Baryonic Weak Decay from Lattice QCD

Arif Sarkar, Protick Mohanta, Sabiar Saikh, Subhasish Basak
National Institute of Science Education and Research Bhubaneswar
Jatni, Khurda, Odisha, India.

The Simulation of Heavy Quark

We have used the Anisotropic Clover Lattice Action is used to simulate the quarks, which is given as:

$$S_{lat} = a^4 \sum_{x,x'} \bar{\psi}(x') \Big(m_0 + \gamma_0 D_0 + \xi \vec{\gamma} \cdot \vec{D} - \frac{a}{2}(D^0)^2 - \frac{a}{2}\xi (\vec{D})^2 + \sum_{\mu\nu} \frac{ia}{4} c_P \sigma_{\mu,\nu} F_{\mu\nu} \Big)_{xx'} \psi(x)$$

For the simulation, the following parameters: the bare mass (m_0), the anisotropic parameter (ξ) and the clover coefficient (c_P) are to be tuned with some guess values.

This tuning removes errors in the on-shell Green's function, ensuring that the resulting action satisfies $m_0 a \geq 1$. As a result, this becomes the effective action for relativistic heavy quark action.

The Hadronic Two Point Correlations:

We consider the following as the two point correlation for Hadrons:

$$C(t) = \sum_{\vec{x}} \langle 0 | \mathcal{O}_H(\vec{x}, t) \mathcal{O}_H^{\dagger}(\vec{0}, 0) | 0 \rangle$$

$\mathcal{O}(\vec{x}, t)$ is the Hadronic interpolation operator. Considering the time evolution of this operator at large euclidean time, the coorelation takes the following form:

$$C(t) \underset{t \to 0}{=} |\langle 0 | \mathcal{O}_H | H \rangle|^2 [e^{-Et} + e^{-(T-t)E}]$$

Here, T is the lattice extent in the Euclidean time direction, and E is the energy of the lowest contributing state and it can also be considered as the mass of the Hadron. Therefore, in the $T \to \infty$ limit, the *Effective Mass* of the hadrons can be obtained as:

$$M_{\text{eff}} = \ln \left\{ \frac{\mathrm{Re}[C(t)]}{\mathrm{Re}[C(t+1)]} \right\}$$

Consider the case of Light Light Meson. The Hadron interpolation operator can be written as: $\mathcal{O}_{l_1, l_2} = \bar{l}^{f_1} \Gamma l^{f_2}$. Considering the Γ to be γ_5 and γ_k (for $k \in \{1, 2, 3\}$), the operator for psudoscalar meson and vector meson can be constructed and l^f is any of the light quarks with flavour f.

For a light light Meson, the Two Point Correlation function can be written as:

$$C_{l_1, l_2}(\vec{p}, t) = \sum_{\vec{x}} e^{i\vec{p}\cdot\vec{x}} \langle 0 | \bar{l}^{f_1}_{c1,s1}(\vec{x}, t) \Gamma_{s1,s2} l^{f_2}_{c1,s2}(\vec{x}, t) \bar{l}^{f_2}_{c2,s3}(\vec{0}, 0) \Gamma_{s3,s4} l^{f_1}_{c2,s4} | 0 \rangle = -\sum_{\vec{x}} e^{i\vec{p}\cdot\vec{x}} \mathrm{Tr}[M^{f_1}(0, x) \Gamma M^{f_2}(x, 0)]$$

This form of the correlation function is utilized in simulations, where the propagators are simulated using the action S_{lat}.

Non Perturbative Tuning of Mass:

By suitably tuning the parameters, we extract the masses of the light-light vector meson and the pseudoscalar meson. A plot for a given momentum is shown below:

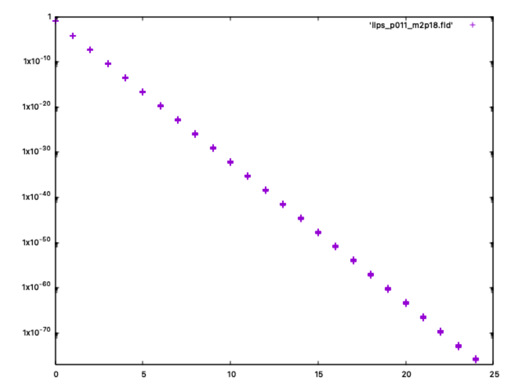

Figure: This plot is for the extraction of mass for light light psudoscalar meson of spatial momentum $\vec{p} = (0, 1, 1)$

After extracting the masses of both the mesons, we can calculate the following quantities:

- **Spin Average Mass($\bar{M}$:)** $\frac{1}{4}(M_{psudoscalar} + 3M_{vector})$
- **Hyperfine Splitting(ΔM):** $M_{vector} - M_{psudoscaler}$
- **Velocity of Light(c):** $c^2 = \frac{M^2_{\vec{p}=\vec{p}} - M^2_{\vec{p}=0}}{|\vec{p}|^2}$

By tuning we aim to match the $\bar{M}$ and ΔM to the values of PDG and set c to 1.

Further Discussion Non Perturbative Tuning of Mass:

Upon suitable tuning once the output parameters ($\bar{M}, \Delta M, \frac{M_1}{M_2}$) are linearly dependent with input parameters ($m_0 a, c_P, \Xi$), we can construct the relation:

$$[\bar{M}, \Delta\bar{M}, \frac{M_1}{M_2}] = J \cdot [m_0 a, c_P, \xi] + A \quad (M_1 \text{ and } M_2 \text{ are rest mass and kinetic mass}).$$

For a deviation in input parameters $\sigma_{\{m_0 a, c_P, \xi\}}$ and we can construct, **seven** set of such vectors as:

$$\begin{bmatrix} m_0 a \\ c_P \\ \xi \end{bmatrix}, \quad \begin{bmatrix} m_0 a - \sigma_{m_0 a} \\ c_P \\ \xi \end{bmatrix}, \quad \begin{bmatrix} m_0 a + \sigma_{m_0 a} \\ c_P \\ \xi \end{bmatrix}$$

For each of this set of input parameters, there is an output $\{\bar{M}_B, \Delta\bar{M}, M_1/M_2\}_i^T \equiv Y_i$. Hence '$J$' and '$A$' can be calculated as:

$$J = \left[\frac{Y_3 - Y_2}{2\sigma_{m_0 a}}, \frac{Y_5 - Y_4}{2\sigma_{c_P}}, \frac{Y_7 - Y_6}{2\sigma_{\xi}} \right]$$

$$A = Y_1 - J \times [m_0 a, c_P, \xi]^T$$

Finally the RHQ parameters:

$$[m_0 a, c_P, \xi]_{RHQ} = J^{-1} \times [\bar{M}, \Delta\bar{M}, \frac{M_1}{M_2}]_{PDG} - A$$

The Three Point Function:

Using three point function we will study the process: $\Xi_C \to \Xi$. For that we need to simulate the following three point function which is:

$$C^{3\mu}_{\alpha\delta}(T, \tau) = \sum_{\vec{x}, \vec{y}} \langle 0 | \{\Xi_S(\vec{x}, T)\}_{\alpha} V^{\mu}(\vec{y}, \tau) \Xi_C^{\dagger}(\vec{0}, 0)_{\delta} | 0 \rangle$$

For the local rest frame of the Ξ, the three point function can be written as:

$$\frac{1}{4E_{P_S} E_{P_C}} \langle 0 | \Xi_S(\vec{0}, 0) | M_{\Xi_S} \rangle \langle M_{\Xi_S} | V^{\mu}(\vec{0}, 0) | M_{\Xi_C} \rangle \times \langle M_{\Xi_C} | V^{\mu}(\vec{0}, 0) | 0 \rangle e^{E_S(T-\tau)} e^{-\tau E}$$

And the form that is to be simulated can be given as:

$$\mathcal{F}\mathcal{E} \epsilon_{abc} \epsilon_{a'b'c'} D^u(x, 0)^{aa'}_{\lambda\rho} D^c(y, 0)^{wc}_{\kappa\delta} \times \left[D^s(x, y)^{bw}_{\sigma\beta} D^s(x, 0)^{cb}_{\alpha\gamma} - D^s(x, y)^{cw}_{\alpha\beta} D^s(y, 0)^{bb'}_{\sigma\gamma} \right]$$

where we have considered the form of Ξ_S, Ξ_C and $V^{\mu}(\vec{y}, \tau)$ as:

$$\Xi = \epsilon_{abc} [u_a^{\lambda}(\vec{x}, T)(C\gamma_5) s_b^{\sigma}] s_c^{\alpha} \quad \text{and} \quad v^{\mu}(\vec{y}, \tau) = \bar{s}_w^{\beta} \gamma^{\mu} c_w^{\kappa}$$

We will use the RHQ action to simulate D^c and the clover action for D^s and D^u.

Future Directions

- Once we calculate the three-point function, we can then derive the double ratios, which will ultimately enable us to determine the form factors for a given process.

Conclusions:

- For further calculation of Three Point function, we need to need to calculate the propagators. For that we need the masses of those following quarks(u, s, c) to be tuned for that we have to follow the aforementioned formalism.
- if we use two component NRQCD for this calculation, it suffers from tuning problem. Whereas, treating these quaks relativistically using RHQ and cloveraction we can have a better tuning.

References:

- Y. Aoki et al. *Nonperturbative tuning of an improved relativistic heavy quark action with application to bottom specstroscopy*. Physical Review D, 86, 116003. 2012.
- Christ, Li, Lin *Relativistic heavy quark effective action*. Physical Review D, 76, 074505. 2007.