairs; no genuine multipole effects

Next: QED matrix-element corrections & applications to QED corrections in B decays

$u\bar{u} \rightarrow Z/\gamma^* \rightarrow e^+e^- \gamma$ (Dressed, no QCD, $p_{\perp,\gamma} < 5 \text{ GeV}$)



$$\cos \theta_{CS}^* = 2 \frac{p_{ee}^z}{|p_{ee}^z|} \frac{p_{e^+}^+ p_{e^-}^- - p_{e^+}^- p_{e^-}^+}{m_{ee} \sqrt{m_{ee}^2 + p_{\perp,ee}^2}},$$

Angle between the incoming quark and the outgoing electron in the Collins-Soper frame, using longitudinal boost of ee pair as stand-in for ambiguous quark direction

Weak Showers

Real corrections: **EW gauge bosons, tops, Higgs part of jets**

Virtual corrections: **Universal incorporation of Sudakov logs** $\frac{\alpha}{\pi} \ln^2(s/Q_{EW}^2)$

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Features of VINCIA's EW Shower [Brooks, PS, Verheyen (2022)]

Chiral → **Helicity showers** Larkoski, Lopez-Villarejo, PS (2013);
Fischer, Lifson, PS (2017)

EW-scale mass corrections & exact massive phase spaces

Longitudinal polarisations / Goldstone bosons

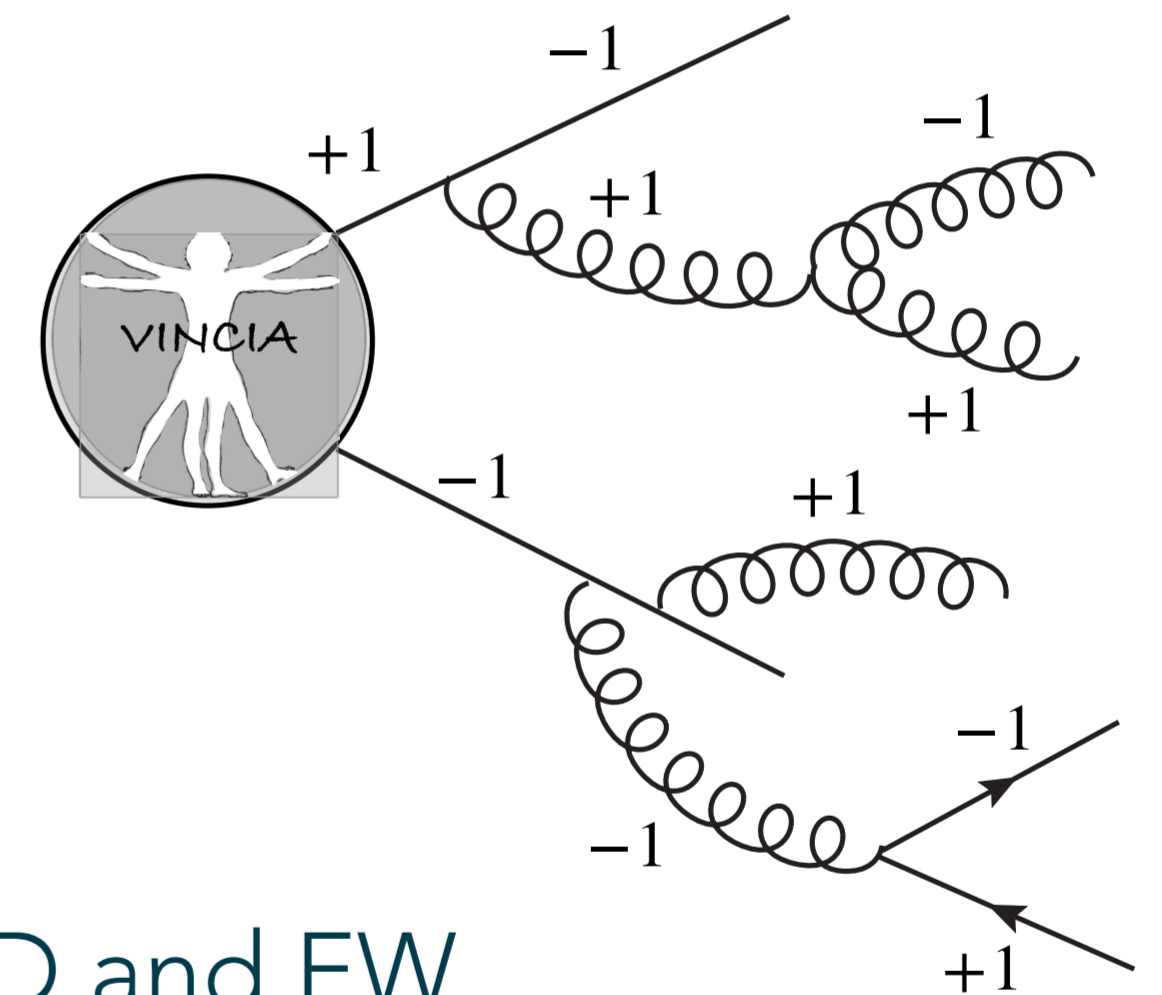
Treatment of neutral boson interference

Overlap vetos to eliminate double-counting between QCD and EW

Resonance-decay like branchings → **Interleaved Resonance Decays**

Caveat: Our EW antenna functions constructed from collinear limits (~DGLAP)

Soft multipole coherence so far only for pure QED, not full EW



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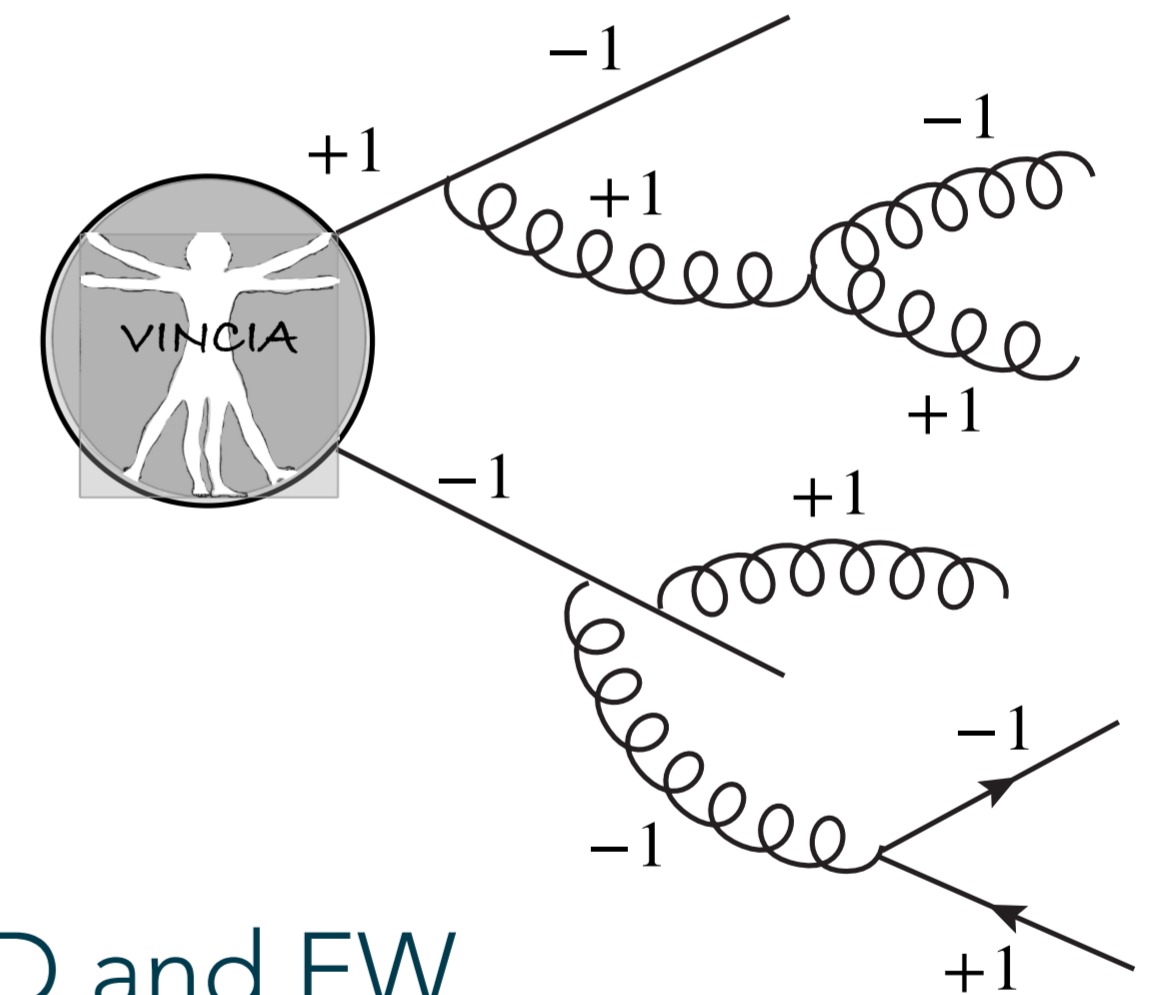
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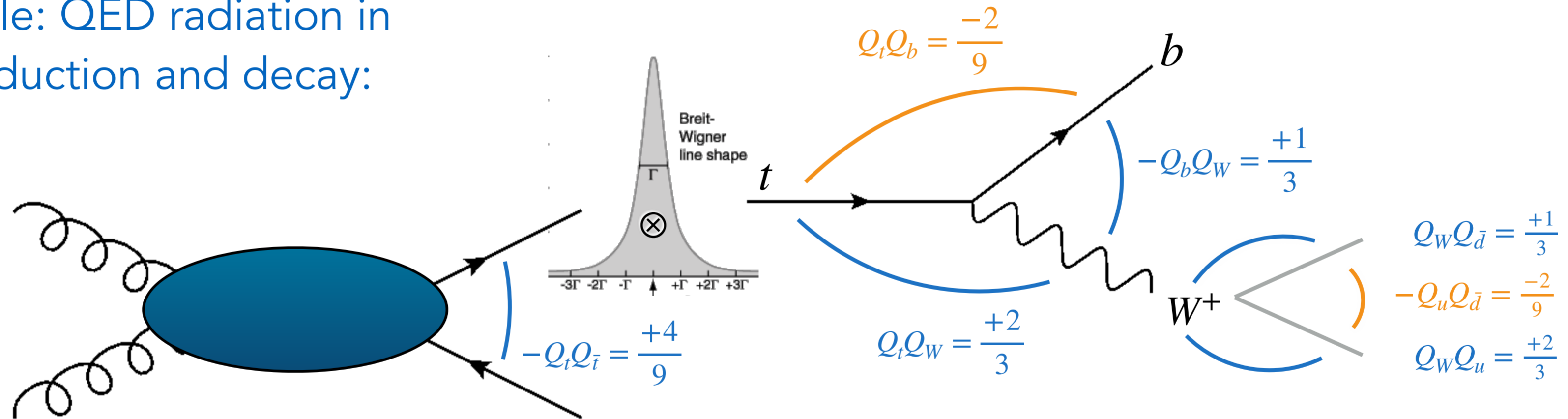


Radiation in Decays

Narrow-Width Limit \Leftrightarrow Conventional “sequential” treatment

Treat each decay (sequentially) as if alone in the universe

Example: QED radiation in $t\bar{t}$ production and decay:



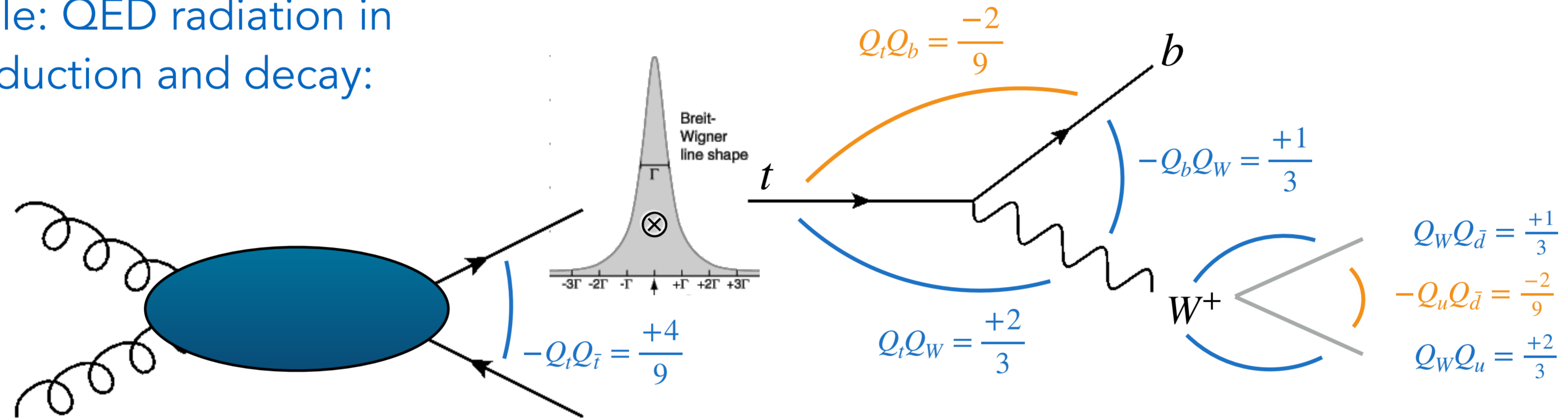
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Beyond Narrow-Width Limit:

Expect interferences to become important for $E_\gamma \lesssim \Gamma_t$ (and $E_\gamma \lesssim \Gamma_W$)

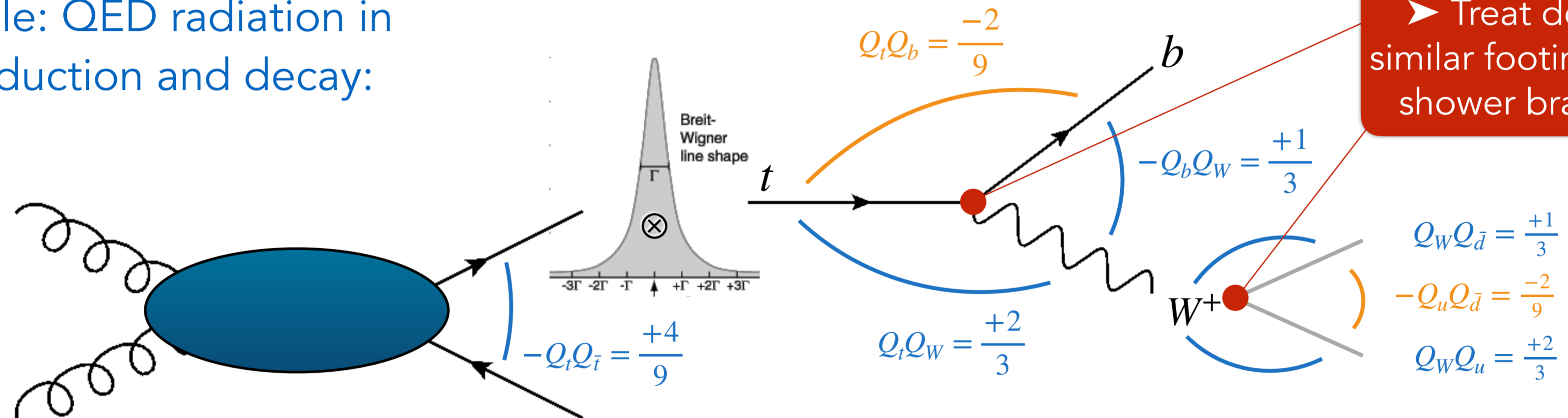
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Radiation in Decays

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Treat each decay (sequentially) as if alone in the universe

Example: QED radiation in $t\bar{t}$ production and decay:



Observation: these are also EW vertices.

► Treat decays on similar footing as other shower branchings.

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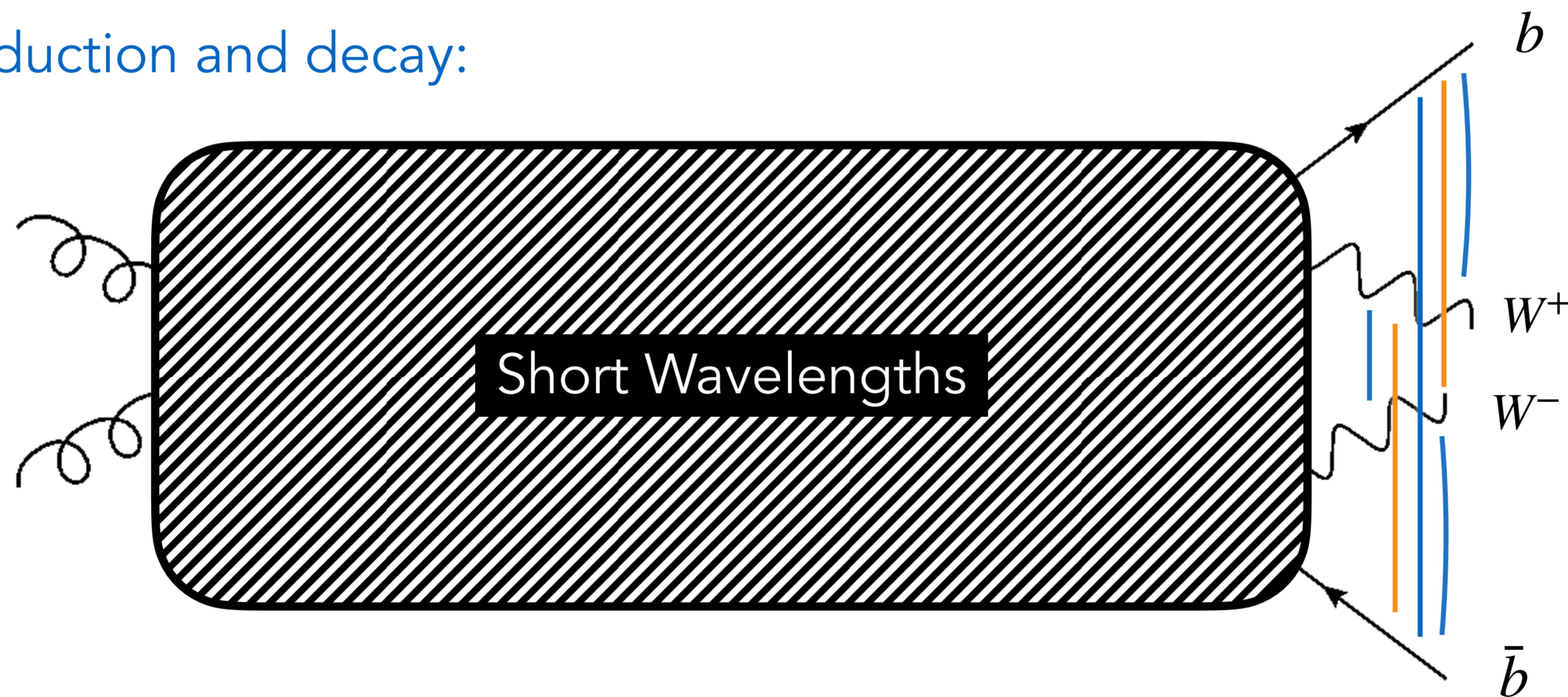
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Physics Motivation for Interleaved Resonance Decays

Long-wavelength radiation should **not** be able to resolve short-lived intermediate states

For **long wavelengths** $\lambda \gtrsim (\hbar c)/\Gamma$ expect interferences (& recoils) *between* decays

Example: QED radiation in $t\bar{t}$ production and decay:



Long Wavelengths

QED quadrupole:

$$-Q_b Q_{W^+} = \frac{+1}{3}$$

$$-Q_b Q_{W^-} = \frac{-1}{3}$$

$$-Q_b Q_{\bar{b}} = \frac{+1}{9}$$

$$-Q_{W^+} Q_{W^-} = +1$$

$$-Q_{W^+} Q_{\bar{b}} = \frac{-1}{3}$$

$$-Q_{W^-} Q_{\bar{b}} = \frac{+1}{3}$$

Affects radiation spectrum, for energies $E_\gamma \lesssim \Gamma$

+ Interferences and recoils *between* systems => **non-local BW modifications**



→ Interleaved Resonance Decays (VINCIA)

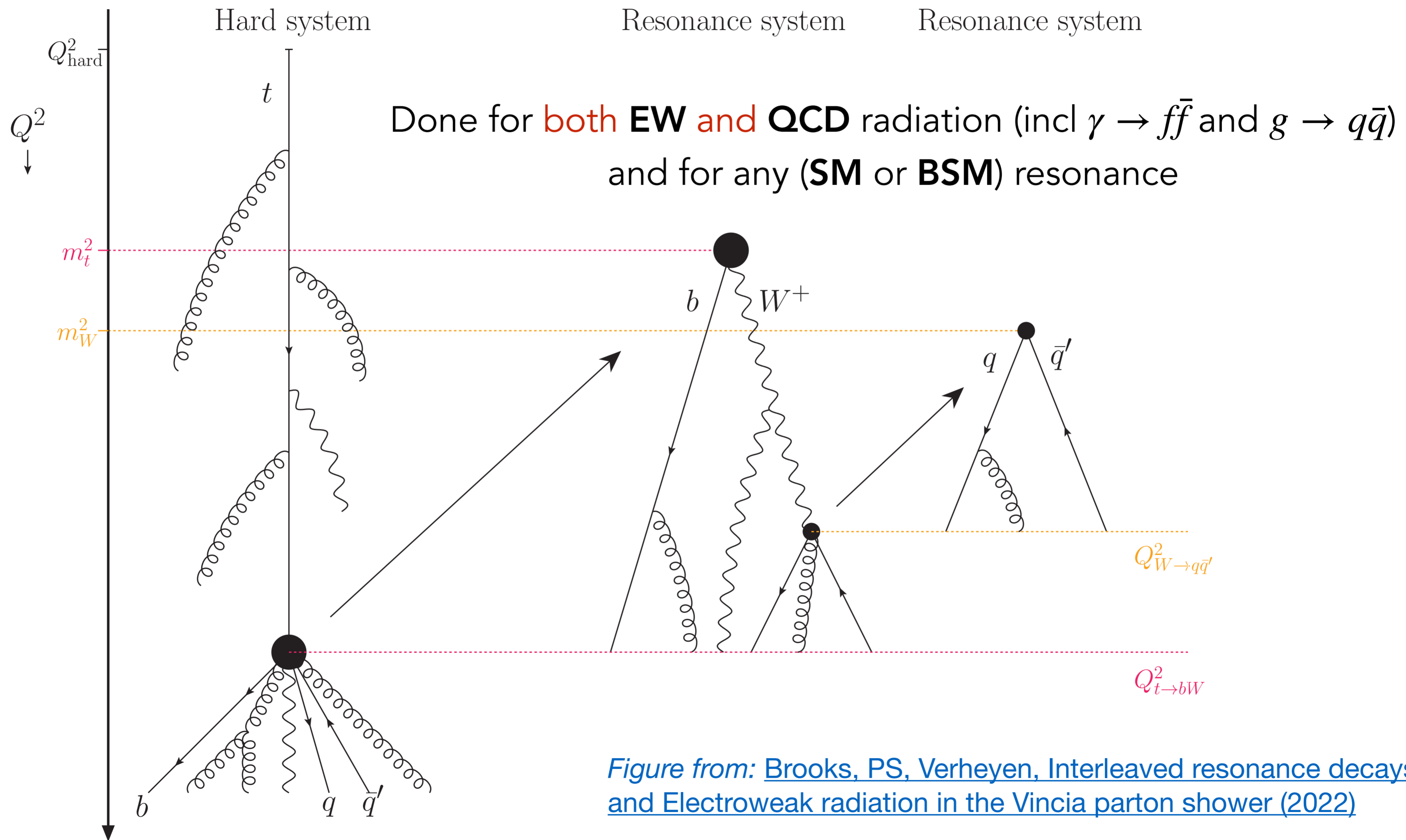


Figure from: [Brooks, PS, Verheyen, Interleaved resonance decays and Electroweak radiation in the Vincia parton shower \(2022\)](#)

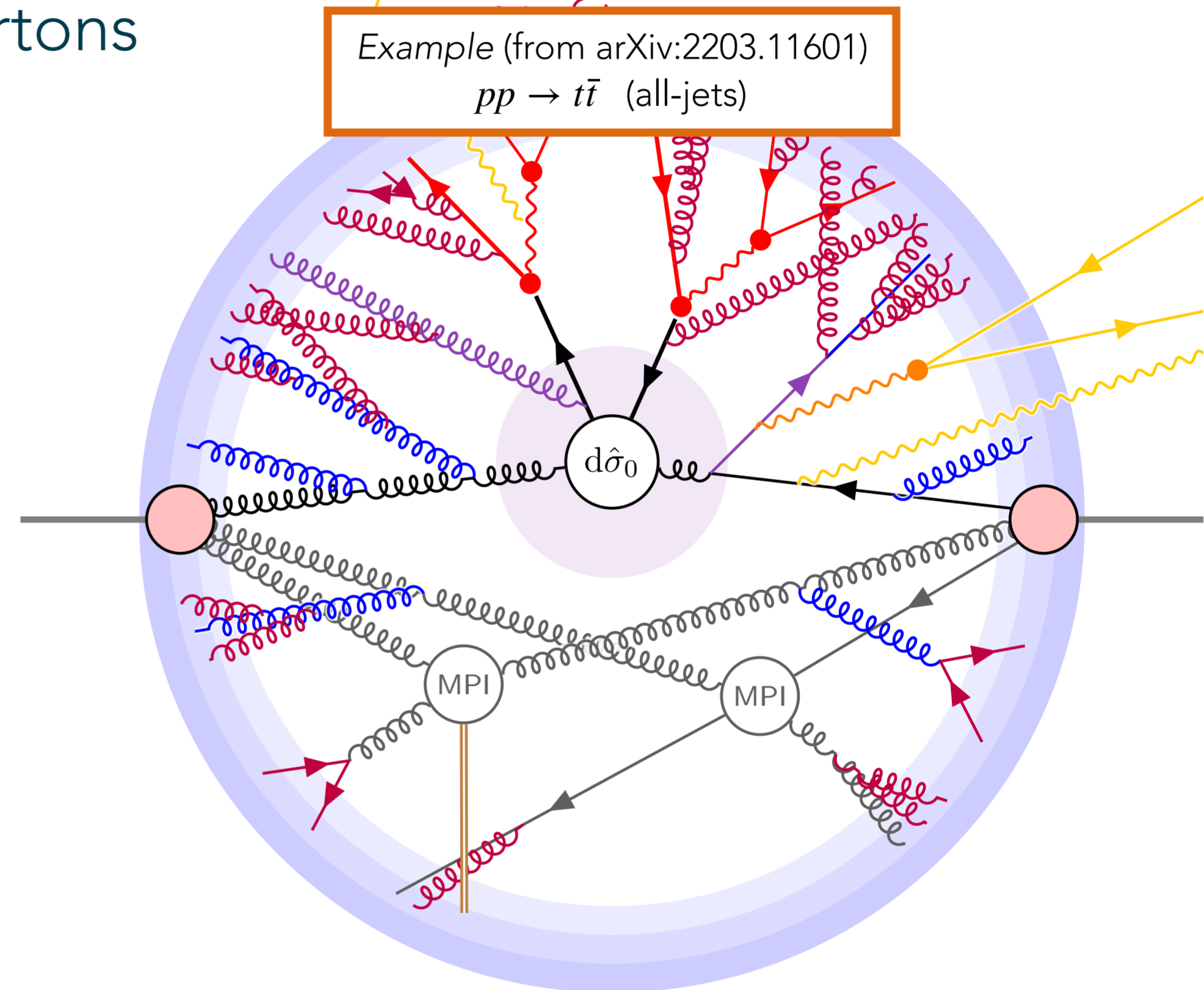
④ After the Shower

High-energy pp collisions — with ISR, Multi-Parton Interactions, and Beam Remnants

Final states with **very many** coloured partons

With significant overlaps in phase space

Who gets confined with whom?



“Parton Level”

(Event structure before confinement)

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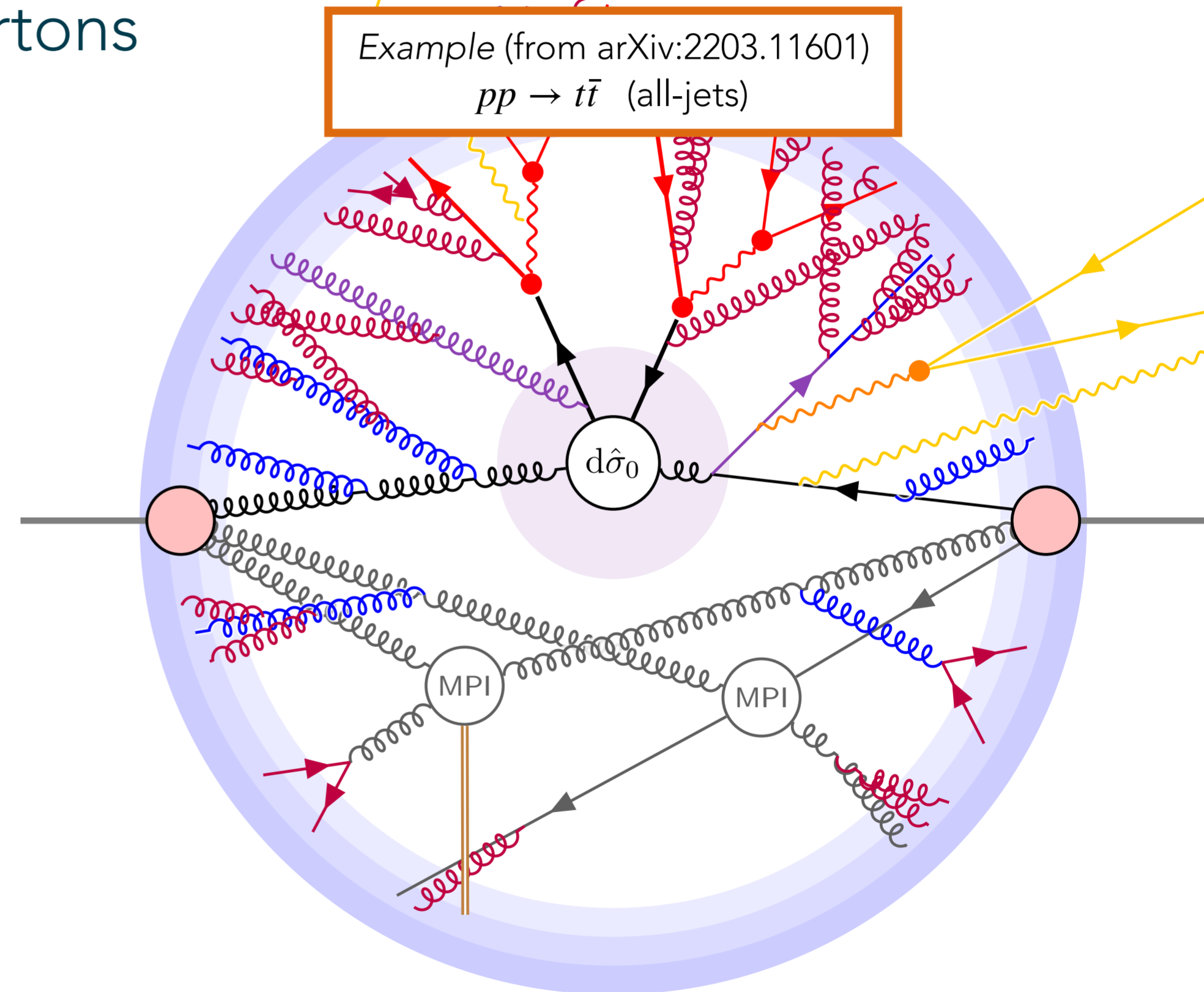
Each has a colour ambiguity $\sim 1/N_C^2 \sim 10\%$

E.g.: **random triplet** charge has 1/9 chance to be in **singlet** state with **random antitriplet**:

$$3 \otimes \bar{3} = 8 \oplus 1$$

$$3 \otimes 3 = 6 \oplus \bar{3} \quad ; \quad 3 \otimes 8 = 15 \oplus 6 \oplus 3$$

$$8 \otimes 8 = 27 \oplus 10 \oplus \bar{10} \oplus 8_S \oplus 8_A \oplus 1$$



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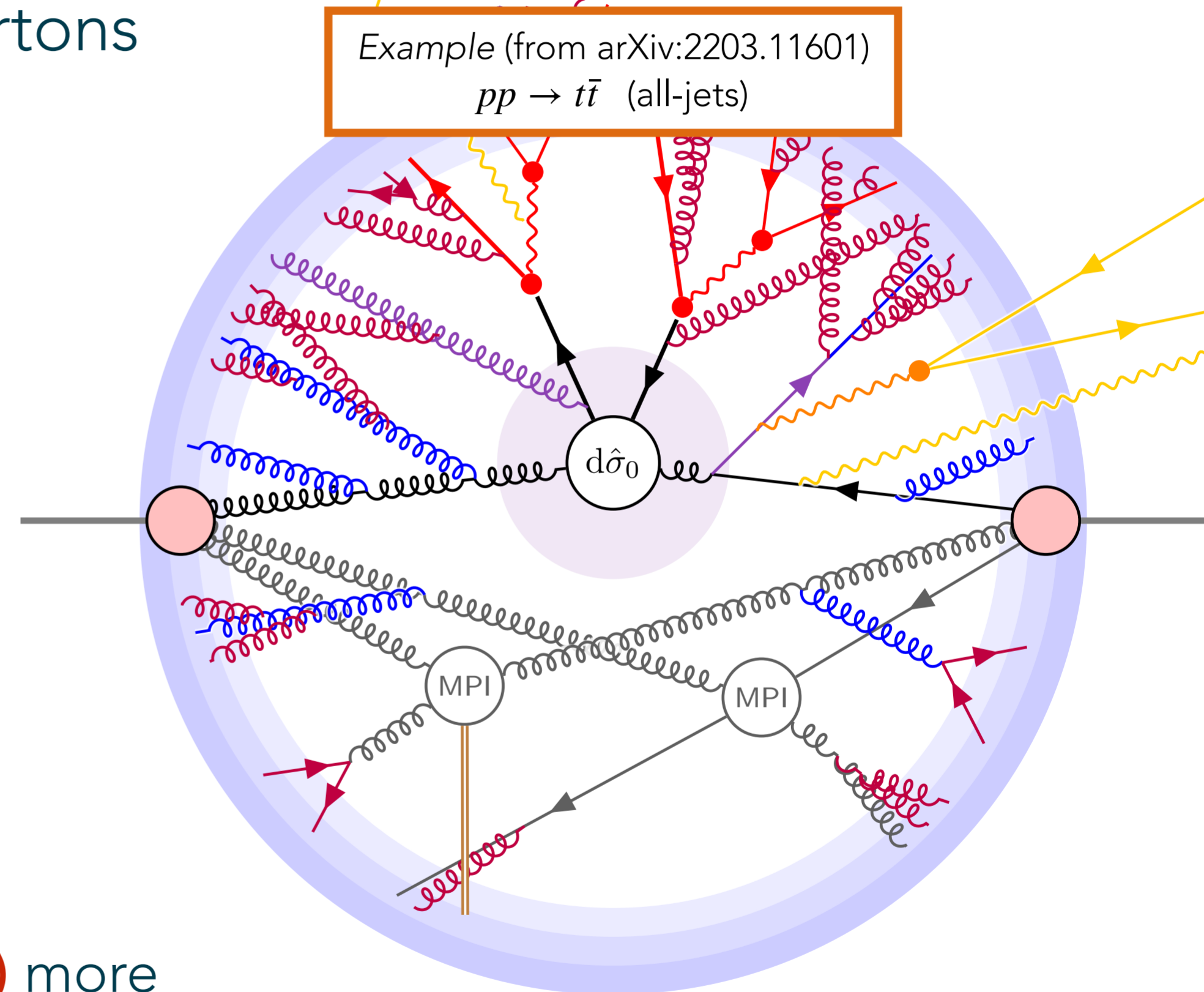
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Many charges \rightarrow Colour Reconnections* (CR) more likely than not — “Colour Promiscuity!” [J. Huston]

*) in this context, QCD CR simply refers to an ambiguity beyond Leading N_C , known to exist. Note the term “CR” can also be used more broadly to incorporate further physics concepts.

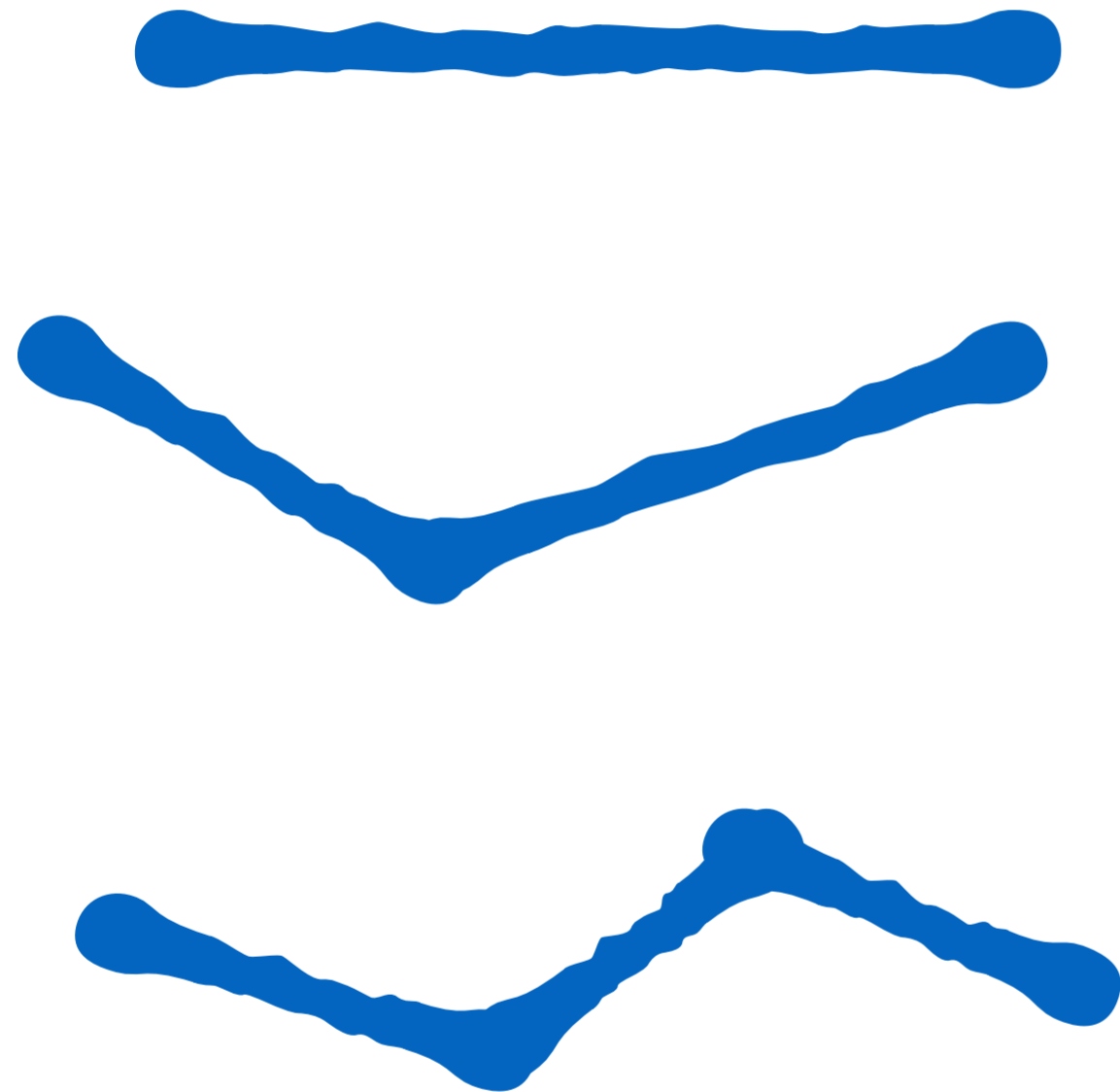


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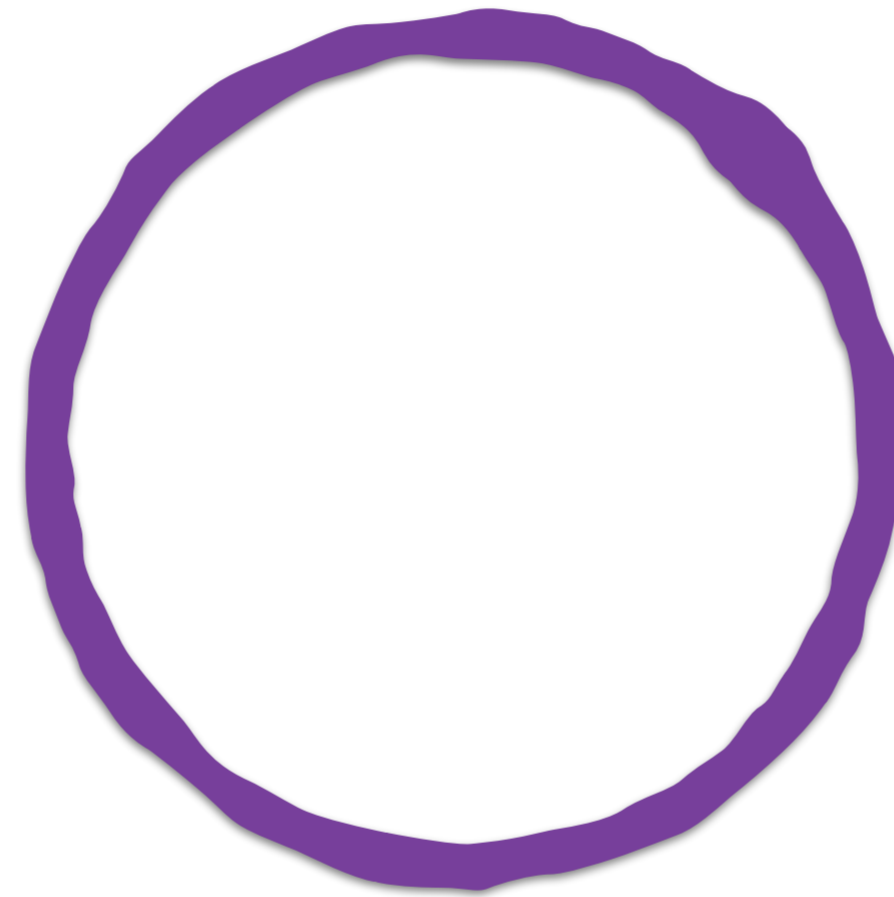
QCD Colour Reconnections \longleftrightarrow String Junctions

Open Strings



$q\bar{q}$ strings (with gluon kinks)
E.g., $Z \rightarrow q\bar{q} + \text{shower}$
 $H \rightarrow b\bar{b} + \text{shower}$

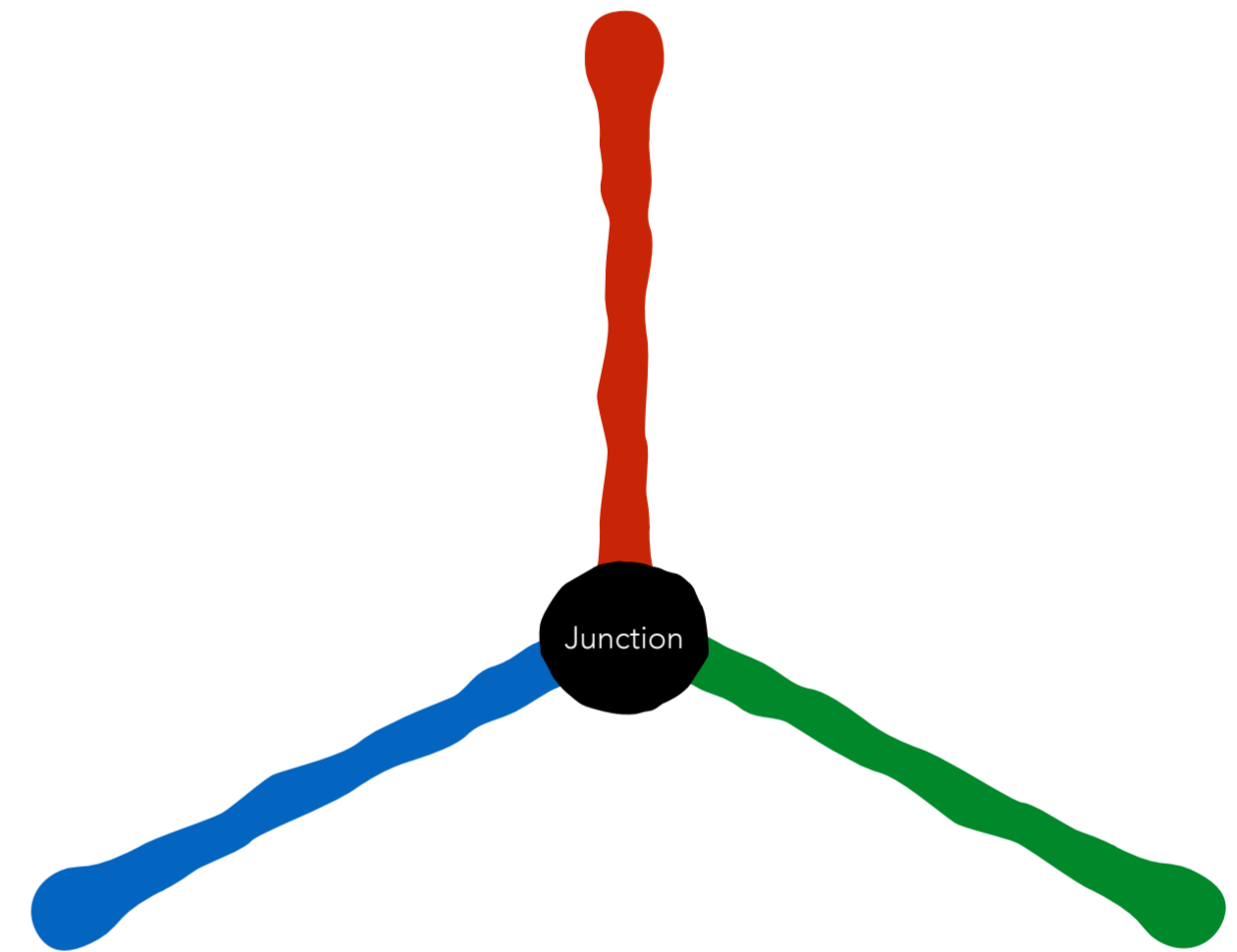
Closed Strings



Gluon rings

E.g., $H \rightarrow gg + \text{shower}$
 $\Upsilon \rightarrow ggg + \text{shower}$

SU(3) String Junction

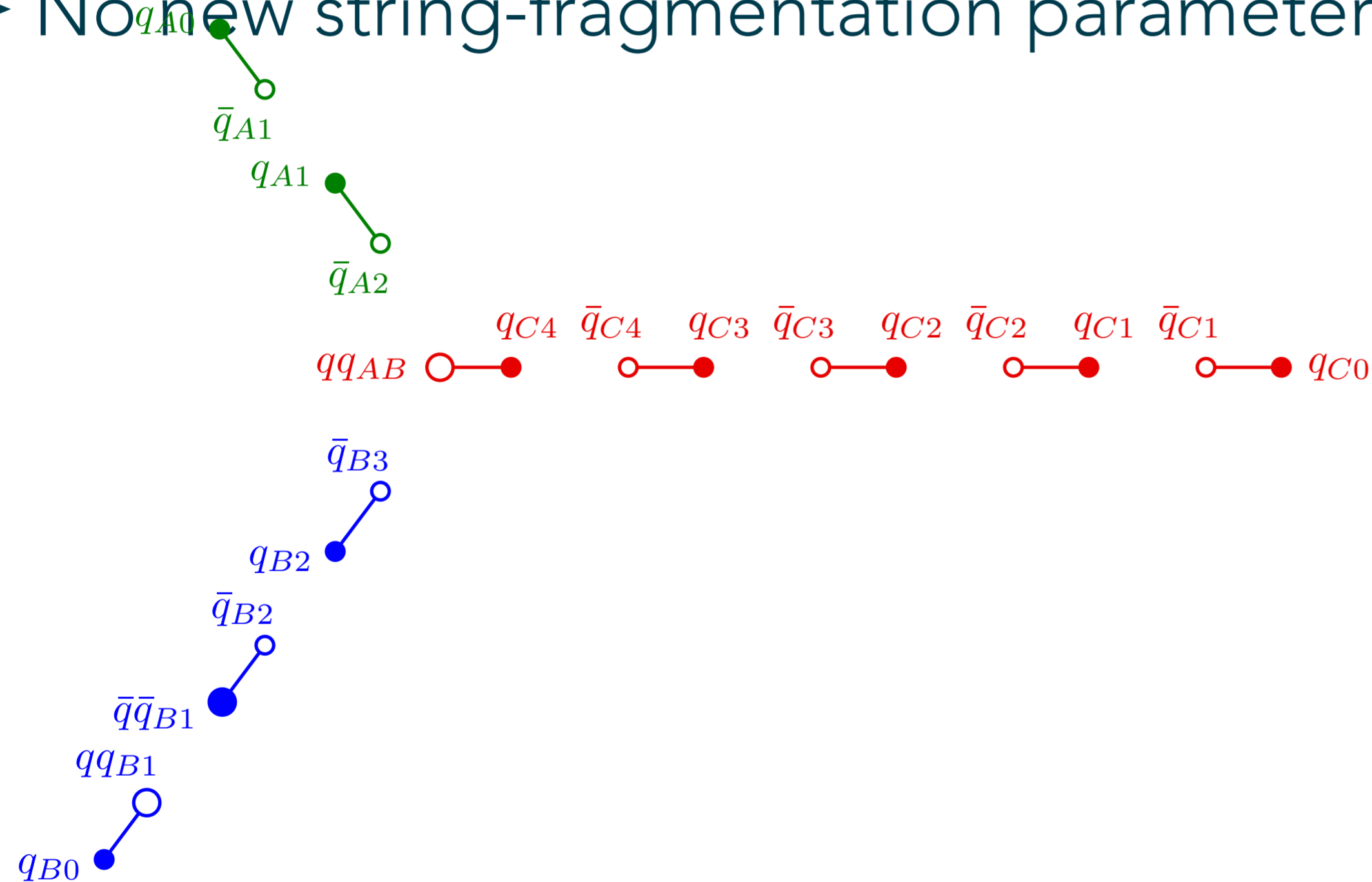


Open strings with $N_C = 3$ endpoints
E.g., Baryon-Number violating
neutralino decay $\tilde{\chi}^0 \rightarrow qqq + \text{shower}$

Fragmentation of String Junctions

Assume Junction Strings have same properties as ordinary ones
(u:d:s, Schwinger p_T , etc)

➤ No new string-fragmentation parameters

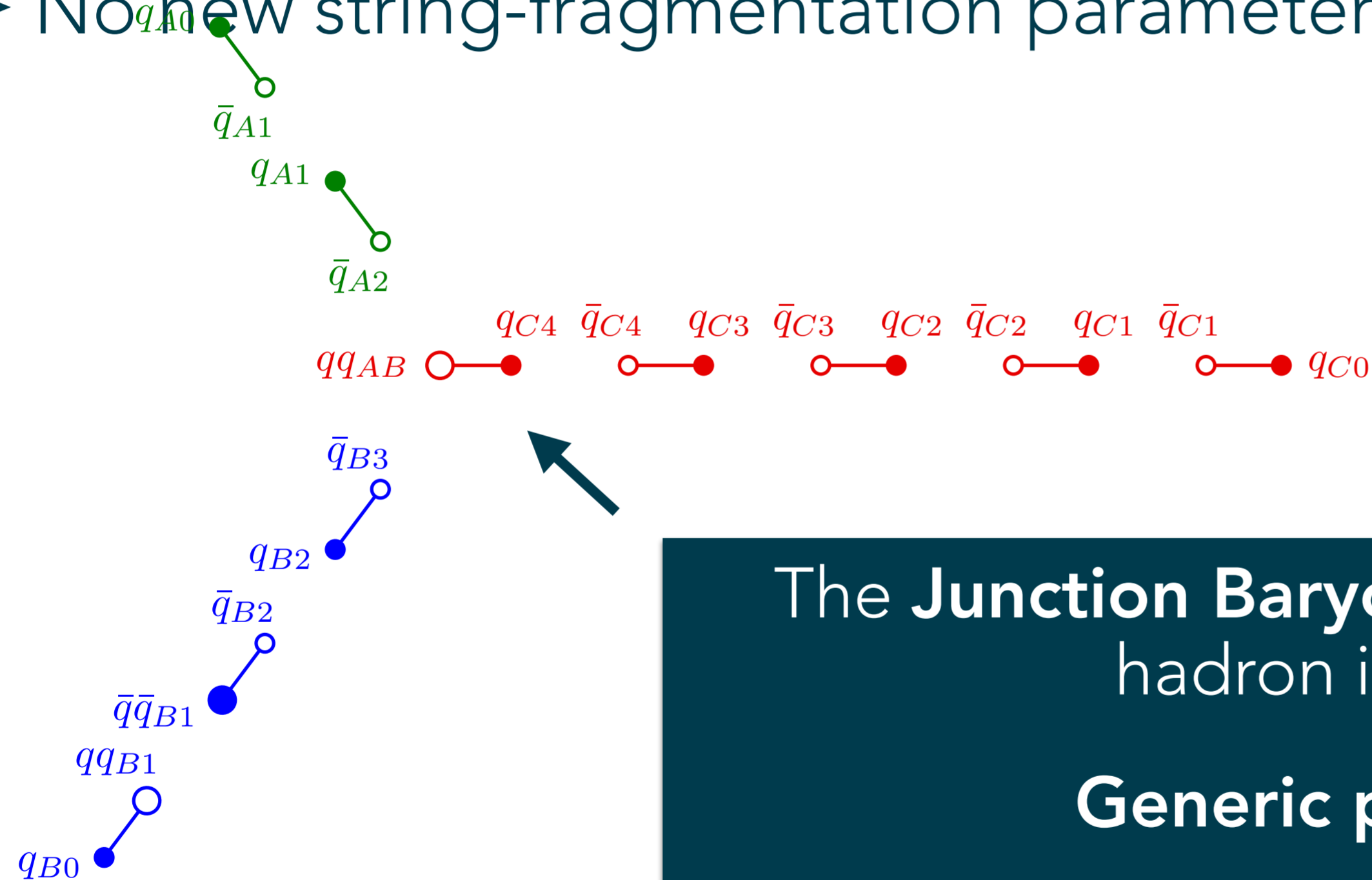


[Sjöstrand & **PS**, [NPB 659 \(2003\) 243](#)]
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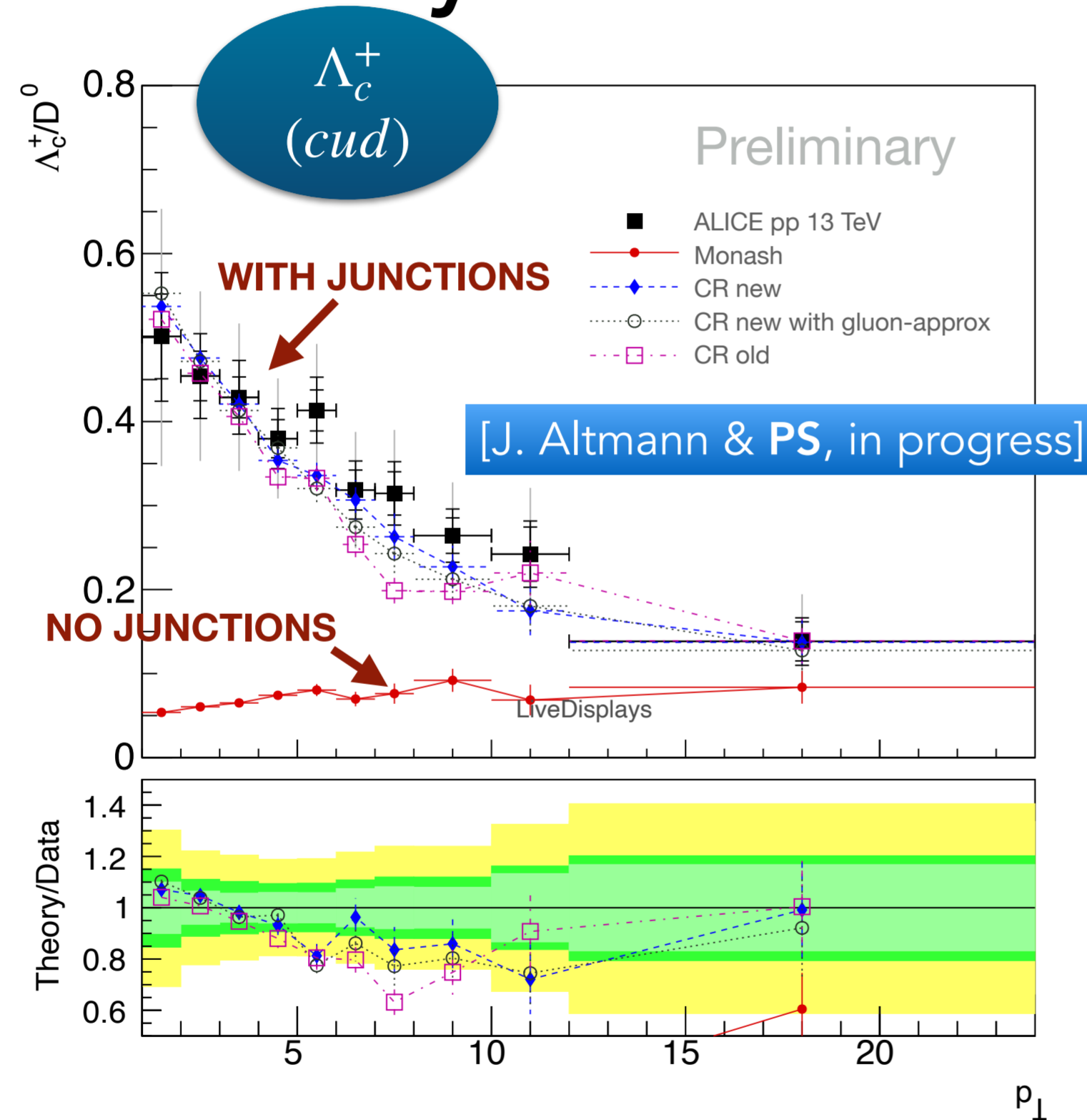
The **Junction Baryon** is the most "subleading" hadron in all three "jets".

Generic prediction: low p_T

A Smoking Gun for String Junctions: Baryon enhancements at low p_T

Confront with Measurements

LHC experiments report very large (**factor-10**) enhancements in **heavy-flavour** baryon-to-meson ratios **at low p_T** !



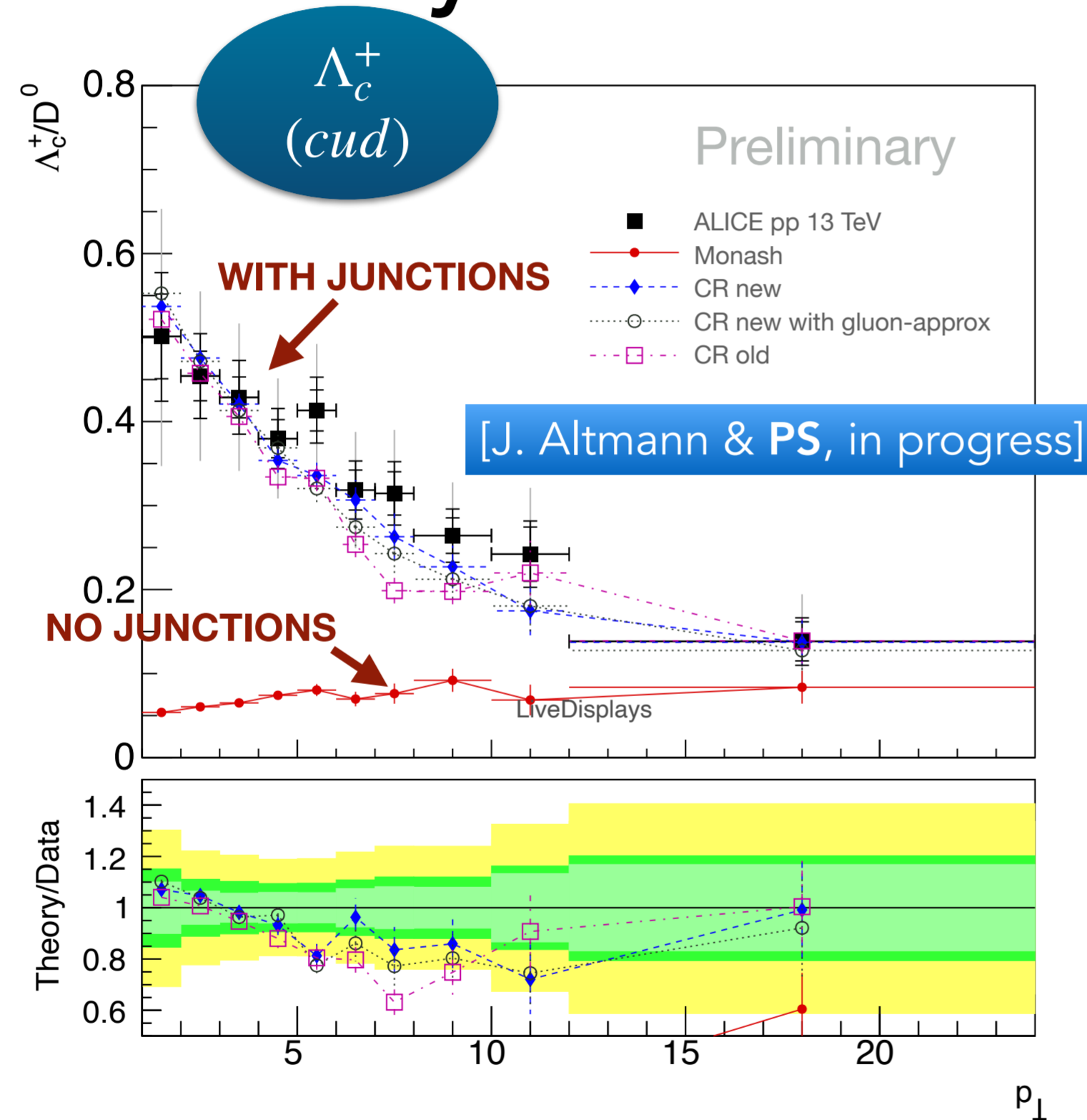
+ Lots of interesting new measurements showing changes in **strange** vs nonstrange strange **hadrons**

& evidence of **flow-like effects** in pp collisions
→ modifications to p_T spectra

Not reproduced by baseline string/cluster models

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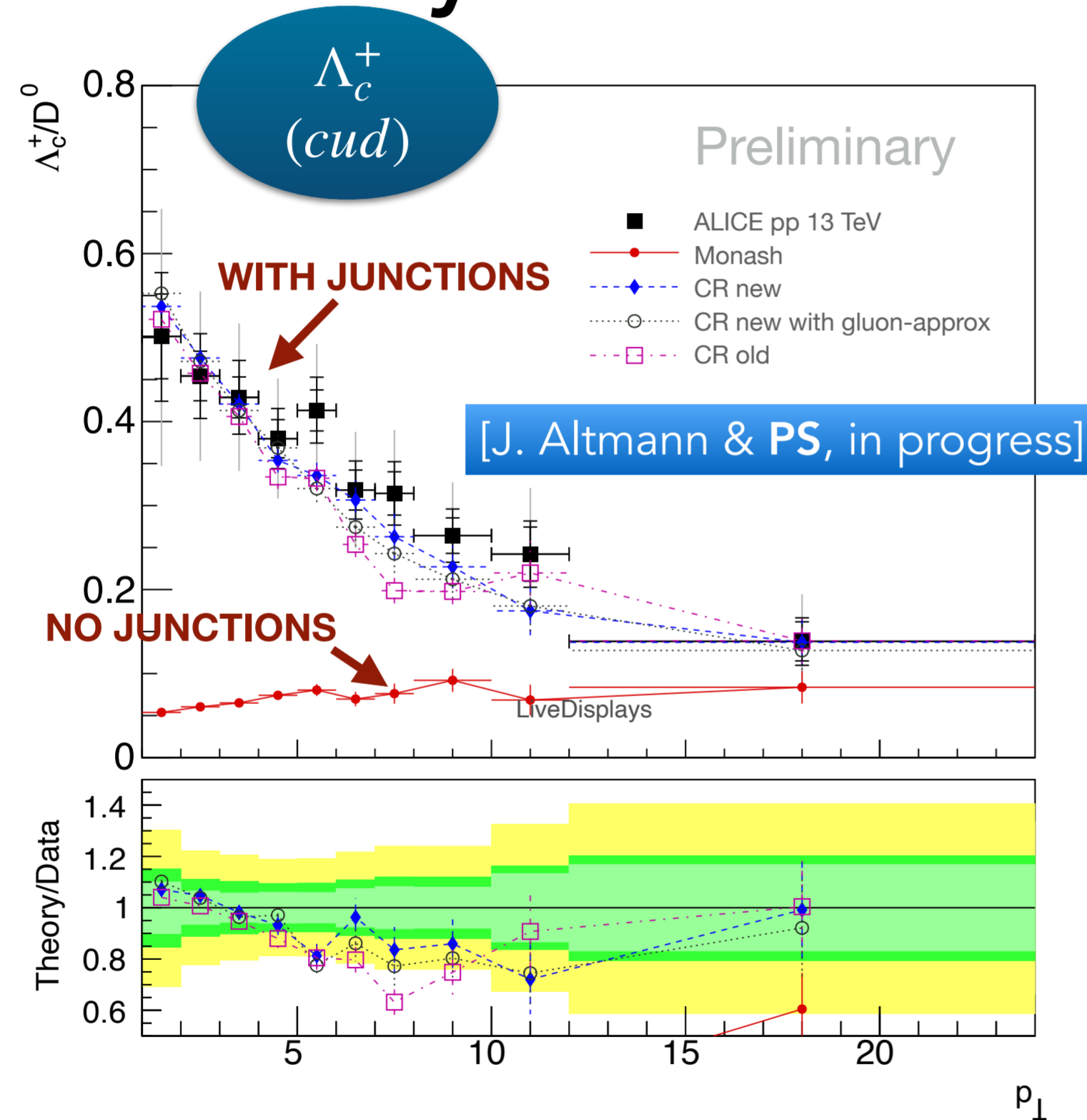
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Very exciting! Lots of Activity

Particle Composition: Impact on Jet Energy Scale



ATLAS PUB Note

ATL-PHYS-PUB-2022-021

29th April 2022



Dependence of the **Jet Energy Scale** on the **Particle Content of Hadronic Jets** in the ATLAS Detector Simulation

The dependence of the ATLAS jet energy measurement on the modelling in Monte Carlo simulations of the particle types and spectra within jets is investigated. **It is found that the hadronic jet response, i.e. the ratio of the reconstructed jet energy to the true jet energy, varies by $\sim 1-2\%$ depending on the hadronisation model used in the simulation. This effect is mainly due to differences in the average energy carried by **kaons and baryons** in the jet.** Model differences observed for jets initiated by *quarks* or *gluons* produced in the hard scattering process are dominated by the differences in these hadron energy fractions indicating that **measurements of the hadron content of jets and improved tuning of hadronization models can result in an improvement in the precision of the knowledge of the ATLAS jet energy scale.**

Variation largest for gluon jets

For $E_T = [30, 100, 200] \text{ GeV}$

Max JES variation = **$[3\%, 2\%, 1.2\%]$**

Fraction of jet E_T carried by baryons (and kaons) varies significantly

Reweighting to force similar baryon and kaon fractions

Max variation \rightarrow **$[1.2\%, 0.8\%, 0.5\%]$**

Significant potential for improved Jet Energy Scale uncertainties!

Motivates Careful Models & Careful Constraints

Interplay with advanced UE models

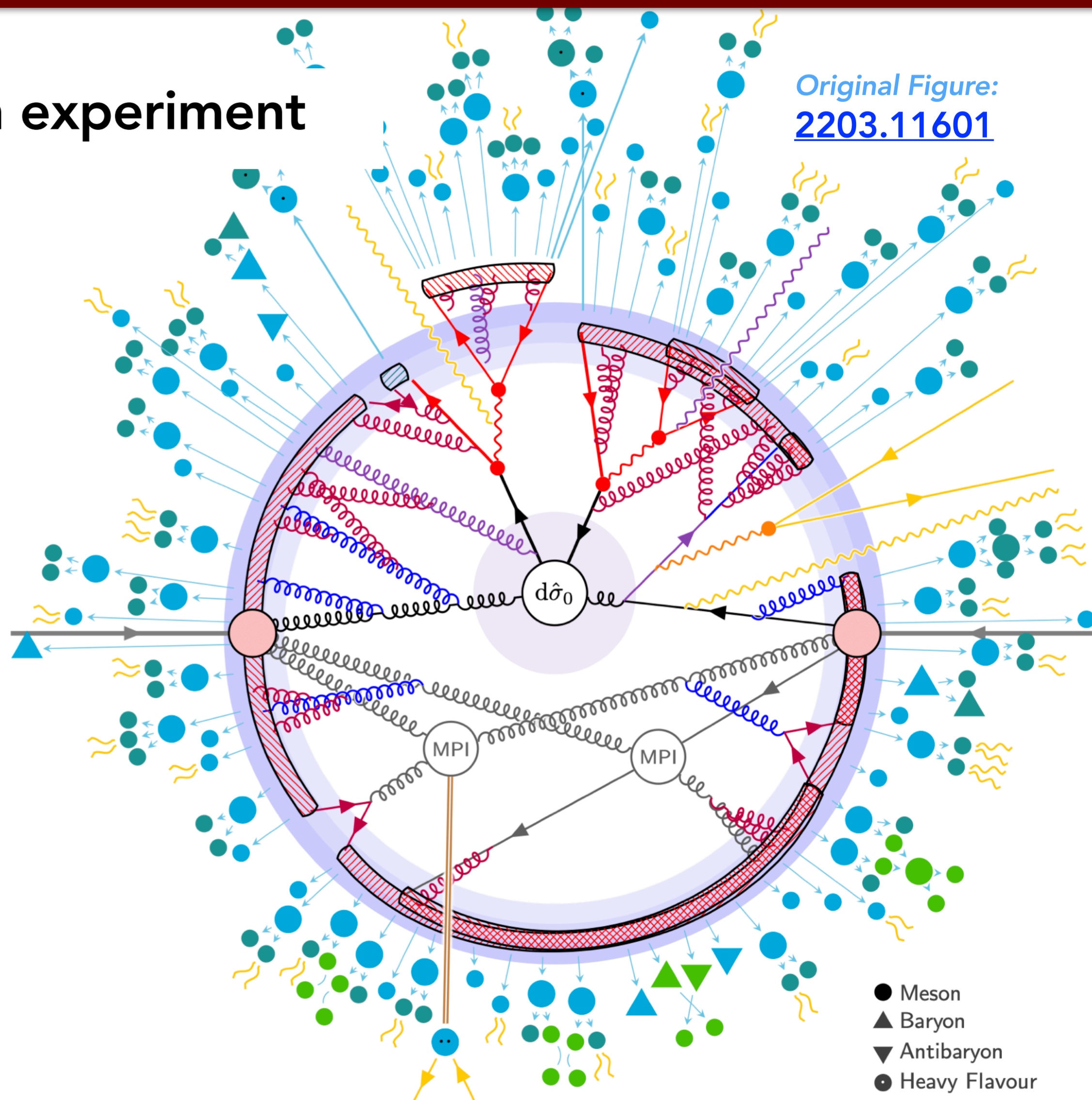
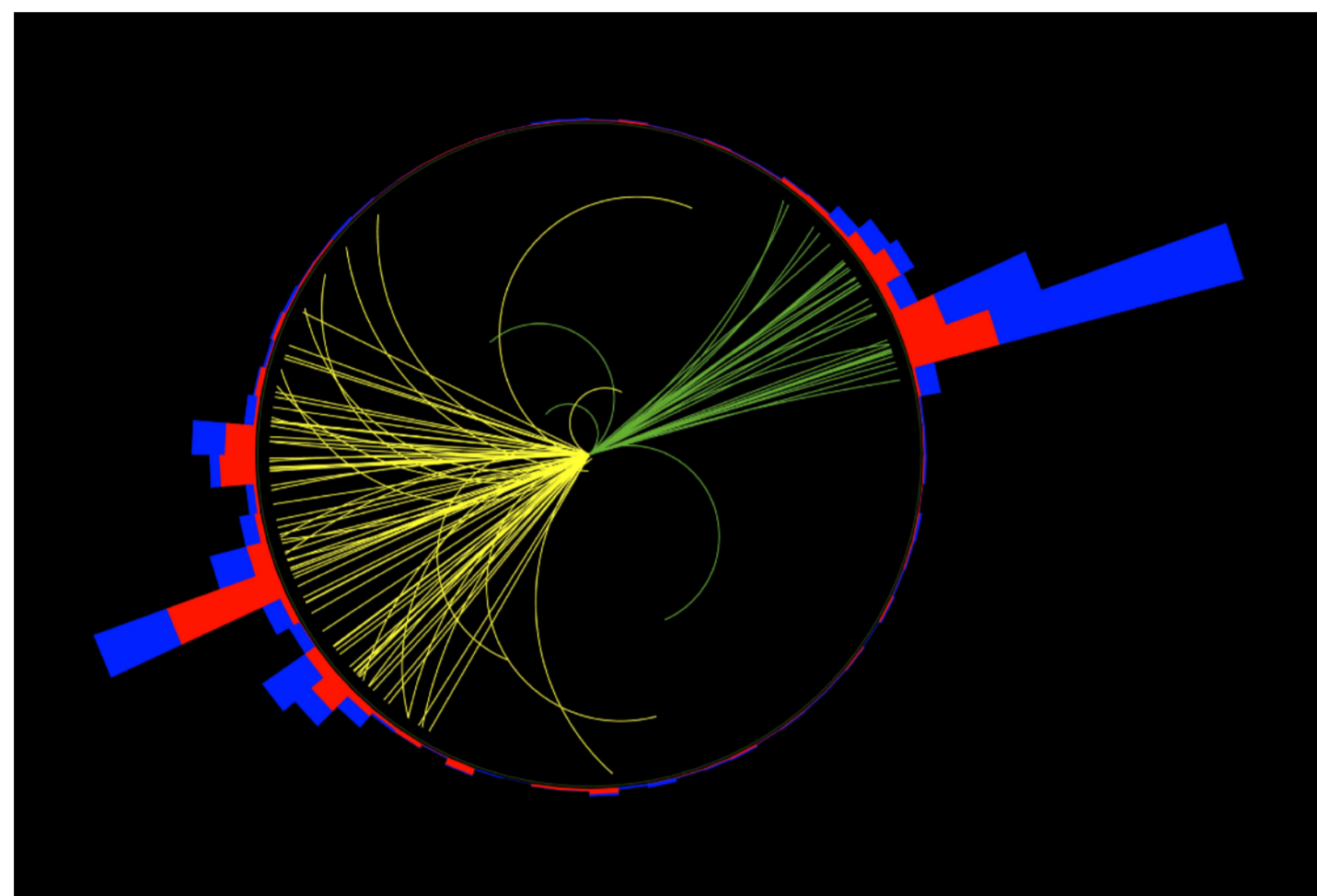
In-situ constraints from LHC data

Revisit comparisons to LEP data



Summary

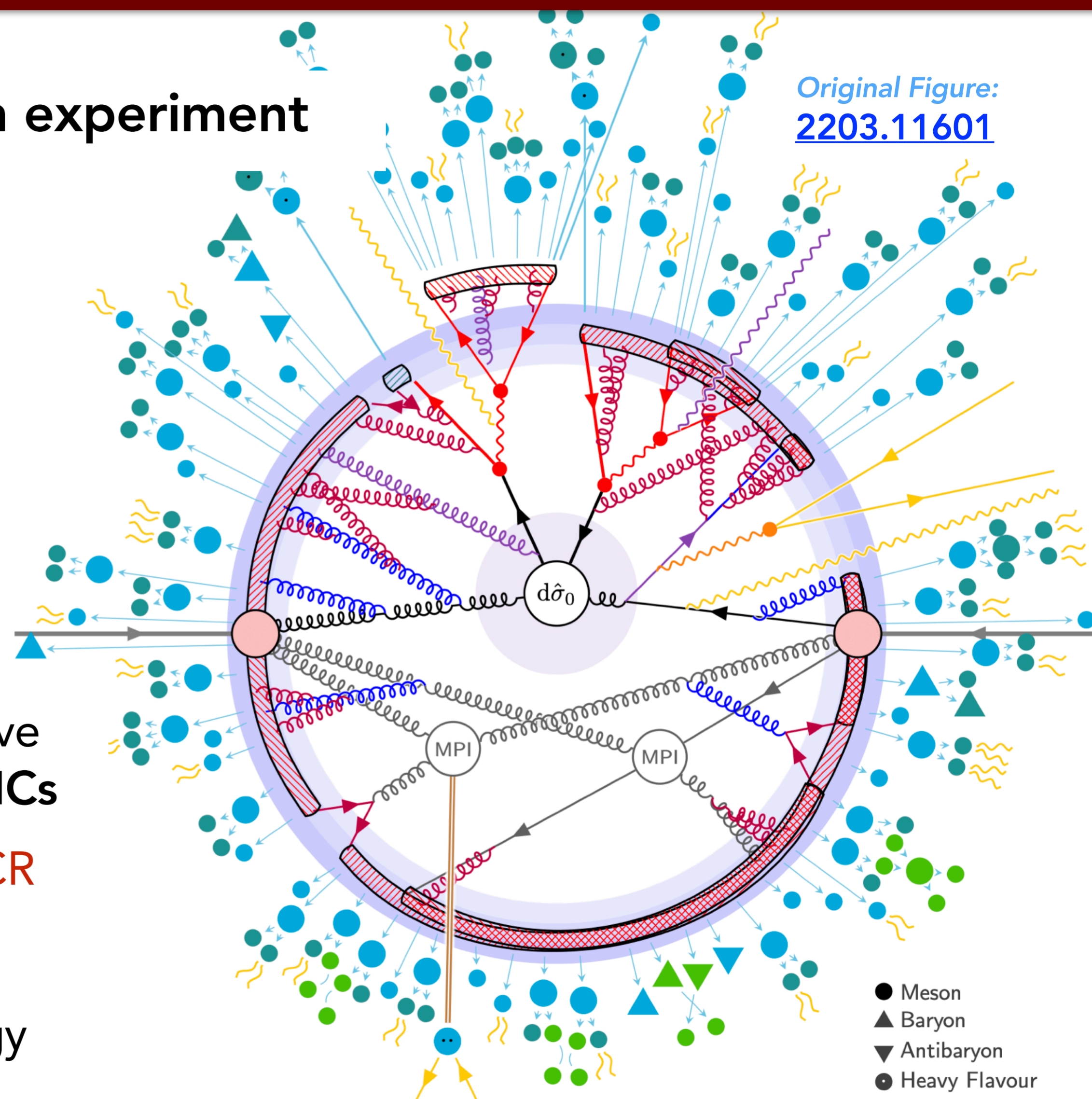
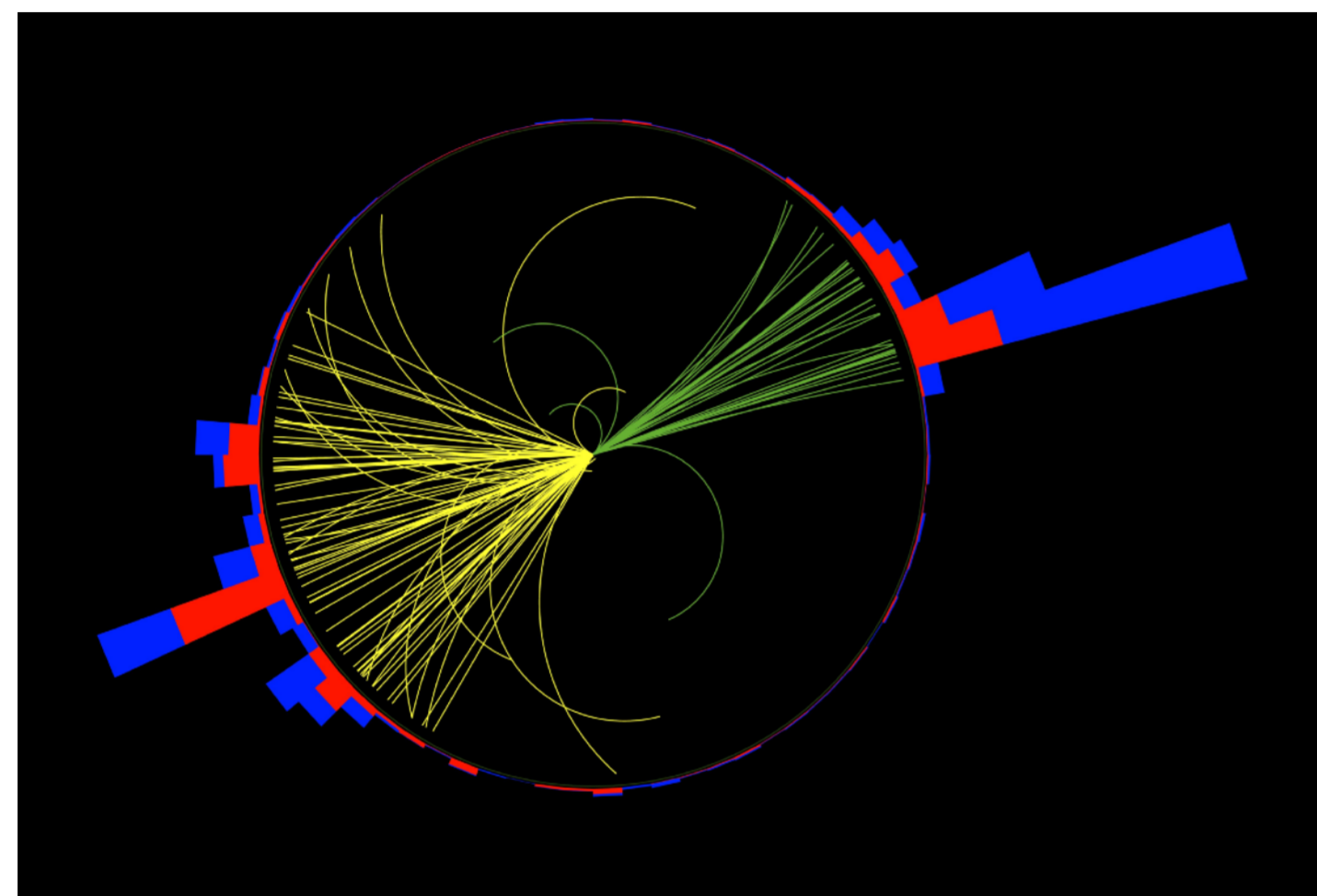
MC generators connect theory with experiment





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Entering era of percent-level perturbative accuracy, with **NNLO+N⁽ⁿ⁾LL accurate MCs**

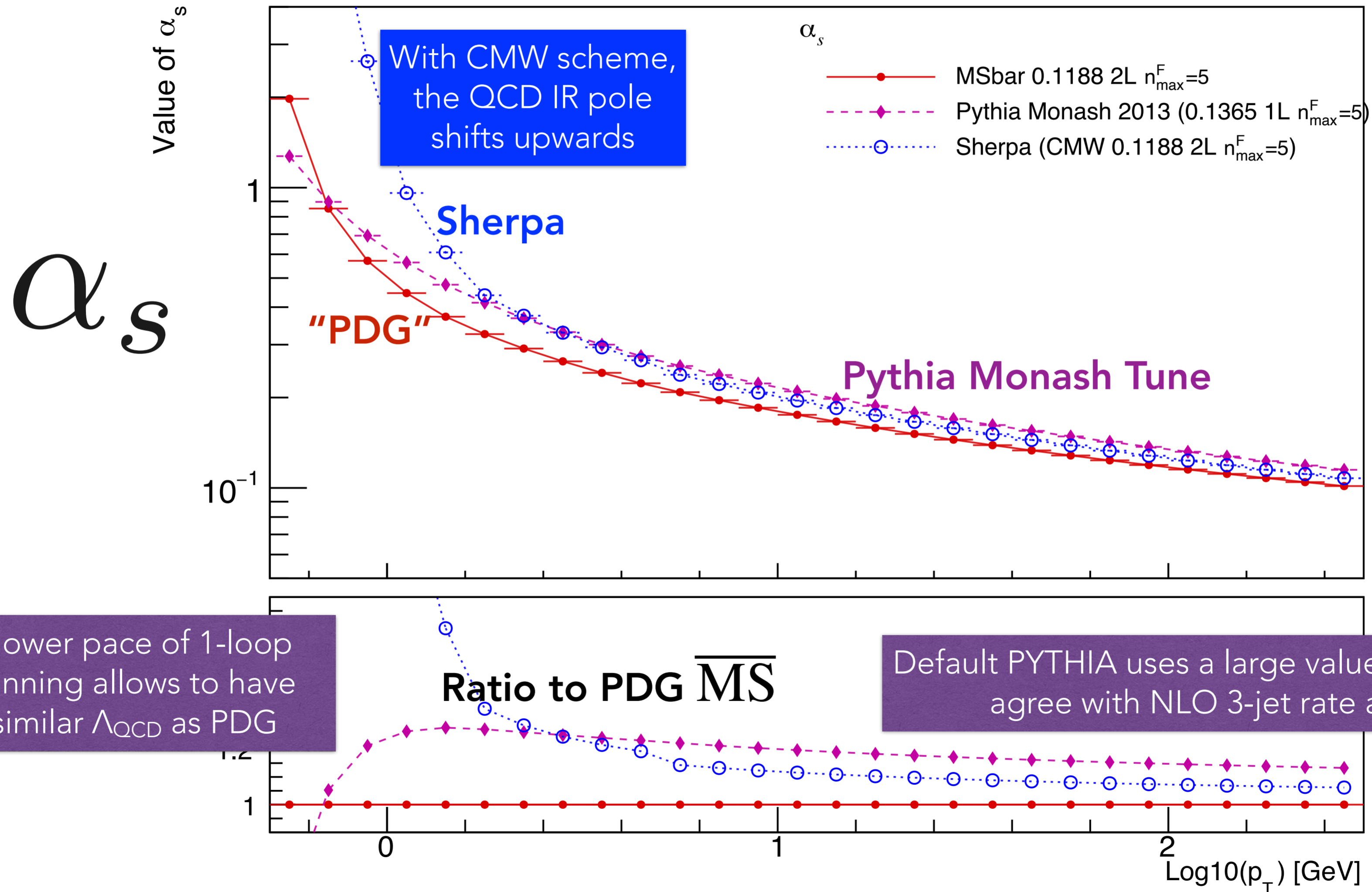
+ much **new work** on **hadronization & CR**

Driven by **LHC physics program**

But ee often used as test bed \leftrightarrow synergy

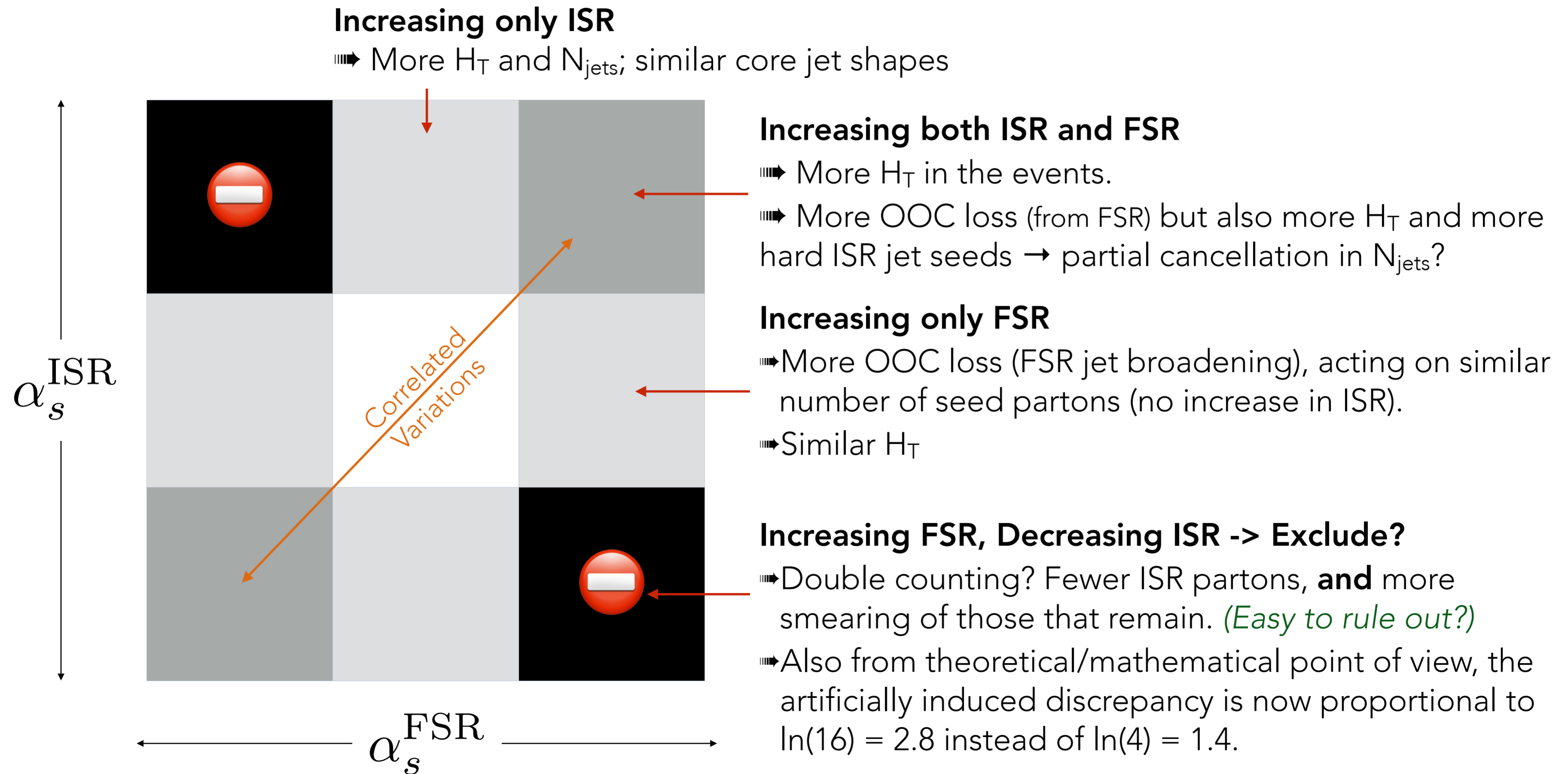
Extra Slides

Note on Different alpha(S) Choices



Correlated or Uncorrelated?

What I would do: **7-point variation** (resources permitting → use the automated bands?)

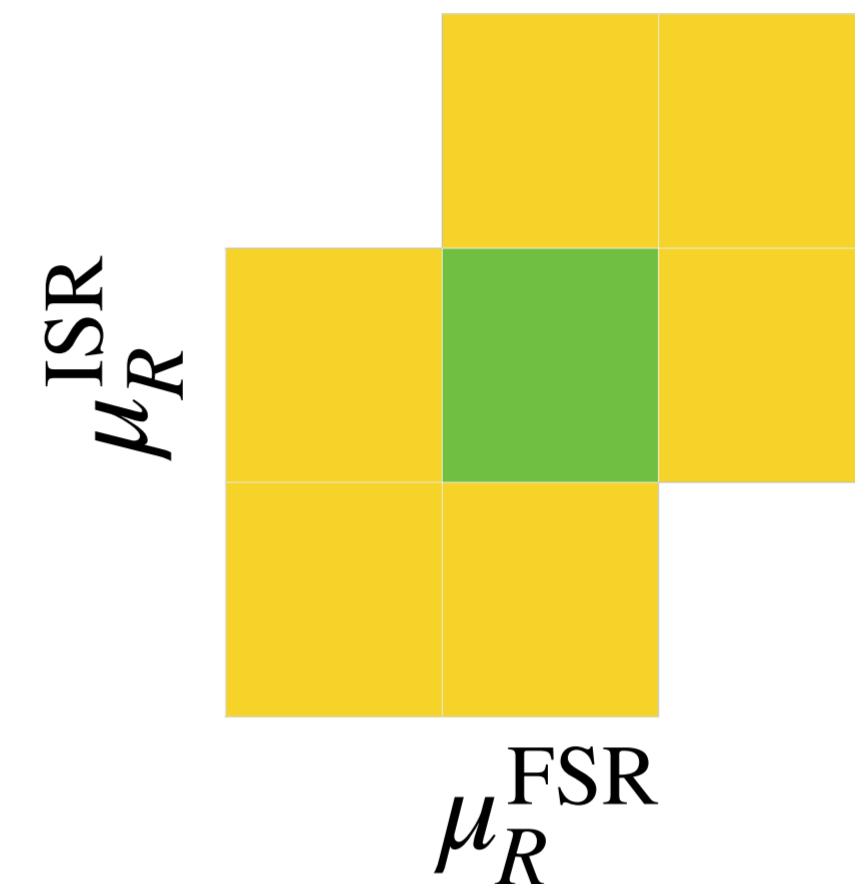


① Perturbative Uncertainties in Showers

First guess: renormalisation-scale variations,

$$\mu_R^2 \rightarrow k_\mu \mu_R^2, \text{ with constant } k_\mu \in [0.5, 2] \text{ or } [0.25, 4], \dots$$

+ e.g., do for ISR and FSR separately \rightarrow **7-point variations** \rightarrow

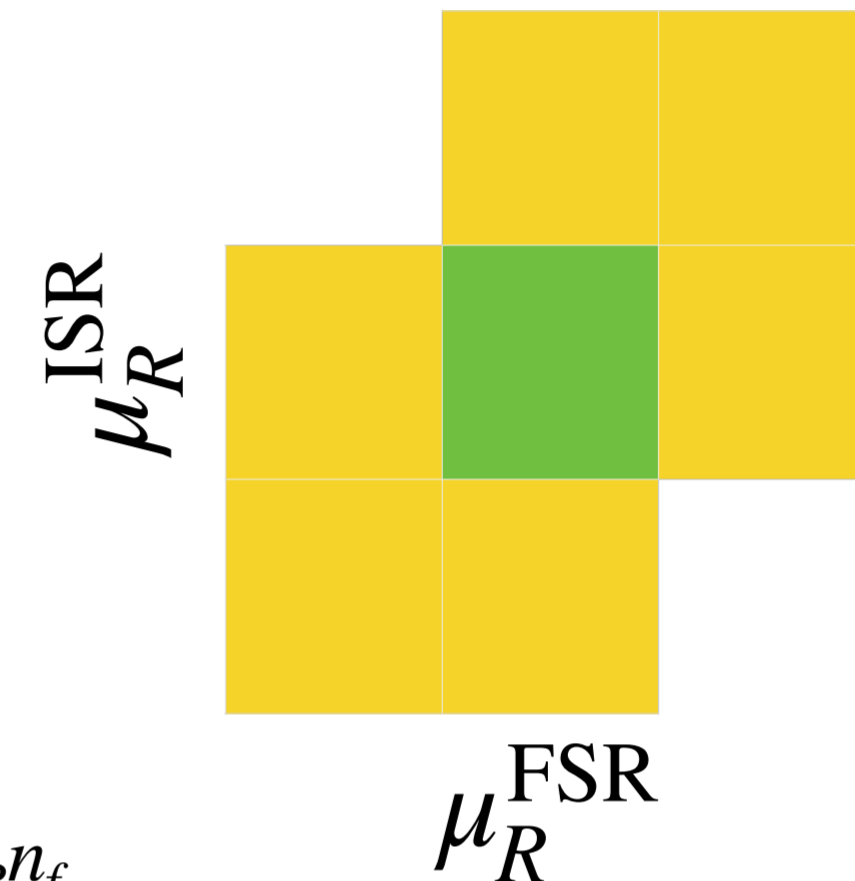


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Induces “nuisance” terms beyond calculated orders

$$\text{Running of } \alpha_s(k\mu^2) = \alpha_s(\mu^2) \frac{1}{1 + b_0 \alpha_s(\mu^2) \ln(k)} \quad \text{with } b_0 = \frac{11N_C - 4T_R n_f}{12\pi} \sim 0.6$$

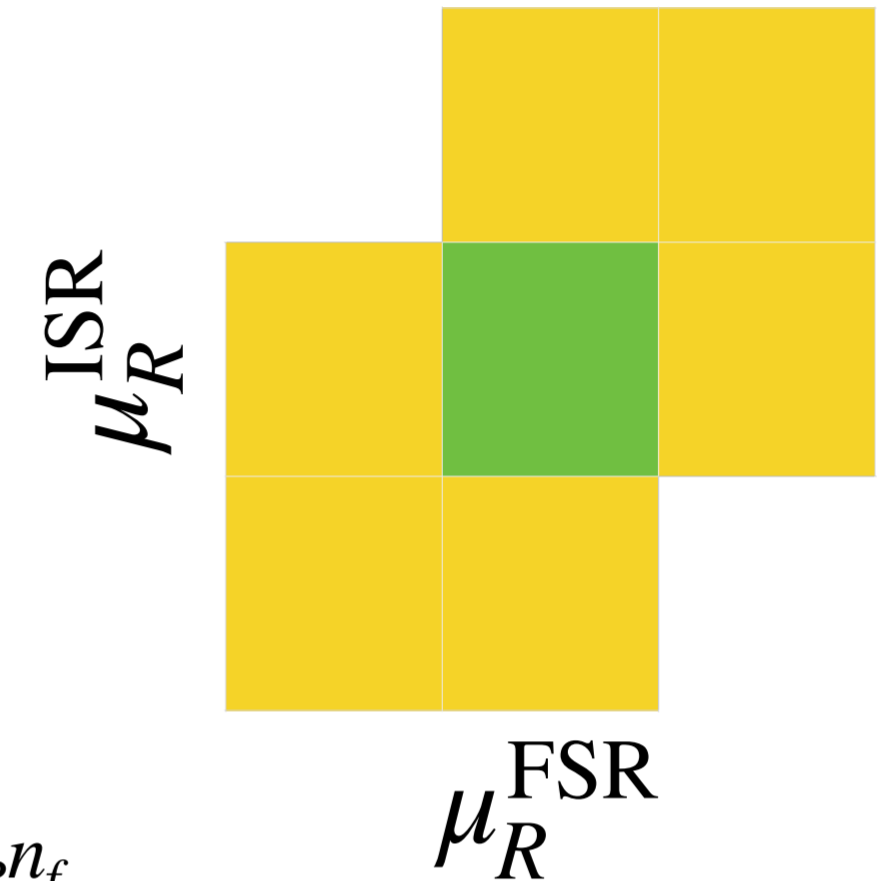
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I think many of us suspect this is unsatisfactory and unreliable

Problem: little guidance on what else to do ...

Invitation for Discussions (after talk)

Issue #1: Multiscale Problems (e.g., a couple of bosons + a couple of jets)

Not well captured by **any** variation k_μ around any **single** scale

More of an issue for hard-ME calculations than for showers (which are intrinsically multiscale)

Best single-scale approximation = **geometric mean of relevant** (nested) **QCD scales**

My recommendation: vary which scales enter this geometric mean

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Issue #2: Terms that are **not proportional to the lower orders**

Renormalization-scale variations always proportional to what you already:

$$\mu_R \text{ variations} \implies d\sigma \rightarrow (1 \pm \Delta\alpha_s) d\sigma$$

No new kinematic dependence

But full higher-order matrix elements will also contain **genuinely new terms** at each order, not proportional to previous orders:

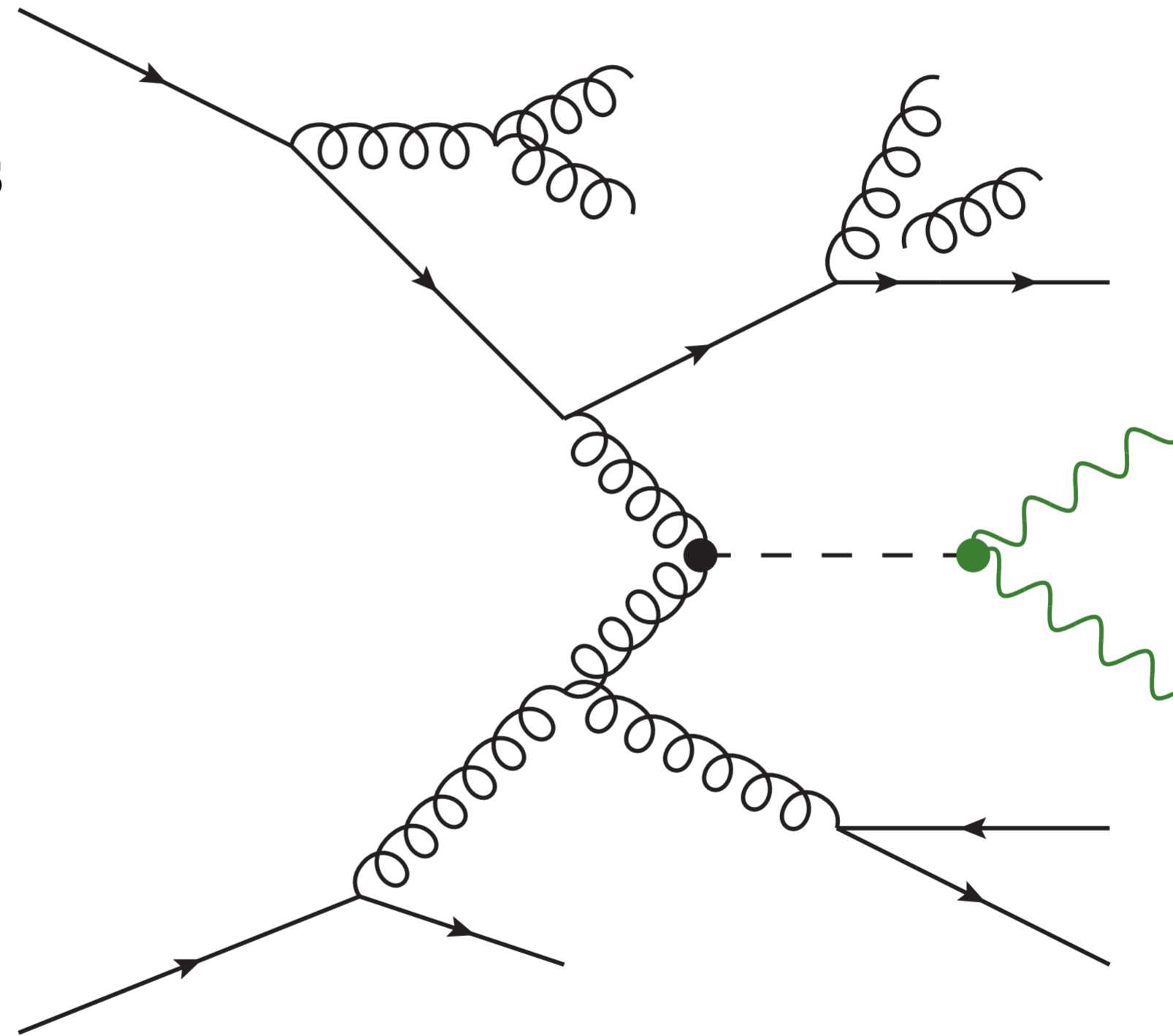
$$\text{More general} \implies d\sigma \rightarrow d\sigma \pm \Delta d\sigma$$

Parton Showers: Theory

see e.g PS, *Introduction to QCD*, TASI 2012, arXiv:1207.2389

Most bremsstrahlung is driven by **divergent propagators** → simple structure

Mathematically, **gauge amplitudes factorize** in **singular limits**

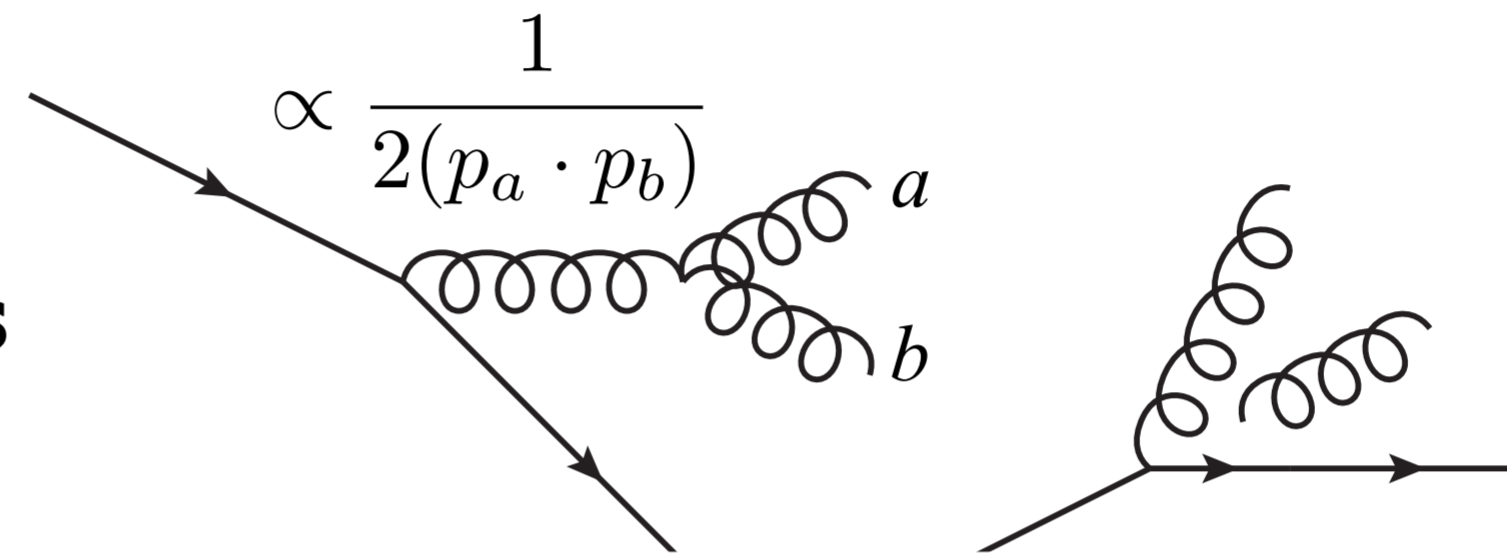


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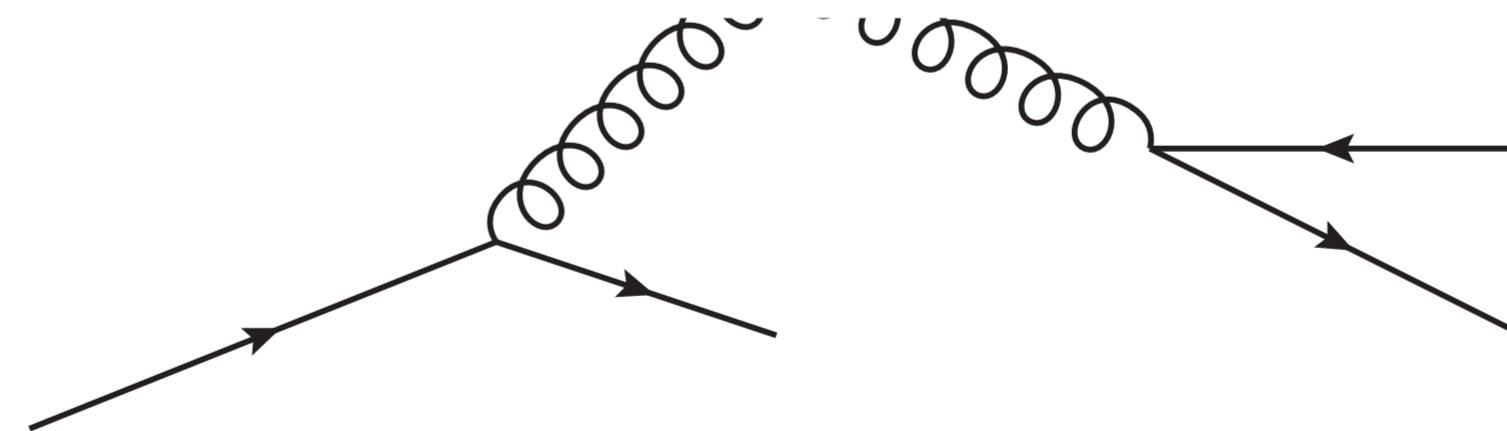
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Partons ab
→ **collinear**:

$$|\mathcal{M}_{F+1}(\dots, a, b, \dots)|^2 \xrightarrow{a||b} g_s^2 \mathcal{C} \frac{P(z)}{2(p_a \cdot p_b)} |\mathcal{M}_F(\dots, a+b, \dots)|^2$$

$P(z)$ = **DGLAP splitting kernels**", with $z = E_a/(E_a + E_b)$

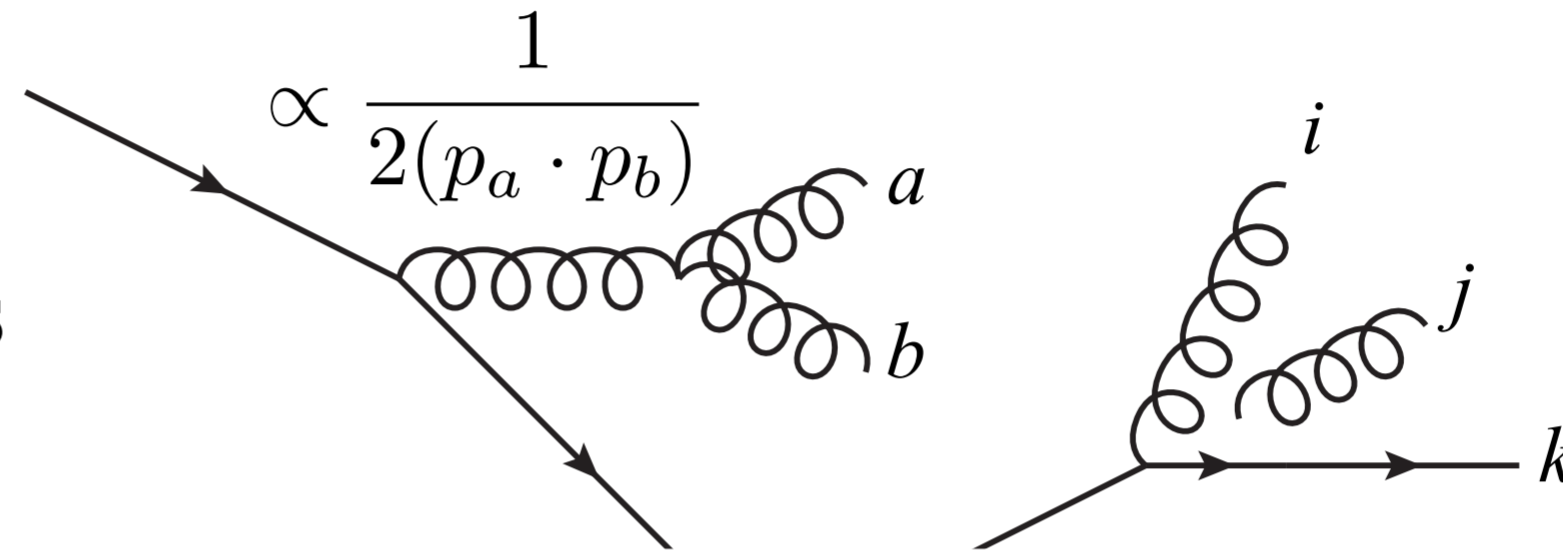


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Gluon j
→ **soft:**

$$|\mathcal{M}_{F+1}(\dots, i, j, k, \dots)|^2 \xrightarrow{j_g \rightarrow 0} g_s^2 \mathcal{C} \frac{(p_i \cdot p_k)}{(p_i \cdot p_j)(p_j \cdot p_k)} |\mathcal{M}_F(\dots, i, k, \dots)|^2$$

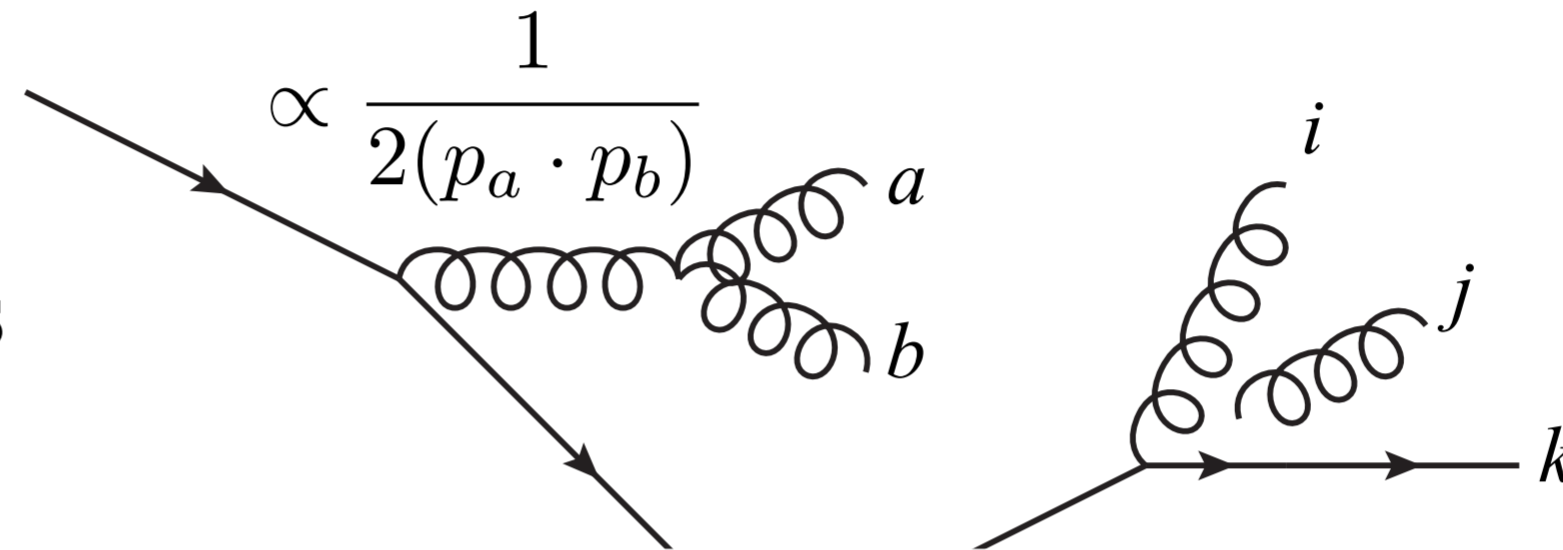
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Coherence → Parton j really emitted by (i,k) "dipole" or "**antenna**" (**eikonal factors**)

These are the **building blocks of parton showers** (DGLAP, dipole, antenna, ...) (+ running coupling, unitarity, and explicit energy-momentum conservation.)

Scale Variations: How big?

What do parton showers do?

In principle, LO shower kernels proportional to α_s

Naively: do the analogous factor-2 variations of μ_{PS} .

There are at least 3 reasons this could be **too** conservative

1. **For soft gluon emissions**, we know what the NLO term is

→ even if you do not use explicit NLO kernels, you are effectively NLO (in the soft gluon limit) **if** you are coherent and use $\mu_{PS} = (k_{CMW} p_T)$, with 2-loop running and $k_{CMW} \sim 0.65$ (somewhat n_f -dependent). *[Though there are many ways to skin that cat; see next slides.]*

Ignoring this, a **brute-force** scale variation **destroys** the NLO-level agreement.

2. Although hard to quantify, showers typically achieve better-than-LL accuracy by accounting for **further physical effects** like (E,p) conservation

3. We see empirically that (well-tuned) showers tend to stay inside the envelope spanned by factor-2 variations in **comparison to data**

(Illustration of the "Magic Trick")

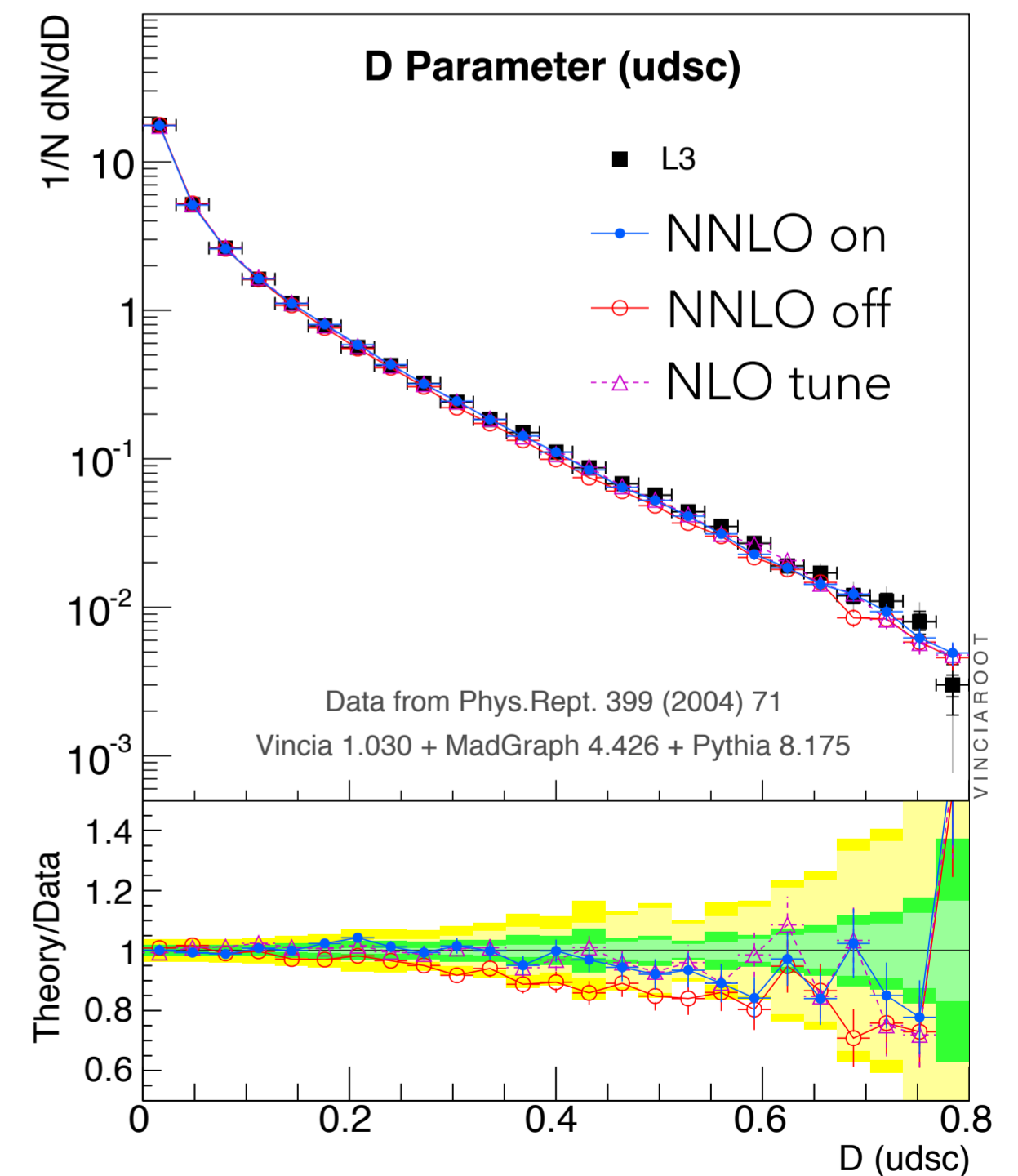
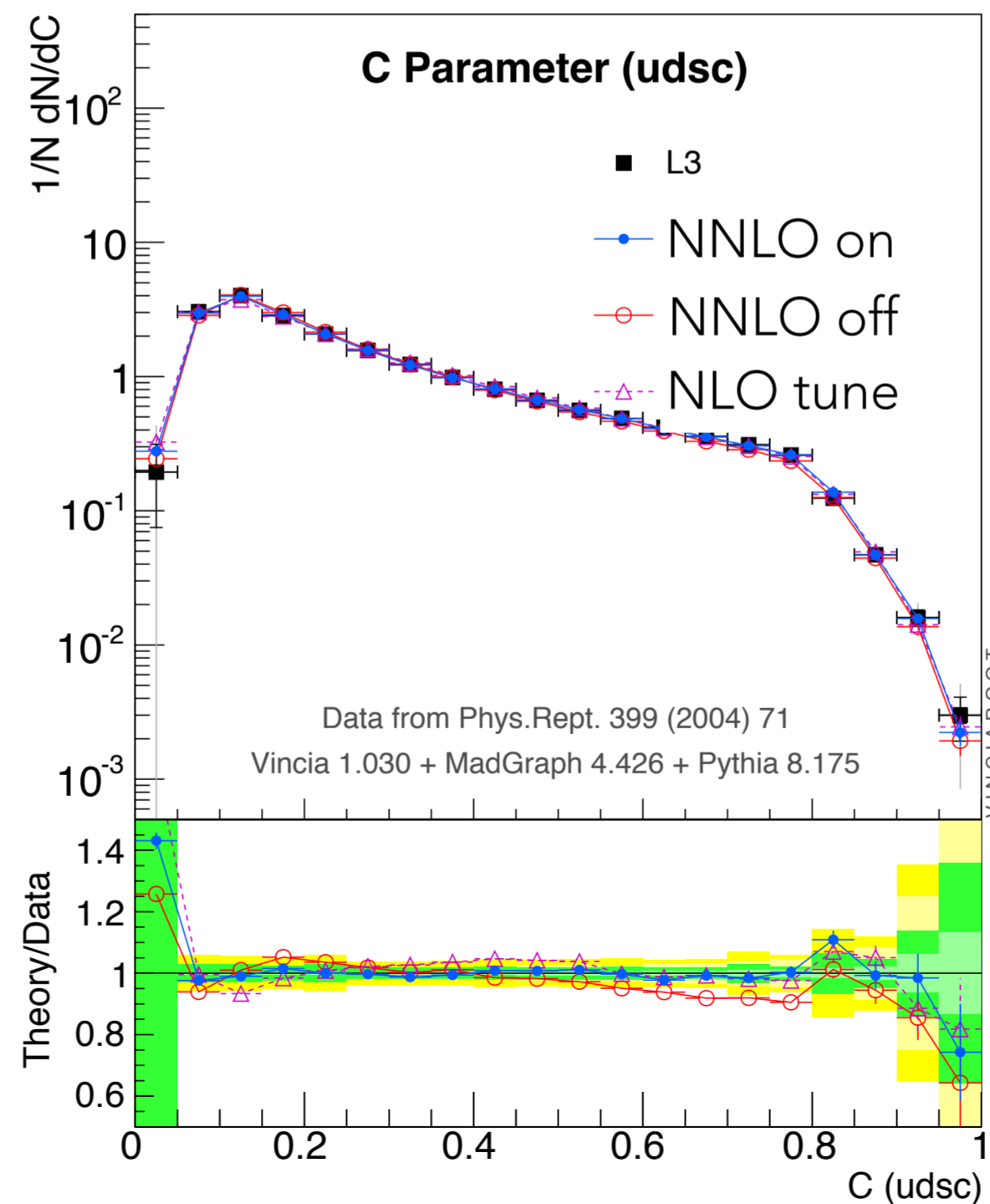
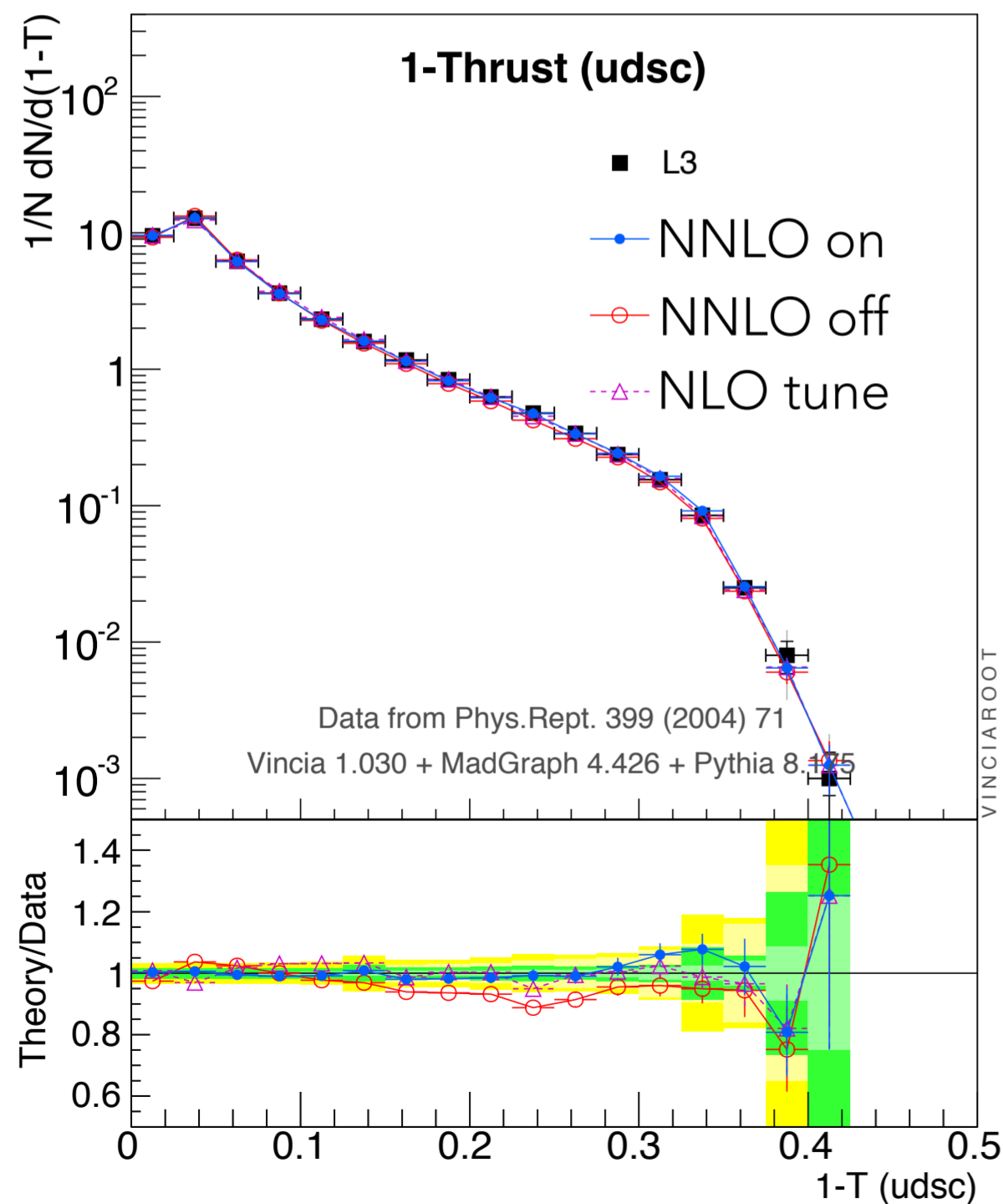
Hartgring, Laenen, **PS**, [arXiv:1303.4974](https://arxiv.org/abs/1303.4974)

Proof-of-Concept NNLO LEP tune (NNLO Z Decay, ie with NLO 3-jet corrections — using VINCIA)

NNLO tune (3-jet NLO) with $\alpha_s(M_Z) = 0.122$ (2-loop running, CMW)

NLO tune ~ Monash (3-jet LO) with $\alpha_s(M_Z) = 0.139$ (1-loop running, MSbar)

Comparable values for Λ_{QCD}



Scale variations: How Big?

Poor man's recipe: Use $\sqrt{2}$ instead?

Sure ... but still somewhat arbitrary

Instead: add compensation term to preserve soft-gluon limit at $O(\alpha_s^2)$

Still allowing full factor-2 outside that limit.

Pythia includes such a compensation term, at least in context of automated uncertainty bands

Since aggressive definitions can lead to overcompensation / **extremely** optimistic predictions → very small uncertainty bands, we chose a rather conservative definition for PYTHIA → larger bands.

$$P'(t, z) = \frac{\alpha_s(kp_\perp)}{2\pi} \left(1 + (1 - \zeta) \frac{\alpha_s(\mu_{\max})}{2\pi} \beta_0 \ln k \right) \frac{P(z)}{t}$$

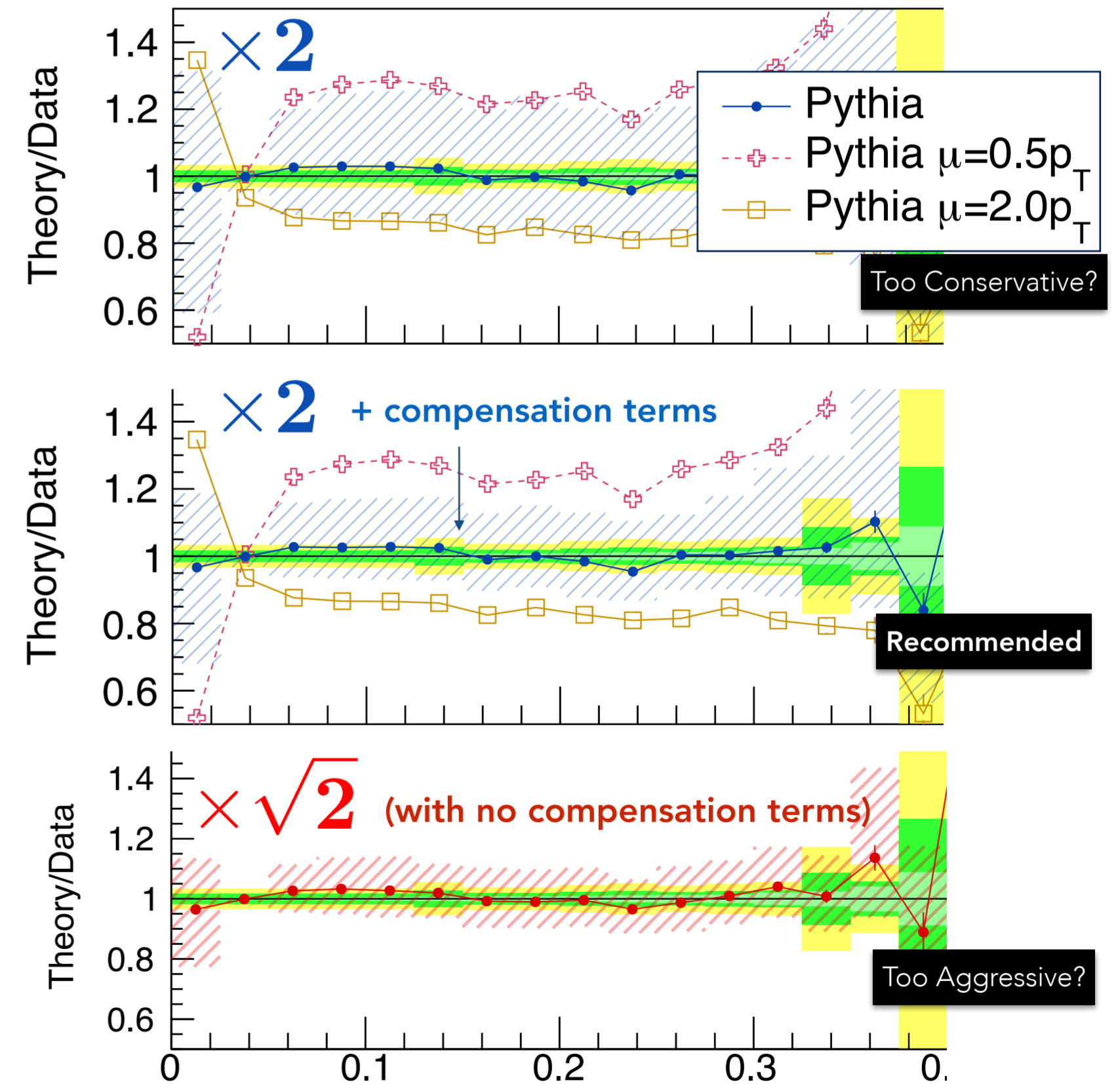
Kills the compensation outside the soft limit

Small absolute size of compensation

$$\zeta = \begin{cases} z & \text{for splittings with a } 1/z \text{ singularity} \\ 1 - z & \text{for splittings with a } 1/(1 - z) \text{ singularity} \\ \min(z, 1 - z) & \text{for splittings with a } 1/(z(1 - z)) \text{ singularity} \end{cases}$$

ee → hadrons 91.2 GeV

1-Thrust (udsc)



S. Mrenna & PS: PRD94(2016)074005; arXiv:1605.08352

Matrix-Element Merging — The Complexity Bottleneck

For CKKW-L style merging: (incl UMEPS, NL3, UNLOPS, ...)

Need to take **all contributing shower histories into account.**

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Each phase-space point receives contributions from many possible branching “histories” (aka “clusterings”)

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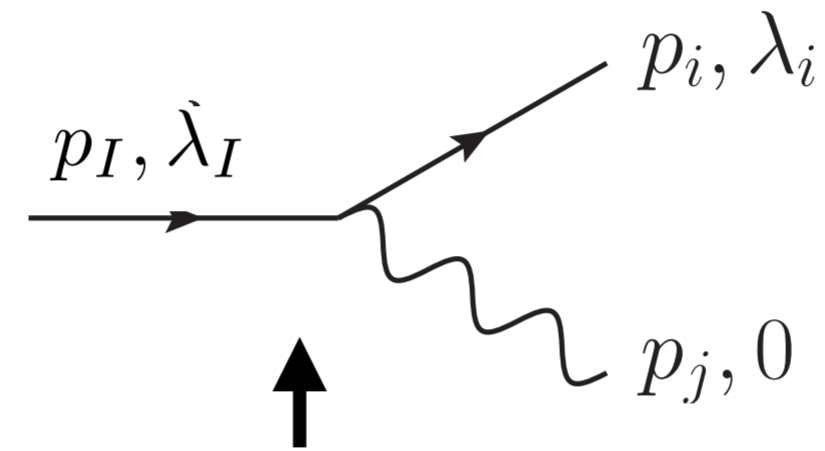
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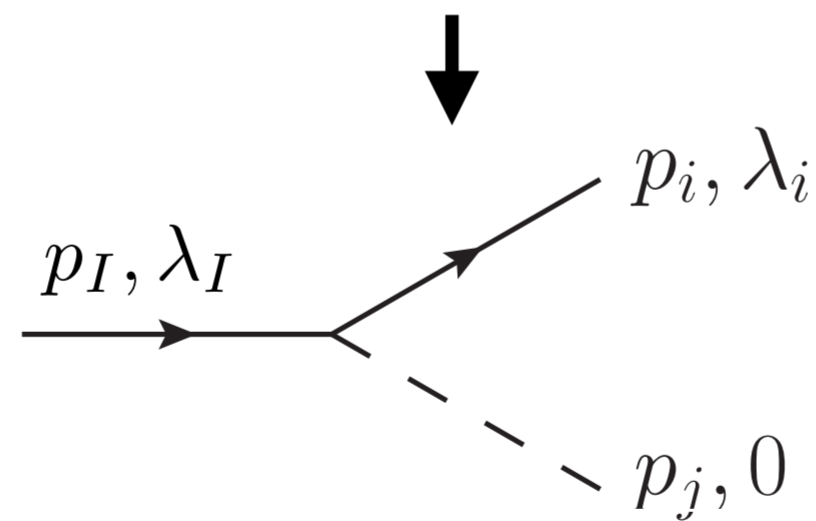
Number of Histories for n Branchings							
Starting from a single $q\bar{q}$ pair	$n = 1$	$n = 2$	$n = 3$	$n = 4$	$n = 5$	$n = 6$	$n = 7$
CS Dipole	2	8	48	384	3840	46080	645120

Bottleneck for merging at high multiplicities (+ high code complexity)

EW Showers: Longitudinal Polarizations / Goldstone bosons



$$\epsilon_0^\mu(p) = \frac{1}{m} \left(p^\mu - \frac{m^2}{p \cdot k} k^\mu \right)$$



Lots of Antenna Functions

$$a_{f_{\lambda} \mapsto f_{\lambda} V_{\lambda}}^{FF} = 2(v - \lambda a)^2 \frac{\tilde{m}_{ij}^2}{(m_{ij}^2 - m_I^2)^2} \frac{1}{x_j}$$

$$a_{f_{\lambda} \mapsto f_{\lambda} V_{-\lambda}}^{FF} = 2(v - \lambda a)^2 \frac{\tilde{m}_{ij}^2}{(m_{ij}^2 - m_I^2)^2} \frac{x_i^2}{x_j}$$

$$a_{f_{\lambda} \mapsto f_{-\lambda} V_{\lambda}}^{FF} = 2 \frac{1}{(m_{ij}^2 - m_I^2)^2} \left((v - \lambda a) m_i \frac{1}{\sqrt{x_i}} - (v + \lambda a) m_I \sqrt{x_i} \right)^2$$

$$a_{f_{\lambda} \mapsto f_{\lambda} V_0}^{FF} = \frac{1}{(m_{ij}^2 - m_I^2)^2} \left[(v - \lambda a) \left(\frac{m_I^2}{m_j} \sqrt{x_i} - \frac{m_i^2}{m_j} \frac{1}{\sqrt{x_i}} - 2m_j \frac{\sqrt{x_i}}{x_j} \right) + (v + \lambda a) \frac{m_I m_i}{m_j} \frac{x_j}{\sqrt{x_i}} \right]^2$$

$$a_{f_{\lambda} \mapsto f_{-\lambda} V_0}^{FF} = \frac{(m_I(v + \lambda a) - m_i(v - \lambda a))^2}{m_j^2} \frac{\tilde{m}_{ij}^2}{(m_{ij}^2 - m_I^2)^2} x_j.$$

$$a_{f_{\lambda} f_{\lambda} H}^{FF} = \frac{e^2}{4s_w^2} \frac{m_i^4}{s_w^2} \frac{1}{(m_{ij}^2 - m_I^2)^2} \left(\sqrt{x_i} + \frac{1}{\sqrt{x_i}} \right)^2$$

$$a_{f_{\lambda} f_{-\lambda} H}^{FF} = \frac{e^2}{4s_w^2} \frac{m_i^2}{s_w^2} \frac{\tilde{m}_{ij}^2}{(m_{ij}^2 - m_I^2)^2} x_j.$$

$$a_{V_{\lambda} \mapsto V_{\lambda} H}^{FF} = \frac{e^2}{s_w^2} \frac{m_v^4}{m_w^2} \frac{1}{(m_{ij}^2 - m_I^2)^2}$$

$$a_{V_{\lambda} \mapsto V_0 H}^{FF} = \frac{e^2}{2s_w^2} \frac{m_v^2}{m_w^2} \frac{\tilde{m}_{ij}^2}{(m_{ij}^2 - m_I^2)^2} x_i x_j$$

$$a_{V_0 \mapsto V_{\lambda} H}^{FF} = \frac{e^2}{2s_w^2} \frac{m_v^2}{m_w^2} \frac{\tilde{m}_{ij}^2}{(m_{ij}^2 - m_I^2)^2} \frac{x_j}{x_i}$$

$$a_{V_0 \mapsto V_0 H}^{FF} = \frac{e^2}{4s_w^2} \frac{1}{m_w^2} \frac{1}{(m_{ij}^2 - m_I^2)^2} \left(m_I^2 - 2m_i^2 \left(x_i + \frac{1}{x_i} \right) \right)^2.$$

$$a_{V_{\lambda} \mapsto f_{\lambda} \bar{f}_{-\lambda}}^{FF} = 2(v - \lambda a)^2 \frac{\tilde{m}_{ij}^2}{(m_{ij}^2 - m_I^2)^2} x_j^2$$

$$a_{V_{\lambda} \mapsto f_{-\lambda} \bar{f}_{\lambda}}^{FF} = 2(v + \lambda a)^2 \frac{\tilde{m}_{ij}^2}{(m_{ij}^2 - m_I^2)^2} x_i^2$$

$$a_{V_{\lambda} \mapsto f_{-\lambda} \bar{f}_{-\lambda}}^{FF} = 2 \frac{1}{(m_{ij}^2 - m_I^2)^2} \left((v + \lambda a) m_i \sqrt{\frac{x_j}{x_i}} + (v - \lambda a) m_j \sqrt{\frac{x_i}{x_j}} \right)^2$$

$$a_{V_0 \mapsto f_{\lambda} \bar{f}_{\lambda}}^{FF} = \frac{((v + \lambda a) m_i - (v - \lambda a) m_j)^2}{m_I^2} \frac{\tilde{m}_{ij}^2}{(m_{ij}^2 - m_I^2)^2}$$

$$a_{V_0 \mapsto f_{\lambda} \bar{f}_{-\lambda}}^{FF} = \frac{1}{(m_{ij}^2 - m_I^2)^2}$$

$$\times \left[(v - \lambda a) \left(2m_I \sqrt{x_i x_j} - \frac{m_i^2}{m_I} \sqrt{\frac{x_j}{x_i}} - \frac{m_j^2}{m_I} \sqrt{\frac{x_i}{x_j}} \right) + (v + \lambda a) \frac{m_i m_j}{m} \frac{1}{\sqrt{x_i x_j}} \right]^2.$$

$$a_{V_{\lambda} \mapsto V_{\lambda} V_{\lambda}}^{FF} = 2g_v^2 \frac{\tilde{m}_{ij}^2}{(m_{ij}^2 - m_I^2)^2} \frac{1}{x_i x_j}$$

$$a_{V_{\lambda} \mapsto V_{\lambda} V_{-\lambda}}^{FF} = 2g_v^2 \frac{\tilde{m}_{ij}^2}{(m_{ij}^2 - m_I^2)^2} \frac{x_i^3}{x_j}$$

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$$a_{V_{\lambda} \mapsto V_{\lambda} V_0}^{FF} = g_v^2 \frac{1}{(m_{ij}^2 - m_I^2)^2} \frac{(m_I^2 - m_i^2 - \frac{1+x_i}{x_j} m_j^2)^2}{m_j^2}$$

$$a_{V_{\lambda} \mapsto V_0 V_{\lambda}}^{FF} = g_v^2 \frac{1}{(m_{ij}^2 - m_I^2)^2} \frac{(m_I^2 - m_j^2 - \frac{1+x_j}{x_i} m_i^2)^2}{m_i^2}$$

$$a_{V_{\lambda} \mapsto V_0 V_0}^{FF} = \frac{g_v^2}{2} \frac{(m_I^2 - m_i^2 - m_j^2)^2}{m_i^2 m_j^2} \frac{\tilde{m}_{ij}^2}{(m_{ij}^2 - m_I^2)^2} x_i x_j.$$

$$a_{H \mapsto f_{\lambda} \bar{f}_{\lambda}}^{FF} = \frac{e^2}{4s_w^2} \frac{m_i^2}{s_w^2} \frac{\tilde{m}_{ij}^2}{(m_{ij}^2 - m_I^2)^2}$$

$$a_{H \mapsto f_{\lambda} \bar{f}_{-\lambda}}^{FF} = \frac{e^2}{4s_w^2} \frac{m_i^4}{s_w^2} \frac{1}{(m_{ij}^2 - m_I^2)^2} \left(\sqrt{\frac{x_i}{x_j}} - \sqrt{\frac{x_j}{x_i}} \right)^2.$$

Collinear Limits

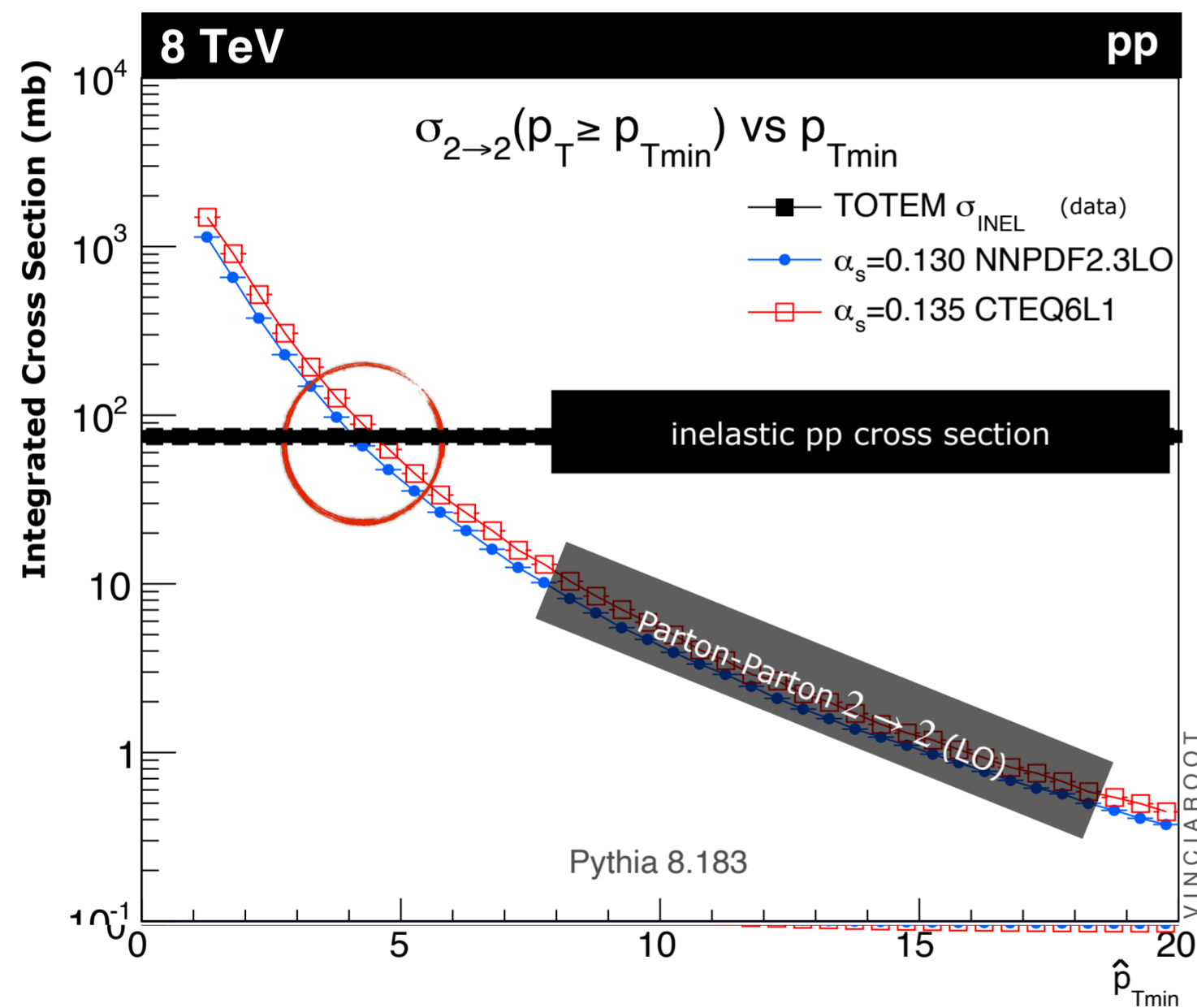
$$\tilde{m}_{ij}^2 = m_{ij}^2 - \frac{m_i^2}{z^2} - \frac{m_j^2}{(1-z)^2}$$

λ_I	λ_i	λ_j	$f \rightarrow f'V$		
λ	λ	λ	$2(v - \lambda a)^2 \frac{\tilde{m}_{ij}^2}{(m_{ij}^2 - m_I^2)^2} \frac{1}{1-z}$	$P(z) \propto \frac{\tilde{m}_{ij}^2}{(m_{ij}^2 - m_I^2)^2} \frac{1+z^2}{1-z}$	Pure vector
λ	λ	$-\lambda$	$2(v - \lambda a)^2 \frac{\tilde{m}_{ij}^2}{(m_{ij}^2 - m_I^2)^2} \frac{z^2}{1-z}$		
λ	$-\lambda$	λ	$2 \frac{1}{(m_{ij}^2 - m_I^2)^2} \left(m_I(v - \lambda a) \sqrt{z} - m_i(v + \lambda a) \frac{1}{\sqrt{z}} \right)^2$		Pure vector
λ	$-\lambda$	$-\lambda$	0	$P(z) \propto \frac{m^2}{(m_{ij}^2 - m_I^2)^2}$	
λ	λ	0	$\frac{1}{(m_{ij}^2 - m_I^2)^2} \left[(v - \lambda a) \left(\frac{m_I^2}{m_j} \sqrt{z} - \frac{m_i^2}{m_j} \frac{1}{\sqrt{z}} - 2m_j \frac{\sqrt{z}}{1-z} \right) + (v + \lambda a) \frac{m_i m_I}{m_j} \frac{1-z}{\sqrt{z}} \right]^2$		Vector + Scalar
λ	$-\lambda$	0	$\frac{\tilde{m}_{ij}^2}{(m_{ij}^2 - m_I^2)^2} (1-z) \left(\frac{m_i}{m_j} (v - \lambda a) - \frac{m_I}{m_j} (v + \lambda a) \right)^2$	$P(z) \propto \frac{\tilde{m}_{ij}^2}{(m_{ij}^2 - m_I^2)^2} (1-z)$	Pure scalar

A Brief History of MPI in PYTHIA

$$\frac{\sigma_{\text{parton-parton}}(\hat{p}_{\perp})}{\sigma_{\text{hadron-hadron}}} > 1$$

⇒ several parton-parton interactions *per* hadron-hadron interaction



Sjöstrand & van Zijl, 1985:

Cast as **Sudakov-style evolution equation**, analogous to the $\sigma_{\chi+\text{jet}}(p_{\perp})/\sigma_{\chi}$ one of showers

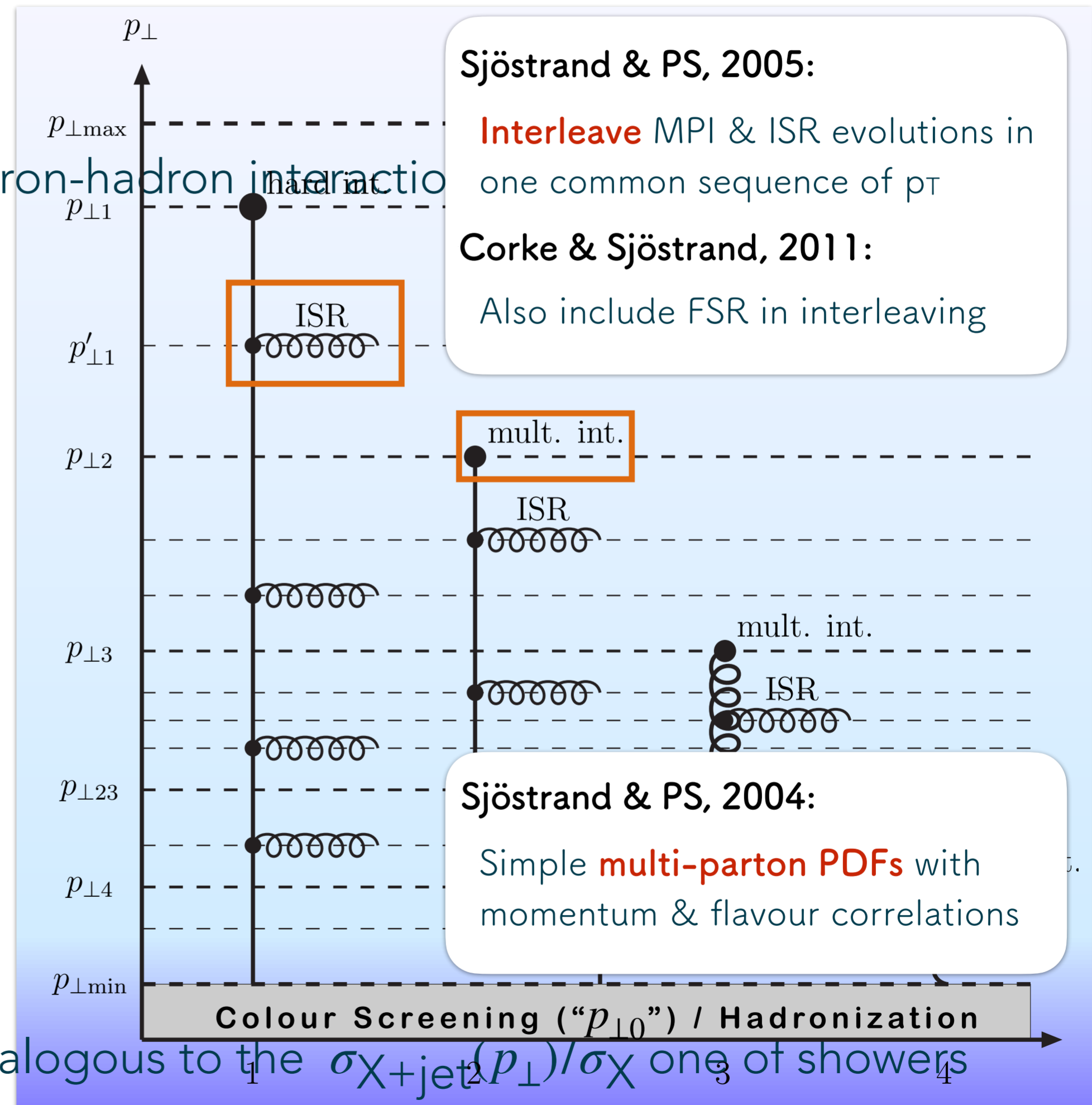


Figure from Sjöstrand & PS, 2005

QCD Colour Reconnections \longleftrightarrow String Junctions

Stochastically restores colour-space ambiguities according to **SU(3) algebra**

➤ Allows for reconnections to minimise string lengths

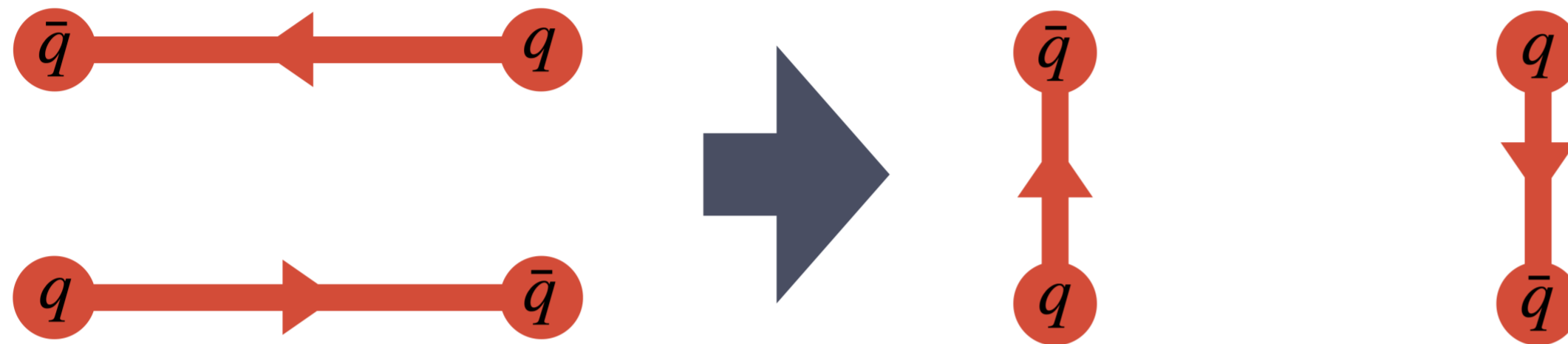


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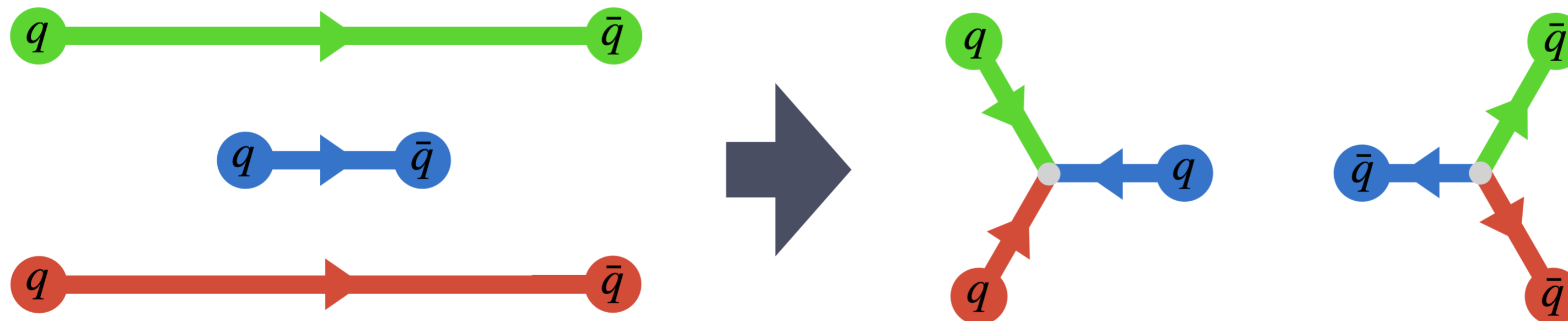
[Christiansen & PS
JHEP 08 (2015) 003]

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Dipole-type reconnection

What about the **red-green-blue** colour singlet state?

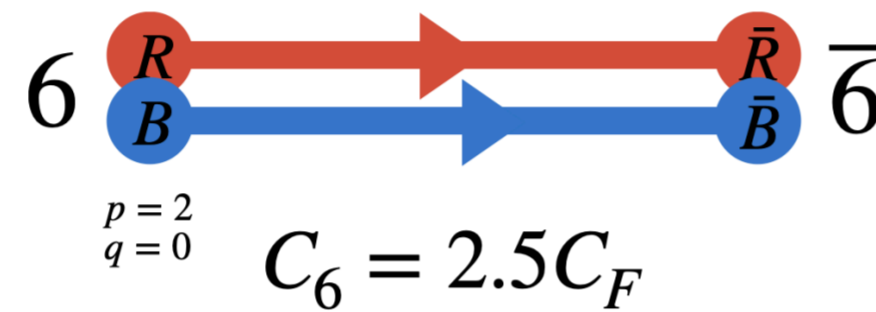
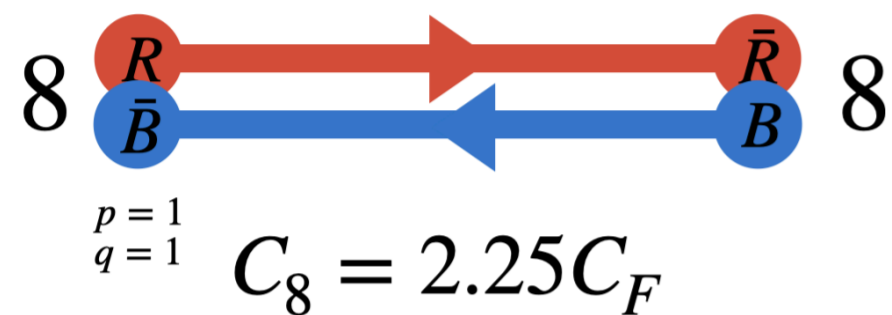


Junctions!

In Progress: Strangeness Enhancement from Close-Packing

Idea: each string exists in an effective background produced by the

Close-packing



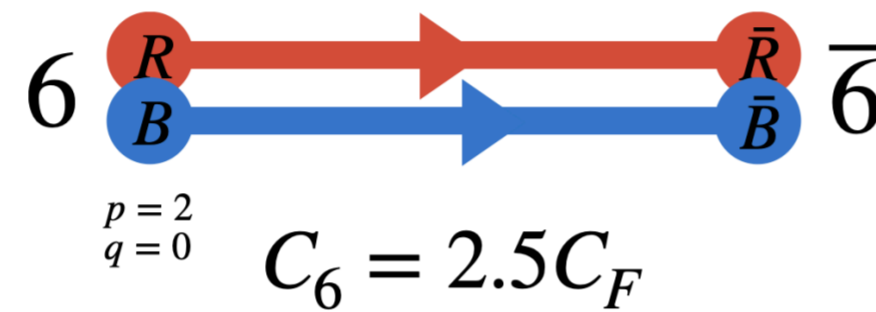
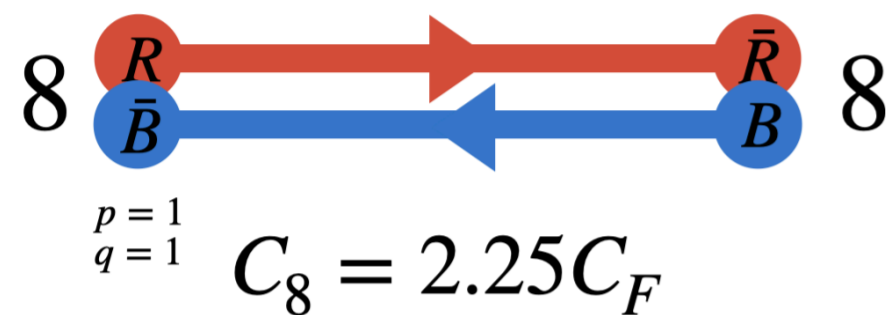
Dense string environments

- Casimir scaling of **effective string tension**
- Higher probability of strange quarks

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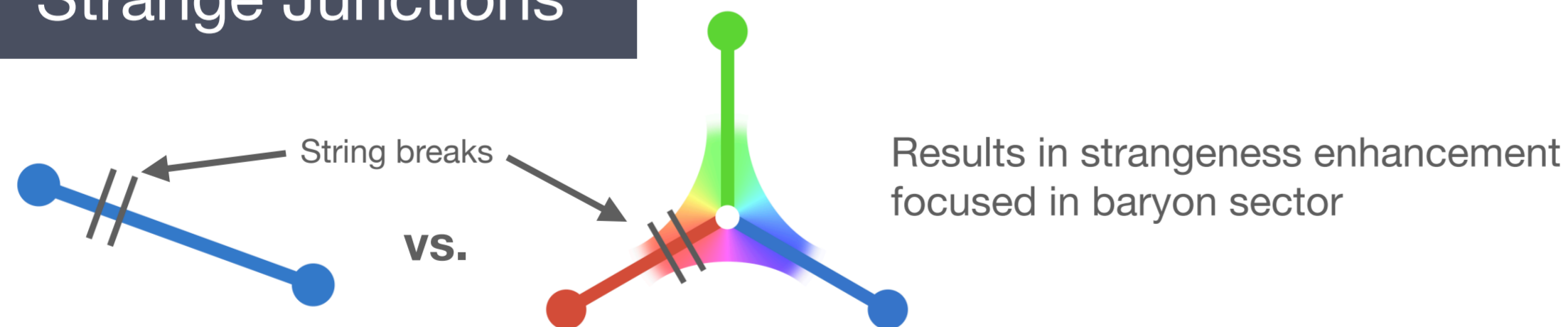


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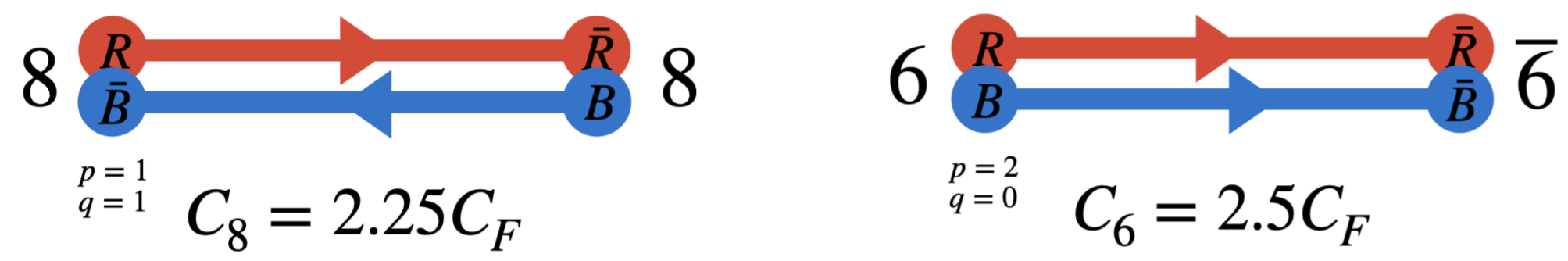


String tension could be different from the vacuum case compared to near a junction

NEW In Progress: **Strangeness Enhancement from Close-Packing**

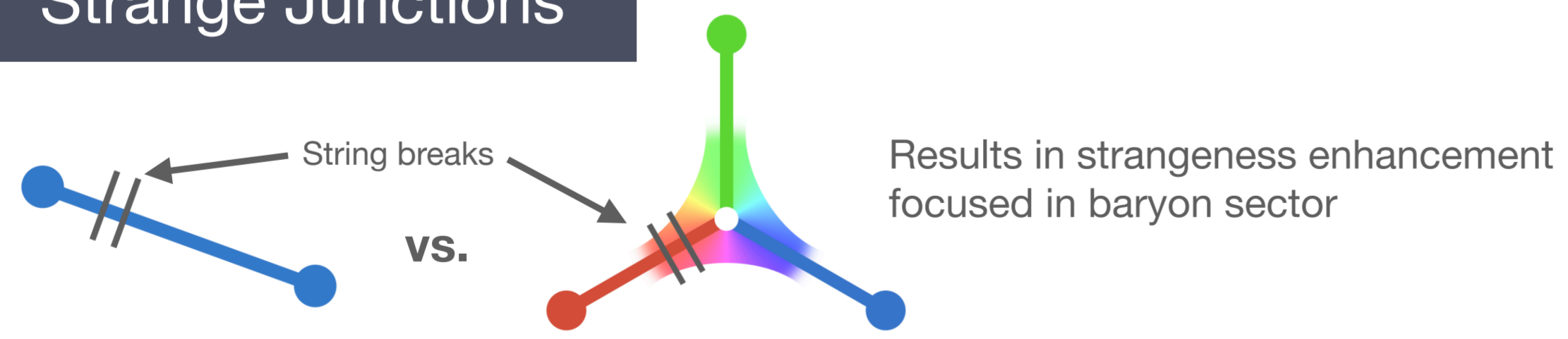
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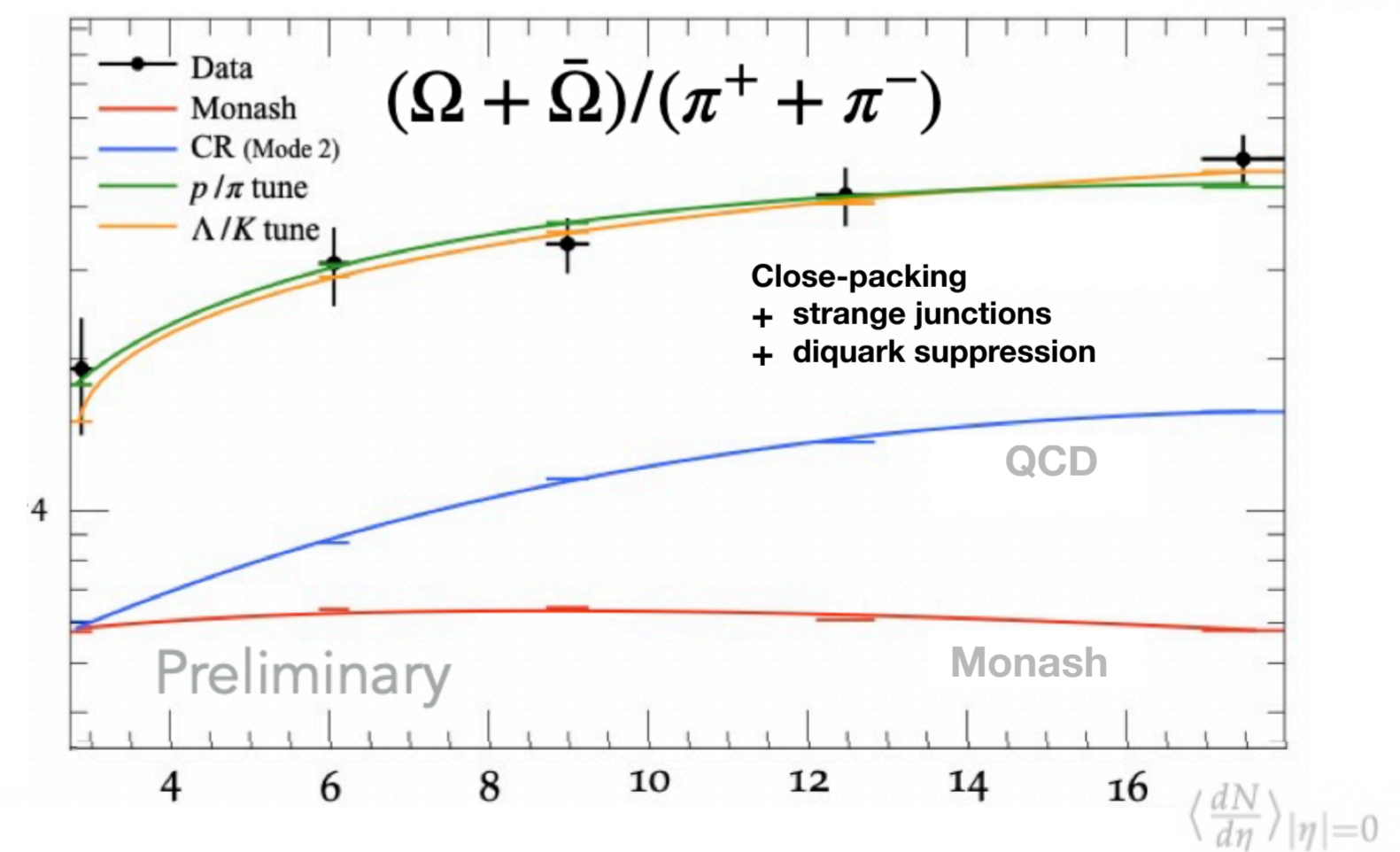
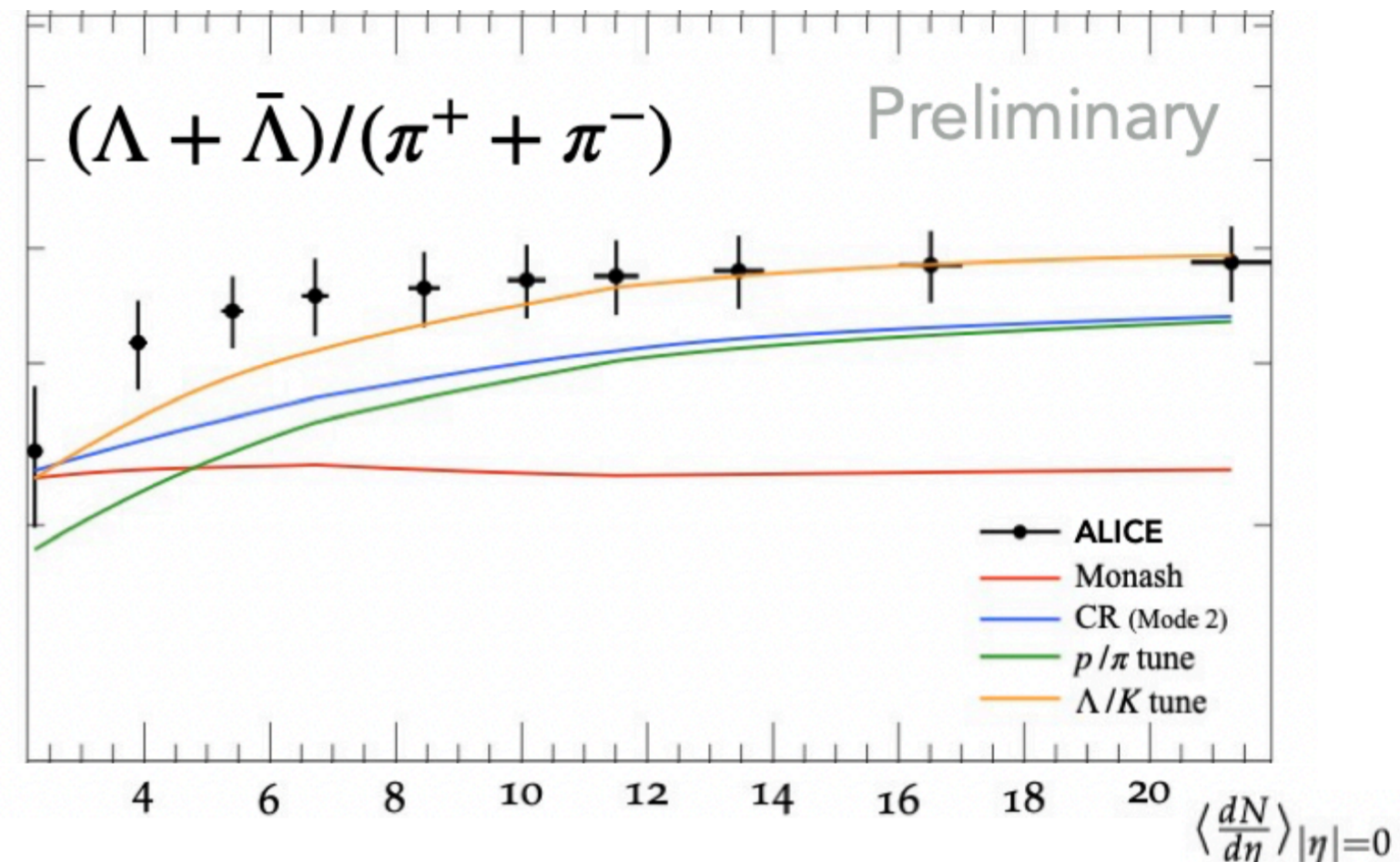


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LHCb: also in Bottom

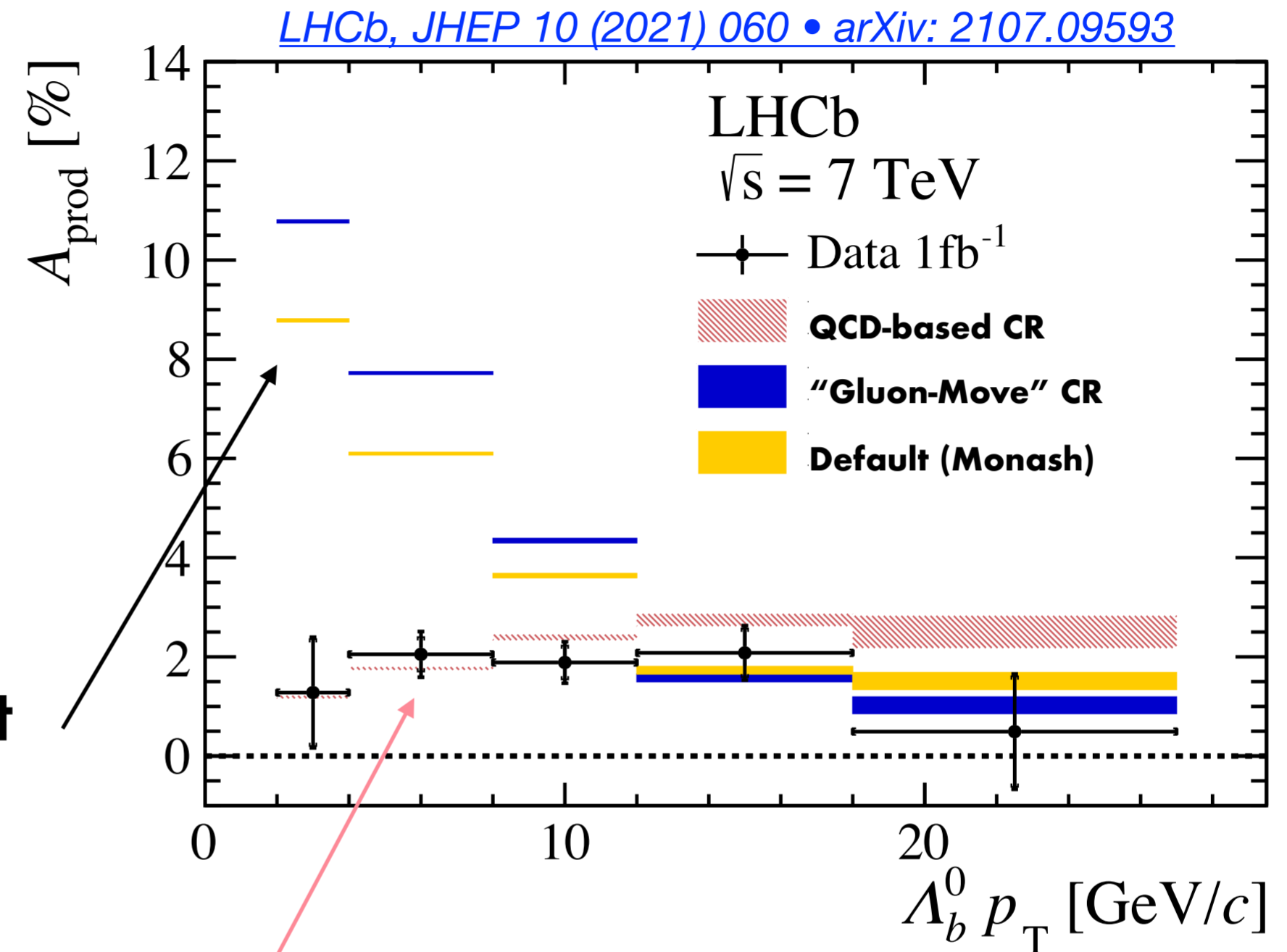
Λ_b asymmetry

$$A = \frac{\sigma(\Lambda_b^0) - \sigma(\bar{\Lambda}_b^0)}{\sigma(\Lambda_b^0) + \sigma(\bar{\Lambda}_b^0)}$$

Without junction CR, an important source of low- p_T Λ_b production is when a b quark combines with the proton beam remnant.

Not possible for $\bar{\Lambda}_b$ (no \bar{p} remnant at LHC)

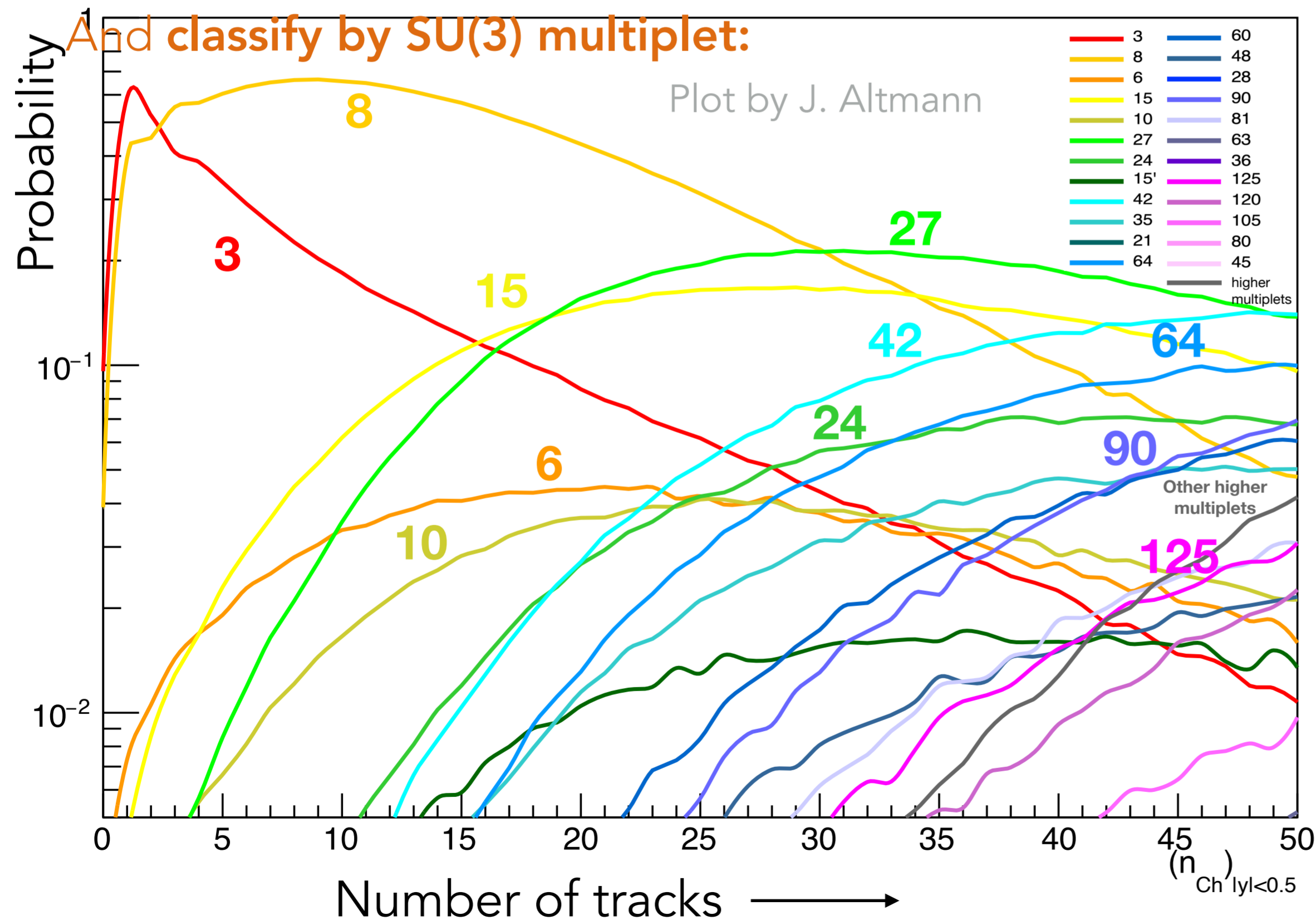
QCD CR adds large amount of low- p_T junction Λ_b and $\bar{\Lambda}_b$, in equal amounts. Dilutes asymmetry!



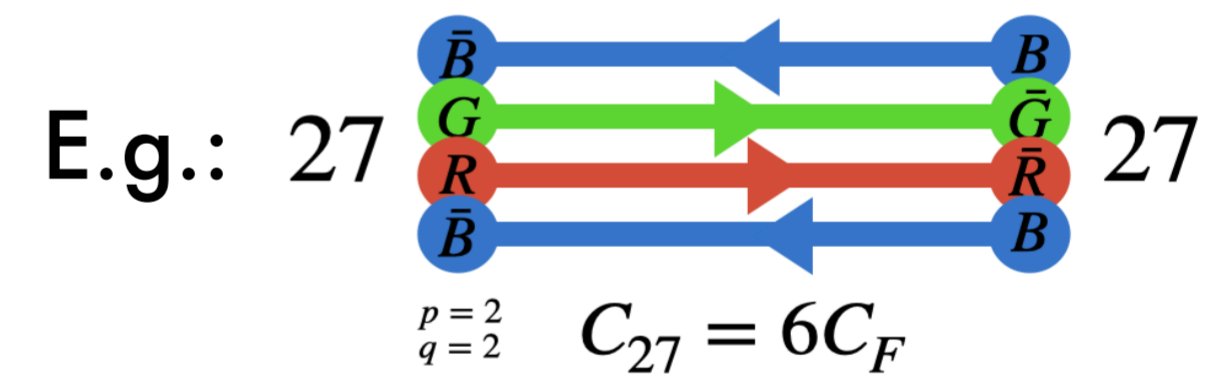
Non-Linear String Dynamics?

MPI \implies **lots** of coloured partons scattered into the final states

Count # of (oriented) flux lines crossing $y = 0$ in pp collisions (according to PYTHIA)



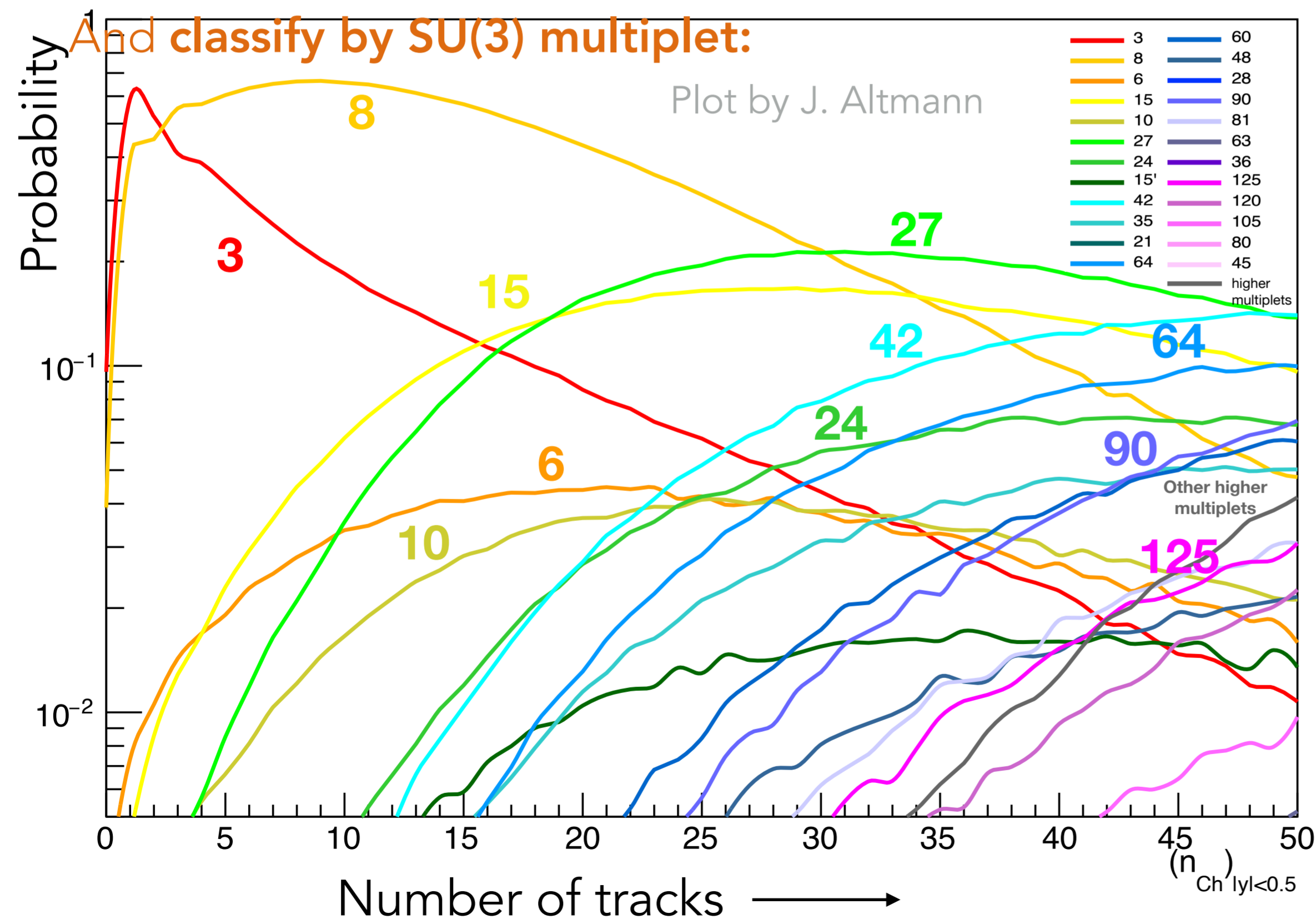
Confining fields may be reaching **higher effective representations** than simple quark-antiquark (3) ones.



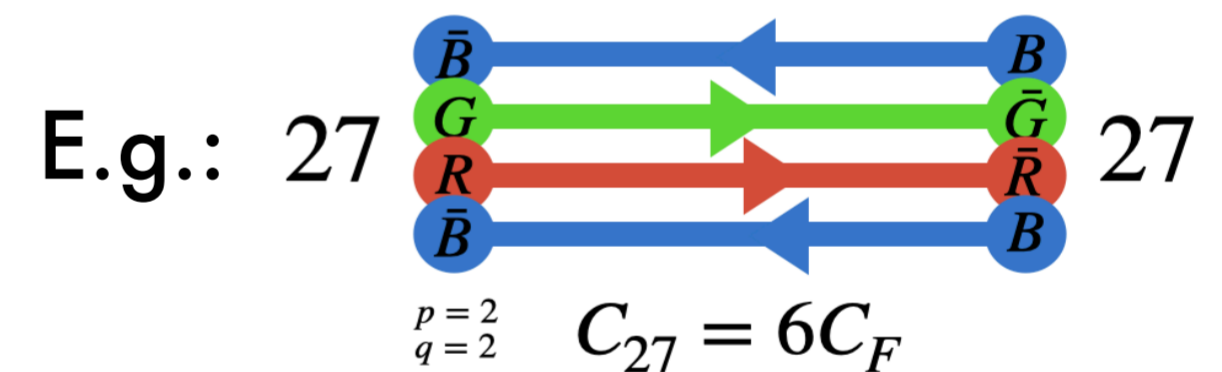
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Two approaches in PYTHIA:

- 1) Colour Ropes (Lund)
- 2) Close-Packing (Monash)