Lattice determination of the NLO HVP contributions to the $(g-2)_\mu$

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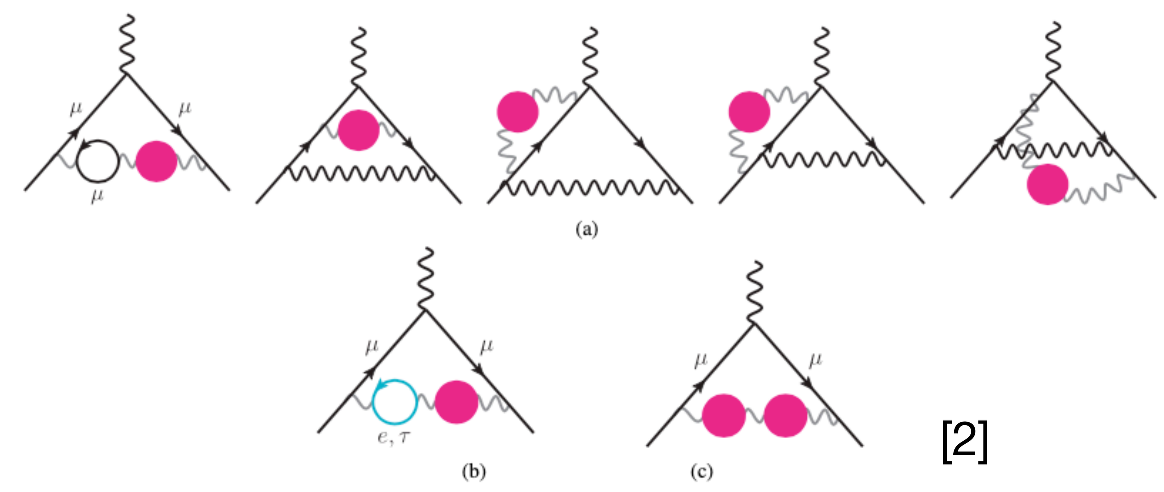
In this work, we present a full computation of the NLO HVP contributions to the muon $g-2$. Starting with a study of the Time-Momentum Representation (TMR) of the corresponding Kernels for the relevant contributions to the numerical implementation of QCD correlators obtained from lattice simulations.



[2]

There are essentially three different types of α^{em} subleading HVP contributions to the $(g-2)_\mu$. When comparing with the LO contribution, these can be classified in diagrams **(a)** containing extra photon or muon lines, **(b)** containing a leptonic (*electron or tauon*) loop and **(c)** with an additional QCD insertion.
The TMR integral representation of these Kernels is

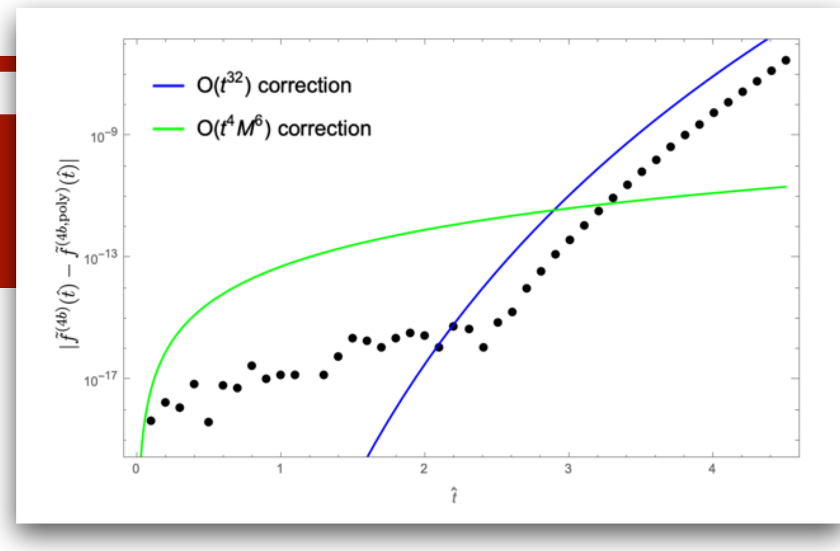
$$\tilde{f}^{(4a)}\left(t;m_\mu\right) = \int_0^\infty d\omega^2 \frac{4\pi^2 f_4(\omega^2;m_\mu)}{\omega^2}\left[\omega^2t^2 - 4\sin^2\frac{\omega t}{2}\right], \qquad \tilde{f}^{(4b)}_l\left(t;m_\mu,m_l\right) = 2\int_0^\infty d\omega^2 \frac{4\pi^2 f_2(\omega^2;m_\mu)}{\omega^2}\left[\omega^2t^2 - 4\sin^2\frac{\omega t}{2}\right] F^l\left(\omega^2;m_l^2\right),$$

$$\tilde{f}^{(4c)}\left(t,\tau;m_\mu\right) = \int_0^\infty d\omega^2 \frac{16\pi^4 f_2(\omega^2;m_\mu^2)}{\omega^4}\left[\omega^2t^2 - 4\sin^2\frac{\omega t}{2}\right]\left[\omega^2\tau^2 - 4\sin^2\frac{\omega \tau}{2}\right],$$

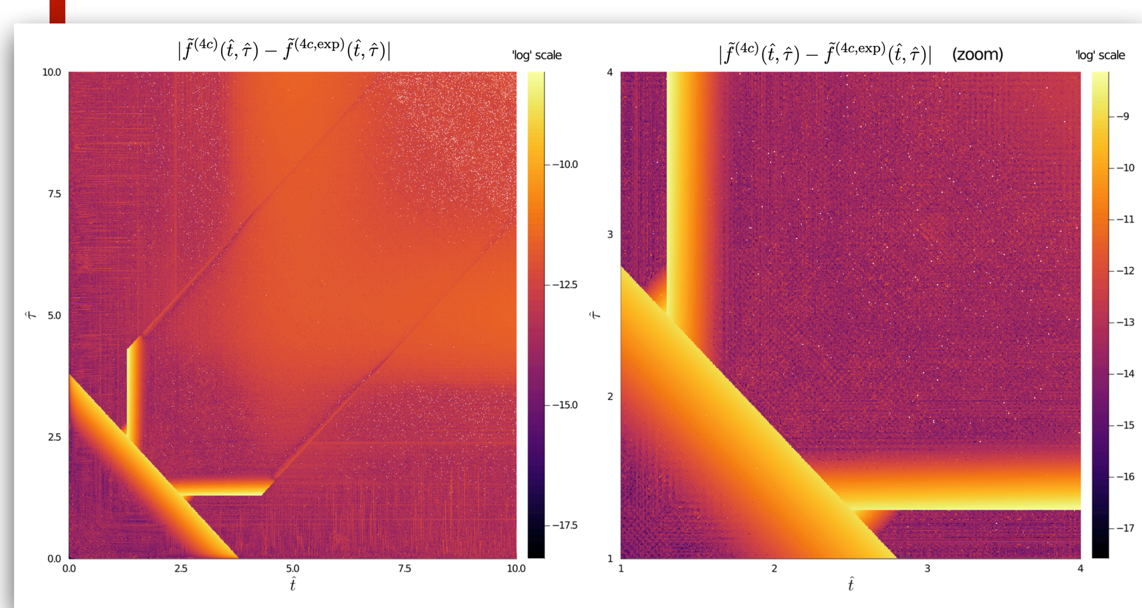
where f_2 and f_4 are the time-like LO and NLO Kernels [1] and F^l is the Lepton loop function [2].

These Kernels are then to be combined with a lattice calculation of the electromagnetic correlator $G(t) = -\frac{1}{3}\sum_{\mu=1}^{3}\sum_{\mathbf{x}\in\Lambda}\langle j_\mu(\mathbf{x},t)j_\mu(0)\rangle$.

Suitable representation for the NLO Kernels



It is convenient to work with a simplified version of the TMR Kernels, a polynomial representation is best suited for the task. $\tilde{f}^{(4c)}$ can be analytically solved and then expanded around 5 different regions depending on the behaviour of the two Euclidean-time coordinates. For the (a) and (b) contributions, it is not possible to analytically solve the integral and another approach is required.



In [1] Balzani, Laporta and Passera developed an analytic method to obtain a polynomial expansion for contribution (a). First, the integral is conveniently split into small and large ω values which allow for different expansions around $t\sim 0$. For high t values, this approximation fails and another expansion is required.
We follow a similar approach for the electronic (b) contribution $l=e$, where one must first apply the method around a small leptonic mass ratio $M=m_e/m_\mu$ and then expand in the same way around $t\sim 0$.

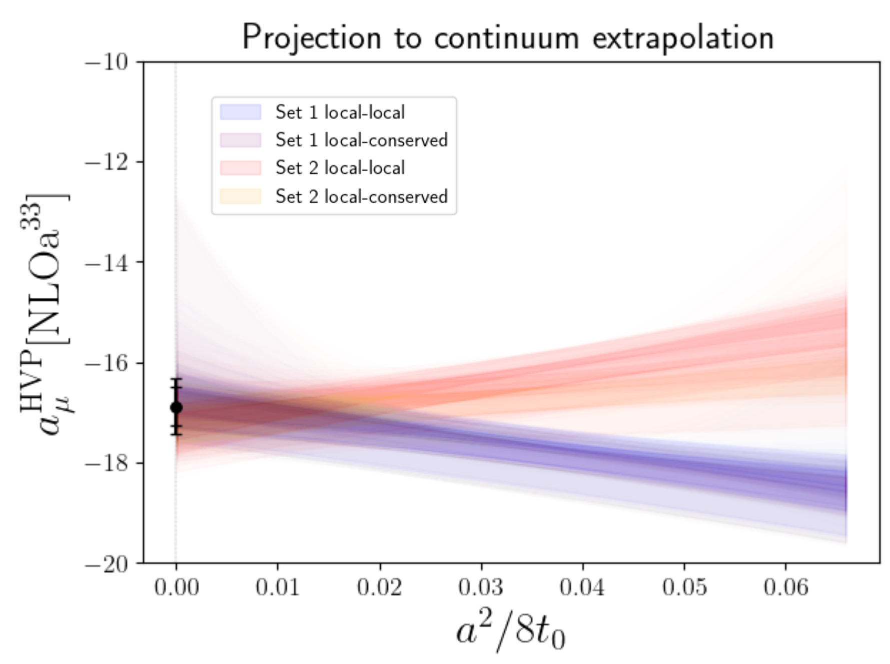
These polynomial representations are valid for all values of the Euclidean time up to a precision of 10^{-8} and can be tested against their numerical representation (shown for (b) in the upper-right corner and for (c) in the bottom-left).

First preliminary results

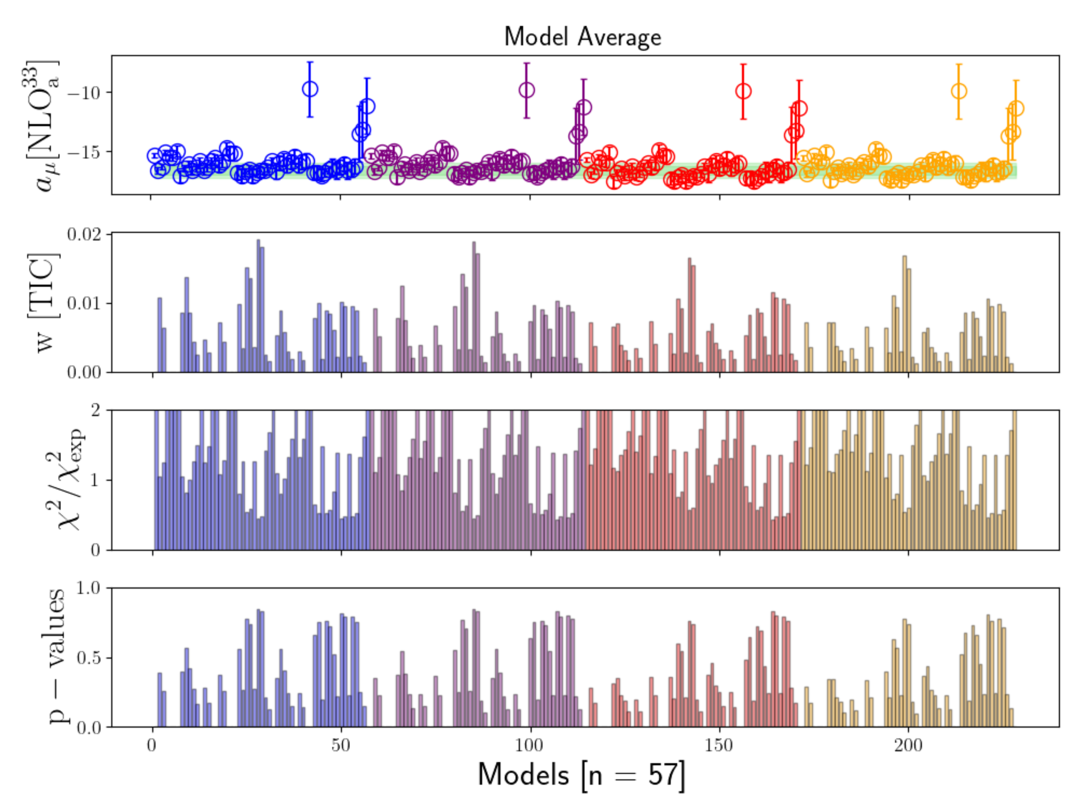
Eventually, the new Kernel representations are combined with lattice data from 12 CLS ensembles [3] with $N_f=2+1$ flavours of $\mathcal{O}(a)$-improved Wilson quarks, to obtain a full preliminary determination of the sub-leading hadronic contribution to the $(g-2)_\mu$.
Following a decomposition in the iso-spin basis to simplify renormalisation, the total contribution can be symbolically expressed as

$$a_\mu^{\text{hvp}}[\text{NLO}_{\text{a\&b}}] = [\text{Iso}-\text{vector}] + \frac{1}{3}[\text{Iso}-\text{scalar}] + \frac{4}{9}[\text{charm conn.} + \text{charm disc.}] + \frac{2}{3\sqrt{3}}[\text{Iso-scalar} \leftrightarrow \text{charm}] + \ldots,$$

where the dots correspond to contributions from the bottom and top quarks, negligible at the current statistical precision.



We have applied 2 different sets of improvement coefficients with two discretizations each to constrain the continuum limit. Finite-volume effects are corrected for using the Hansen-Patella method [4]. To better control the signal-to-noise problem in the long distance regime of the correlators we have made use of the Bounding Method [3].



Lastly, we perform combined chiral and continuum extrapolations considering multiple different models. The results from different models are then combined into a weighted average using the Takeuchi Information Criterion (TIC) [5], in order to estimate the systematics arising from the spread of results from different models.

The preliminary results presented here still have a relatively large uncertainty due to the limited parameter space coverage. In the future, we will add more ensembles and increase statistics significantly.

$$a_\mu^{\text{hvp}}[\text{NLO}_\text{a}] = -16.60(37)(55) + \frac{1}{3}[-10.33(27)(51)] + \frac{4}{9}[-1.986(26)(78) + 67.0(0.0)(7.0)\times 10^{-5}] + \frac{2}{3\sqrt{3}}[3.0(6.0)(2.0)\times 10^{-5}] = -\mathbf{20.92(42)(57)} \quad [3.38\%]$$

$$a_\mu^{\text{hvp}}[\text{NLO}_\text{b}] = 8.64(25)(34) + \frac{1}{3}[4.83(19)(29)] + \frac{4}{9}[0.556(07)(22) - 20.2(0.0)(5.7)\times 10^{-5}] + \frac{2}{3\sqrt{3}}[-1.6(4.0)(4.4)\times 10^{-5}] = \mathbf{10.49(28)(35)} \quad [4.27\%]$$

$$a_\mu^{\text{hvp}}[\text{NLO}_\text{c}^*] = 0.226(09)(12) + \frac{1}{9}[0.0810(52)(87)] + \frac{16}{81}[0.00213(50)(17)] + \frac{2}{3}[0.1253(60)(95)] + \frac{8}{9}[0.0180(05)(10)] + \frac{8}{27}[0.0087(19)(38)] = \mathbf{0.338(13)(14)} \quad [5.65\%]$$

$$a_\mu^{\text{hvp}}[\text{NLO}] = a_\mu^{\text{hvp}}[\text{NLO}_\text{a}] + a_\mu^{\text{hvp}}[\text{NLO}_\text{b}] + a_\mu^{\text{hvp}}[\text{NLO}_\text{c}] = -\mathbf{10.09(23)(67)} = -\mathbf{10.09(71)} \quad [7.02\%]$$

* Notice a different structure for the (c) contribution, this is caused by the $G(t)\times G(\tau)$ product in the integrand.

[1] Elisa Balzani, Stefano Laporta, and Massimo Passera. "Time-kernel for lattice determinations of NLO hadronic vacuum polarization contributions to the muon g-2". In: (June 2024). arXiv: 2406.17940 [hep-ph].
[2] B. Chakraborty et al. "Higher-order hadronic-vacuum-polarization contribution to the muon g-2 from lattice QCD". In: Physical Review D 98.9 (Nov. 2018). issn: 2470-0029.
[3] Antoine Gérardin et al. "Leading hadronic contribution to (g – 2)μ from lattice QCD with Nf = 2 + 1 flavours of O(a) improved Wilson quarks". In: Physical Review D 100.1 (July 2019). issn: 2470-0029.
[4] Maxwell T. Hansen and Agostino Patella. "Finite-Volume Effects in $(g-2)_\mu^{\text{HVP,LO}}$". In: Physical Review Letters 123.17 (Oct. 2019). issn: 1079-7114.
[5] Ethan T. Neil and Jacob W. Sitison. "Improved information criteria for Bayesian model averaging in lattice field theory". In: Physical Review D 109.1 (Jan. 2024). issn: 2470-0029.