# Overview of the Numerical Tools and Related Challenges for Modeling of Beam-Beam Effects

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**Beam-Beam Workshop**

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BERKELEY LAB

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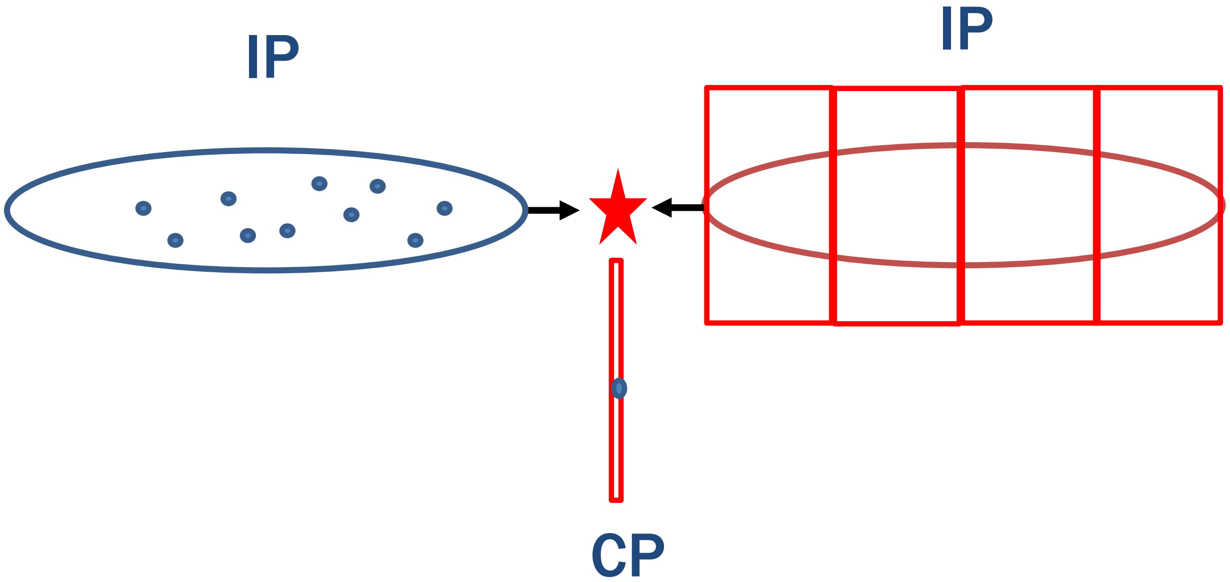
ACCELERATOR TECHNOLOGY & APPLIED PHYSICS DIVISION ATAP

# Outline

- Introduction to weak-strong beam-beam model
- Strong-strong beam-beam model and challenges
- Some currently used beam-beam codes
- Outlook for modeling beam-beam effects

# Weak-Strong Beam-Beam Model at IP (cont'd)



- Strong beam is not affected by the weak beam
- Particles in the weak beam drift to the collision point
- Beam-beam forces from the strong beam are applied to the weak beam particles
- Weak beam particles drift back to its original locations

# Weak-Strong Beam-Beam Model at IP

$$
\begin{aligned}
x^{new} &= x + S(z, z_*) f_X(X, Y; Z), \\
p_x^{new} &= p_x - f_X(X, Y; Z), \\
y^{new} &= y + S(z, z_*) f_Y(X, Y; Z), \\
p_y^{new} &= p_y - f_Y(X, Y; Z), \\
z^{new} &= z, \\
\epsilon^{new} &= \epsilon - \frac{1}{2} f_X(X, Y; Z)\left[p_x - \frac{1}{2} f_X(X, Y; Z)\right] \\
&\quad - \frac{1}{2} f_Y(X, Y; Z)\left[p_y - \frac{1}{2} f_Y(X, Y; Z)\right]
\end{aligned}
\qquad
\begin{aligned}
X &= x + p_x S(z, z_*), & P_X &= p_x, \\
Y &= y + p_y S(z, z_*), & P_Y &= p_y, \\
Z &= z, & P_Z &= \epsilon - \frac{p_x^2 + p_y^2}{4}
\end{aligned}
.
$$

$$
f_y(x, y, \sigma_x, \sigma_y) + i f_x(x, y, \sigma_x, \sigma_y) = \frac{N_* r_e}{\gamma_0} \sqrt{\frac{2\pi}{\sigma_x^2 - \sigma_y^2}}
$$

$$
\times \left[ \mathrm{w}\left( \frac{x + iy}{\sqrt{2(\sigma_x^2 - \sigma_y^2)}} \right) - \exp\left( -\frac{x^2}{2\sigma_x^2} - \frac{y^2}{2\sigma_y^2} \right) \mathrm{w}\left( \frac{\frac{\sigma_y}{\sigma_x} x + i \frac{\sigma_x}{\sigma_y} y}{\sqrt{2(\sigma_x^2 - \sigma_y^2)}} \right) \right]
$$

K. Hirata, H. Moshammer, F. Ruggiero, Particle Accelerator, 1993, vol. 40, p. 205.

# Weak-Strong Beam-Beam Model

$$\begin{pmatrix} Q \\ P \end{pmatrix} \longrightarrow U(2\pi\nu_x) \begin{pmatrix} Q \\ P \end{pmatrix},$$

$$\begin{pmatrix} Z \\ \mathcal{E} \end{pmatrix} \longrightarrow U(-2\pi\nu_s) \begin{pmatrix} Z \\ \mathcal{E} \end{pmatrix},$$

$$U(\alpha) = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix}.$$

$$Q \longrightarrow \lambda_x Q + \sqrt{1-\lambda_x^2}\,\hat{r}_1,$$

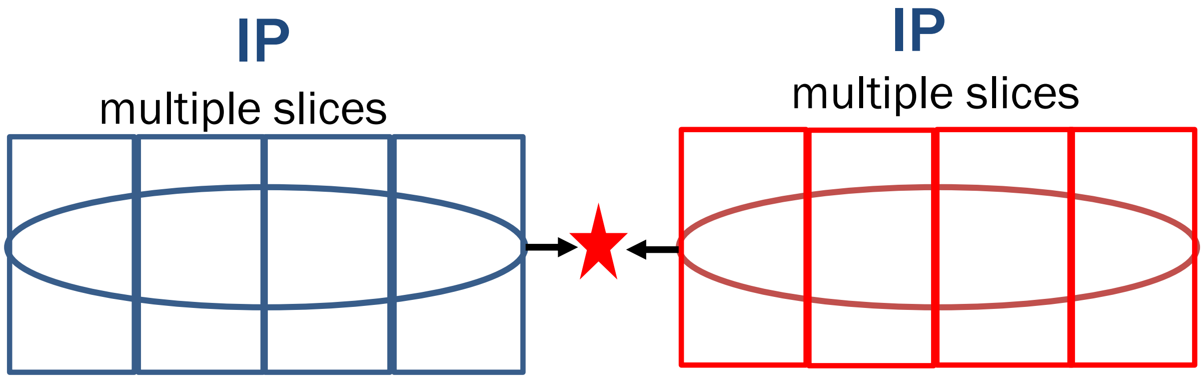
$$P \longrightarrow \lambda_x P + \sqrt{1-\lambda_x^2}\,\hat{r}_2,$$

$$\mathcal{E} \longrightarrow \lambda_s^2 \mathcal{E} + \sqrt{1-\lambda_s^4}\,\hat{r}_3,$$

$$\lambda_x = \exp(-1/T_x), \quad \text{and} \quad \lambda_s = \exp(-1/T_\epsilon),$$

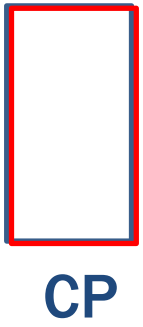
- Codes including weak-strong beam-beam only: MAD-X, BMAD, Lifetrack…
- Weak-strong beam-beam model is fast but lacks accuracy:
  → Not self-consistent
  → Cannot model coherent motion of two beams

# Strong-Strong Beam-Beam Model at IP



$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = -2\pi\rho(x, y),$$

$$\mathbf{E} = -\nabla\phi,$$



$$\phi(x, y) = \int G(x, \bar{x}, y, \bar{y})\rho(\bar{x}, \bar{y})d\bar{x}d\bar{y},$$

1st Challenge: Speed

- Each collision needs $N^2$ Poisson Solutions
- Strong-strong beam-beam model is self-consistent but much slower than the weak-strong model

# Fast Efficient Poisson Solver Needed to Improve Speed

$$\phi(r) = \int G(r,r')\rho(r')dr'$$

$$\phi(r_i) = h\sum_{i'=1}^{N} G(r_i - r_{i'})\rho(r_{i'})$$

$$\boxed{G(x,y) = -\frac{1}{2}\log(x^2 + y^2)}$$

Direct summation of the convolution scales as $N^2$ !!!!
N – total number of grid points

# Green's Function Convolution Can Be Computed Effectively

*Hockney's Algorithm:- scales as $(2N)^2\log(2N)$*

- Ref: Hockney and Easwood, ***Computer Simulation using Particles***, McGraw-Hill Book Company, New York, 1985.

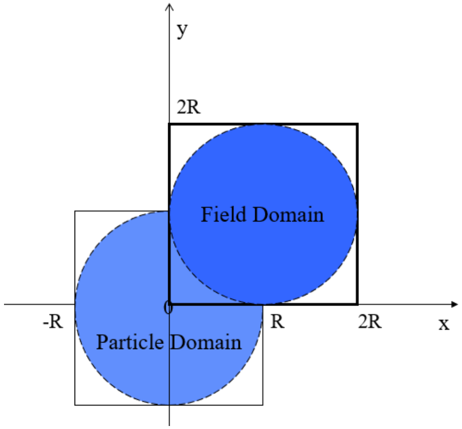
$$\phi_c(r_i) = h\sum_{i'=1}^{2N} G_c(r_i - r_{i'})\rho_c(r_{i'})$$

$$\phi(r_i) = \phi_c(r_i) \quad \text{for i} \; = \; 1, \text{N}$$

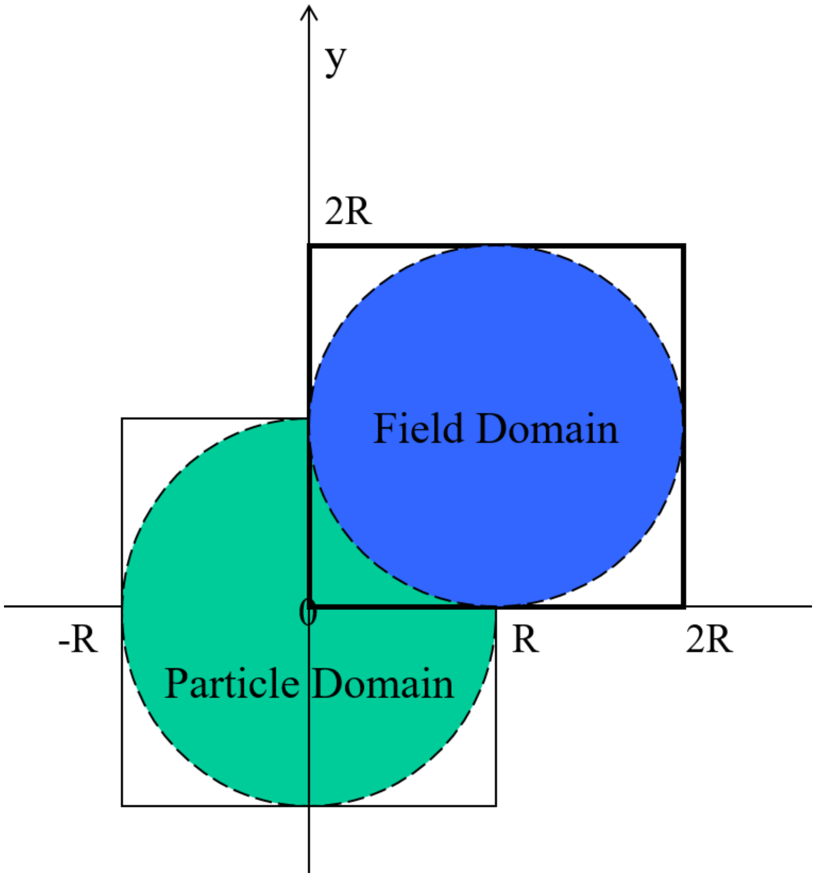
*Shifted Green function Algorithm:*

$$\phi_F(r) = \int G_s(r,r')\rho(r')dr'$$

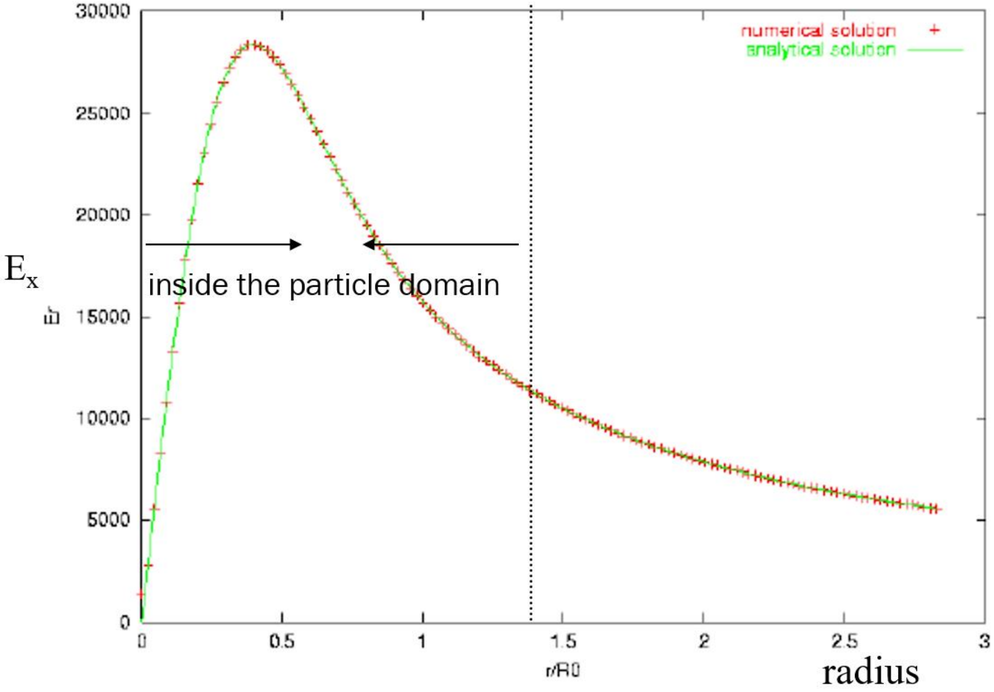
$$G_s(r,r') = G(r + r_s, r')$$



# Good Agreement between the Numerical Solution from the Shifted Green Function and the Analytical Solution



**beam-beam field vs. radius**

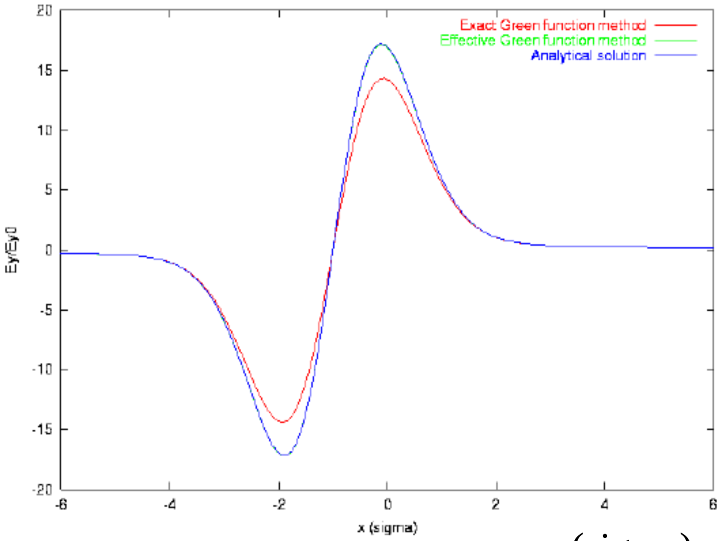


# Integrated Green Function Method Is More Effective to Handle Large Aspect Ratio Beam

*Integrated Green function Algorithm for large aspect ratio:*

$$\phi_c(r_i) = \sum_{i'=1}^{2N} G_i(r_i - r_{i'})\rho_c(r_{i'})$$

$$\boxed{G_i(r,r') = \oint G_s(r,r')\,dr'}$$



$E_y$

x (sigma)

# Parallel Computing Implementation Is Needed to Improve Speed

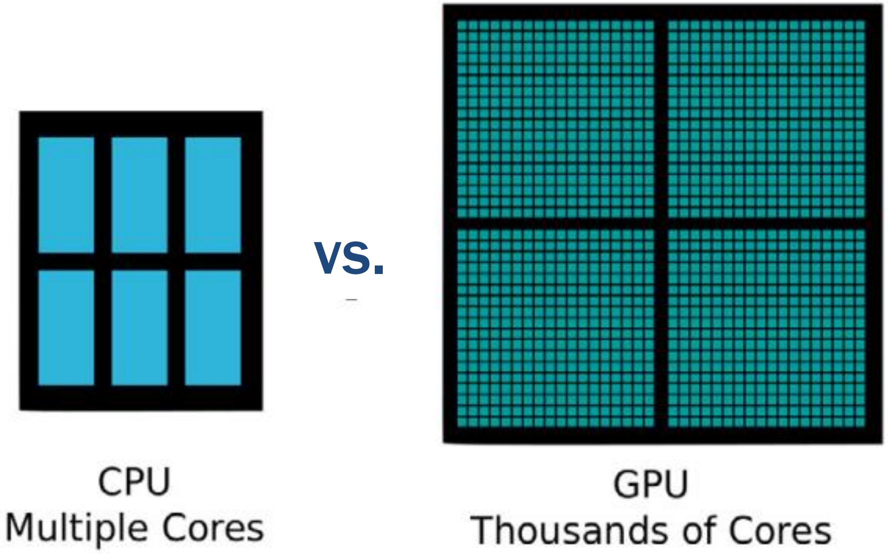
Frontier uses 9,472 AMD Epyc 7713 "Trento" 64 core 2 GHz CPUs (606,208 cores) and 37,888 Instinct MI250X GPUs (8,335,360 cores).

# Parallel Strong Scaling of a Strong-Strong Beam-Beam Model Using Both MPI and OpenMP

| Time(s) | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| MPI tasks<br>OMP in each MPI | 4 | 8 | 16 | 32 | 64 | 128 | 256 | 512 | 1024 |
| 1 | 780.07 | 448.05 | 286.25 | 221.53 | 158.12 | 129.03 | 137.88 | 165.39 | |
| 2 | 447.51 | 285.59 | 219.9 | 160.6 | 98.45 | 93.23 | 131.08 | 137.7 | |
| 4 | 402.98 | 271.21 | 135.55 | 88.29 | 62.1 | 74.91 | 91.24 | | |
| 8 | 273.83 | 148.53 | 76.12 | 54.68 | 60.75 | 74.12 | | | |
| 16 | 150.49 | 74.29 | 49.85 | 56.85 | 58.44 | | | | |
| 32 | 87.19 | 65.47 | 59.51 | 67.8 | | | | | |
| 64 | 117.94 | 94.86 | 103.24 | | | | | | |

- **Message Passing Interface (MPI) is a distributed memory parallel programing paradigm**
- **OpenMP is a shared memory parallel programming paradigm**
- **A hybrid MPI and OpenMP provides the best performance**

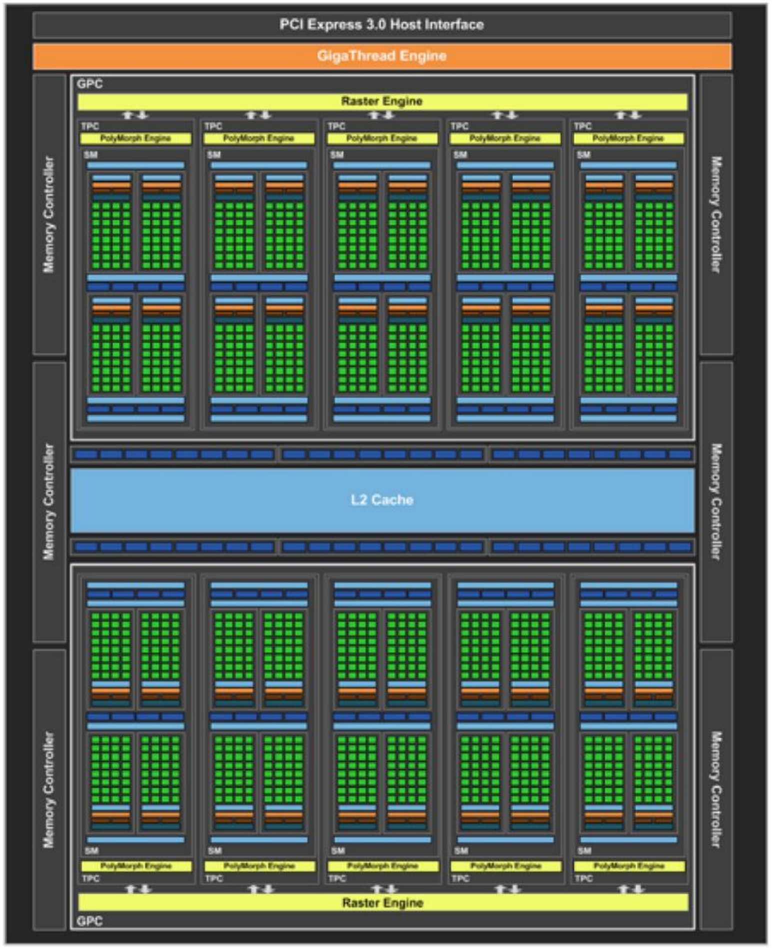
# Central Processing Unit (CPU) versus Graphics Processing Unit (GPU)



CPU
Multiple Cores

GPU
Thousands of Cores

The architecture of the GeForce GTX 1060 GPU processor

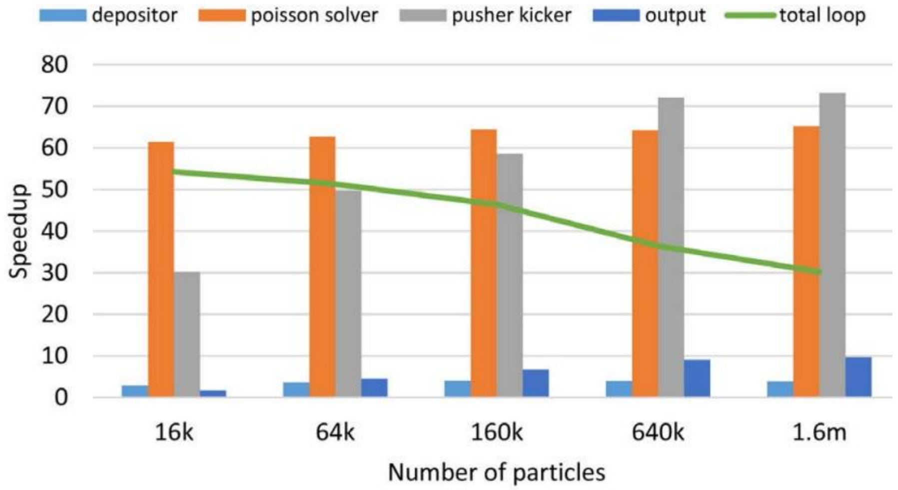


**CPU: fewer number of cores, each core is more powerful, slower communication**

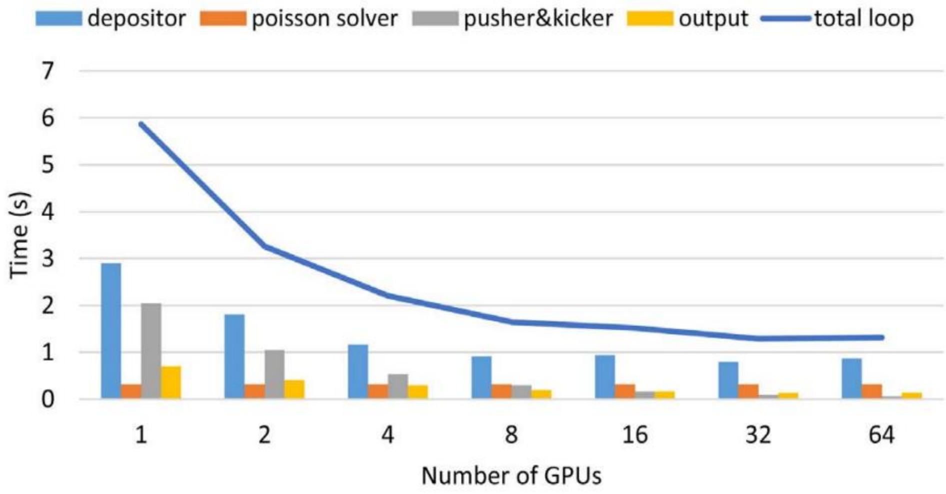
**GPU: large number of cores, each core is less powerful, faster communication**

# Speedup of a PIC Code on GPUs

Speedup of the beam dynamics GPU PIC Code on a single GPU versus the number of particles



Scalability of the PIC code using 64 64 64 × × grid points and 1.6M particles on Titan.



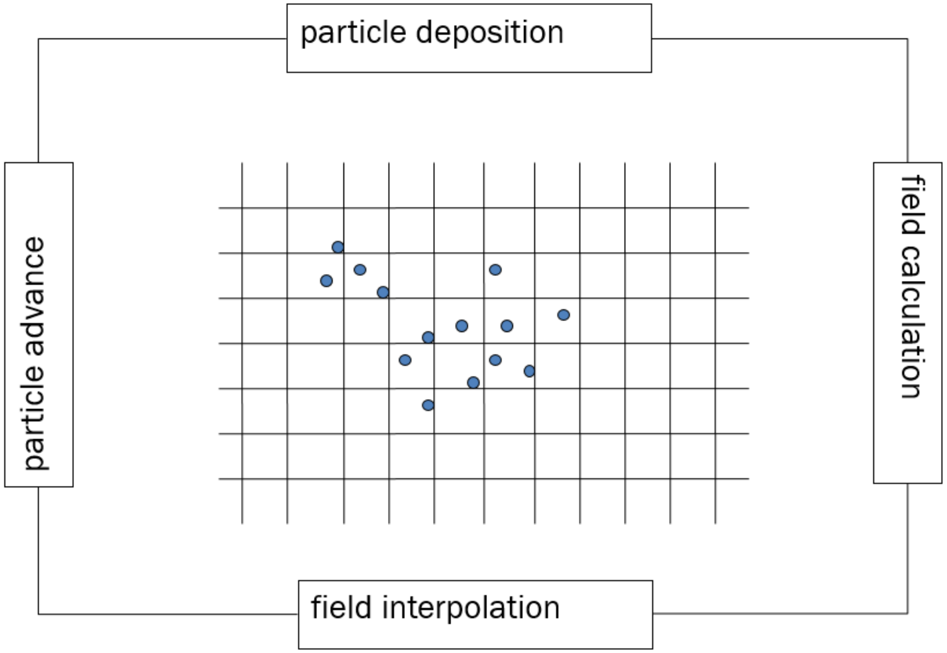
- For small problem sizes, a single GPU can be more than 50 faster than a CPU core
- For a larger problem size, a single GPU can be more than 30 faster than a CPU core
- The PIC code does not scale well beyond 8 GPU for the fixed problem size

Z. Liu and J. Qiang, Journal of Software Engineering and Applications 12, p. 321 (2019).

# Self-Consistent PIC Based Strong-Strong Beam-Beam Model is Subect to Numerical Noise

## 2nd Challenge: Accuracy

$$\rho_p = \sum_{i=1}^{n} q_i w(x_i - x_p)$$



particle deposition

field calculation

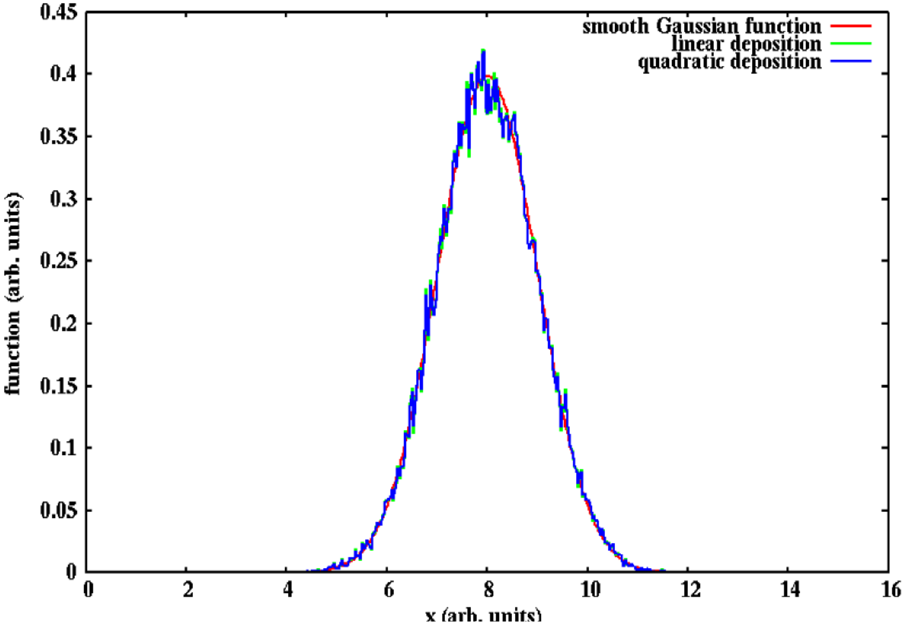
field interpolation

particle advance

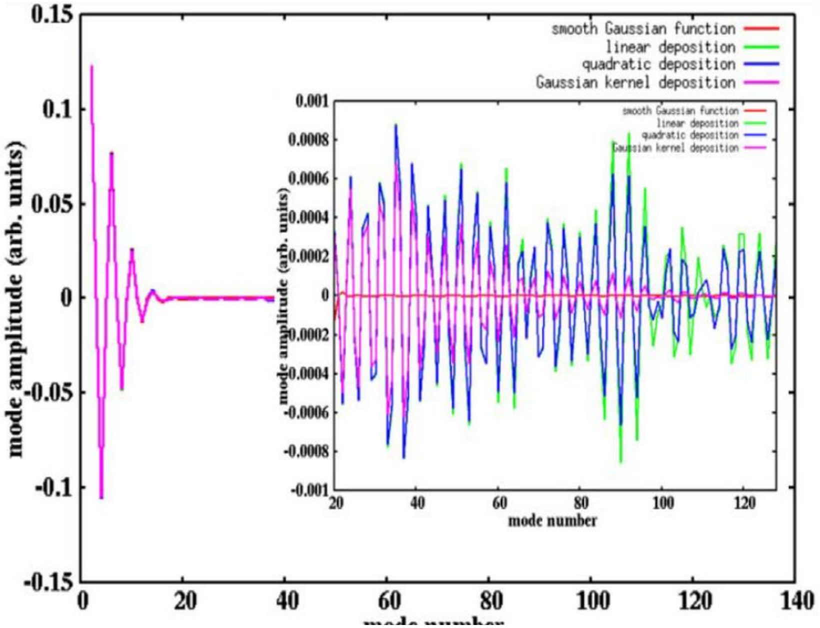
$$E_i = \sum_{i=1}^{M} E_p w(x_i - x_p)$$

# 1D Gaussian function from macroparticle sampling deposition and from the function itself

1D Gaussian Function



Spectrum of  the Gaussian Function



$$\mathrm{var}(\rho^l) = \frac{1}{N_p}\left(\frac{2}{L}\right)^2 \mathrm{var}\left(\sum_{I=1}^{N_g} S(x_I - x_i)\sin(\alpha_l x_I)\right)$$

$$\frac{\Delta\epsilon/\epsilon}{\tau} \approx \frac{1}{2}\frac{\langle x^2\rangle\langle(\delta F)^2\rangle}{\epsilon^2}$$

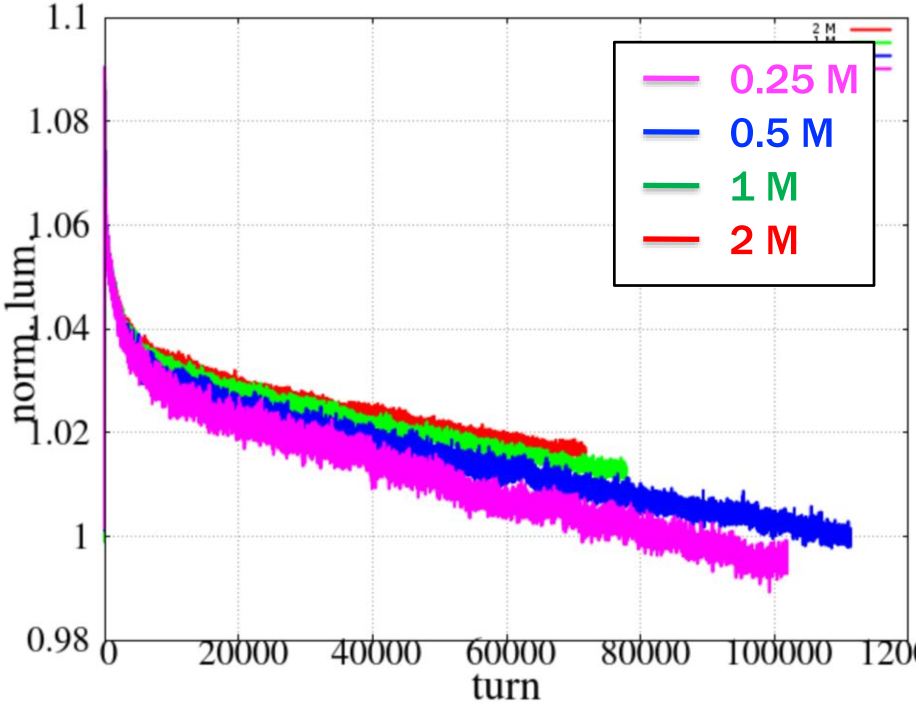
- Numerical noise results from finite macroparticle sampling
- Such noise causes fluctuation in beam-beam forces → numerical emittance growt

J. Qiang, “Advances in the simulation of space-charge effects,” J. of Instrumentation **15** P07028, 2020.

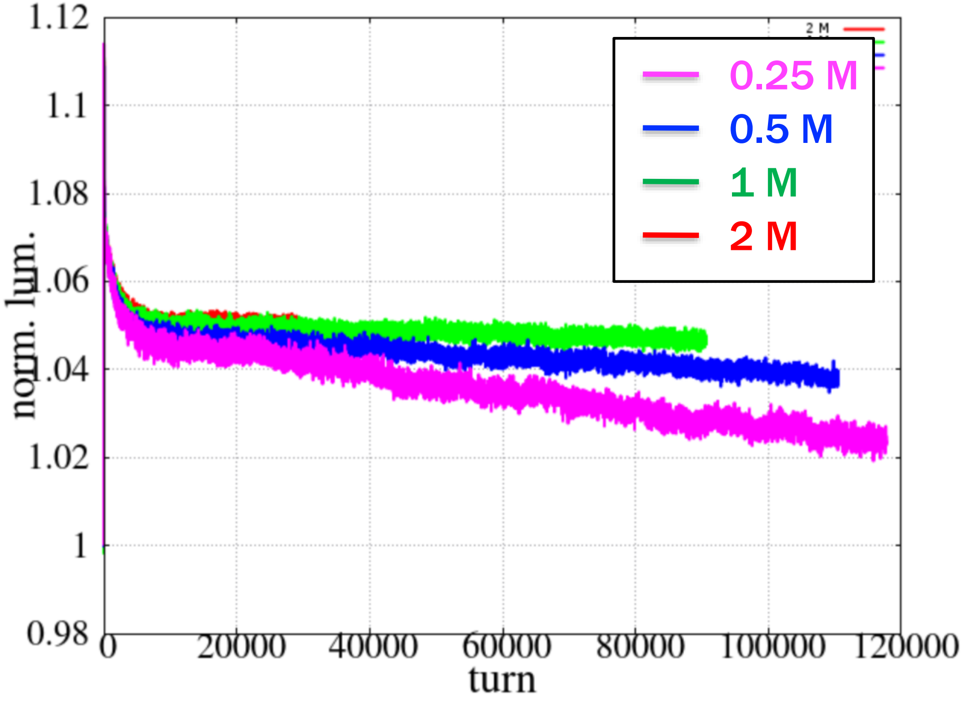
# Predicted Luminosity Degradation from Beam-Beam Simulation Depends on the Number of Macroparticles

- **Strong-strong beam-beam simulation subject to numerical noise driven emittance growth and luminosity degradation**
- **Increase of macroparticle number helps reduce numerical noise effects**

**with crossing angle/crab cavity**

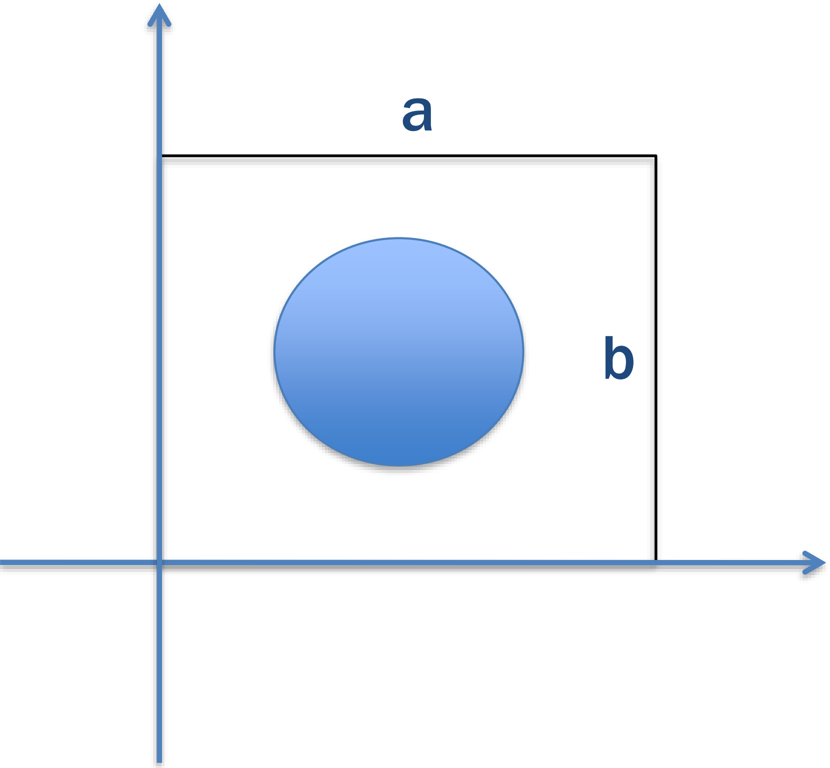


**0 crossing angle/crab cavity**



# Reducing the Numerical Noise Effects through a Spectral Method

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = -4\pi\rho,$$



$$\rho(x,y) = \sum_{l=1}^{N_l} \sum_{m=1}^{N_m} \rho^{lm} \sin(\alpha_l x) \sin(\beta_m y)$$

$$\phi(x,y) = \sum_{l=1}^{N_l} \sum_{m=1}^{N_m} \phi^{lm} \sin(\alpha_l x) \sin(\beta_m y),$$

$$\rho^{lm} = \frac{4}{ab} \int_0^a \int_0^b \rho(x,y) \sin(\alpha_l x) \sin(\beta_m y) dxdy$$

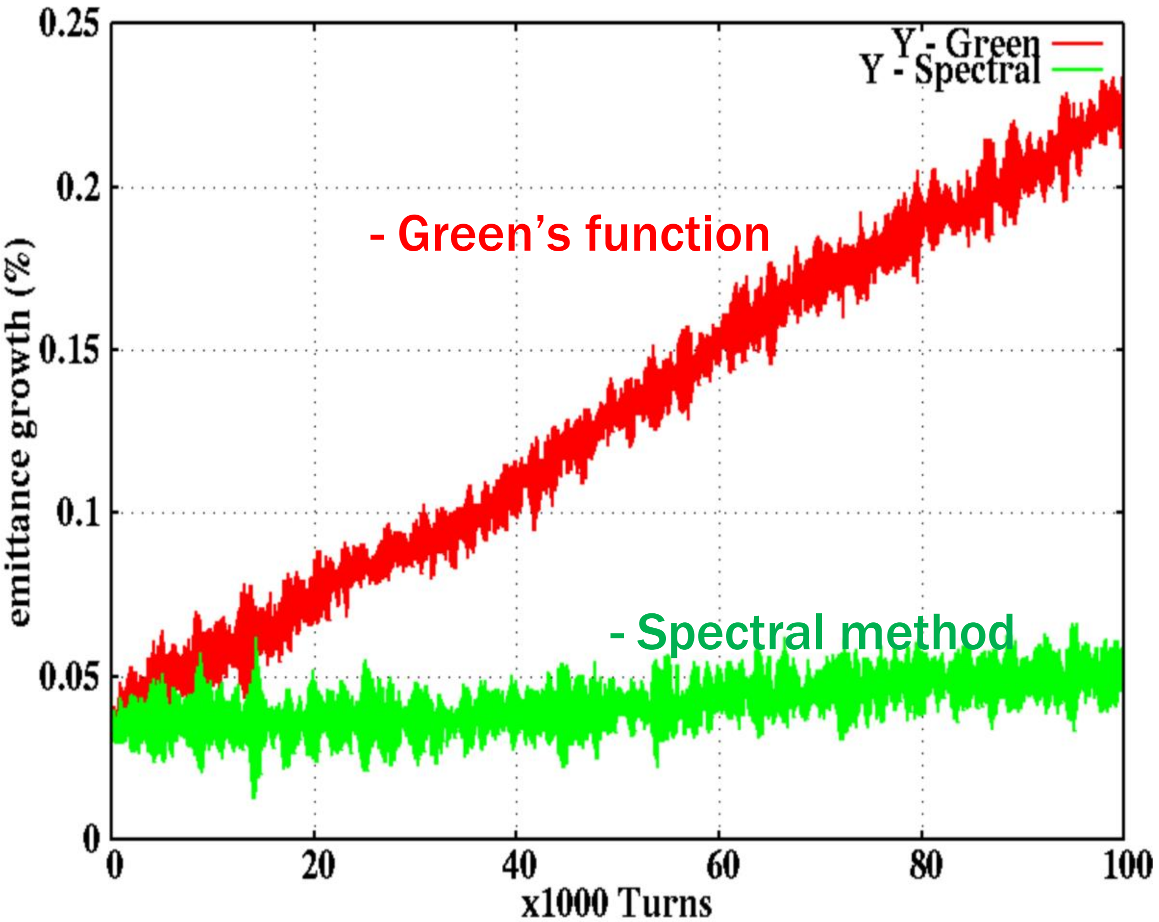
$$\phi^{lm} = \frac{4}{ab} \int_0^a \int_0^b \phi(x,y) \sin(\alpha_l x) \sin(\beta_m y) dxdy,$$

where $\alpha_l = l\pi/a$ and $\beta_m = m\pi/b$.

$$\phi^{lm} = \frac{4\pi\rho^{lm}}{\gamma_{lm}^2}$$

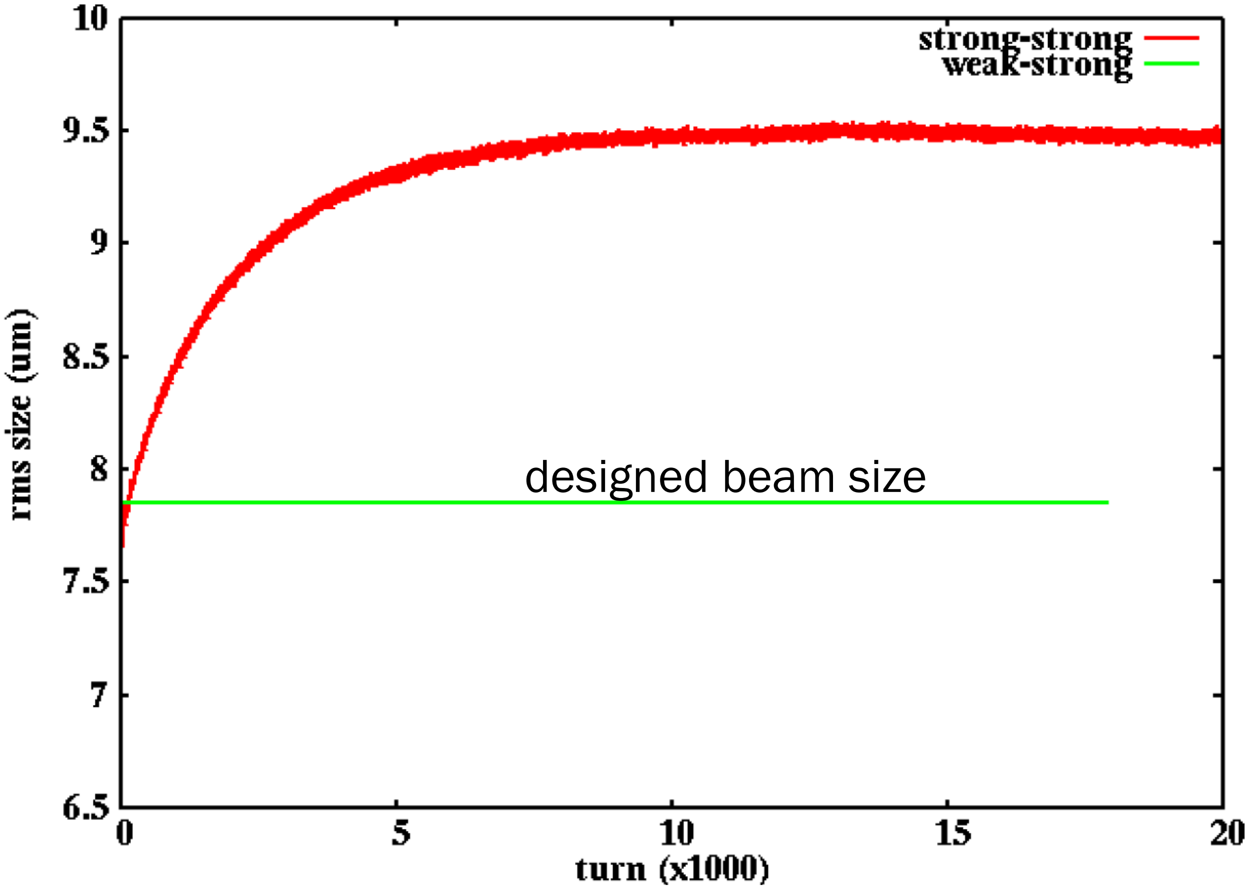
where $\gamma_{lm}^2 = \alpha_l^2 + \beta_m^2$.

# A Spectral Method Might Be Used to Mitigate the Numerical Noise Driven Emittance Growth



- Much smaller numerical noise driven emittance growth using the spectral method in a LHC application

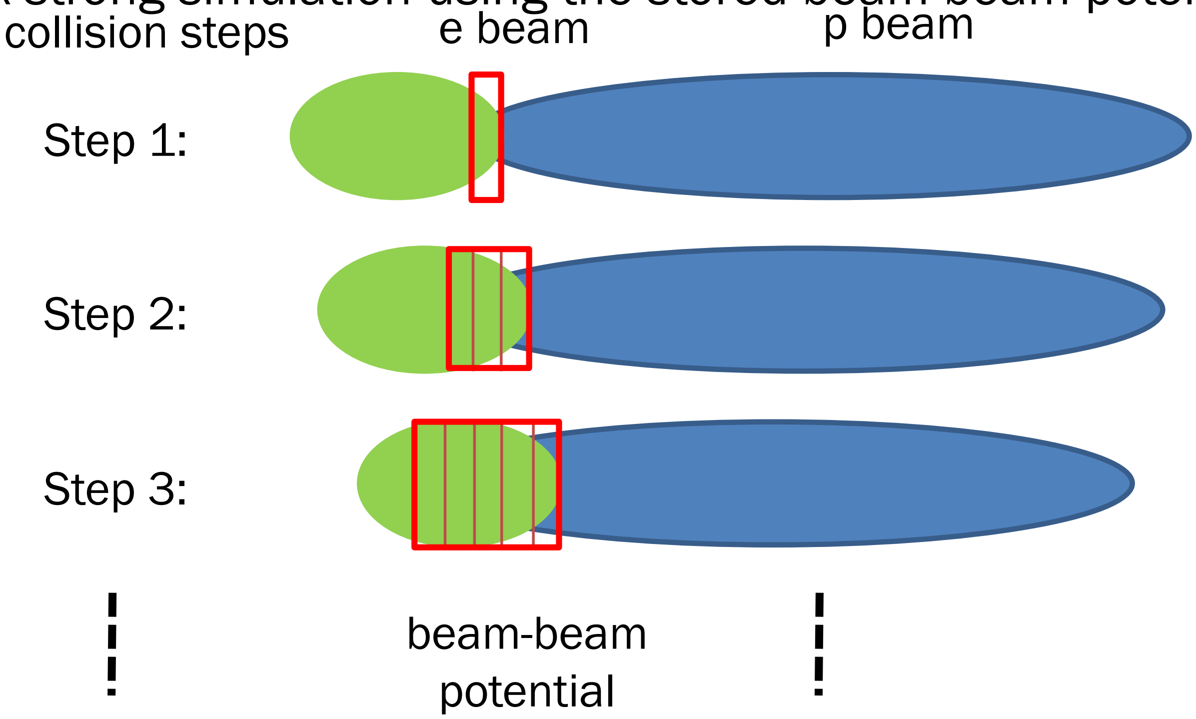
# Electron Beam Vertical RMS Size Evolution from Strong-Strong Simulation in an EIC Design



- Electron beam blow up due to strong coherent beam-beam effects seen in strong-strong simulation

# A Hybrid Strong-Strong and Weak-Strong Model

- Run fully strong-strong beam-beam simulation for a number of turns
- Store the beam-beam interaction potentials during the electron and proton collision
- Switch to weak-strong simulation using the stored beam-beam potentials



collision steps

e beam

p beam

Step 1:

Step 2:

Step 3:

⋮

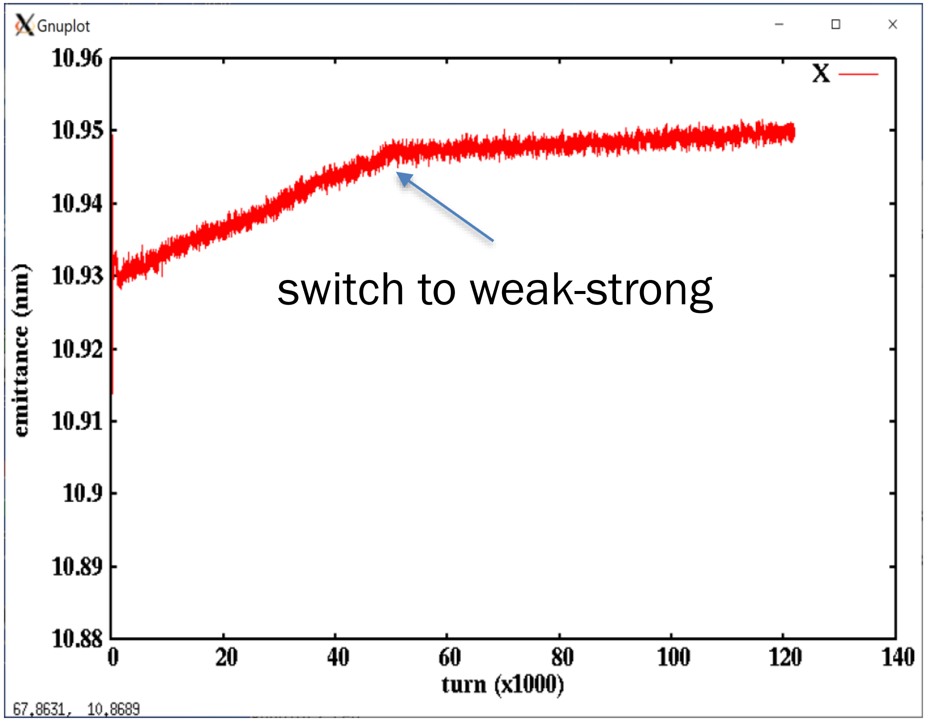
beam-beam potential

⋮

# Less Numerical Emittance Growth in Proton Beam with the Faster Strong-Strong and Weak-Strong Simulation

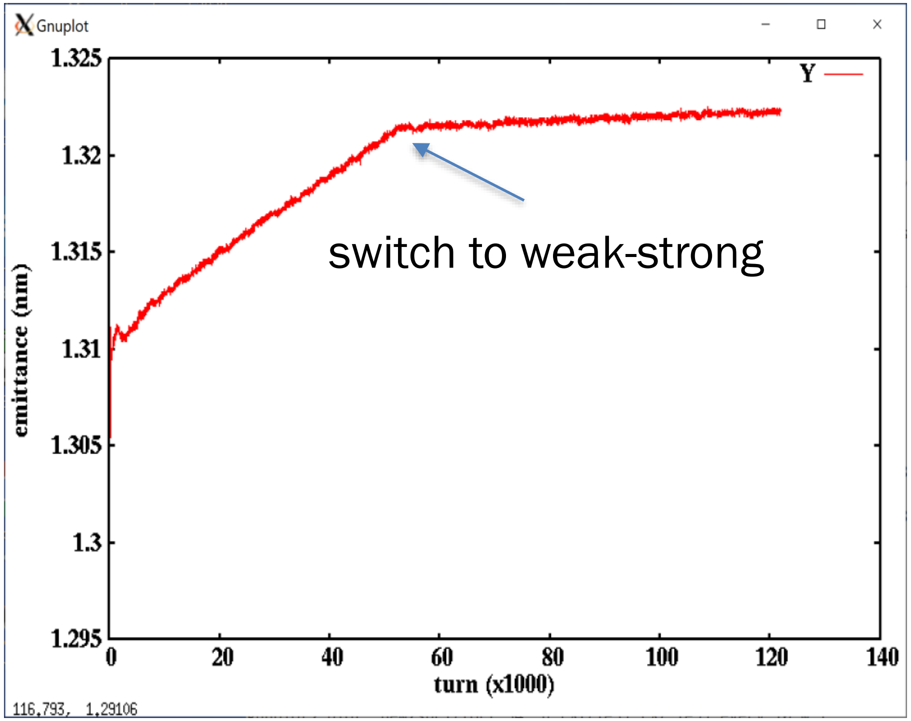
## Horizontal and Vertical RMS Emittance Evolution

horizontal



switch to weak-strong

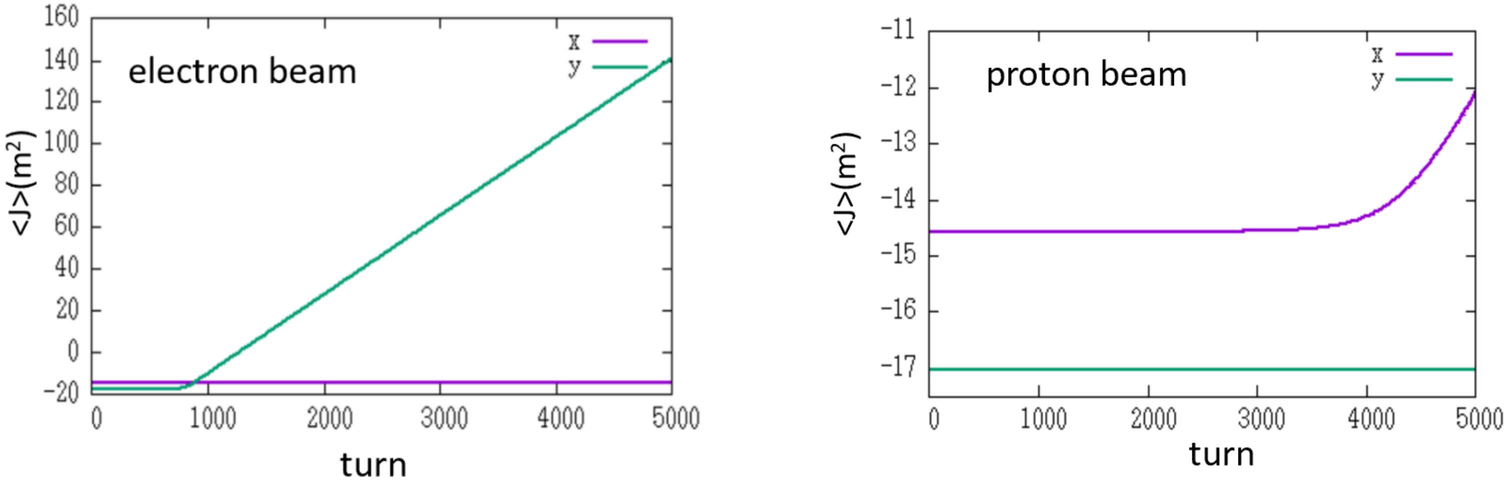
vertical



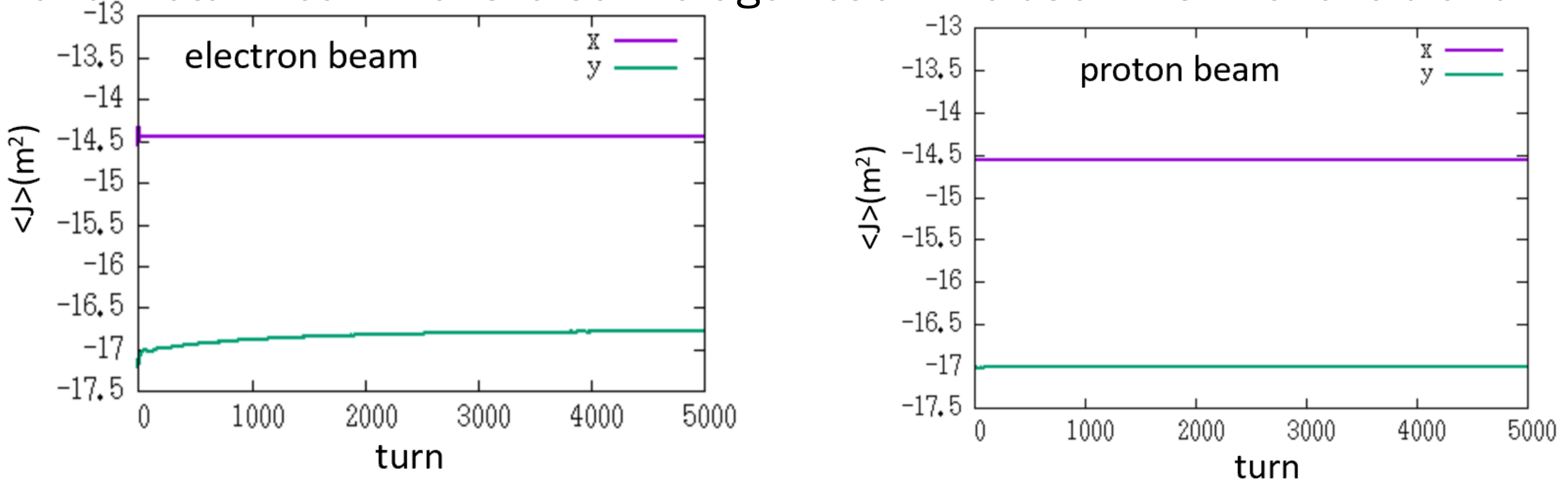
switch to weak-strong

# 3rd Challenge: Beyond Beam-Beam Only Model: Inclusion of Wakefield, Space-Charge, Intrabeam Scattering, etc

Electron and Proton Beam Transverse Average Action Evolution with Wakefield Only in the EIC

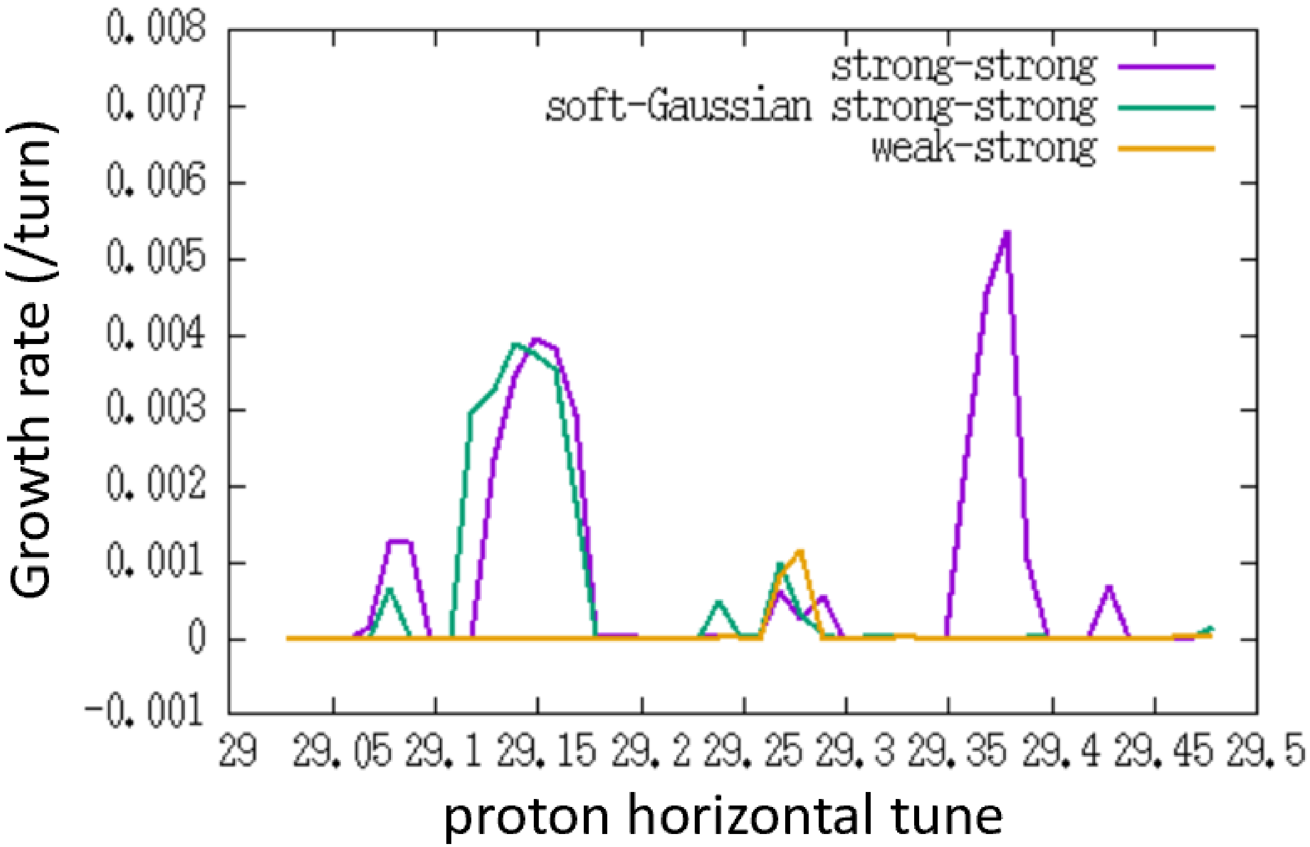


Electron and Proton Beam Transverse Average Action Evolution with Wakefield and Beam-Beam



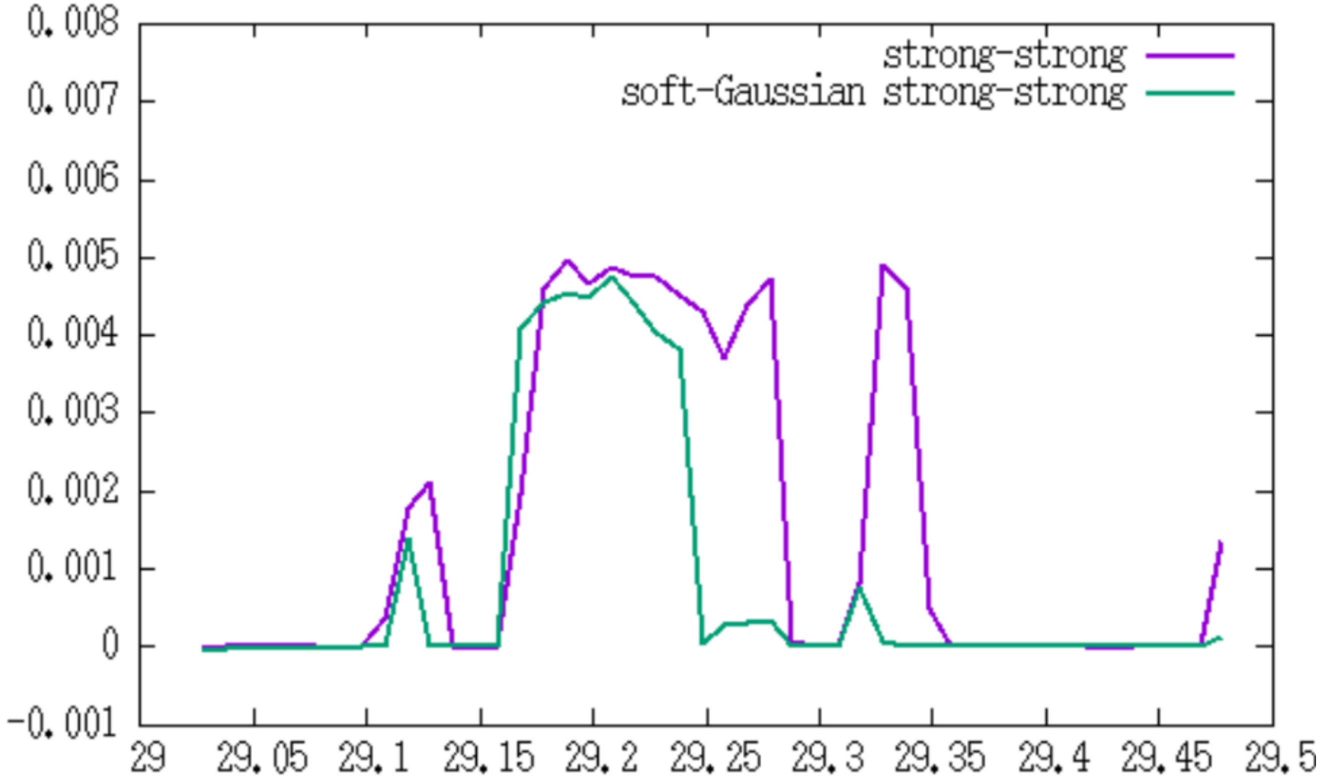
# Strong-Strong Beam-Beam and Wakefield Model Shows More Instability Stopband than Weak-Strong Model

CS Parameter Growth Rate vs. Proton Beam Horizontal Tune with
Fixed EIC CDR Electron Beam Tunes (0.08,0.06) and Vertical Tune (0.21)



# Instability Stopbands Move with the Electron Beam Tune

CS Parameter Growth Rate vs. Proton Beam Horizontal Tune with Fixed New Electron Beam Tunes (0.12,0.06) and Vertical Tune (0.21)



# List of Beam-Beam Codes

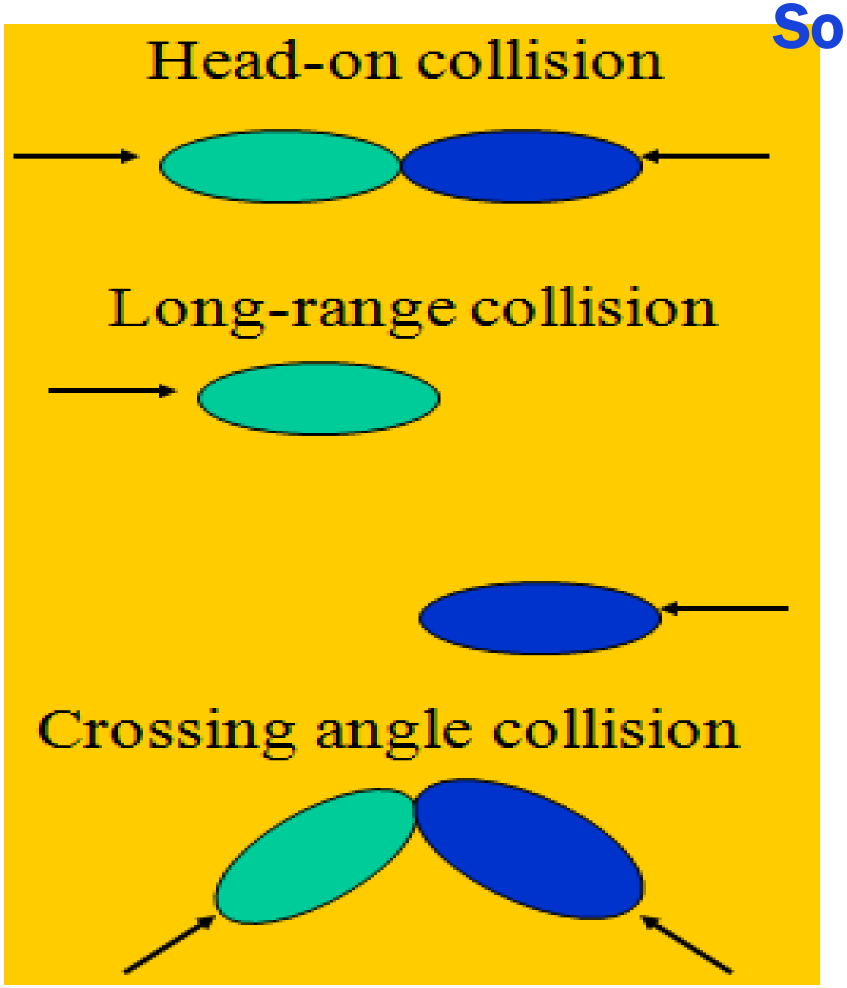
| | Weak-strong 6D | Quasi-strong-strong 6D | Strong-strong 6D SG | Strong-strong 6D PIC | Beamstrahlung | Bhabha-scattering | Transverse wakefields | Longitudinal wakefields | Linear tracking | Lattice tracking | Open source | Runs on GPU |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| GUINEA-PIG [2] | 🟨 | ⬛ | ⬛ | 🟨 | 🟨 | 🟨 | ⬛ | ⬛ | ⬛ | ⬛ | 🟨 | ⬛ |
| COMBI [3] | 🟨 | 🟨 | 🟨 | 🟨 | ⬛ | ⬛ | 🟨 | ⬛ | 🟨 | ⬛ | 🟨 | ⬛ |
| BBWS [4] | 🟨 | ⬛ | ⬛ | ⬛ | 🟨 | ⬛ | 🟨 | 🟨 | 🟨 | 🟨 | ⬛ | ⬛ |
| BBSS [5] | ⬛ | ⬛ | 🟨 | 🟨 | 🟨 | ⬛ | 🟨 | 🟨 | 🟨 | ⬛ | ⬛ | ⬛ |
| SCTR [6] | ⬛ | ⬛ | 🟨 | 🟨 | 🟨 | ⬛ | 🟨 | 🟨 | 🟨 | 🟨 | ⬛ | 🟨 |
| IBB [7] | ⬛ | ⬛ | 🟨 | ⬛ | 🟨 | ⬛ | 🟨 | 🟨 | 🟨 | ⬛ | ⬛ | ⬛ |
| LIFETRAC [8] | 🟨 | 🟨 | ⬛ | ⬛ | 🟨 | ⬛ | ⬛ | ⬛ | 🟨 | 🟨 | ⬛ | ⬛ |
| BeamBeam3D [9] | 🟨 | ⬛ | 🟨 | 🟨 | 🟨 | ⬛ | 🟨 | 🟨 | 🟨 | ⬛ | 🟨 | ⬛ |
| Xsuite [10] | 🟨 | 🟨 | 🟨 | ⬛ | 🟨 | 🟨 | 🟨 | 🟨 | 🟨 | 🟨 | 🟨 | 🟨 |

🟨 Available

⬛ Not available

Ref: P. Kicsiny et al., FCC week, 11 June 2024.

# **BeamBeam3D:** A Parallel Colliding Beam Simulation Code (https://github.com/beam-beam/BeamBeam3D)



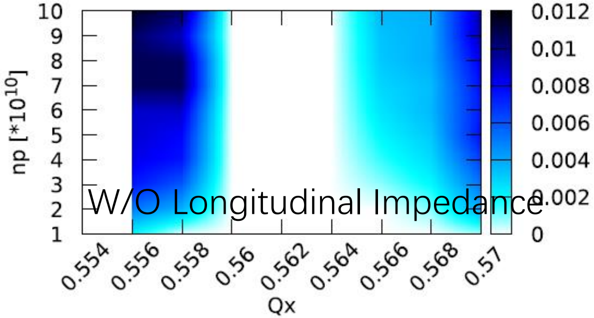
## Some key features of the BeamBeam3D

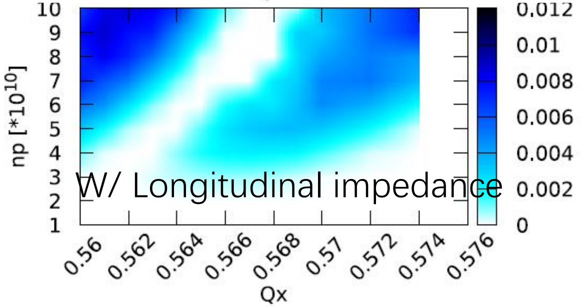
- Multiple-slice model for finite bunch length
- New algorithm -- shifted Green function -- efficiently models long-range collisions
- Parallel particle-field based decomposition to achieve perfect load balance
- Lorentz boost to handle crossing angle
- Arbitrary closed-orbit separation
- Multiple bunches, multiple collision points
- Linear transfer matrix + one turn chromaticity+amplitude dependent tune
- Read-in 2nd order + 3rd order transfer maps
- Conducting wire, crab cavity, e-lens compensation model
- Feedback model
- Impedance model (short-range+long-range, x+y+z)
- Beamstrahlung model

J. Qiang et al., "A Parallel Particle-In-Cell Model for Beam-Beam Interactions in High Energy Ring Colliders," J. Comp. Phys. vol. 198, 278 (2004).

# Introduction of IBB

- **Linear Arc Map with SR radiation**
- **One turn map including general chromaticity**
- **Horizontal crossing angle: Lorentz boost map**
- **Bunch slice number is about 10 times Piwinski angle**
- **Slice-Slice collision:**
  - **Synchro-beam mapping method**
  - **Integral of Green function (flat beam)**
  - **PIC: FACR -> FFT (shift Green method)**
- **Beamstrahlung: Synchrotron radiation during collision**
- **Longitudinal wakefield**
  - **Impact of Potential-well-distortion on X-Z instability**
- **Transverse wakefield**
  - **Impact of vertical beam-beam impedance on TMCI instabilit**
- **Multiple IPs and Multiple Bunches**
- **Combination of lattice and beam-beam: IBB->APES-T**





Y. Zhang et al., PRST-AB, 8, 074402 (2005)

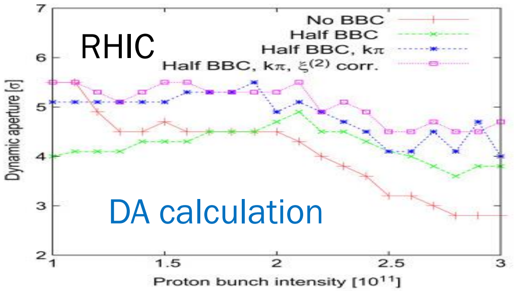
# SimTrack

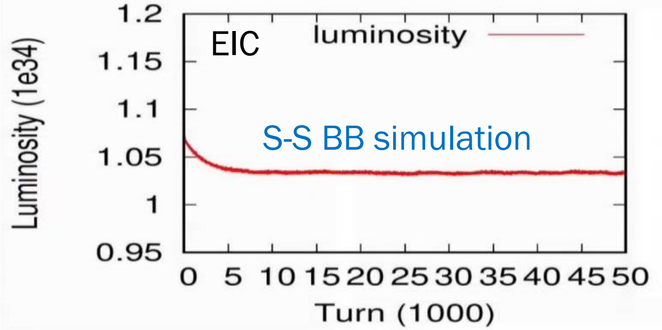
- **A c++ library for 6-d symplectic element-by-element particle tracking in circular accelerators. It includes 4th symplectic integration through magnets and 6-d synchro-beam map for weak-strong beam-beam.**

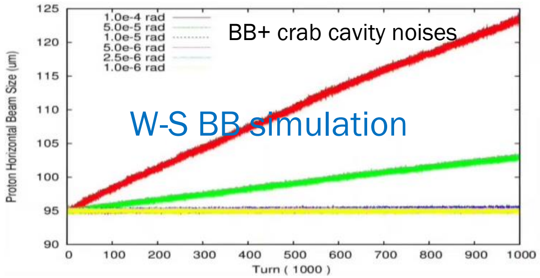
- **Since its inception in 2009, SimTrack has been intensively used for dynamic aperture calculations with beam–beam interaction for RHIC. Recently a strong-strong beam-beam code (OMP) was built on SimTrack for the EIC beam-beam simulations.**

- **Features of SimTrack:**
  - **1) c++ class based: easy to define new element types**
  - **2) element & line manipulations: insert, delete, revert, rewind**
  - **3) linear optics calculation, together with limited nonlinear optics calculation and limited optimization methods**
  - **4) element parameters can be changed during tracking**
  - **4) track hadrons and electrons**
  - **5) spin tracking included**

Ref: Y. Luo, “SimTrack: A compact c++ code for particle orbit and spin tracking in accelerators‘’, NIMA, v801, pp.95-103, 2015.



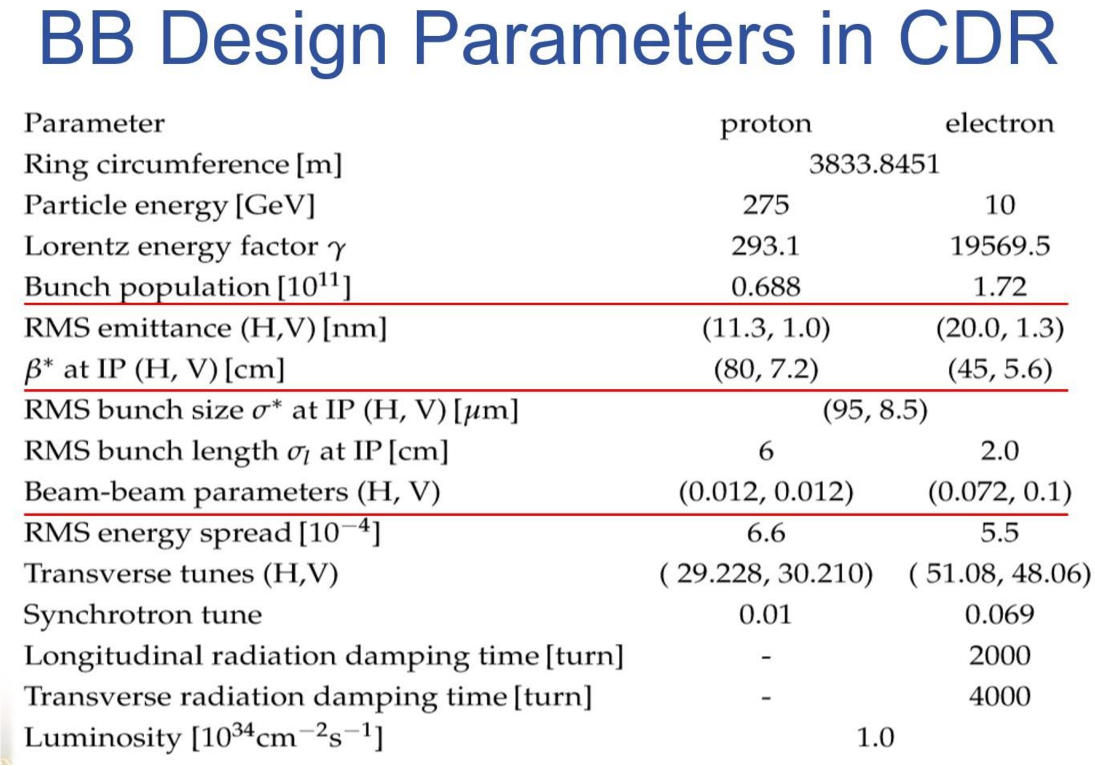




# Outlook for Modeling Beam-Beam Effects

- **Incorporate AI/ML into beam-beam modeling**
- **Apply differentiable simulation to the beam-beam modeling**
- **Integrate beam-beam simulation with optimization**

# Differentiable Simulation

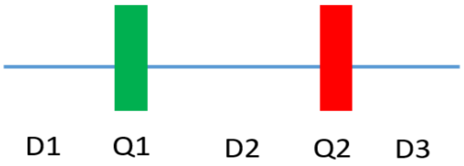


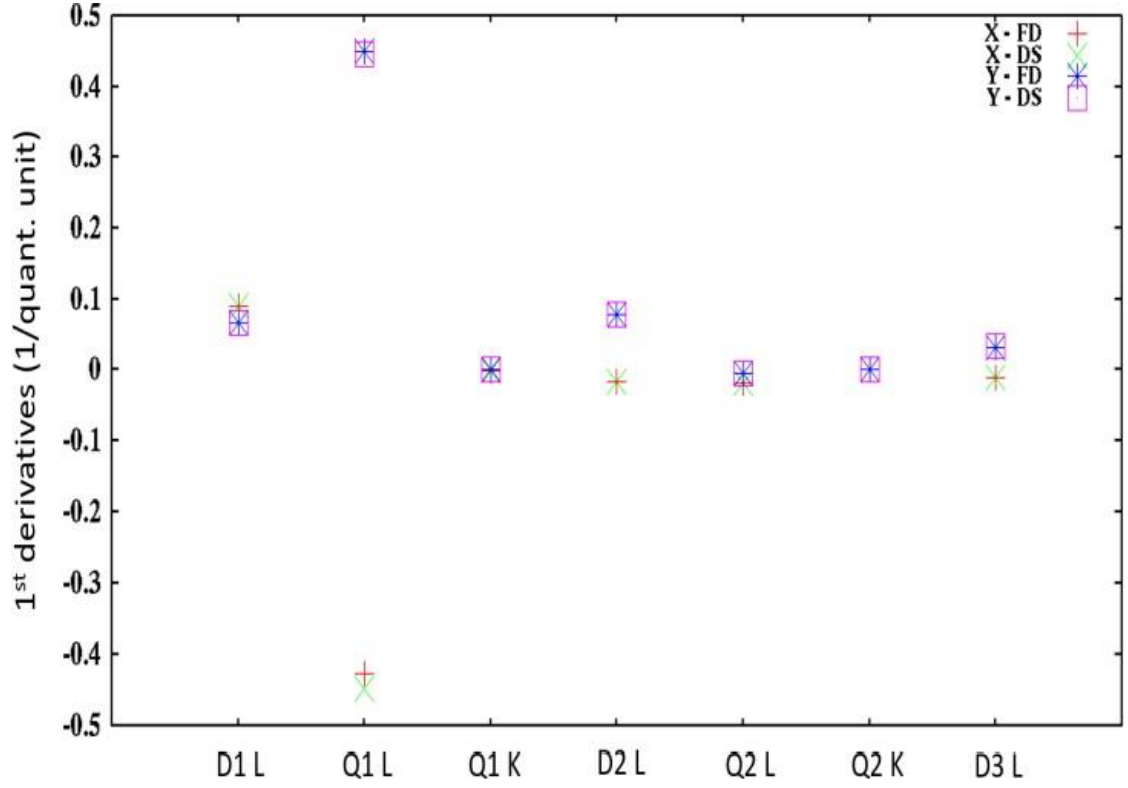
## BB Design Parameters in CDR

| Parameter | proton | electron |
|---|---|---|
| Ring circumference [m] | 3833.8451 | |
| Particle energy [GeV] | 275 | 10 |
| Lorentz energy factor $\gamma$ | 293.1 | 19569.5 |
| Bunch population [$10^{11}$] | 0.688 | 1.72 |
| RMS emittance (H,V) [nm] | (11.3, 1.0) | (20.0, 1.3) |
| $\beta^*$ at IP (H, V) [cm] | (80, 7.2) | (45, 5.6) |
| RMS bunch size $\sigma^*$ at IP (H, V) [$\mu$m] | (95, 8.5) | |
| RMS bunch length $\sigma_l$ at IP [cm] | 6 | 2.0 |
| Beam-beam parameters (H, V) | (0.012, 0.012) | (0.072, 0.1) |
| RMS energy spread [$10^{-4}$] | 6.6 | 5.5 |
| Transverse tunes (H,V) | ( 29.228, 30.210) | ( 51.08, 48.06) |
| Synchrotron tune | 0.01 | 0.069 |
| Longitudinal radiation damping time [turn] | - | 2000 |
| Transverse radiation damping time [turn] | - | 4000 |
| Luminosity [$10^{34}$cm$^{-2}$s$^{-1}$] | 1.0 | |

- How sensitive is the luminosity w.r.t. these design parameters (~30)?
- *The differentiable simulation is a simulation that can automatically compute derivatives of the simulation result with respect to its input parameters.*

# Derivatives of the X and Y Emittances w.r.t. 7 Lattice Parameters from 1 Differentiable Simulation and from Finite Difference Approximation with Multiple Simulations Shows Good Agreement





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*Thank You!*