



Motivations

Why leptonic B decays ?

– Direct determination of the CKM element $|V_{ub}|$.

– **Chirality-suppressed** in the SM \rightarrow powerful probe of (pseudo) scalar new physics.

$$\mathcal{B}(B^- \rightarrow \ell^- \bar{\nu}_\ell) = \tau_B G_F^2 f_B^2 |V_{ub}|^2 \frac{m_B m_\ell^2}{8\pi} \left(1 - \frac{m_\ell^2}{m_B^2}\right)^2$$

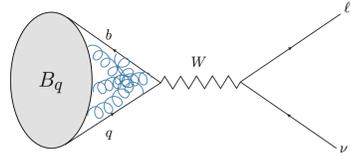
– Testing **flavor universality** in charged current : Belle II will measure the $\ell = \tau, \mu$ channels with 5 – 7 % uncertainties. [Belle II Physics Book]

Why QED corrections are needed?

– Pure hadronic effects are simple and well understood :

$$\langle 0 | \bar{q} \gamma^\mu \gamma_5 b | B_q(p) \rangle = i f_{B_q} p^\mu$$

[FNAL/MILC 1712.09262]



f_{B_q} is known with $\mathcal{O}(1\%)$ precision:

$$f_{B_q} = 189.4 \pm 1.4 \text{ MeV}$$

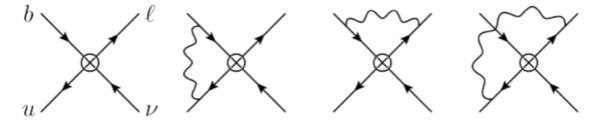
– **BUT** QED corrections are sensitive to the lepton mass and the restriction on additional radiation, yielding large (double) **logarithmic corrections** !

In the exclusive channel, with strong cuts on additional soft radiation, QED corrections can be **sizeable** and compete with QCD uncertainties

\rightarrow need a **precise** estimation

A multi-scale problem

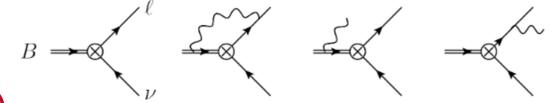
\rightarrow Focusing on $\ell = \mu$ case, QED introduces new scales (both static and dynamic) to which $B^- \rightarrow \mu^- \bar{\nu}_\mu$ is sensitive :



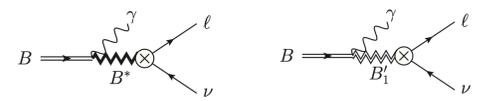
B-meson described as a superposition of Fock-states:
 $|\bar{u}b\rangle \oplus |\bar{u}bg\rangle \oplus \dots$ [Beneke et al(2019),JHEP10232]

\rightarrow virtual corrections can resolve the partonic substructure !

B-meson described as a point-like pseudo-scalar boson...



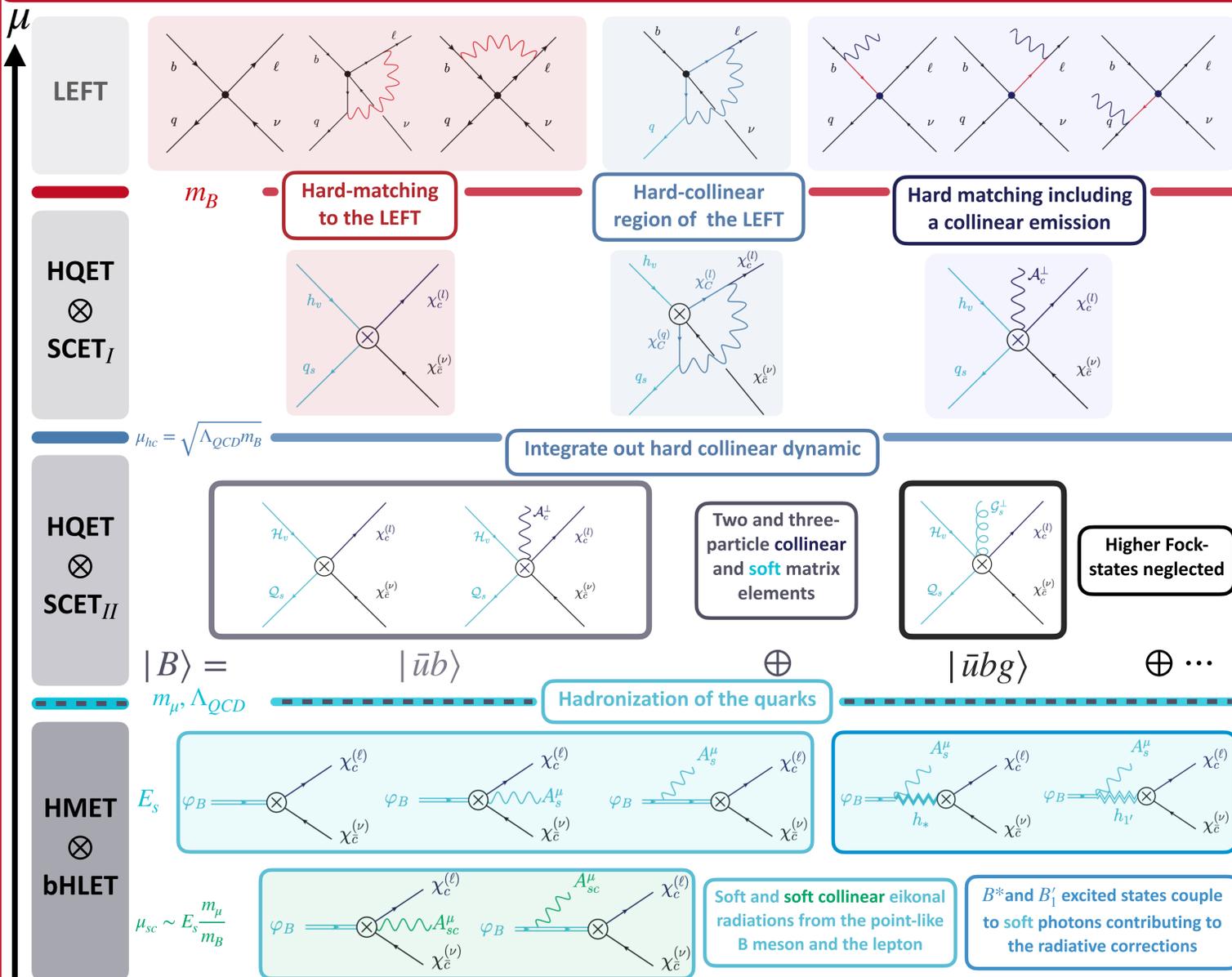
...including its first (pseudo) vector excited states as off-shell intermediate propagator ! [Becirevic et al (0907.1845)]



\rightarrow Scales dictated by the experimental energy cut on the additional final radiation.
 \rightarrow FCC and Belle II could in principle achieve a resolution with $E_s \sim \mathcal{O}(\Lambda_{QCD}^2)$...
...BUT $E_s \sim \mathcal{O}(\Lambda_{QCD})$ is needed for Belle II to improve the statistics of the measurement !

We need to find the **appropriate effective description** at each scale including all the relevant scales. The aim is to evaluate each object at its **natural scale**, turning a multi-scale problem to a product of single scale objects. \rightarrow **Factorization theorem**

The plan : embark in an EFT journey running down with the scale !



EFT toolbox

Power counting : $\lambda = \frac{m_\mu}{m_B} \sim \frac{\Lambda_{QCD}}{m_b}$

Light-cone directions :
 $n^\mu = (1,0,0,1)$, $\bar{n}^\mu = (1,0,0,-1)$

Relevant scaling :
 $p^\mu = (\bar{n} \cdot p, n \cdot p, p_\perp)$

Hard : $p^\mu \sim (1,1,1)m_b$
 $p^2 \sim m_b^2 \sim \mathcal{O}(1) \rightarrow$ **b-quark scale**

Hard-collinear : $p^\mu \sim (1,\lambda,\sqrt{\lambda})m_b$
 $p^2 \sim m_\mu m_b \sim \mathcal{O}(\lambda)$
 \rightarrow **Soft and collinear quark X-talk**

Collinear : $p^\mu \sim (1,\lambda^2,\lambda)m_b$
 $p^2 \sim m_\mu^2 \sim \mathcal{O}(\lambda^2)$
 \rightarrow **Lepton virtuality**

Soft : $p^\mu \sim (\lambda,\lambda,\lambda)m_b$
 $p^2 \sim \Lambda_{QCD}^2 \sim E_s^2 \sim \mathcal{O}(\lambda^2)$
 \rightarrow **Spectator virtuality and cut on soft radiations**

Soft-collinear : $p^\mu \sim (\lambda,\lambda^3,\lambda^2)m_b$
 $p^2 \sim E_s^2 (m_\mu^2/m_B^2) \sim \mathcal{O}(\lambda^4)$
 \rightarrow **Soft photon to the muon boosted in the B frame**

Conclusion : factorization formula for the decay rate ...

$$\Gamma = \Gamma_0 \left(|y_B(\mu)|^2 |K(\mu)|^2 S(E_s, \mu) + |y_B^*|^2 S_{B^*}(E_s) + |y_{B'_1}|^2 S_{B'_1}(E_s) \right)$$

Virtual corrections Real corrections Structure-dependent radiative corrections

Virtual corrections are factorized at the level of the amplitude : appear as an **effective Yukawa coupling**

$$y_B(\mu) = C_L(\mu) \times H(\mu) \times \mathcal{F}(\mu)$$

Real corrections factorize at the decay rate level : **convolution of radiative functions**

$$S(E_s, \mu) = W_s(E_s, \mu) \otimes W_{sc}(E_s, \mu)$$

...and estimation of the QED corrections !

$$|y_B(\mu)|^2 S(E_s, \mu) \sim m_\mu^2 \log \frac{E_s}{m_B} \log \frac{m_\mu}{m_B}$$

$$|y_B^*|^2 S_{B^*}(E_s) \sim E_s^2$$

$$|y_{B'_1}|^2 S_{B'_1}(E_s) : \text{(numerical cancellation)}$$

Structure-dependent radiative corrections enter via **additional Yukawa coupling** multiplied by a **soft radiative function**. They dominate for looser cuts!

