Localization of Dirac modes in the SU(2)-Higgs model at finite temperature

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Motivation

- The connection between deconfinement and chiral symmetry restoration at the finite temperature QCD transition is still not fully understood.
- Low Dirac modes could be key in understanding this connection.
- Chiral symmetry breaking is controlled by the density $\rho(\lambda)$ of low modes according to the Banks-Casher relation

$$\left|\langle\bar{\psi}(x)\psi(x)\rangle\right| \overset{m\to 0}{=} \pi\rho(0).$$

- Deconfinement is signalled by the ordering of Polyakov loops

$$P(\vec{x}) = \mathrm{tr}\,\mathrm{Pexp}\left\{ig\int_0^{\frac{1}{T}} \mathrm{d}t A_4(t,\vec{x})\right\}.$$

- "Sea/islands" picture: Islands of fluctuations in the sea of ordered Polyakov loops are "energetically" favorable for Dirac modes ⇒ low-lying Dirac modes localize [1].
- The "sea/islands" mechanism is general and requires only the ordering of the Polyakov loop [2]: test it in gauge theories with a deconfinement transition other than QCD, e.g., the SU(2)-Higgs model in the infinite coupling limit [3].

SU(2)-Higgs model

The model on a hypercubic $N_s^3 \times N_t$ lattice is defined by the action

$$S = \frac{\beta}{2}\sum_n\sum_{\mu<\nu}\mathrm{tr}U_{\mu\nu}(n) - \frac{\kappa}{2}\sum_n\sum_\mu \mathrm{tr}G_\mu(n)\,,$$

$$U_{\mu\nu}(n) = U_\mu(n)U_\nu(n+\hat{\mu})U_\mu(n+\hat{\nu})^\dagger U_\nu(n)^\dagger\,,$$

$$G_\mu(n) = \phi(n)^\dagger U_\mu(n)\phi(n+\hat{\mu})\,,$$

with ϕ the unit-length Higgs field and $U_\mu = e^{ia^2 A_\mu}$ with A_μ the SU(2) gauge field. We work at finite temperature $T = 1/(aN_t)$, keeping N_t fixed ($N_t = 4$) and changing a by changing β and κ.

Localization

Localized (resp. delocalized) modes occupy a finite amount (resp. a finite fraction) of modes. Localization can be studied [2]

- by looking at the participation ratio (PR) of the eigenvectors ψ_l

$$\mathrm{PR}_l = \frac{1}{N_t N_s^3}\mathrm{IPR}_l^{-1}\,,\quad \mathrm{IPR}_l = \sum_n \|\psi_l(n)\|^4\,,$$

with $\mathrm{PR} \sim N_s^{\alpha-3}$ at large N_s, with α the fractal dimension of modes;

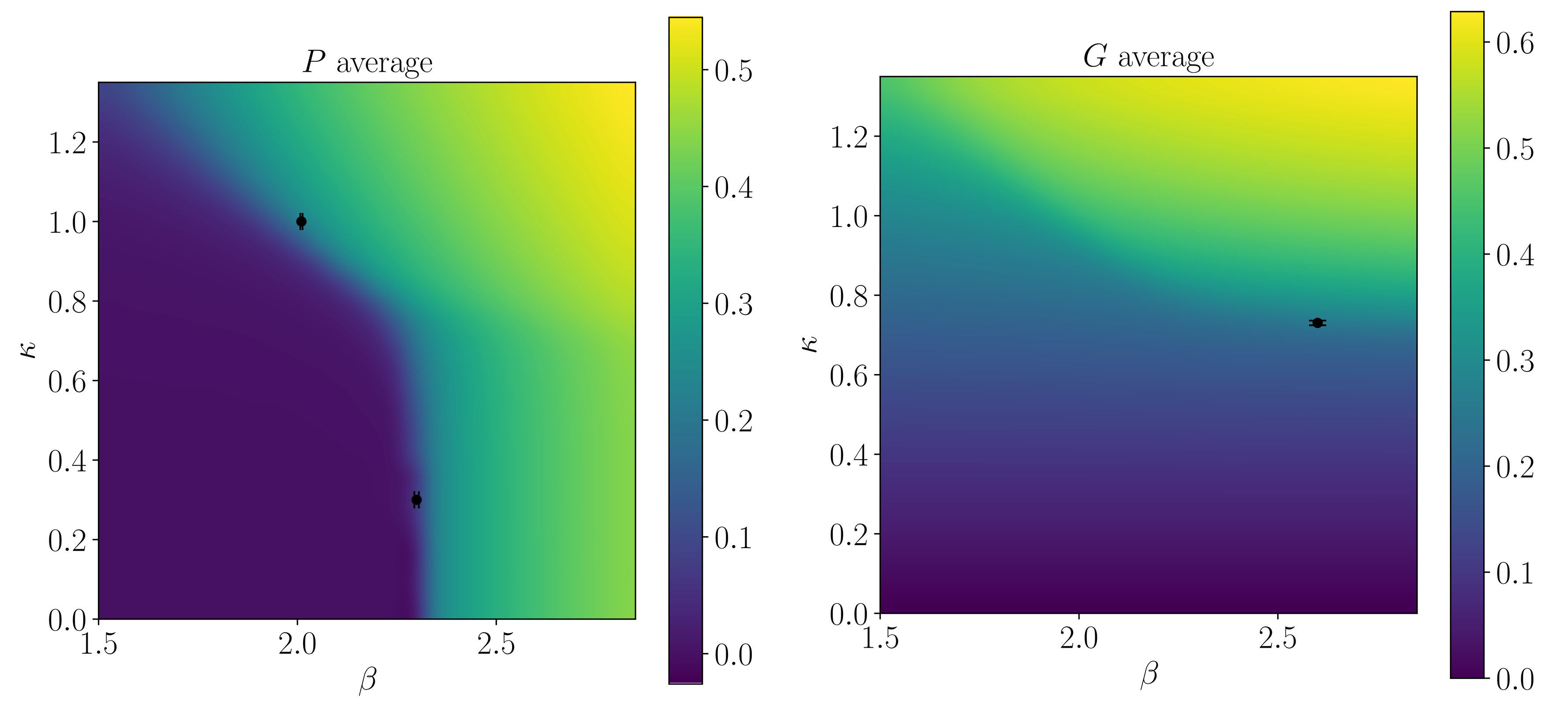
- or by studying the probability distribution of the unfolded eigenvalue level spacing

$$I_{s_0} = \int_0^{s_0} p(s)\mathrm{d}s,\quad s = (\lambda_{i+1}-\lambda_i)\rho(\lambda_i).$$

with known $p_{\mathrm{Poisson}}(s)$ for localized modes, and $p_{\mathrm{RMT}}(s)$ for delocalized modes.

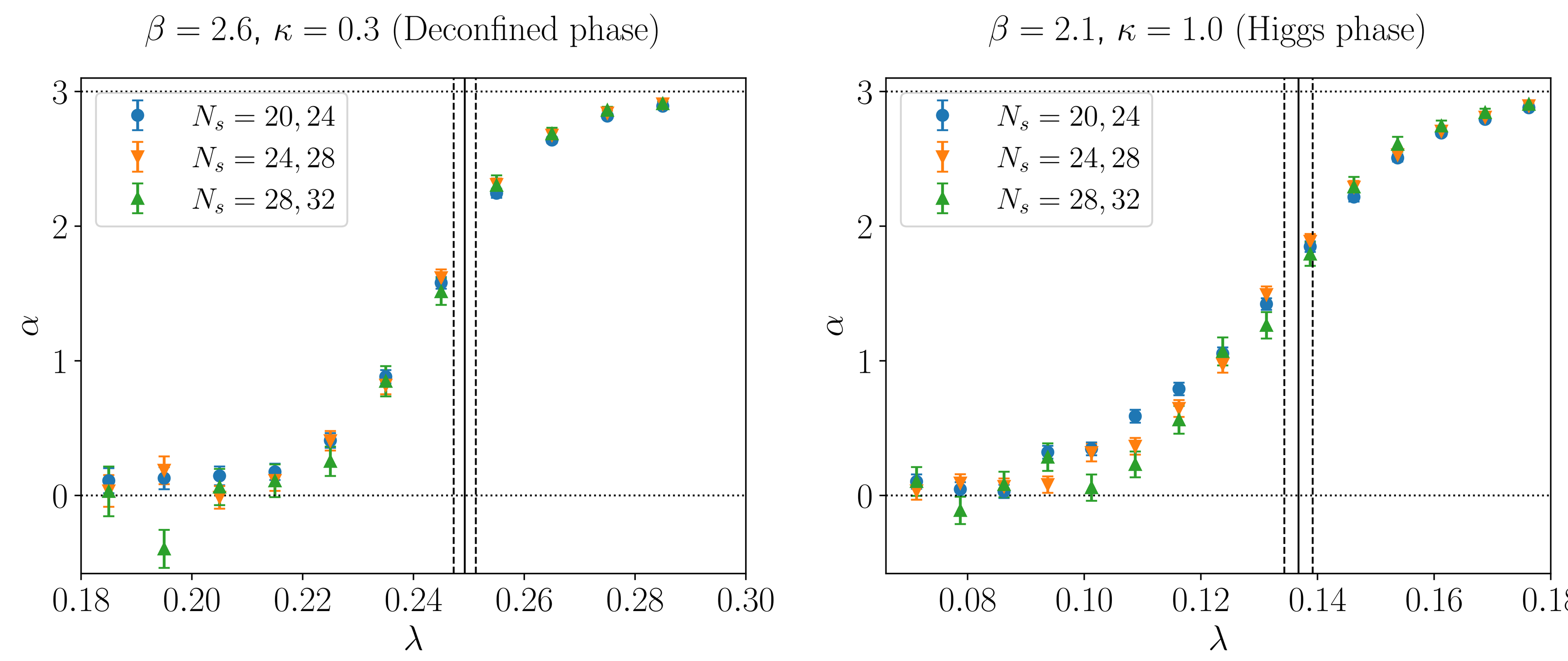
Phase diagram at finite temperature

The model displays three phases: a confined phase at low β and κ, a deconfined phase at large β and low κ, and a Higgs phase at large κ, distinguished by the average Polyakov loop and gauge-Higgs term.

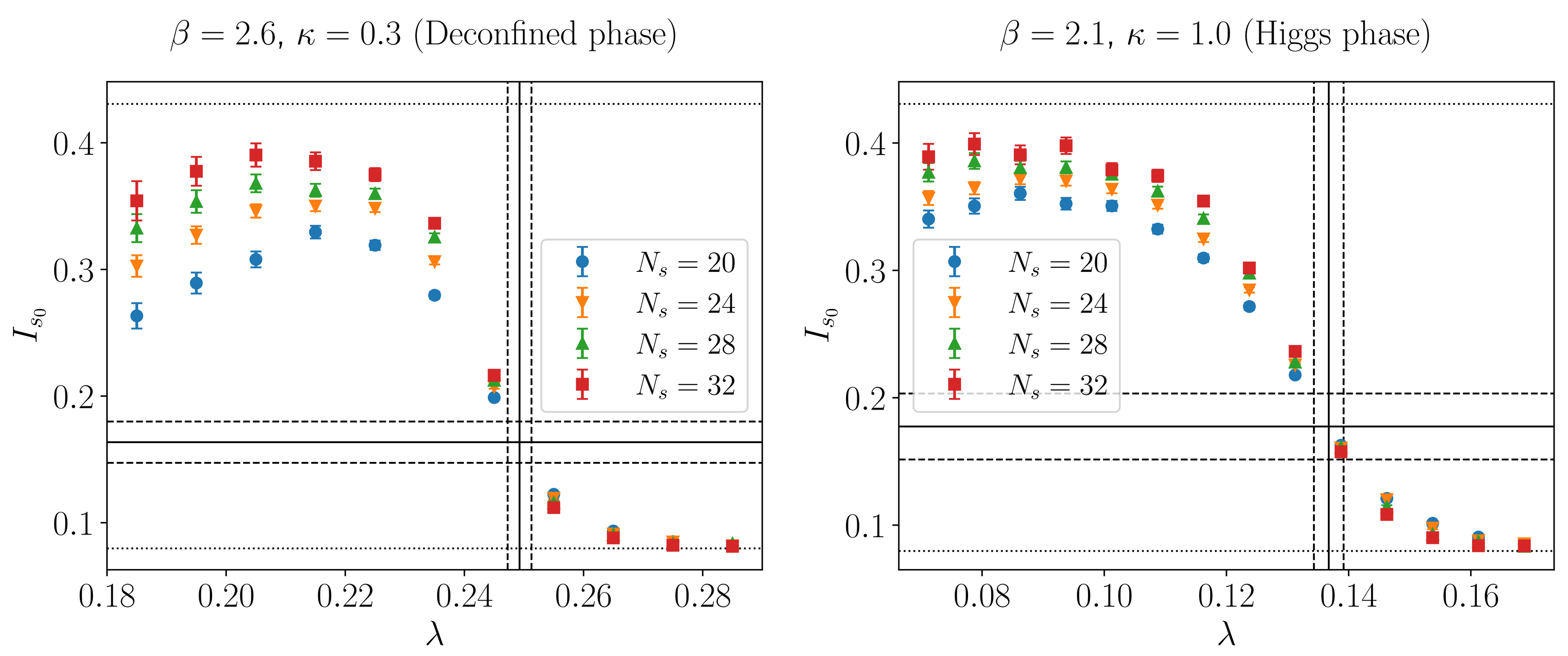


Localization properties of Dirac eigenmodes

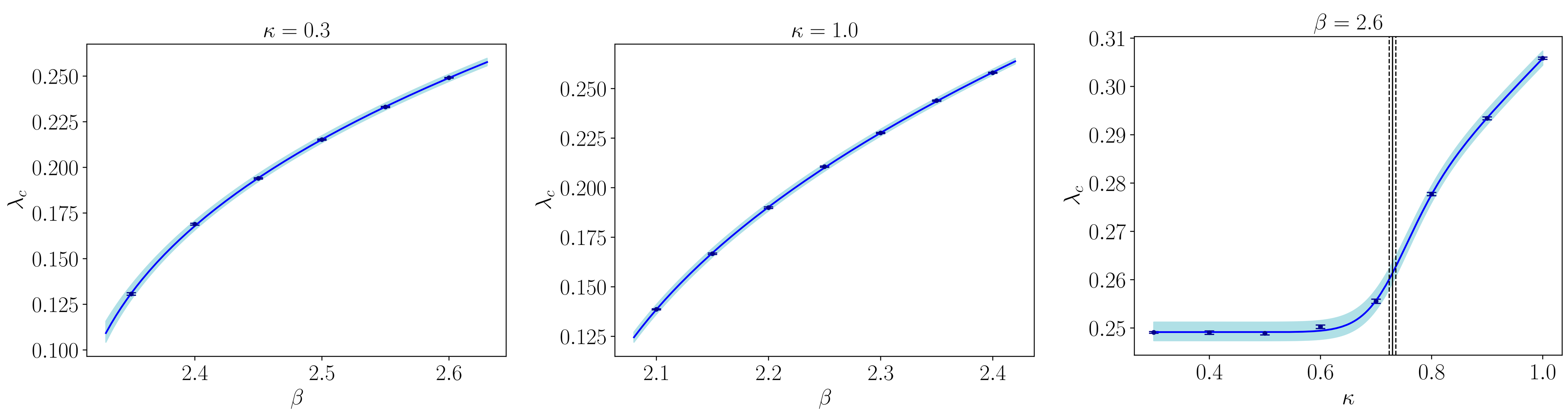
In the confined phase the fractal dimension of low modes hovers around 3, while in the deconfined and Higgs phases it is 0 up to a "mobility edge", λ_c, and 3 above.



In the confined phase $I_{s_0} \approx I_{s_0,\mathrm{RMT}}$, while in the deconfined and Higgs phases $I_{s_0} \approx I_{s_0,\mathrm{RMT}}$ above λ_c, and $I_{s_0} \approx I_{s_0,\mathrm{Poisson}}$ below.



The mobility edge can be identified as the point where I_{s_0} takes its critical value[2]. As one approaches the confined phase, it decreases, and $\lambda_c \to 0$ in the crossover region (black dots in the phase diagram). At the transition from the deconfined to the Higgs phase, the dependence of λ_c on κ changes.



Our results confirm the universality of the "sea/islands" picture and the close connection between deconfinement and localization of the low Dirac modes.

References

[1] F. Bruckmann, T. G. Kovács, and S. Schierenberg, *Phys.Rev. D* **84**, 034505 (2011)
[2] M. Giordano and T. G. Kovács, *Universe* **7**, 194 (2021)
[3] G. Baranka and M. Giordano, arXiv:2310.03542 (2023)