Incoherent solar neutrino scattering off Thallium isotopes

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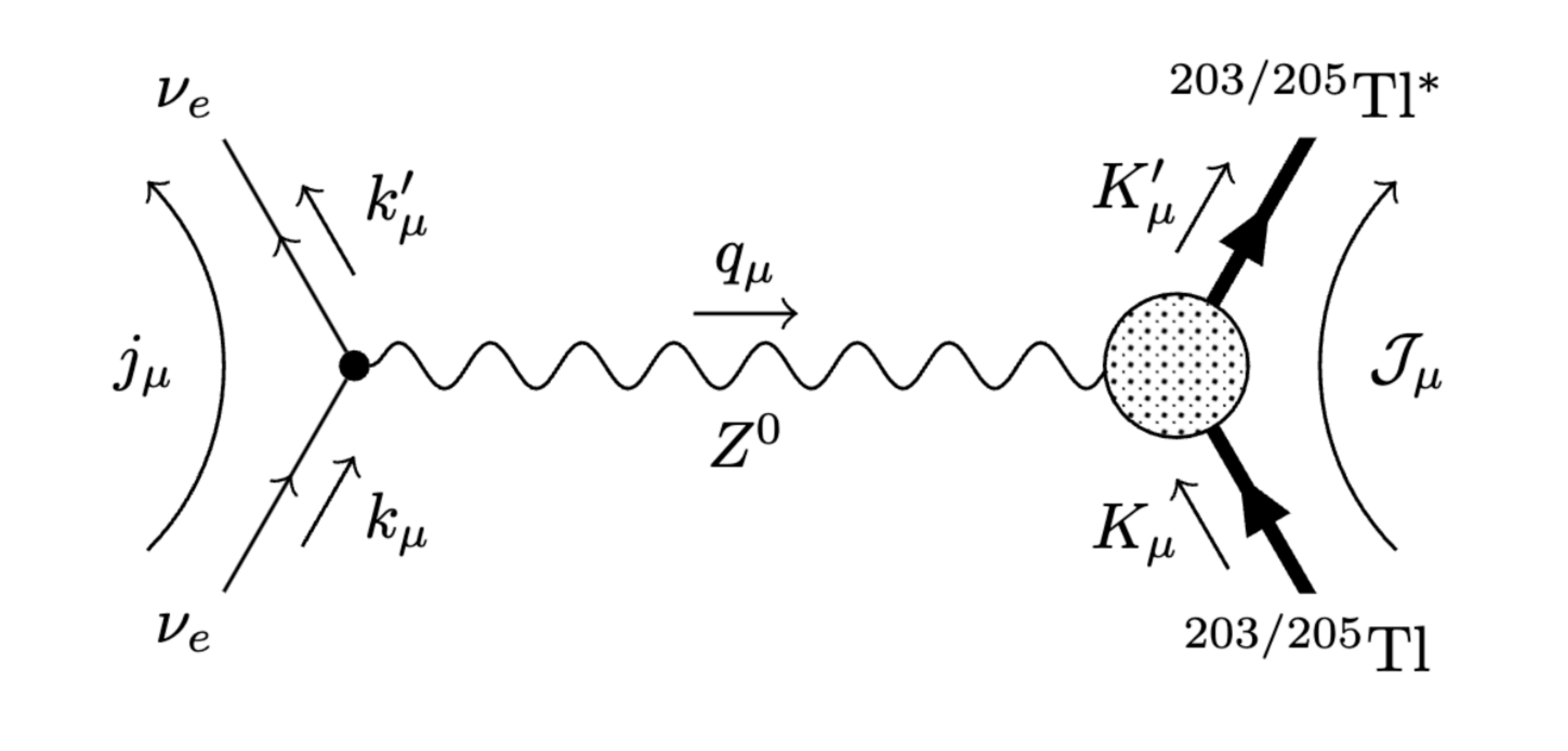
Outline

- Overview of inelastic neutrino-nucleus scattering
  - Multipole decomposition of the hadronic current
  - Irreducible tensor operators
- Formalism in terms of nuclear recoil energy
  - Lepton traces
  - Inelastic cross section
- Shell Model calculations
  - Inelastic neutrino scattering off Thallium isotopes
  - Inelastic event rates induced by solar neutrinos

Inelastic neutrino-nucleus scattering

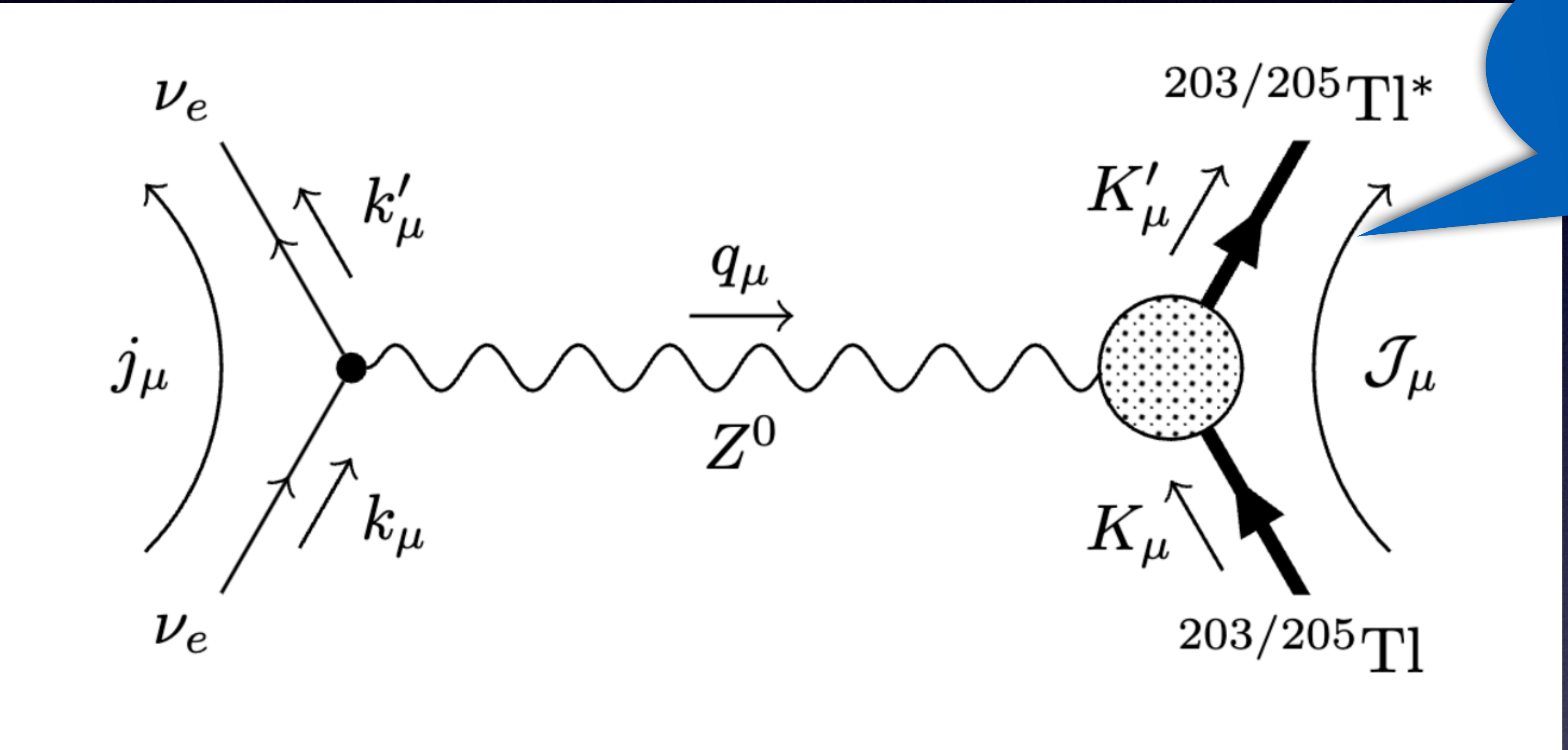
$$\nu_e + {}^{203/205}\mathrm{Tl}(\text{ground state}) \longrightarrow \nu_e + {}^{203/205}\mathrm{Tl}(\text{excited state})$$



$$\hat{H}_{\text{eff}} = \frac{G}{\sqrt{2}} \int \mathrm{d}^3\mathbf{x} j_\mu(\mathbf{x}) \mathcal{J}^\mu(\mathbf{x}),$$

Inelastic neutrino-nucleus scattering

$$\nu_e + {}^{203/205}\mathrm{Tl(ground\ state)} \longrightarrow \nu_e + {}^{203/205}\mathrm{Tl(excited\ state)}$$



Final state nucleus not in the ground state

ω : **excitation energy**

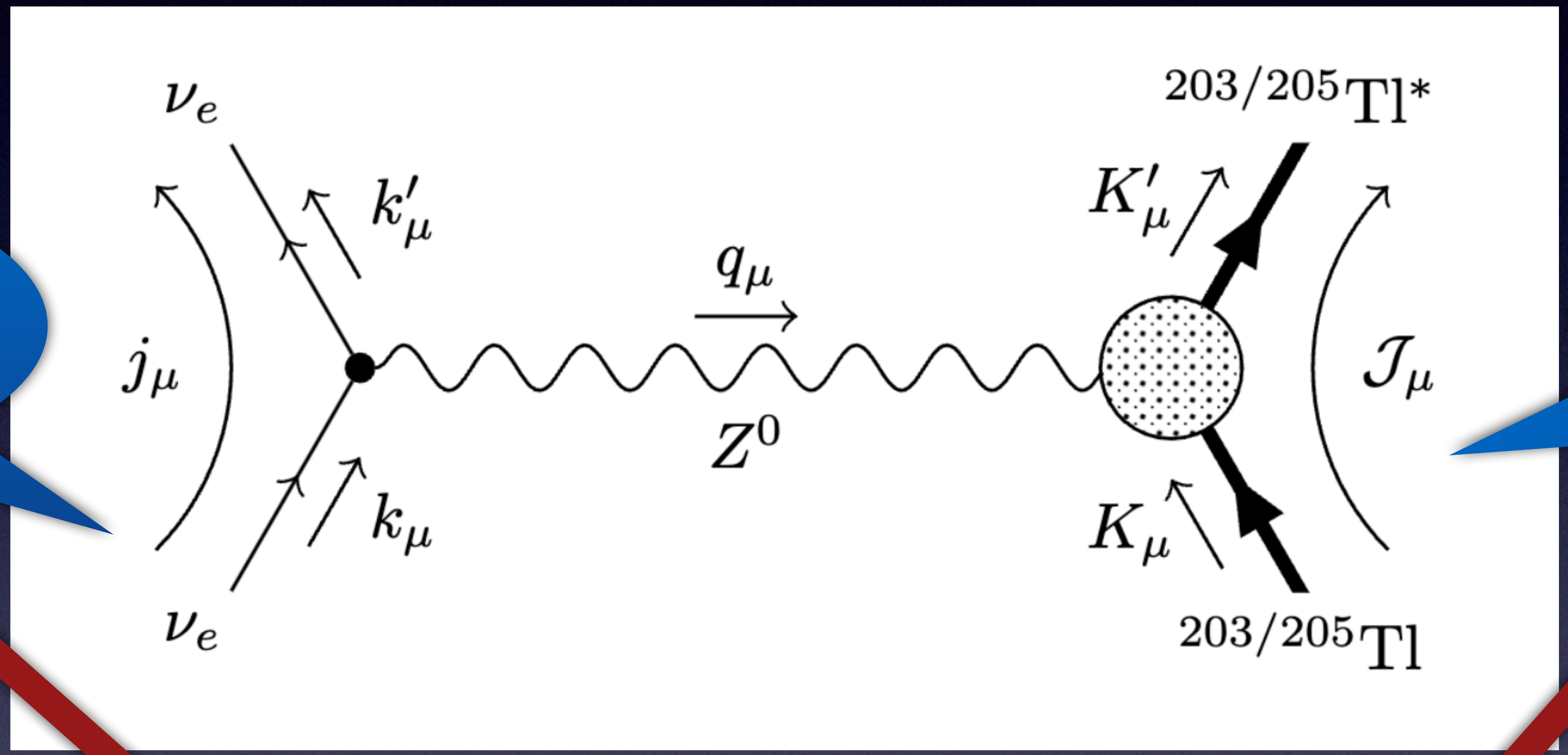
$$q_\mu = k_\mu - k'_\mu = K'_\mu - K_\mu$$

$$\hat{H}_{\mathrm{eff}} = \frac{G}{\sqrt{2}} \int \mathrm{d}^3\mathbf{x} j_\mu(\mathbf{x}) \mathcal{J}^\mu(\mathbf{x}),$$

Inelastic neutrino-nucleus scattering

$$\nu_e + {}^{203/205}\mathrm{Tl}(\text{ground state}) \quad \longrightarrow \quad \nu_e + {}^{203/205}\mathrm{Tl}(\text{excited state})$$

Leptonic current



Hadronic current

$$\hat{H}_{\text{eff}} = \frac{G}{\sqrt{2}} \int \mathrm{d}^3\mathbf{x}\, j_\mu(\mathbf{x}) \mathcal{J}^\mu(\mathbf{x}),$$

Leptonic and hadronic currents

The cross section from an initial $|i\rangle$ to a final $|f\rangle$ nuclear state will be proportional to

$$\sigma \propto \left| \langle f | \hat{H}_{\text{eff}} | i \rangle \right|^2$$

Leptonic and hadronic currents

The cross section from an initial $|i\rangle$ to a final $|f\rangle$ nuclear state will be proportional to

$$\sigma \propto \left| \langle f | \hat{H}_{\text{eff}} | i \rangle \right|^2$$

With the matrix element being

$$\langle f | \hat{H}_{\text{eff}} | i \rangle = \int \langle f | \mathcal{H}_{\text{eff}} | i \rangle \, \mathrm{d}^3\boldsymbol{x} = \frac{G}{\sqrt{2}} \int \langle f | j_\mu(\boldsymbol{x}) | i \rangle \langle f | \mathcal{J}^\mu(\boldsymbol{x}) | i \rangle \, \mathrm{d}^3\boldsymbol{x}.$$

The hadronic current can be written as a sum over nucleons

$$\mathcal{J}_\mu(\mathbf{x}) = \sum_{i=1}^{A} \mathcal{J}_\mu(\mathbf{x}_i)\delta^{(3)}(\mathbf{x} - \mathbf{x}_i) = \sum_{i=1}^{A} [J_\mu(\mathbf{x}_i) + J_{\mu 5}(\mathbf{x}_i)]\delta^{(3)}(\mathbf{x} - \mathbf{x}_i)$$

The leptonic current is

$$j_\mu(\boldsymbol{x}) = \overline{\psi}_{l'}\gamma_\mu(1 - \gamma_5)\psi_l,$$

$$\psi(x) = \frac{1}{\sqrt{V}} \sum_{p\lambda} \left[a_{p\lambda} u(\boldsymbol{p}\lambda) e^{ip\cdot x} + b^\dagger_{p\lambda} v(-\boldsymbol{p}\lambda) e^{-ip\cdot x} \right]$$

Leptonic and hadronic matrix elements

$$\langle f|\,\hat{H}_{\text{eff}}\,|i\rangle = \int \langle f|\,\mathcal{H}_{\text{eff}}\,|i\rangle \,\mathrm{d}^3\boldsymbol{x} = \frac{G}{\sqrt{2}} \int \langle f|\, j_\mu(\boldsymbol{x})\,|i\rangle \,\langle f|\,\mathcal{J}^\mu(\boldsymbol{x})\,|i\rangle \,\mathrm{d}^3\boldsymbol{x}.$$

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leptonic matrix element

$$\langle f|\,j_\mu(\boldsymbol{x})\,|i\rangle = l_\mu e^{-i\boldsymbol{q}\cdot\boldsymbol{x}} = (l_0,\,-\boldsymbol{l})e^{-i\boldsymbol{q}\cdot\boldsymbol{x}},$$

$$l_\mu = \frac{1}{V}\cdot\begin{cases}\overline{u}(\boldsymbol{k}')\gamma_\mu(1-\gamma_5)u(\boldsymbol{k}), & \text{for neutrino reactions,}\\ \overline{v}(-\boldsymbol{k})\gamma_\mu(1-\gamma_5)v(\boldsymbol{k}'), & \text{for antineutrino reactions.}\end{cases}$$

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hadronic matrix element

$$\langle f|\,\mathcal{J}_\mu(\boldsymbol{x})\,|i\rangle = \langle f|\,(\mathcal{J}_0(\boldsymbol{x}), -\boldsymbol{\mathcal{J}}(\boldsymbol{x}))\,|i\rangle = (\langle f|\,\mathcal{J}_0(\boldsymbol{x})\,|i\rangle\,, -\langle f|\,\boldsymbol{\mathcal{J}}(\boldsymbol{x})\,|i\rangle) \equiv (\mathcal{J}_0(\boldsymbol{x})_{fi}, -\boldsymbol{\mathcal{J}}(\boldsymbol{x})_{fi})$$

Leptonic and hadronic matrix elements

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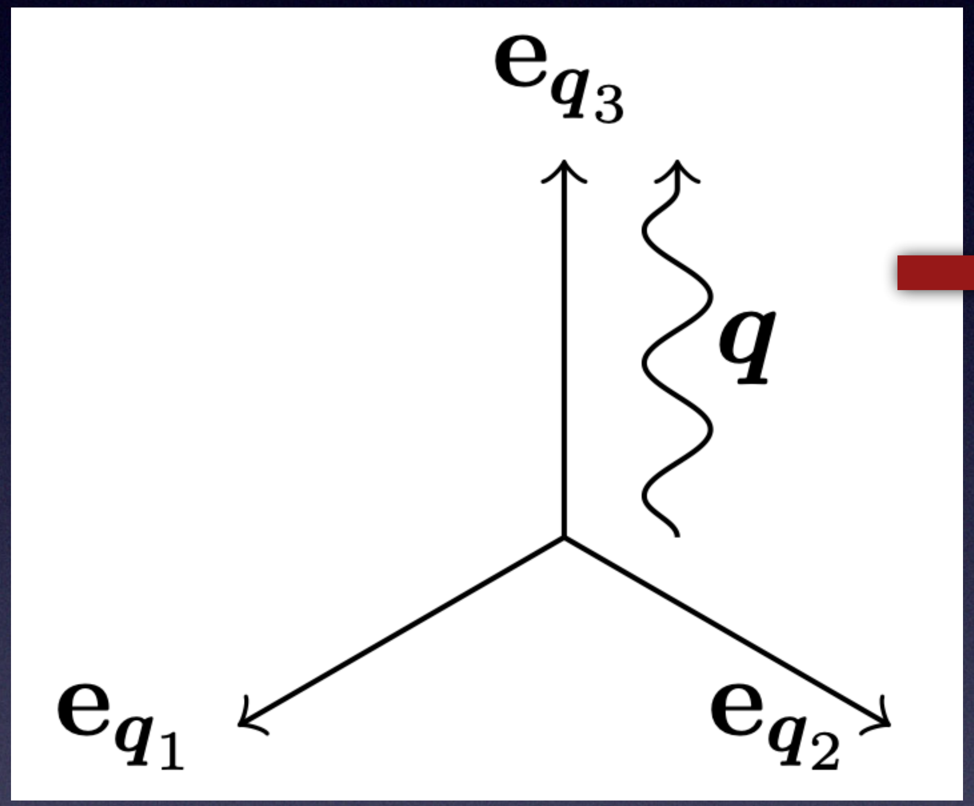
Putting everything together

$$H_{fi} = \frac{G}{\sqrt{2}} \int l_\mu e^{-i\boldsymbol{q}\cdot\boldsymbol{x}} \begin{pmatrix} \mathcal{J}_0(\boldsymbol{x})_{fi} \\ \boldsymbol{\mathcal{J}}(\boldsymbol{x})_{fi} \end{pmatrix} \mathrm{d}^3\boldsymbol{x} = \frac{G}{\sqrt{2}} \int e^{-i\boldsymbol{q}\cdot\boldsymbol{x}} \left[l_0\mathcal{J}_0(\boldsymbol{x})_{fi} - \boldsymbol{l}\cdot\boldsymbol{\mathcal{J}}(\boldsymbol{x})_{fi}\right] \mathrm{d}^3\boldsymbol{x}.$$

Donnelly-Walecka multipole decomposition

$$H_{fi} = \frac{G}{\sqrt{2}} \int l_\mu e^{-i\boldsymbol{q}\cdot\boldsymbol{x}} \begin{pmatrix} \mathcal{J}_0(\boldsymbol{x})_{fi} \\ \boldsymbol{\mathcal{J}}(\boldsymbol{x})_{fi} \end{pmatrix} \mathrm{d}^3\boldsymbol{x} = \frac{G}{\sqrt{2}} \int e^{-i\boldsymbol{q}\cdot\boldsymbol{x}} \left[l_0 \mathcal{J}_0(\boldsymbol{x})_{fi} - \boldsymbol{l} \cdot \boldsymbol{\mathcal{J}}(\boldsymbol{x})_{fi} \right] \mathrm{d}^3\boldsymbol{x}.$$

Define a complete orthonormal set of unit spatial vectors



Then any 3-vector can be written in this basis as

$$\mathbf{l} = \sum_{\lambda=0,\pm1} l_\lambda \mathbf{e}_\lambda^\dagger = l_1 \mathbf{e}_1^\dagger + l_{-1} \mathbf{e}_{-1}^\dagger + l_{\lambda=0} \mathbf{e}_0^\dagger = l_1 \mathbf{e}_1^\dagger + l_{-1} \mathbf{e}_{-1}^\dagger + l_3 \mathbf{e}_0^\dagger,$$

need to expand in plane waves the following quantities

$$l_0 e^{-i\mathbf{q}\cdot\mathbf{x}}, \quad l_+ e^{-i\mathbf{q}\cdot\mathbf{x}}, \quad l_- e^{-i\mathbf{q}\cdot\mathbf{x}}, \quad l_3 e^{-i\mathbf{q}\cdot\mathbf{x}}.$$

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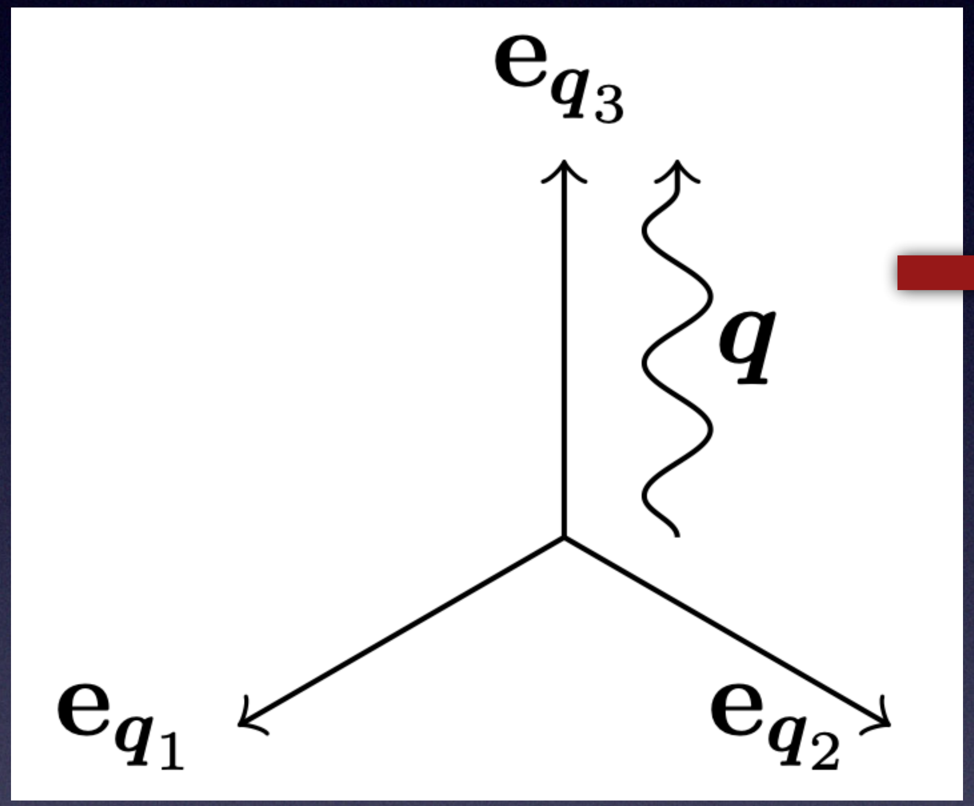

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Tensor operators

The matrix element of the interaction Hamiltonian finally becomes

$$\langle f|\hat{H}_{eff}|i\rangle = -\frac{G}{\sqrt{2}}\langle f|\Big\{\sum_{J\geq 0}\sqrt{4\pi(2J+1)}(-i)^J\Big(l_3\hat{\mathcal{L}}_{J0}(\kappa) - l_0\hat{\mathcal{M}}_{J0}(\kappa)\Big)$$

$$+\sum_{\lambda=\pm 1}\sum_{J\geq 1}\sqrt{2\pi(2J+1)}(-i)^J l_\lambda\Big(\lambda\hat{\mathcal{T}}^{mag}_{J-\lambda}(\kappa) + \hat{\mathcal{T}}^{el}_{J-\lambda}(\kappa)\Big)\Big\}|i\rangle\,.$$

Eight irreducible tensor operators

$$\begin{aligned}
\hat{\mathcal{M}}_{JM}(\kappa) &= \hat{M}^{coul}_{JM} - \hat{M}^{coul5}_{JM} = \int d\mathbf{r} M^J_M(\kappa\mathbf{r})\hat{\mathcal{J}}_0(\mathbf{r}),\\
\hat{\mathcal{L}}_{JM}(\kappa) &= \hat{L}_{JM} - \hat{L}^5_{JM} = i\int d\mathbf{r}\left(\frac{1}{\kappa}\nabla M^J_M(\kappa\mathbf{r})\right)\cdot\hat{\mathcal{J}}(\mathbf{r}),\\
\hat{\mathcal{T}}^{el}_{JM}(\kappa) &= \hat{T}^{el}_{JM} - \hat{T}^{el5}_{JM} = \int d\mathbf{r}\left(\frac{1}{q}\nabla\times\mathbf{M}^{JJ}_M(\kappa\mathbf{r})\right)\cdot\hat{\mathcal{J}}(\mathbf{r}),\\
\hat{\mathcal{T}}^{mag}_{JM}(\kappa) &= \hat{T}^{mag}_{JM} - \hat{T}^{mag5}_{JM} = \int d\mathbf{r}\mathbf{M}^{JJ}_M(\kappa\mathbf{r})\cdot\hat{\mathcal{J}}(\mathbf{r})\,,
\end{aligned}$$

$$\begin{aligned}
\hat{\mathcal{M}}_{JM}(\kappa r) &= \hat{M}^{coul}_{JM} + \hat{M}^{coul5}_{JM}\\
&= F^V_1 M^J_M(\kappa r) - i\frac{\kappa}{M_N}[F_A\Omega^J_M(\kappa r) + \frac{1}{2}(F_A + q_0 F_P)\Sigma''^J_M(\kappa r)]\,,\\
\hat{\mathcal{L}}_{JM}(\kappa r) &= \hat{L}_{JM} + \hat{L}^5_{JM}\\
&= \frac{q_0}{\kappa}F^V_1 M^J_M(\kappa r) + iF_A\Sigma''^J_M(\kappa r)]\,,\\
\hat{\mathcal{T}}^{el}_{JM}(\kappa r) &= \hat{T}^{el}_{JM} + \hat{T}^{el5}_{JM}\\
&= \frac{\kappa}{M_N}[F^V_1\Delta'^J_M(\kappa r) + \frac{1}{2}\mu^V\Sigma^J_M(\kappa r)] + iF_A\Sigma'^J_M(\kappa r)]\,,\\
\hat{\mathcal{T}}^{mag}_{JM}(\kappa r) &= \hat{T}^{mag}_{JM} + \hat{T}^{magn5}_{JM}\\
&= -\frac{q}{M_N}[F^V_1\Delta^J_M(\kappa r) - \frac{1}{2}\mu^V\Sigma'^J_M(\kappa r)] + iF_A\Sigma^J_M(\kappa r)]\,,
\end{aligned}$$

Describing nuclear transitions in semileptonic processes

$$\frac{1}{2J_i+1}\sum_{M_i,M_f}|\langle f|\hat{H}_{\text{eff}}|i\rangle|^2 =$$
$$\frac{G^2}{2}\frac{4\pi}{2J_i+1}\Bigg\{\sum_{J\geq 1}\Bigg[\frac{\mathbf{l}\cdot\mathbf{l}^* - l_3 l_3^*}{2}\left(|\langle J_f|\hat{\mathcal{T}}_J^{\text{mag}}(q)|J_i\rangle|^2 + |\langle J_f|\hat{\mathcal{T}}_J^{\text{el}}(q)|J_i\rangle|^2\right)$$
$$-i\frac{\mathbf{l}\times\mathbf{l}^*}{2}\left(2\text{Re}\,\langle J_f|\hat{\mathcal{T}}_J^{\text{mag}}(q)|J_i\rangle\langle J_f|\hat{\mathcal{T}}_J^{\text{el}}(q)|J_i\rangle^*\right)\Bigg] + \sum_{J\geq 0}\Bigg[l_3 l_3^*\left(|\langle J_f|\hat{\mathcal{L}}_J(q)|J_i\rangle|^2\right.$$
$$\left.+l_0 l_0^*|\langle J_f|\hat{\mathcal{M}}_J(q)|J_i\rangle|^2\right) - 2\text{Re}\left(l_3 l_0^*\langle J_f|\hat{\mathcal{L}}_J(q)|J_i\rangle\langle J_f|\hat{\mathcal{M}}_J(q)|J_i\rangle^*\right)\Bigg]\Bigg\},$$

$l_i l_j^*$: are the lepton traces

J_i : is the spin of the initial nuclear state

$M_{i,f}$: magnetic quantum numbers of the nuclear state

Irreducible tensor operators (Calculated using Shell Model)

- Coulomb,
- Longitudinal,
- Transverse electric
- Transverse magnetic

$\hat{\mathcal{M}}_J$, $\hat{\mathcal{L}}_J$, $\hat{\mathcal{T}}_J^{\text{el}}$ and $\hat{\mathcal{T}}_J^{\text{mag}}$

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$$-i\frac{\mathbf{l}\times\mathbf{l}^*}{2}\left(2\text{Re}\,\langle J_f|\hat{\mathcal{T}}_J^{\text{mag}}(q)|J_i\rangle\langle J_f|\hat{\mathcal{T}}_J^{\text{el}}(q)|J_i\rangle^*\right)\Bigg] + \sum_{J\geq 0}\Bigg[l_3 l_3^*\left(|\langle J_f|\hat{\mathcal{L}}_J(q)|J_i\rangle|^2\right.$$
$$\left.+l_0 l_0^*|\langle J_f|\hat{\mathcal{M}}_J(q)|J_i\rangle|^2\right) - 2\text{Re}\left(l_3 l_0^*\langle J_f|\hat{\mathcal{L}}_J(q)|J_i\rangle\langle J_f|\hat{\mathcal{M}}_J(q)|J_i\rangle^*\right)\Bigg]\Bigg\},$$

Lepton traces for neutrino-nucleus scattering

$$\sum_{\text{spins}} l_0 l_0^* = 1+\cos\theta$$
$$\sum_{\text{spins}} l_3 l_0^* = \frac{E_\nu - E_{\nu'}}{|\mathbf{q}|}(1+\cos\theta)$$
$$\sum_{\text{spins}} l_3 l_3^* = (1+\cos\theta) - 2\frac{E_\nu E_{\nu'}}{|\mathbf{q}|^2}\sin^2\theta$$
$$\sum_{\text{spins}} \frac{1}{2}(\mathbf{l}\cdot\mathbf{l} - l_3 l_3^*) = (1-\cos\theta) + \frac{E_\nu E_{\nu'}}{|\mathbf{q}|^2}\sin^2\theta$$
$$\frac{-i}{2}\sum_{\text{spins}}(\mathbf{l}\times\mathbf{l}^*)_3 = -\frac{E_\nu + E_{\nu'}}{|\mathbf{q}|}(1-\cos\theta)$$

Irreducible tensor operators (Calculated using Shell Model)

- Coulomb,
- Longitudinal,
- Transverse electric
- Transverse magnetic

$$\hat{\mathcal{M}}_J,\ \hat{\mathcal{L}}_J,\ \hat{\mathcal{T}}_J^{\text{el}} \text{ and } \hat{\mathcal{T}}_J^{\text{mag}}$$

Inelastic neutrino-nucleus scattering

$$\frac{\mathrm{d}^2\sigma_{i\to f}}{\mathrm{d}\Omega\mathrm{d}E_{\mathrm{exc}}} = \frac{G^2|\mathbf{k}'|E_{k'}}{\pi(2J_i+1)}\left(\sum_{J\geq 0}\sigma_{\mathrm{CL}}^J + \sum_{J\geq 1}\sigma_{\mathrm{T}}^J\right),$$

Coulomb-Longitudinal contribution

$$\sigma_{\mathrm{CL}}^J = (1+\cos\theta)|(J_f||\mathcal{M}_J(q)||J_i)|^2 + \left(1+\cos\theta - 2\frac{E_kE_{k'}}{q^2}\sin^2\theta\right)|(J_f||\mathcal{L}_J(q)||J_i)|^2 + \frac{E_k - E_{k'}}{q}(1+\cos\theta)2\mathrm{Re}\left[(J_f||\mathcal{L}_J(q)||J_i)(J_f||\mathcal{M}_J(q)||J_i)^*\right]$$

Transverse Electric/Magnetic contribution

$$\sigma_{\mathrm{T}}^J = \left(1-\cos\theta + \frac{E_kE_{k'}}{q^2}\sin^2\theta\right)\left[|(J_f||\mathcal{T}_J^{\mathrm{el}}(q)||J_i)|^2 + |(J_f||\mathcal{T}_J^{\mathrm{mag}}(q)||J_i)|^2\right] - \frac{(E_k - E_{k'})}{q}(1-\cos\theta)2\mathrm{Re}\left[(J_f||\mathcal{T}_J^{\mathrm{mag}}(q)||J_i)(J_f||\mathcal{T}_J^{\mathrm{el}}(q)||J_i)^*\right]$$

Inelastic neutrino-nucleus scattering

$$\frac{d^2\sigma_{i\to f}}{d\Omega dE_{\rm exc}} = \frac{G^2|\mathbf{k}'|E_{k'}}{\pi(2J_i+1)}\left(\sum_{J\geq 0}\sigma^J_{\rm CL} + \sum_{J\geq 1}\sigma^J_{\rm T}\right),$$

If $\omega = 0$ => CEvNS

CEvNS!
(Vector)

Coulomb-Longitudinal contribution

$$\sigma^J_{\rm CL} = (1+\cos\theta)|(J_f||\mathcal{M}_J(q)||J_i)|^2 + \left(1+\cos\theta - 2\frac{E_k E_{k'}}{q^2}\sin^2\theta\right)|(J_f||\mathcal{L}_J(q)||J_i)|^2 + \frac{E_k - E_{k'}}{q}(1+\cos\theta)2\mathrm{Re}\left[(J_f||\mathcal{L}_J(q)||J_i)(J_f||\mathcal{M}_J(q)||J_i)^*\right]$$

CEvNS!
(Axial vector)

Transverse Electric/Magnetic contribution

$$\sigma^J_{\rm T} = \left(1-\cos\theta + \frac{E_k E_{k'}}{q^2}\sin^2\theta\right)\left[|(J_f||\mathcal{T}^{\rm el}_J(q)||J_i)|^2 + |(J_f||\mathcal{T}^{\rm mag}_J(q)||J_i)|^2\right] - \frac{(E_k - E_{k'})}{q}(1-\cos\theta)2\mathrm{Re}\left[(J_f||\mathcal{T}^{\rm mag}_J(q)||J_i)(J_f||\mathcal{T}^{\rm el}_J(q)||J_i)^*\right]$$

Expressions in terms of the nuclear recoil energy T

$$\frac{d^2\sigma_{i\to f}}{d\Omega dE_{\rm exc}} = \frac{G^2|\mathbf{k}'|E_{k'}}{\pi(2J_i+1)}\left(\sum_{J\geq 0}\sigma_{\rm CL}^J + \sum_{J\geq 1}\sigma_{\rm T}^J\right),$$

Change of variables

$$\frac{d\sigma}{dT} = \frac{d\sigma}{d\cos\theta}\frac{M}{E_\nu(E_\nu-\omega)}$$

kinematics $\omega \equiv E_{\rm exc}$

$E_\nu - E_{\nu'} = \omega + T$, with ω being the excitation energy

$$T \approx \frac{E_\nu(E_\nu-\omega)(1-\cos\theta)+\omega^2/2}{M}.$$

$$T_{\rm min} = \frac{\omega^2}{2M}, \qquad T_{\rm max} = \frac{(2E_\nu-\omega)^2}{2M}.$$

Lepton traces in terms of T, ω

$$\sum_{\rm spins} l_0 l_0^* = \frac{4E_\nu^2 - 4E_\nu(T+\omega) - 2MT + \omega(2T+\omega)}{2E_\nu(E_\nu-\omega)},$$

$$\sum_{\rm spins} l_3 l_0^* = \frac{(T+\omega)\left(4E_\nu^2 - 2MT - 4E_\nu(T+\omega) + \omega(2T+\omega)\right)}{2\sqrt{2}E_\nu\left(E_\nu-\omega\right)\sqrt{MT}},$$

$$\sum_{\rm spins} l_3 l_3^* = \frac{(T+\omega)^2\left(4E_\nu^2 - 2MT - 4E_\nu(T+\omega) + \omega(2T+\omega)\right)}{4E_\nu MT\left(E_\nu-\omega\right)}$$

$$\sum_{\rm spins}\frac{1}{2}(\mathbf{l}\cdot\mathbf{l}^* - l_3 l_3^*) = \frac{\left(2MT - \omega(2T+\omega)\right)\left(4E_\nu^2 + 2MT - 4E_\nu(T+\omega) + \omega(2T+\omega)\right)}{8E_\nu MT\left(E_\nu-\omega\right)}$$

$$\sum_{\rm spins}\frac{-i}{2}(\mathbf{l}\times\mathbf{l}^*)_3 = \frac{(2E_\nu-\omega)\left(2MT-\omega(2T+\omega)\right)}{2\sqrt{2}E_\nu\left(E_\nu-\omega\right)\sqrt{MT}}.$$

Expressions in terms of the nuclear recoil energy T

$$\frac{1}{2J_i+1}\sum_{M_i,M_f}|\langle f|\hat{H}_{\rm eff}|i\rangle|^2 =$$
$$\frac{G^2}{2}\frac{4\pi}{2J_i+1}\left\{\sum_{J\geq 1}\left[\frac{\mathbf{l}\cdot\mathbf{l}^* - l_3 l_3^*}{2}\left(|\langle J_f|\hat{\mathcal{T}}_J^{\rm mag}(q)|J_i\rangle|^2 + |\langle J_f|\hat{\mathcal{T}}_J^{\rm el}(q)|J_i\rangle|^2\right)\right.\right.$$
$$\left. -i\frac{\mathbf{l}\times\mathbf{l}^*}{2}\left(2{\rm Re}\langle J_f|\hat{\mathcal{T}}_J^{\rm mag}(q)|J_i\rangle\langle J_f|\hat{\mathcal{T}}_J^{\rm el}(q)|J_i\rangle^*\right)\right] + \sum_{J\geq 0}\left[l_3 l_3^*\left(|\langle J_f|\hat{\mathcal{L}}_J(q)|J_i\rangle|^2\right.\right.$$
$$\left.\left.\left. + l_0 l_0^*|\langle J_f|\hat{\mathcal{M}}_J(q)|J_i\rangle|^2\right) - 2{\rm Re}\left(l_3 l_0^*\langle J_f|\hat{\mathcal{L}}_J(q)|J_i\rangle\langle J_f|\hat{\mathcal{M}}_J(q)|J_i\rangle^*\right)\right]\right\},$$

$$l_3 l_0^* = \frac{T+\omega}{\sqrt{2MT}}\,l_0 l_0^* \approx \frac{T+\omega}{|\mathbf{q}|}\,l_0 l_0^*,$$
$$l_3 l_3^* = \frac{(T+\omega)^2}{2MT}\,l_0 l_0^* \approx \left(\frac{T+\omega}{|\mathbf{q}|}\right)^2 l_0 l_0^*.$$

$$(\mathbf{l}\cdot\mathbf{l}^* - l_3 l_3^*) \approx \left(1-\frac{\omega^2}{|\mathbf{q}|^2}\right)\left(l_0 l_0^* + \frac{|\mathbf{q}|^2}{E_\nu(E_\nu-\omega)}\right)$$

Lepton traces in terms of T, ω

$$\sum_{\rm spins} l_0 l_0^* = \frac{4E_\nu^2 - 4E_\nu(T+\omega) - 2MT + \omega(2T+\omega)}{2E_\nu(E_\nu-\omega)},$$
$$\sum_{\rm spins} l_3 l_0^* = \frac{(T+\omega)\left(4E_\nu^2 - 2MT - 4E_\nu(T+\omega) + \omega(2T+\omega)\right)}{2\sqrt{2}E_\nu(E_\nu-\omega)\sqrt{MT}},$$
$$\sum_{\rm spins} l_3 l_3^* = \frac{(T+\omega)^2\left(4E_\nu^2 - 2MT - 4E_\nu(T+\omega) + \omega(2T+\omega)\right)}{4E_\nu MT(E_\nu-\omega)}$$
$$\sum_{\rm spins}\frac{1}{2}(\mathbf{l}\cdot\mathbf{l}^* - l_3 l_3^*) = \frac{(2MT-\omega(2T+\omega))\left(4E_\nu^2 + 2MT - 4E_\nu(T+\omega) + \omega(2T+\omega)\right)}{8E_\nu MT(E_\nu-\omega)}$$
$$\sum_{\rm spins}\frac{-i}{2}(\mathbf{l}\times\mathbf{l}^*)_3 = \frac{(2E_\nu-\omega)(2MT-\omega(2T+\omega))}{2\sqrt{2}E_\nu(E_\nu-\omega)\sqrt{MT}}.$$

Expressions in terms of the nuclear recoil energy T

$$\frac{1}{2J_i+1}\sum_{M_i,M_f}|\langle f|\hat{H}_{\rm eff}|i\rangle|^2 =$$
$$\frac{G^2}{2}\frac{4\pi}{2J_i+1}\Bigg\{\sum_{J\geq 1}\Bigg[\frac{\mathbf{l}\cdot\mathbf{l}^* - l_3 l_3^*}{2}\left(|\langle J_f|\hat{\mathcal{T}}_J^{\rm mag}(q)|J_i\rangle|^2 + |\langle J_f|\hat{\mathcal{T}}_J^{\rm el}(q)|J_i\rangle|^2\right)$$
$$-i\frac{\mathbf{l}\times\mathbf{l}^*}{2}\left(2{\rm Re}\,\langle J_f|\hat{\mathcal{T}}_J^{\rm mag}(q)|J_i\rangle\langle J_f|\hat{\mathcal{T}}_J^{\rm el}(q)|J_i\rangle^*\right)\Bigg] + \sum_{J\geq 0}\Bigg[l_3 l_3^*\left(|\langle J_f|\hat{\mathcal{L}}_J(q)|J_i\rangle|^2\right.$$
$$\left.+l_0 l_0^*|\langle J_f|\hat{\mathcal{M}}_J(q)|J_i\rangle|^2\right) - 2{\rm Re}\left(l_3 l_0^*\langle J_f|\hat{\mathcal{L}}_J(q)|J_i\rangle\langle J_f|\hat{\mathcal{M}}_J(q)|J_i\rangle^*\right)\Bigg]\Bigg\},$$

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$$l_3 l_3^* = \frac{(T+\omega)^2}{2MT}\, l_0 l_0^* \approx \left(\frac{T+\omega}{|\mathbf{q}|}\right)^2 l_0 l_0^*.$$

$$(\mathbf{l}\cdot\mathbf{l}^* - l_3 l_3^*) \approx \left(1-\frac{\omega^2}{|\mathbf{q}|^2}\right)\left(l_0 l_0^* + \frac{|\mathbf{q}|^2}{E_\nu(E_\nu-\omega)}\right)$$

$T \ll \omega$ and $\omega/|\mathbf{q}| \sim 10\%$ or less

Lepton traces in terms of T, ω

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$$\left. -i\frac{\mathbf{l}\times\mathbf{l}^*}{2}\left(2{\rm Re}\,\langle J_f|\hat{\mathcal{T}}_J^{\rm mag}(q)|J_i\rangle\langle J_f|\hat{\mathcal{T}}_J^{\rm el}(q)|J_i\rangle^*\right)\right] + \sum_{J\geq 0}\left[l_3 l_3^*\left(|\langle J_f|\hat{\mathcal{L}}_J(q)|J_i\rangle|^2\right.\right.$$

$$\left.\left.\left. + l_0 l_0^* |\langle J_f|\hat{\mathcal{M}}_J(q)|J_i\rangle|^2\right) - 2{\rm Re}\left(l_3 l_0^*\langle J_f|\hat{\mathcal{L}}_J(q)|J_i\rangle\langle J_f|\hat{\mathcal{M}}_J(q)|J_i\rangle^*\right)\right]\right\},$$

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$$l_3 l_3^* = \frac{(T+\omega)^2}{2MT}\, l_0 l_0^* \approx \left(\frac{T+\omega}{|\mathbf{q}|}\right)^2 l_0 l_0^*.$$

Suppressed

$$(\mathbf{l}\cdot\mathbf{l}^* - l_3 l_3^*) \approx \left(1 - \frac{\omega^2}{|\mathbf{q}|^2}\right)\left(l_0 l_0^* + \frac{|\mathbf{q}|^2}{E_\nu(E_\nu-\omega)}\right)$$

$T \ll \omega$ and $\omega/|\mathbf{q}| \sim 10\%$ or less

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$$-i\frac{\mathbf{l}\times\mathbf{l}^*}{2}\Big(2{\rm Re}\,\langle J_f|\,\hat{\mathcal{T}}_J^{\rm mag}(q)\,|J_i\rangle\,\langle J_f|\,\hat{\mathcal{T}}_J^{\rm el}(q)\,|J_i\rangle^*\Big)\Bigg] + \sum_{J\geq 0}\Big[l_3 l_3^*\Big(|\langle J_f|\,\hat{\mathcal{L}}_J(q)\,|J_i\rangle|^2$$

$$+l_0 l_0^*|\langle J_f|\,\hat{\mathcal{M}}_J(q)\,|J_i\rangle|^2\Big) - 2{\rm Re}\Big(l_3 l_0^*\,\langle J_f|\,\hat{\mathcal{L}}_J(q)\,|J_i\rangle\,\langle J_f|\,\hat{\mathcal{M}}_J(q)\,|J_i\rangle^*\Big)\Big]\Bigg\},$$

Let's understand these a bit better!

$$l_3 l_0^* = \frac{T+\omega}{\sqrt{2MT}}\, l_0 l_0^* \approx \frac{T+\omega}{|\mathbf{q}|}\, l_0 l_0^*\,,$$

Suppressed

$$l_3 l_3^* = \frac{(T+\omega)^2}{2MT}\, l_0 l_0^* \approx \left(\frac{T+\omega}{|\mathbf{q}|}\right)^2 l_0 l_0^*\,.$$

doubly Suppressed

$$(\mathbf{l}\cdot\mathbf{l}^* - l_3 l_3^*) \approx \left(1 - \frac{\omega^2}{|\mathbf{q}|^2}\right)\left(l_0 l_0^* + \frac{|\mathbf{q}|^2}{E_\nu\,(E_\nu-\omega)}\right)$$

$T \ll \omega$ and $\omega/|\mathbf{q}| \sim 10\%$ or less

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$$\sum_{\rm spins} l_0 l_0^* = \frac{4E_\nu^2 - 4E_\nu(T+\omega) - 2MT + \omega(2T+\omega)}{2E_\nu(E_\nu-\omega)}\,,$$

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Expressions in terms of the nuclear recoil energy T

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$$\left.-i\frac{\mathbf{l}\times\mathbf{l}^*}{2}\left(2\text{Re}\,\langle J_f|\hat{\mathcal{T}}_J^{\text{mag}}(q)|J_i\rangle\langle J_f|\hat{\mathcal{T}}_J^{\text{el}}(q)|J_i\rangle^*\right)\right]+\sum_{J\geq0}\left[l_3l_3^*\left(|\langle J_f|\hat{\mathcal{L}}_J(q)|J_i\rangle|^2\right.\right.$$
$$\left.\left.\left.+l_0l_0^*|\langle J_f|\hat{\mathcal{M}}_J(q)|J_i\rangle|^2\right)-2\text{Re}\left(l_3l_0^*\langle J_f|\hat{\mathcal{L}}_J(q)|J_i\rangle\langle J_f|\hat{\mathcal{M}}_J(q)|J_i\rangle^*\right)\right]\right\},$$

Let's understand these a bit better!

$$l_3l_0^*=\frac{T+\omega}{\sqrt{2MT}}\,l_0l_0^*\approx\frac{T+\omega}{|\mathbf{q}|}\,l_0l_0^*,$$
$$l_3l_3^*=\frac{(T+\omega)^2}{2MT}\,l_0l_0^*\approx\left(\frac{T+\omega}{|\mathbf{q}|}\right)^2 l_0l_0^*.$$

Suppressed

doubly Suppressed

$$(\mathbf{l}\cdot\mathbf{l}^*-l_3l_3^*)\approx\left(1-\frac{\omega^2}{|\mathbf{q}|^2}\right)\left(l_0l_0^*+\frac{|\mathbf{q}|^2}{E_\nu\,(E_\nu-\omega)}\right)$$

Always larger

$T\ll\omega$ and $\omega/|\mathbf{q}|\sim10\%$ or less

Lepton traces in terms of T,ω

$$\sum_{\text{spins}}l_0l_0^*=\frac{4E_\nu^2-4E_\nu(T+\omega)-2MT+\omega(2T+\omega)}{2E_\nu(E_\nu-\omega)},$$
$$\sum_{\text{spins}}l_3l_0^*=\frac{(T+\omega)\,(4E_\nu^2-2MT-4E_\nu(T+\omega)+\omega(2T+\omega))}{2\sqrt{2}E_\nu\,(E_\nu-\omega)\sqrt{MT}},$$
$$\sum_{\text{spins}}l_3l_3^*=\frac{(T+\omega)^2\,(4E_\nu^2-2MT-4E_\nu(T+\omega)+\omega(2T+\omega))}{4E_\nu MT\,(E_\nu-\omega)}$$
$$\sum_{\text{spins}}\frac{1}{2}(\mathbf{l}\cdot\mathbf{l}^*-l_3l_3^*)=\frac{(2MT-\omega(2T+\omega))\,(4E_\nu^2+2MT-4E_\nu(T+\omega)+\omega(2T+\omega))}{8E_\nu MT\,(E_\nu-\omega)}$$
$$\sum_{\text{spins}}\frac{-i}{2}(\mathbf{l}\times\mathbf{l}^*)_3=\frac{(2E_\nu-\omega)\,(2MT-\omega(2T+\omega))}{2\sqrt{2}E_\nu\,(E_\nu-\omega)\sqrt{MT}}.$$

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$$\left.-i\frac{\mathbf{l}\times\mathbf{l}^*}{2}\left(2{\rm Re}\,\langle J_f|\hat{\mathcal{T}}_J^{\rm mag}(q)|J_i\rangle\langle J_f|\hat{\mathcal{T}}_J^{\rm el}(q)|J_i\rangle^*\right)\right] + \sum_{J\geq 0}\left[l_3 l_3^*\left(|\langle J_f|\hat{\mathcal{L}}_J(q)|J_i\rangle|^2\right.\right.$$
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On the other hand…

if $T \to T_{\rm min} => \omega \approx |\mathbf{q}|$

$$l_3 l_0^* = \frac{T+\omega}{\sqrt{2MT}}\, l_0 l_0^* \approx \frac{T+\omega}{|\mathbf{q}|}\, l_0 l_0^*,$$
$$l_3 l_3^* = \frac{(T+\omega)^2}{2MT}\, l_0 l_0^* \approx \left(\frac{T+\omega}{|\mathbf{q}|}\right)^2 l_0 l_0^*.$$

$$(\mathbf{l}\cdot\mathbf{l}^* - l_3 l_3^*) \approx \left(1 - \frac{\omega^2}{|\mathbf{q}|^2}\right)\left(l_0 l_0^* + \frac{|\mathbf{q}|^2}{E_\nu(E_\nu - \omega)}\right)$$

$l_0 l_0^* \approx l_3 l_0^* \approx l_3 l_3^*$ **and**

$(\mathbf{l}\cdot\mathbf{l} - l_3 l_3^*) \ll l_0 l_0^*$

Lepton traces in terms of T, ω

$$\sum_{\rm spins} l_0 l_0^* = \frac{4E_\nu^2 - 4E_\nu(T+\omega) - 2MT + \omega(2T+\omega)}{2E_\nu(E_\nu - \omega)},$$

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$$\sum_{\rm spins} l_3 l_3^* = \frac{(T+\omega)^2\left(4E_\nu^2 - 2MT - 4E_\nu(T+\omega) + \omega(2T+\omega)\right)}{4E_\nu MT\,(E_\nu-\omega)}$$

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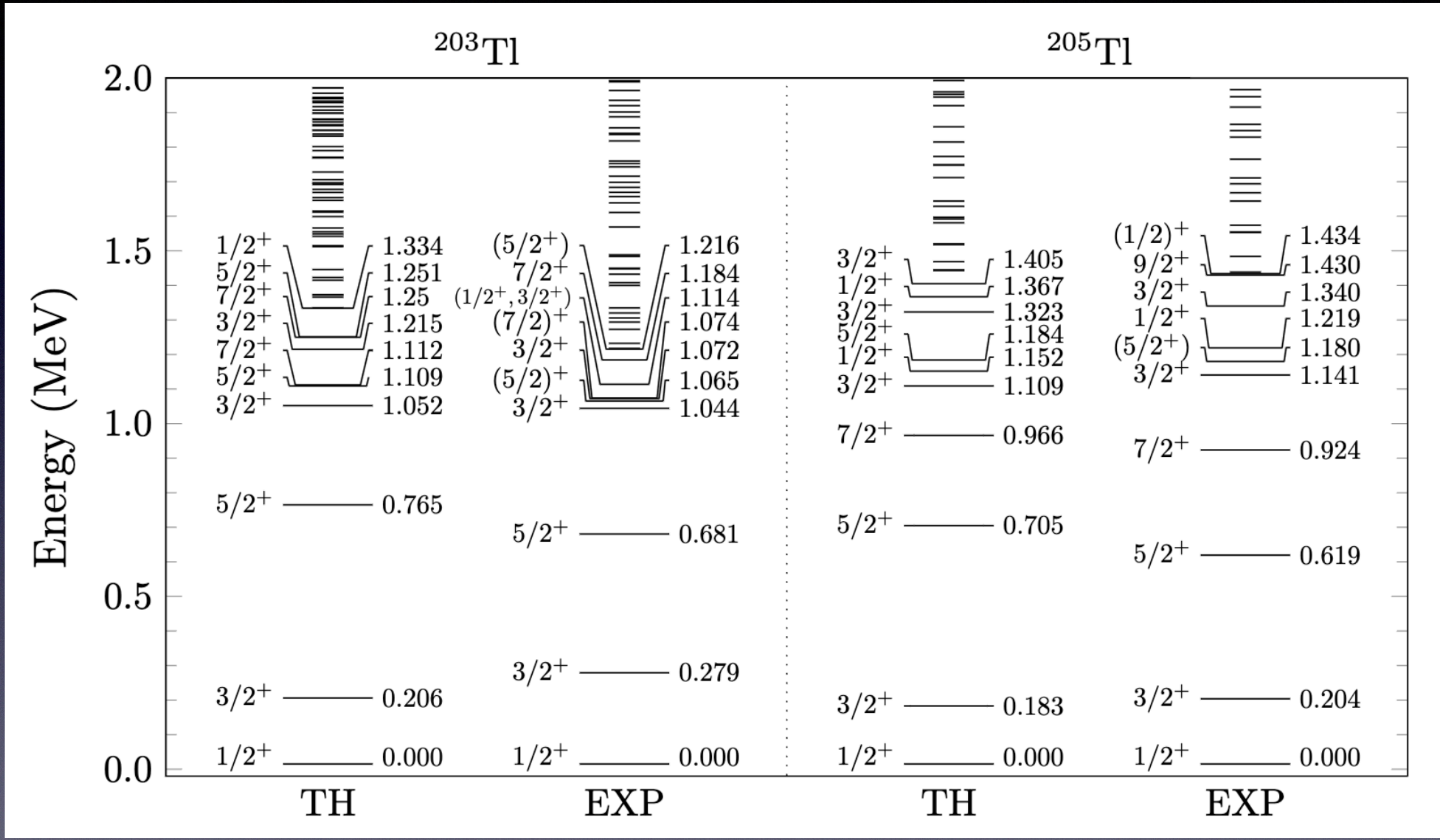
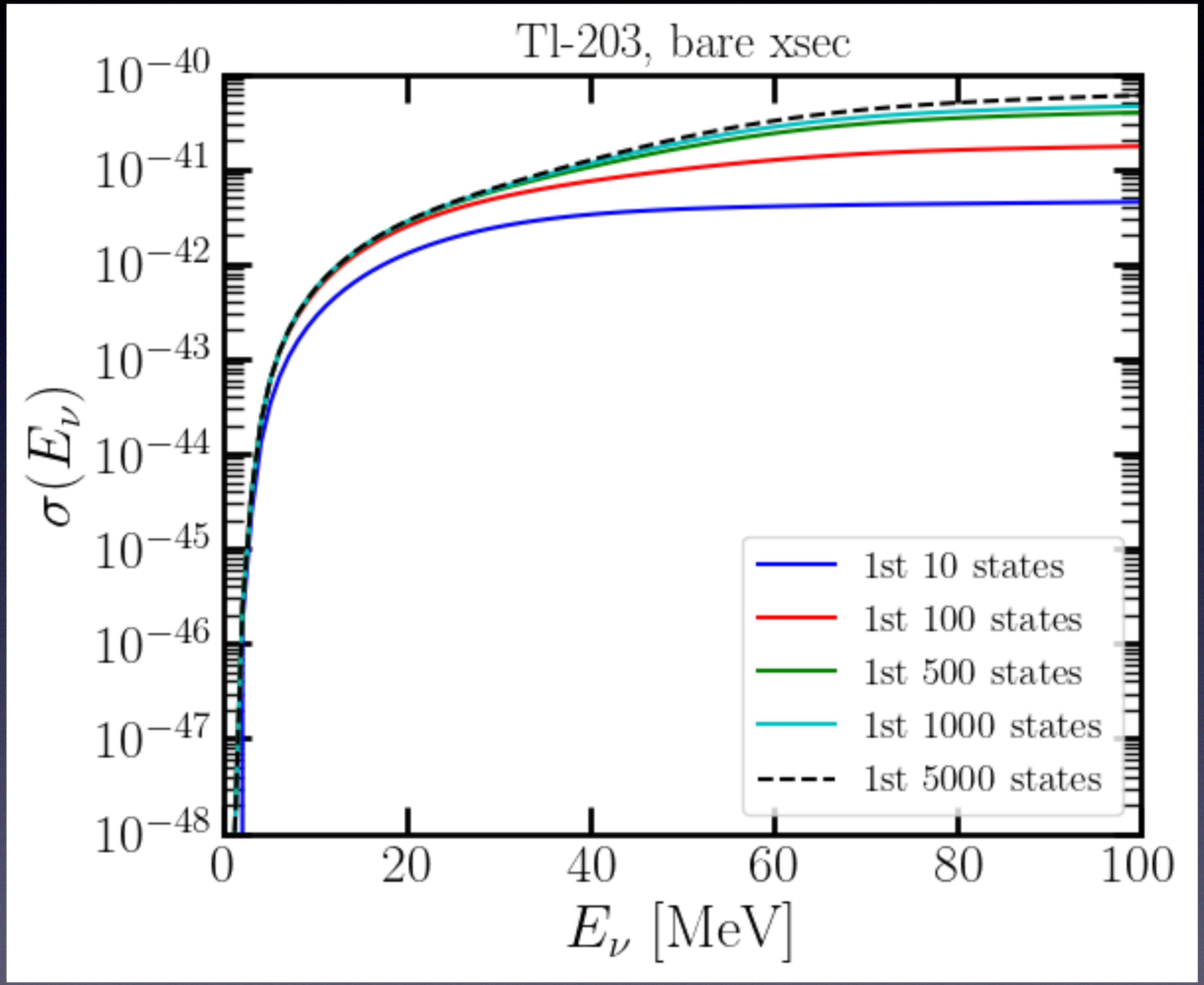
Shell Model Spectra

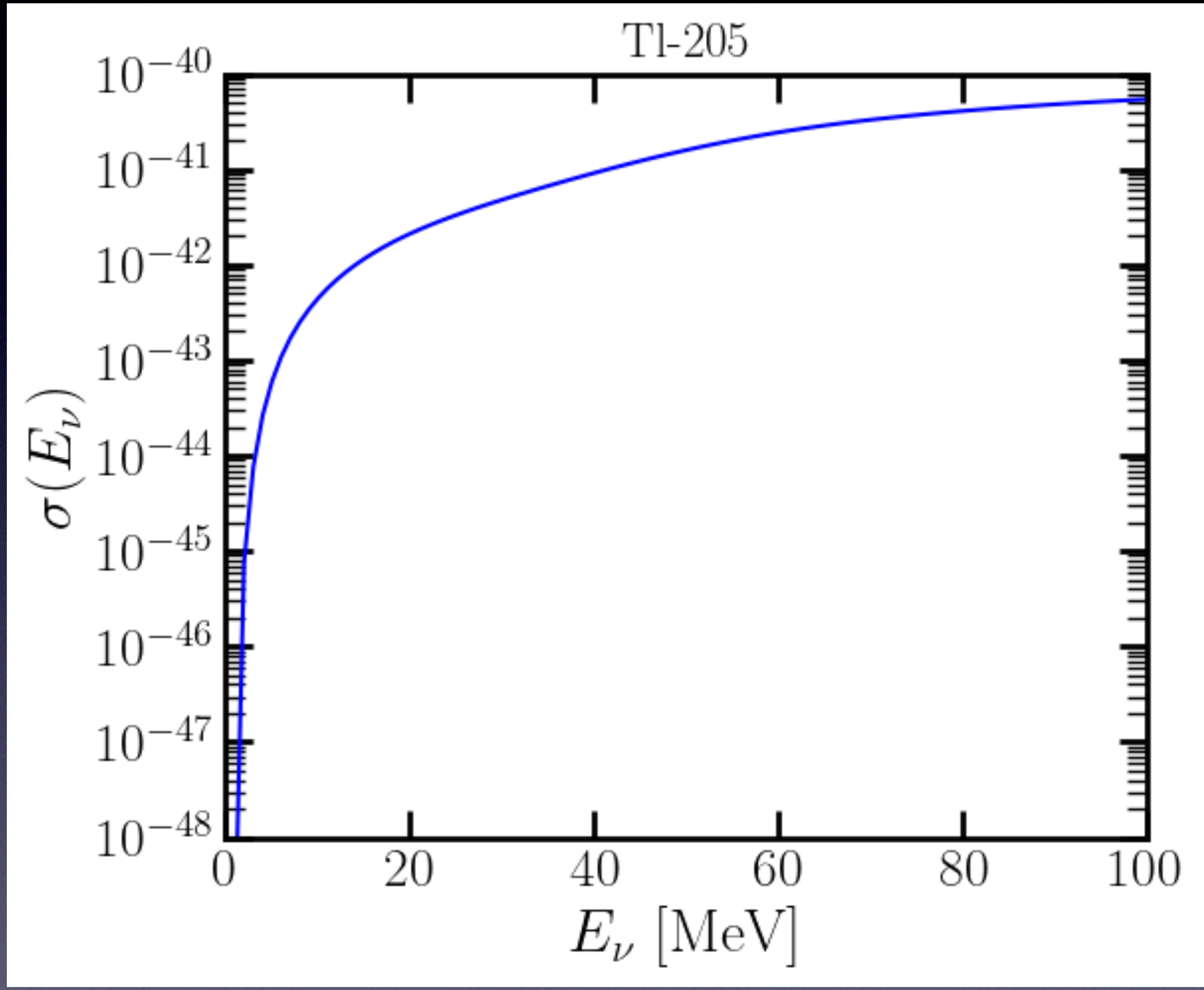


Calculated using NuShellX@MSU assuming the jj56pn model space and khhe interaction

About 100K states for Tl-203
1393 states for Tl-205

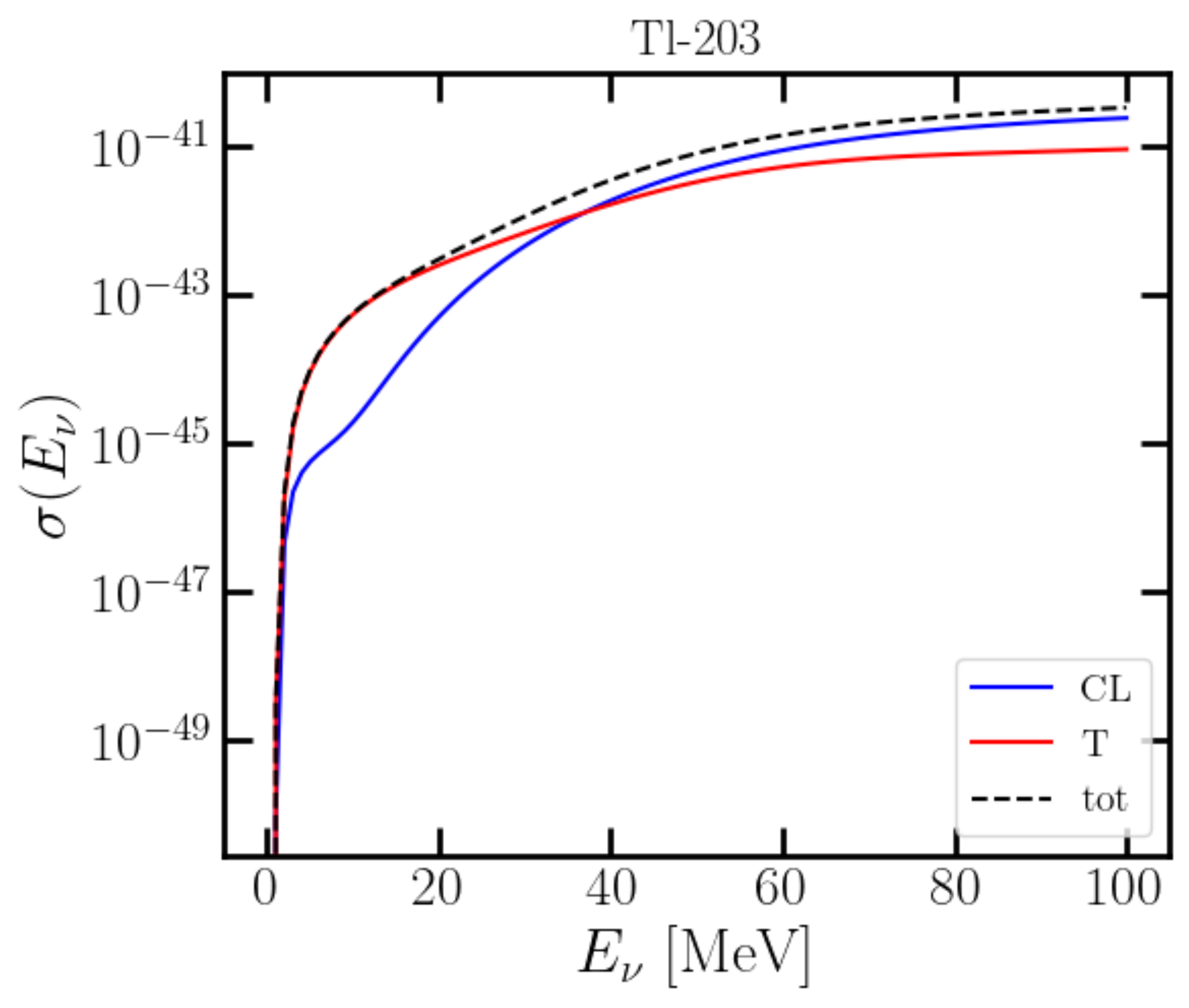
Cross sections results

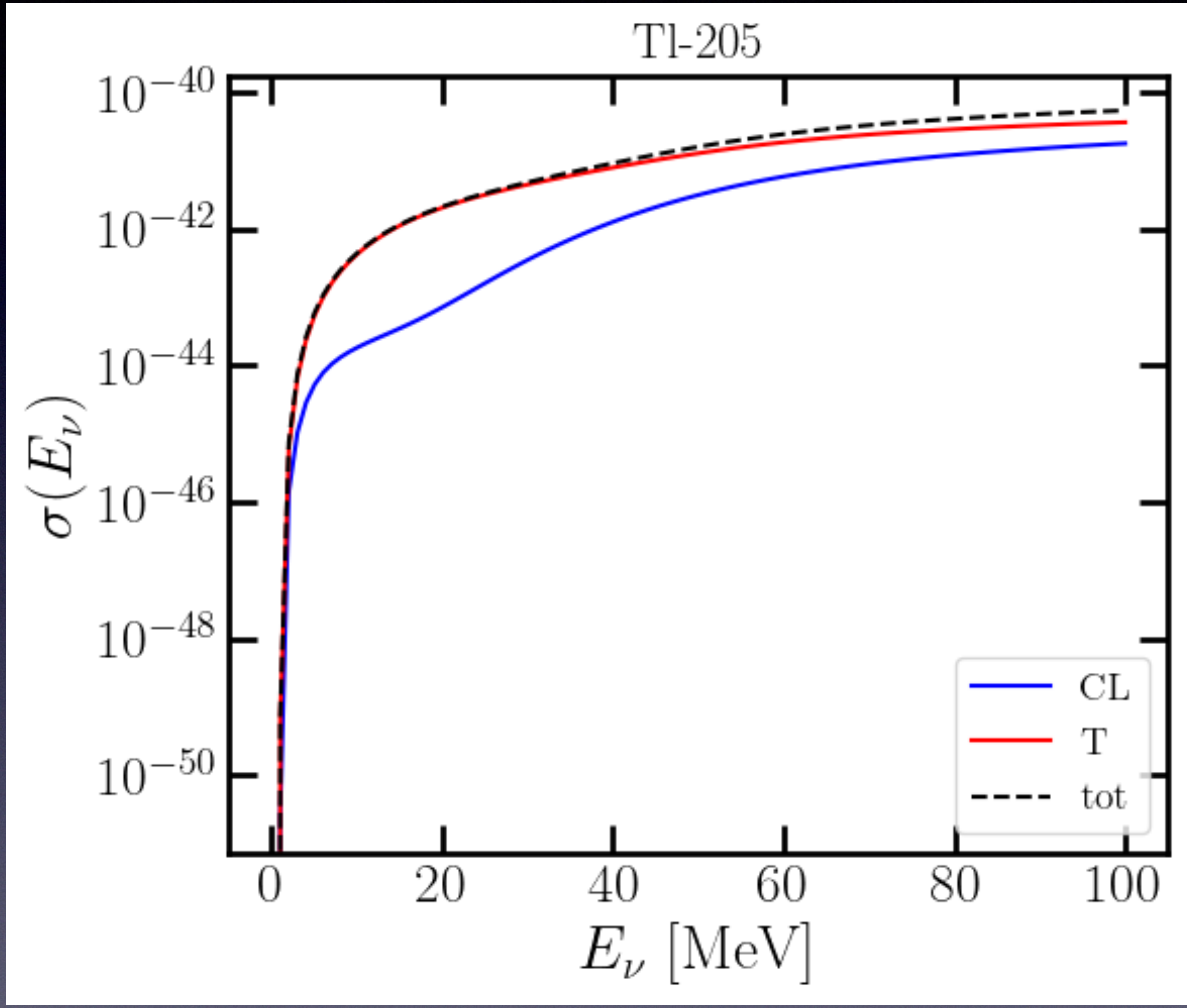




$T_{\text{thres}} = 0$ is assumed

Relative contributions to the cross section

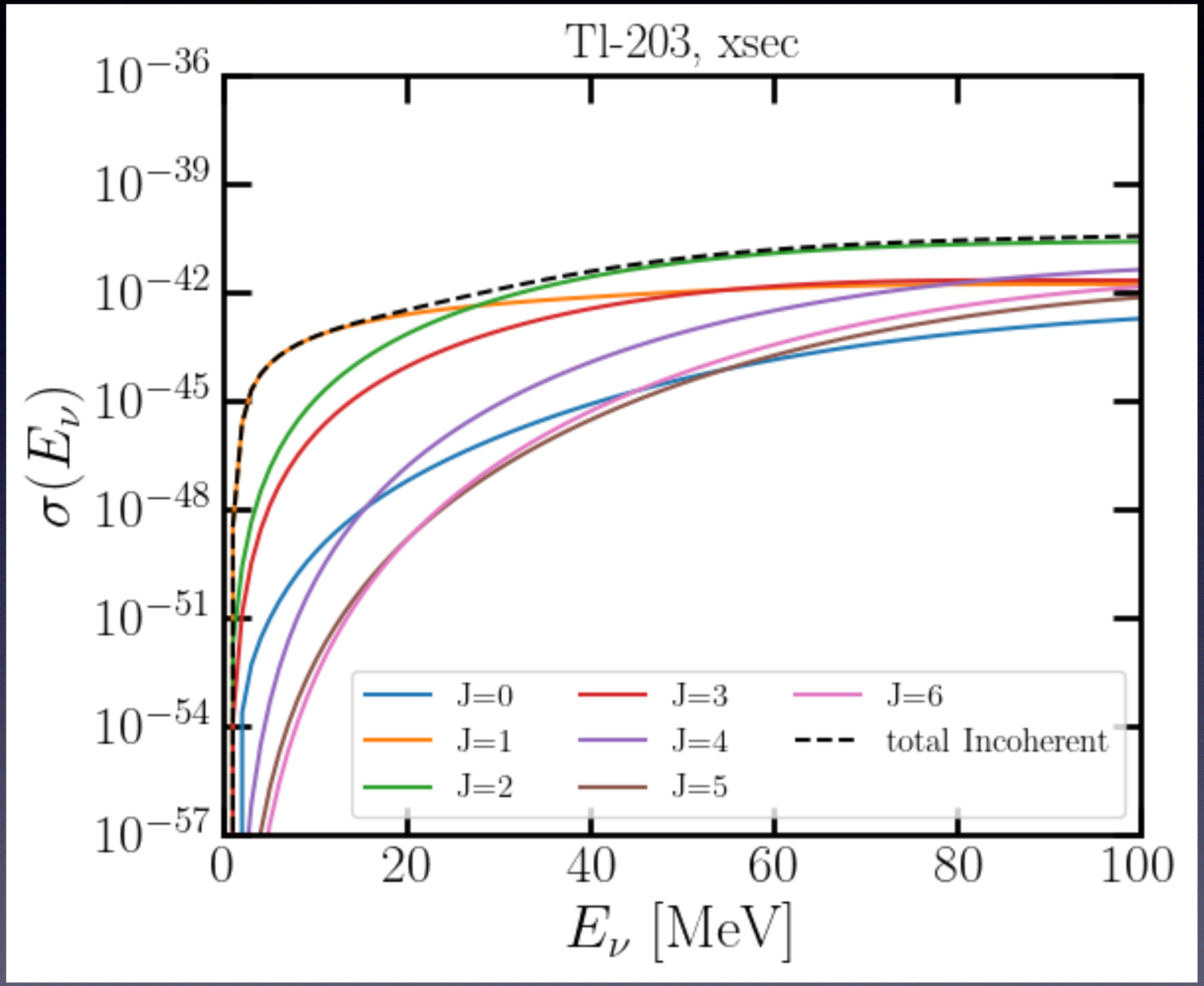


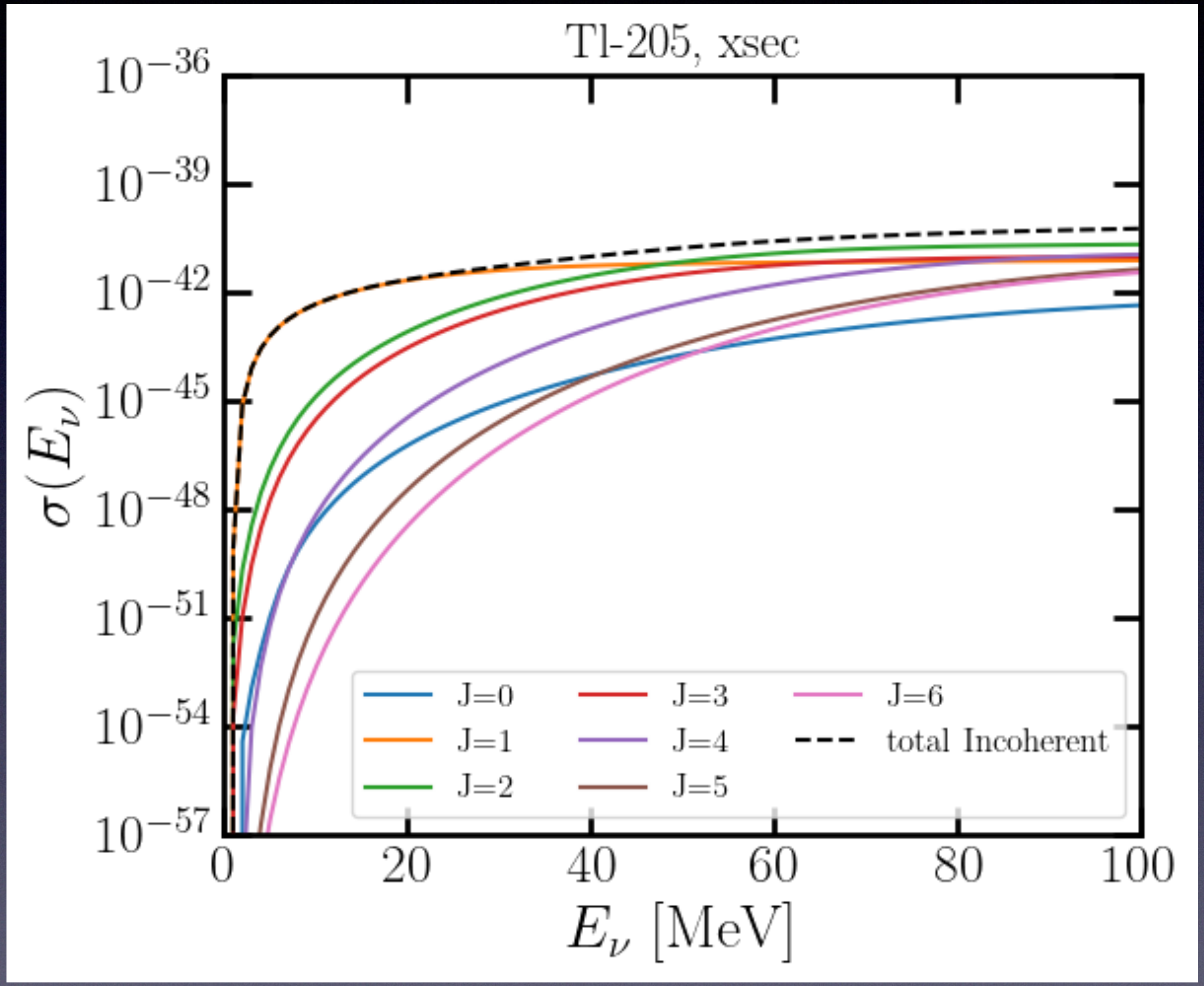


At about $E_\nu \sim 35$ MeV CL becomes dominant

Transverse is always dominant

Multipole contributions to the cross section

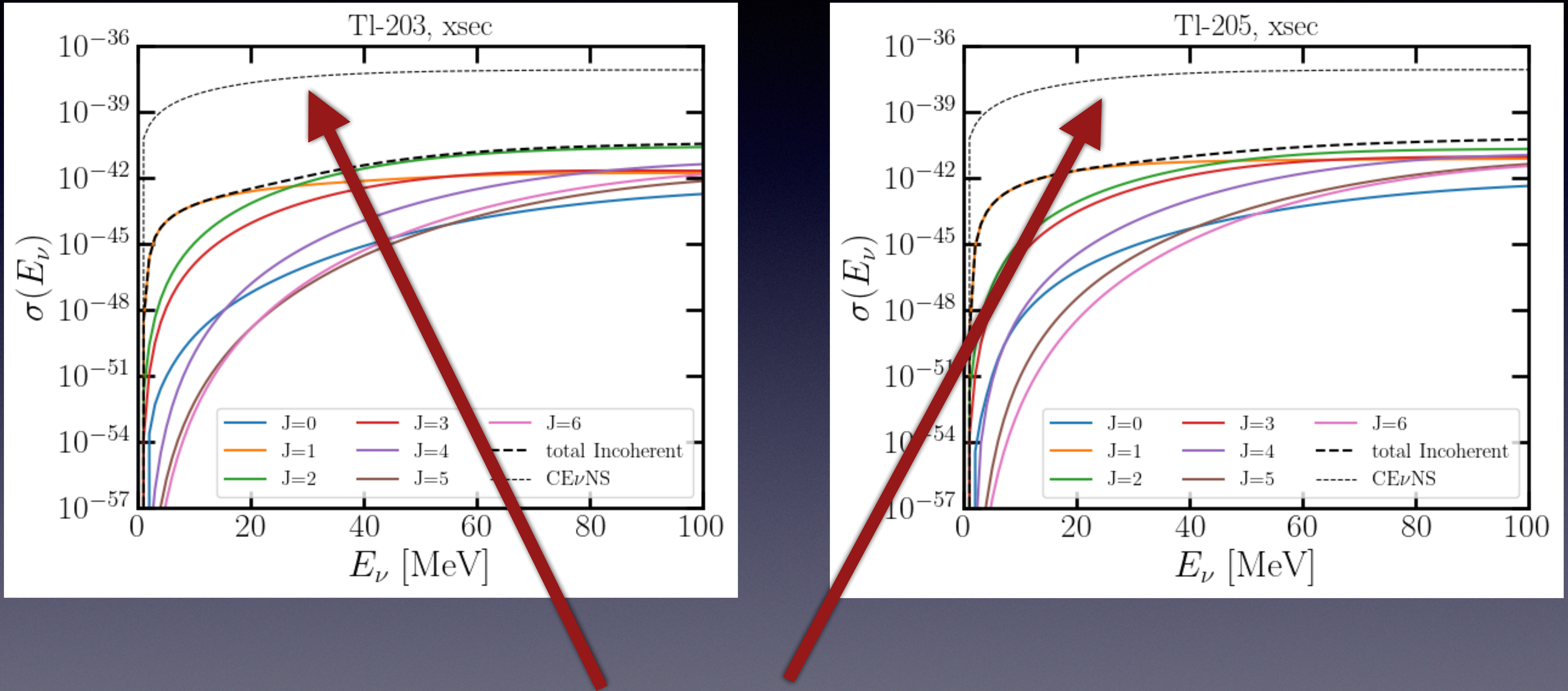




- $J = 1$ transitions dominate up to $E_\nu \sim 20$ MeV
- For $E_\nu > 20$ MeV, $J = 2$ transitions dominate

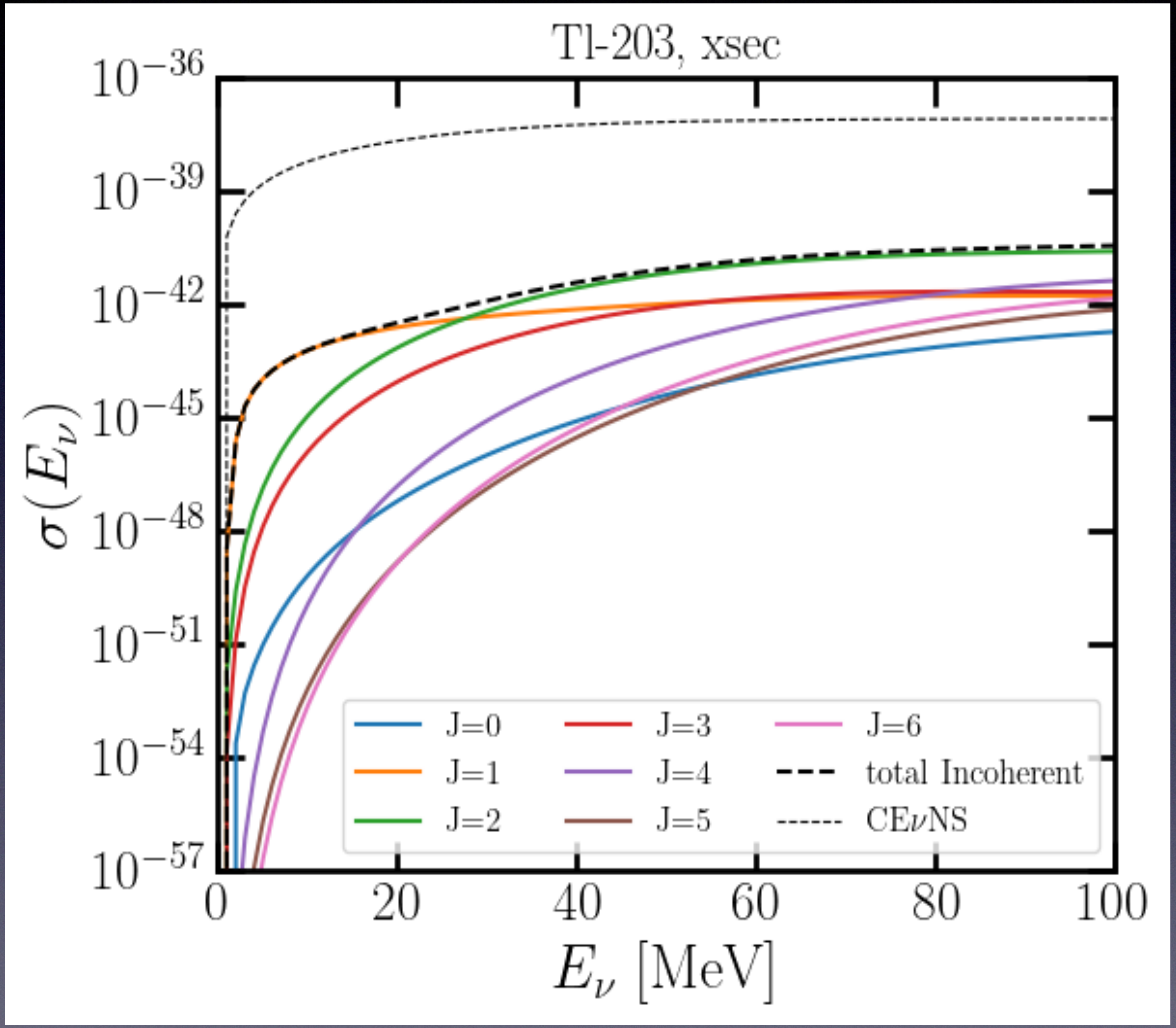
- $J = 1$ transitions dominate up to $E_\nu \sim 40$ MeV
- For $E_\nu > 40$ MeV, $J = 2$ transitions dominate

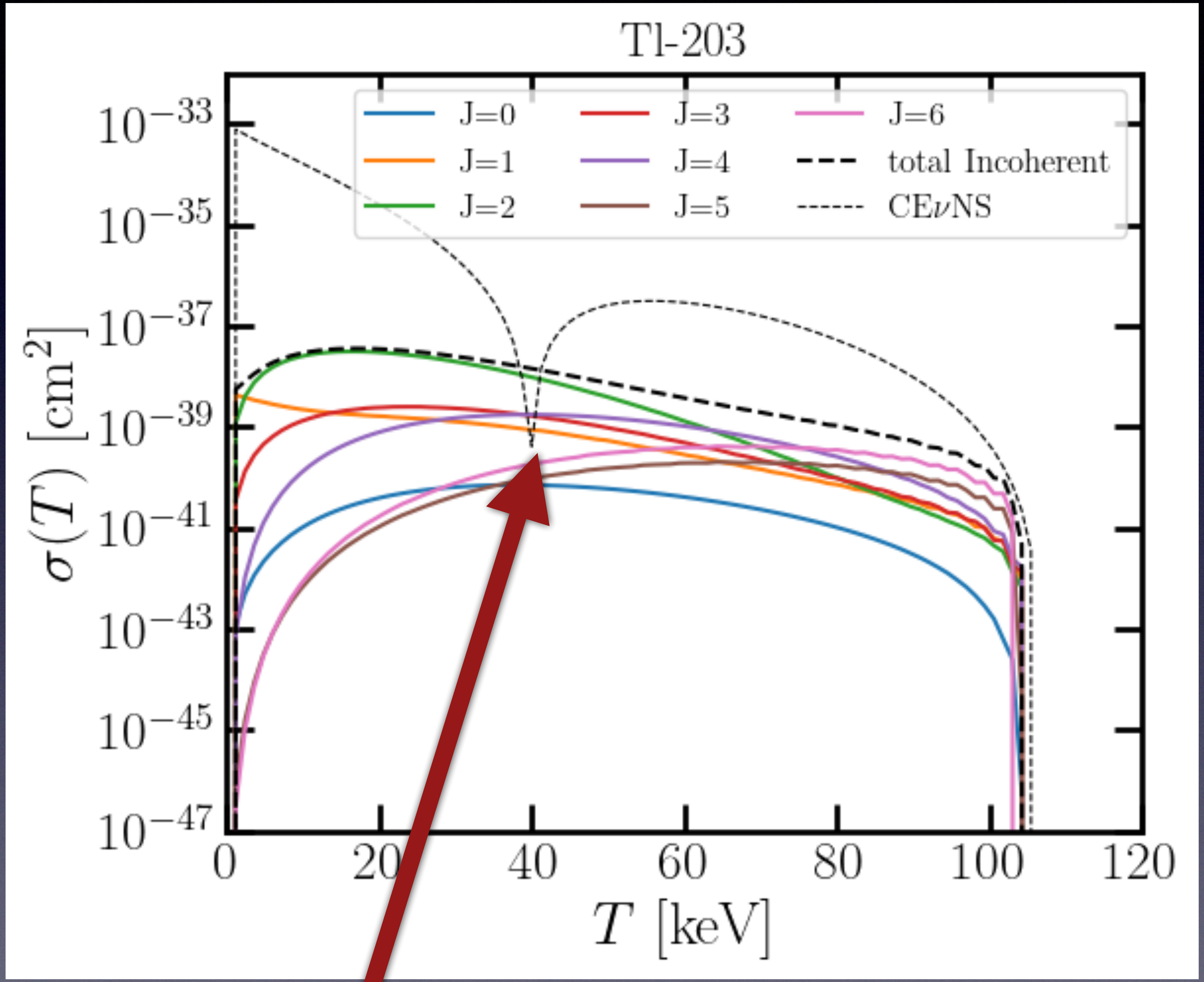
Inelastic vs CEvNS



CEvNS dominates by far

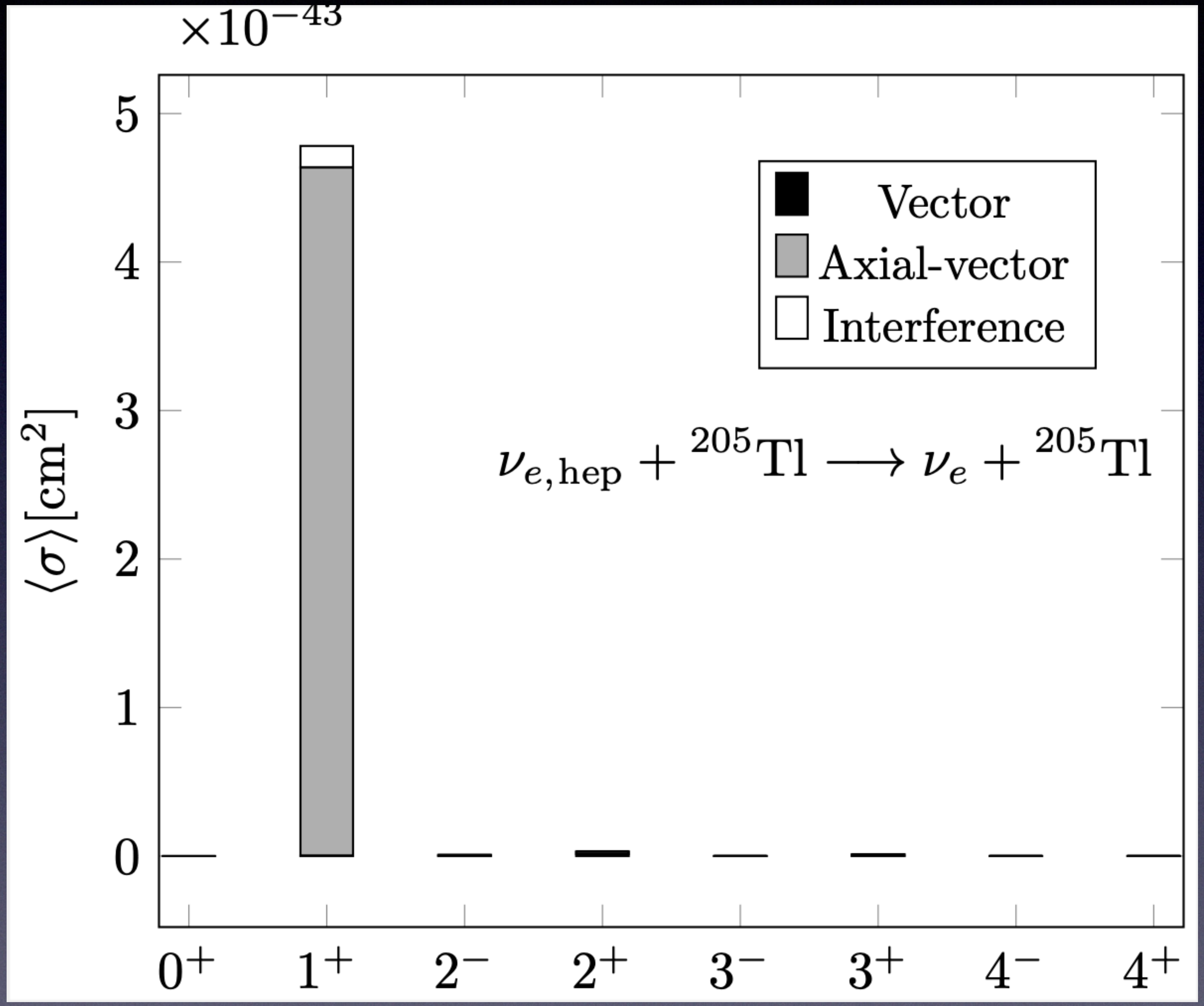
Inelastic vs CEvNS

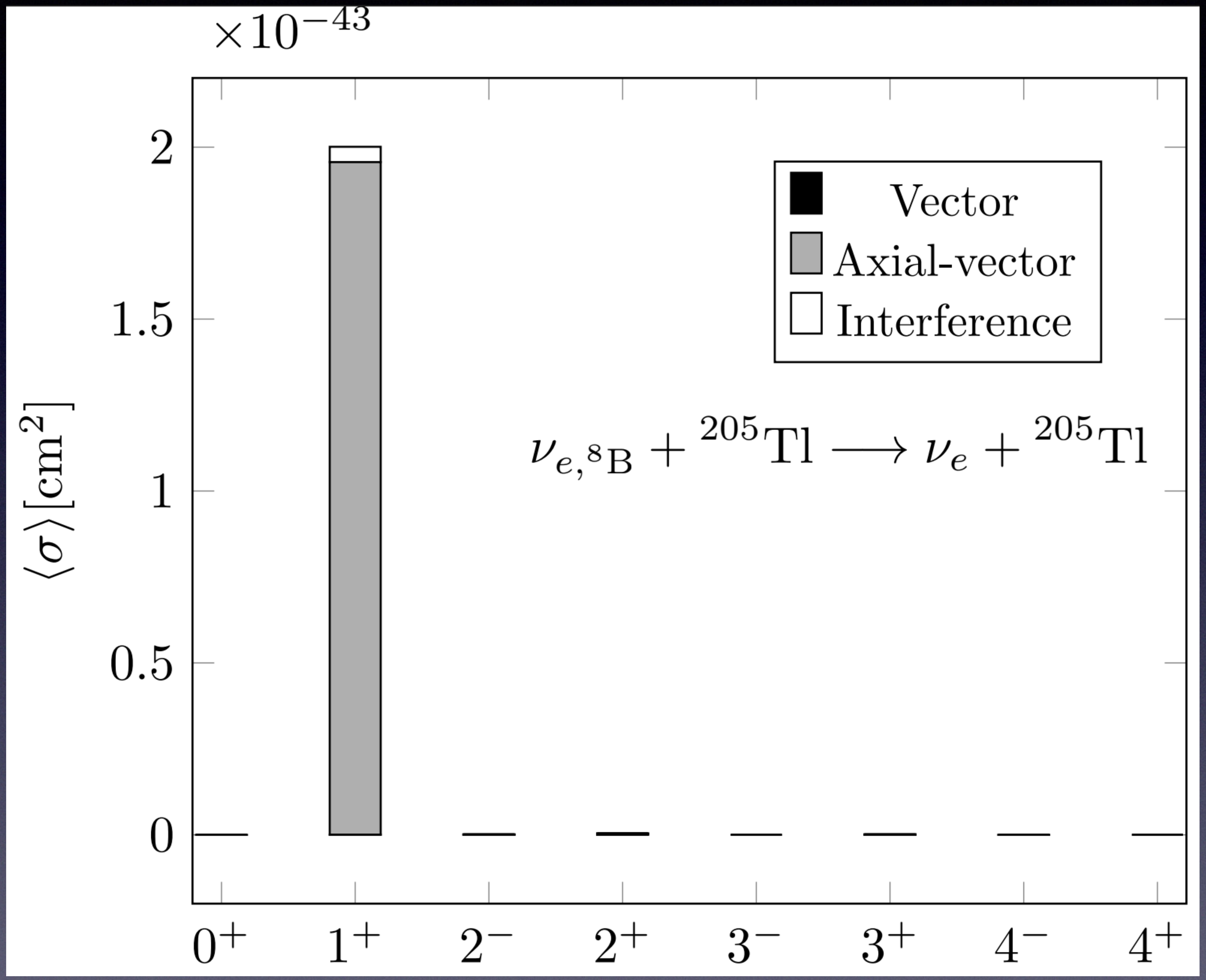




CEvNS and inelastic scattering become comparable

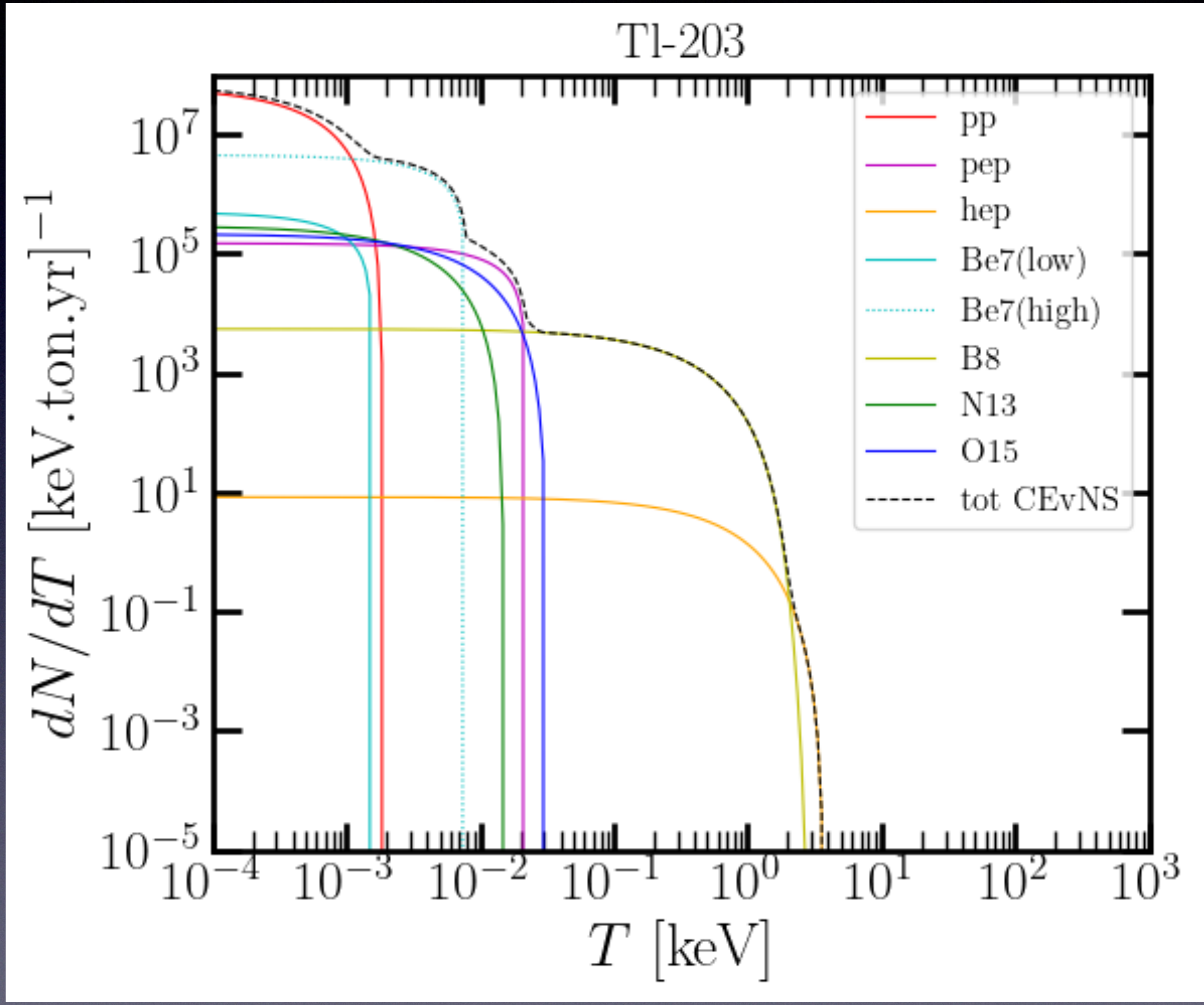
Vector vs axial vector





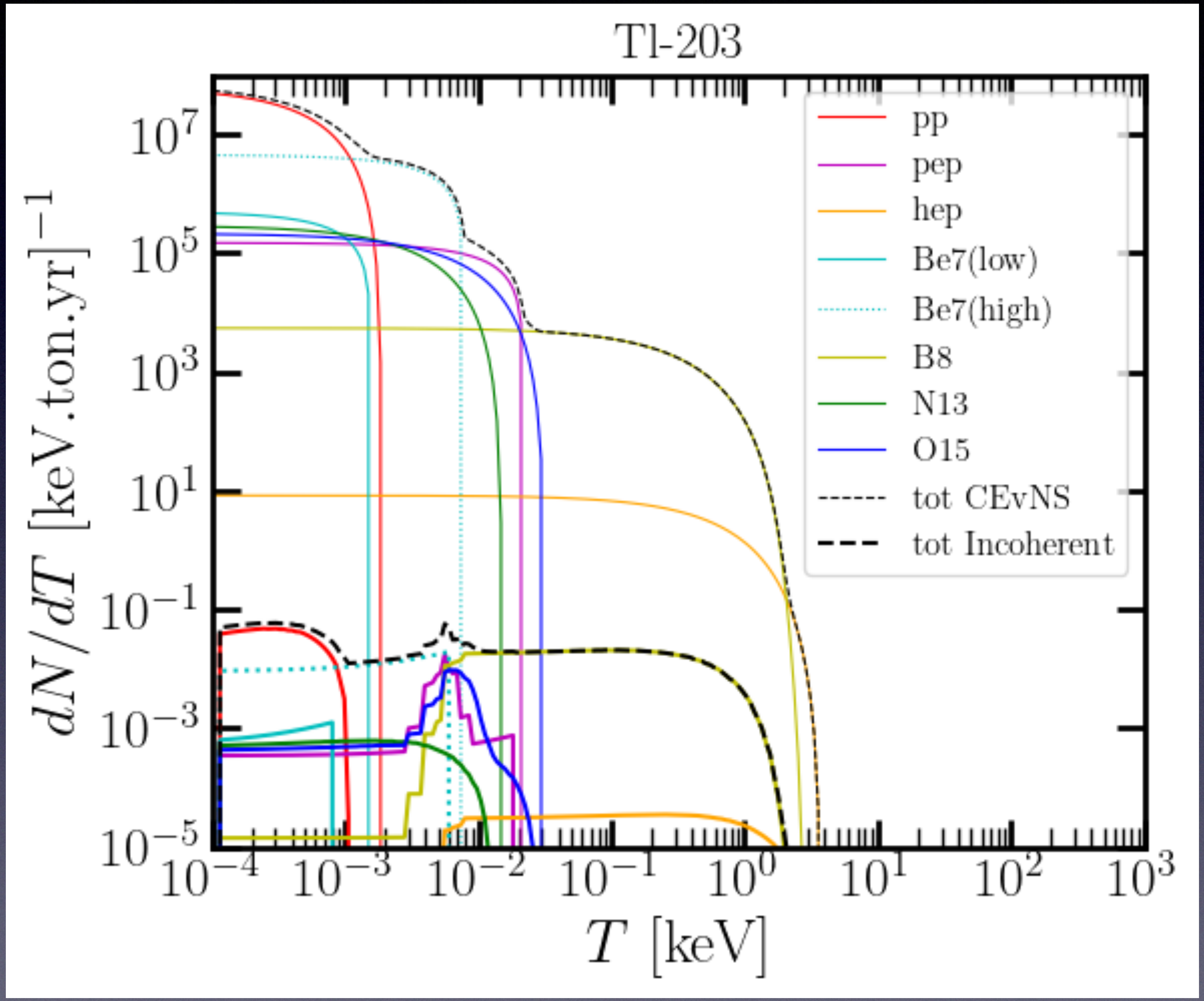
Inelastic cross section dominated
by axial vector contribution and $J = 1$ transitions

Solar neutrino-nucleus scattering spectra



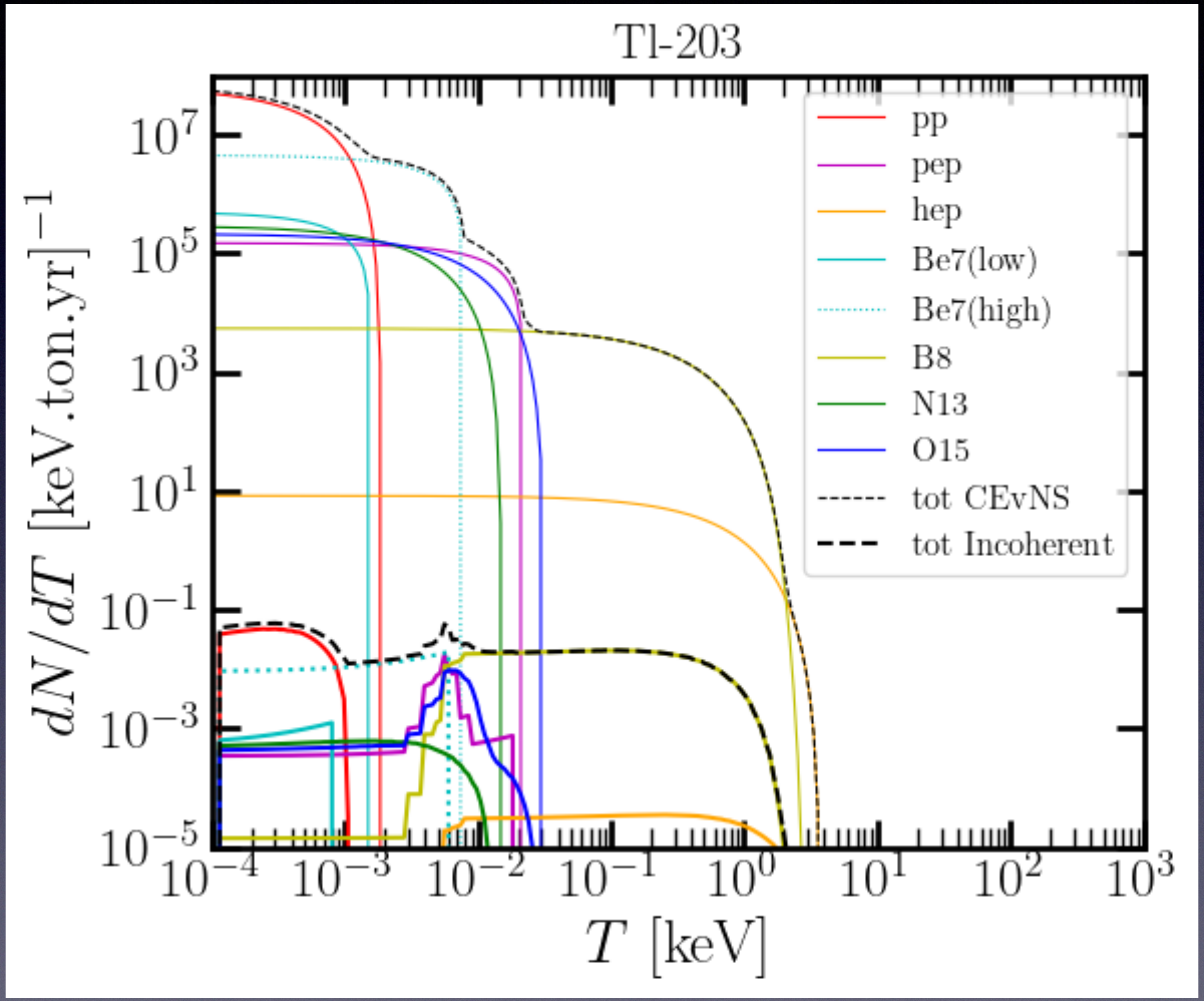
Usual CEvNS spectra

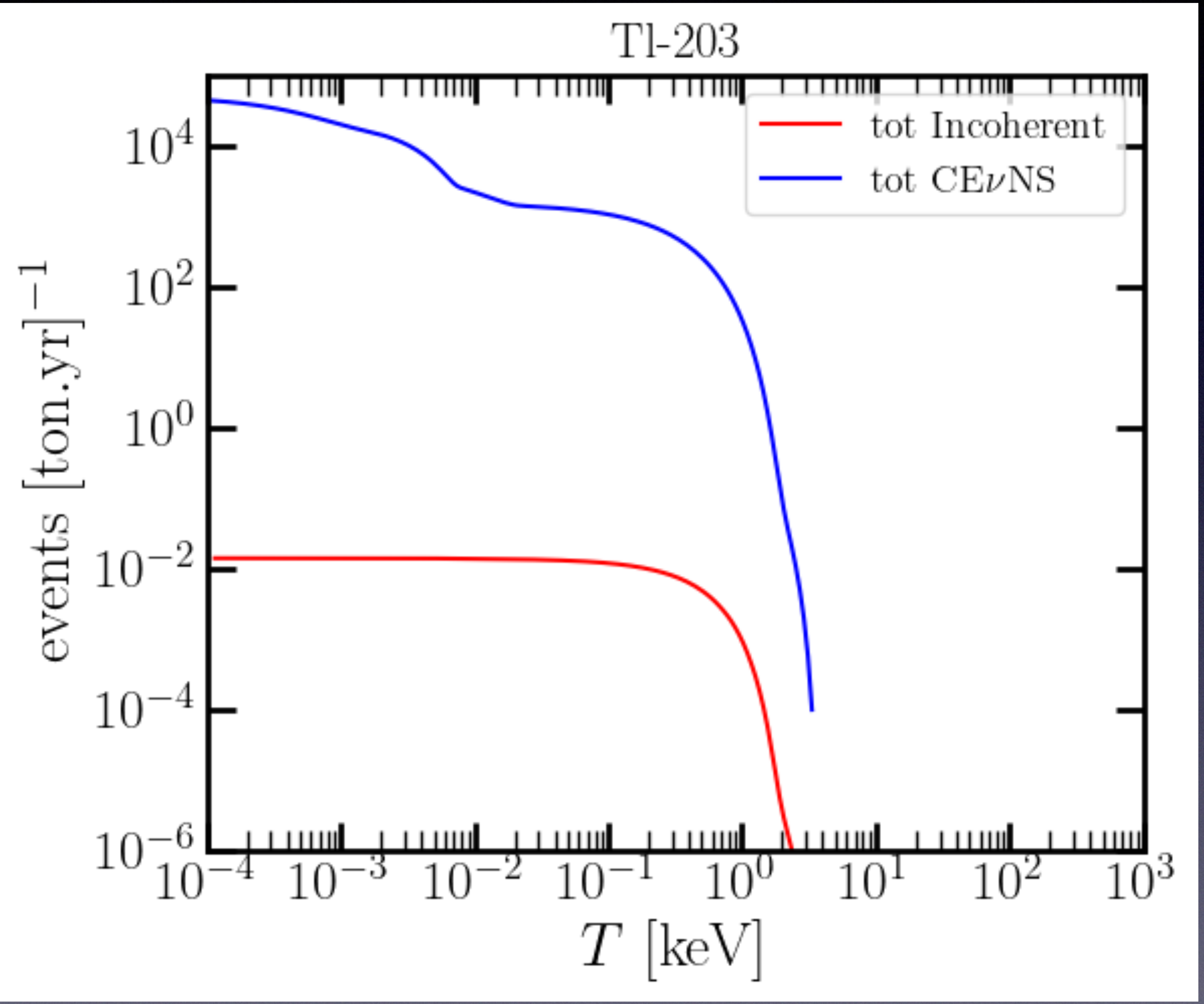
Solar neutrino-nucleus scattering spectra



CEvNS vs Inelastic spectra

Solar neutrino-nucleus scattering spectra





CEvNS vs Inelastic spectra

Summary

- Overview of Donnelly Walecka spherical decomposition method
- Shell Model calculations for evaluating the nuclear matrix elements
- Formalism in terms of recoil energy
- Inelastic solar neutrino scattering of stable Thallium isotopes
- Compared to CEvNS, inelastic contributions are found to be rather suppressed (this might be different for other nuclei)

Thank you for your attention

H.F.R.I.
Hellenic Foundation for Research & Innovation

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(Project Number: 7036)

Back up slides

Multipole expansion of the hadronic current

- At low and intermediate energies, any semi-leptonic process is described by an effective interaction Hamiltonian, written in terms of the leptonic $\hat{j}_\mu^{lept}$ and hadronic $\hat{\mathcal{J}}^\mu$ currents as

$$\hat{H}_{eff} = \frac{G}{\sqrt{2}} \int d^3\mathbf{x}\, \hat{j}_\mu^{lept}(\mathbf{x}) \hat{\mathcal{J}}^\mu(\mathbf{x}),$$

- Leptonic current ME, between an initial $|\ell_i\rangle$ and a final state $|\ell_f\rangle$

$$\langle \ell_f | \hat{j}_\mu^{\mathrm{lept}} | \ell_i \rangle = \ell_\mu \, e^{-i\mathbf{q}\cdot\mathbf{x}} .$$

- Define a complete orthonormal set of spatial unit vectors: $\mathbf{l} = \sum_{\lambda=0,\pm1} l_\lambda \mathbf{e}_\lambda^\dagger$
- Expand the plane wave as:

$$e^{i\mathbf{q}\cdot\mathbf{x}} = \sum_l i^l \sqrt{4\pi(2l+1)} j_l(\rho) Y_{l0}(\Omega_x), \qquad \rho = \kappa|\mathbf{x}|, \kappa = |\mathbf{q}|$$

- The Clebsch-Gordan coefficient limits the sum on l to three terms, $l = J$ and $J \pm 1$. Evaluating for $\lambda = \pm1$, one finds

$$\mathbf{e}_{\mathbf{q}\lambda} e^{i\mathbf{q}\cdot\mathbf{x}} = -\sum_{J\geq1}^{\infty} \sqrt{2\pi(2J+1)} i^J \left\{ \lambda j_J(\rho) \mathbf{Y}_{JJ1}^\lambda + \frac{1}{\kappa} \boldsymbol{\nabla} \times \left[j_J(\rho) \mathbf{Y}_{JJ1}^\lambda \right] \right\},$$

and for $\lambda = 0$

$$\mathbf{e}_{\mathbf{q}0} e^{i\mathbf{q}\cdot\mathbf{x}} = \frac{-i}{\kappa} \sum_{J\geq0}^{\infty} \sqrt{4\pi(2J+1)} i^J \boldsymbol{\nabla} \left[j_J(\rho) Y_{J0} \right] .$$

T.W. Donnelly and J.D.Walecka, Nucl. Phys. **A 274** (1976) 368

Tensor operators

- Substituting one finds

$$\langle f|\hat{H}_{eff}|i\rangle = -\frac{G}{\sqrt{2}}\langle f|\Bigg\{\sum_{J\geq 0}\sqrt{4\pi(2J+1)}(-i)^J\Big(l_3\hat{\mathcal{L}}_{J0}(\kappa) - l_0\hat{\mathcal{M}}_{J0}(\kappa)\Big)$$

$$+\sum_{\lambda=\pm 1}\sum_{J\geq 1}\sqrt{2\pi(2J+1)}(-i)^J l_\lambda\Big(\lambda\hat{\mathcal{T}}^{mag}_{J-\lambda}(\kappa) + \hat{\mathcal{T}}^{el}_{J-\lambda}(\kappa)\Big)\Bigg\}|i\rangle\,.$$

- The multipole expansion procedure gives 8 independent irreducible tensor multipole operators, acting on the nuclear Hilbert space and having rank J
- four operators are defined for the polar vector component $\hat{J}_\lambda = (\hat{\rho}, \hat{\mathbf{J}})$ and
- four for the the axial vector component $\hat{J}^5_\lambda = (\hat{\rho}^5, \hat{\mathbf{J}}^5)$ of the hadronic current

$$\begin{aligned}
\hat{\mathcal{M}}_{JM}(\kappa) &= \hat{M}^{coul}_{JM} - \hat{M}^{coul5}_{JM} = \int d\mathbf{r}\, M^J_M(\kappa\mathbf{r})\hat{\mathcal{J}}_0(\mathbf{r}),\\
\hat{\mathcal{L}}_{JM}(\kappa) &= \hat{L}_{JM} - \hat{L}^5_{JM} = i\int d\mathbf{r}\left(\frac{1}{\kappa}\boldsymbol{\nabla} M^J_M(\kappa\mathbf{r})\right)\cdot\hat{\mathcal{J}}(\mathbf{r}),\\
\hat{\mathcal{T}}^{el}_{JM}(\kappa) &= \hat{T}^{el}_{JM} - \hat{T}^{el5}_{JM} = \int d\mathbf{r}\left(\frac{1}{q}\boldsymbol{\nabla}\times\mathbf{M}^{JJ}_M(\kappa\mathbf{r})\right)\cdot\hat{\mathcal{J}}(\mathbf{r}),\\
\hat{\mathcal{T}}^{mag}_{JM}(\kappa) &= \hat{T}^{mag}_{JM} - \hat{T}^{mag5}_{JM} = \int d\mathbf{r}\mathbf{M}^{JJ}_M(\kappa\mathbf{r})\cdot\hat{\mathcal{J}}(\mathbf{r})\,,
\end{aligned}$$

the V-A structure of the weak interaction is adopted: $\hat{\mathcal{J}}_\mu = \hat{J}_\mu - \hat{J}^5_\mu = (\hat{\rho}, \hat{\mathbf{J}}) - (\hat{\rho}^5, \hat{\mathbf{J}}^5)\,.$

Required nuclear matrix elements

We proceed by defining

$$
\begin{aligned}
\hat{\mathcal{M}}_{JM}(\kappa r) &= \hat{M}^{coul}_{JM} + \hat{M}^{coul5}_{JM} \\
&= F_1^V M_M^J(\kappa r) - i\frac{\kappa}{M_N}[F_A \Omega_M^J(\kappa r) + \frac{1}{2}(F_A + q_0 F_P)\Sigma_M^{''J}(\kappa r)]\,, \\
\hat{\mathcal{L}}_{JM}(\kappa r) &= \hat{L}_{JM} + \hat{L}^5_{JM} \\
&= \frac{q_0}{\kappa} F_1^V M_M^J(\kappa r) + iF_A \Sigma_M^{''J}(\kappa r)]\,, \\
\hat{\mathcal{T}}^{el}_{JM}(\kappa r) &= \hat{T}^{el}_{JM} + \hat{T}^{el5}_{JM} \\
&= \frac{\kappa}{M_N}[F_1^V \Delta_M^{'J}(\kappa r) + \frac{1}{2}\mu^V \Sigma_M^J(\kappa r)] + iF_A \Sigma_M^{'J}(\kappa r)]\,, \\
\hat{\mathcal{T}}^{mag}_{JM}(\kappa r) &= \hat{T}^{mag}_{JM} + \hat{T}^{magn5}_{JM} \\
&= -\frac{q}{M_N}[F_1^V \Delta_M^J(\kappa r) - \frac{1}{2}\mu^V \Sigma_M^{'J}(\kappa r)] + iF_A \Sigma_M^J(\kappa r)]\,,
\end{aligned}
$$

with $F_X(Q^2)$, X=1,A,P and $\mu^V(Q^2)$ being the free nucleon form factors

- CVC Theory: only seven operators are linearly independent
- Polar-vector: Coulomb M^{coul}_{JM}, longitudinal L_{JM}, transverse electric T^{el}_{JM} [with normal parity $\pi = (-)^J$] and transverse magnetic T^{mag}_{JM} [with abnormal parity $\pi = (-)^{J+1}$].
- Axial-vector: M^{coul5}_{JM}, L^5_{JM}, T^{el5}_{JM} (with abnormal parity) and T^{mag5}_{JM} (with normal parity).

J. D. Walecka, Theoretical Nuclear And Subnuclear Physics, World Scientific, Imperial College Press

Multipole decomposition

- Interaction Hamiltonian for neutral-current (NC) neutrino-nucleus scattering

$$\langle f|\hat{H}_{eff}|i\rangle = \frac{G_F}{\sqrt{2}} \int d^3\mathbf{x}\, \langle \ell_f | \hat{j}^{lept}_{\mu}(\mathbf{x})|\ell_i\rangle \langle J_f | \hat{\mathcal{J}}^{\mu}(\mathbf{x})|J_i\rangle$$

with $\langle \ell_f|\hat{j}^{lept}_{\mu}|\ell_i\rangle = \bar{\nu}_\alpha \gamma_\mu (1-\gamma_5)\nu_\alpha\, e^{-i\mathbf{q}\cdot\mathbf{x}}$, $\quad \mathbf{q}: 3-\text{momentum transfer}$

- In the Donnelly-Walecka multipole decomposition method, the NC, double diff. SM cross section from an initial $|J_i\rangle$ to a final $|J_f\rangle$ nuclear state (constructed explicitly through QRPA realistic nuclear structure calculations), reads

$$\frac{d^2\sigma_{i\to f}}{d\Omega\, d\omega} = \frac{G_F^2}{\pi} \frac{\varepsilon_i \varepsilon_f}{(2J_i+1)} \left(\sum_{J=0}^{\infty} \sigma^J_{\mathrm{CL}} + \sum_{J=1}^{\infty} \sigma^J_{\mathrm{T}} \right),$$

ε_i (ε_f) is the initial (final) neutrino energy and ω is the nucleus excitation energy.

- Contributions to σ^J_{CL} (Coulomb-longitudinal) and σ^J_{T} (transverse electric-magnetic) components T. W. Donnelly and R. D. Peccei, Phys. Rept. **50** (1979) 1

$$\sigma^J_{\mathrm{CL}} = (1 + a\cos\theta)|\langle J_f||\hat{\mathcal{M}}_J(\kappa)||J_i\rangle|^2 + (1 + a\cos\theta - 2b\sin^2\theta)|\langle J_f||\hat{\mathcal{L}}_J(\kappa)||J_i\rangle|^2$$
$$+ \left[\frac{\omega}{\kappa}(1 + a\cos\theta) + d\right] 2\Re e|\langle J_f||\hat{\mathcal{L}}_J(\kappa)||J_i\rangle||\langle J_f||\hat{\mathcal{M}}_J(\kappa)||J_i\rangle|^*,$$

$$\sigma^J_{\mathrm{T}} = (1 - a\cos\theta + b\sin^2\theta)\left[|\langle J_f||\hat{\mathcal{T}}^{mag}_J(\kappa)||J_i\rangle|^2 + |\langle J_f||\hat{\mathcal{T}}^{el}_J(\kappa)||J_i\rangle|^2\right]$$
$$\mp \left[\frac{(\varepsilon_i+\varepsilon_f)}{\kappa}(1 - a\cos\theta) - d\right] 2\Re e|\langle J_f||\hat{\mathcal{T}}^{mag}_J(\kappa)||J_i\rangle||\langle J_f||\hat{\mathcal{T}}^{el}_J(\kappa)||J_i\rangle|^*$$

where the parameters $a = 1$, $b = \varepsilon_i\varepsilon_f/\kappa^2$, $d = 0$ are obtained from the kinematics and $\kappa = |\mathbf{q}|$

The seven basic nuclear operators

- Seven new operators are defined (proton-neutron representation) as

$$
\begin{aligned}
T_1^{JM} &\equiv M_M^J(\kappa \mathbf{r}) = \delta_{LJ}\, j_L(\kappa r) Y_M^L(\hat{r}), \\
T_2^{JM} &\equiv \Sigma_M^J(\kappa \mathbf{r}) = \mathbf{M}_M^{JJ} \cdot \boldsymbol{\sigma}, \\
T_3^{JM} &\equiv \Sigma'^J_M(\kappa \mathbf{r}) = -i\left[\frac{1}{\kappa}\boldsymbol{\nabla} \times \mathbf{M}_M^{JJ}(\kappa \mathbf{r})\right] \cdot \boldsymbol{\sigma}, \\
T_4^{JM} &\equiv \Sigma''^J_M(\kappa \mathbf{r}) = \left[\frac{1}{\kappa}\boldsymbol{\nabla} M_M^J(\kappa \mathbf{r})\right] \cdot \boldsymbol{\sigma}, \\
T_5^{JM} &\equiv \Delta_M^J(\kappa \mathbf{r}) = \mathbf{M}_M^{JJ}(\kappa \mathbf{r}) \cdot \frac{1}{\kappa}\boldsymbol{\nabla}, \\
T_6^{JM} &\equiv \Delta'^J_M(\kappa \mathbf{r}) = -i\left[\frac{1}{\kappa}\boldsymbol{\nabla} \times \mathbf{M}_M^{JJ}(\kappa \mathbf{r})\right] \cdot \frac{1}{\kappa}\boldsymbol{\nabla}, \\
T_7^{JM} &\equiv \Omega_M^J(\kappa \mathbf{r}) = M_M^J(\kappa \mathbf{r})\boldsymbol{\sigma} \cdot \frac{1}{\kappa}\boldsymbol{\nabla}.
\end{aligned}
$$

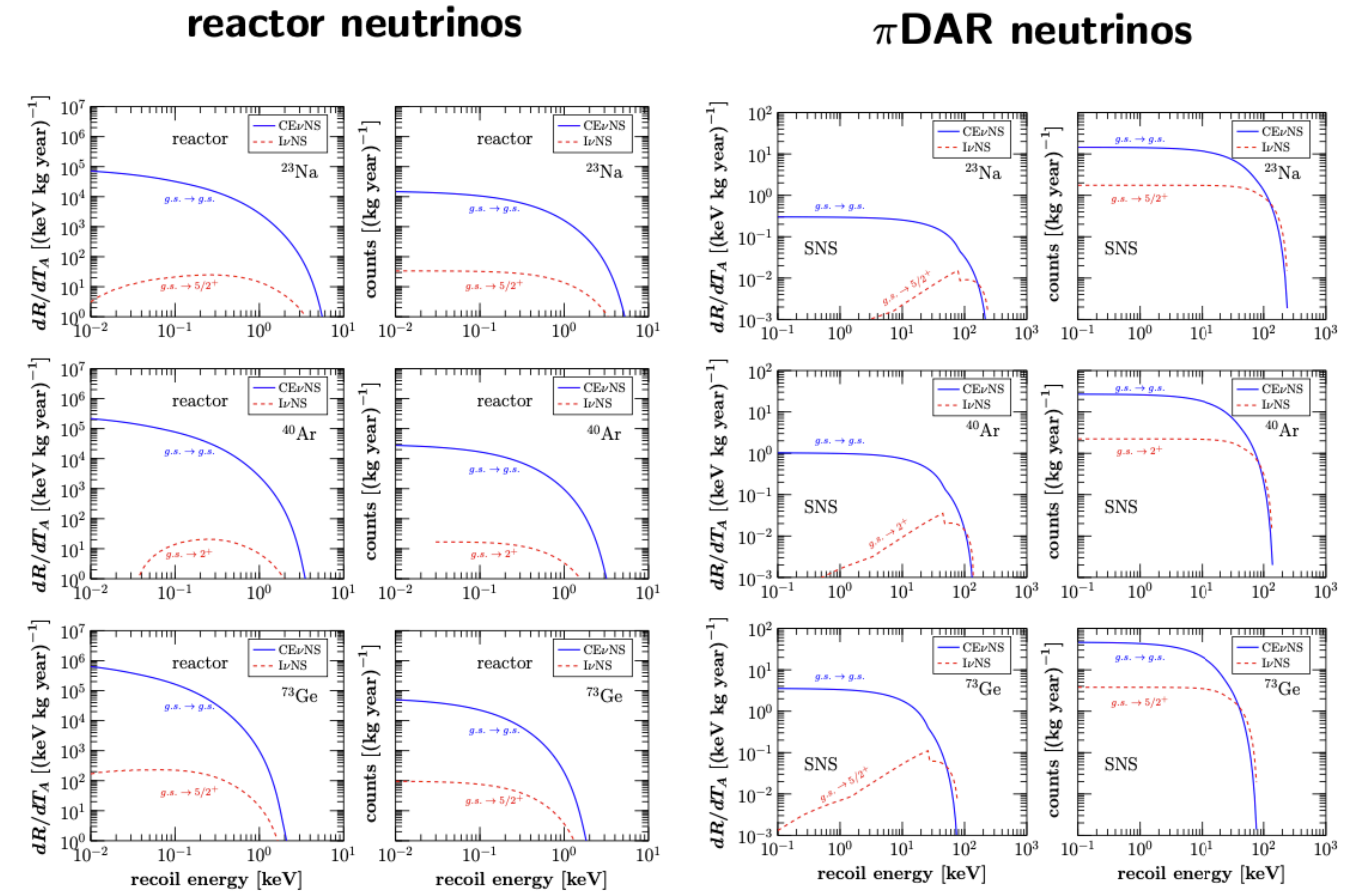
Closed compact analytic formulae for the single-particle reduced ME (upper) and many-body reduced ME (lower) for QRPA calculations, are deduced.

$$
\langle (n_1\ell_1)j_1 || T_i^J || (n_2\ell_2)j_2 \rangle = e^{-y} y^{\beta/2} \sum_{\mu=0}^{n_{max}} \mathcal{P}_\mu^{i,\,J} y^\mu, \quad y = (\kappa b/2)^2, \quad n_{max} = (N_1 + N_2 - \beta)/2, \quad N_i = 2n_i + \ell_i
$$

$$
\langle f \| \widehat{T}^J \| 0_{gs}^+ \rangle = \sum_{j_2 \geq j_1} \frac{\langle j_2 \| \widehat{T}^J \| j_1 \rangle}{\hat{J}} \left[X_{j_2 j_1} u_{j_2}^{p(n)} v_{j_1}^{p(n)} + Y_{j_2 j_1} v_{j_2}^{p(n)} u_{j_1}^{p(n)} \right]
$$

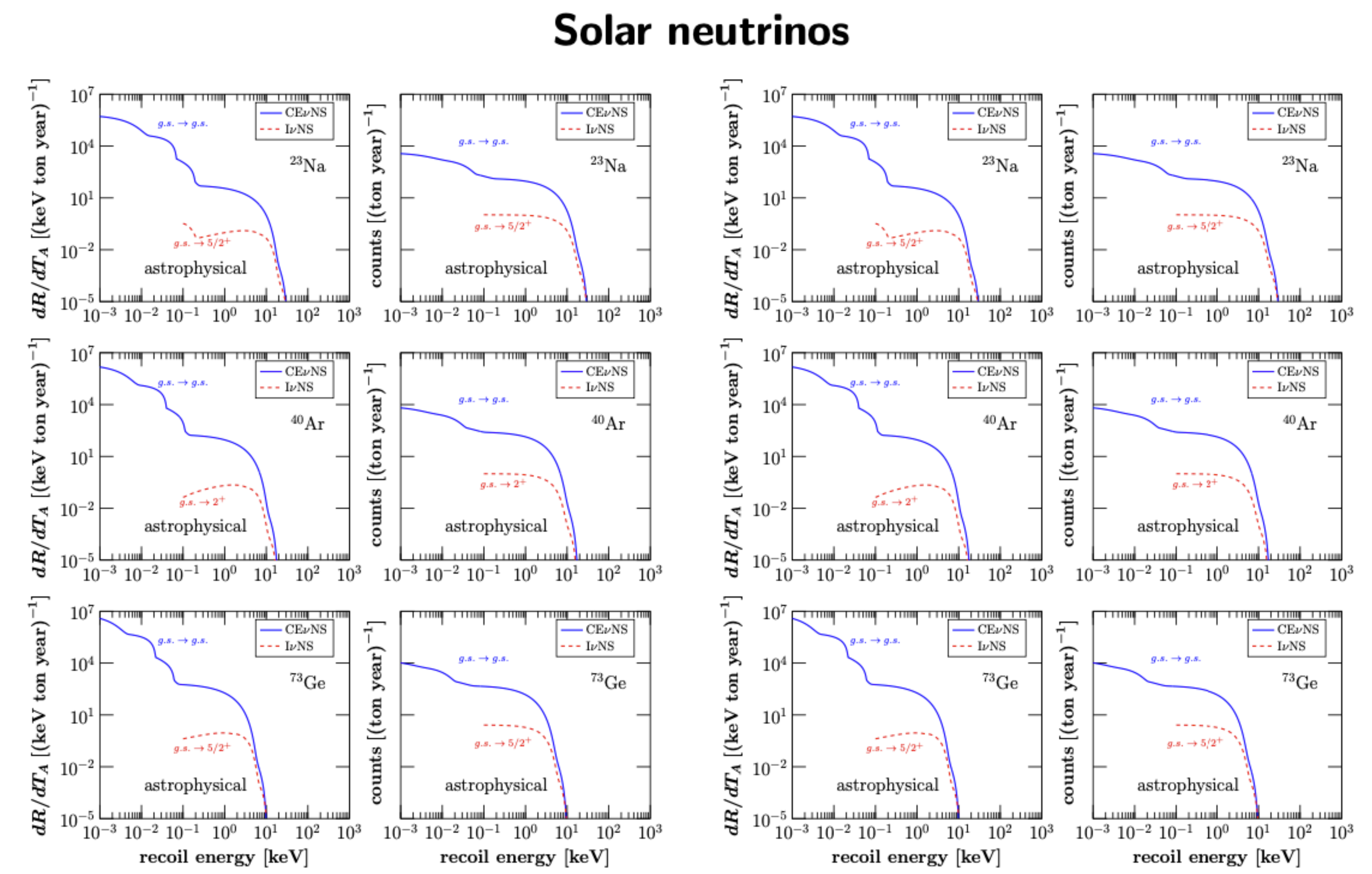
V.Ch. Chasioti and T.S. Kosmas, Nucl. Phys. **A 829** (2009) 234
P.G. Giannaka, D.K. Papoulias, T.S. Kosmas, unpublished (for any configuration $(j_1, j_2)J$)

Coherent vs incoherent rates



[Sahu, DKP, Kota, Kosmas, PRC 102 (2020) 3, 035501]

Coherent vs incoherent rates



[Sahu, DKP, Kota, Kosmas, PRC 102 (2020) 3, 035501]