Nuclear structure theory for CEνNS

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Magnificent CEνNS 2024 workshop

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ICCUB

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Nuclear matrix elements for new-physics searches

Neutrinos, dark matter studied in experiments using nuclei

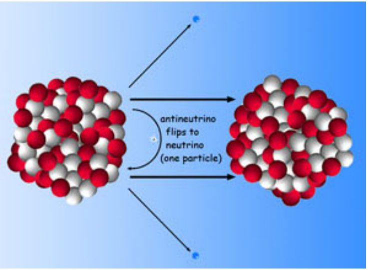
Nuclear structure physics
encoded in nuclear matrix elements
key to plan, fully exploit experiments

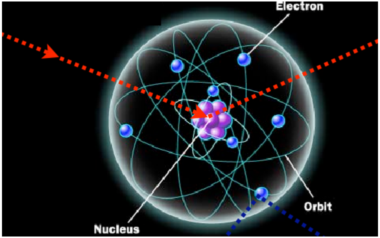
$$0\nu\beta\beta: \left(T_{1/2}^{0\nu\beta\beta}\right)^{-1} \propto g_A^4 \left|M^{0\nu\beta\beta}\right|^2 m_{\beta\beta}^2$$

$$\text{Dark matter: } \frac{d\sigma_{\chi\mathcal{N}}}{d\boldsymbol{q}^2} \propto \left|\sum_i c_i\,\zeta_i\,\mathcal{F}_i\right|^2$$

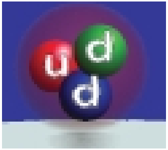
$$\text{CE}\nu\text{NS: } \frac{d\sigma_{\nu\mathcal{N}}}{d\boldsymbol{q}^2} \propto \left|\sum_i c_i\,\zeta_i\,\mathcal{F}_i\right|^2$$

$M^{0\nu\beta\beta}$: Nuclear matrix element
$\mathcal{F}_i$: Nuclear structure factor



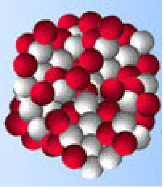


Particle, hadronic and nuclear physics



ν scattering off nuclei
interplay of particle, hadronic and nuclear physics:

ν's: interaction with quarks and gluons
Quarks and gluons: embedded in the nucleon
Nucleons: form complex, many-nucleon nuclei



General ν-nucleus scattering cross-section:

$$\frac{d\sigma_{\nu\mathcal{N}}}{d\boldsymbol{q}^2} \propto \left| \sum_i c_i \, \zeta_i \, \mathcal{F}_i \right|^2$$

ζ: kinematics $(q^2, \cdots)$

c coefficients:
ν couplings to quark, gluons (Wilson coefficients), particle physics
convoluted with hadronic matrix elements, hadronic physics

$\mathcal{F}$ functions: $\mathcal{F}^2 \sim$ structure factor, nuclear structure physics

Couple mostly to neutrons: $\sigma \propto F_w^2 \propto N^2$: not very well known in nuclei

Neutral current ν scattering off nuclei

ν-nucleus scattering detailed cross-section:

$$\frac{d\sigma_A}{dT} = \frac{G_F^2 m_A}{4\pi}\left(1 - \frac{m_A T}{2E_\nu^2} - \frac{T}{E_\nu}\right) Q_w^2 \left|F_w(\mathbf{q}^2)\right|^2 + \frac{G_F^2 m_A}{4\pi}\left(1 + \frac{m_A T}{2E_\nu^2} - \frac{T}{E_\nu}\right) F_A(\mathbf{q}^2)$$

Dominated by the first term, proportional to the weak form factor:

$$\begin{aligned} F_w(\mathbf{q}^2) = \frac{1}{Q_w}\Bigg[& \left(Q_w^p\left(1 + \frac{\langle r_E^2\rangle^p}{6} t + \frac{1}{8m_N^2} t\right) + Q_w^n \frac{\langle r_E^2\rangle^n + \langle r_{E,s}^2\rangle^N}{6} t\right)\mathcal{F}_p^M(\mathbf{q}^2) \\ & + \left(Q_w^n\left(1 + \frac{\langle r_E^2\rangle^p + \langle r_{E,s}^2\rangle^N}{6} t + \frac{1}{8m_N^2} t\right) + Q_w^p \frac{\langle r_E^2\rangle^n}{6} t\right)\mathcal{F}_n^M(\mathbf{q}^2) \\ & - \frac{Q_w^p(1+2\kappa^p) + 2Q_w^n(\kappa^n + \kappa_s^N)}{4m_N^2} t\mathcal{F}_p^{\Phi''}(\mathbf{q}^2) \\ & - \frac{Q_w^n(1+2\kappa^p+2\kappa_s^N) + 2Q_w^p\kappa^n}{4m_N^2} t\mathcal{F}_n^{\Phi''}(\mathbf{q}^2)\Bigg], \qquad t = q^2 \end{aligned}$$

which depends on the nuclear responses $\mathcal{F}_p^M$, $\mathcal{F}_n^M$, $\mathcal{F}_n^{\Phi''}$, $\mathcal{F}_p^{\Phi''}$

To a first approximation:

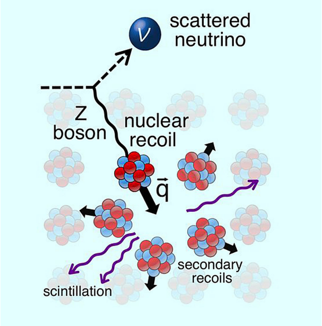
$$R_w \approx R_n$$

Nuclear structure factors

Nuclear matrix elements and nuclear structure factors
needed in low-energy new-physics searches

$$\langle \text{Final} \,|\mathcal{L}_{\text{leptons}-\text{nucleons}}|\, \text{Initial} \,\rangle = \langle \text{Final} \,| \int dx\, j^{\mu}(x) J_{\mu}(x) \,|\, \text{Initial} \,\rangle$$

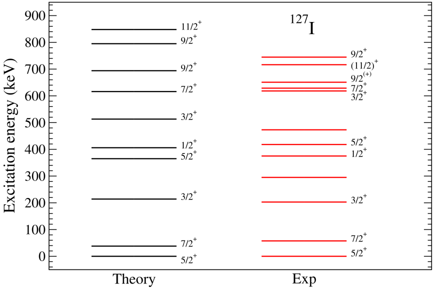
- Nuclear structure calculation of the initial and final states:
  Shell model
  Energy-density functional
  Ab initio many-body theory
  QMC, Coupled-cluster, IMSRG...
- Lepton-nucleus interaction:
  Hadronic current in nucleus:
  phenomenological,
  effective theory of QCD

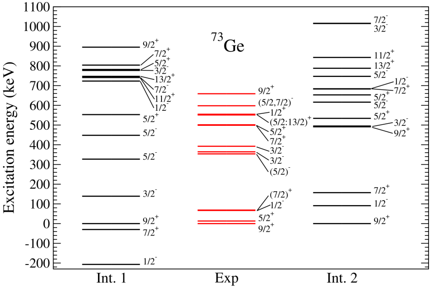


Shell-model spectra for heavy nuclei

Very good general agreement
between the properties of low-energy nuclear states
and nuclear shell-model calculations

However, some nuclei present challenging features
such as ^{73}Ge ground and first-excited state, likely related to deformation



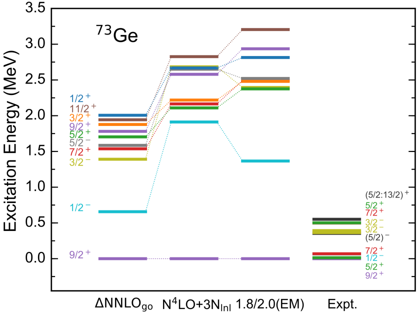


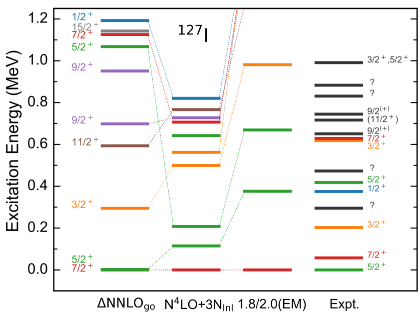
Klos, JM, Gazit, Schwenk, PRD 88, 083516 (2013)

Ab initio spectra for heavy nuclei

While VS-IMSRG calculations high quality in light nuclei (eg Na) challenges remain in heavier systems, such as ^{73}Ge

Interesting sensitivity to the chiral nuclear Hamiltonian used for ^{127}I

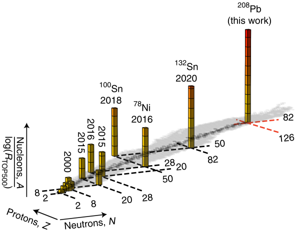




Hu et al. PRL 128, 072502 (2022)

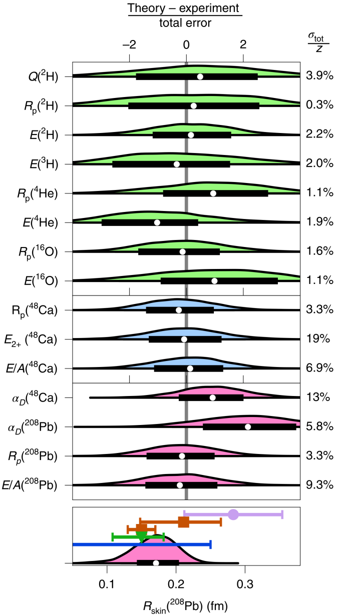
Ab initio predictions for neutron skin

Very notable progress
ab initio calculations of
(relatively uncorrelated) heavy nuclei
reaching ^{208}Pb



Determine ^{208}Pb neutron skin
using Bayesian approach
based on sampling of 10^9
(parameters of) nuclear Hamiltonians
obtained with chiral EFT (rooted in QCD)

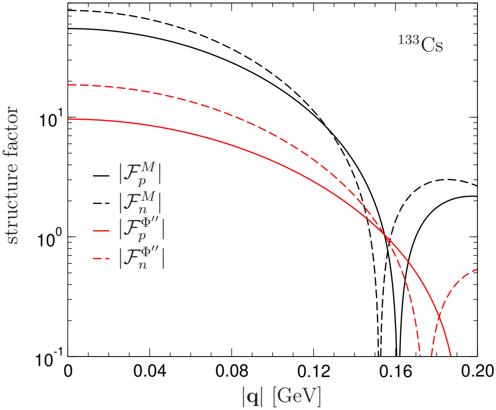
Hu, Jiang, Miyagi et al.
Nature Phys. 18, 1196 (2022)



Shell-model response functions for heavy nuclei

Coherent response functions correspond to M, Φ'' operators

Shell-model calculation as function of momentum transfer $\mathbf{q}$



Hoferichter, JM, Schwenk, PRD102 074018 (2020)

$\mathcal{F}_{\pm}^{M} \longrightarrow 1$

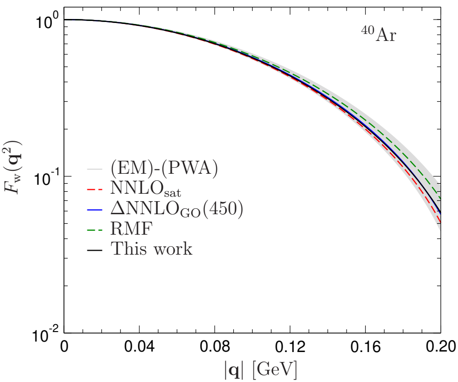
coherent (charge)

$\mathcal{F}_{\pm}^{\Phi''} \longrightarrow \mathbf{S}_N \cdot (\mathbf{q} \times \mathbf{P}) \sim \mathbf{S}_N \cdot \mathbf{l}_N$

semi-coherent
(spin-orbit,
attractive *mean field*
in nuclear potential)

CEνNS off argon

Nuclear structure factors for CEνNS off ^{40}Ar



Ab initio band from Hamiltonians built with no information about charge radii of nuclei

Other calculations include somehow this information in their Hamiltonian/parameters

Hoferichter, JM, Schwenk PRD102 074018 (2020)

Payne et al. PRC100 061304 (2019)

Yang et al. PRC100 054301 (2019)

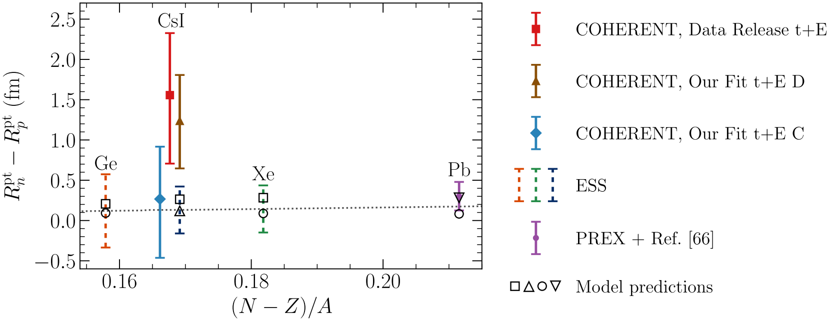
Abdel Khaleq et al. arXiv:2405.20060

Good agreement within uncertainties between calculations nuclear shell model, ab initio coupled cluster and relativistic mean field

Nuclear neutron radius from CEνNS

Use sensitivity to nuclear weak (nucleon) radius
to determine the distribution of neutrons in nuclei

Difficult to obtain from nuclear reactions because of model dependence (reaction theory) in extracting results from experimental data



Coloma, Esteban, JM, Gonzalez-Garcia, JHEP 08, 030 (2020)

It may be difficult to tell apart neutron radii from different nuclear structure calculations with expected sensitivity of ESS measurements

Modified "weak" structure factor with new physics

ν-nucleus scattering can probe new physics as well

With vector and axial currents
different Wilson coefficients than Standard Model ones
lead to a modified "weak" structure factor

$$\frac{d\sigma_A}{dT} = \frac{m_A}{2\pi}\left(1 - \frac{m_A T}{2E_\nu^2} - \frac{T}{E_\nu}\right)\tilde{Q}_\mathrm{w}^2\left|\tilde{F}_\mathrm{w}(\mathbf{q}^2)\right|^2 + \frac{m_A}{2\pi}\left(1 + \frac{m_A T}{2E_\nu^2} - \frac{T}{E_\nu}\right)\tilde{F}_A(\mathbf{q}^2),$$

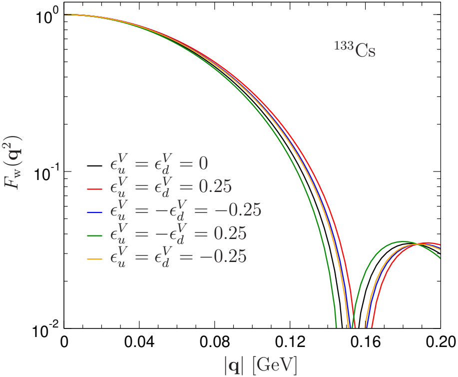
which depends on the same nuclear responses $\mathcal{F}_p^M$, $\mathcal{F}_n^M$, $\mathcal{F}_n^{\Phi''}$, $\mathcal{F}_p^{\Phi''}$
but that is distinguishable from the Standard Model one
because of the different couplings

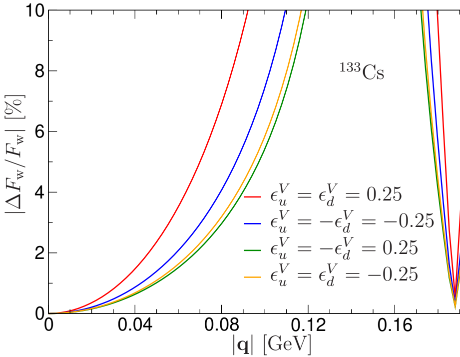
$$\tilde{F}_\mathrm{w}(\mathbf{q}^2) = \frac{1}{\tilde{Q}_\mathrm{w}}\Bigg[\left(g_V^p + \dot{g}_V^p t + \frac{g_V^p + 2g_{V,2}^p}{8m_N^2}t\right)\mathcal{F}_p^M(\mathbf{q}^2) + \left(g_V^n + \dot{g}_V^n t + \frac{g_V^n + 2g_{V,2}^n}{8m_N^2}t\right)\mathcal{F}_n^M(\mathbf{q}^2)$$
$$- \frac{g_V^p + 2g_{V,2}^p}{4m_N^2}t\mathcal{F}_p^{\Phi''}(\mathbf{q}^2) - \frac{g_V^n + 2g_{V,2}^n}{4m_N^2}t\mathcal{F}_n^{\Phi''}(\mathbf{q}^2)\Bigg].$$

Hoferichter, JM, Schwenk, PRD102 074018(2020)

Modified “weak” structure factors

Different Standard Model and Beyond Standard Model structure factors





Relatively small difference for possibly large BSM parameters comparable to nuclear structure uncertainties between calculations unless close to the diffraction minimum

Other BSM couplings (eg tensor) lead to different responses similar to axial-axial SM ones

Axial contribution to CEνNS

Precision studies such as BSM searches require correction to Standard-Model cross-section from non coherent axial-axial interaction

$$\frac{d\sigma_A}{dT} = \frac{G_F^2 m_A}{4\pi}\left(1 - \frac{m_A T}{2E_\nu^2} - \frac{T}{E_\nu}\right) Q_w^2 \left|F_w(\mathbf{q}^2)\right|^2 + \frac{G_F^2 m_A}{4\pi}\left(1 + \frac{m_A T}{2E_\nu^2} - \frac{T}{E_\nu}\right) F_A(\mathbf{q}^2)$$

$$F_A(\mathbf{q}^2) = \frac{8\pi}{2J+1} \times \left(\left(g_A^{s,N}\right)^2 S_{00}^{\mathcal{T}}(\mathbf{q}^2) - g_A g_A^{s,N} S_{01}^{\mathcal{T}}(\mathbf{q}^2) + \left(g_A\right)^2 S_{11}^{\mathcal{T}}(\mathbf{q}^2)\right)$$

$$F_A(0) = \frac{4}{3} g_A^2 \frac{J+1}{J}\left(\langle \mathbf{S}_p \rangle - \langle \mathbf{S}_n \rangle\right)^2,$$

which is transverse and (dominated by) isovector response proportional to expectation value of spin of protons/neutrons in the nucleus

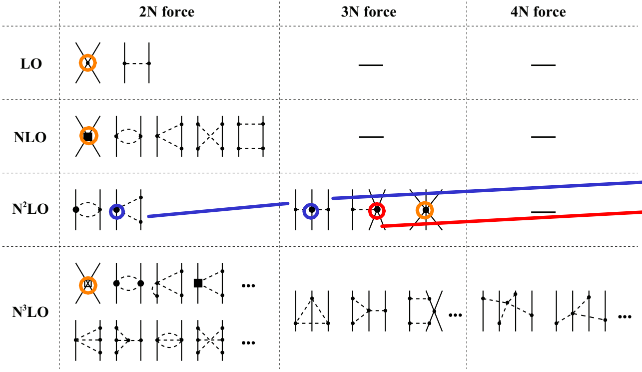
No direct proble of spin distribution of nucleons in nuclei

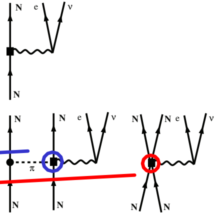
Vanishes for even-even systems

Chiral EFT: low energy approach to QCD, nuclear structure energies

Approximate chiral symmetry: pion exchanges, contact interactions

Systematic expansion: nuclear forces and electroweak currents





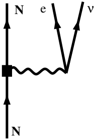
Weinberg, van Kolck, Kaplan, Savage, Wise, Meißner, Epelbaum...

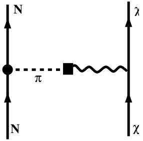
Park, Krebs, Baroni...

Short-range couplings fitted to experiment once

Axial 1b and 2b currents

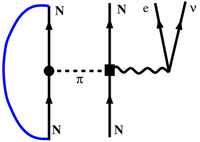
Axial one-body currents complemented with two-body currents

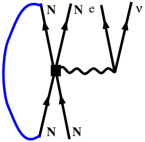




$$\mathbf{J}^3_{i,1\mathrm{b}} = \frac{1}{2}\,\tau_i^3 \left(G_A^3(\mathbf{q}^2)\,\boldsymbol{\sigma}_i - \frac{G_P^3(\mathbf{q}^2)}{4m_N^2}\,(\mathbf{q}\cdot\boldsymbol{\sigma}_i)\,\mathbf{q} \right)$$

Approximate in medium-mass nuclei: normal-ordering wrt Fermi gas





$$\mathbf{J}^{\mathrm{eff}}_{i,2\mathrm{b}}(\rho,\mathbf{q}) = g_A\,\frac{\tau_i^3}{2}\left[\delta a(\mathbf{q}^2)\,\boldsymbol{\sigma}_i + \frac{\delta a^P(\mathbf{q}^2)}{\mathbf{q}^2}(\mathbf{q}\cdot\boldsymbol{\sigma}_i)\mathbf{q}\right]$$

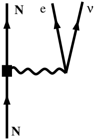
Normal-ordered two-body currents modify structure factor

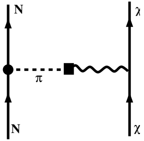
$$S_{11} = S_{11}^{\mathcal{T}} + S_{11}^{\mathcal{L}} = \sum_L \left[[1+\delta'(\mathbf{q}^2)]\mathcal{F}_-^{\Sigma'_L}(\mathbf{q}^2)\right]^2 + \sum_L \left[[1+\delta''(\mathbf{q}^2)]\mathcal{F}_-^{\Sigma''_L}(\mathbf{q}^2)\right]^2$$

$$\delta'(\delta a), \qquad \delta''(\delta a, \delta a^P)$$

Axial 1b and 2b currents

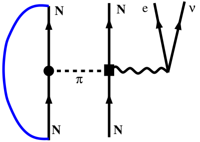
Axial one-body currents complemented with two-body currents

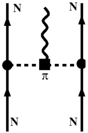




$$\mathbf{J}^3_{i,1\mathrm{b}} = \frac{1}{2}\,\tau^3_i \left(G^3_A(\mathbf{q}^2)\,\boldsymbol{\sigma}_i - \frac{G^3_P(\mathbf{q}^2)}{4m_N^2}\,(\mathbf{q}\cdot\boldsymbol{\sigma}_i)\,\mathbf{q} \right)$$

Approximate in medium-mass nuclei: normal-ordering wrt Fermi gas





$$\mathbf{J}^{\mathrm{eff}}_{i,2\mathrm{b}}(\rho,\mathbf{q}) = g_A\,\frac{\tau^3_i}{2}\left[\delta a(\mathbf{q}^2)\,\boldsymbol{\sigma}_i + \frac{\delta a^P(\mathbf{q}^2)}{\mathbf{q}^2}(\mathbf{q}\cdot\boldsymbol{\sigma}_i)\mathbf{q}\right]$$

Normal-ordered two-body currents modify structure factor

$$S_{11} = S^{\mathcal{T}}_{11} + S^{\mathcal{L}}_{11} = \sum_L \left[[1+\delta'(\mathbf{q}^2)]\mathcal{F}^{\Sigma'_L}_{-}(\mathbf{q}^2) \right]^2 + \sum_L \left[[1+\delta''(\mathbf{q}^2)]\mathcal{F}^{\Sigma''_L}_{-}(\mathbf{q}^2) \right]^2$$

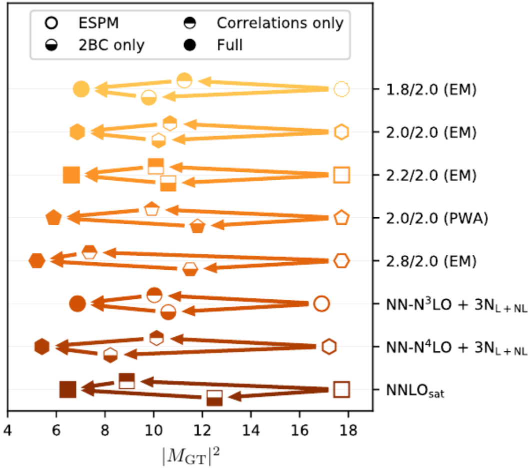
$$\delta'(\delta a), \qquad \delta''(\delta a, \delta a^P)$$

Similar (non-coherent) vector two-body currents!

Miyagi, Brase, Schwenk, JM, in progress

β decay "quenching"

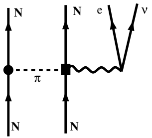
Which are main effects missing in conventional β-decay calculations?
Test case: GT decay of ^{100}Sn



Relatively similar and complementary impact of

- nuclear correlations
- meson-exchange currents

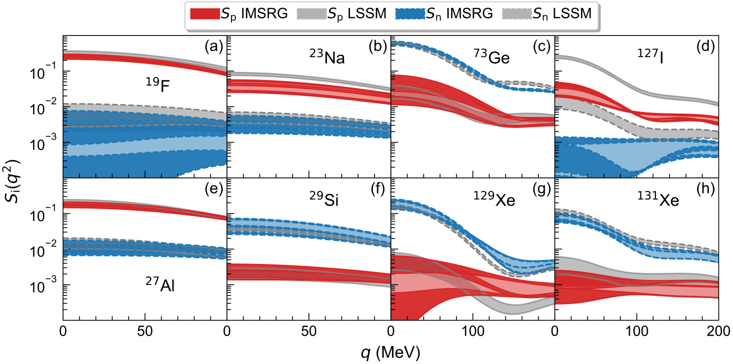
Gysbers et al.
Nature Phys. 15 428 (2019)



Axial-axial neutrino scattering off nuclei

Recent ab initio calculation using VS-IMSRG
(valence-space in-medium similarity renormalization group method)

Consistent with nuclear shell model results
still show larger uncertainties, interesting discrepancy in ^{127}I



Hoferichter, JM, Schwenk, PRD102 074018(2020)

Hu et al. PRL 128 072502 (2022)

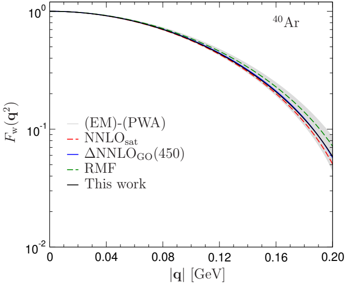
Summary

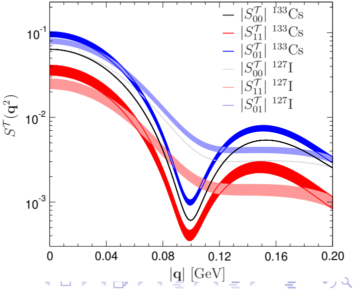
CEνNS cross-section depends on nuclear structure factors than need to be calculated with nuclear structure theory

Ab initio and more phenonomelogical approaches good agreement for dominant coherent structure factor: sensitivity to radius of neutrons in nuclei

BSM searches in general sensitive to different structure factors due to different Wilson coefficients

Precision studies need to take into account axial-axial cross-section as well: 1b+2b currents

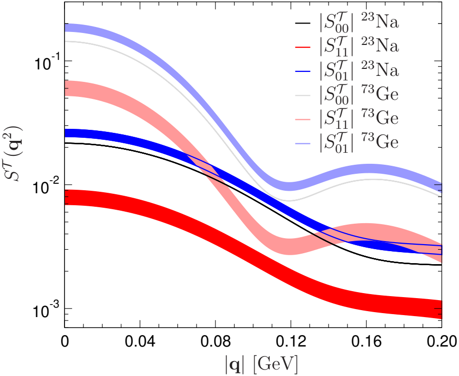


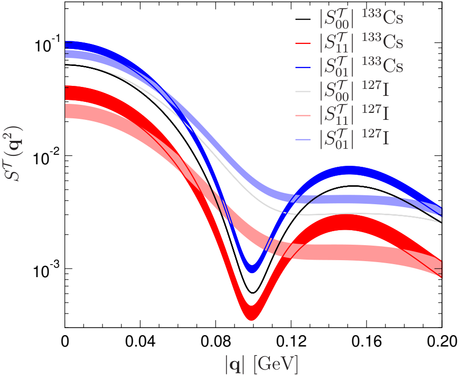


Thank you very much!

Axial-axial neutrino scattering off nuclei

Calculation of nuclear structure factors
for axial-axial elastic ν scattering off CsI, Ar, F, Na, Ge, Xe:

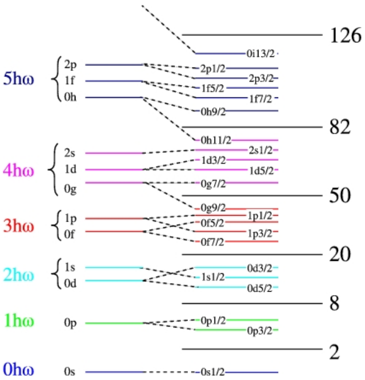




Hoferichter, JM, Schwenk, PRD102 074018(2020)

Uncertainty bands from uncertainty in chiral EFT couplings
needed to describe shell-model quenching

Nuclear shell model



Nuclear shell model configuration space
only keep essential degrees of freedom

- High-energy orbitals: always empty
- Valence space:
  where many-body problem is solved
- Inert core: always filled

$$H\,|\Psi\rangle = E\,|\Psi\rangle \;\rightarrow H_{eff}\,|\Psi\rangle_{eff} = E\,|\Psi\rangle_{eff}$$

$$|\Psi\rangle_{eff} = \sum_{\alpha} c_\alpha\,|\phi_\alpha\rangle\,, \quad |\phi_\alpha\rangle = a^+_{i1} a^+_{i2} ... a^+_{iA}\,|0\rangle$$

Shell model diagonalization:

$\sim 10^{10}$ Slater dets. Caurier et al. RMP77 (2005)

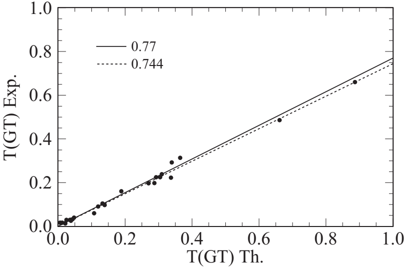
$\gtrsim 10^{24}$ Slater dets. with Monte Carlo SM
Otsuka, Shimizu, Y.Tsunoda
Phys. Scr. 92 063001 (2017)

H_{eff} includes effects of

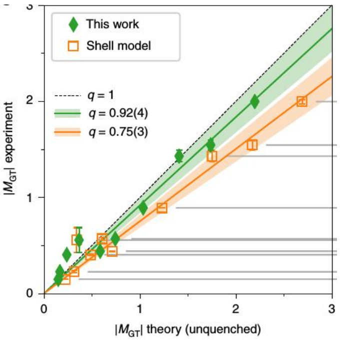
- inert core
- high-energy orbitals

β–decay Gamow-Teller transitions: "quenching"

β decays (e^- capture): phenomenology vs ab initio



Martinez-Pinedo et al. PRC53 2602(1996)



Gysbers et al. Nature Phys. 15 428 (2019)

$$\langle F | \sum_i [g_A\, \boldsymbol{\sigma}_i \tau_i^-]^{\text{eff}} | I \rangle\,, \quad [\boldsymbol{\sigma}_i \tau]^{\text{eff}} \approx 0.7 \boldsymbol{\sigma}_i \tau$$

Standard shell model needs $\boldsymbol{\sigma}_i \tau$ "quenching"

Ab initio calculations including meson-exchange currents and additional nuclear correlations do not need any "quenching"

Ab initio many-body methods

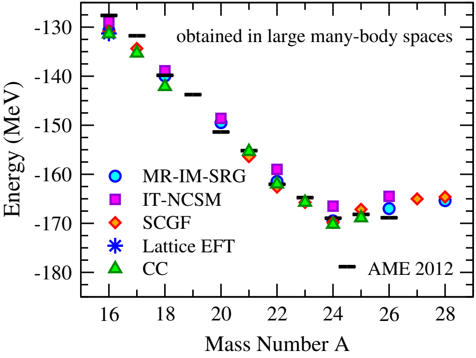
Oxygen dripline using chiral NN+3N forces correctly reproduced
ab-initio calculations treating explicitly all nucleons
excellent agreement between different approaches

No-core shell model
(Importance-truncated)

In-medium SRG
Hergert et al. PRL110 242501(2013)

Self-consistent Green's function
Cipollone et al. PRL111 062501(2013)

Coupled-clusters
Jansen et al. PRL113 142502(2014)



Recent application to ^{208}Pb
Hu, Jiang, Miyagi et al. Nature Phys. 18, 1196 (2022)

Nuclear weak radius

ν-nucleus scattering thus sensitive to weak radii of nuclei
similar to e-nucleus scattering sensitive to charge radii:

$$R_\mathrm{w}^2 = \frac{ZQ_\mathrm{w}^p}{Q_\mathrm{w}}\left(R_p^2 + \langle r_E^2\rangle^p + \frac{Q_\mathrm{w}^n}{Q_\mathrm{w}^p}\left(\langle r_E^2\rangle^n + \langle r_{E,s}^2\rangle^N\right)\right) + \frac{NQ_\mathrm{w}^n}{Q_\mathrm{w}}\left(R_n^2 + \langle r_E^2\rangle^p + \langle r_{E,s}^2\rangle^N + \frac{Q_\mathrm{w}^p}{Q_\mathrm{w}^n}\langle r_E^2\rangle^n\right) + \frac{3}{4m_N^2} + \langle \tilde{r}^2\rangle_\mathrm{so},$$

$$\langle \tilde{r}^2\rangle_\mathrm{so} = -\frac{3Q_\mathrm{w}^p}{2m_N^2 Q_\mathrm{w}}\left(1 + 2\kappa^p + 2\frac{Q_\mathrm{w}^n}{Q_\mathrm{w}^p}(\kappa^n + \kappa_s^N)\right)\mathcal{F}_p^{\Phi''}(0) - \frac{3Q_\mathrm{w}^n}{2m_N^2 Q_\mathrm{w}}\left(1 + 2\kappa^p + 2\kappa_s^N + 2\frac{Q_\mathrm{w}^p}{Q_\mathrm{w}^n}\kappa^n\right)\mathcal{F}_n^{\Phi''}(0).$$

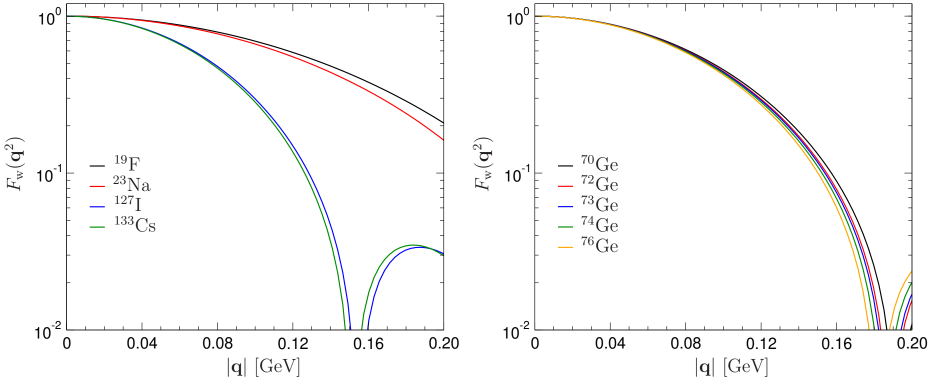
To a first approximaction

$$R_\mathrm{w} \approx R_n,$$

Nuclear weak radius also probed in pariry-violating electron scattering usually measured at a single kinenamical point ($\mathbf{q}^2$ value)

Elastic neutrino scattering off nuclei

Calculation of nuclear structure factors
for coherent elastic ν scattering off CsI, Ar, F, Na, Ge, Xe



Hoferichter, JM, Schwenk, PRD102 074018(2020)

These are similar to structure factors for beyond Standard Model interactions

Also similar to dark matter-nucleus (WIMP-nucleus) structure factors
relativistic ν's instead of nonrelativistic WIMPs

2b currents in $0\nu\beta\beta$ decay

In $0\nu\beta\beta$ decay, two weak currents lead to four-body operator when including the product of two 2b currents: computational challenge

Approximate 2b current as effective 1b current normal ordering with respect to a Fermi gas

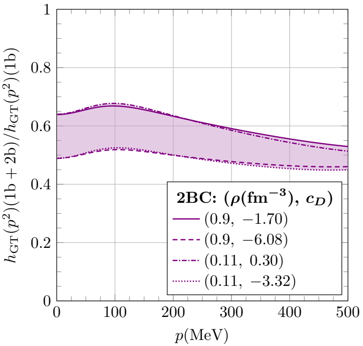
JM, Gazit, Schwenk, PRL107 062501(2011)

Normal-odering approximation works remarkably well for β decay ($q = 0$)

Gysbers et al. Nature Phys. 15 428 (2019)

Some reduction of quenching due to 2b currents at $p \sim m_\pi$ relevant for $0\nu\beta\beta$ decay

Hoferichter, JM, Schwenk PRD102 074018 (2020)



Jokiniemi, Romeo, Soriano, JM, PRC 107 044305 (2023)

Effective shell-model interactions

Coupled Cluster:

Solve coupled-cluster equations for
core (reference state $|\Phi\rangle$), $A+1$ and $A+2$ systems

Project the coupled-cluster solution into valence space
(Okubo-Lee-Suzuki transformation)

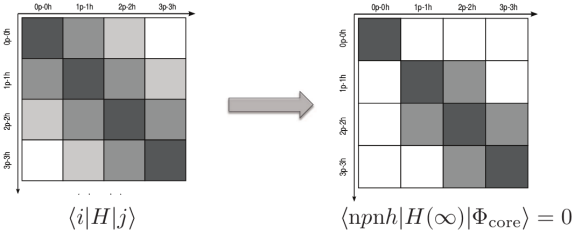
Jansen et al. Phys. Rev. Lett. 113, 142502 (2014)

In-medium similarity
renormalization group

decouple
core from excitations

decouple A particles in
valence space from rest

Stroberg et al.
Annu. Rev. Nucl. Part. Sci. 69, 307 (2019)



In addition to H_{eff}, these non-perturbative methods provide the core energy

Low-energy states nuclear properties

Very good general agreement
between the properties of low-energy nuclear states

Charge radii, quadrupole and magnetic moments
electric quadrupole and magnetic dipole transitions

		$\langle r^2\rangle_{ch}^{1/2}$ [fm]		Q [efm^2]		μ [n.m.]		B(E2) [e^2fm^4]		B(M1) [n.m.^{2}]	
Nucleus	State /Transition	Th	Exp	Th	Exp	Th	Exp	Th	Exp	Th	Exp
^{40}Ar	0^+_{gs}	3.43	3.427(3)	⋯	⋯	⋯	⋯				
	2^+_1			+2.6	+1(4)	−0.54	−0.04(6)				
	$2^+_1 \to 0^+_{gs}$							50	73(3)	⋯	⋯
	$0^+_2 \to 2^+_1$							29	43(7)	⋯	⋯
	$2^+_2 \to 0^+_{gs}$							0.7	10(2)	⋯	⋯
	$2^+_2 \to 2^+_1$							55	150(50)	0.016	0.07(1)
	$4^+_1 \to 2^+_1$							36	43(8)	⋯	⋯
	$6^+_1 \to 4^+_1$							16	13.6(5)	⋯	⋯
^{70}Ge	0^+_{gs}	4.05	4.0414(12)	⋯	⋯	⋯	⋯				
	2^+_1			+23	+4(3)	0.96	0.91(5)				
	$2^+_1 \to 0^+_{gs}$							240	360(7)	⋯	⋯
	$0^+_2 \to 2^+_1$							36	820(120)	⋯	⋯
	$2^+_2 \to 0^+_{gs}$							8.0	9(1)	⋯	⋯
	$2^+_2 \to 2^+_1$							16	1100(190)	0.022	0.003(2)
	$2^+_2 \to 0^+_2$							270	270(50)	⋯	⋯
	$4^+_1 \to 2^+_1$							370	430(90)	⋯	⋯
^{72}Ge	0^+_{gs}	4.07	4.0576(12)	⋯	⋯	⋯	⋯				
	2^+_1			+16	−13(6)	0.55	0.77(5)				
	$2^+_1 \to 0^+_{gs}$							260	418(7)	⋯	⋯
	$2^+_1 \to 0^+_2$							60	317(5)	⋯	⋯
	$2^+_2 \to 0^+_{gs}$							29	2.3(4)	⋯	⋯
	$2^+_2 \to 0^+_2$							15	0.5(1)	⋯	⋯
	$2^+_2 \to 2^+_1$							360	1100(180)	0.023	$29(9) \times 10^{-5}$