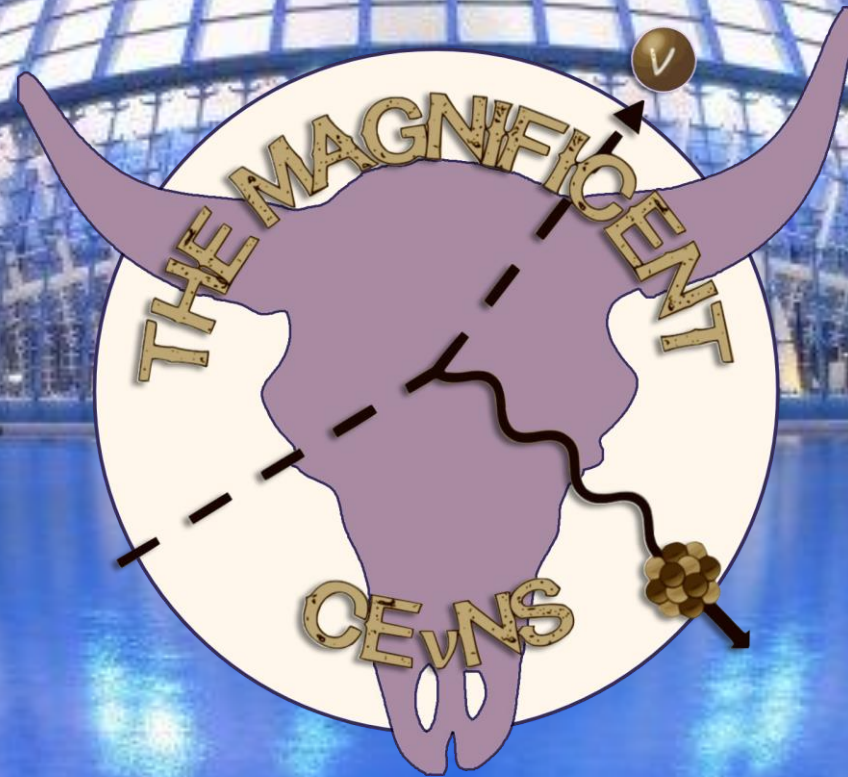


Exploring the Migdal Effect in $CE\nu NS$ searches at low energies



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13/06/2024

In collaboration with

M. Cadeddu, N. Cargioli, F. Dordei, C. Giunti



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Istituto Nazionale di Fisica Nucleare

Exploring the Migdal Effect in CEvNS searches at low energies

Outline of the talk

Contents lists available at ScienceDirect
Physics Letters B
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Letter
On the impact of the Migdal effect in reactor CEvNS experiments
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ARTICLE INFO ABSTRACT

Editor: B. Balantekin

The search for coherent elastic neutrino nucleus scattering (CEvNS) using reactor antineutrinos represents a formidable experimental challenge, recently boosted by the observation of such a process at the Dresden-II reactor site using a germanium detector. This observation relies on an unexpected enhancement at low energies of the measured quenching factor with respect to the theoretical Lindhard model prediction, which implies an extra observable ionization signal produced after the nuclear recoil. A possible explanation for this additional contribution could be provided by the so-called Migdal effect, which however has never been observed. Here, we study in detail the impact of the Migdal contribution to the standard CEvNS signal calculated with the Lindhard quenching factor, finding that the former is completely negligible for observed energies below ~ 0.3 keV where the signal is detectable, and thus unable to provide any contribution to CEvNS searches in this energy regime. To this purpose, we compare different formalisms used to describe the Migdal effect that intriguingly show a perfect agreement, making our findings robust.

1. Introduction

Coherent elastic neutrino-nucleus scattering (CEvNS) is a pure weak neutral current low-energy process predicted by Freedman in 1973 [1] where the neutrino interacts with the nucleus as a whole. Namely, all the nucleons within the nucleus respond coherently to the neutrino interaction, leading to a higher cross section than other low-energy processes involving neutrinos. The remarkable observations of CEvNS from the COHERENT Collaboration [2–4] have opened a new era to test our knowledge of the Standard Model (SM) of particle physics and has posed an exciting technological challenge to develop innovative detectors capable of spotting the extremely tiny nuclear recoils produced as a single outcome of the interaction [5–20]. These experiments play a crucial role in advancing our knowledge of neutrino interactions and for their implications for fundamental physics [21–36].

CEvNS requires a high neutrino flux to produce signal events above experimental backgrounds. Among the different neutrino sources available, in this study, we focus on CEvNS produced from reactor antineutrinos and observed with germanium detectors. There are three main germanium detectors currently operating, namely NCC-1701 [13,14] (also referred to as Dresden-II), CONUS [37] and vGEN [12], located 10.39 m, 17.1 m and 11 m away from 2.96 GW_{th}, 3.9 GW_{th}, 3.1 GW_{th} commercial reactors, respectively. Interestingly, the Ricochet experiment at the ILL site, 8.8 m away from the core of the 58.3 MW_{th} research nuclear reactor, is also aiming to measure CEvNS down to the sub-100 eV nuclear energy recoil regime [38].

In particular, the recent first observation of CEvNS at the Dresden-II reactor [14] has gained a lot of attention due to the broad impact of such a result on current and future CEvNS searches, and the physics that can be extracted within the SM and beyond [39–42]. As a matter of fact, this measurement is highly affected by the knowledge of the germanium quenching factor (QF) at low nuclear recoil energies. The QF quantifies the reduction of the ionization yield produced by a nuclear recoil with respect to an electron recoil of the same energy. Indeed, the CEvNS observation by Dresden-II depends crucially on the two new QF measurements reported in Ref. [43]. They have been obtained from photon source measurements, so-called YBe, and from iron-filtered monochromatic neutrons, so-called Fef [43]. However, these two QF determinations are in contrast with and significantly higher than the standard Lindhard prediction with the parameter $k = 0.157$ [44] and other independent experimental measurements (see e.g. Ref. [45] for a recent measurement and the summary plot in Fig. 1). Moreover, CONUS

1

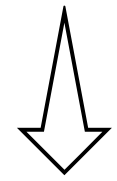
The hunt for CEvNS from reactor $\bar{\nu}_e$

2

The quenching factor puzzle on Ge

3

The Migdal Effect

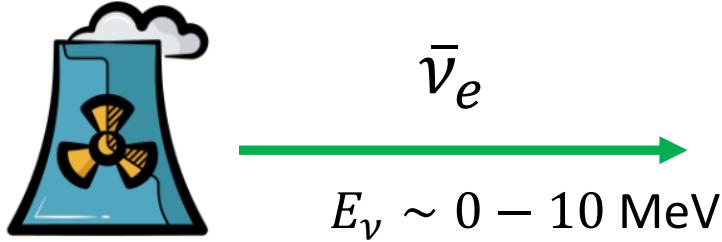


4

Impact of the Migdal Effect in the Dresden-II observation

The Hunt for CEνNS from reactor $\bar{\nu}_e$

Reactor antineutrinos



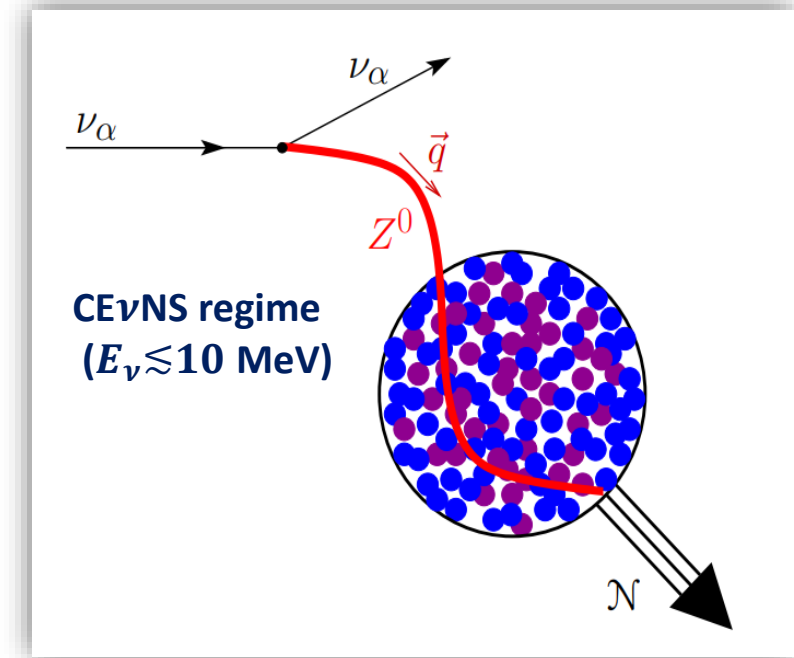
World-wide effort to spot this tiny interaction

Neutrinos are produced through β -decay at a reactor power plant. These low energetic neutrinos produce small nuclear recoils.

Coherent elastic neutrino nucleus scattering (CEνNS) is pure weak neutral current process when the De-Broglie wavelength for the Z^0 boson is bigger than the size of the nucleus.

$$\lambda_{Z^0} \simeq \frac{h}{|q|} \geq 2R \quad |q| \sim E_\nu$$

To fully exploit the coherency in the interaction, it is beneficial to consider low-energy neutrinos



The Hunt for CE ν NS from reactor $\bar{\nu}_e$

Pro of CE ν NS search from using reactor anti-neutrinos:

- ✓ Full Coherency
- ? No dependence on the nuclear structure
- ✓ Very high neutrino flux
- ✓ Extract weak mixing angle and neutrino charge radius at unprobed energy regime



See [M. Cadeddu](#), [F. Dordei](#) and [O. Miranda](#) talks!

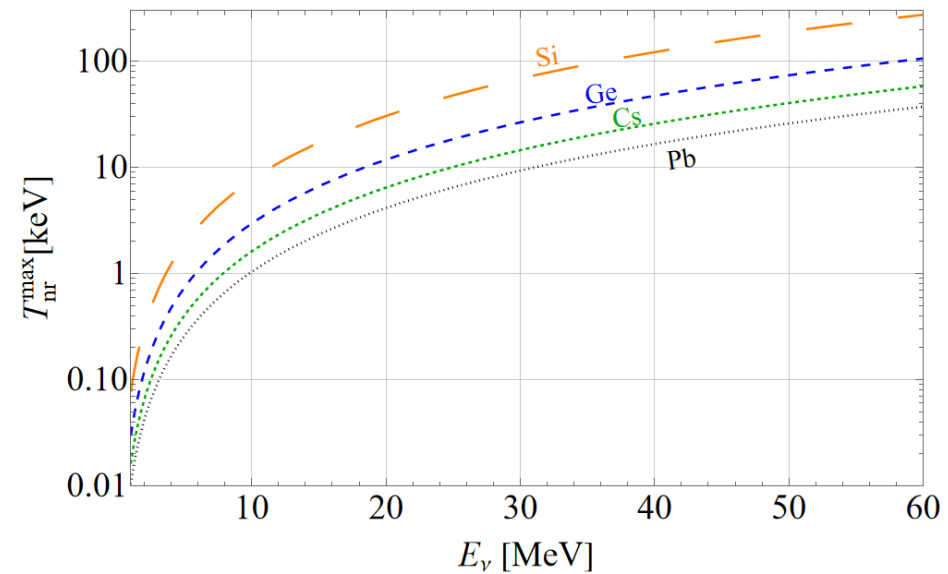
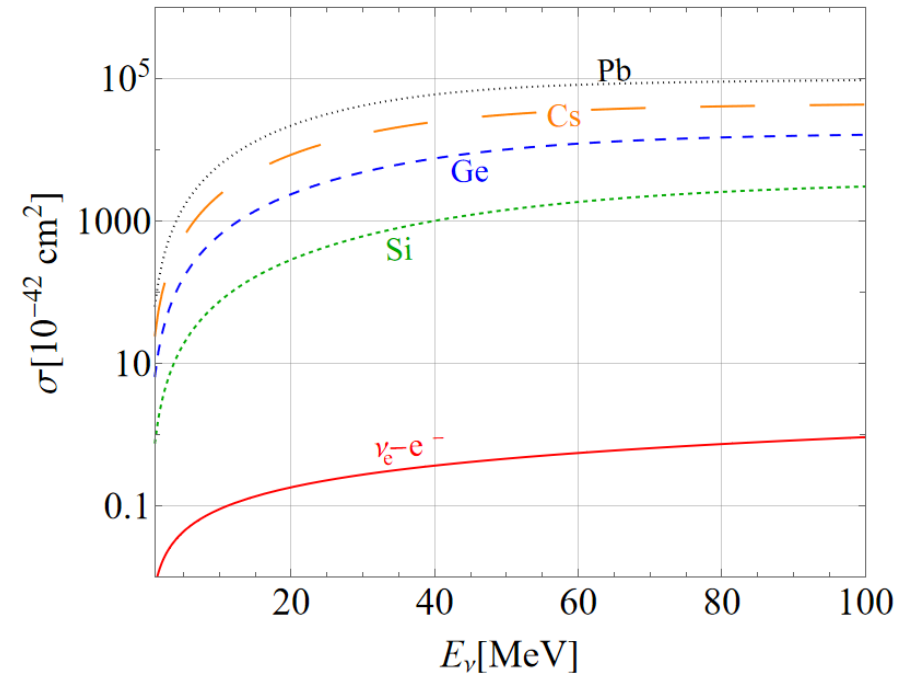
- ✓ Light mediators and ν EM properties 'explode' at low energies



See [C. A. Ternes](#) and [Y. Li](#) talks

Cons

- ✗ CE ν NS cross section is smaller for lower neutrino energy
- ✗ Very low threshold is required
- ✗ Pioneering low energy physics (new effects might occur)



The world-wide hunt



The NCC-1701 detector at the Dresden-II reactor

- 96.4 day (**Rx-ON**) exposure of a 3 kg ultra-low noise germanium detector.
- 10.39 m away from the Dresden-II boiling water reactor ($P = 2.96 \text{ GW}_{\text{th}}$).
- **Strong preference** ($p < 1.2 \times 10^{-3}$) for the presence of $\text{CE}\nu\text{NS}$.
- This observation was under debate within the $\text{CE}\nu\text{NS}$ Community.

It relies on two main hypotheses:

1. Deep understanding of the **background model**.

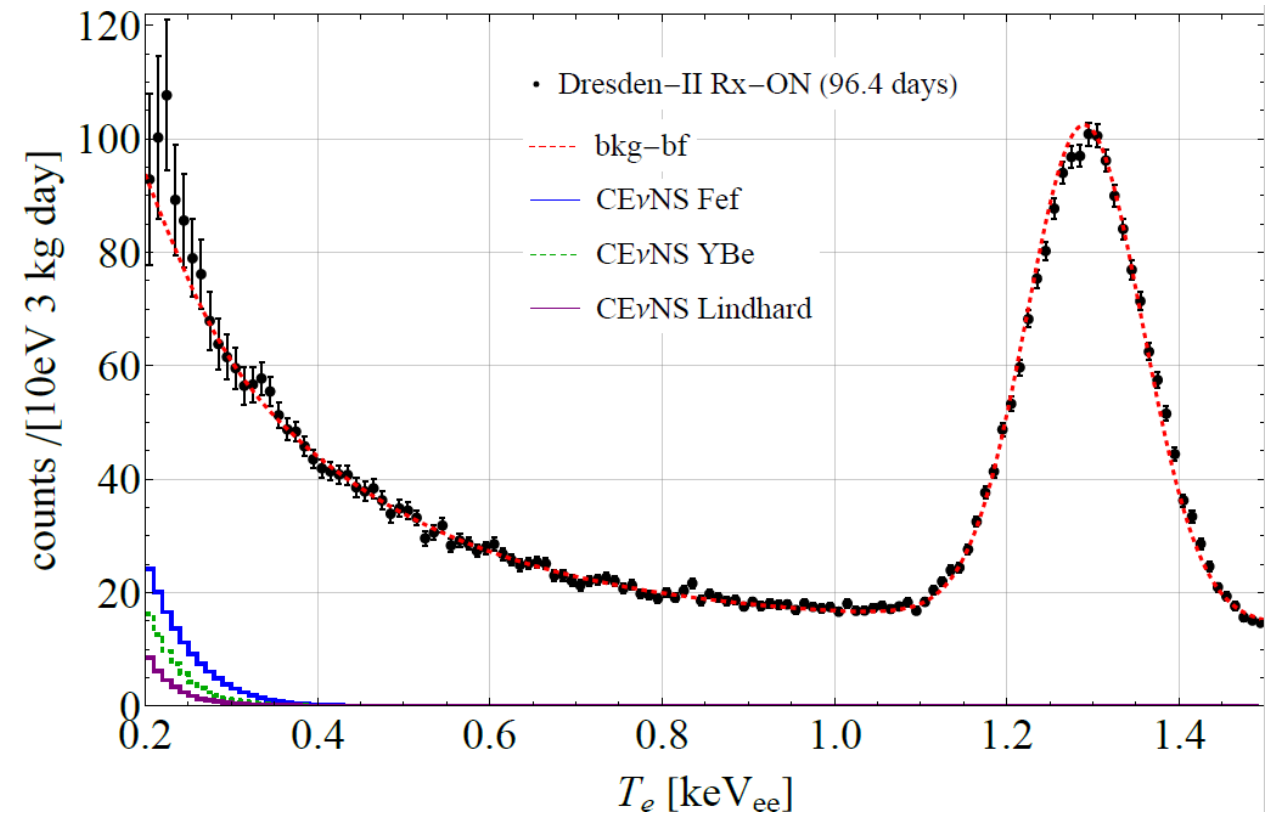
$$\frac{dN^{\text{bkg}}}{dT_e} = N_{\text{epith}} + A_{\text{epith}} e^{-T_e/T_{\text{epith}}} + \sum_{i=L1,L2,M} \frac{A_i}{\sqrt{2\pi}\sigma_i} e^{-\frac{(T_e - T_i)^2}{2\sigma_i^2}}$$

2. Deep understanding of the **predicted $\text{CE}\nu\text{NS}$ signal**.

PHYSICAL REVIEW LETTERS **129**, 211802 (2022)

Measurement of Coherent Elastic Neutrino-Nucleus Scattering
from Reactor Antineutrinos

J. Colaresi,¹ J. I. Collar^{2,*}, T. W. Hossbach³, C. M. Lewis², and K. M. Yocum¹



Formation of the CEνNS signal

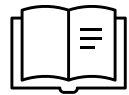
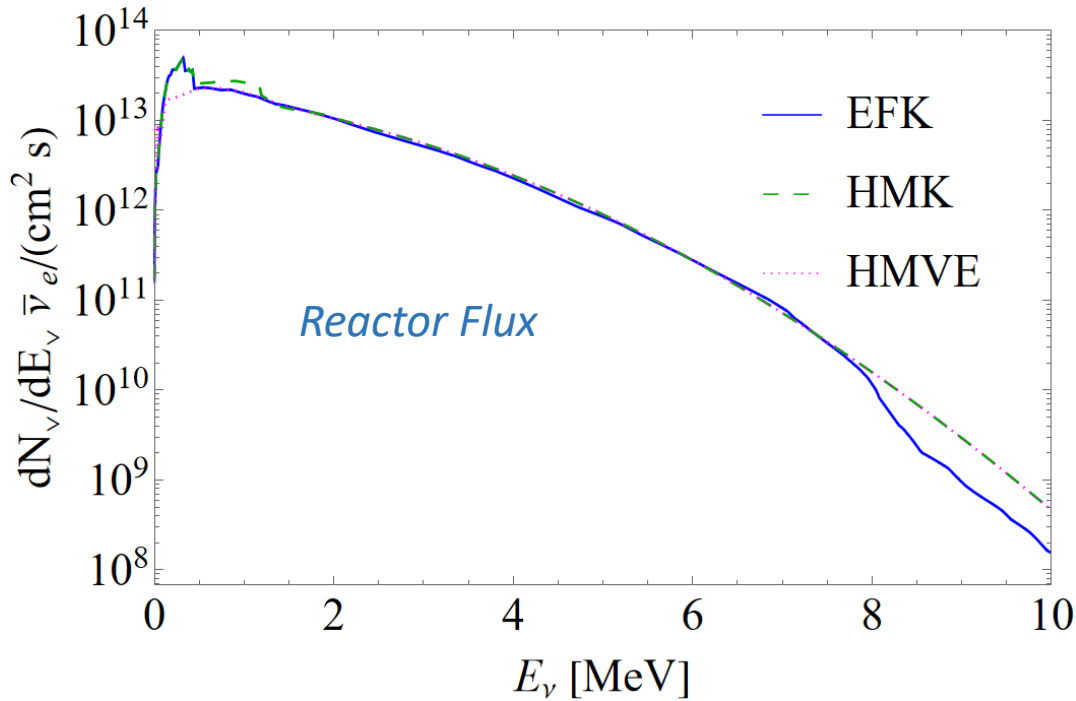
$$\text{CE}\nu\text{NS Event rate} = \int dE_\nu \frac{d\sigma}{dT_{\text{nr}}} \times \frac{d\phi}{dE_\nu} \times R$$

✓ CEνNS cross section. Very well known thanks also to the COHERENT measurements.

✓ Neutrino Flux is under control.
 ✓ Different parameterizations lead to very similar predictions!

Detector response. Include different effects:
 ? Energy efficiency effects
 ? Energy smearing effects
 ? Quenching factor

} Beyond our control.



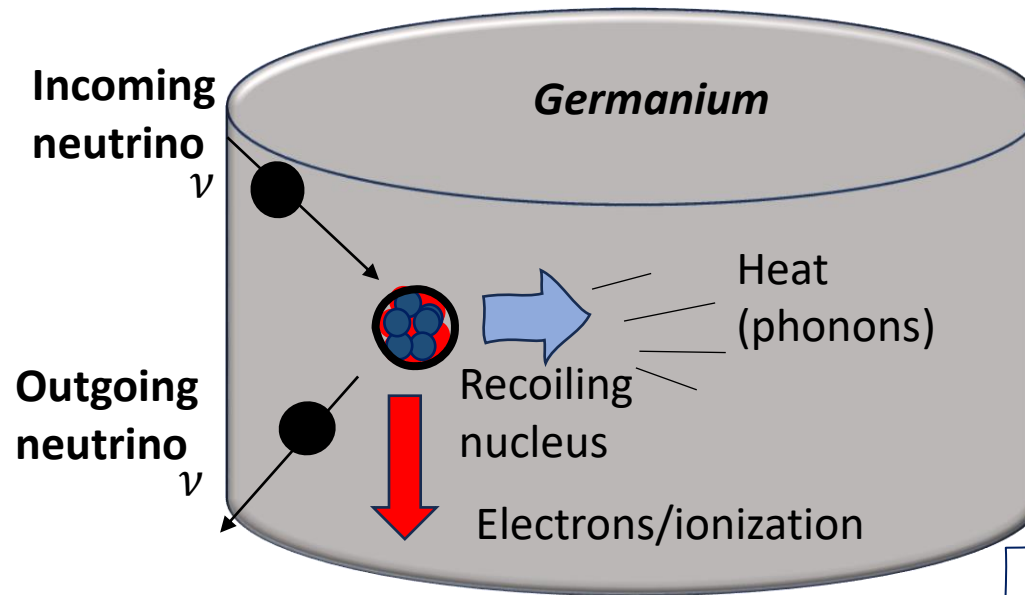
See JHEP 09 (2022) 164 for a description of the reactor fluxes used

Defined as
 ‘the reduction of the ionization yield produced by a nuclear recoil with respect to an electron recoil of the same energy’

The quenching factor

When a neutral particle (neutron, neutrino, dark matter) hits a nucleus, the energy is splitted in two components:

- i. **Heat** (does not produce observable signal in the Dresden-II detector)
- ii. **Ionization energy** keV_{ee}



We have a theory that predicts this effect \rightarrow **Lindhard theory**

Quenching factor $\rightarrow f_Q(T_{nr}) = \frac{T_e}{T_{nr}}$

Observed energy in the detector $\leftarrow T_e$

Total energy deposit due to the nuclear recoil $\leftarrow T_{nr}$

Do we really know its behaviour at low energies? $?$

The quenching factor puzzle

PHYSICAL REVIEW D **103**, 122003 (2021)

Editors' Suggestion

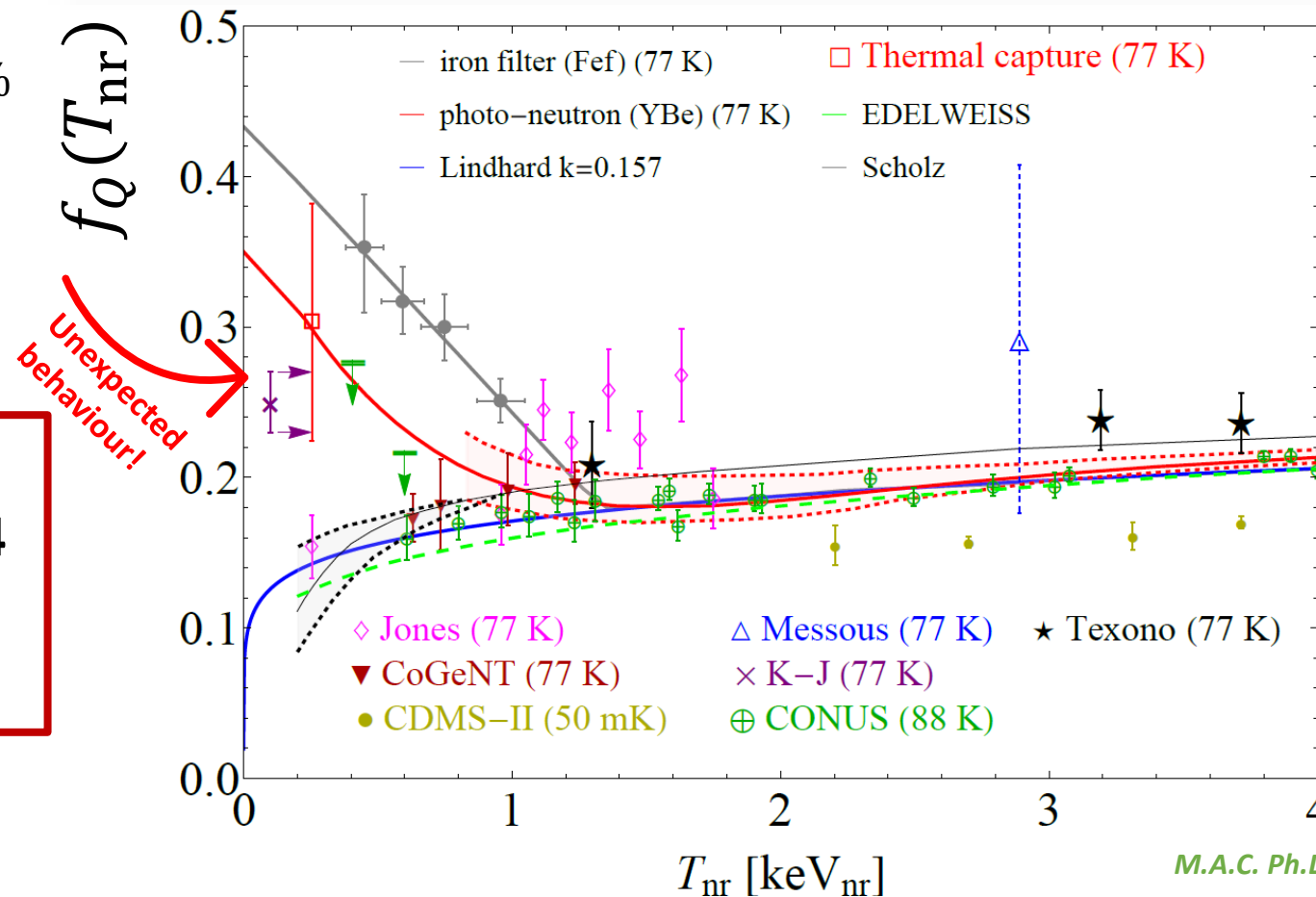
Germanium response to sub-keV nuclear recoils:
A multipronged experimental characterization

New actor in the game
arXiv: 2405.10405

$$f_Q(T_{nr} = 0.254 \text{ eV}_{nr}) = 25 \pm 2\%$$

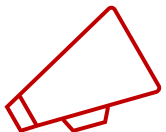
greater than the 14 % predicted
by Lindhard Model!
Pointing to a deviation from the
Lindhard theory.

Tension with CONUS
recent result **2401.07684**
(No Excess observed
compatible with CEνNS)



Key Point

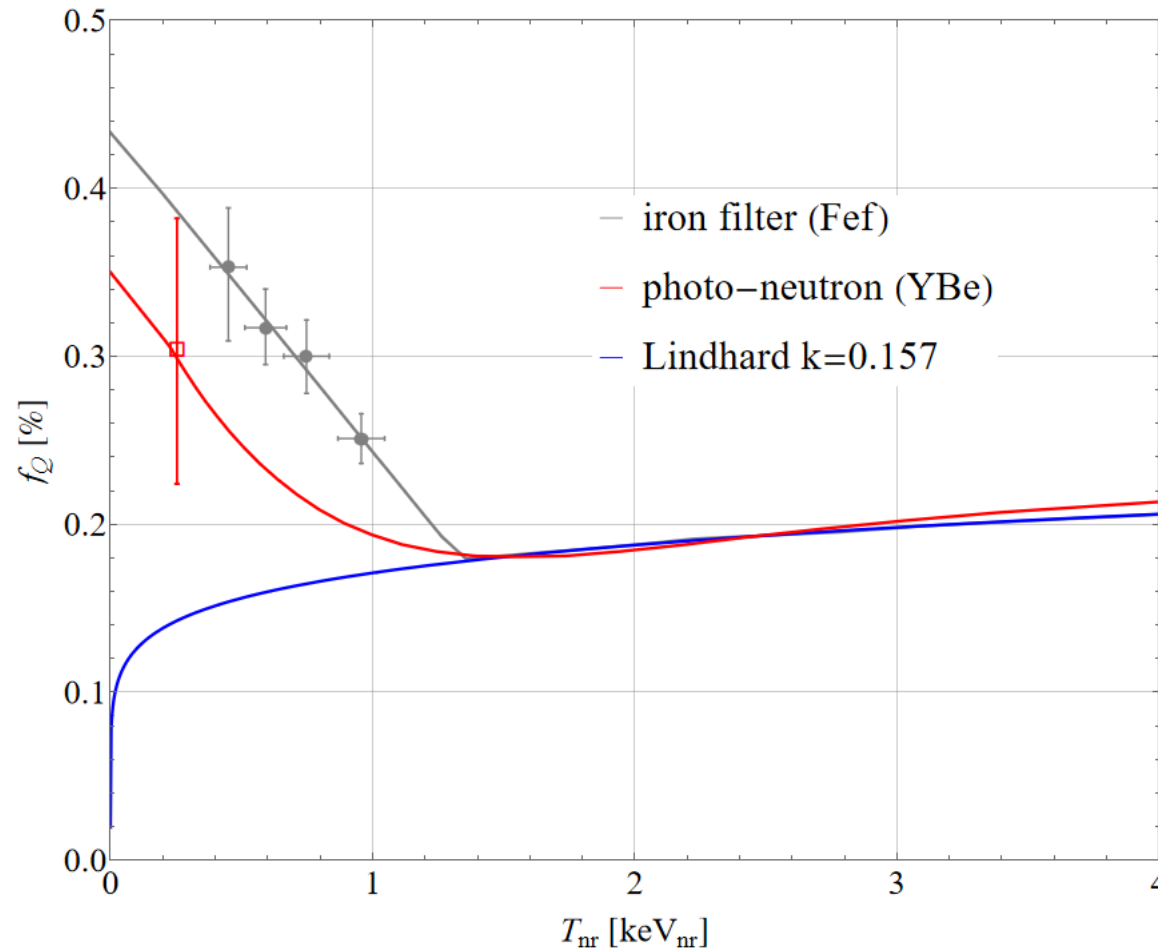
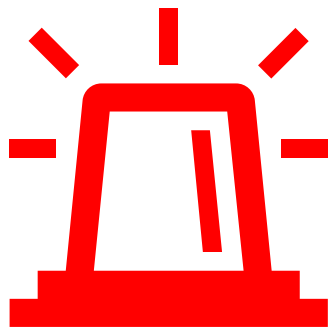
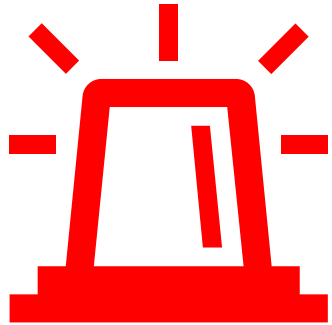
- The new measurements are **much higher** than expected.
- The new quenching results in a **much higher** CEνNS signal.



J. Hakenmüller
Talk

M.A.C. Ph.D. thesis (in preparation)

The quenching factor puzzle

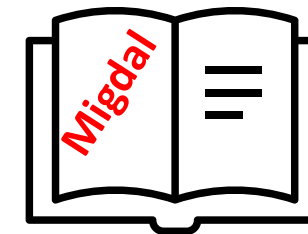


PRD 103, 12003 (2021)

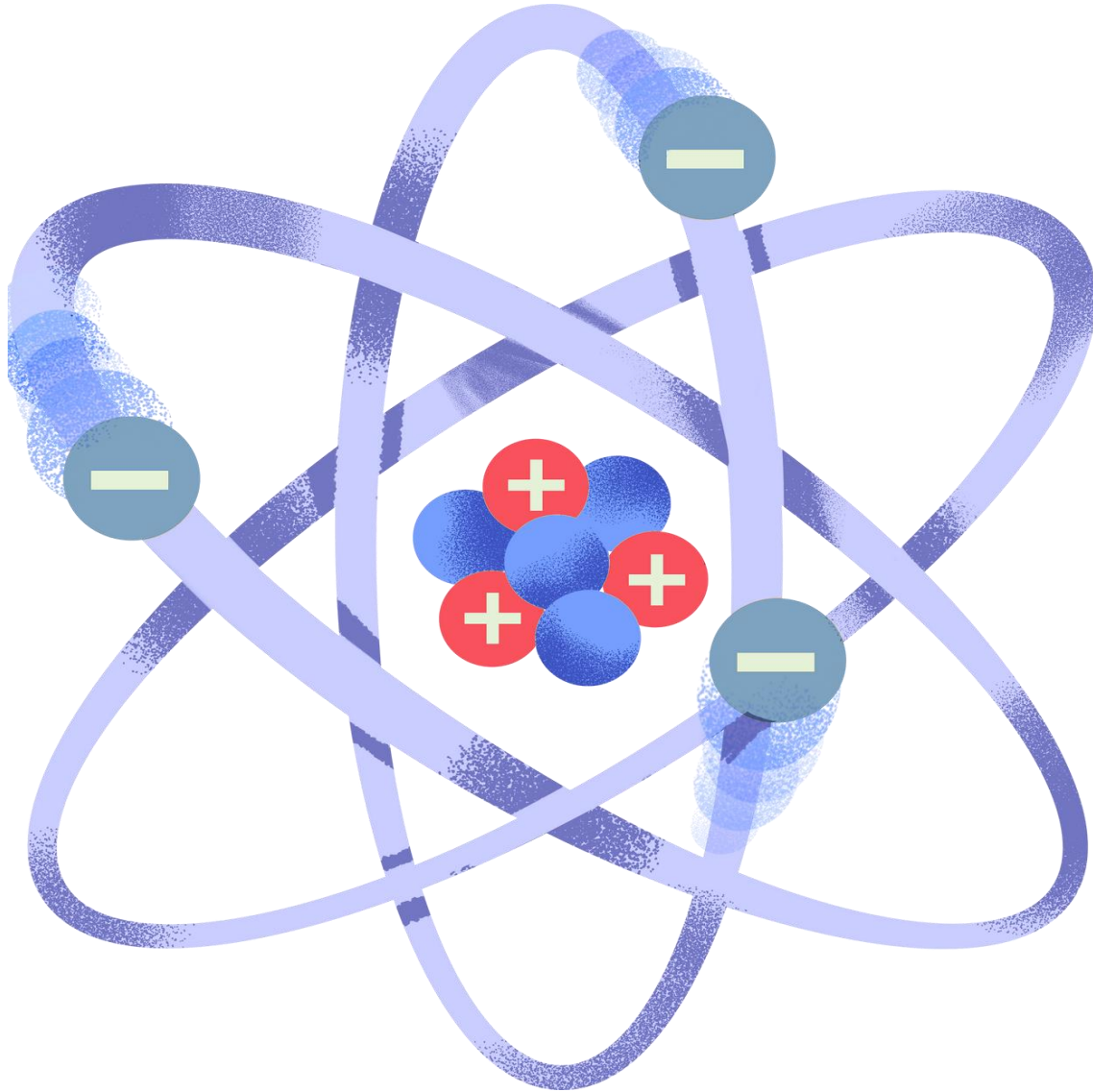
Our results point to a marked **deviation** from the predictions of the Lindhard model in this mostly unexplored energy range.

[...]

We comment on the compatibility of our data with **low-energy processes such as the Migdal effect** [...]



The Migdal Effect



The *Migdal effect* refers to a phenomenon occurring after a nuclear process:

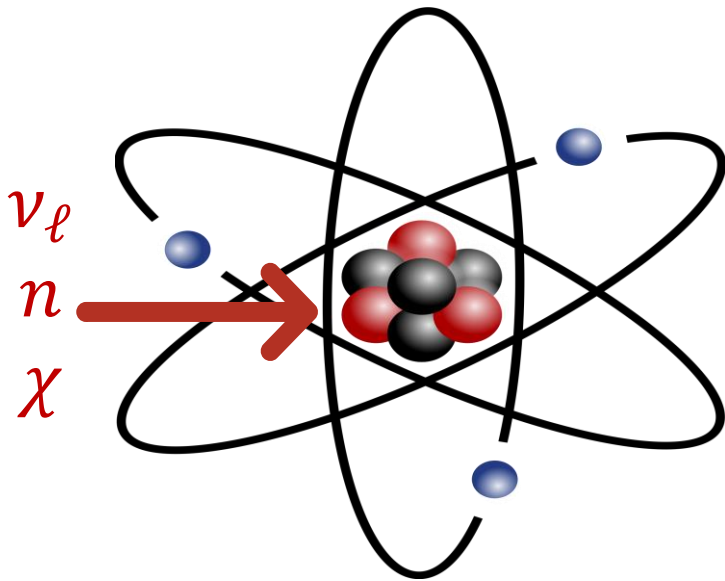
- *α and β decays*
- **Neutral particle scattering**

This results in extra radiation being emitted from the atom.

The Migdal effect induced by NRs

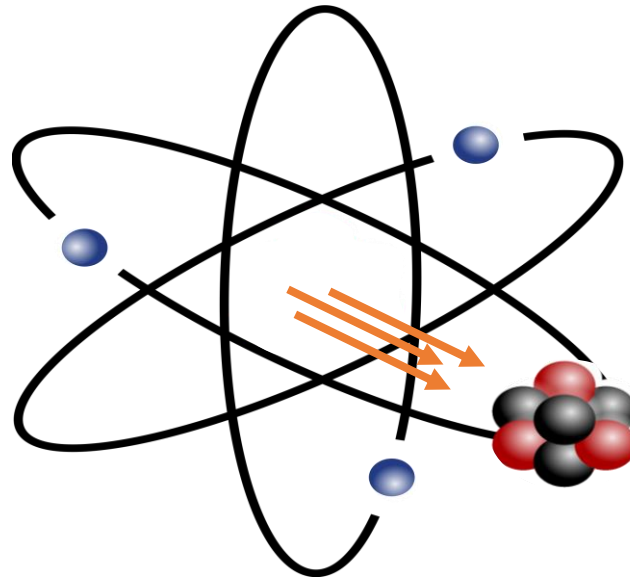
Yet-to-be-observed quantum mechanical effect predicted by A. Migdal in 1939.

Step 1



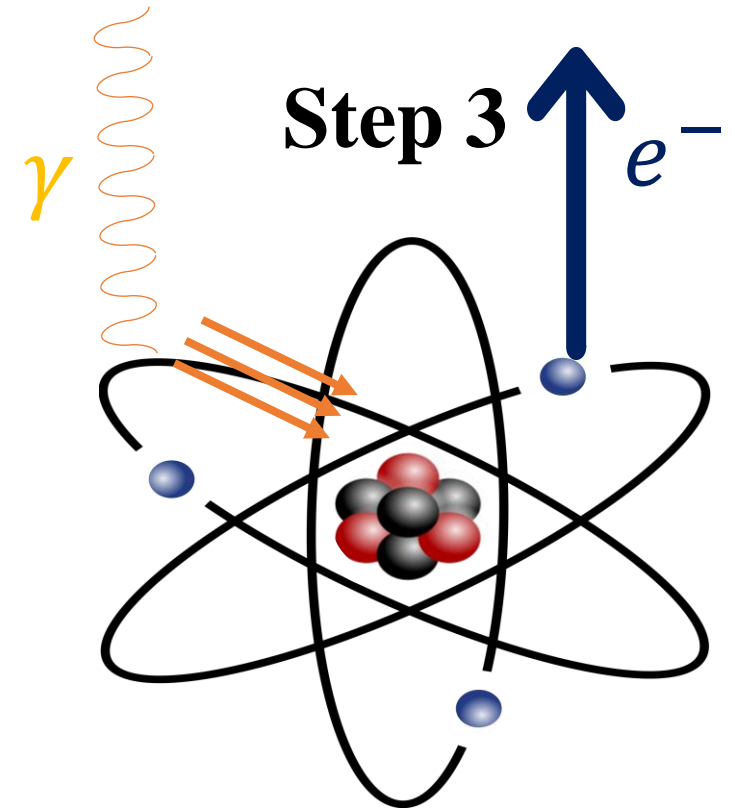
Incoming neutral particle that induces a **nuclear recoil**.

Step 2



The nuclear recoil happens. **The electronic cloud** does not follow **immediately** the recoiling nucleus.

Step 3



The electron cloud follows the nucleus. In this process, **extra ionization can be produced**.

Migdal effect formalisms

Ibe et al. formalism

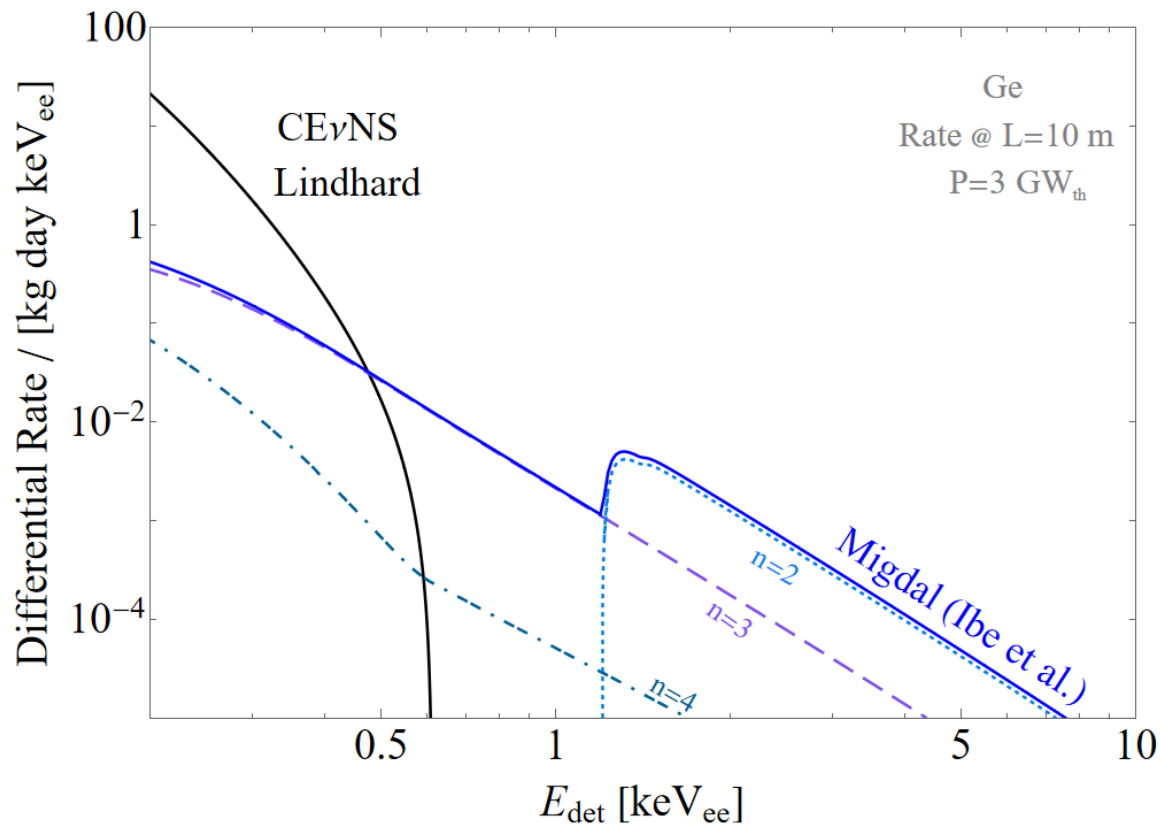
Ibe et al. *JHEP* 03 (2018) 194

- ✓ Evaluate the ionization rate for the electron in the initial quantum state (n, ℓ)
- ✓ Ionization probabilities are evaluated using the Dirac-Hartree-Fock method.

- ✗ Treats the atom as isolated
- ✗ Large theoretical uncertainties
- ✗ Computationally expensive

$$\left(\frac{d^2 \sigma_{\bar{\nu}_e - \mathcal{N}}}{dT_{\text{nr}} dT_e} \right)_{nl}^{\text{Ibe et al.}} = \frac{d\sigma_{\nu \ell - \mathcal{N}}}{dT_{\text{nr}}} \frac{1}{2\pi} \frac{d}{dT_e} p_{qe}^c(n\ell \rightarrow T_e)$$

Ionization probabilities provided in
Ibe et al. *JHEP* 03 (2018) 194



$$E_{\text{det}} = f_Q(T_{\text{nr}})T_{\text{nr}}$$

Migdal effect formalisms

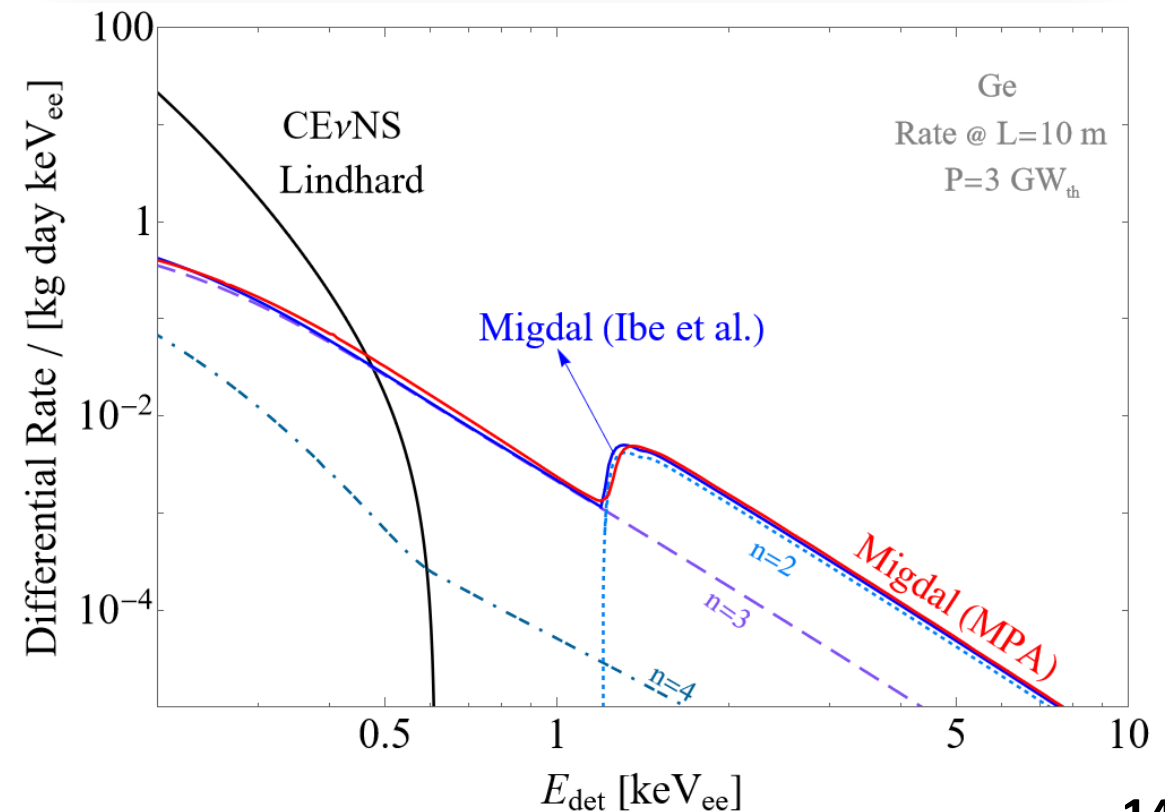
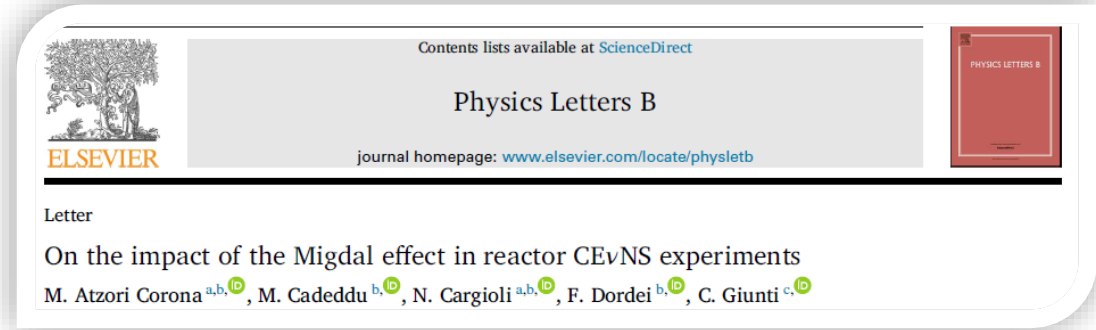
Migdal Photo Absorption (MPA)

Liu *et al.* *Phys.Rev.D* 102 (2020) 12, 121303

- ✓ Relates the Migdal ionization rate to the experimentally well-known **photoabsorption cross section**.
- ✓ Data-driven approach
- ✓ Accounts for many-body effects
- ✓ Computationally easy
- ✗ It is not possible to evaluate the single atomic state (n, ℓ) contribution

$$\left(\frac{d^2 \sigma_{\bar{\nu}_e - \mathcal{N}}}{dT_{\text{nr}} dE_r} \right)_{\text{Migdal}}^{\text{MPA}} = \frac{d\sigma_{\nu \ell - \mathcal{N}}}{dT_{\text{nr}}} \frac{1}{2\pi^2 \alpha_{\text{EM}}} \frac{m_e^2}{M} \frac{T_{\text{nr}}}{E_r} \sigma_{\gamma}^{\text{Ge}}(E_r)$$

Surprisingly, the two approaches lead to very consistent results



Dresden-II and the Migdal effect

Contents lists available at ScienceDirect






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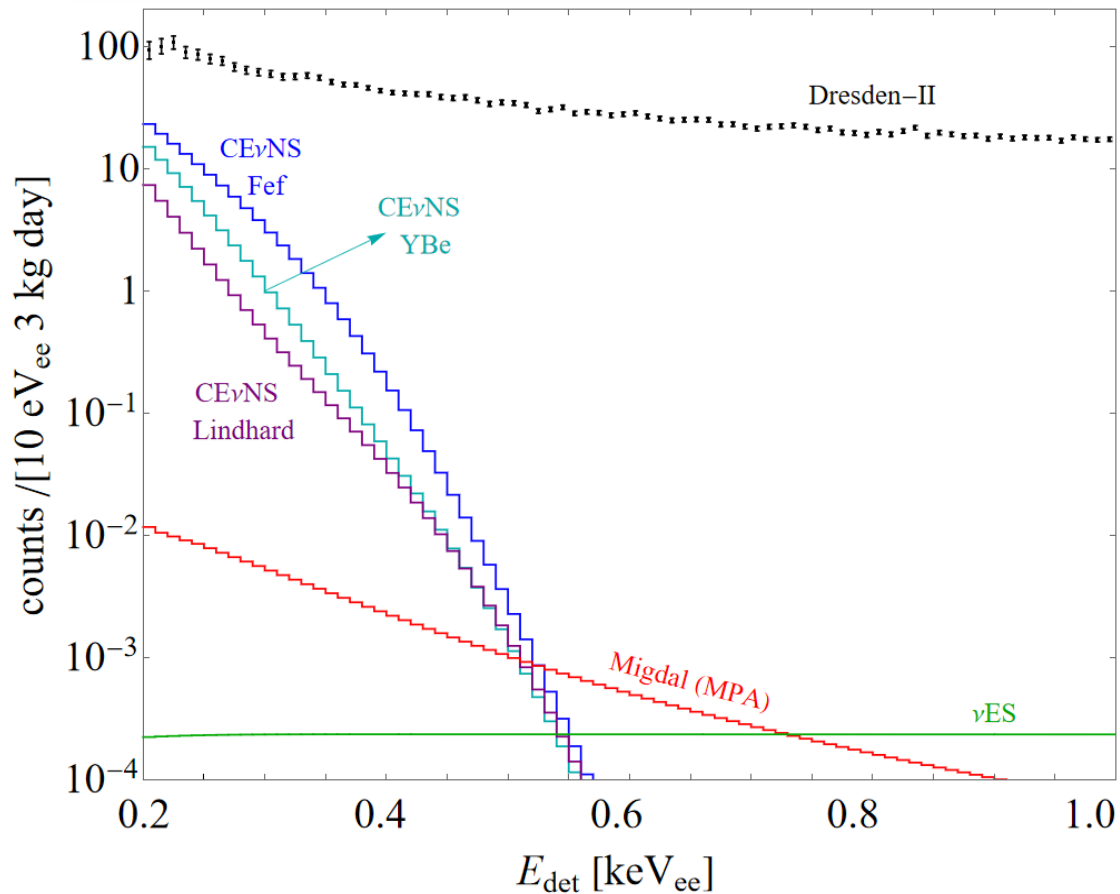
Physics Letters B

journal homepage: www.elsevier.com/locate/physletb

Letter

On the impact of the Migdal effect in reactor CEνNS experiments

M. Atzori Corona ^{a,b, }, M. Cadeddu ^{b, }, N. Cargioli ^{a,b, }, F. Dordej ^{b, }, C. Giunti ^{c, }



✓ The Migdal contribution **adds** to the standard Lindhard prediction.

! If the enhanced signal (Fef or YBe) is due to Migdal, it should have been retrieved as: **Migdal + CEνNS Lindhard.**

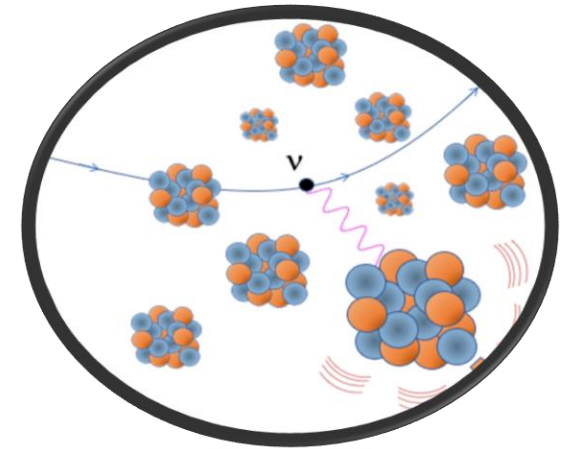
! The Migdal contribution is **completely subdominant wrt CEνNS!**



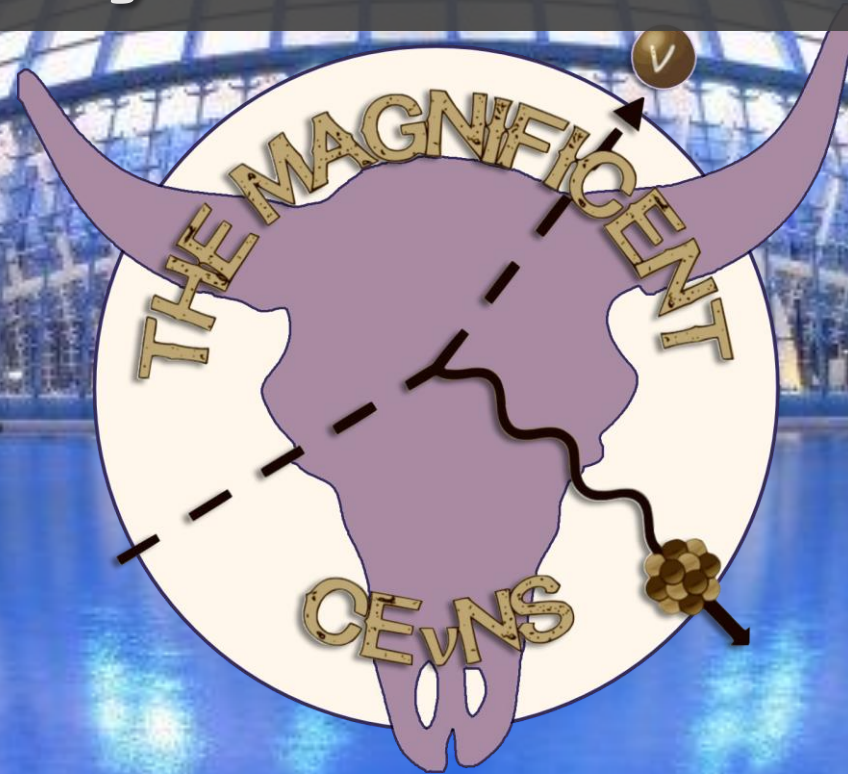
✗ The standard Migdal effect induced by neutrinos cannot be the source of the **enhancement** at low energies of the **new quenching factor measured for Germanium!**

Conclusions

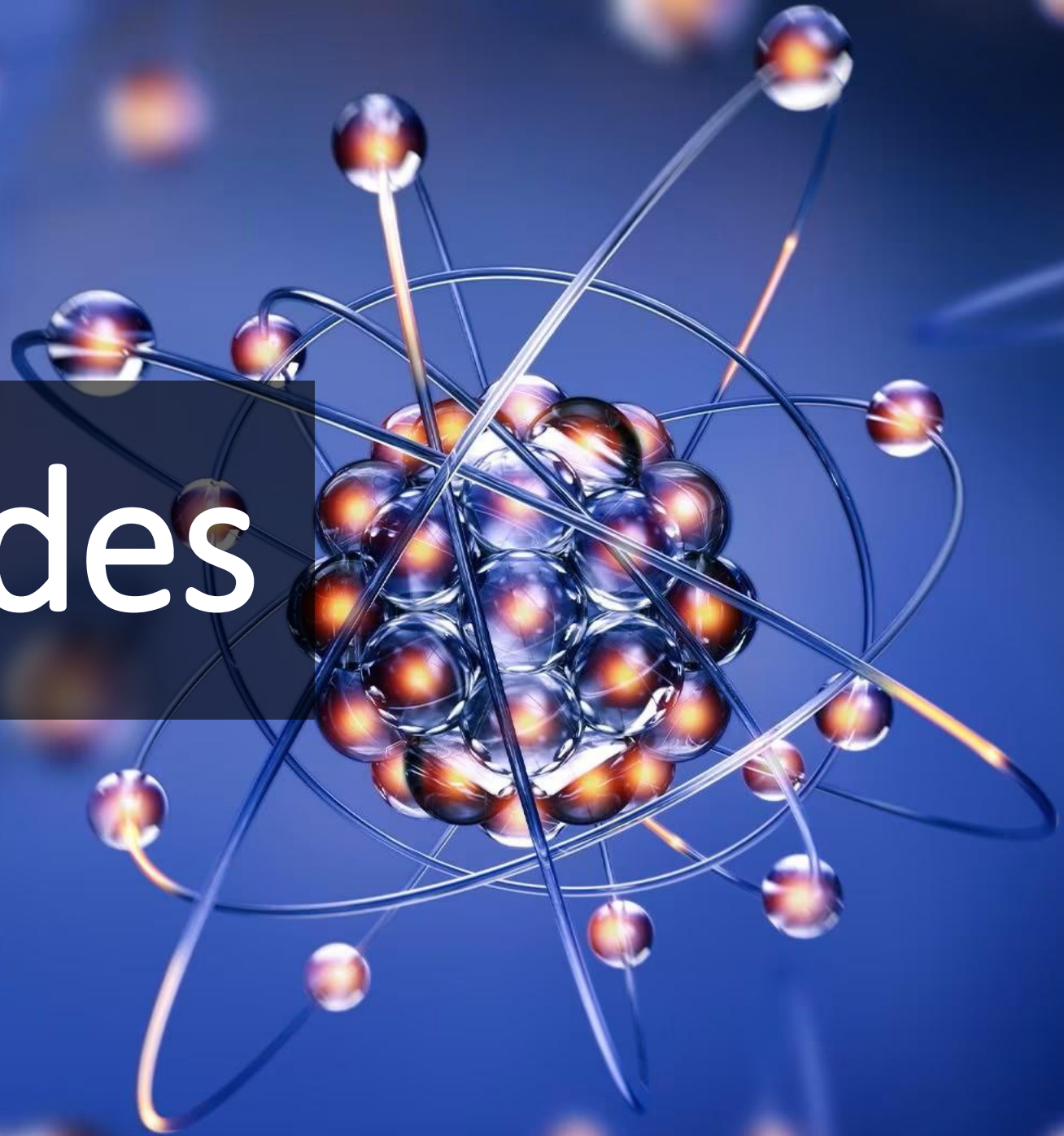
- The search for CE ν NS from reactor $\bar{\nu}_e$ remains a **formidable experimental challenge**.
- The recent results from the **NCC-1701 germanium detector** boosted experimental and theoretical investigations
- The higher quenching factor at low energy is crucial to interpret the Dresden-II excess in terms of a CE ν NS signal (different experimental campaigns are underway to explore this poorly known energy regime)
- We discussed the **Migdal effect** following the NR in the neutrino scattering
- Using different formalisms, we showed that the amount of ionization provided by the Migdal effect is **negligible compared to CE ν NS assuming the Lindhard theory**. The Fef or YBe QF cannot be explained by the standard Migdal effect.



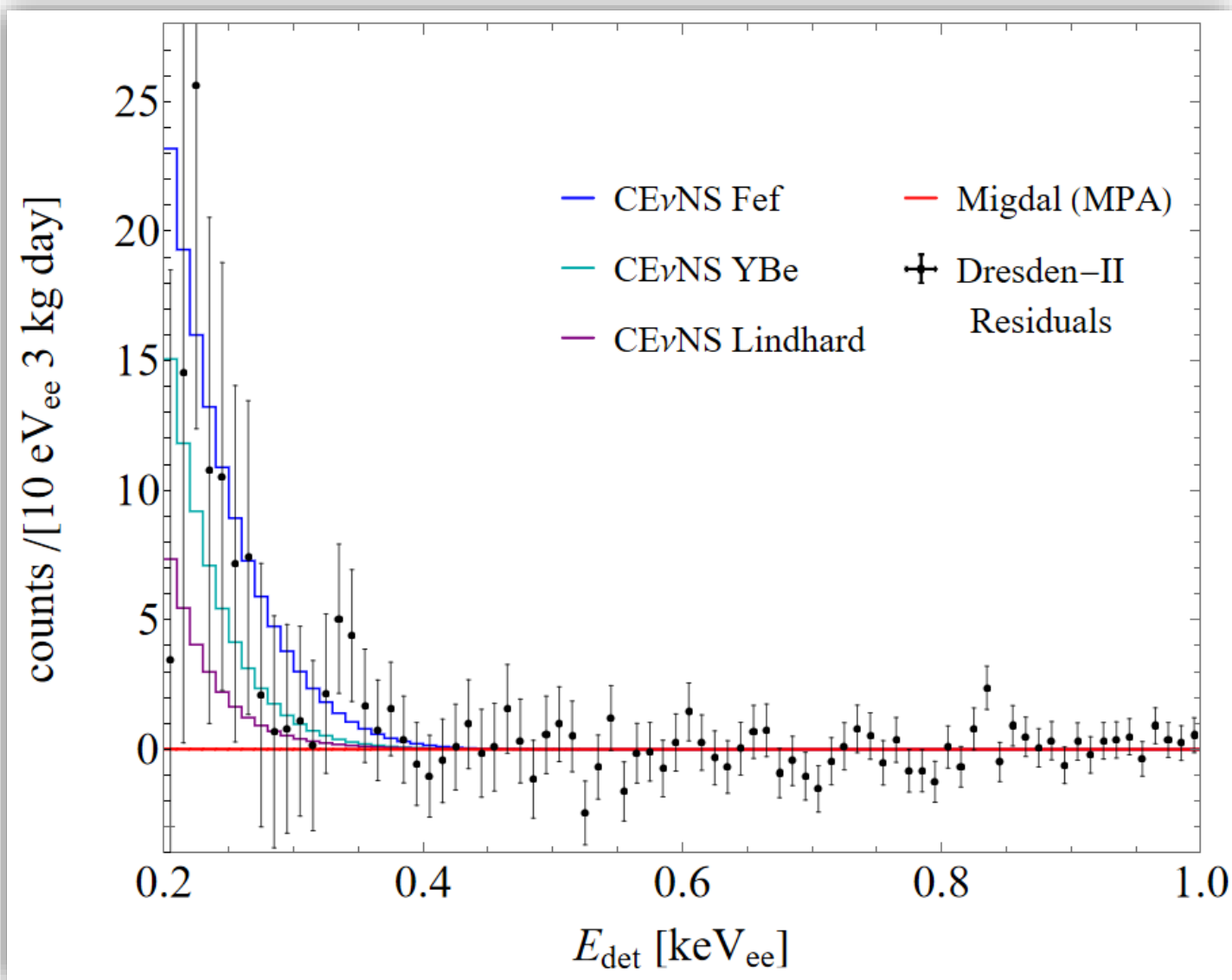
*Thanks
for your attention!*



Backup slides



Migdal vs CE ν NS rate



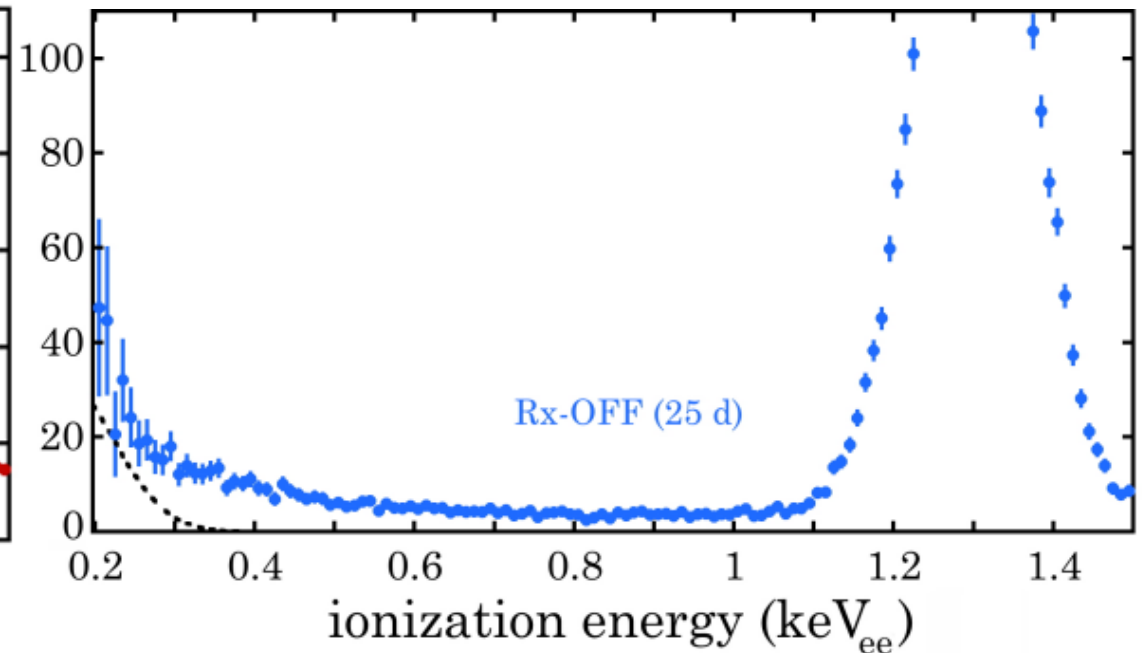
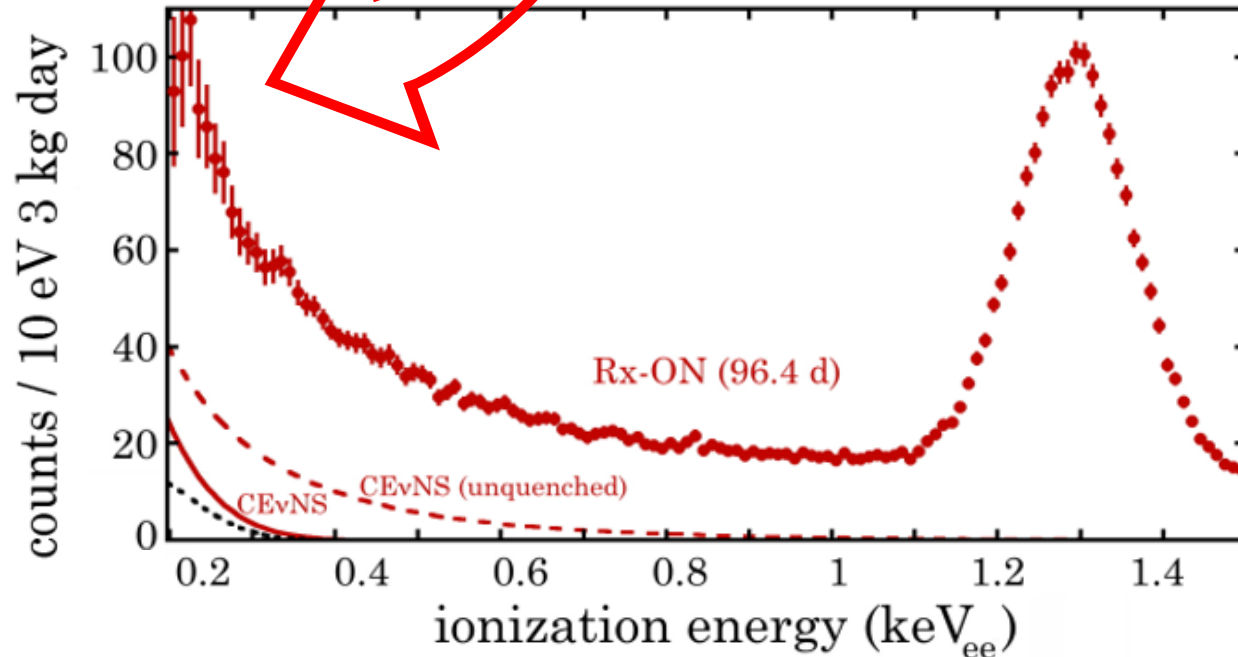
Background for Dresden-II

$$\frac{dN^{\text{bkg}}}{dT_e} = \underbrace{N_{\text{epith}} + A_{\text{epith}} e^{-T_e/T_{\text{epith}}}}_{\text{Reactor related background (mainly slow neutrons)}} + \underbrace{\sum_{i=L1,L2,M} \frac{A_i}{\sqrt{2\pi}\sigma_i} e^{-\frac{(T_e-T_i)^2}{2\sigma_i^2}}}_{\text{Detector related background}}$$

*Reactor related background
(mainly slow neutrons)*

Detector related background

L1-shell Electron Capture peak of 1.297 keV of ^{71}Ge
<https://atom.kaeri.re.kr/cgi-bin/decay?Ge-71%20EC>
 $^{71}_{32}\text{Ge} + {}^0_{-1}\text{e} \rightarrow ^{71}_{31}\text{Ga}^* + \nu_e$

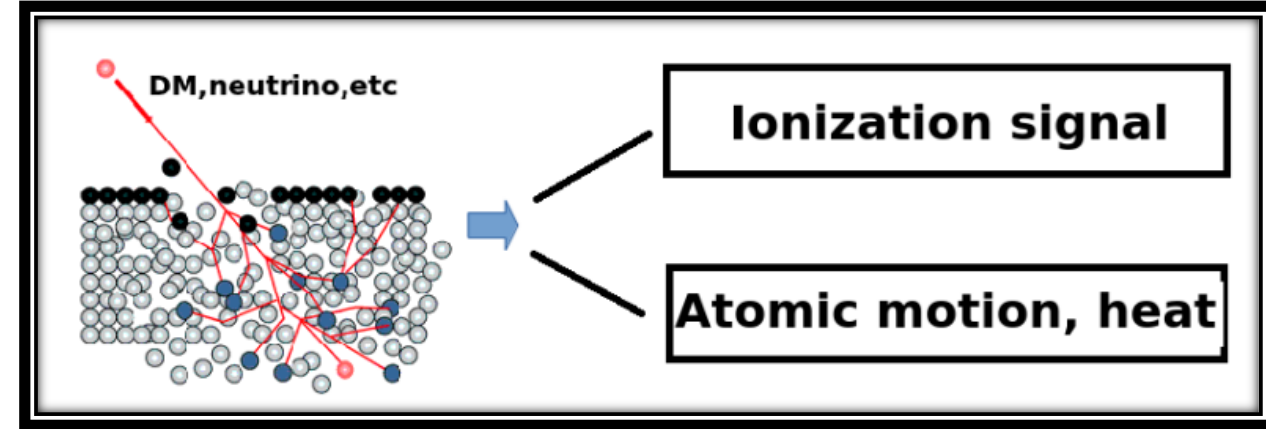


The standard Lindhard Theory for the Quenching Factor

The moving ion sets off a cascade of slowing-down processes that dissipate the energy E throughout the medium.

Underlying assumptions within this formalism:

- i. The contribution of the moving electrons is negligible
- ii. The electronic cloud follow immediately the nucleus
- iii. The energy transferred to ionized electrons is small compared to that transferred to recoiling ions.
- iv. The effects of electronic and atomic collisions can be treated separately.
- v. $T_{nr} < E_{inc}$



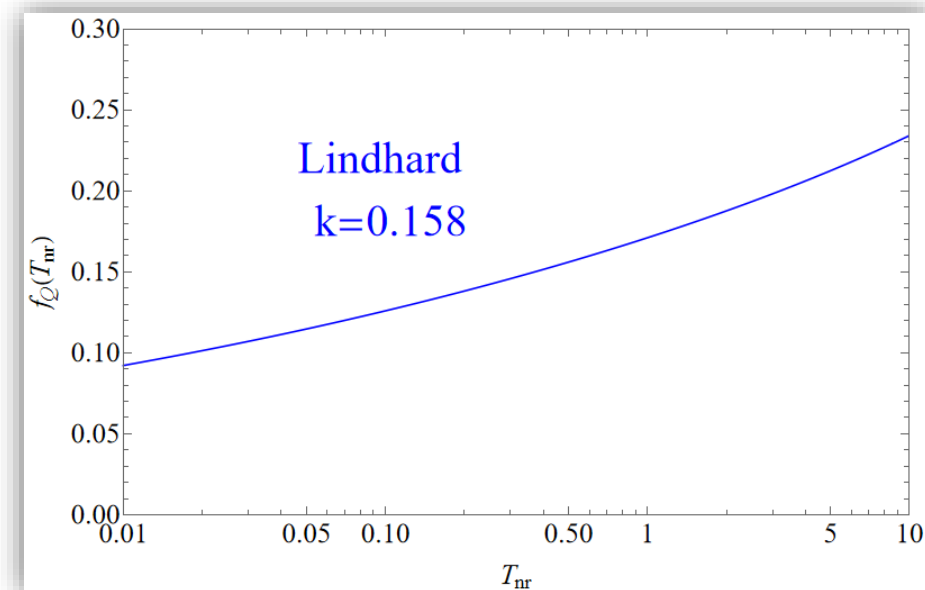
The Lindhard QF is defined as

$$f_Q(T_{nr}) = \frac{k g(\epsilon)}{1 + k g(\epsilon)}$$

where $g(\epsilon) = 3\epsilon^{0.15} + 0.7\epsilon^{0.6} + \epsilon$

with $\epsilon = 11.5 Z^{-7/3} T_{nr}$.

k is found to be $k = 0.133 Z^{2/3} A^{-1/3} \sim 0.158$ for Ge



New measurements of the Quenching factor

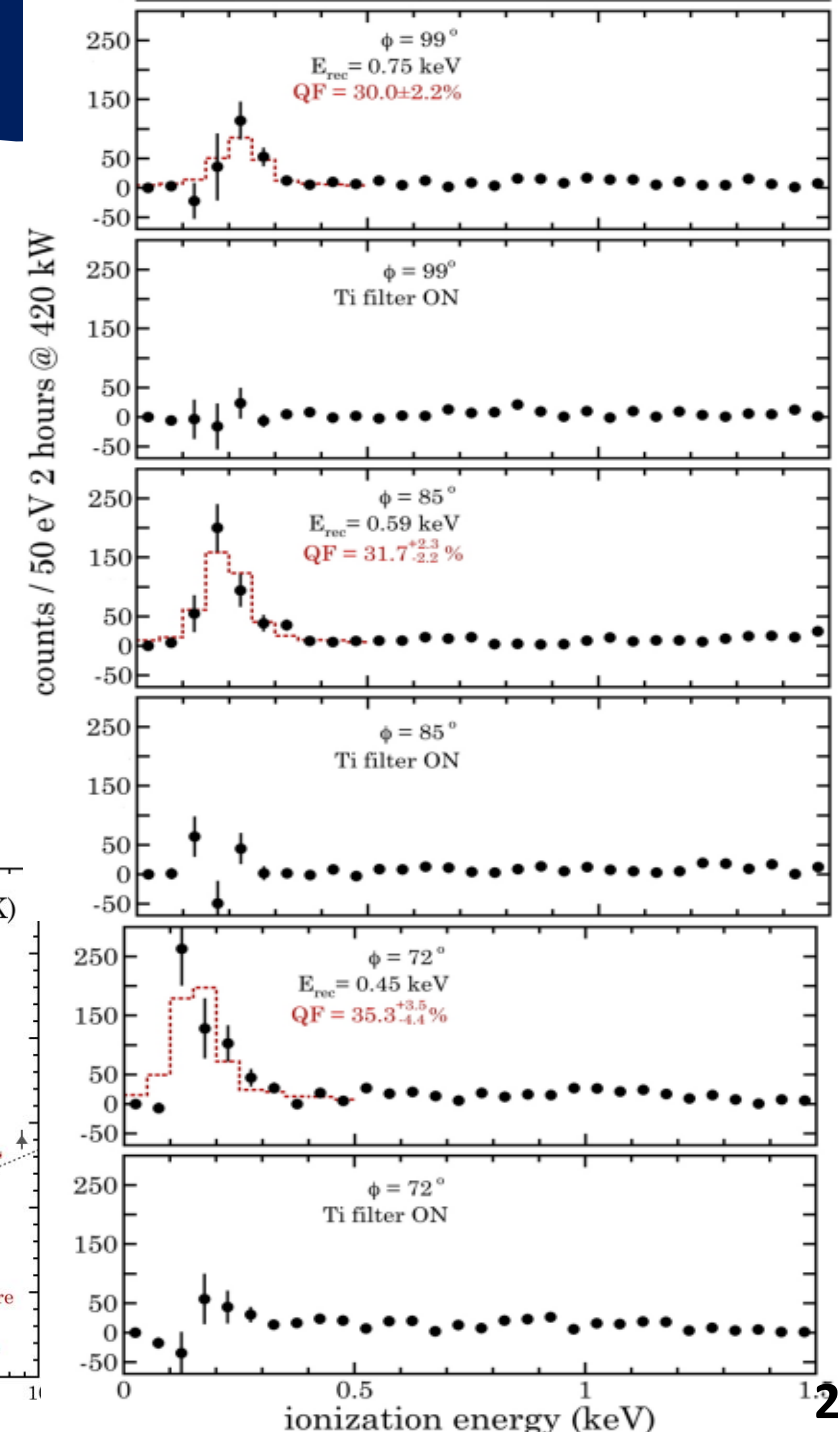
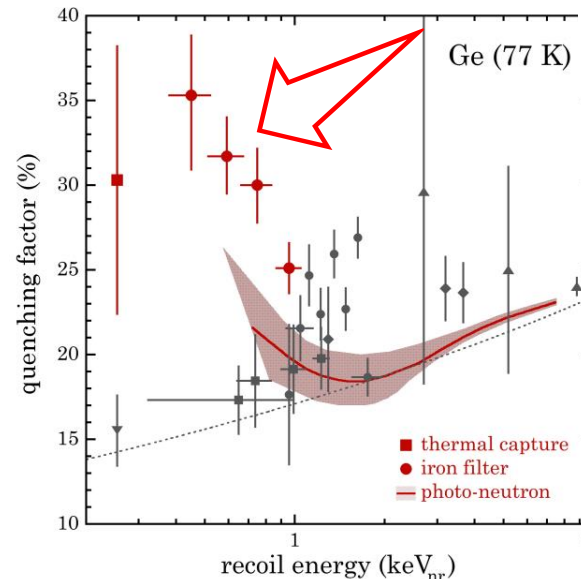
Calibration QF measurements are performed with neutrons.

Three techniques have been exploited:

1. Iron filter (Fef)
2. Photo-neutron source (YBe)
3. Neutron capture

Iron filter (Fef)

- The germanium detector was exposed to a monochromatic (~ 24 keV) iron-filtered neutron beam produced at the Kansas State University (KSU) experimental reactor.
- This technique exploits a dip in the elastic scattering neutron cross-section for iron at 24 keV, where it is reduced by three orders of magnitude over a narrow energy region. For ultra-pure iron filters ~ 1 m in length, designed to reduce capture gamma backgrounds, a high beam purity is achievable.
- By changing the scattering angle it is possible to obtain measurements for different T_{nr} , fixed by the kinematics.



New QF measurements of the Quenching factor

Calibration QF measurements are performed with neutrons.

Three techniques have been exploited:

1. Iron filter (Fef)
2. **Photo-neutron source (YBe)**
3. Neutron capture

Photo-disintegration process with the production of neutrons.

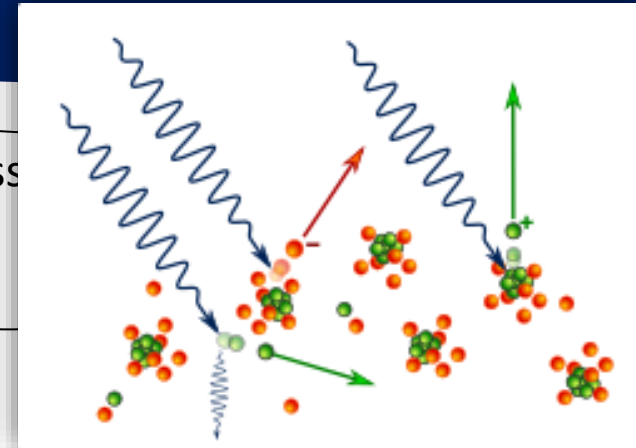
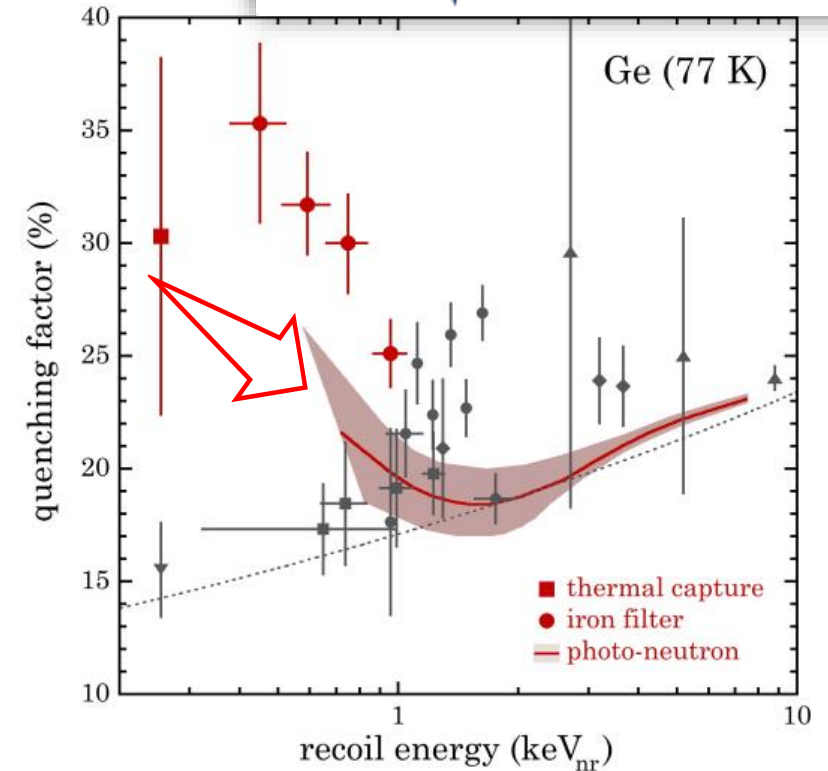


Photo-neutron source (YBe)

- Photo-disintegration is a nuclear process where a nucleus absorbs a high-energy photon, enters an excited state, and immediately decays emitting subatomic particles, including **neutrons**.
- A photo-neutron YBe source generates monochromatic **152 keV** neutrons from beryllium photo-disintegration.
- The intense high-energy gamma emission is stopped by 15 cm of lead.
- Average neutron yield 848 n/s.



$$E_{nr}^{\max} = (4MmE_n)/(M + m)^2 = 8.5 \text{ keV}_{nr}$$

New QF measurements of the Quenching factor

Calibration QF measurements are performed with neutrons.

Three techniques have been exploited:

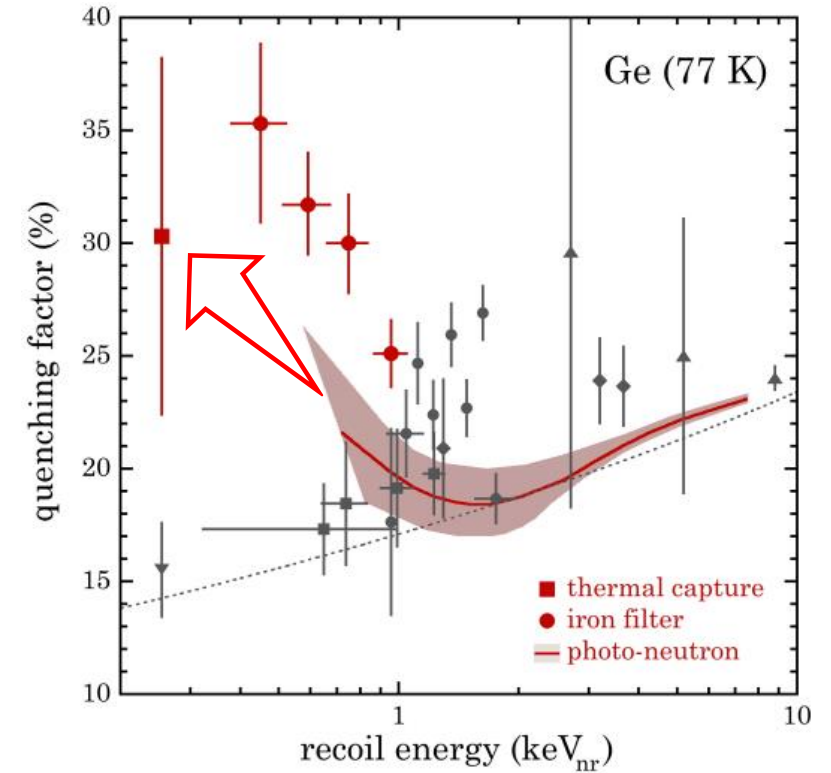
1. Iron filter (Fef)
2. Photo-neutron source (YBe)
3. **Neutron capture**

Neutron Capture

- The third technique exploits recoils from gamma emission following **thermal neutron capture**.
- This techniques (that backs to 1975!) is able to produce fixed-energy germanium recoils carrying a mere $0.254 \pm 0.001 \text{ keV}_{\text{nr}}$.
- $^{72}\text{Ge} + \text{n} \rightarrow ^{73}\text{Ge} + \gamma$

The γ is observed with a SiPM.

$$E_R = E_\gamma^2 / 2Mc^2$$



Migdal formalism

The Migdal ionization rate is determined by the matrix element for the transition of the electron in the initial state ψ_i into the final state ψ_f .

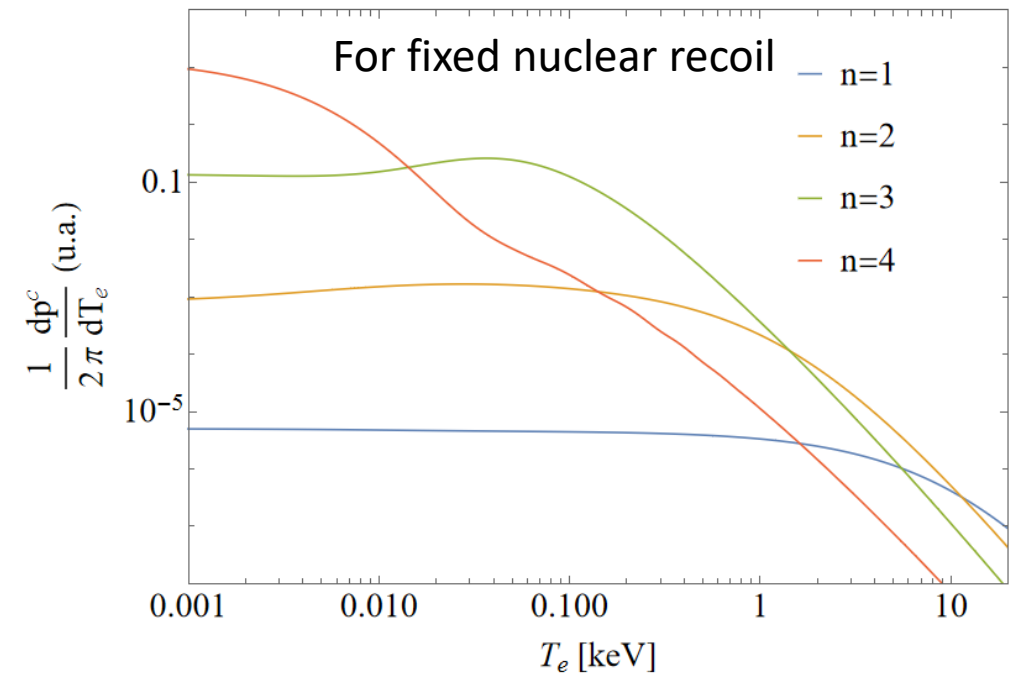
$$\begin{aligned} M_{fi} &= \langle \psi_f | e^{-im_e \vec{v} \cdot \sum_{i=1}^Z \vec{r}_i} | \psi_i \rangle \simeq \\ &\simeq -im_e \vec{v} \cdot \langle \psi_f | \sum_{i=1}^Z \vec{r}_i | \psi_i \rangle \\ &\equiv -im_e \vec{v} \cdot \boxed{\vec{D}_{fi}}, \quad \text{Dipole matrix element} \end{aligned}$$

Ibe et al. approach

- The final state wavefunctions are boosted to the rest frame of the recoiling nucleus by a Galilean transformation and are computed using the Dirac-Hartree-Fock method in order to compute \vec{D}_{fi} .
- The ionization rate is expressed in terms of $p^c(n\ell \rightarrow T_e)$, which gives the probabilities for an atomic electron, with quantum numbers n and ℓ , to be ionized with a final energy T_e .

The total energy absorbed in the detector is

$$E_{\text{det}} = f_Q T_{nr} + E_{n\ell} + T_e$$



Migdal Photo Absorption Formalism

It is based on the relation between the dipole matrix element and the photoabsorption cross section.

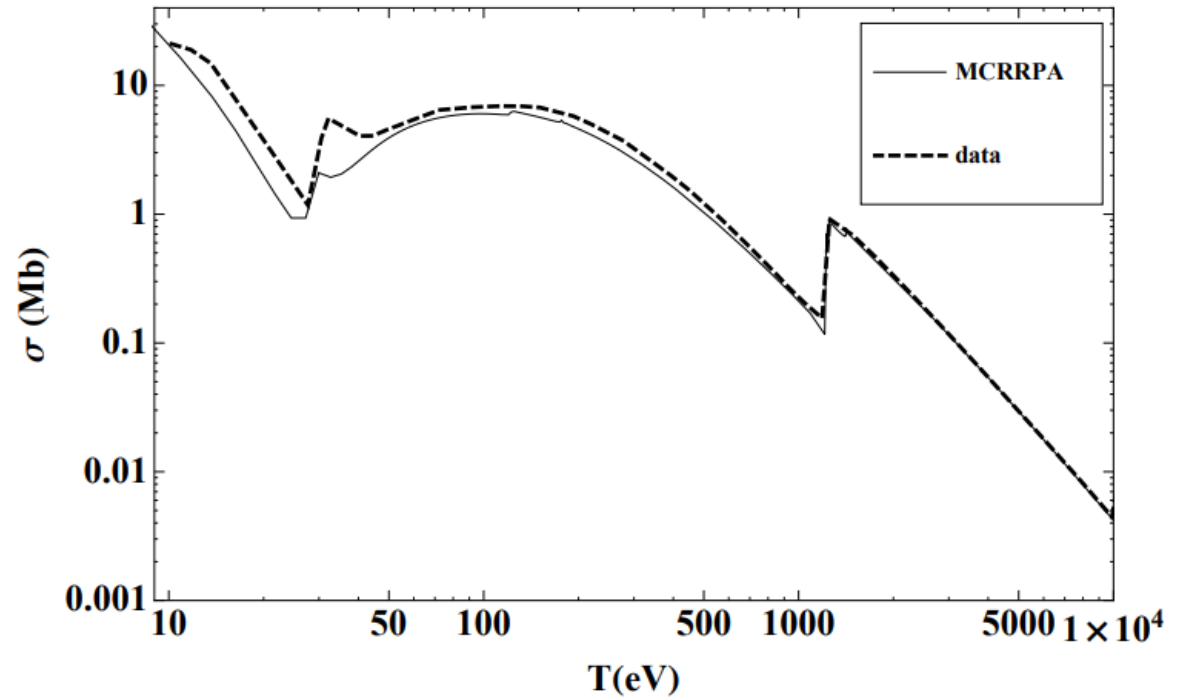
- $\vec{D}_{fi} = \frac{1}{4\pi^2 \alpha_{EM} E_r} \sigma(E_r)$

E_r is the total energy deposit due to ionisation or excitation.

$$E_{det} = f_Q T_{nr} + E_r$$

$$\left(\frac{d^2 \sigma_{\bar{\nu}_e - \mathcal{N}}}{dT_{nr} dE_r} \right)_{\text{Migdal}}^{\text{MPA}} = \frac{G_F^2 M}{\pi} \left(1 - \frac{MT_{nr}}{2E_\nu^2} \right) Q_W^2 \times \frac{1}{2\pi^2 \alpha_{EM}} \frac{m_e^2}{M} \frac{T_{nr}}{E_r} \sigma_\gamma^{\text{Ge}}(E_r), \quad (11)$$

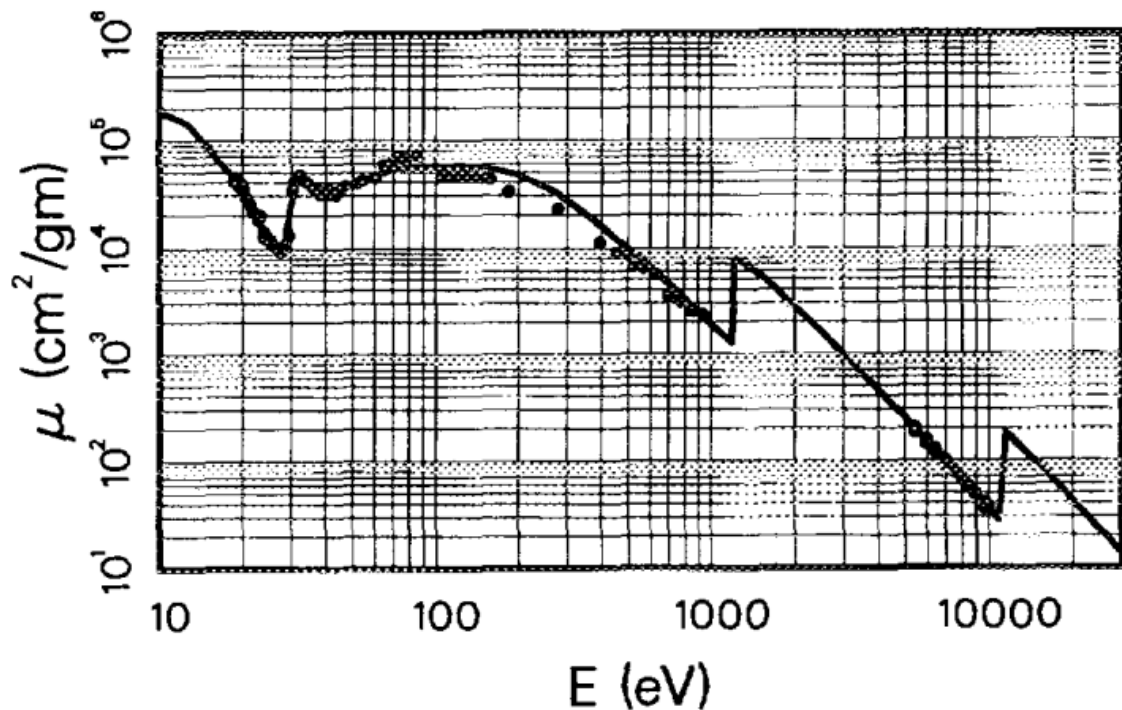
Phys. Rev. D 91, 013005 (2015)



The photoabsorption cross section reflects the atomic shell structure.

Germanium photoabsorption cross sections

Atomic Data and Nuclear Data Tables Volume 54, Issue 2, July 1993, Pages 181-342



Edge Energies

| | | | | | |
|---|------------|------------------|------------------------|------------------|-----------------------|
| K | 11103.1 eV | L _I | 1414.6 eV ^a | M _I | 180.1 eV ^a |
| | | L _{II} | 1248.1 eV ^a | M _{II} | 124.9 eV ^a |
| | | L _{III} | 1217.0 eV ^a | M _{III} | 120.8 eV ^a |
| | | | | M _{IV} | 29.9 eV ^a |
| | | | | M _V | 29.3 eV ^a |

Ge ($q_e = m_e \times 10^{-3}$)

| (n, ℓ) | $\mathcal{P}_{\rightarrow 4p}$ | $\mathcal{P}_{\rightarrow 4d}$ | $\mathcal{P}_{\rightarrow 5s}$ | $\mathcal{P}_{\rightarrow 5p}$ | $\mathcal{P}_{\rightarrow 6s}$ | E_{nl} [eV] | $\frac{1}{2\pi} \int dE_e \frac{dp^c}{dE_e}$ |
|-------------|--------------------------------|--------------------------------|--------------------------------|--------------------------------|--------------------------------|-------------------|--|
| 1s | 5.0×10^{-8} | – | – | 7.9×10^{-9} | – | 1.1×10^4 | 1.8×10^{-5} |
| 2s | 1.8×10^{-6} | – | – | 2.8×10^{-7} | – | 1.4×10^3 | 1.3×10^{-4} |
| 2p | – | 3.3×10^{-7} | 1.1×10^{-7} | – | 3.4×10^{-8} | 1.2×10^3 | 7.3×10^{-4} |
| 3s | 3.7×10^{-5} | – | – | 5.6×10^{-6} | – | 1.7×10^2 | 5.5×10^{-4} |
| 3p | – | 6.0×10^{-9} | 2.8×10^{-5} | – | 8.3×10^{-6} | 1.2×10^2 | 2.4×10^{-3} |
| 3d | 2.3×10^{-3} | – | – | 2.3×10^{-4} | – | 3.5×10 | 2.8×10^{-2} |
| 4s | 4.0×10^{-2} | – | – | 3.9×10^{-4} | – | 1.5×10 | 6.1×10^{-4} |
| 4p | – | 2.7×10^{-2} | 1.6×10^{-2} | – | 1.5×10^{-3} | 6.5 | 2.6×10^{-2} |

| (n, ℓ) | 4d | 5s | 5p | 6s |
|---------------|-----|-----|-----|-----|
| E_{nl} [eV] | 1.6 | 3.0 | 2.0 | 1.4 |

See also

https://xdb.lbl.gov/Section1/Table_1-1.pdf

IONIZATION OF ATOMS ACCOMPANYING α - and β -DECAY

By A. MIGDAL

(Received November 15, 1940)

The probability of ionization of the inner electron shells accompanying α - and β -decay is calculated. Also an estimation of the order of magnitude of ionization of the outer shells is given.

1. Ionization accompanying β -decay

1. The probability of ionization of an atom as a result of the β -decay can be without difficulty calculated if one makes use of the fact that the velocity of a β -electron is usually great as compared with velocities of atomic electrons.

It is easily seen that in this case one can neglect the direct interaction of the β -decay electron with the atomic ones. The ionization is due to the fact that the nuclear charge is changed within a time interval which is short comparing to atomic periods.

The following estimation shows that the direct interaction can be actually neglected. The probability of an electron transition due to the direct interaction is according to perturbation theory:

$$W = \frac{|\int_0^\infty V_{\alpha\beta} e^{i\omega_{\alpha\beta}t} dt|^2}{\hbar^2} \quad (1)$$

$V_{\alpha\beta}$ is here the matrix element of the perturbation energy; $\omega_{\alpha\beta} = (E_\alpha - E_\beta)/\hbar$ —the frequency corresponding to the electron transition; it is of the order of atomic frequencies. The time interval τ within which the decay electron traverses electron shells is much smaller than the atomic periods.

Hence the transition probability is of the order

$$W \sim \frac{V^2 \tau^2}{\hbar^2} \sim \frac{1}{\hbar^2} \left(\frac{\gamma e^2}{a} \cdot \frac{a}{\gamma c} \right)^2 = \left(\frac{e^2}{\hbar c} \right)^2$$

(the quantity $\gamma = E/mc^2$ disappears because the Lorentz contraction of the field is compensated by an increase of the latter.)

On the other hand, the probability of ionization by a «sudden» change of nuclear charge, as will be shown, is of the order of $1/Z_{\text{eff}}^2$. Hence the condition for the direct interaction to be small

$$\left(\frac{Z_{\text{eff}} e^2}{\hbar c} \right)^2 \ll 1. \quad (2)$$

The condition (2) has a simple meaning in the case of a K -electron, because $(Ze^2/\hbar c)^2 = (V_K/c)^2$. Therefore, the direct interaction is to be considered as a relativistic correction. The condition (2) is approximately valid even for K -electrons of uranium.

2. One can calculate the probability of ionization by means of a sudden change of the nuclear charge in the following manner. The above estimation shows that the Ψ -function of atomic electrons does not change when the decay electron is emitted. Therefore, the transition probability is equal to the square of the coefficient of expansion of the Ψ -function cor-

Migdal effect storyline

• 1939 Ionization in nuclear reactions *ZhETF*, 11, 207-212 (1941).

• 1941 Ionization in α and β decays *ZhETF*, 11, 207-212 (1941).

• 1975 First observation of the Migdal effect in α -decay *PRC* 11, 1740-1745 (1975), *PRC* 11, 1746-1754 (1975)

• 2012 First observation in β -decay *PRL* 108, 243201 (2012)

• 2017 Ibe et al. Paper *JHEP* 03 (2018) 194

• 2018 First observation in β^+ -decay *PRA*, 97, 023402 (2018)

• **20XX** – first observation in nuclear recoil?