OUTLINE OF PART III: DIGITAL QUANTUM COMPUTING TIME EVOLUTION IN LGTs

i) A general algorithmic strategy
ii) Time evolution in the Schwinger model
- In purely fermionic formulation
- In fermion-boson formulation

iii) Outlining the differences between Abelian and non-Abelian algorithms
iv) Finally…what we did not cover

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SIMULATION STRATEGIES: FEW QUESTIONS

Q1: What is the best breakdown of Hamiltonian term to H_i terms such that:

i) each term can be simulated with the least resources,
ii) the number of terms to be simulated is minimized,
iii) the Trotter error is minimized,
iv) as many symmetries as possible are retained?

We may not be able to simultaneously satisfy all these conditions so we need to seek a balance. The last condition may or may not matter!

Q2: How to simulate each e^{-itH_i} ? This amounts to:

i) finding the unitary transformation that diagonalizes e^{-itH_i} in the computational basis, i.e., $e^{-itH_i} = \mathcal{U}_i e^{-it\mathcal{D}_i} \mathcal{U}_i^\dagger$.
ii) circuitizing the unitary transformation $\mathcal{U}_i$,
iii) circuitizing the diagonal form $e^{-it\mathcal{D}_i}$.

If e^{-itH_i} is already diagonal, steps i) and ii) are not needed.

Q3: What quantum resources should we minimize given those choices in the previous Qs?

i) In the near-term scenario,
- the hardware systems are small so the **less ancillary qubits** the better,
- single-qubit gates are almost free but **two-qubit gates (CNOT)** are of low fidelity.

ii) In the far-term scenario,
- we likely do not have qubit-resource constraints,
- compilation of all Clifford gates (including CNOT) is less costly but non-Clifford (**T gates**) have high fault-tolerant implementation cost.

Q4: Given all these consideration, which Hamiltonian formulation and basis states of the theory are most suitable? We may need to consider formulations that:

i) give the desired continuum physics faster with the least resources,
ii) have the least encoding overhead,
ii) have less complex terms,
iii) respect more symmetries by construction.

We are not considering state preparation and measurements here, but those often enter our considerations of what is the most suitable formulation given the observable of interest.

RESOURCE ANALYSIS

$$||V_p(t) - e^{-itH}|| \leq \epsilon$$

Given the accuracy ϵ on the time evolution operator, how many ancilla qubits and costly gates are needed for simulating a Hamiltonian with given parameters for time t using the p-th order product formula?

For a LGT Hamiltonian, these are volume, lattice spacing, couplings, masses, and truncation scale of the bosonic fields.

The errors that accumulate to add up to the total error ϵ are:

i) Trotter error,
ii) function-evaluation approximation error,
iii) gate-synthesis error.

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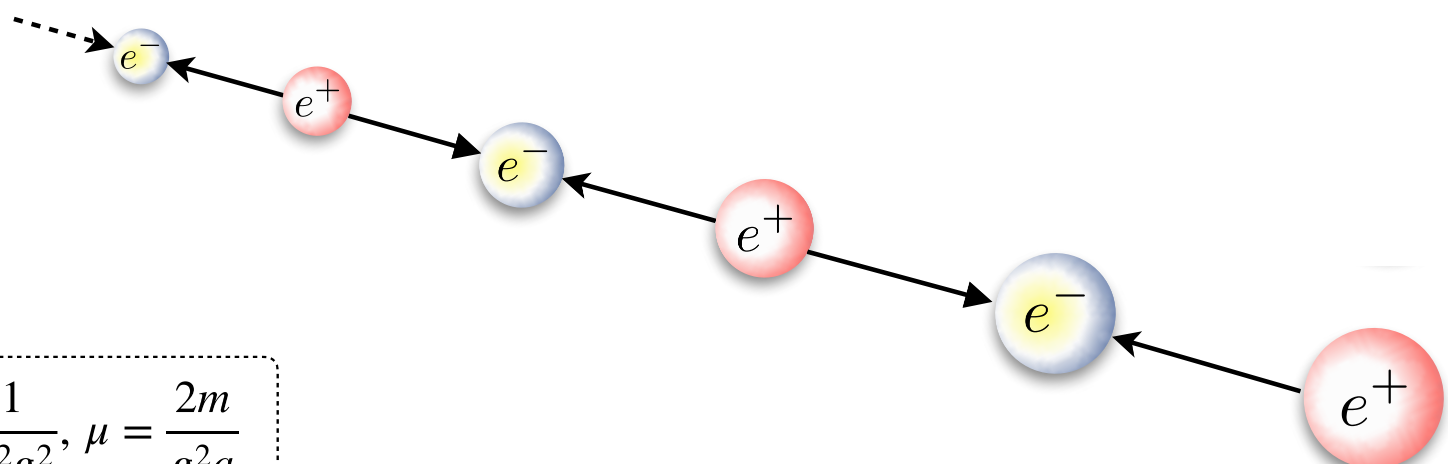
(RESCALED HENCE DIMENSIONLESS)
LATTICE SCHWINGER MODEL HAMILTONIAN

$$H = \mathrm{x} \sum_x \left[\psi^\dagger(x)\psi(x+1) + \mathrm{h.c.}\right] + \sum_x \left\{ \varepsilon_0 - \sum_{y=0}^{x} \left[\psi^\dagger(y)\psi(y) - \frac{1-(-1)^y}{2} \right] \right\}^2 + \mu \sum_x (-1)^x \psi^\dagger(x)\psi(x)$$

Staggered mass term

Fermion hopping via gauge links

Electric field energy



$$\mathrm{x} = \frac{1}{a^2 g^2}, \ \mu = \frac{2m}{g^2 a}$$

(RESCALED HENCE DIMENSIONLESS)
LATTICE SCHWINGER MODEL HAMILTONIAN

$$H = \mathrm{x} \sum_x \left[\psi^\dagger(x)\psi(x+1) + \mathrm{h.c.}\right] + \sum_x \left\{ \varepsilon_0 - \sum_{y=0}^{x} \left[\psi^\dagger(y)\psi(y) - \frac{1-(-1)^y}{2} \right] \right\}^2 + \mu \sum_x (-1)^x \psi^\dagger(x)\psi(x)$$

Staggered mass term

Fermion hopping via gauge links

Electric field energy

Jordan-Wigner
transformation

$$H = \mathrm{x} \sum_x \left[\sigma^+(x)\sigma^-(x+1) + \mathrm{h.c.}\right] + \sum_x \left\{ \varepsilon_0 + \frac{1}{2} \sum_{y=0}^{x} \left[\sigma^z(y) + (-1)^y\right] \right\}^2 + \frac{\mu}{2} \sum_x (-1)^x \sigma^z(x)$$

Nearest neighbor
spin-spin interactions

Long range spin-spin
interactions plus an
effective magnetic field

An effective
magnetic field

LET'S START SIMPLE: THE **FULLY FERMIONIC** REPRESENTATION WITH **FIRTS-ORDER** PRODUCT FORMULA.

$$H = \mathrm{x}\sum_x \left[\sigma^+(x)\sigma^-(x+1) + \mathrm{h.c.}\right] + \sum_x \left\{\varepsilon_0 + \frac{1}{2}\sum_{y=0}^{x}\left[\sigma^z(y) + (-1)^y\right]\right\}^2 + \frac{\mu}{2}\sum_x (-1)^x \sigma^z(x)$$

$$= H^{\mathrm{x}} + H^{ZZ} + H^Z \quad \text{or} \quad H^{(XX)} + H^{(YY)} + H^{ZZ} + H^Z$$

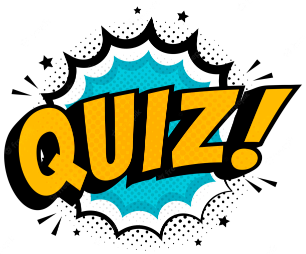
Two time orderings, one that respects the global charge conservation:

$$V_1(\delta t) = e^{-i\delta t\,\hat{H}^Z} e^{-i\delta t\,\hat{H}^{ZZ}} \prod_{k=1}^{(N/2)-1} e^{-i\delta t\,\hat{H}^x_{2k,2k+1}} \prod_{k=1}^{N/2} e^{-i\delta t\,\hat{H}^x_{2k-1,2k}}$$

and one that breaks it!

$$V_1'(\delta t) = e^{-i\delta t\,\hat{H}^Z} e^{-i\delta t\,\hat{H}^{ZZ}} \prod_{k=1}^{N-1} e^{-i\delta t\,\hat{H}^{(YY)}_{k,k+1}} \prod_{k=1}^{N-1} e^{-i\delta t\,\hat{H}^{(XX)}_{k,k+1}}$$

and many more!

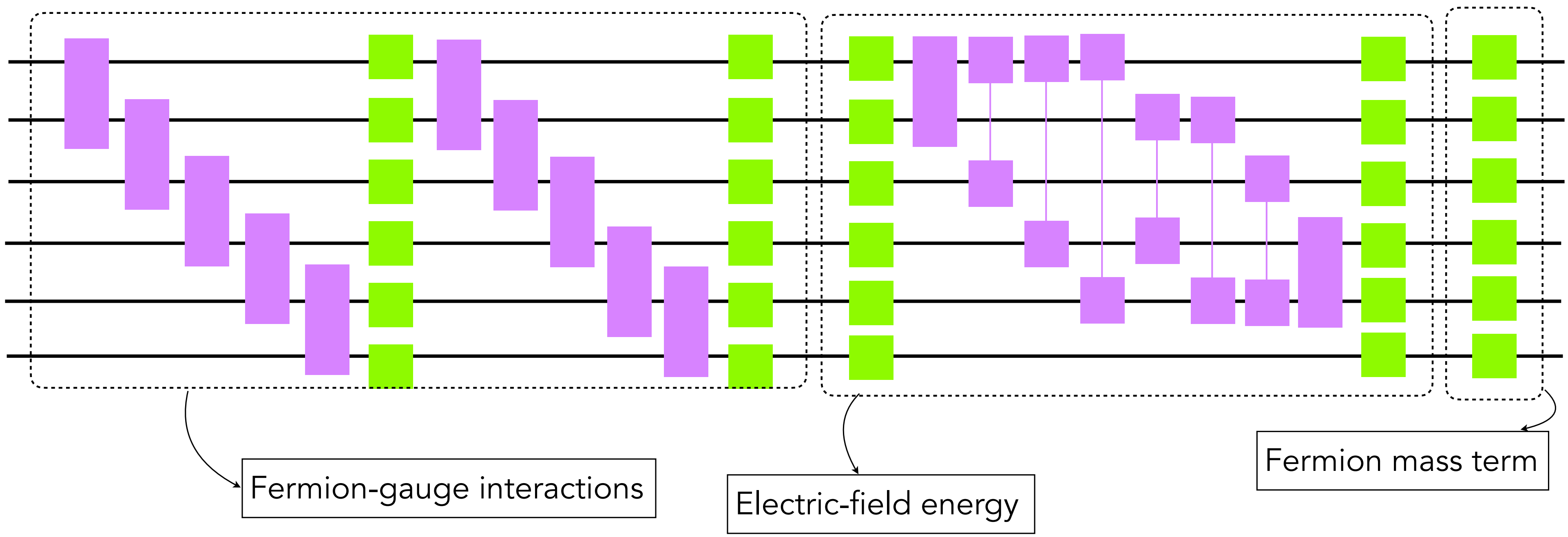


What is the global conserved charge in the Schwinger-model Hamiltonian?
Why is one of the schemes in the previous slide conserves the global charge and the other does not?

- $R_{ij}^{zz} \equiv e^{-i\theta\sigma_i^z\sigma_j^z}$ can be implemented either directly (like in trapped ions) or by two CNOTs and one single-qubit rotation since $e^{-i\theta\sigma_i^z\sigma_j^z} = \mathrm{CNOT}_{ij}\, R_i^z(\theta)\, \mathrm{CNOT}_{ij}$.
- $e^{-i\theta\sigma_i^x\sigma_j^x}$ and $e^{-i\theta\sigma_i^y\sigma_j^y}$ can be implemented similarly by rotating to the eigenstates of σ^z.
- $e^{-i\theta\sigma_i^z}$ is already an elementary gate and can be applied directly.

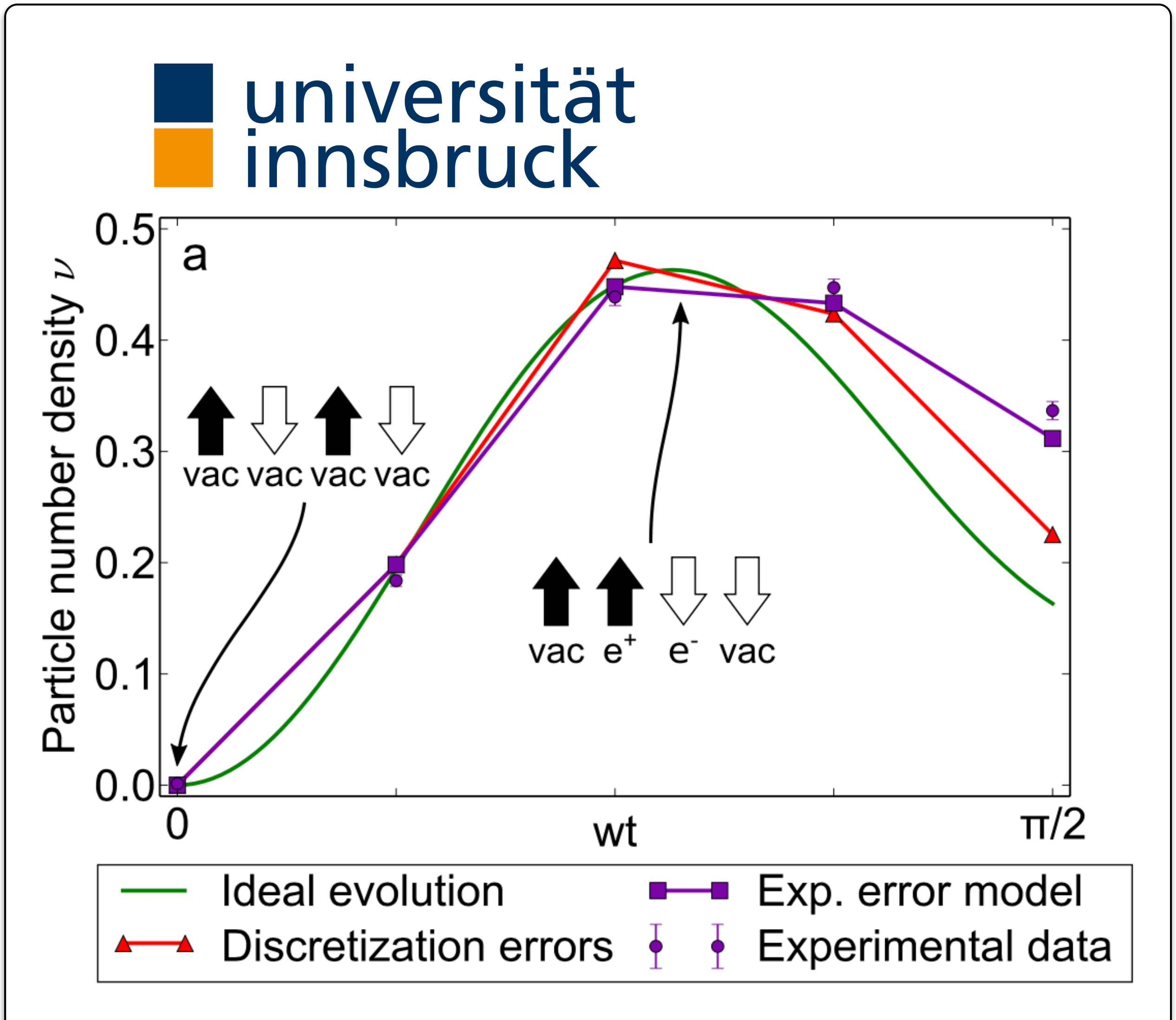
$V_1'(\delta t)$

Example of circuit structure for a six-site theory in each Trotter step:



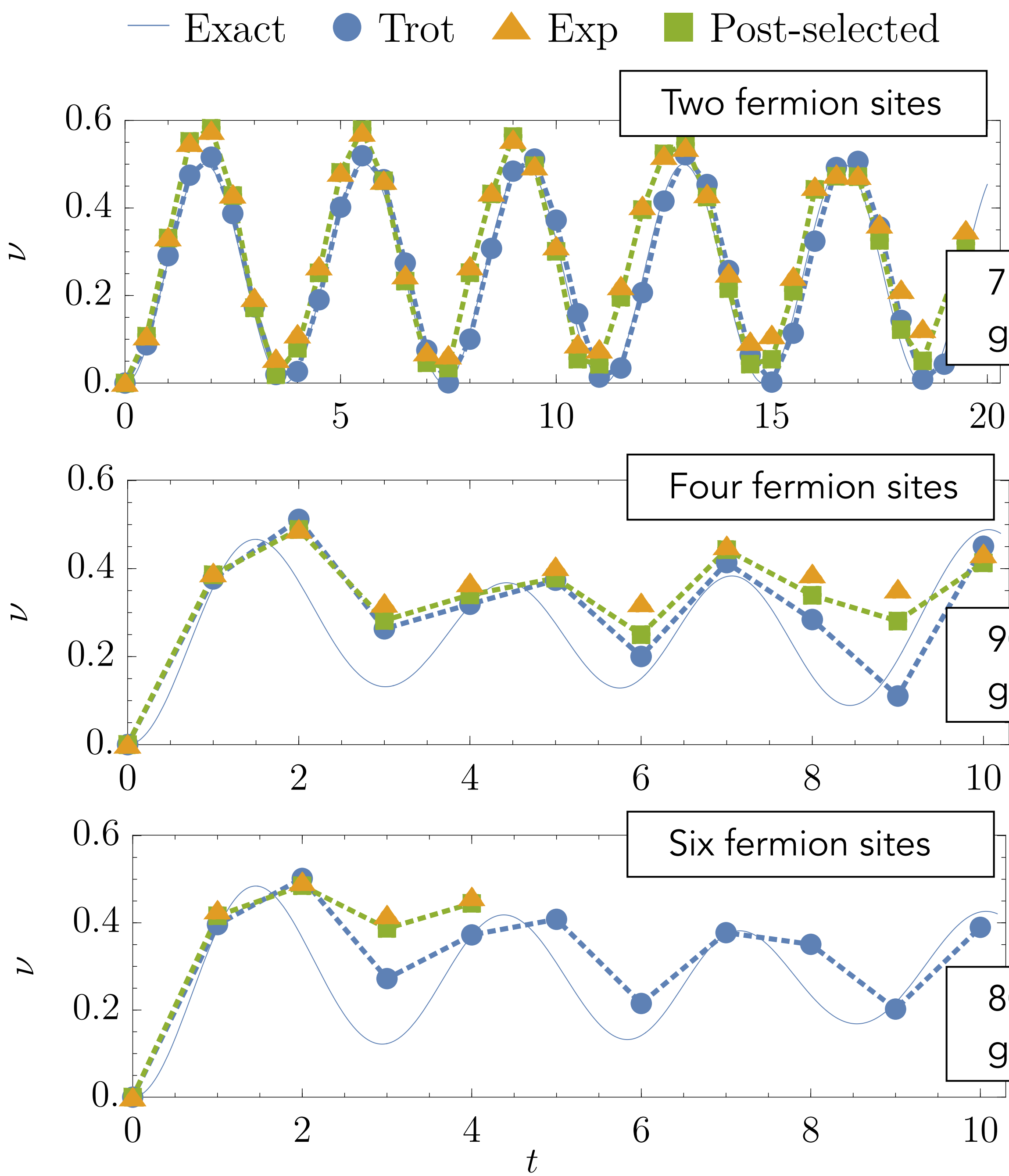
This Trotter block will be repeated $N_T = t/\delta t$ times.

Martinez et al, Nature 534, 516 EP (2016).



RELA-DEVICE IMPLEMENTATIONS

Nguyen, Tran, Zhu, Green, Huerta Alderete, ZD, Linke, *PRX Quantum* 3 (2022) 2, 020324.

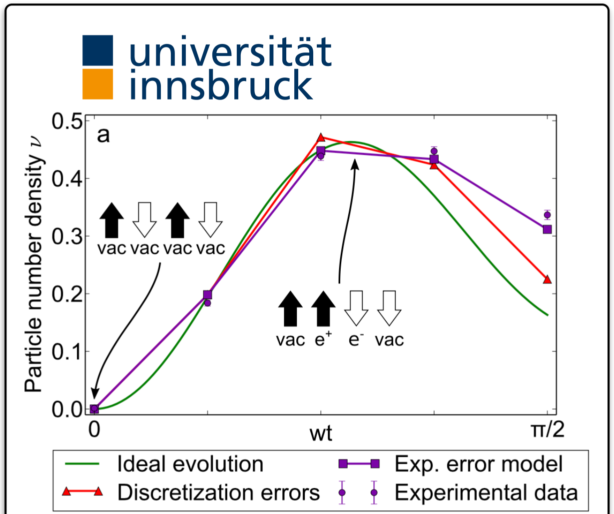


76 entangling gates!

90 entangling gates!

80 entangling gates!

Martinez et al, Nature 534, 516 EP (2016).



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NOW WHAT ABOUT **FERMIONIC-BOSONIC** REPRESENTATION WITH THE **SECOND-ORDER** PRODUCT FORMULA?

$$H = \mathrm{x}\sum_x \left[\sigma^+(x)U(x)\sigma^-(x+1) + \mathrm{h.c.}\right] + \sum_x E(x)^2 + \frac{\mu}{2}\sum_x (-1)^x \sigma^z(x)$$

One can split the terms in the Hamiltonian as:

Shaw, Lougovski, Stryker, Wiebe, Quantum 4, 306 (2020).

$$H = \sum_x (T_x + D_x)\ , \quad \text{with} \quad D_x := D_x^{(M)} + D_x^{(E)}.$$

$$T_x := \mathrm{x}\left(\frac{1}{4}(U_x + U_x^\dagger)(X_x X_{x+1} + Y_x Y_{x+1}) + \frac{i}{4}(U_x - U_x^\dagger)(X_x Y_{x+1} - Y_x X_{x+1})\right)$$

$$D_x^{(M)} := \frac{\mu}{2}(-1)^x Z_x \quad \text{and} \quad D_x^{(E)} := E_x^2$$

and do the following ordering of the terms

$$V_2(t) = \prod_x \left(\prod_{k\in\{M,E\}} e^{-iD_x^{(k)}t/2} \prod_{j=1}^{4} e^{-iT_x^{(j)}t/2}\right) e^{-iD_N^{(M)}t} \prod_x \left(\prod_{j=4}^{1} e^{-iT_x^{(j)}t/2} \prod_{k\in\{E,M\}} e^{-iD_x^{(k)}t/2}\right)$$

In reverse order

Example

This example concerns finding a quantum circuit for implementing

$$U^{(E)} = \prod_{i-1}^{N} U_i^{(E)} = e^{-it\sum_{i=1}^{N} E_i^2}$$

in the time-evolution of lattice Schwinger model in a near-term scenario that avoids introducing any ancilla qubits. Consider $E_i \in [-\Lambda, \Lambda]$ and encode the electric-field Hilbert space on each link i into $\eta \equiv \lceil \log_2(2\Lambda) + 1 \rceil$ qubits. Given this, find a circuit representation for $U_i^{(E)}$ in terms of only single-qubit rotations around the z axis of Bloch sphere as well as two-qubit CNOT gates. Verify your answer by explicitly working out a small example.

It is easy to show that the electric-field operator at each link acting on the computational (binary) basis is:

$$E = -\Lambda\,\mathbb{I} + \frac{1}{2}\left[(2^\eta - 1)\mathbb{I} - \sum_{j=0}^{\eta-1} 2^j\,\sigma_j^z\right]$$

Therefore,

$$E^2 = \Lambda^2\,\mathbb{I} - \Lambda\left[(2^\eta - 1)\mathbb{I} - \sum_{j=0}^{\eta-1} 2^j\,\sigma_j^z\right] + \frac{1}{4}\left[(2^\eta - 1)^2\mathbb{I} - 2(2^\eta - 1)\sum_{j=0}^{\eta-1} 2^j\,\sigma_j^z + \sum_{j,j'=0}^{\eta-1} 2^{j+j'}\,\sigma_j^z\sigma_{j'}^z\right]$$

Consequently, the operator $U^{(E)}$ can be written as a product of $N\eta\; R^z$ rotations and $N\eta(\eta-1)/2\; R^{zz}$ rotations with rotation angles that can be read off from the expression above. Note that each R^{zz} gate amounts to two CNOT gates and one R^z gate.

Example

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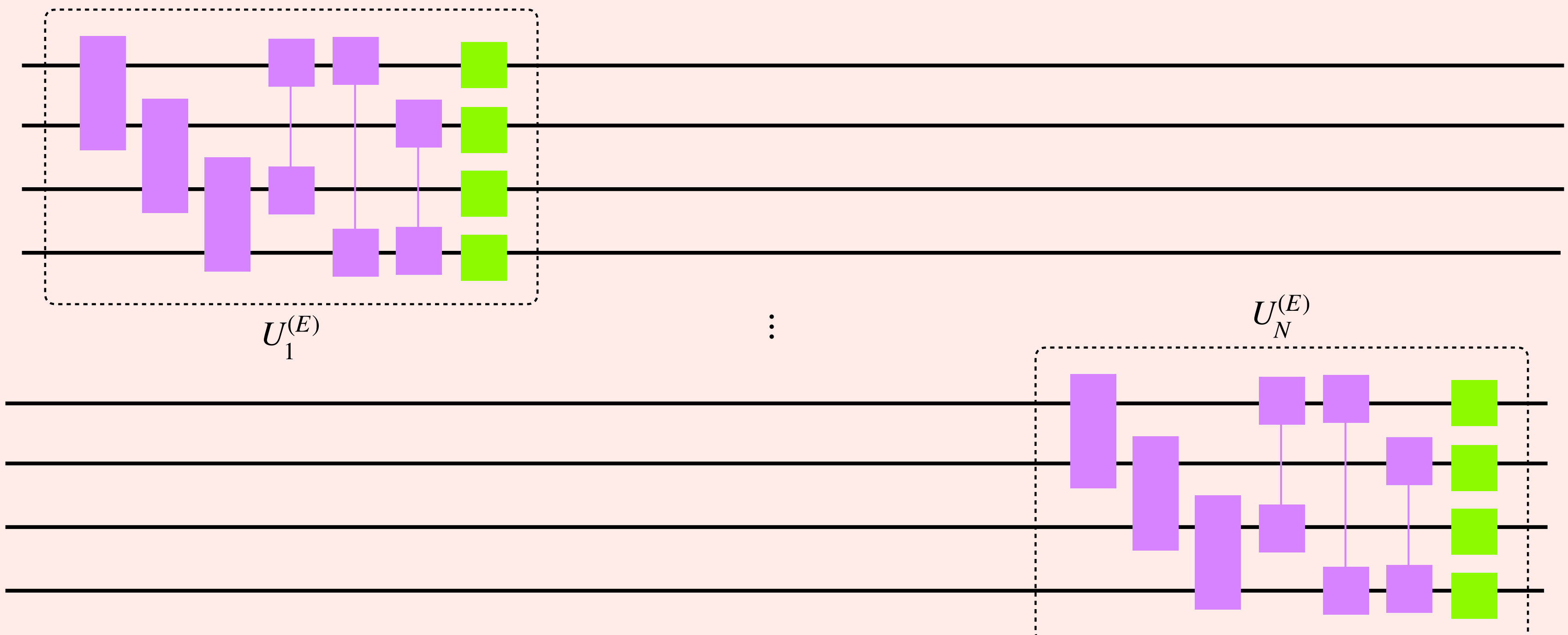
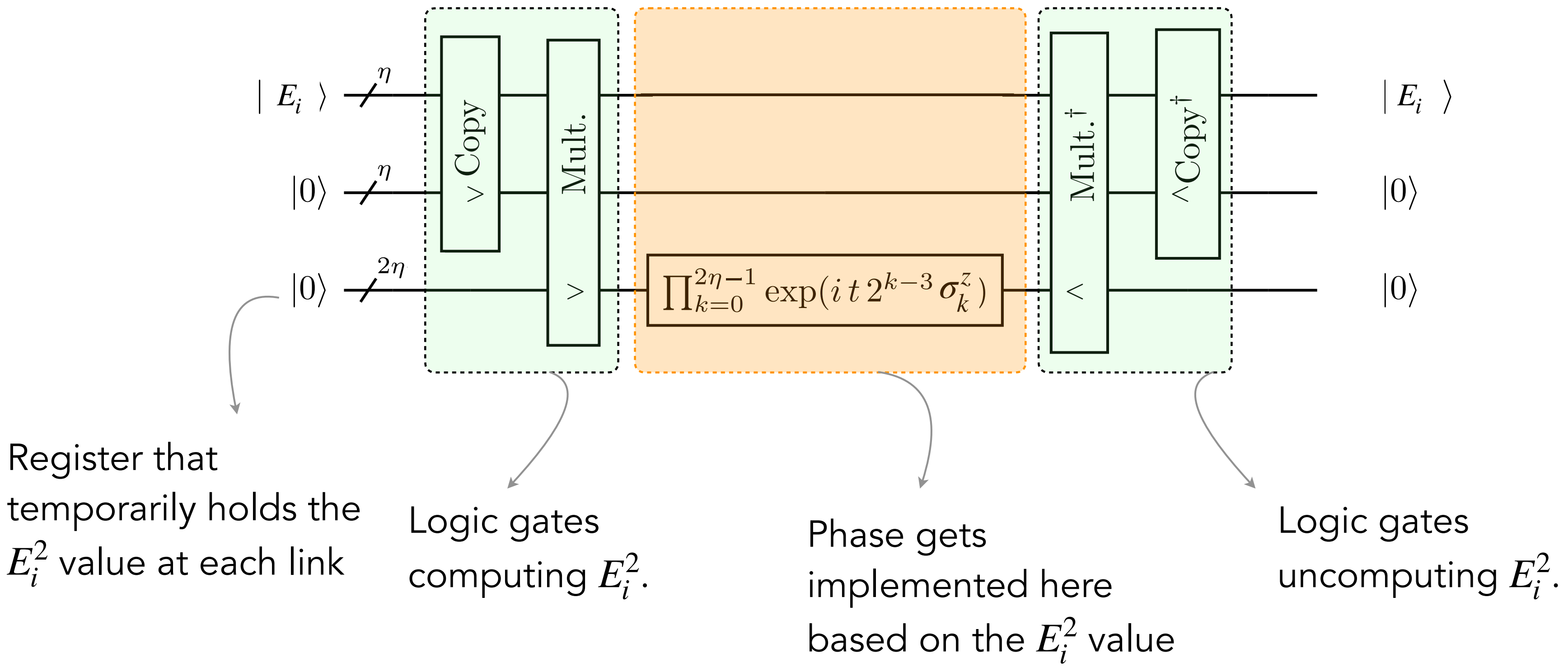
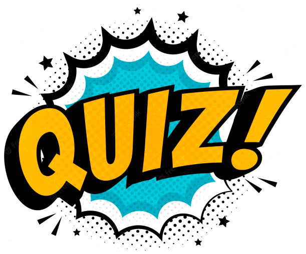
An $\eta = 4$ example:



The previous example requires $O(N\eta^2)$ number of R^z gates, which are costly operations in the fault-tolerant regime as they need to be synthesized up to accuracy ϵ using roughly $\log(1/\epsilon)$ T gates. Can one reduce the R^z cost of electric-field evolution to $O(N\eta)$? The answer is yes, but at the cost of extra $O(\eta)$ ancillas that are, nonetheless, available in the fault-tolerant era. One such circuit can be constructed using the so-called phase-kickback routine. For each $U_i^{(E)}$:



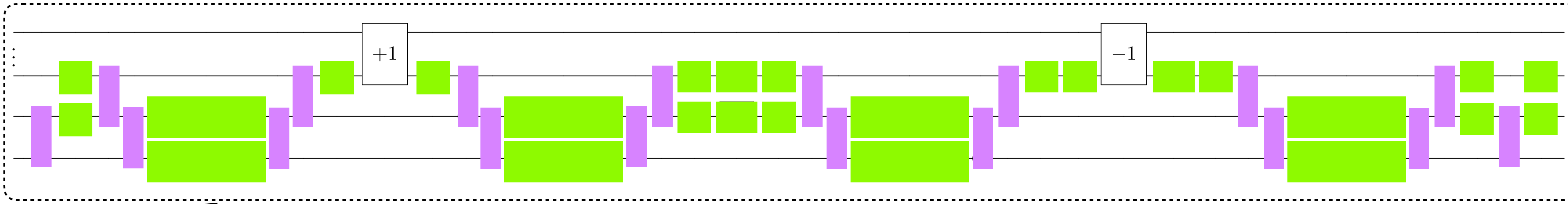
The logical copy and multiplication routines are known circuits and overall cost $O(\eta^2)$ T gates. The ancilla qubits are reset in the end and can be used in the remainder of the circuit.



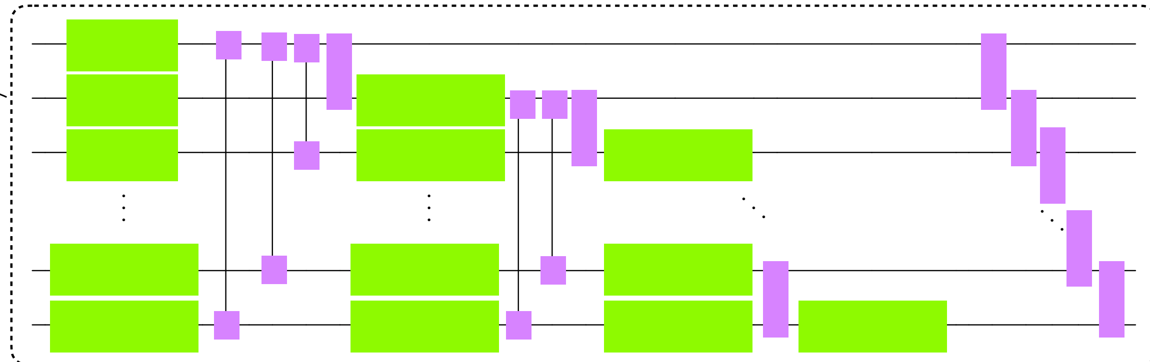
How do you implement arbitrary diagonal operator $e^{-it\mathscr{D}}$ in the computational basis? [Think about two examples: i) $\mathscr{D}|n\rangle = n|n\rangle$ and ii) $\mathscr{D}|n\rangle = \sqrt{\frac{n+1}{n-1}}|n\rangle$!]

Circuit and recourse analysis

Shaw, Lougovski, Stryker, Wiebe, Quantum 4, 306 (2020).



Sample gauge-fermion interaction block



Part of electric field interactions acting on gauge DOF registers

Near term cost

	$\delta_g = 10^{-3}$		$\delta_g = 10^{-4}$		$\delta_g = 10^{-5}$		$\delta_g = 10^{-6}$		$\delta_g = 10^{-7}$	
	$\tilde{\epsilon}^2$	CNOT	$\tilde{\epsilon}^2$	CNOT	$\tilde{\epsilon}^2$	CNOT	$\tilde{\epsilon}^2$	CNOT	$\tilde{\epsilon}^2$	CNOT
$x = 10^{-2}$	—	7.3e4	—	1.6e5	—	3.4e5	—	7.3e5	5.6e-2	1.6e6
$x = 10^{-1}$	—	1.6e4	—	3.5e4	—	7.5e4	5.9e-2	1.6e5	2.7e-3	3.5e5
$x = 1$	—	4.6e3	—	9.9e3	1.0e-1	2.1e4	4.7e-3	4.6e4	2.2e-4	9.9e4
$x = 10^2$	—	2.8e3	8.3e-1	6.1e3	3.8e-2	1.3e4	1.8e-3	2.8e4	8.2e-5	6.0e4

COMPARISON BETWEEN THE TWO FORMULATIONS FOR THE SECOND-ORDER FORMULA

	Purely fermionic (non-local)	Fermionic-bosonic (local)
Qubit cost	N	$N + N\log_2(N)$
Gate complexity*	$\mathcal{O}(N^{9/2}t^{3/2})$	$\mathcal{O}(N^{5/2}t^{3/2})$

*Defined at the required number of Trotter steps for simulation time t, system size $N \sim \Lambda$, and at fixed x and μ, given a fixed error tolerance.

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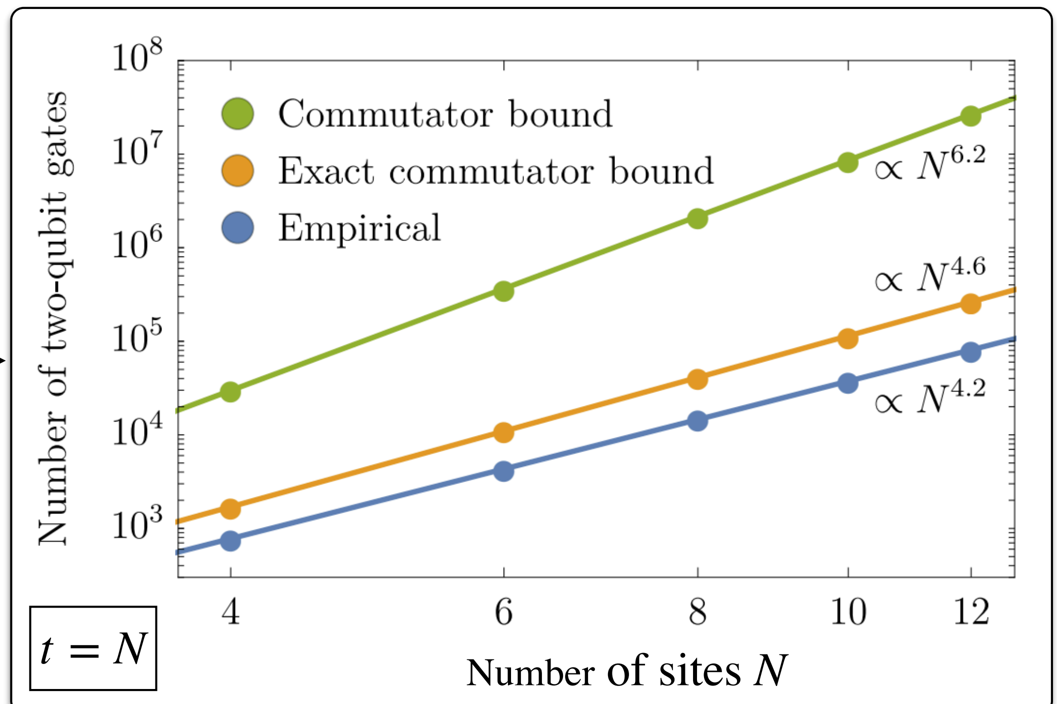
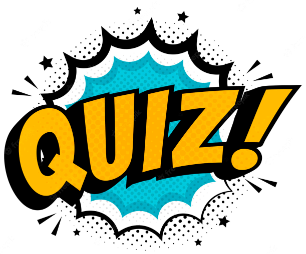
*Defined at the required number of Trotter steps for simulation time t, system size $N \sim \Lambda$, and at fixed x and μ, given a fixed error tolerance.

Nonetheless, empirically it seems like the non-local formulation performs as well as the bound on the local formulation! →



Nguyen, Tran, Zhu, Green, Huerta Alderete, ZD, Linke, *PRX Quantum* 3 (2022) 2, 020324.



Explain the qubit and gate scalings of the second-order Trotter simulation of the lattice Schwinger model in both formulations, as given in the previous slide.

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Abelian vs. non-Abelian

Since we do not have the option of removing the gauge links generally, let us focus on the fermionic-bosonic formulations in the electric-field basis. So what are the major differences between simulating digitally Abelian and non-Abelian LGTs? Let is compare U(1) and SU(N) LGTs.

i) There are more degrees of freedom involved for SU(N) LGTs. For example, at each site, there are N-component fermions, and at each link there are multiple bosonic variables.

ii) As a result, there are more terms that need to be simulated, hence more complexity and generally more Trotter error.

iii) The diagnozalization procedure for hopping and magnetic terms generally follow the same rules but is more gate-intensive for SU(N).

iv) The diagonal operators in an Abelian theory like U(1) are trivial while for SU(N), they require evaluating phases that are non-trivial functions of bosonic occupation-number operators. These require expensive function-evaluation routines (in the E basis).

Algorithmic progress for U(1), SU(2), and SU(3) theories can be found in:

Shaw, Lougovski, Stryker, Wiebe, Quantum 4, 306 (2020).
Ciavarella, Klco, and Savage, Phys. Rev. D 103, 094501 (2021).
Kan and Nam, arXiv:2107.12769 [quant-ph].
ZD, Shaw, and Stryker, Quantum 7, 1213 (2023),
Rhodes, Kreshchuk, Pathak, arXiv:2405.10416 [quant-ph]

What about the ultimate theory for us?
Quantum Chromodynamics, a SU(3) LGT in 3+1 coupled to 6 flavors of quarks

		10^3 lattice at fixed paramts.
Kan and Nam:	• Kogut and Susskind in E basis, no Gauss-law implementation *a priori* • Evaluates matrix elements quantumly • Uses product formulas. Breaks all bosonic ladder ops. to even/odd space	$O(10^{50})$ T gates
ZD and Stryker:	• Kogut and Susskind in E basis, no Gauss-law implementation *a priori* • Evaluates matrix elements quantumly • Uses PFs. Breaks only some of the bosonic ladder ops. to even/odd space	PRELIMINARY $O(10^{30})$ T gates
Rhodes, Kreshchuk, Pathak	• Kogut and Susskind in E basis, no Gauss-law implementation a priori • Uses QROM to access matrix elements evaluated calssically • Uses block encoding of time evolution. No even-odd breaking.	$O(10^{25})$ T gates
Ciavarella, Klco, Savage:	• Kogut and Susskind in E basis, some Gauss-law implementation *a priori* • Uses controlled operations to access matrix elements evaluated calssically • Not a full algorithm in 3+1 D with error analysis	-
Lamm et al:	• Kogut and Susskind in U basis, no Gauss-law implementation *a priori* • Matrix elements simple (no Clebsch–Gordan coeff. in this basis) • Uses block encoding, no full error analysis for SU(3) subgroups yet	- [For SU(2), $O(10^{13})$ T gates]

How far can we continue to improve? Will this problem become reasonably doable in the fault-tolerant era?

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WHILE WE HAVE COVERED SOME BASICS, A LOT OF IMPORTANT TOPICS WERE LEFT OUT…

- A variety of Hamiltonian formulations of gauge theories and in various bases, systematic uncertainties, renormalization and continuum limit, etc.
- Detailed discussions of non-Abelian gauge theories and higher-dimensional models
- State-preparation strategies for in quantum (gauge) field theories including for thermal states
- The so-called near-optimal time-evolution algorithms beyond product formulas
- Observables, e.g., scattering amplitudes, transport coefficients, structure functions, nonequilibrium dynamics, thermodynamics
- Error correction and error mitigations, including in the context of quantum (gauge) field theories
- Quantum-hardware architecture and other analog and hybrid proposals for simulating gauge theories

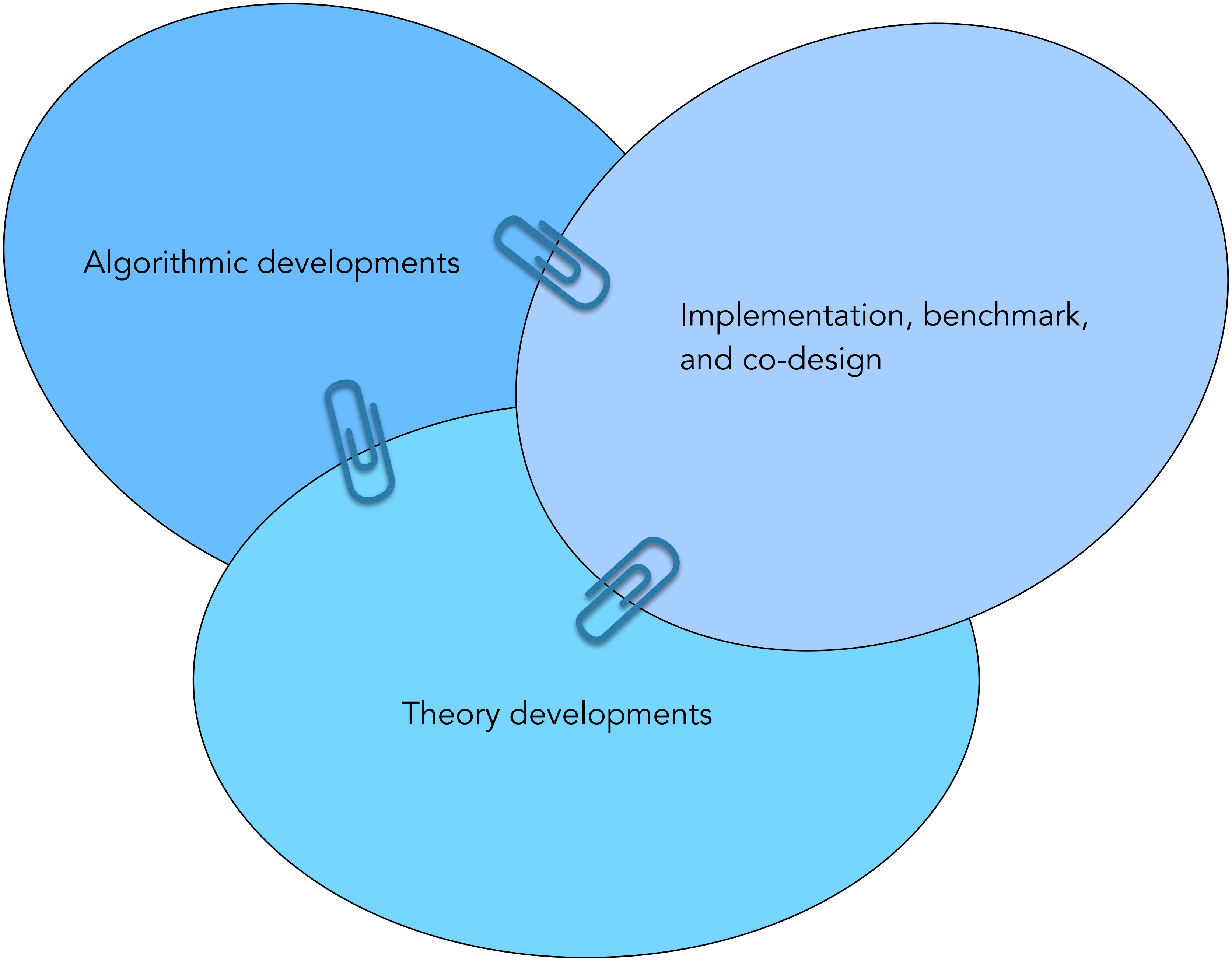
For a review and perspective See Bauer, ZD, et al, “Quantum Simulation for High Energy Physics”, PRX Quantum 4 (2023) 2, 027001.

NONETHELESS, YOU MUST BE SUFFICIENTLY EQUIPPED NOW GIVEN THESE LECTURES TO START EXPLORING THIS EXCITING AND FASTLY-EVOLVING FIELD OF RESEARCH IF YOU DESIRE.

POST-LECTURE [TO CONCLUDE]

QUANTUM SIMULATION OF FUNDAMENTAL PARTICLES AND FORCES, WHERE ARE WE NOW AND WHERE ARE WE GOING?

QUANTUM SIMULATION OF GAUGE FIELD THEORIES: A MULTI-PRONG EFFORT



HAMILTONIAN FORMULATIONS OF GAUGE THEORIES CONTINUES TO BE DEVELOPED.

Gauge-field theories (Abelian and non-Abelian) starting from the seminal work of Kugot and Susskind:

Group-element representation
Zohar et al; Lamm et al

Prepotential formulation
Mathur, Raychowdhury et al

Loop-String-Hadron basis
Raychowdhury and Stryker

Link models, qubitization
Chandrasekharan, Wiese et al,
Alexandru, Bedaque, et al.

Fermionic basis
Hamer et al; Martinez et al; Banuls et al

Bosonic basis
Cirac and Zohar

Light-front quantization
Kreshchuk, Love, Goldstien,
Vary et al.; Ortega at al

Local irreducible representations
Byrnes and Yamamoto;
Ciavarella, Klco, and Savage

Manifold lattices
Buser et al

Dual plaquette (magnetic) basis
Bender, Zohar et al; Kaplan and Styker; Unmuth-Yockey; Hasse et al; Bauer and Grabowska

Spin-dual representation
Mathur et al

Scalar field theory

Field basis
Jordan, Lee, and Preskill

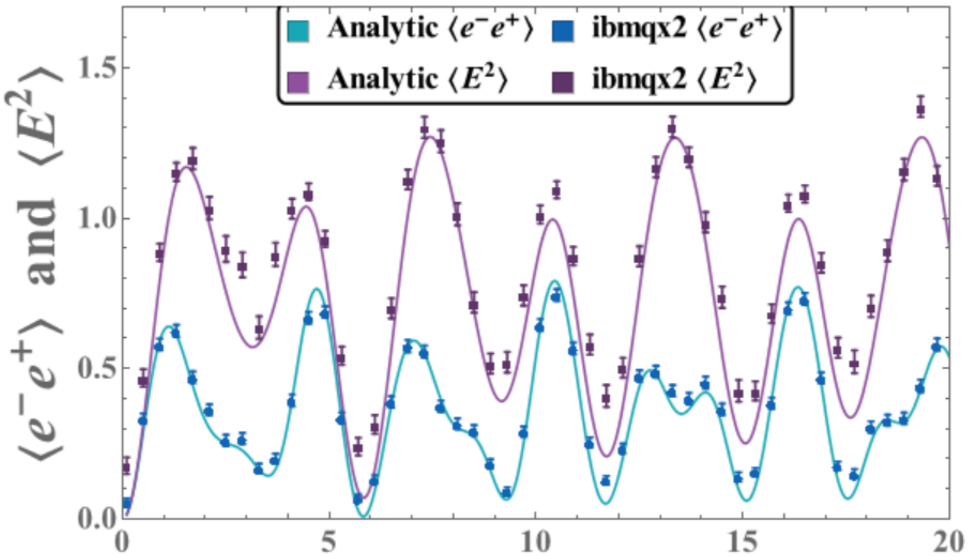
Continuous-variable basis
Pooser, Siopsis et al

Harmonic-oscillator basis
Klco and Savage

Single-particle basis
Barata , Mueller, Tarasov, and Venugopalan.

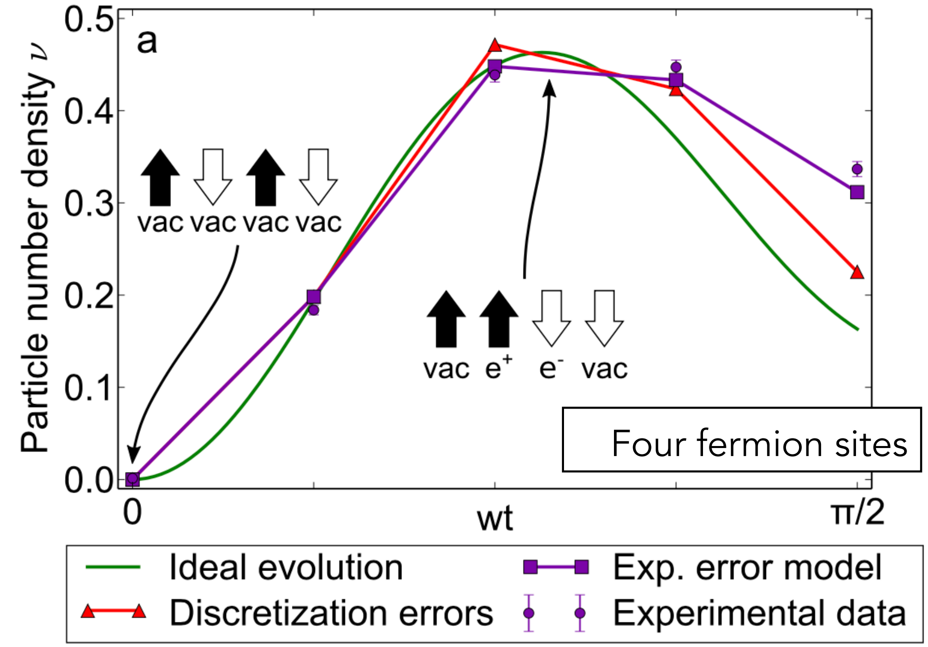
DIGITAL COMPUTATIONS OF ABELIAN LGTs

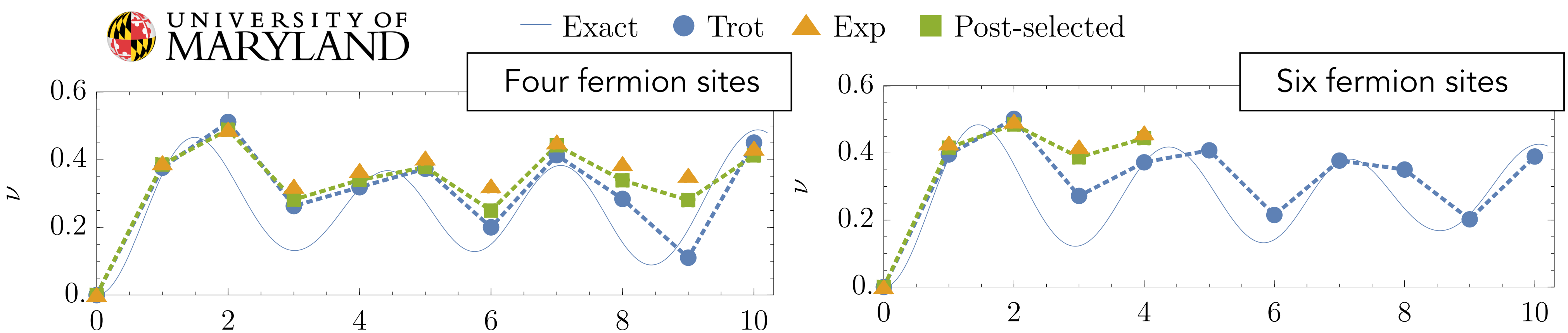
Klco, Savage, et al, Phys. Rev. A 98, 032331 (2018).



A hybrid classical-quantum approach allows a 2-qubit reduction of 4-qubit simulation.

Martinez et al, Nature 534, 516 EP (2016).

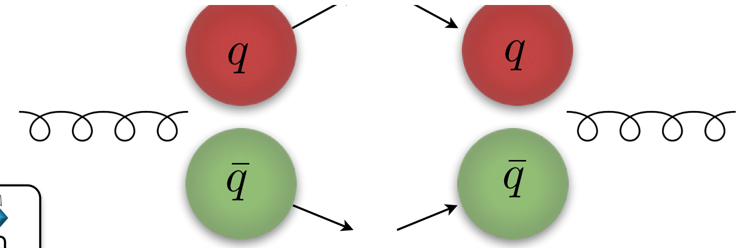




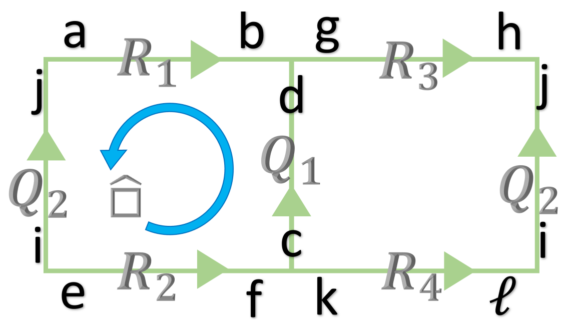
80 entangling gates!

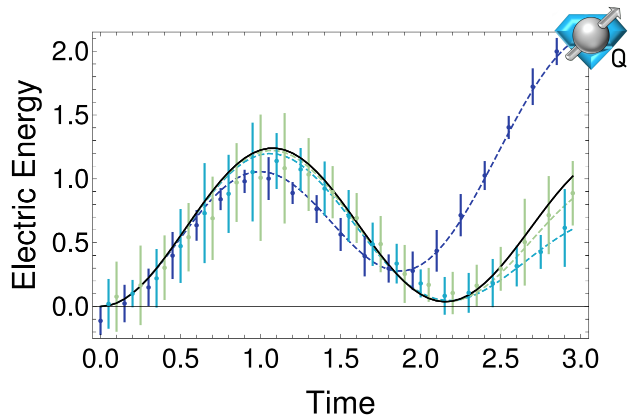
90 entangling gates!

Nguyen, Tran, Zhu, Green, Huerta Alderete, ZD, Linke, *PRX Quantum* 3 (2022) 2, 020324.



DIGITAL COMPUTATIONS OF NON-ABELIAN LGTs

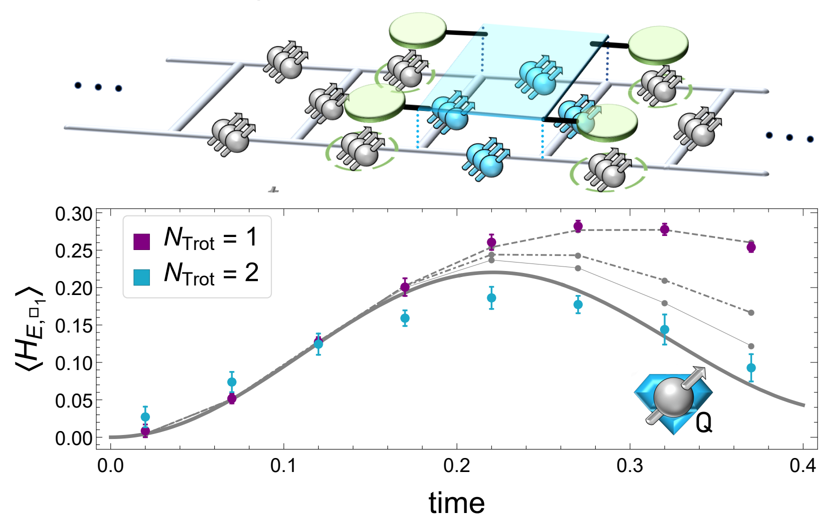




Real-time dynamic of pure SU(3) with global irrupts on IBM

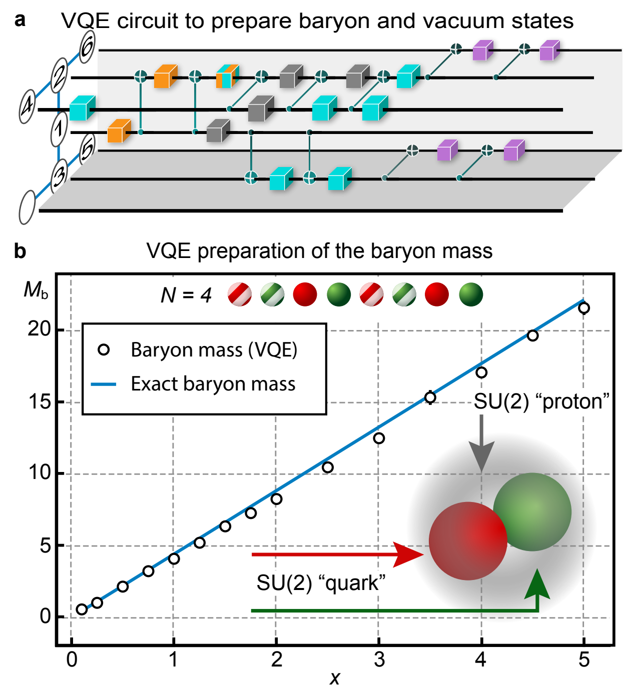
Ciavarella, Klco, and Savage, Phys. Rev. D 103, 094501 (2021).

Real-time dynamic of pure SU(2) with global irreps on IBM



Klco, Savage, and Stryker, Phys. Rev. D 101, 074512 (2020).

Low-lying spectrum of SU(2) with matter in 1+1 D on IBM

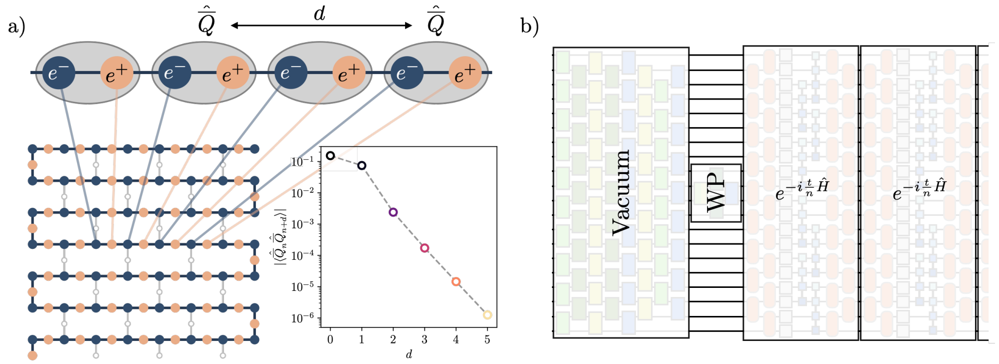


Atas et al, Nature Communications 12, 6499 (2021). SU(3) example: Atas et al: arXiv:2207.03473 [quant-ph].

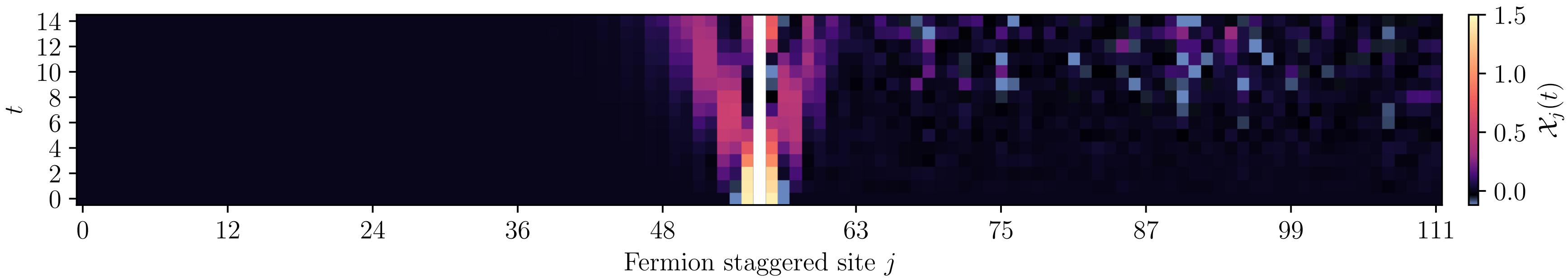
See also studies on D-wave annealers: Rahman et al, Phys. Rev. D 104, 034501 (2021), Illa and Savage, arXiv:2202.12340 [quant-ph], Farrel et al, arXiv:2207.01731 [quant-ph].

FIRST STEPS TOWARD HADRONIC WAVEPACKETS FOR COLLISION PROCESSES

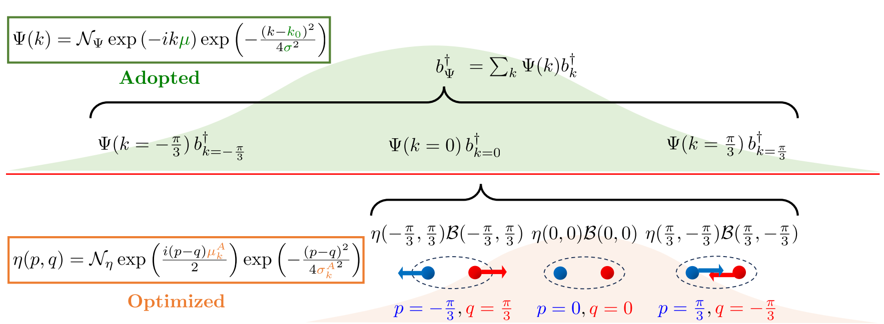
Hadron wavepacket evolution in the Schwinger model (112 staggered sites with IBM with noise mitigation):

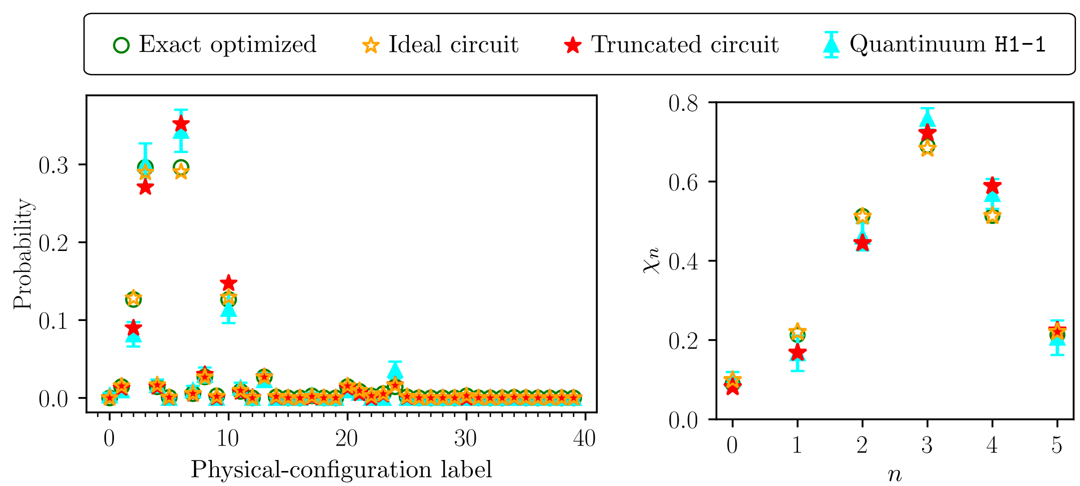


Farrell, Illa, Ciavarella, Savage, Phys.Rev. D 109 (2024) 11, 114510.



Hadron wavepacket in the Z_2 gauge theory (12 staggered sites with Quantinuum, minimal noise mitigation):

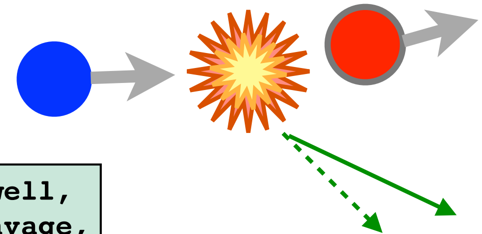




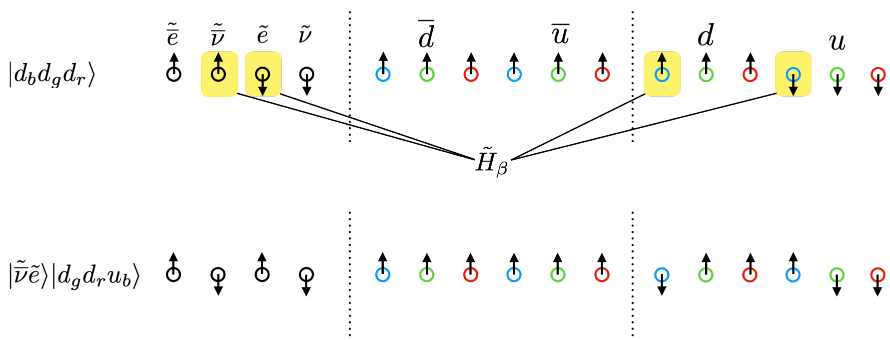
ZD, Hsieh, and Kadam, arXiv:2402.00840 [quant-ph].

DECAY AMPLITUDES, PARTON SHOWER, PARTON DISTRIBUTION FUNCTIONS

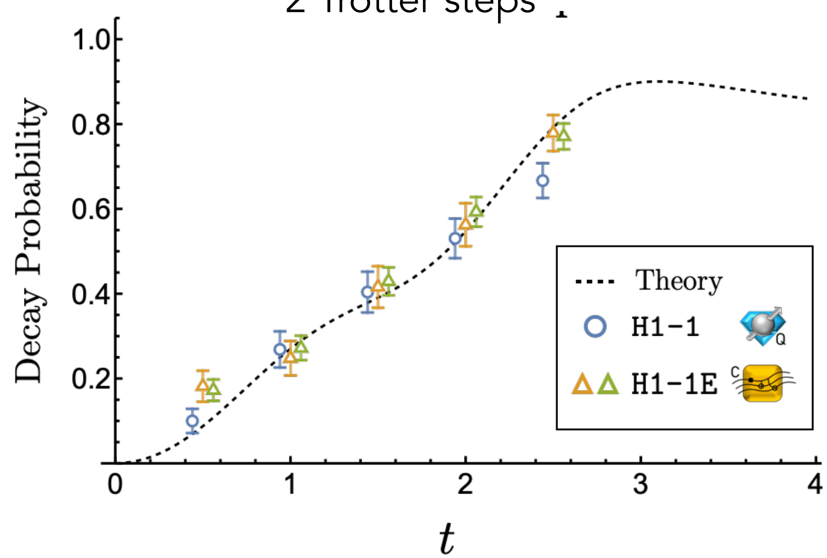
Quantum computing β decay in 1+1 D QCD



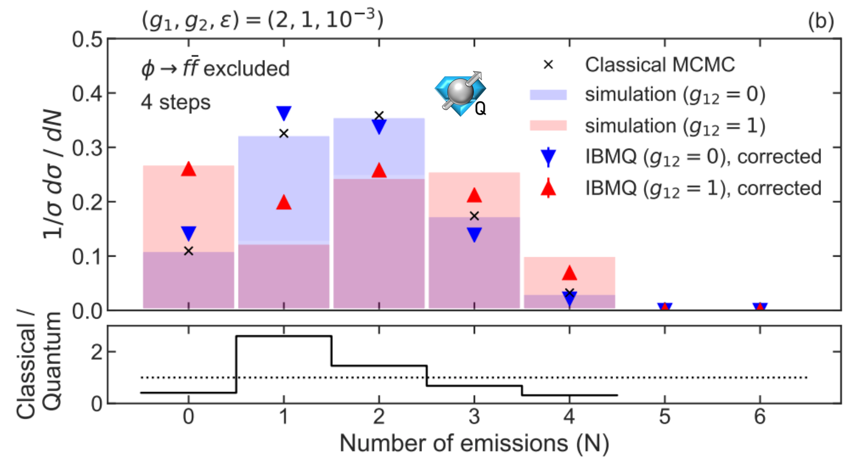
Farrell, Chernyshev, Powell, Zemlevskiy, Illa, and Savage, Phys. Rev. D 107, 054513 (2023).



2 Trotter steps

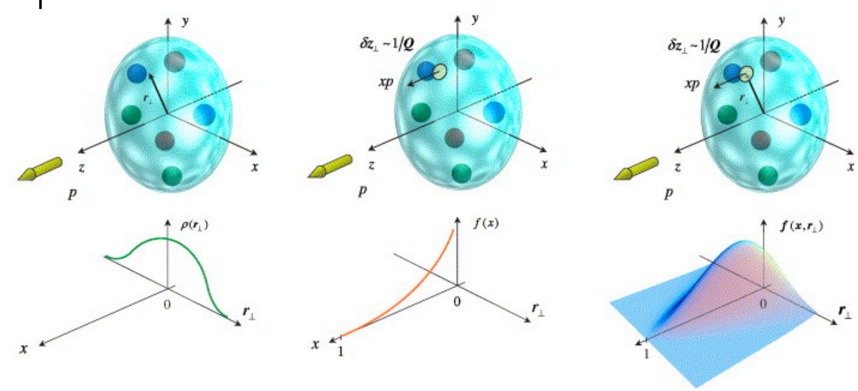


A quantum algorithm for parton shower:



Nachman, Provasoli, and Bauer, *Phys. Rev. Lett.* 126 (2021) 6, 062001.

PDFs from non-equal time amplitudes on quantum computers:

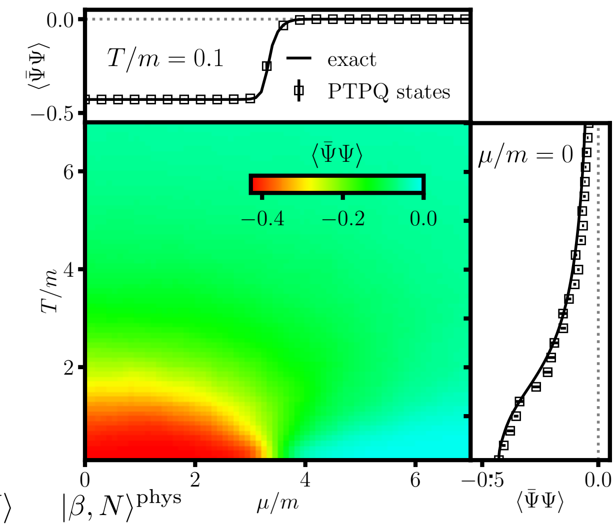


Belitskya, Radyushkinbc, Phys. Rep. 418 (2005), 1-387.

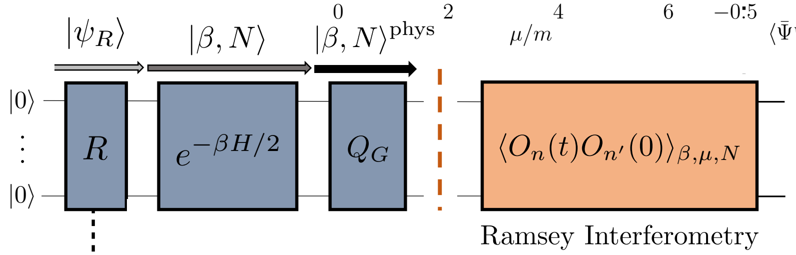
Mueller, Tarasov, and Raju Venugopalan, PRD 102, 016007 (2020), Lamm, Lawrence, and Yamauchi, Phys. Rev. Res. 2, 013272 (2020), Echevarria, Egusquiza, Rico, and G Schnell, PRD 104, 014512 (2021).

FINITE-DENSITY AND NON-EQUILIBRIUM PHYSICS IN STRONGLY-INTERACTING SYSTEMS

Toward Quantum Computing Phase Diagrams of Gauge Theories with Thermal Pure Quantum States, ZD, Mueller, Powers, Phys. Rev. Lett. 131, 081901 (2023).

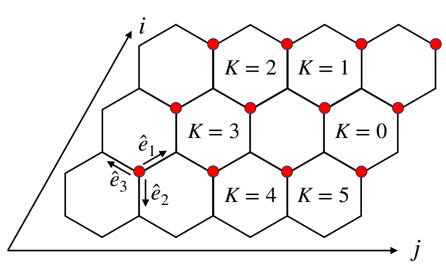


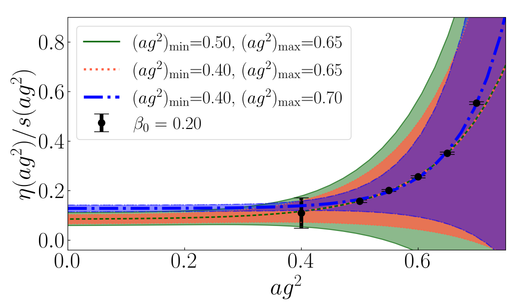
Phase diagram of Z_2^{1+1} with fermions



Shear viscosity in SU(2) LGT in 2+1 D with $j_{\text{max}} = 1/2$

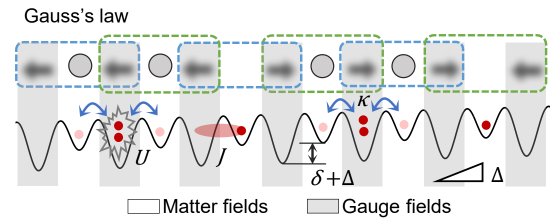
Turro, Ciavarella, Yao, Phys. Rev. D 109, 114511 (2024).



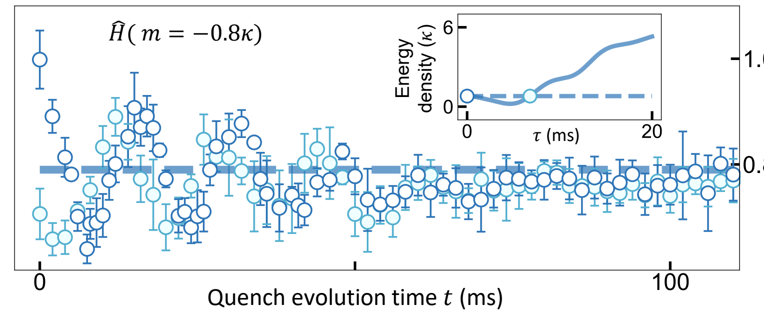


See also: Farrell, Illa, Savage, arXiv:2405.06620 [quant-ph].

Thermalization dynamics of U(1) Quantum Link Model in a 71-site analog simulator

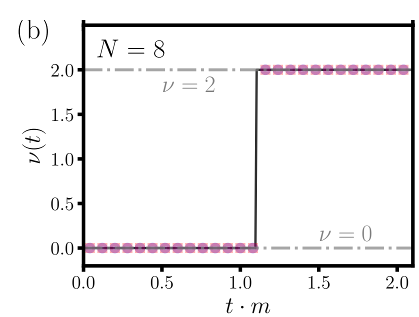


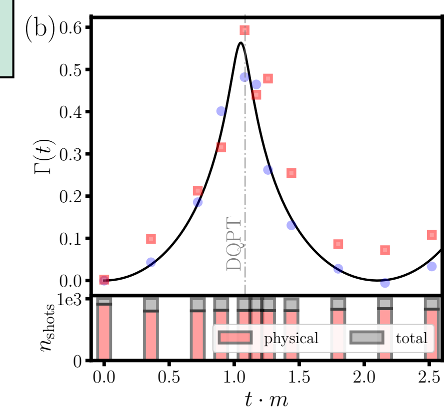
Zhou et al, *Science* 377 (2022) 6603.



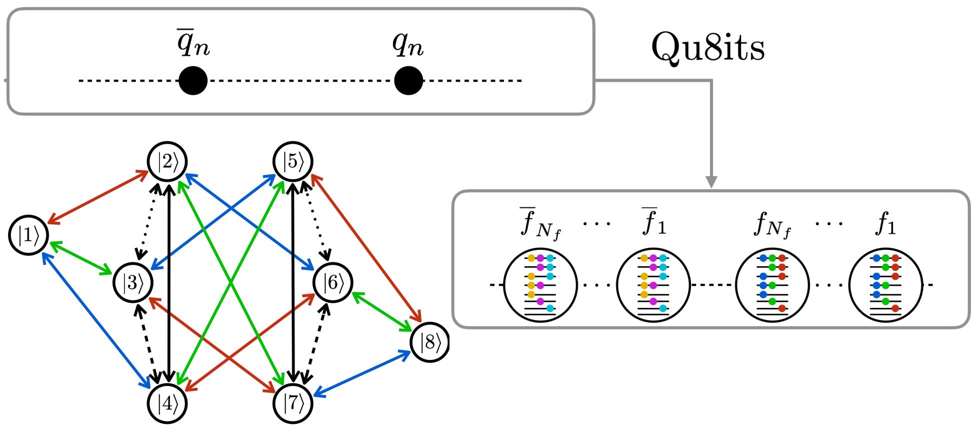
A dynamical phase transition and topological order in lattice Schwinger model with an IonQ quantum computer:

Mueller, Carolan, Connelly, ZD, Dumitrescu, Mueller, Yeter-Aydeniz, *PRX Quantum* 4 (2023) 3, 030323.

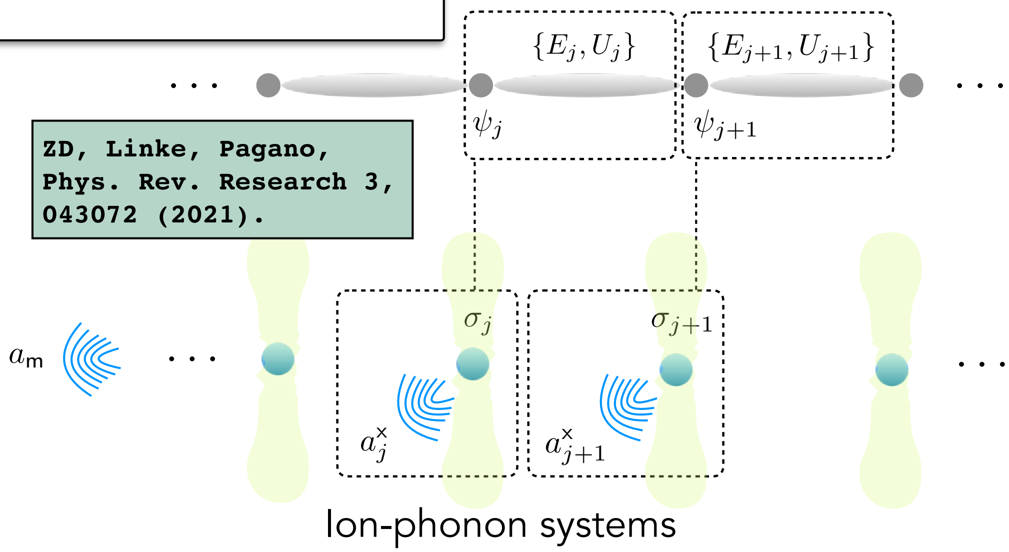




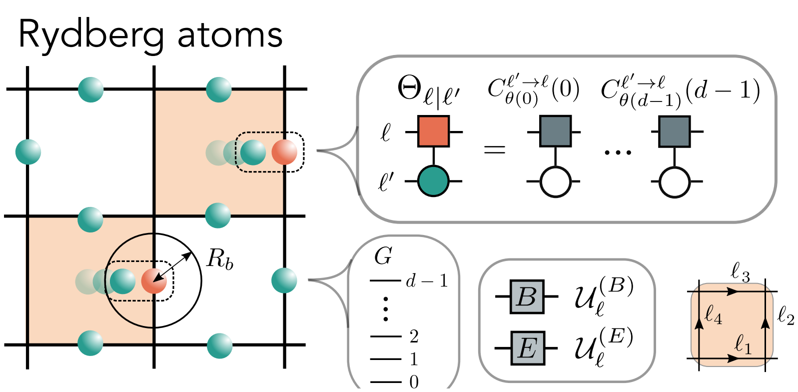
SOME CO-DESIGN EXAMPLES: LEVERAGING MULTI-DIMENSIONAL LOCAL HILBERT SPACES AND MULTI-MODE INTERACTIONS



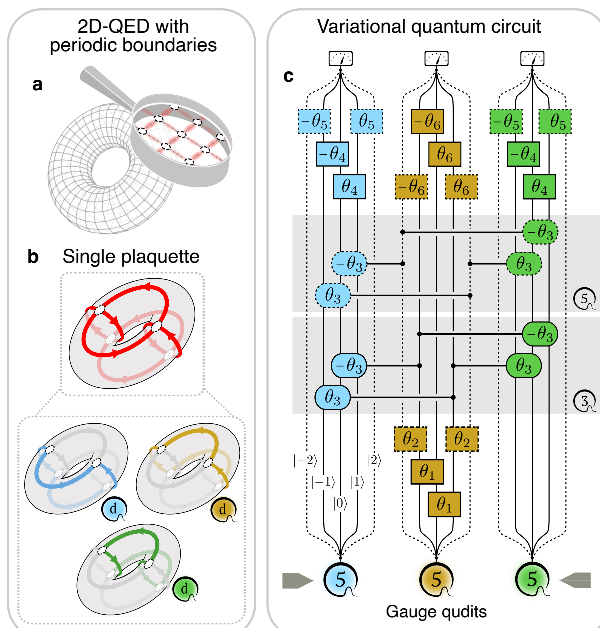
Illa, Robin, Savage, Phys. Rev. D 110 (2024), 014507.



ZD, Linke, Pagano, Phys. Rev. Research 3, 043072 (2021).



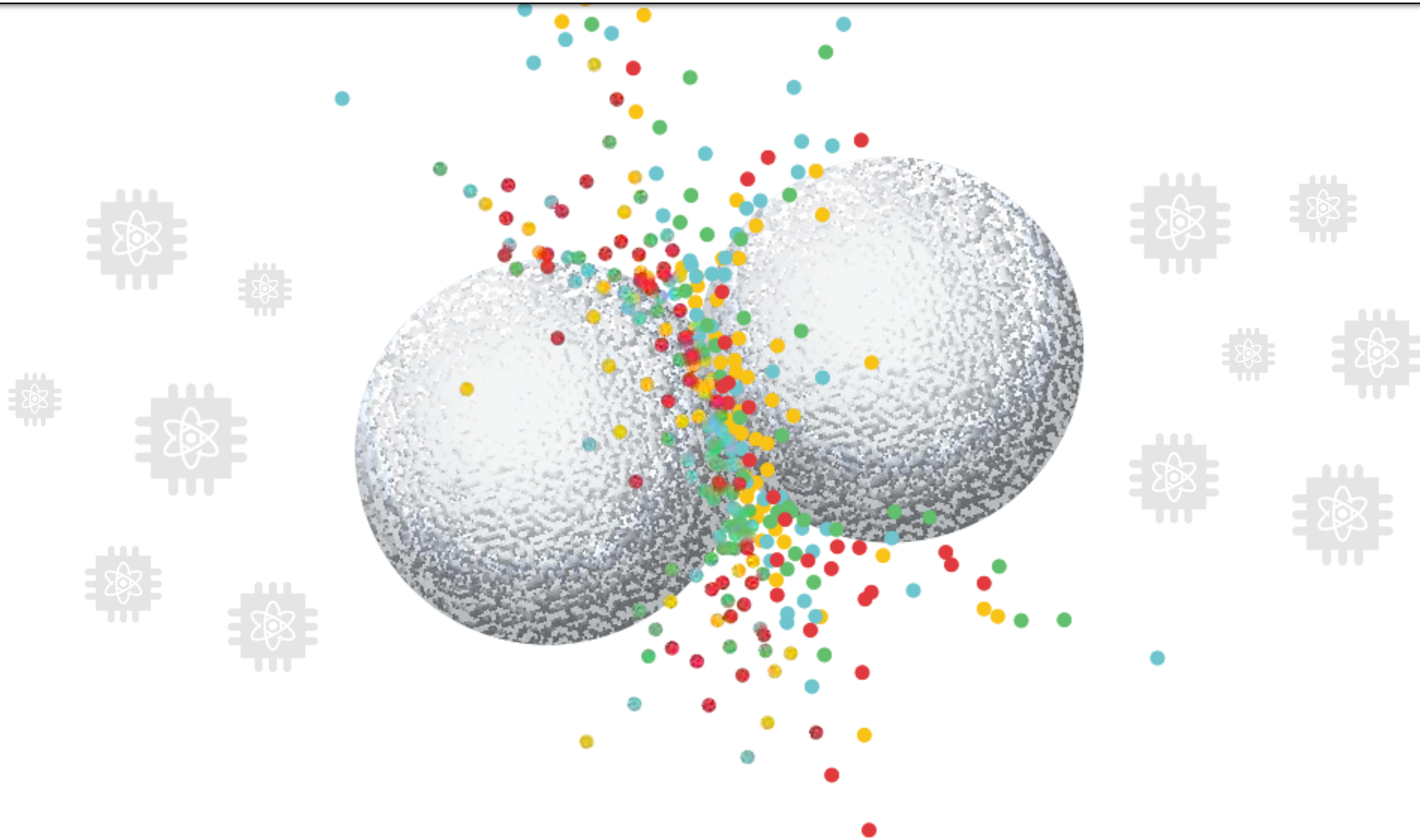
González-Cuadra, Zache, Carrasco, Kraus, Zoller, Phys. Rev. Lett. 129, 160501 (2022).



Ion qudits

Meth, Haase, Zhang, Edmunds, Postler, Jena, Steiner, Dellantonio, Blatt, Zoller, Monz, Schindler, Muschik, and Ringbauer, arXiv:2310.12110 [quant-ph].

We've got a long way to fully and reliably simulate **the Standard Model** but we know what to do! **Theory/algorithm/experiment** collaborations will be the key. It is even more important in the quantum-computing era since our computers are themselves physical systems! This is an active field with lots to work on and develop! It is time to get involved if you are interested!



QUESTIONS?