Gauge field digitization in the Hamiltonian limit

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Abstract

The use of quantum computers could circumvent the complex action problem hampering first-principles studies of gauge theories in real time or at finite density. One of the main bottlenecks of quantum computers is the limited number of available qubits. One approach to mitigate this bottleneck is the discretization of continuous gauge groups to their discrete subgroups, which introduces systematic uncertainties. Previously, discrete subgroups and dense subsets of gauge groups had been investigated, but only with isotropic Euclidean lattices. In this work, we take the first steps in studying the systematics associated with digitization by performing anisotropic Euclidean simulations and taking the Hamiltonian limit, where the temporal lattice spacing approaches zero while the spatial lattice spacing is kept fixed.

Complex action problem

The partition function as a path integral reads

$$\mathcal{Z} = \int \mathcal{D}\phi \, e^{-S[\phi]} = \int \mathcal{D}\phi \, w[\phi] \, .$$

If $w[\phi] \notin \mathbb{R}^+$, Markov chain Monte Carlo methods with usual importance sampling are not applicable to compute expectation values of observables: this is the **complex action problem** (CAP).

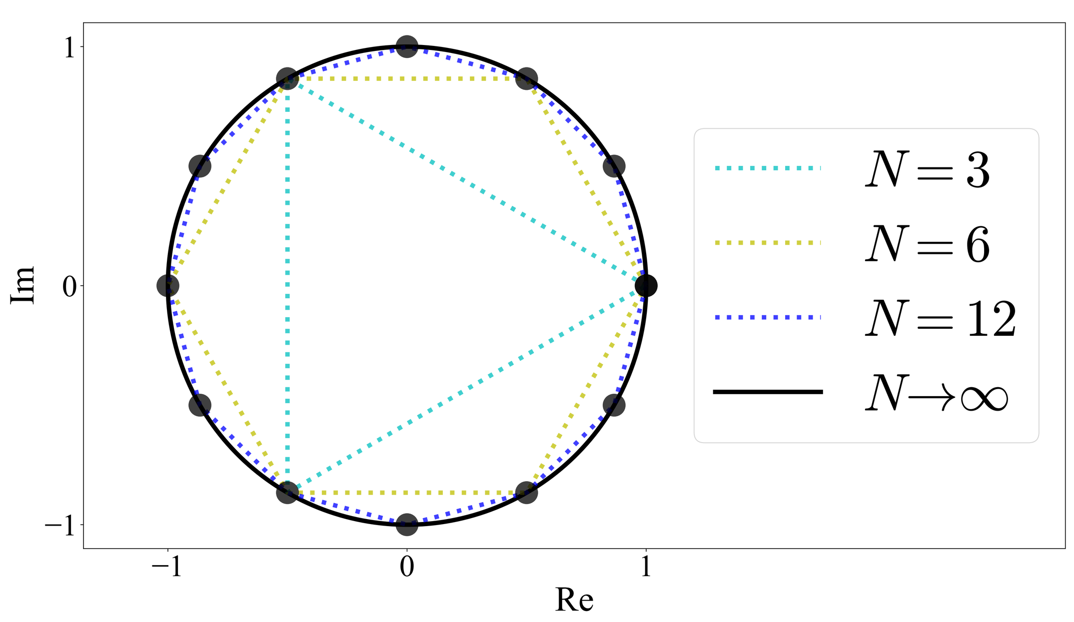
In principle, CAP can be bypassed with the help of **quantum computing** (QC) which are inherently capable of complex operations [1].

Digitization of gauge groups

In the NISQ era, one of the main bottlenecks of QC is the **limited number of qubits**, hence continuous variables cannot be mapped onto them faithfully. In a gauge theory the corresponding Hilbert space for gauge fields is infinite dimensional, i.e., it shall be made discrete and finite via an appropriate **digitization** scheme [2, 3].

Simplest example: U(1) discretized to Z(N)

$$g_\infty(\varphi \in \mathbb{R}) = e^{i\varphi} \quad \mapsto \quad g_N(n \in \mathbb{Z}^+) = e^{2\pi i n/N}$$



The Hamiltonian limit

To derive the Kogut-Susskind Hamiltonian one shall consider the following [4, 5]:

1. **Zero-temperature limit** due to the choice of temporal gauge.
2. **Anisotropic lattice** with action

$$S^{\mathrm{a}} = \sum_x \Big[-\beta_S \sum_{k<l} \mathrm{Re}\mathrm{Tr}\,\mathcal{U}_{kl}(x) - \beta_T \sum_k \mathrm{Re}\mathrm{Tr}\big[U_k(x+\hat{0})U_k^\dagger(x)\big] \Big] \, .$$

3. Taking the **Hamiltonian limit** as

$$\begin{cases} a_s = \text{fixed} & \sim \text{ discrete space} \\ a_t \to 0 & \sim \text{ continuous time} \end{cases}$$

References

[1] D. C. Hackett et. al.: `quant-ph/1811.03629`
[2] A. Alexandru et. al.: `hep-lat/1906.11213`
[3] T. Hartung et. al.: `hep-lat/2201.09625`
[4] J. B. Kogut, L. Susskind: Phys.Rev.D 11.395
[5] M. Creutz: *Quarks, Gluons and Lattices*

Scaling laws of gauge couplings in the Hamiltonian limit

In the Hamiltonian limit $\beta_S \to 0$ and $\beta_T \to \infty$ from dimensional analysis. *How?*

The Hamiltonian $\mathcal{H}$ is constructed from the **transfer matrix T** as

$$\mathcal{Z} = \mathrm{Tr}\,\mathbf{T}^{N_t} \quad \text{from which} \quad \mathbf{T} = \mathbf{1} - a_t\mathcal{H} + \mathcal{O}(a_t^2) \, .$$

To have a non-trivial Hamiltonian in the continuous time limit, scaling laws of coupling constants are imposed.

- **Temporal coupling**: different scaling laws arise in the cases of continuous and discrete gauge groups.

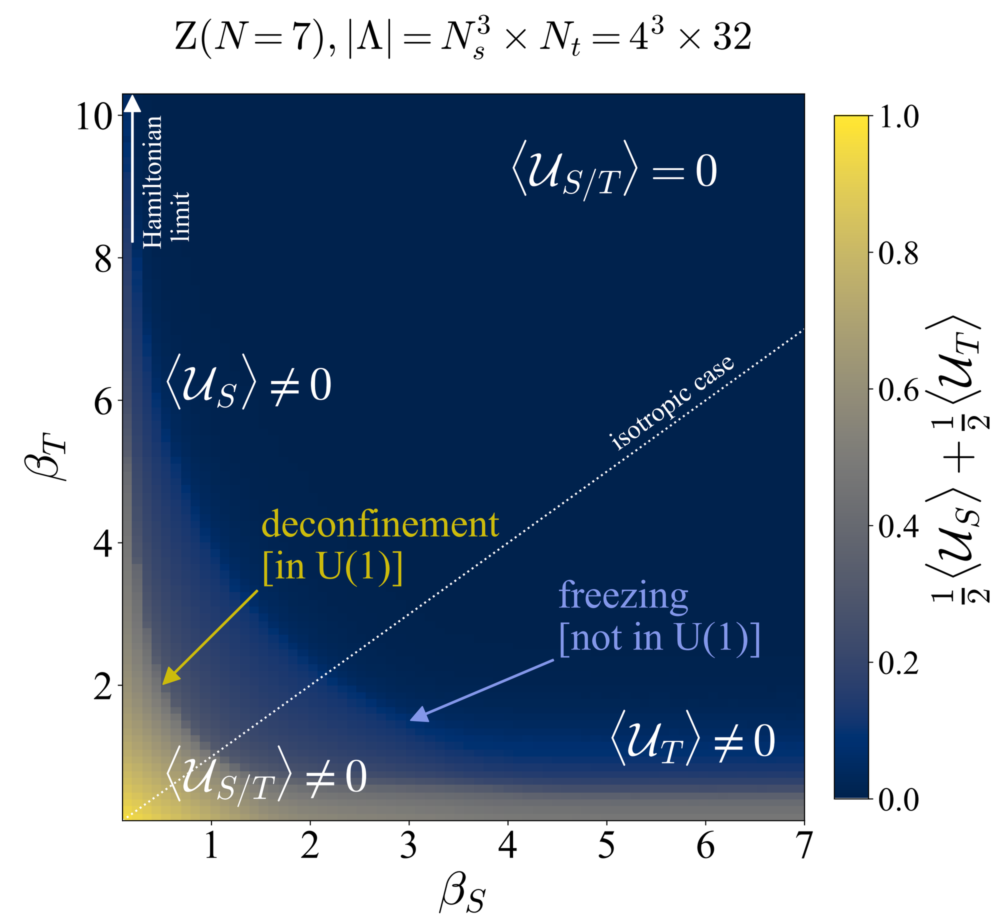
$$a_t\mathcal{H}_{\infty,\mathrm{kin}} = \frac{1}{\beta_T}\Delta_\infty \quad \Longrightarrow \quad \beta_T \sim \frac{1}{a_t}$$

$$a_t\mathcal{H}_{N,\mathrm{kin}} = e^{-c\beta_T}\Delta_N \quad \Longrightarrow \quad \beta_T \sim \log(1/a_t)$$

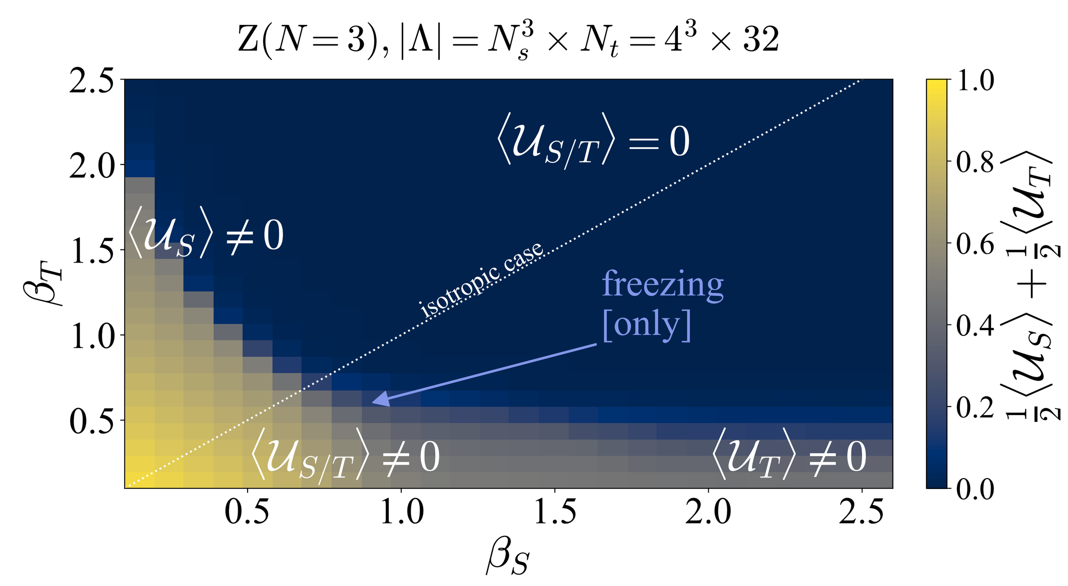
- **Spatial coupling**: $\beta_S \sim a_t$ (regardless).

Phase transitions in U(1) and Z(N) lattice gauge theories

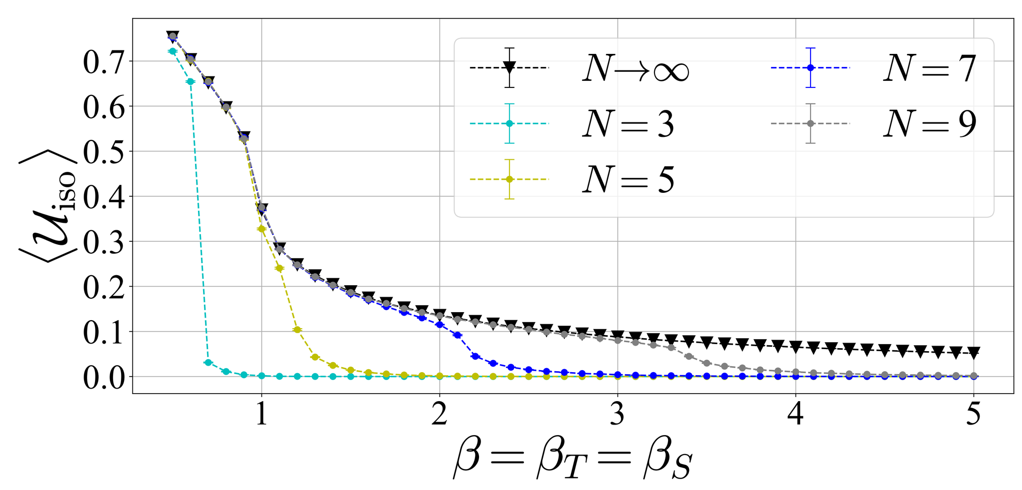
Anisotropic scan in (β_S, β_T):



For $N \lesssim 5$ freezing washes away deconfinement:



Freezing goes away as $N \to \infty$:

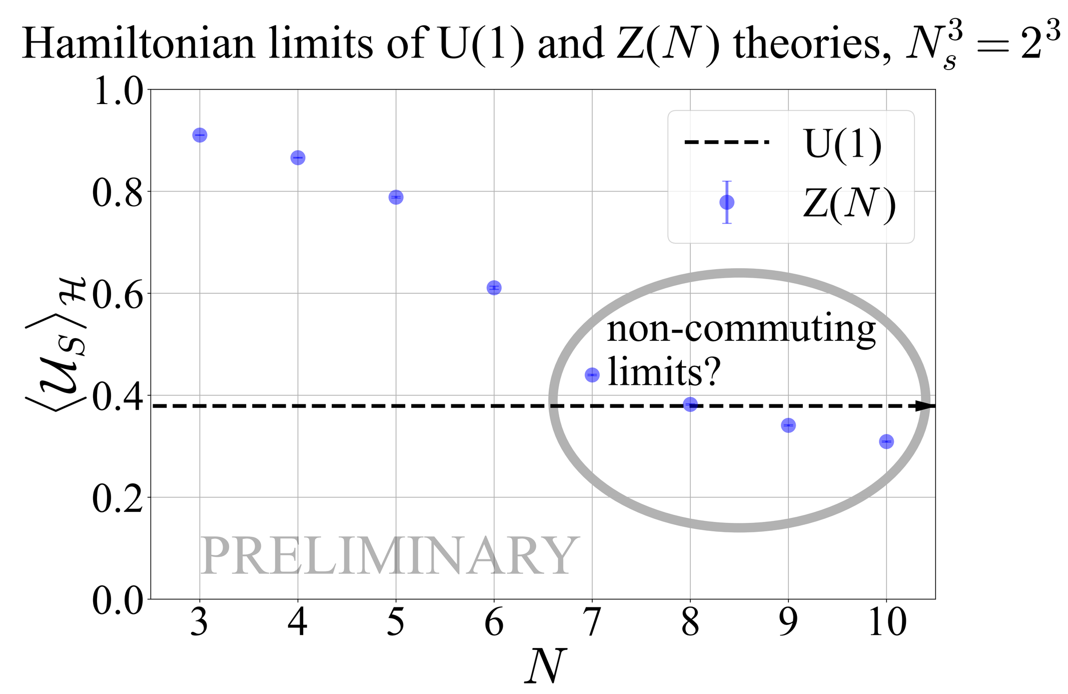


Hamiltonian limit from Euclidean simulations

Do the Hamiltonian limit and the $N \to \infty$ limit commute?

For a fixed spatial extent **2(+1) numerical limits** are to be computed for U(1) or Z(N):

1. Zero-temperature limit, i.e., $N_t \to \infty$,
2. $\beta_S \to 0$ and $\beta_T \to \infty$ (done simultaneously) based on scaling laws,
3. $N \to \infty$ for Z(N).



Summary

- **Freezing transition** due to discrete nature of the group Z(N), absent in U(1).
- Studying the **freezing transition with anisotropic lattices** for the first time.
- Scans in the (β_S, β_T) coupling space reveals **rich phase diagram** for Z(N) gauge theories.

Plans: further analysis of (non-)commuting limits, extend analysis to SU(2) and SU(3) gauge theories.

- **Deconfinement transition** also in Z($\gtrsim 5$) and U(1) theories. Otherwise flushed away by freezing.
- **Partial freezing transitions** if β_S small (large) and β_T large (small).
- **Remaining question**: what happens with the freezing transition in the Hamiltonian limit as $N \to \infty$, i.e., the discretization is taken to be finer?