Standard Model

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Issues concerning the Standard Model of particle physics:

Even though we call it a model it is actually the candidate for **the** 'theory' of the fundamental particles and interactions among them!

Built, brick by brick, over the last 50-60 years, combining information from a lot of different types of experiments and many many innovative theoretical ideas.

The basic mathematical framework is that of quantum field theories (QFT) which possess some special properties (symmetries). Some aspects of these will be covered in lectures by Prof. Deredinger.

Using this information I intend then to cover the following :

- How did we find out about the fundamental constituents and interactions among them.
- How did we arrive at an understanding of the symmetries and hence a gauge theory description of the same: how was the SM built?
- What is the significance of the different families of quarks and leptons: flavour physics.
- What is the piece of the SM still left to be checked and how does the theory guide us about how and where to look for the missing piece.

SM - II

Different interactions and their symmetries

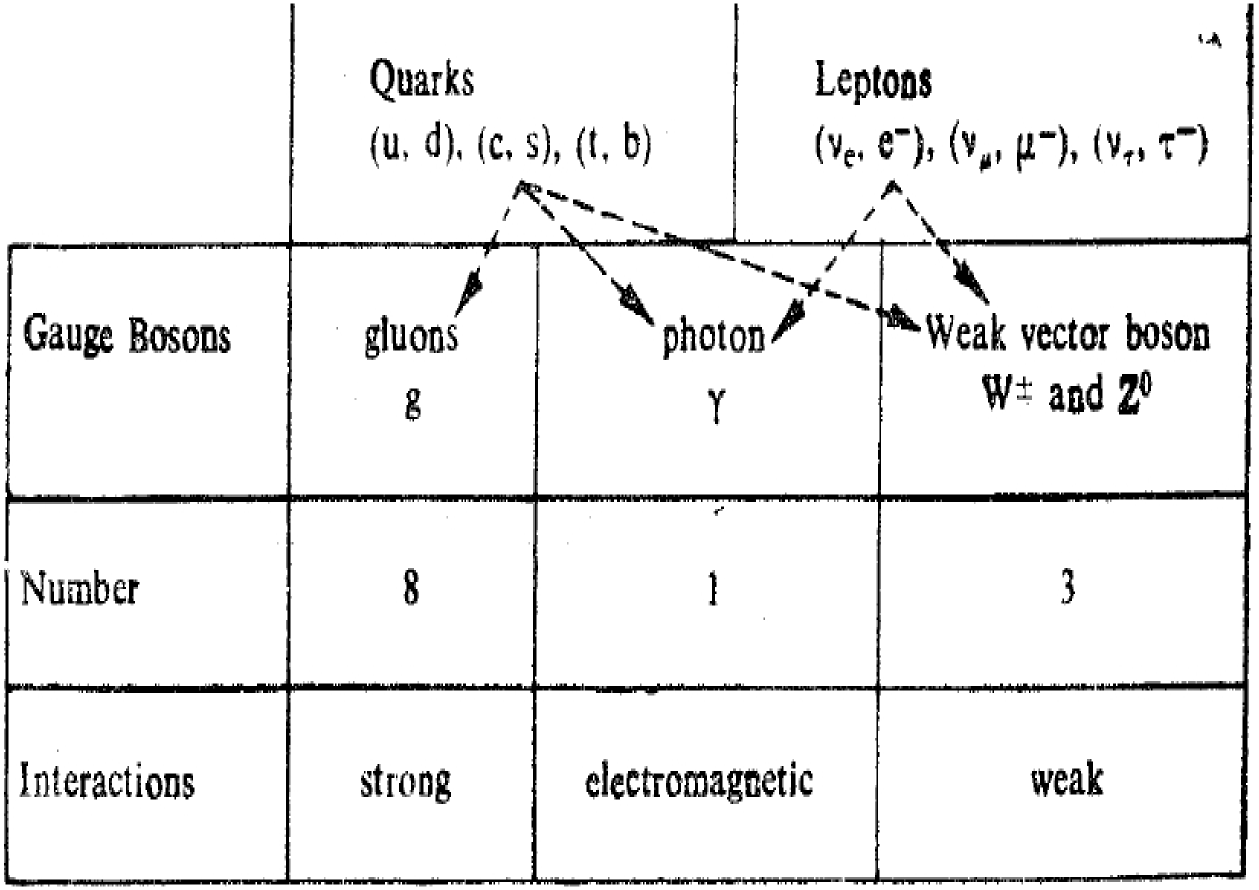
1. Take a look once again at the table of quarks and leptons and interactions among them.

2. Connection between Symmetries and conservation laws.

3. Discuss differences among different interactions, viz. different conservation laws they obey.

4. Summarise experimentally observed features of Weak Interactions in weak decays of nuclei, strange particles and pre-gauge theory description of weak interaction.

5. $SU(2)_L \times U(1)_Y$ model , prediction of a new interaction (weak neutral currents) and a new particle (charm quark).

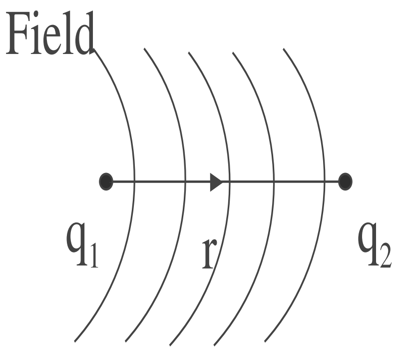


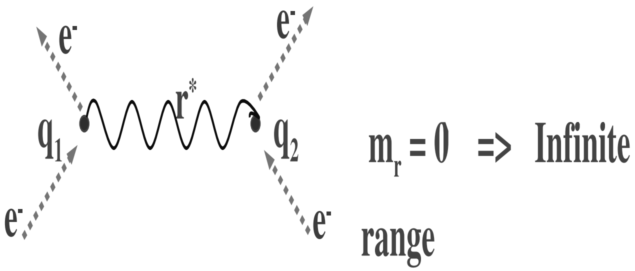
	Quarks (u, d), (c, s), (t, b)		Leptons (ν_e, e^-), (ν_μ, μ^-), (ν_τ, τ^-)
Gauge Bosons	gluons g	photon γ	Weak vector boson $W^\pm$ and Z^0
Number	8	1	3
Interactions	strong	electromagnetic	weak

Electromagnetic interactions: known for a long time, QED constructed first, best tested.

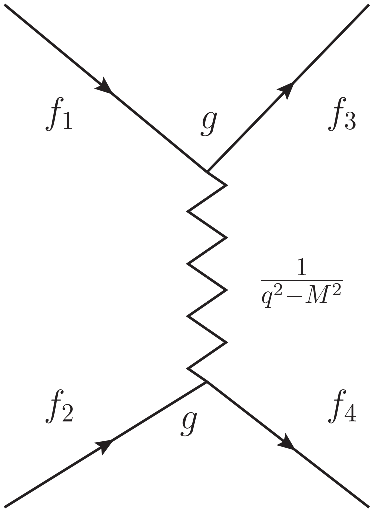
Nuclear strong interaction (α decay), weak interaction (β decay) known simultaneously.

Development of gauge theory of strong and weak interactions also happened simultaneously, but QCD now completely established. EW still has one piece to be settled.





$$V(r) = \frac{q_1 q_2}{r}$$



$$V(r) \propto \frac{e^{-Mr}}{r}$$

Force between two particles can be understood either in terms of the field or in terms of exchange of a field quantum!

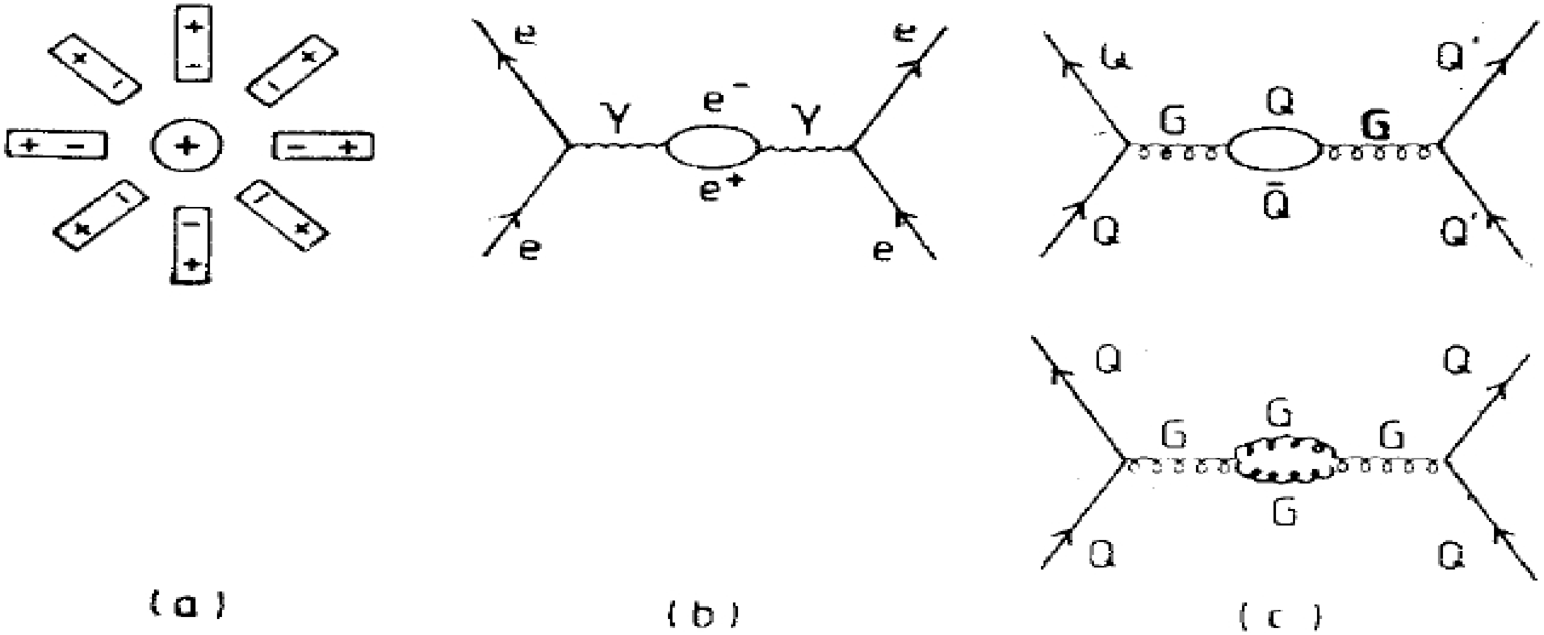
The properties of the quantum such as spin, mass govern the nature of the force such as range, dependence on $\vec{r}$ between the particles etc.

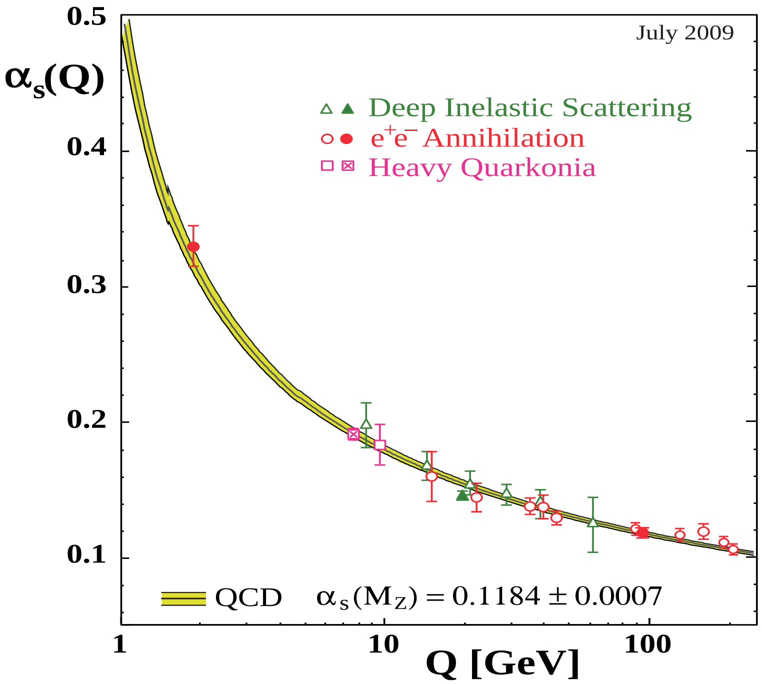
Analyse differences between the strong, weak and electromagnetic interactions, to appreciate differences between $g, W^{\pm}, Z^0$ and γ. All are gauge interactions but have different conservation laws.

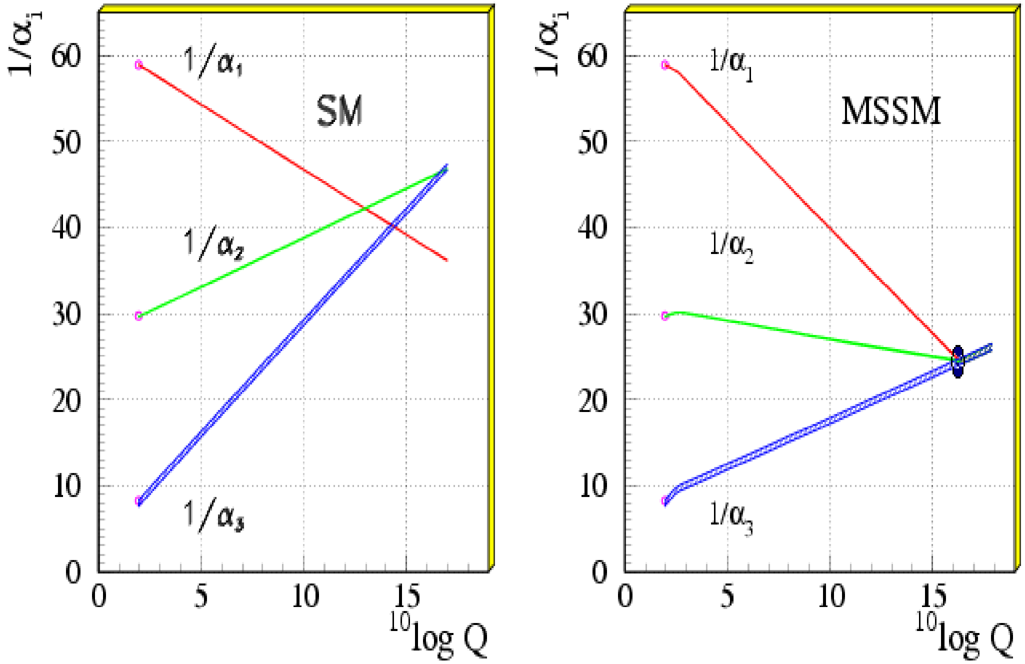
Force	Carrier	Range	Mass	Strength
Weak	W/Z_0	$< 10^{-18}$ m	$\simeq$ 100 GeV	$G_F \simeq \frac{10^{-5}}{M_P^2}$
Electromag.	γ	∞	0	$\alpha_{em} = \frac{1}{137}$
Nuclear	π	$< 10^{-15}$ m	140 MeV	$\frac{g_N^2}{4\pi} \simeq 1$
Strong	g	confining	0	$\alpha_s(M_P^2) \simeq 1.0$

Both cases mass of the gauge boson is zero. But the behaviour of the potential is very different.

This is technically due to the fact that gluons themselves have colour charges (recall $SU(3)_C$) whereas photons do not. The effective charges have different energy dependence in QED and QCD.







Calculating inter quark potential in QCD is a nonperturbative problem. Solved on Lattice. Calculations do seem to give rise to a confining potential. Very much a case of work in progress.

$$\alpha_s(Q^2) = \frac{g_s^2(Q^2)}{4\pi} = \frac{12\pi}{(33-2N_f)\log(Q^2/\Lambda^2_{QCD})}$$

Along with QFT another tool that Particle Physicists used heavily was the connection between conserved quantities and symmetries. (Noether charges: lectures by Prof. Deredinger)

Space-time symmetries:

The form of laws of physics can not depend on the choice of coordinate axes or origin..

$$\vec{F} = m\vec{a} = m\frac{d^2\vec{x}}{dt^2}$$

In particular it HAS to be invariant under a coordinate translation or time translation. Choice of origin for coordinates or time.

$\vec{x}' = \vec{x} + \vec{a}$, and $t' = t + t_0$

Such an invariance corresponds to conservation of linear momentum and energy. (In classical mechanics we construct integrals of motion)

Form of this equation remains the same i.e. is covariant with respect to a simple coordinate rotation.

$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

If now $\vec{F}(\vec{x}) = \vec{F}(r)$ where $r = \sqrt{(x^2 + y^2 + z^2)}$ then we say the system is invariant under a rotation of coordinate system. We know that a central force ⇒ conservation of orbital Angular Momentum $(\bar{L})$

Rotational symmetry implies Conservation pf total angular momentum $\bar{J} = \bar{L} + \bar{S}$, $\bar{S}$ being the spin angular momentum.

Consider two frames which move at a constant velocity with respect to each other.

A static charge q will exert only electrostatic field, where as a charge in motion (current) will also produce magnetic field.

I.e. the values of electric and magnetic fields depend on the frame where you study the motion, but laws of motion of course do not. Expression for the Lorentz force

$$\vec{F} = (q\vec{E} + \frac{q}{c}\vec{v} \times \vec{B})$$

no matter in which frame you do the experiments.

In fact Maxwell's equations can be written in a form that is 'covariant' under Lorentz transformations.

You saw in the other lectures that Maxwell's equations implied that velocity of electromagnetic waves is the same in *all* frames of references.

Further, one knows from electromagnetism that this constant velocity c is in vacuum is given by:

$$c = \frac{1}{4\pi\sqrt{\epsilon_0 \mu_0}}$$

where ϵ_0, μ_0 are the values of dielectric constant and permeability of vacuum.

Unification of Electricity and Magnetism $\Rightarrow$ value of c in terms of ϵ_0, μ_0 measured in laboratory.

We will like to see counterpart of such predictions for Electroweak Unification.

In the other lectures you were already told that to describe behaviour of elementary particles like electrons, we need to use both Quantum Mechanics and special theory of relativity. I.e. the equations of quantum mechanics describing the electron HAVE to be 'covariant' under a Lorentz transformation.

This was achieved by Dirac in the famous 'Dirac' equation

$$\left(\sum_{\mu=1,4} i\hbar\partial_\mu(\gamma^\mu)_{\alpha\beta} + m\delta_{\alpha\beta}\right)\psi_\alpha = 0$$

This equation predicted

1) Gyromagnetic ratio for electron is 2

2)A particle with the same mass and spin as the electron **BUT** opposite electric charge must exist.

I.e. QM and Special theory of relativity predicts existence of a positron.

Confirmed by Anderson in 1932.

This thus proved that a description of electromagnetic interactions of electron based on a picture consistent with Relativity and Quantum Mechanics is perhaps correct.

The studies led theoretical physicists further to QED which has been already discussed in the other lectures.

Antiproton was found in 1956.!

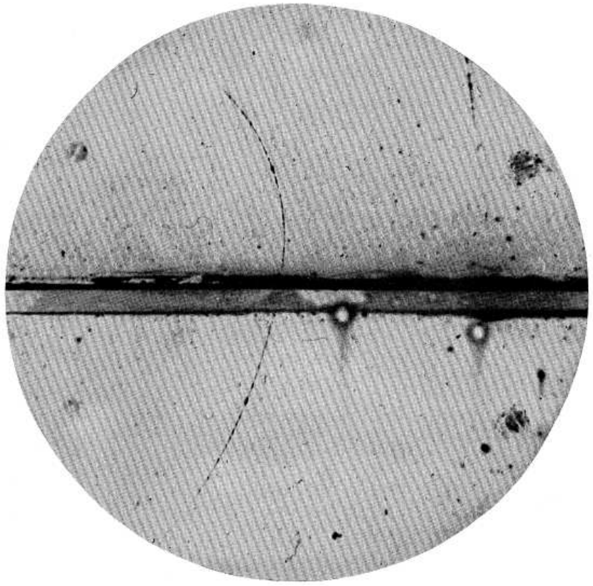


FIG. 1. A 63 million volt positron ($H\rho = 2.1 \times 10^5$ gauss-cm) passing through a 6 mm lead plate and emerging as a 23 million volt positron ($H\rho = 7.5 \times 10^4$ gauss-cm). The length of this latter path is at least ten times greater than the possible length of a proton path of this curvature.

This whole idea led to a realisation of the symmetry of laws governing fundamental particles, called charge conjugation $\mathcal{C}$.

Consider electromagnetism:

$|particle> \rightarrow |(anti)particle>$ AND change signs of $\vec{E}, \vec{B}$ Nothing will change.

Call this operation Charge Conjugation : $\mathcal{C}$. Looking at relationships between vector potential $A_\mu = (\phi, \vec{A})$ and $\vec{E}, \vec{B}$ one can then conclude that the photon described by the field A_μ is odd under $\mathcal{C}$.

$|e^- > \xrightarrow{\mathcal{C}} |e^+ >, \quad |p > \xrightarrow{\mathcal{C}} |\bar{p} >,$

$e^\pm$ can not be eigenstates of $\mathcal{C}$

On the other hand,

$|\gamma > \xrightarrow{\mathcal{C}}$ (-) $|\gamma >$.

Eigenvalues of $\mathcal{C}$ can be only ± 1 (strictly speaking phase).

But not all electrically neutral particles are eigenstates of $\mathcal{C}$.

For spin 1/2 neutral particles, fermion number changes sign, i.e. neutron too has an anti-particle and for baryons it will change the sign of the baryon number. For strange particles $\mathcal{C}$ will change sign of strangeness quantum number.

For a quantum system the fact that nothing changes under charge conjugation will mean $\mathcal{C}$ commutes with the Hamiltonian for electromagnetic interactions.

We know that if an operator commutes with the Hamiltonian then states can be classified according to the eigen value of this operator.

I.e. if the operation of charge conjugation is a symmetry of the system

If an interaction is invariant under charge conjugation, processes mediated by that interaction will conserve the charge conjugation quantum number, called $\mathcal{C}$ parity.

Parity $\mathcal{P}$:

Consider space reflection:

$$\vec{r} \to -\vec{r},\ t \to t,\ \vec{E} \to -\vec{E},\ \vec{B} \to +\vec{B},\ \vec{J} \to \vec{J}.$$

We will find:

$$\vec{F} = m\vec{a} \to -\vec{F} = m(-\vec{a})$$

In the Quantum context if the Hamiltonian is invariant under this operation, states can be classified as having even or odd parity. (Atomic states : $(-1)^l$)

Matter fermions are even under parity (convention). Anti-fermions HAVE to be then odd under parity Theoretical prediction. Necessary that experiments confirm this and they did!

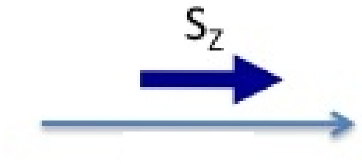
An important concept required in discussing behaviour of weak interaction Hamiltonian under Parity transformation, is that of helicity.

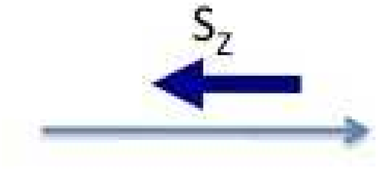
I am using words helicity and handedness interchangeably, strictly true only when particles are relativistic.

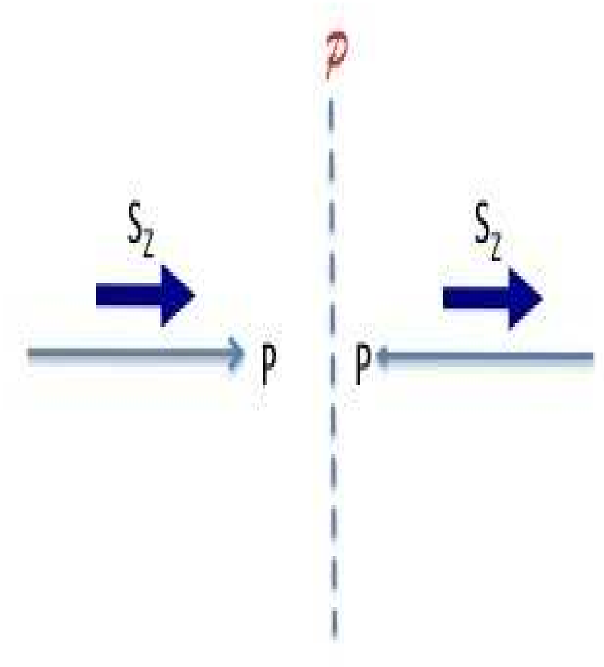
All the particle states thus can be labeled by the mass, and spin. The various invariances imply that a particle with a particular spin $\bar{S}$ will have $2S + 1$ states with $S_z = -S, -S + 1, ..S$.

Two states of (say) spin 1/2 electron, with $S_z = \pm 1/2$, with z as the axis of motion of the particle are called states with helicity ± 1.

Helicity $+1$: right handed Helicity -1 : left handed.







$|f,h=1> \xrightarrow{\mathcal{P}} |f,h=-1>$

Note also that for a massive spin 1/2 particle, relativistic invariance says that states with BOTH helicity MUST exist, as we can change the direction of momentum by going to a different frame of reference.

For a massless particle, in principle, a particle could exist in ONLY one helicity state, but not if Parity is conserved.

These arguments just tell us that invariances (symmetries) indeed are very constraining.

Time reversal: $\mathcal{T}$

$t \rightarrow -t$, $\vec{r} \rightarrow \vec{r}$, $\vec{p} \rightarrow -\vec{p}$, $\vec{J} \rightarrow -\vec{J}$, $\vec{F} \rightarrow \vec{F}$:

Now

$\vec{F} = m\vec{a} \Rightarrow \vec{F} = m\vec{a}$.

In reality we can not reverse time in our experiments, but can study implications of invariance of equations under these operations and implications for certain observables.

Not only the three interactions strong, electromagnetic and weak differ in their strengths, More importantly they have different symmetry properties, particularly under the discrete symmetry operations and follow different conservation laws.

The weak interactions, responsible for the radioactive decays and also the decays of strange particles, involve initial and final states of differing total strangeness, whereas the strong and electromagnetic interactions conserve strangeness.

Weak interactions seem to break strangeness conservation but in a very definite manner, i.e., : $\Delta S = \Delta Q$.

Most importantly Weak interactions were seen to violate parity and that too maximally.

C.N. Yang and T.D. Lee: Phys. Rev. 104, 254, October 1956 : suggested in a theory paper, looking at so called τ–θ puzzle.

C.S.Wu: Tested it experimentally. Phys. Rev. 105, 1413 , February 1957 i.e. The e^- which is emitted in β decay is left handed and the e^+ is right handed.

ν emitted in β decay was always left handed and $\bar{\nu}$ is always right handed (Maurice Goldhaber et al,Phys. Rev. 109, 1015 (1958)).

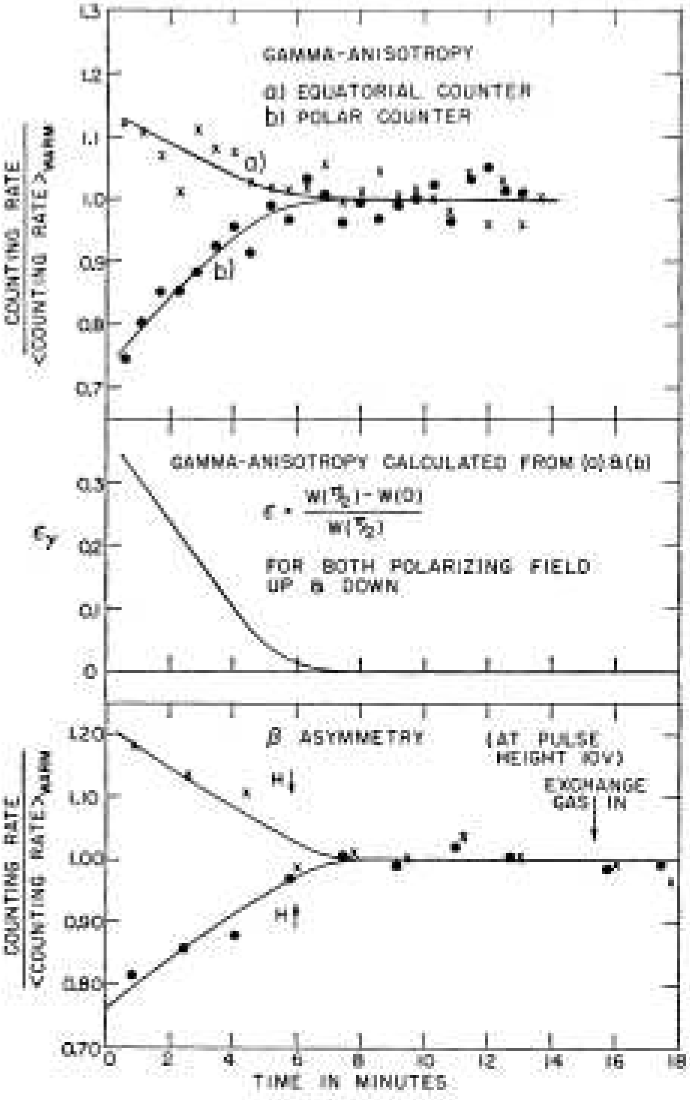
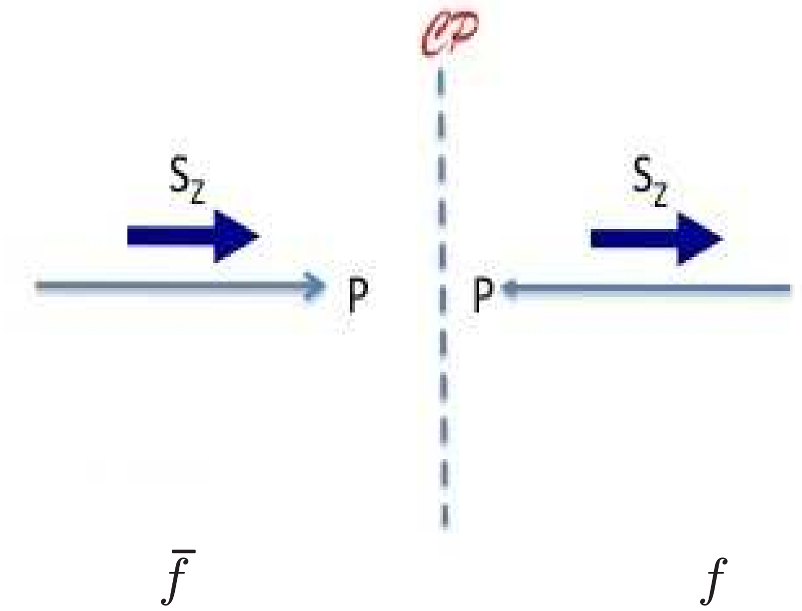


FIG. 2. Gamma anisotropy and beta asymmetry for polarizing field pointing up and pointing down.



In fact since β decay seemed to produce left handed fermions and right handed antifermions and the two transform into each other under the combined operation of $\mathcal{CP}$ it looked like CP was conserved by Weak Interactions even if P was violated.

Somewhat later it was seen that Weak Interactions violate CP as well. Christenson, Cronin, Fitch and Turlay Phys. Rev. 140, B74(1965)

There exist two K−meson states very close in mass but with life times differing by three orders of magnitude.

In case of shorter lived K_S ($\sim 10^{-10}$ sec) the final state has two π. For the longer lived it is K_L ($\sim 10^{-7}$ sec) the final state has three π ($\mathcal{CP}$ parity = -1 of a π is -1).

K-decays always via Weak Interactions.

Pais: K_L and K_S have CP parity -1 and +1, decay interaction conserves CP, forcing K_L to decay into three π. Smaller phase space thus making it longer lived.

Experiment looked whether K_L ever decays into two pions. (Suggested by Pais)

Indeed tiny fraction of K_L decays has two pions in the final state.

Strength of the $\mathcal{CP}$ violation: 1 part in 1000, The strength of $\mathcal{CP}$ violating part responsible for the decay weaker by a factor 1000.

Is this yet another new, weaker interaction? (Wolfenstein: Super-weak!)

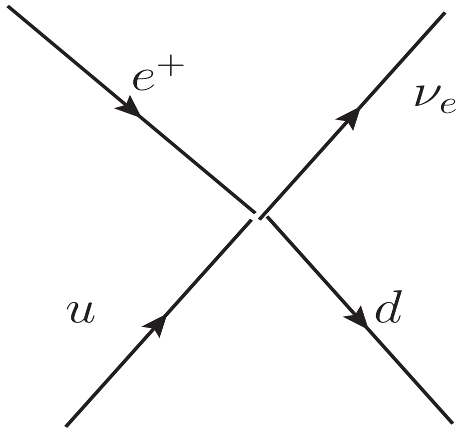
A gauge theory of Weak Interactions (standard model), has all the necessary ingredient to automatically give rise to the observed level of CP violation.

In terms of Cabbibo-Kobayshi-Masakawa(CKM) mixing): Kobayashi-Masakawa Nobel prize 2008.

In the first discussions forget CP violation.

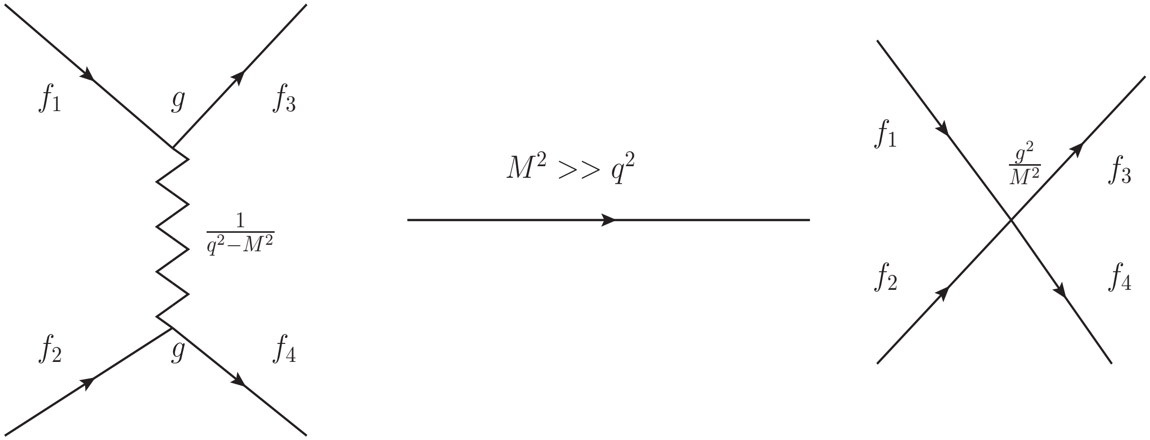
Weak interaction description pre gauge theory: Started from Fermi's (correct) proposal that the e^- and $\bar{\nu}_e$ in β decay are emitted only in $J = 1$ state. Experimental data on allowed transitions and agreement of ft values with that expected from pure phase space, led Fermi to postulate that the amplitude did not have momentum dependence

$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}}(J^W_\mu J^{W\mu\dagger}) = \frac{G_F}{\sqrt{2}}(\bar{\psi}_3\gamma_\mu\psi_1)(\bar{\psi}_4\gamma^\mu\psi_2)$$



But predictions of such a 'local' interaction can be seen to violate unitarity for $e^-\nu_e$ scattering around $E_\nu \propto \frac{1}{\sqrt{G_F}} \sim 250$ GeV.

Hypothesis: The current-current interaction is just an approximation to a real ampiltude with a very heavy boson. Good agreement of Current Current interaction idea with data meant that **IVB** was necessarily heavy. IVB: Klein/Schwinger

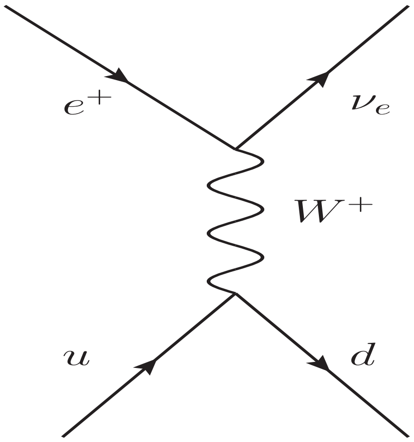


$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}}(J^W_\mu J^{W\mu\dagger}) = \frac{G_F}{\sqrt{2}}(\bar{\psi}_3\gamma_\mu\psi_1)(\bar{\psi}_4\gamma^\mu\psi_2); \qquad \frac{G_F}{\sqrt{2}} = \frac{g^2}{M^2}$$

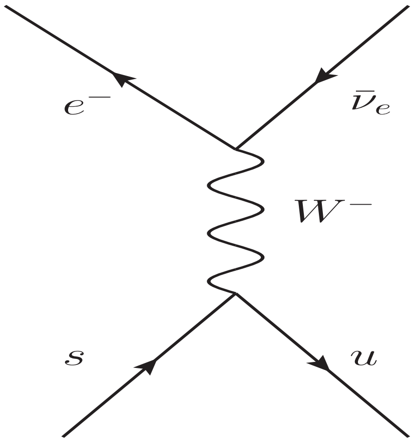
V-A Theory.

E.C.G. Sudarshan, R.E. Marshak Phys. Rev. D 109, 1860, 1958 (Feynman, Gelmann : later)

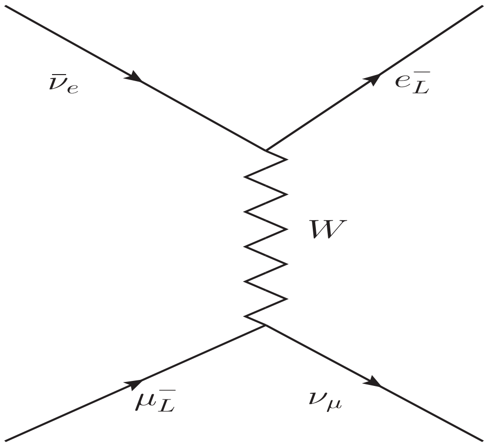
The original Fermi hypothesis 'almost' correct, but replace γ_μ by $\gamma_\mu(1-\gamma_5)$ to describe correctly the handedness of fermions observed in *all* weak processes.



$\Delta S = 0$

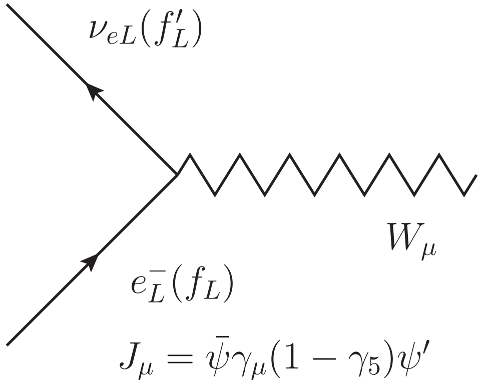


$\Delta S = 1$



Pure leptonic.

The basic unit of charged current weak interactions is a doublet of left handed fermions.



$J_\mu = \bar{\psi}\gamma_\mu(1-\gamma_5)\psi'$

$\Delta S = \Delta Q$ if $f = s, f' = u$

f and f' differ in charge by one unit.

For strange quark case $\Delta S = 1$

Charged Current.

Cabbibo's important observation Phys. rev. Lett. 10, 531 (1963)

The strength of all the three types of weak transitions $\Delta S = 0, \Delta S = 1$ and pure leptonic *iff* appropriate doublet was

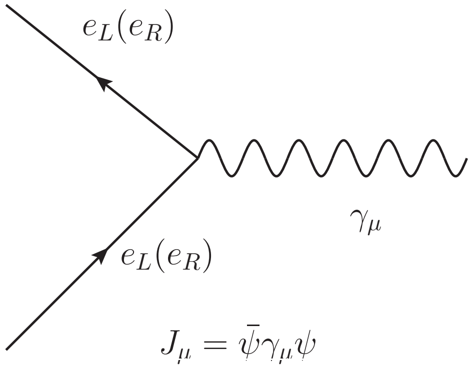
$$\begin{pmatrix} u \\ d\cos\theta_c + s\sin\theta_c \end{pmatrix}$$

θ_c called Cabbibo angle, experimentally determined value : $\sim 12^\circ$.

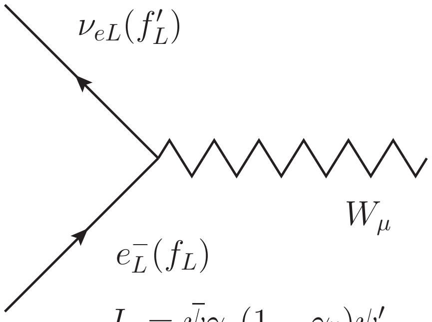
Interaction eignestate: $d\cos\theta_c + s\sin\theta_c$

Bjorken and Glashow (1964) Postulated a new quark c, with same quantum numbers as the u quark , which forms a doublet with orthogonal combination $s\cos\theta_c - d\sin\theta_c$.

(Come back to it later when discussing GIM mechanism)



$$J_\mu = \bar{\psi}\gamma_\mu\psi$$



$$J_\mu = \bar{\psi}\gamma_\mu(1-\gamma_5)\psi'$$

Obvious similarity: Spin one state of initial and final state fermions, ie. spin 1 nature of the exchanged boson.

If $M_W \sim 100$ GeV, the strength of the couplings of fermions for both γ and $W^\pm$ would be similar!

Differences: $M_W >> M_\gamma = 0$

Only (left) handed fermions take part in interactions with the W. The electromagnetic interactions treat f_L, f_R equally.

Issues

1) How to construct a gauge field theory with massive gauge bosons which has good high energy behaviour?

2)Recall a mass term for the gauge boson in the Lagrangian $M_A^2 A_\mu A^\mu$ (for $U(1)$ case obvious) NOT Gauge Invariant.

3)Gauge invariance required for renormalisability.

4)Problems for a unified description of Electromagnetic and Weak Interactions: J_μ^W parity violating, J_μ^{em} parity conserving.

Glashow:

(Two) Doublets of quarks $\mathcal{Q}_i$ and leptons$\mathcal{L}_i$:

Postulated that Left handed quarks and lepton form a doublet of a $SU(2)_L$ the L implying that only the left handed fermions have a nonzero charge, i.e. interactions with the $W^\pm$

Glashow's observation:

$Q = I_W^3 + Y/2$

where$Y = 1/3$ for quark doublets and $Y = -1$ for lepton doublet. Thus Y is the hyper charge of the $U(1)$ symmetry group.

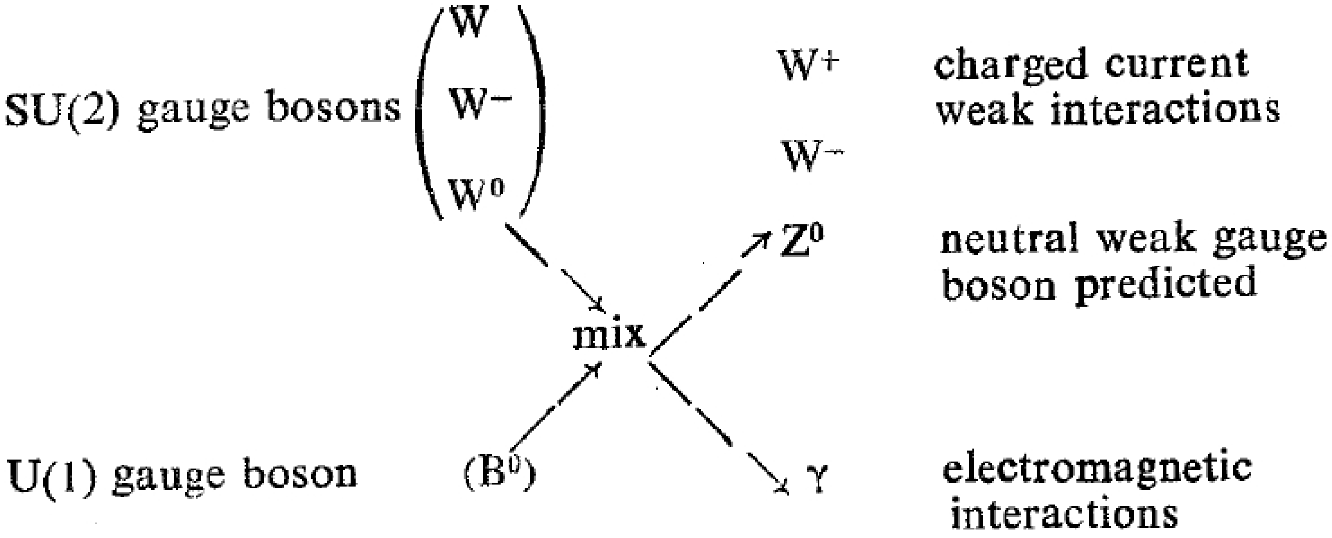
Right handed fermions will have $I_W^3 = 0$ and hence a different Y value than the left handed fermions.

Thus the net symmetry is $SU(2)_L \times U(1)_Y$ g_2 coupling constant for $SU(2)$ and g_1 for $U(1)_Y$.

$SU(2)_L$: Three gauge bosons W^1, W^2, W^3 : all couple only to left handed fermions.

$U(1)_Y$: B couples to all fermions (LH and RH).

B and W^3 mix, giving one zero mass eignestate γ. Identify other with a Neutral vector boson called Z^0.



$$A_\mu = cos\theta_W B_\mu + \sin\theta_W W_\mu^3$$

$$Z_\mu = -\sin\theta_W B_\mu + \cos\theta_W W_\mu^3$$

$$e = g_2 \sin\theta_W = g_1 \cos\theta_W$$

$$\frac{G_F}{\sqrt{2}} = \frac{g_2^2}{M_W^2} = \frac{e^2}{8M_W^2 \sin^2\theta_W}$$

Thus the model predicts

1)new particles: Z^0, c.

2)new interactions: Weak Neutral current analogous to Weak Charged Current mediated by the W.

3)Mass predicted in terms of one parameter θ_W, which will decide the couplings of the various particles to the Z^0.

4)WWZ^0 coupling possible and predicted.

5)A sort of unification: Unless $\sin\theta_W$ unnaturally small (then the whole idea is not sensible) e, g_1, g_2 all of similar order. The difference in strengths of interactions only apparent due to large M_W.

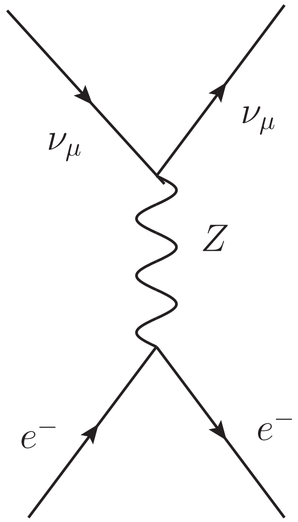
An aside:

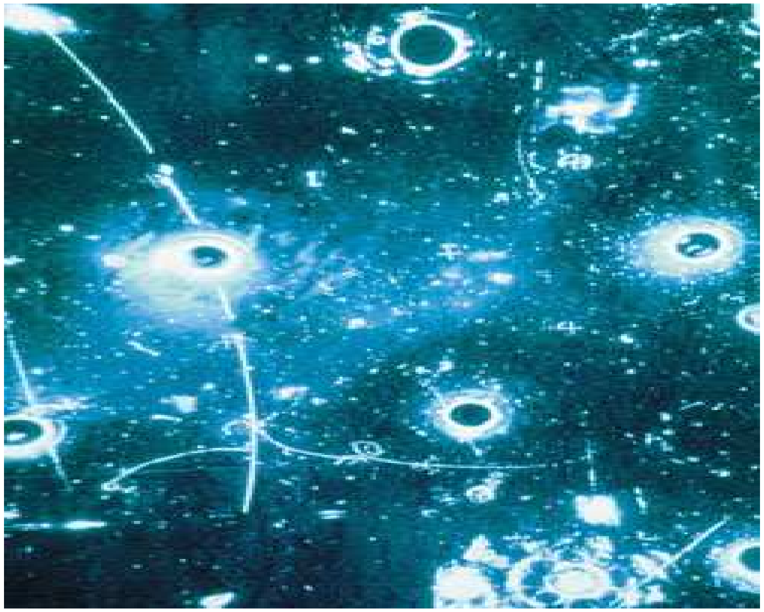
Massive gauge bosons have a problem. Amplitudes like $\nu_e\bar{\nu}_e \to W^+W^-$ grow with energy and can violate unitarity.

One can show that this violation of unitarity can be tamed by adding a neutral spin 1 boson which ZW^+W^- couplings as expected in the $SU(2)_L \times U(1)_Y$ model!

Cornwall, Tiktopolous (1974, 1975); Llewellyn Smith (1973), S.D. Joglekar (1973)

First attempt to see this: D. H. Perkins, Veltman. Failed..They put an upper limit! Discovery of neutral currents: In an experiment at CERN with a Bubble Chamber. (Photograph Courtesy CERN)





1)More about NC and about predictions for M_c,

2) Weinberg-Salam Model which used Higgs Mechanism to predict also M_Z in terms of $\sin\theta_W$,

3)Prediction for M_t from measured $M_Z, M_W \ldots$ ....

Next two lectures!