

Hadronic vacuum-polarization contribution to a_μ

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FOR FUNDAMENTAL PHYSICS

Continuum foundations School, CERN, July 22, 2024

Outline

Introduction: $(g - 2)_\mu$ in the Standard Model

Hadronic Vacuum Polarization contribution

White Paper: data-driven approach

Lattice: the BMW result

Lattice vs data-driven: the window quantity

The new CMD3 measurement of $e^+e^- \rightarrow \pi^+\pi^-$

Conclusions and Outlook

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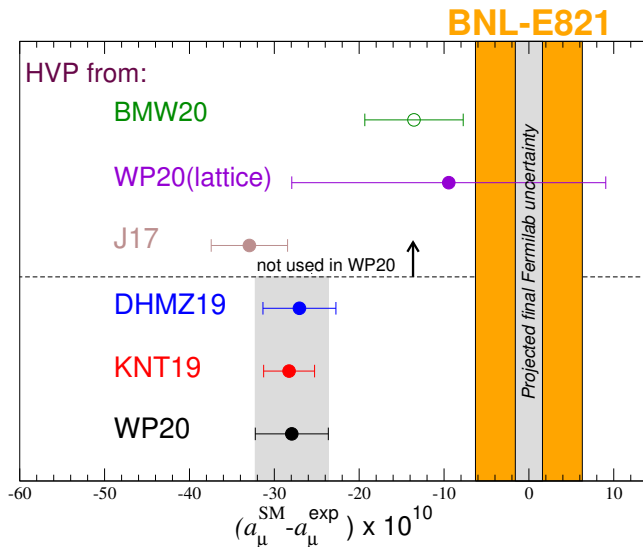
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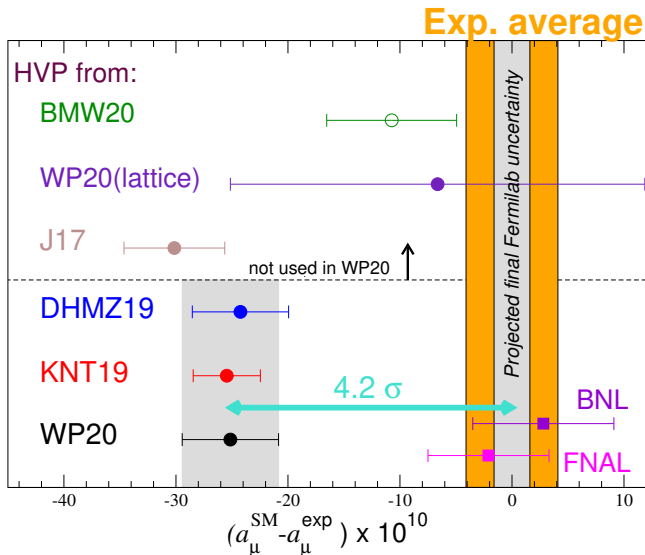
Present status of $(g - 2)_\mu$: experiment vs SM

Before



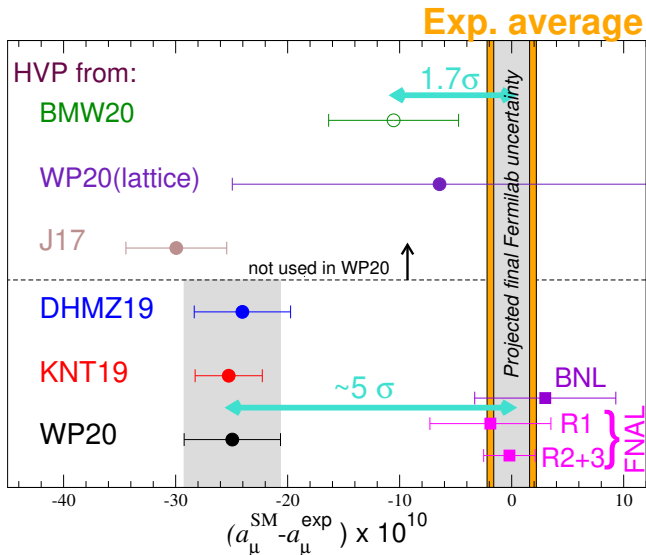
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After the 2021 Fermilab result



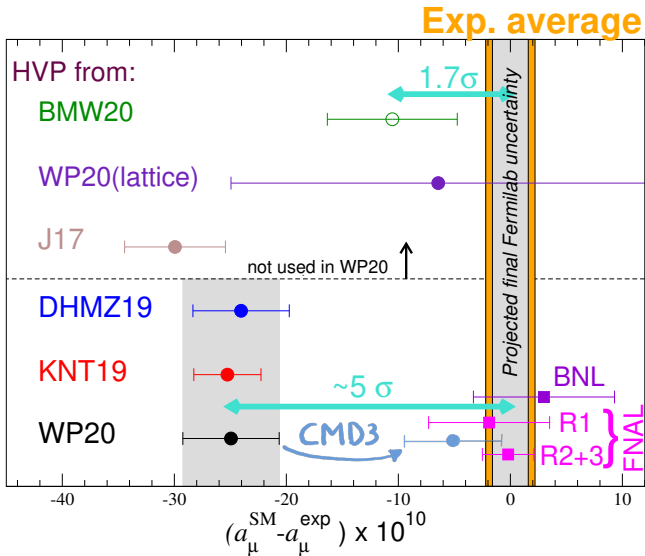
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After the 2023 Fermilab result



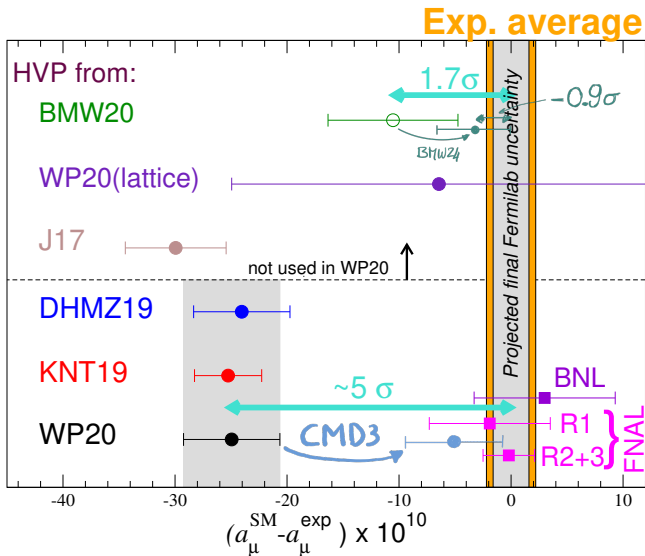
Present status of $(g - 2)_\mu$: experiment vs SM

After the 2023 Fermilab result and $\sigma(e^+e^- \rightarrow \pi^+\pi^-)$ from CMD3



Present status of $(g - 2)_\mu$: experiment vs SM

After



White Paper (2020): $(g - 2)_\mu$, experiment vs SM

Contribution	Value $\times 10^{11}$
HVP LO (e^+e^-)	6931(40)
HVP NLO (e^+e^-)	-98.3(7)
HVP NNLO (e^+e^-)	12.4(1)
HVP LO (lattice, $udsc$)	7116(184)
HLbL (phenomenology)	92(19)
HLbL NLO (phenomenology)	2(1)
HLbL (lattice, uds)	79(35)
HLbL (phenomenology + lattice)	90(17)
QED	116 584 718.931(104)
Electroweak	153.6(1.0)
HVP (e^+e^- , LO + NLO + NNLO)	6845(40)
HLbL (phenomenology + lattice + NLO)	92(18)
Total SM Value	116 591 810(43)
Experiment	116 592 059(22)
Difference: $\Delta a_\mu := a_\mu^{\text{exp}} - a_\mu^{\text{SM}}$	249(48)

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White Paper:

T. Aoyama et al. Phys. Rep. 887 (2020) = WP(20)

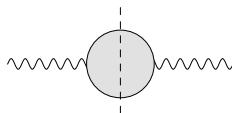
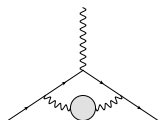
Muon $g - 2$ Theory Initiative

Plenary Workshops:

- ▶ 1st, Q-Center (Fermilab), 3-6 June 2017
- ▶ 2nd, Mainz, 18-22 June 2018
- ▶ 3rd, Seattle, 9-13 September 2019
- ▶ 4th, KEK (virtual), 28 June-02 July 2021
- ▶ 5th, Higgs Center Edinburgh, 5-9 Sept. 2022
- ▶ 6th, Bern, 4-8 Sept. 2023
- ▶ 7th, KEK, 9-13 Sept. 2024

Theory uncertainty comes from hadronic physics

- ▶ Hadronic contributions responsible for most of the theory uncertainty
- ▶ Hadronic vacuum polarization (HVP) is $\mathcal{O}(\alpha^2)$, dominates the total uncertainty, despite being known to $< 1\%$

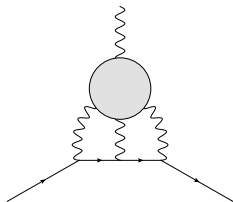


- ▶ unitarity and analyticity \Rightarrow dispersive approach
- ▶ \Rightarrow direct relation to experiment: $\sigma_{\text{tot}}(e^+e^- \rightarrow \text{hadrons})$
- ▶ e^+e^- Exps: BaBar, Belle, BESIII, CMD2/3, KLOE2, SND
- ▶ **alternative approach**: lattice, becoming competitive

(BMW, ETMC, Fermilab, HPQCD, Mainz, MILC, RBC/UKQCD)

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- ▶ Hadronic contributions responsible for most of the theory uncertainty
- ▶ Hadronic vacuum polarization (HVP) is $\mathcal{O}(\alpha^2)$, dominates the total uncertainty, despite being known to $< 1\%$
- ▶ Hadronic light-by-light (HLbL) is $\mathcal{O}(\alpha^3)$, known to $\sim 20\%$, second largest uncertainty (now subdominant)



- ▶ **earlier**: model-based—uncertainties difficult to quantify
- ▶ **recently**: dispersive approach \Rightarrow data-driven, systematic treatment
- ▶ lattice QCD is competitive

(Mainz, RBC/UKQCD)

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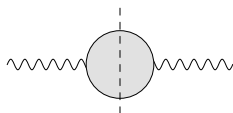
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HVP contribution: Master Formula

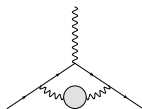
Unitarity relation: **simple**, same for all intermediate states



$$\text{Im}\bar{\Pi}(q^2) \propto \sigma(e^+e^- \rightarrow \text{hadrons}) = \sigma(e^+e^- \rightarrow \mu^+\mu^-)R(q^2)$$

Analyticity $\left[\bar{\Pi}(q^2) = \frac{q^2}{\pi} \int ds \frac{\text{Im}\bar{\Pi}(s)}{s(s-q^2)} \right] \Rightarrow$ **Master formula for HVP**

Bouchiat, Michel (61)

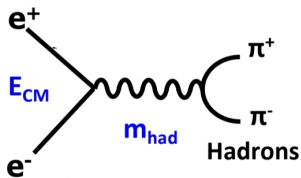


\Leftrightarrow

$$a_{\mu}^{\text{hvp}} = \frac{\alpha^2}{3\pi^2} \int_{s_{\text{th}}}^{\infty} \frac{ds}{s} K(s)R(s)$$

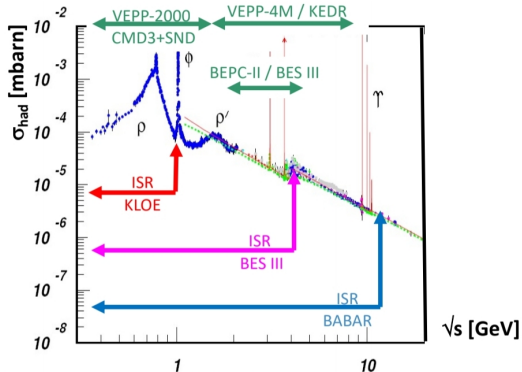
$K(s)$ known, depends on m_{μ} and $K(s) \sim \frac{1}{s}$ for large s

HVP contribution: Master Formula



- No systematic variation of E_{beam}
- High statistics thanks to high luminosity
- Radiative corrections (H_{rad})

PHOKHARA event generator



Comparison between DHMZ19 and KNT19

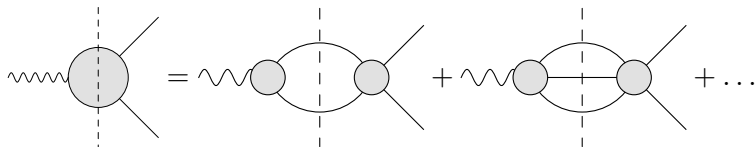
	DHMZ19	KNT19	Difference
$\pi^+\pi^-$	507.85(3.38)	504.23(1.90)	3.62
$\pi^+\pi^-\pi^0$	46.21(1.45)	46.63(94)	-0.42
$\pi^+\pi^-\pi^+\pi^-$	13.68(0.30)	13.99(19)	-0.31
$\pi^+\pi^-\pi^0\pi^0$	18.03(0.55)	18.15(74)	-0.12
K^+K^-	23.08(0.44)	23.00(22)	0.08
$K_S K_L$	12.82(0.24)	13.04(19)	-0.22
$\pi^0\gamma$	4.41(0.10)	4.58(10)	-0.17
Sum of the above	626.08(3.90)	623.62(2.27)	2.46
[1.8, 3.7] GeV (without $c\bar{c}$)	33.45(71)	34.45(56)	-1.00
$J/\psi, \psi(2S)$	7.76(12)	7.84(19)	-0.08
[3.7, ∞) GeV	17.15(31)	16.95(19)	0.20
Total $a_\mu^{\text{HVP, LO}}$	694.0(4.0)	692.8(2.4)	1.2

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For the dominant $\pi\pi$ channel more theory input can be used

Omnès representation including isospin breaking

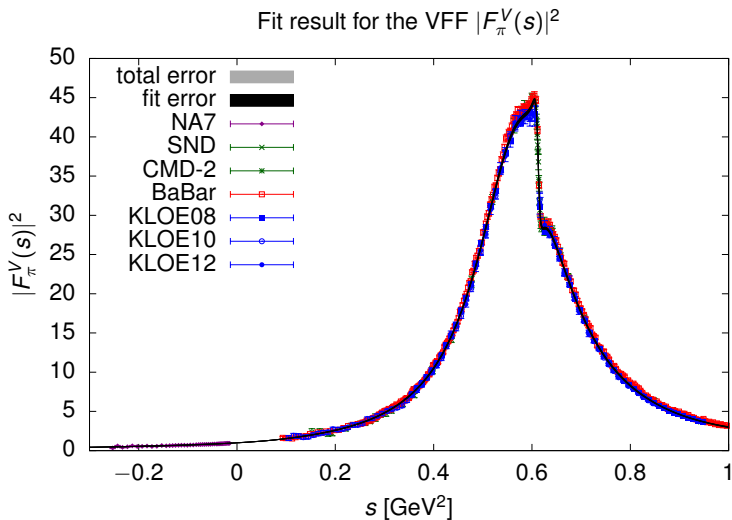


$$F_V(s) = \Omega_{\pi\pi}(s) \cdot G_\omega(s) \cdot \Omega_{\text{in}}(s)$$

main contribution $\Omega_{\pi\pi}(s)$: 2 parameters

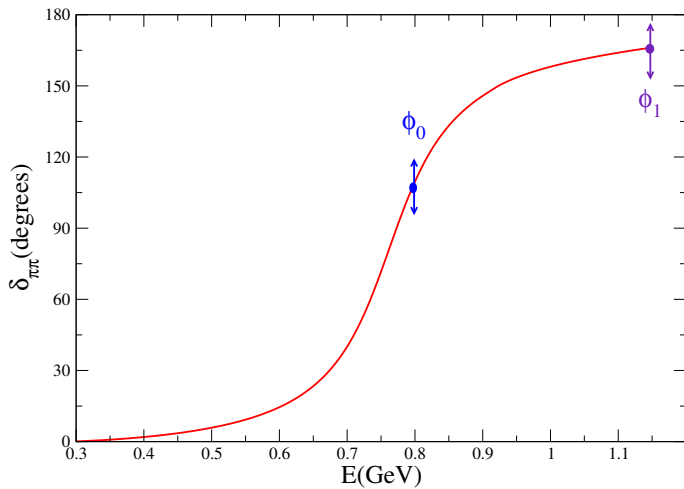
Fit results

GC, Hoferichter, Stoffer (18)



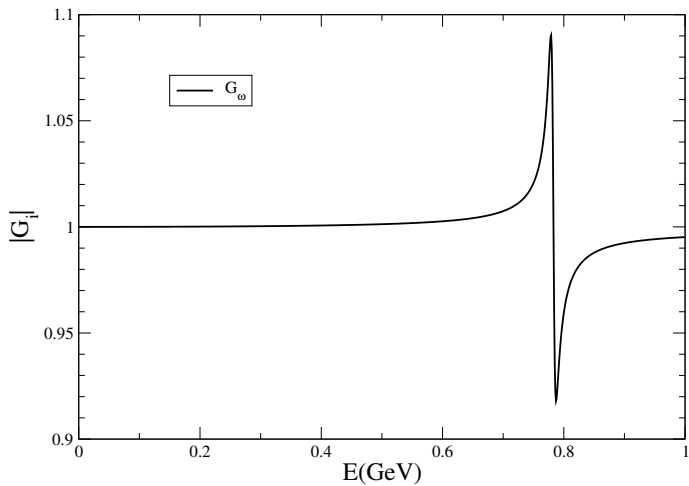
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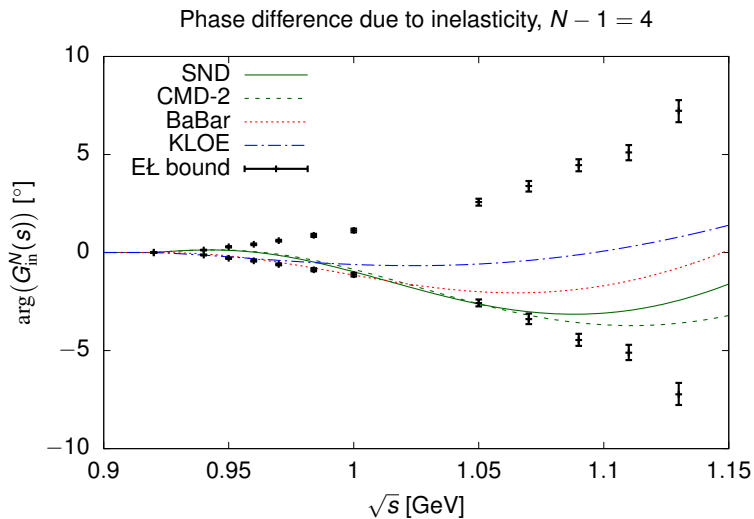
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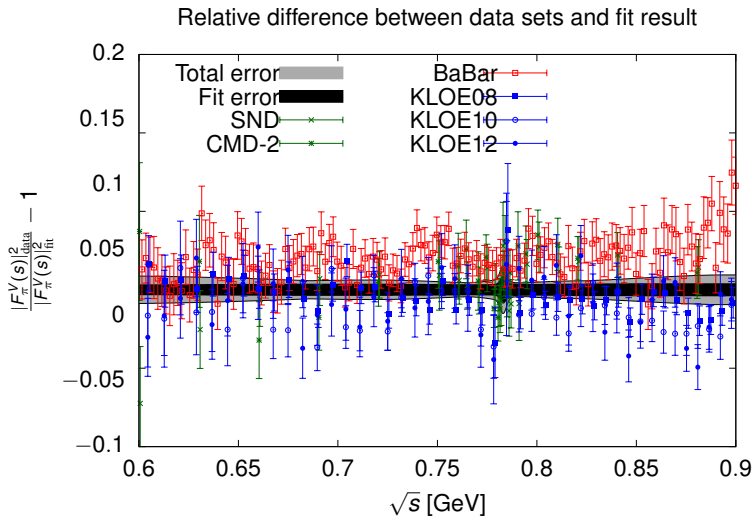
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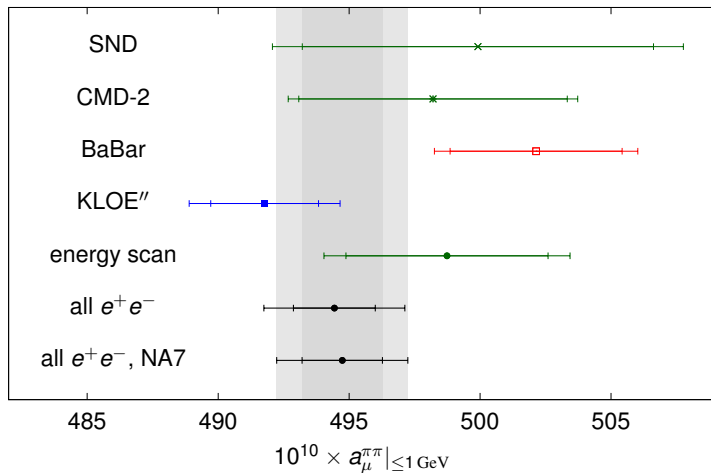
GC, Hoferichter, Stoffer (18)



Fit results

GC, Hoferichter, Stoffer (18)

Result for $a_{\mu}^{\pi\pi}|_{\leq 1 \text{ GeV}}$ from the VFF fits to single experiments and combinations



2π : comparison with the dispersive approach

2π channel described dispersively \Rightarrow more theory constraints

Ananthanarayan, Caprini, Das (19), GC, Hoferichter, Stoffer (18) WP(20)

Energy range	CHS18	DHMZ19	KNT19
≤ 0.6 GeV	110.1(9)	110.4(4)(5)	108.7(9)
≤ 0.7 GeV	214.8(1.7)	214.7(0.8)(1.1)	213.1(1.2)
≤ 0.8 GeV	413.2(2.3)	414.4(1.5)(2.3)	412.0(1.7)
≤ 0.9 GeV	479.8(2.6)	481.9(1.8)(2.9)	478.5(1.8)
≤ 1.0 GeV	495.0(2.6)	497.4(1.8)(3.1)	493.8(1.9)
[0.6, 0.7] GeV	104.7(7)	104.2(5)(5)	104.4(5)
[0.7, 0.8] GeV	198.3(9)	199.8(0.9)(1.2)	198.9(7)
[0.8, 0.9] GeV	66.6(4)	67.5(4)(6)	66.6(3)
[0.9, 1.0] GeV	15.3(1)	15.5(1)(2)	15.3(1)
≤ 0.63 GeV	132.8(1.1)	132.9(5)(6)	131.2(1.0)
[0.6, 0.9] GeV	369.6(1.7)	371.5(1.5)(2.3)	369.8(1.3)
$[\sqrt{0.1}, \sqrt{0.95}]$ GeV	490.7(2.6)	493.1(1.8)(3.1)	489.5(1.9)

Combination method and final result

Complete analyses DHMZ19 and KNT19, as well as CHS19 (2π) and HHK19 (3π), have been so combined:

- ▶ central values are obtained by simple averages (for each channel and mass range)
- ▶ the largest experimental and systematic uncertainty of DHMZ and KNT is taken
- ▶ 1/2 difference DHMZ–KNT (or BABAR–KLOE in the 2π channel, if larger) is added to the uncertainty

Final result:

$$\begin{aligned}
 a_{\mu}^{\text{HVP, LO}} &= 693.1(2.8)_{\text{exp}}(2.8)_{\text{sys}}(0.7)_{\text{DV+QCD}} \times 10^{-10} \\
 &= 693.1(4.0) \times 10^{-10}
 \end{aligned}$$

The BMW result

Borsanyi et al. Nature 2021

State-of-the-art lattice calculation of $a_{\mu}^{\text{HVP, LO}}$ based on

- ▶ current-current correlator, summed over all distances, integrated in time with appropriate kernel function (TMR)
- ▶ using staggered fermions on an $L \sim 6$ fm lattice ($L \sim 11$ fm used for finite volume corrections)
- ▶ at (and around) physical quark masses
- ▶ including isospin-breaking effects

The BMW result

Borsanyi et al. Nature 2021

Isospin-symmetric



Connected light

$$633.7(2.1)_{\text{stat}}(4.2)_{\text{sys}}$$



Connected strange

$$53.393(89)_{\text{stat}}(68)_{\text{sys}}$$



Connected charm

$$14.6(0)_{\text{stat}}(1)_{\text{sys}}$$



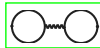
Disconnected

$$-13.36(1.18)_{\text{stat}}(1.36)_{\text{sys}}$$

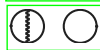
QED isospin breaking: valence



$$\text{Connected } -1.23(40)_{\text{stat}}(31)_{\text{sys}}$$



$$\text{Disconnected } -0.55(15)_{\text{stat}}(10)_{\text{sys}}$$



Strong-isospin breaking



Connected

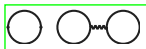
$$6.60(63)_{\text{stat}}(53)_{\text{sys}}$$



Disconnected

$$-4.67(54)_{\text{stat}}(69)_{\text{sys}}$$

QED isospin breaking: sea



$$\text{Connected } 0.37(21)_{\text{stat}}(24)_{\text{sys}}$$



$$\text{Disconnected } -0.040(33)_{\text{stat}}(21)_{\text{sys}}$$



Other

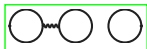
Bottom; higher-order;
perturbative

$$0.11(4)_{\text{tot}}$$

QED isospin breaking: mixed



$$\text{Connected } -0.0093(86)_{\text{stat}}(95)_{\text{sys}}$$



$$\text{Disconnected } 0.011(24)_{\text{stat}}(14)_{\text{sys}}$$

Finite-size effects

Isospin-symmetric

$$18.7(2.5)_{\text{tot}}$$

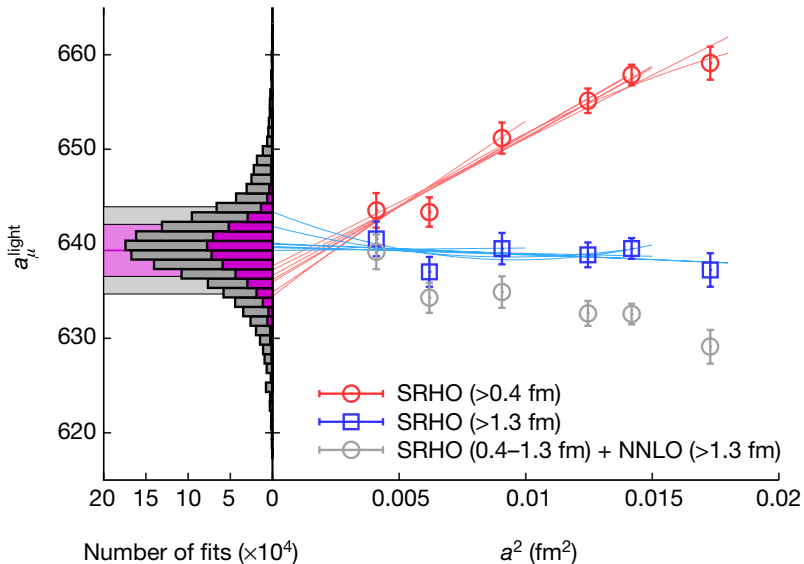
Isospin-breaking

$$0.0(0.1)_{\text{tot}}$$

$$a_{\mu}^{\text{LO-HVP}} (\times 10^{10}) = 707.5(2.3)_{\text{stat}}(5.0)_{\text{sys}}(5.5)_{\text{tot}}$$

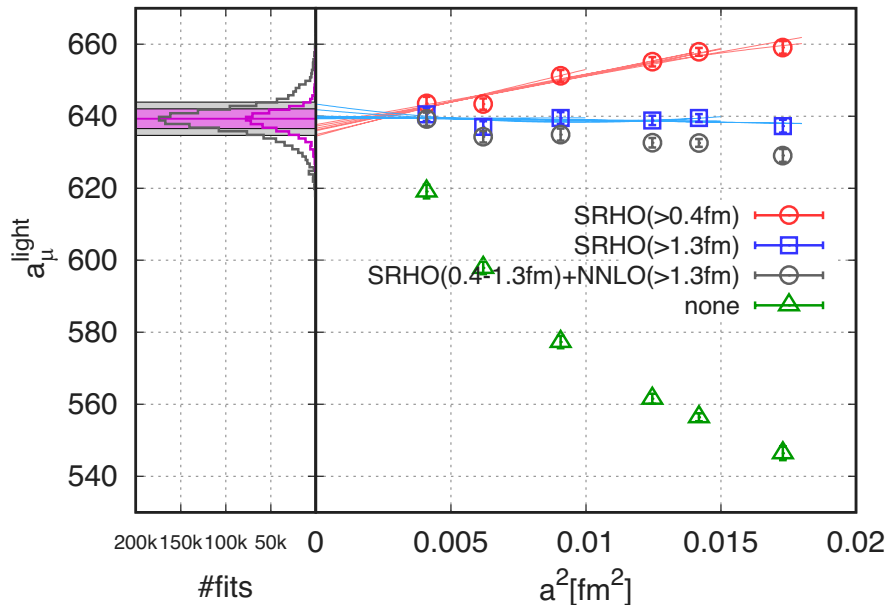
The BMW result

Borsanyi et al. Nature 2021



The BMW result

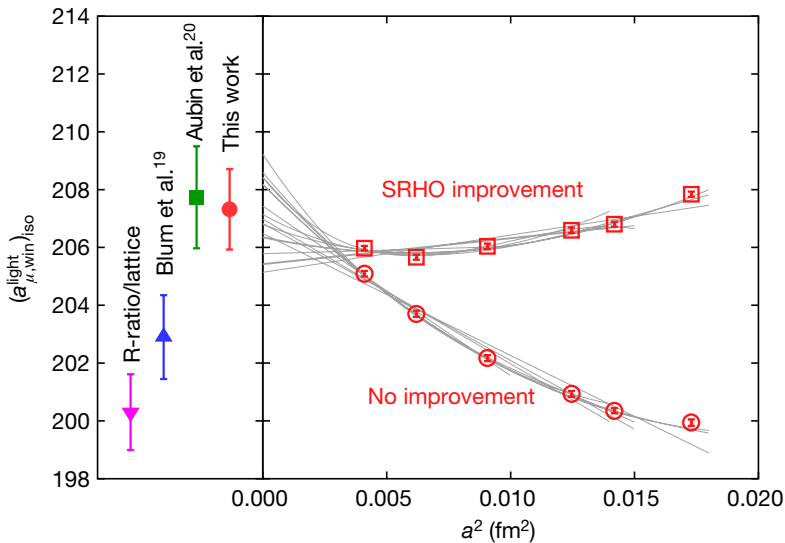
Borsanyi et al. Nature 2021



The BMW result

Borsanyi et al. Nature 2021

Article

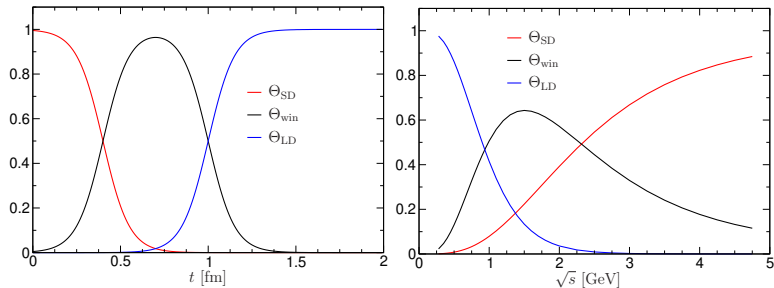


The BMW result

Borsanyi et al. Nature 2021

Weight functions for window quantities

RBC/UKQCD (18)



Consequences of the BMW result

A shift in the value of $a_\mu^{\text{HVP, LO}}$ would have consequences:

- ▶ $\Delta a_\mu^{\text{HVP, LO}} \Leftrightarrow \Delta\sigma(e^+e^- \rightarrow \text{hadrons})$
- ▶ $\Delta\alpha_{\text{had}}(M_Z^2)$ is determined by an integral of the same $\sigma(e^+e^- \rightarrow \text{hadrons})$ (more weight at high energy)
- ▶ changing $a_\mu^{\text{HVP, LO}}$ necessarily implies a shift in $\Delta\alpha_{\text{had}}(M_Z^2)$: size depends on the energy range of $\Delta\sigma(e^+e^- \rightarrow \text{hadrons})$
- ▶ a shift in $\Delta\alpha_{\text{had}}(M_Z^2)$ has an impact on the EW-fit
- ▶ to save the EW-fit $\Delta\sigma(e^+e^- \rightarrow \text{hadrons})$ must occur below ~ 1 (max 2) GeV

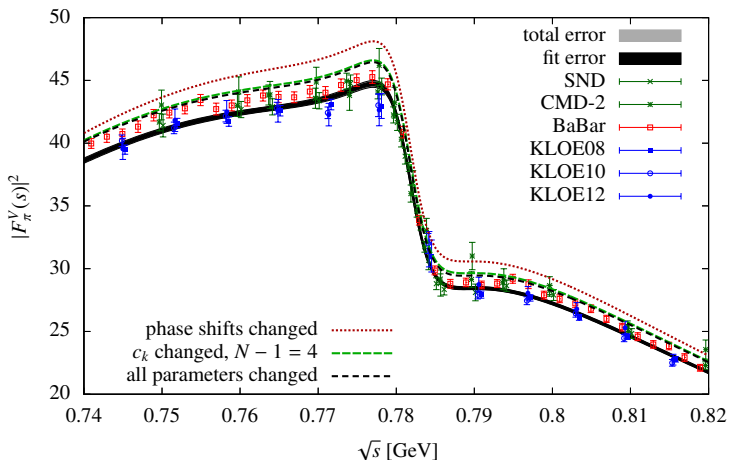
Crivellin, Hoferichter, Manzari, Montull (20)/Keshavarzi, Marciano, Passera, Sirlin (20)/Malaescu, Schott (20)

- ▶ or the need for BSM physics would be moved elsewhere

Changes in $\sigma(e^+e^- \rightarrow \text{hadrons})$ below 1 GeV?

- ▶ Below 1 – 2 GeV only one significant channel: $\pi^+\pi^-$
- ▶ Strongly constrained by analyticity and unitarity ($F_\pi^V(s)$)
- ▶ $F_\pi^V(s)$ parametrization which satisfies these
⇒ small number of parameters GC, Hoferichter, Stoffer (18)
- ▶ $\Delta a_\mu^{\text{HVP, LO}} \Leftrightarrow$ shifts in these parameters
analysis of the corresponding scenarios GC, Hoferichter, Stoffer (21)

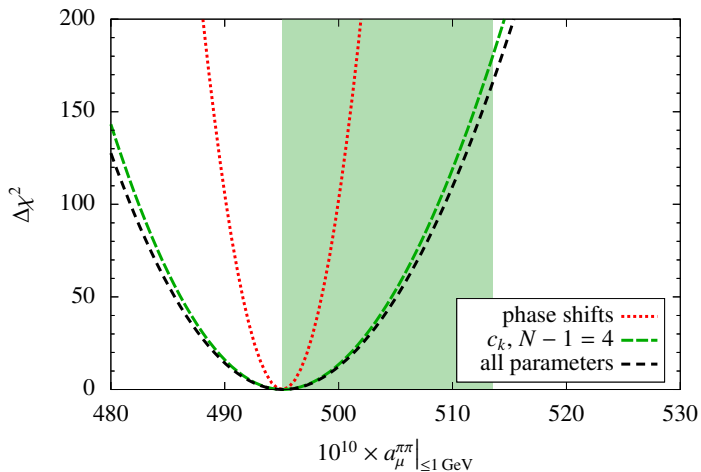
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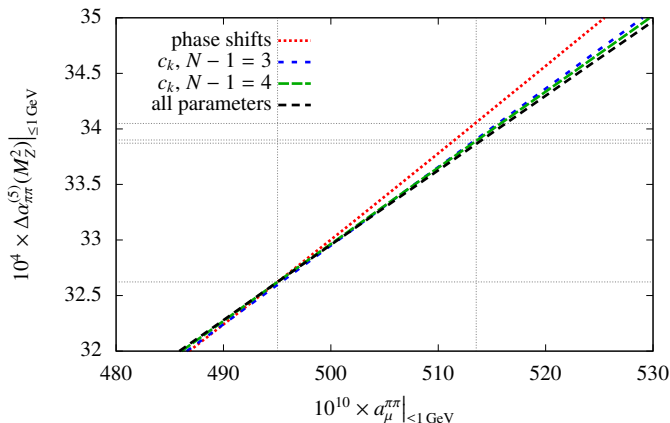
GC, Hoferichter, Stoffer (21)

Tension [BMW20 vs e^+e^- data] stronger for KLOE than for BABAR

Changes in $\sigma(e^+e^- \rightarrow \text{hadrons})$ below 1 GeV?



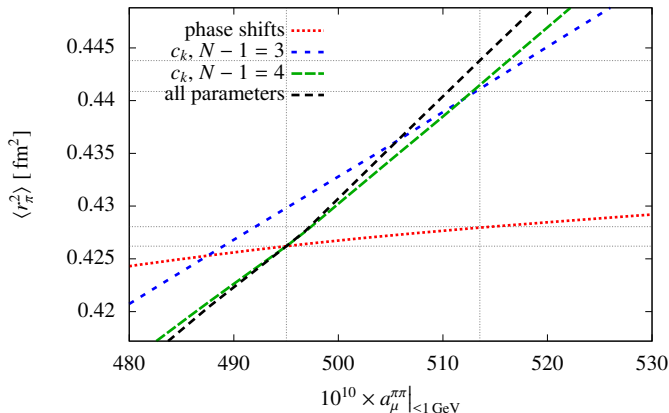
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GC, Hoferichter, Stoffer (21)

$$10^4 \Delta\alpha_{\text{had}}^{(5)}(M_Z^2) = \begin{cases} 272.2(4.1) & \text{EW fit} \\ 276.1(1.1) & \sigma_{\text{had}}(s) \end{cases}$$

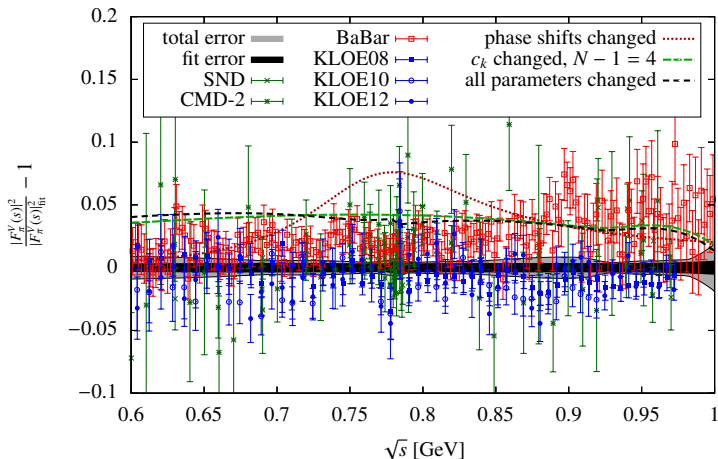
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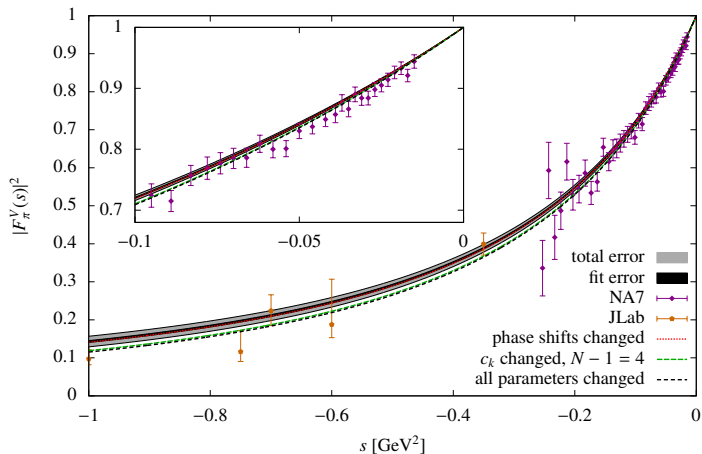
GC, Hoferichter, Stoffer (21)

$$\langle r_\pi^2 \rangle = \begin{cases} 0.429(4)\text{fm}^2 & \text{CHS(18)} \\ 0.436(5)(12)\text{fm}^2 & \chi\text{QCD(20)} \end{cases}$$

Changes in $\sigma(e^+e^- \rightarrow \text{hadrons})$ below 1 GeV?

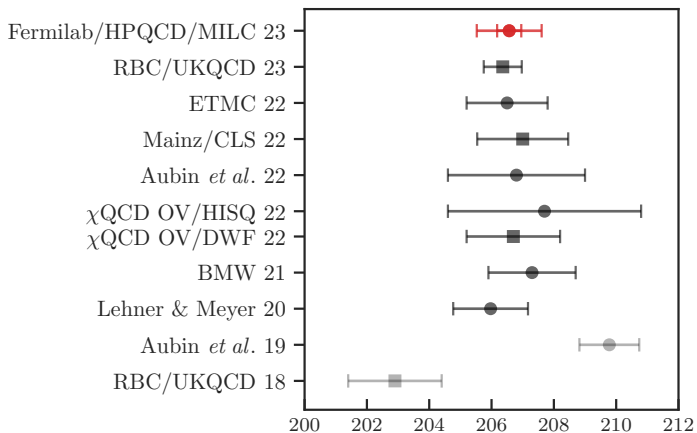


Changes in $\sigma(e^+e^- \rightarrow \text{hadrons})$ below 1 GeV?



Present status of the window quantities

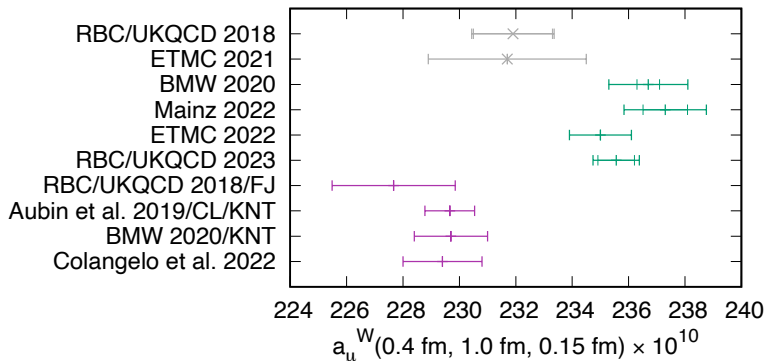
Several lattice calculations have confirmed BMW's result



arXiv:2301.08274, [Fermilab Lattice-HPQCD-MILC \(23\)](#)

Present status of the window quantities

Several lattice calculations have confirmed BMW's result



arXiv:2301.08696 [RBC/UKQCD \(23\)](#)

Individual-channel contributions to a_μ^{win}

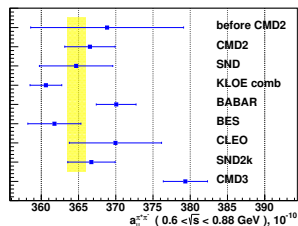
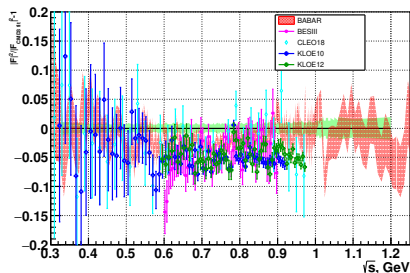
Channel	total	window
$\pi^+\pi^-$	504.23(1.90)	144.08(49)
$\pi^+\pi^-\pi^0$	46.63(94)	18.63(35)
$\pi^+\pi^-\pi^+\pi^-$	13.99(19)	8.88(12)
$\pi^+\pi^-\pi^0\pi^0$	18.15(74)	11.20(46)
K^+K^-	23.00(22)	12.29(12)
$K_S^0K_L^0$	13.04(19)	6.81(10)
$\pi^0\gamma$	4.58(10)	1.58(4)
Sum of the above	623.62(2.27)	203.47(78)
[1.8, 3.7] GeV (without $c\bar{c}$)	34.45(56)	15.93(26)
$J/\psi, \psi(2S)$	7.84(19)	2.27(6)
[3.7, ∞) GeV	16.95(19)	1.56(2)
WP(20) / GC, El-Khadra <i>et al.</i> (22)	693.1(4.0)	229.4(1.4)
BMWc	707.5(5.5)	236.7(1.4)
Mainz/CLS		237.3(1.5)
ETMc		235.0(1.1)
RBC/UKQCD		235.6(0.8)

Numbers for the channels refer to KNT19 — thanks to Alex Keshavarzi for providing them

$$\Delta a_\mu^{\text{HVP, LO}} = 14.4(6.8) (2.1\sigma), \quad \Delta a_\mu^{\text{win}} \sim 6.5(1.5) (\sim 4.3\sigma)$$

CMD-3 measurement of $e^+e^- \rightarrow \pi^+\pi^-$

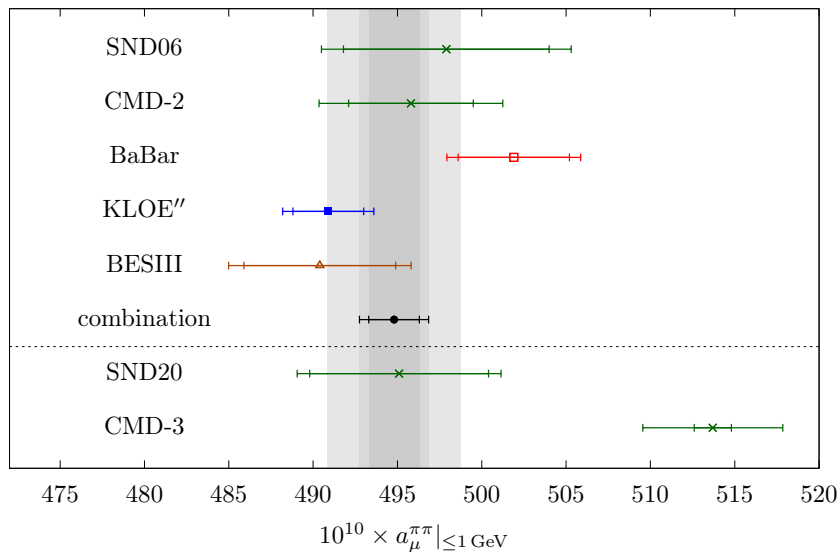
F. Ignatov et al., CMD-3, arXiv:2302.08834



The comparison of pion form factor measured in this work with the most recent ISR experiments (BABAR [21], KLOE [18, 19], BES [22]) is shown in Fig. 34. The comparison with the most precise previous energy scan experiments (CMD-2 [12, 13, 14, 15], SND [16] at the VEPP-2M and SND [23] at the VEPP-2000) is shown in Fig. 35. **The new result generally shows larger pion form factor in the whole energy range under discussion.** The most significant difference to other energy scan measurements, including previous CMD-2 measurement, is observed at the left side of ρ -meson ($\sqrt{s} = 0.6 - 0.75$ GeV), where it reach up to 5%, well beyond the combined systematic and statistical errors of the new and previous results. **The source of this difference is unknown at the moment.**

Preliminary analysis of the CMD-3 measurement

GC, Hoferichter and Stoffer, [arXiv:2308.04217](https://arxiv.org/abs/2308.04217) (thanks for providing the plot)



Preliminary analysis of the CMD-3 measurement

GC, Hoferichter and Stoffer, arXiv:2308.04217

$10^{10} \times$	$a_{\mu}^{\pi\pi} _{\leq 1\text{GeV}}$	$a_{\mu}^{\pi\pi, \text{win}} _{\leq 1\text{GeV}}$	χ^2/dof
SND06	497.9(6.1)(4.2)	139.6(1.8)(1.0)	1.09
CMD-2	495.8(3.7)(4.0)	139.4(1.0)(0.8)	1.01
BaBar	501.9(3.3)(2.2)	140.6(1.0)(0.7)	1.17
KLOE''	490.9(2.1)(1.7)	137.1(0.6)(0.4)	1.13
BESIII	490.4(4.5)(3.0)	137.8(1.3)(0.4)	1.01
SND20	495.1(5.3)(2.9)	139.2(1.5)(0.4)	1.88
CMD-3	513.7(1.1)(4.0)	144.0(0.3)(1.1)	1.09
Combination	494.8(1.5)(1.4)(3.4)	138.3(0.4)(0.3)(1.1)	1.21

Combination: NA7 + all data sets other than SND20 and CMD-3

$$\Delta a_{\mu}^{\text{HVP, LO}}(\text{CMD-3-Comb.}) = 18.9(5.1), \quad \Delta a_{\mu}^{\text{win}}(\text{CMD-3-Comb.}) = 5.7(1.5)$$

$$\Delta a_{\mu}^{\text{HVP, LO}}(\text{BMW-WP20}) = 14.4(6.8), \quad \Delta a_{\mu}^{\text{win}}(\text{Lattice-WP20}) \sim 6.5(1.5)$$

Preliminary analysis of the CMD-3 measurement

GC, Hoferichter and Stoffer, arXiv:2308.04217

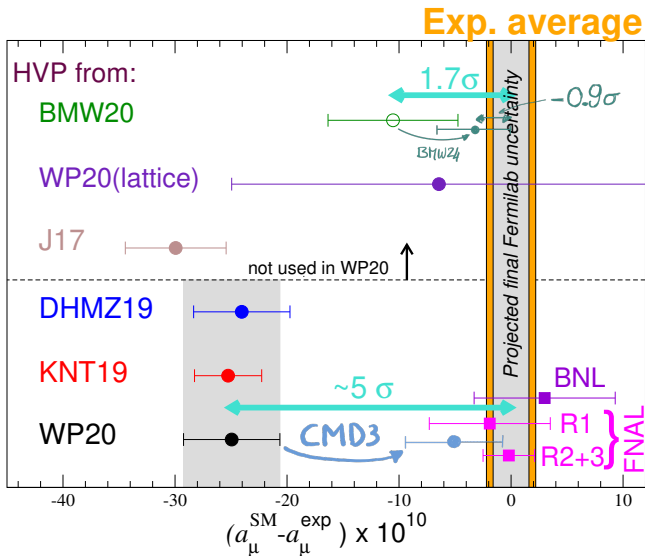
Discrepancy	$a_\mu^{\pi\pi} \Big _{[0.60,0.88] \text{ GeV}}$	$a_\mu^{\pi\pi} \Big _{\leq 1 \text{ GeV}}$	int window
SND06	1.8σ	1.7σ	1.7σ
CMD-2	2.3σ	2.0σ	2.1σ
BaBar	3.3σ	2.9σ	3.1σ
KLOE''	5.6σ	4.8σ	5.4σ
BESIII	3.0σ	2.8σ	3.1σ
SND20	2.2σ	2.1σ	2.2σ
Combination	$4.2\sigma [6.1\sigma]$	$3.7\sigma [5.0\sigma]$	$3.8\sigma [5.7\sigma]$

Combination: NA7 + all data sets other than SND20 and CMD-3

$$\Delta a_\mu^{\text{HVP, LO}}(\text{CMD-3-Comb.}) = 18.9(5.1), \quad \Delta a_\mu^{\text{win}}(\text{CMD-3-Comb.}) = 5.7(1.5)$$

$$\Delta a_\mu^{\text{HVP, LO}}(\text{BMW-WP20}) = 14.4(6.8), \quad \Delta a_\mu^{\text{win}}(\text{Lattice-WP20}) \sim 6.5(1.5)$$

Present status of $(g - 2)_\mu$: experiment vs SM



Outline

Introduction: $(g - 2)_\mu$ in the Standard Model

Hadronic Vacuum Polarization contribution

White Paper: data-driven approach

Lattice: the BMW result

Lattice vs data-driven: the window quantity

The new CMD3 measurement of $e^+e^- \rightarrow \pi^+\pi^-$

Conclusions and Outlook

Conclusions

- ▶ Data-driven evaluation of the HVP contribution (WP20):
0.6% error \Rightarrow dominates the theory uncertainty
- ▶ Dominant contribution to HVP: $\pi\pi$ (<1 GeV). WP20 based on:
CMD-2, SND06, BaBar, KLOE, BES-III
New puzzle: measurement by CMD-3 significantly higher!
- ▶ Recent lattice calculation [BMW(20)] has reached a similar precision
but differs from the dispersive one (=from e^+e^- data).
If confirmed \Rightarrow discrepancy with experiment \searrow below 1σ
- ▶ Intermediate window of BMW has been confirmed by other lattice
collaborations (Aubin et al., Mainz, ETMc, RBC/UKQCD, Fermilab-HPQCD-MILC)
and disagrees with data-driven [other than CMD-3, which would agree]

Outlook

- ▶ The Fermilab experiment aims to reduce the BNL uncertainty by a **factor four** \Rightarrow potential **7σ** discrepancy
- ▶ Improvements on the SM theory/data side:
 - ▶ Situation for HVP data-driven **urgently needs to be clarified**:
 - New **CMD-3** result—after thorough scrutiny—is a puzzle
 - Forthcoming measur./analyses: **BaBar**, **Belle II**, **BESIII**, **KLOE**, **SND**
 - Model-independent evaluation of **RadCorr** underway (but unlikely the culprit)
 - Monte Carlo codes used by experiments: **what is their role?**
 - **MuonE** will provide an alternative way to measure HVP
 - ▶ HVP lattice:
calculations w/ precision \sim **BMW** for $a_\mu^{\text{HVP, LO}}$ expected soon