Estimation of beam-beam blowup and lifetime with lattice and beamstrahlung

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Many thanks to X. Buffat, P. Kicsiny, K. Ohmi, T. Pieloni, P. Raimondi, G. Roy, L.V. Riesen-Haupt, D. Shatilov, F. Zimmermann, and all FCC-ee/FCCIS colleagues

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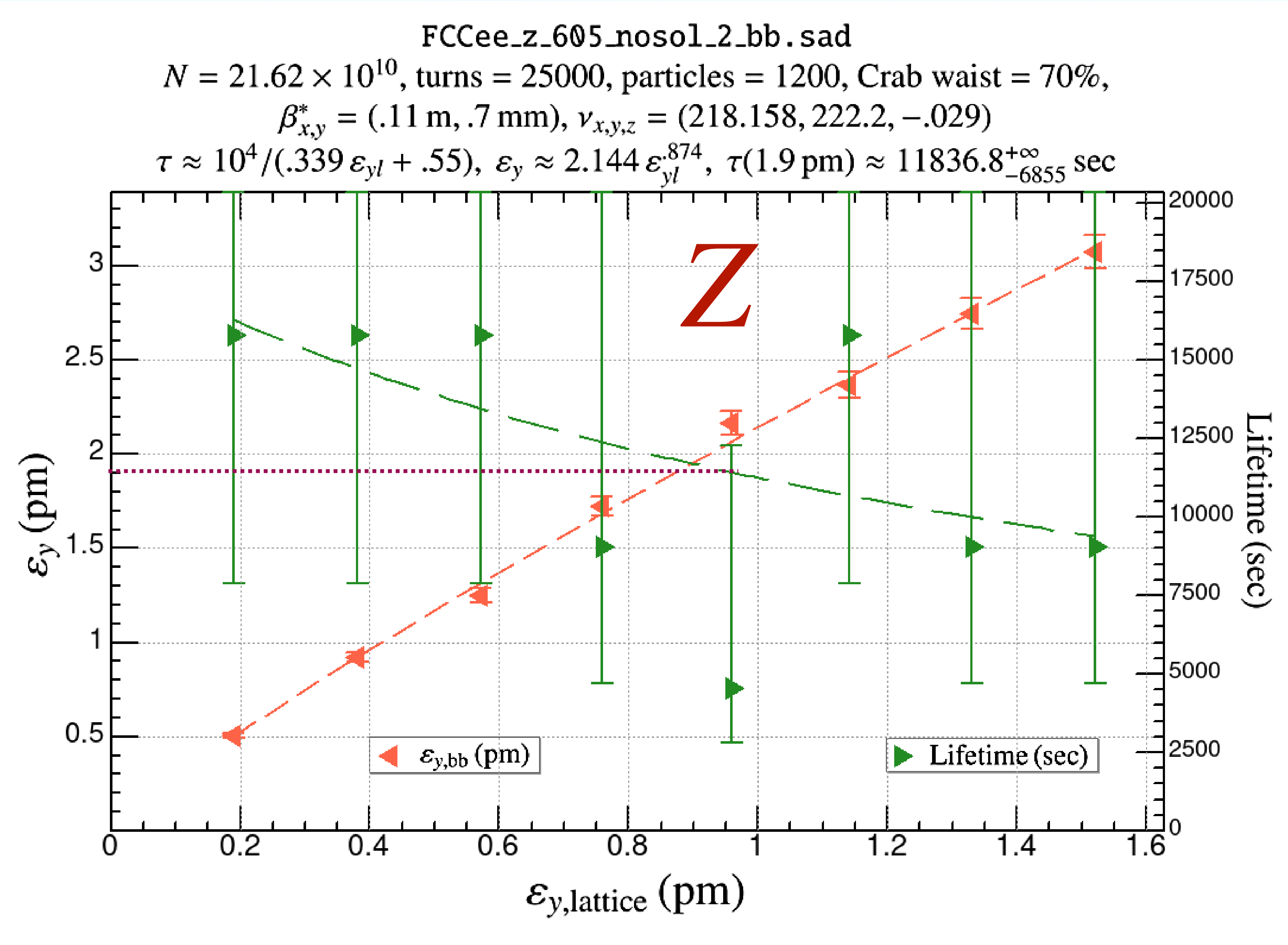
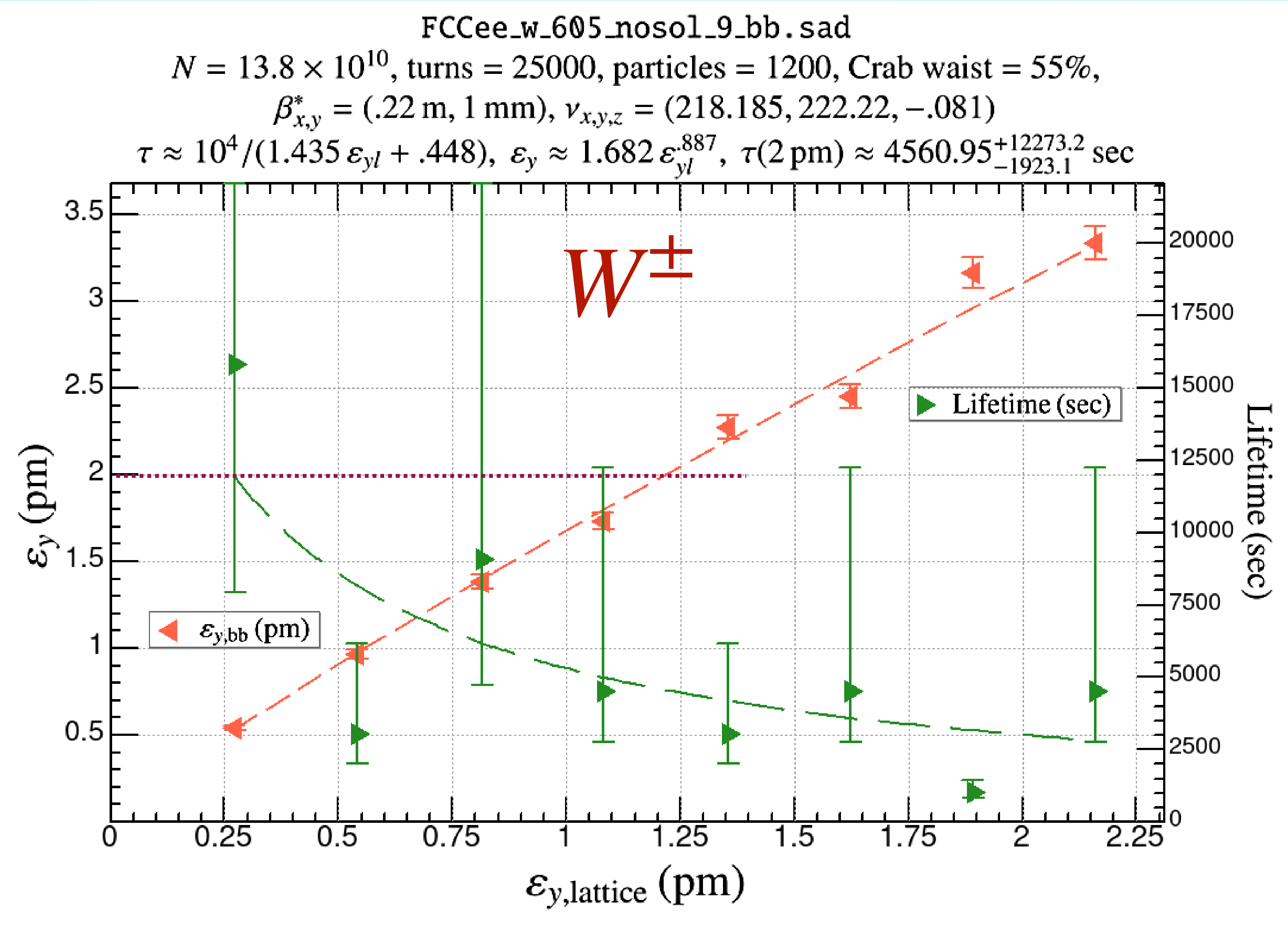
Lifetime & beam blowup with lattice + beam beam & beamstrahlung
Simulation method

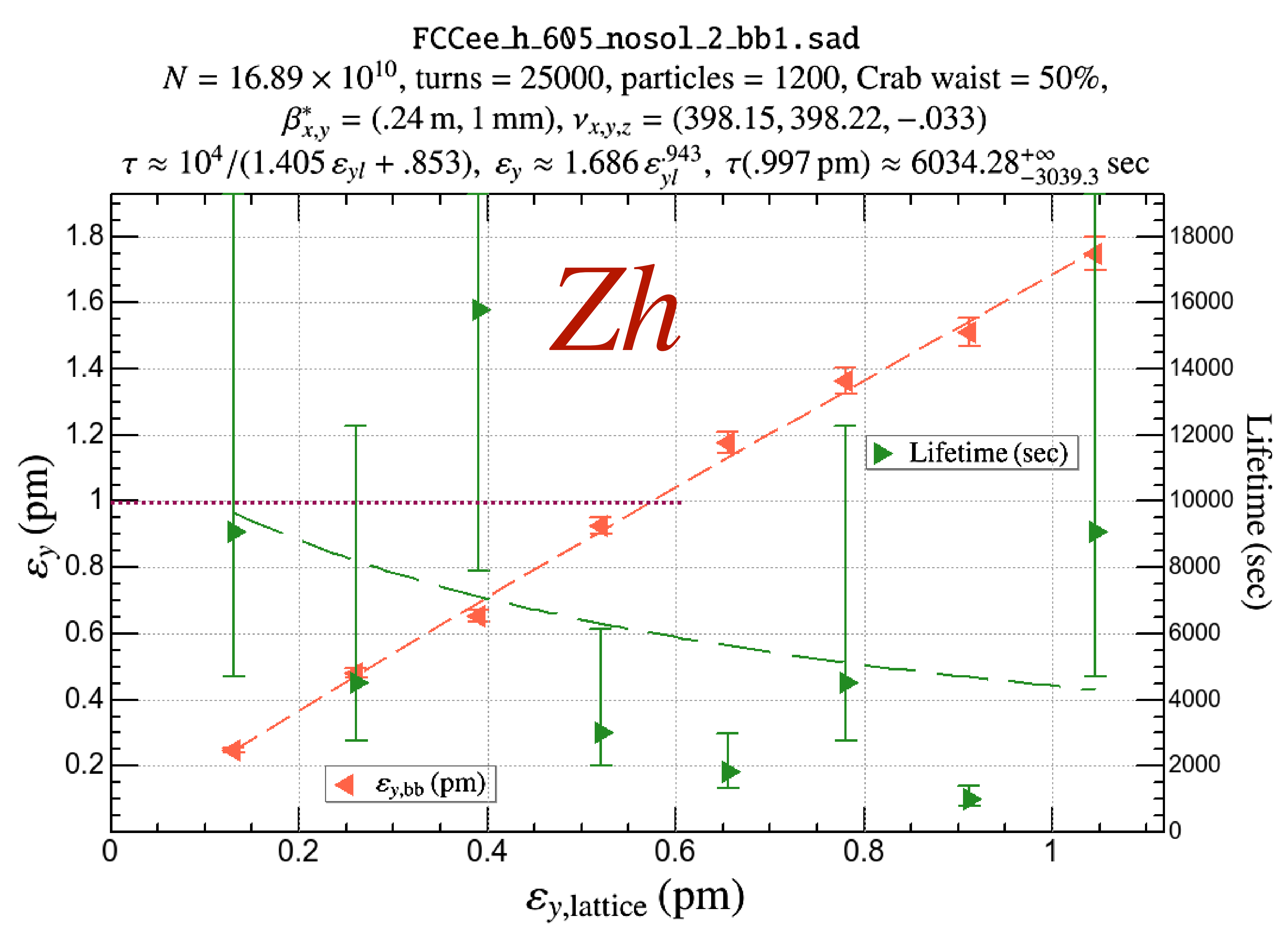
FUTURE CIRCULAR COLLIDER

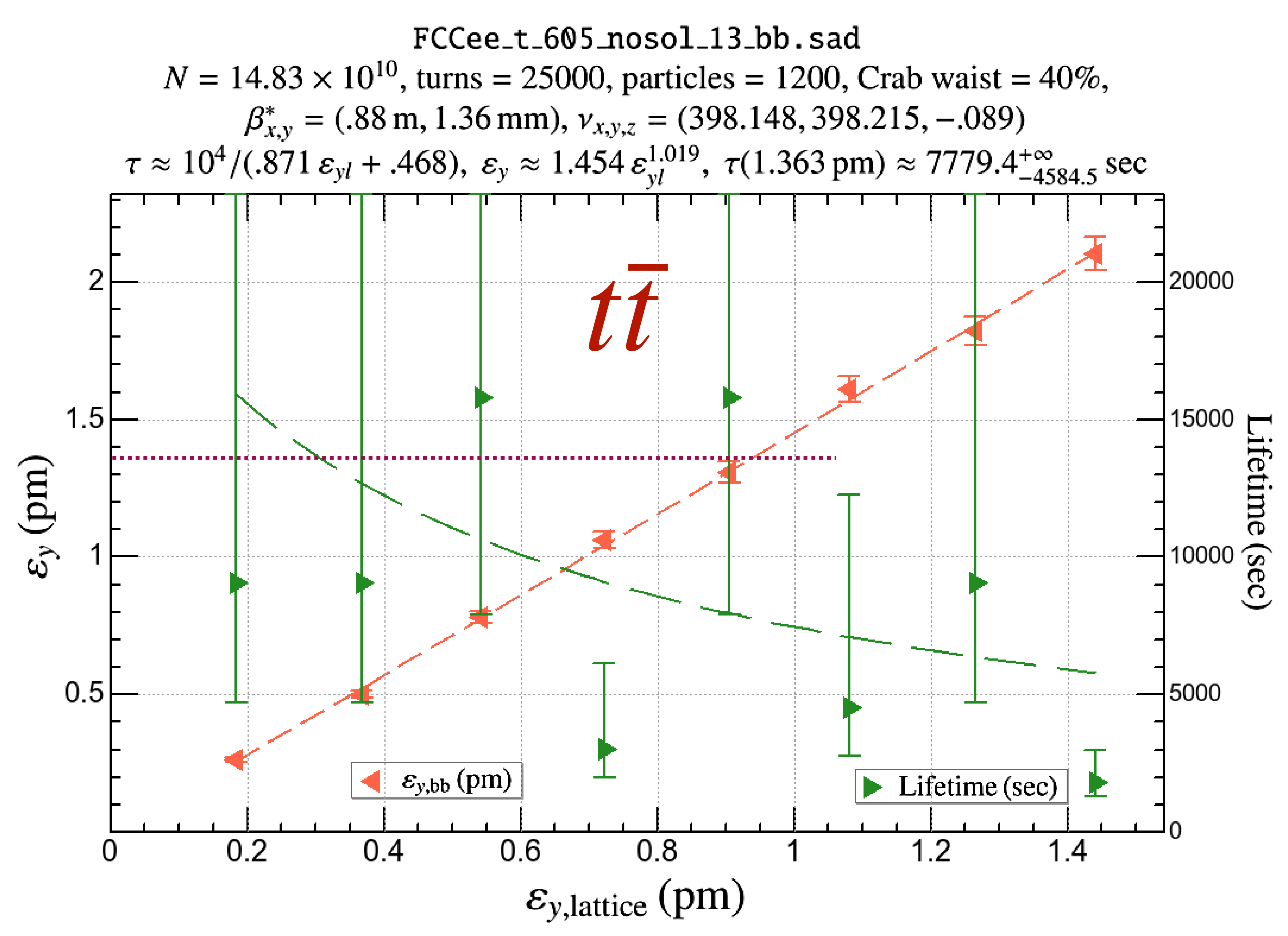
- "GHC" lattice, produced by SAD also converted to MADX.
  - Lattice Repository: https://gitlab.cern.ch/acc-models/fcc/fcc-ee-lattice by G. Roy
  - SAD: https://hep-project-sad.web.cern.ch/SADHelp/SADHelp.html
- A weak-strong model (BBWS by K. Ohmi) at 4 IPs, with beamstrahlung.
  - 200 slices on a bunch for a crossing, with ±15 mrad horizontal crossing angle.
- Synchrotron radiation (SR) in all accelerator elements, including magnet fringe.
  - "Tapering" scheme is applied to compensate the SR effects on orbit and optics.
- No solenoid.
- No machine errors.
- Using the HPC-BATCH "muon" cluster.
  - Full multithread computation, using 96 cores × 2 nodes.
  - Calculation for FCC-ee takes 15 - 20 minutes for 25000 turns with 1200 particles
    - More time needed per turn for higher energy due to the radiation.
    - Lattice tracking takes more time than beam-beam.

Lifetime & beam blowup with lattice + beam beam & beamstrahlung









- The vertical emittance after collision (red) and the lifetime (green) against the lattice vertical emittance for each collision energy.
- The purple horizontal dashed line shows the goal vertical emittance at collision, where the vertical emittance of the strong beam is set at.
- These results, and also the DA, have been reproduced by independent simulations by P. Kicsiny at FCCIS 2024: https://indico.cern.ch/event/1335891/contributions/5632544/attachments/2745020/4776609/pkicsiny_fccee_optics_meeting_2023_11_02.pdf
- except for the lifetime at that time - recently solved by L.V. Riesen-Haupt (talk at 09:50 today).

Parameters

FCC-ee collider parameters for the GHC lattice as of Aug. 2, 2024.

Beam energy	[GeV]	45.6	80	120	182.5
Layout		PA31-3.0			
# of IPs		4			
Circumference	[km]	90.658728			
Bend. radius of arc dipole	[km]	10.021			
Energy loss / turn	[GeV]	0.0390	0.369	1.86	9.94
SR power / beam	[MW]	50			
Beam current	[mA]	1283	135	26.8	5.0
Colliding bunches / beam		11200	1852	300	64
Colliding bunch population	$[10^{11}]$	2.16	1.38	1.69	1.48
Hor. emittance at collision ε_x	[nm]	0.70	2.16	0.66	1.51
Ver. emittance at collision ε_y	[pm]	1.9	2.0	1.0	1.36
Lattice ver. emittance $\varepsilon_{y,\mathrm{lattice}}$	[pm]	0.87	1.20	0.57	0.94
Arc cell		Long 90/90		90/90	
Momentum compaction α_p	$[10^{-6}]$	28.67		7.52	
Arc sext families		75		146	
$\beta^*_{x/y}$	[mm]	110 / 0.7	220 / 1	240 / 1	900 / 1.4
Transverse tunes $Q_{x/y}$		218.158 / 222.220	218.185 / 222.220	398.150 / 398.220	398.148 / 398.215
Chromaticities $Q'_{x/y}$		0 / +5	0 / +5	0 / 0	0 / 0
Energy spread (SR/BS) σ_δ	[%]	0.039 / 0.110	0.069 / 0.105	0.102 / 0.176	0.152 / 0.184
Bunch length (SR/BS) σ_z	[mm]	5.57 / 15.6	3.46 / 5.28	3.26 / 5.59	1.91 / 2.32
RF voltage 400/800 MHz	[GV]	0.079 / 0	1.00 / 0	2.09 / 0	2.1 / 9.20
Harm. number for 400 MHz		121200			
RF frequency (400 MHz)	MHz	400.787129			
Synchrotron tune Q_s		0.0289	0.0809	0.0334	0.0881
Long. damping time	[turns]	1171	218	65.4	19.4
RF acceptance	[%]	1.06	3.32	2.06	3.06
Energy acceptance (DA)	[%]	±1.0	±1.0	±1.9	-2.8/+2.5
Beam crossing angle at IP θ_x	[mrad]	±15			
Crab waist ratio	[%]	70	55	50	40
Beam-beam ξ_x/ξ_y [a]		0.0022 / 0.0977	0.013 / 0.129	0.0108 / 0.130	0.065 / 0.136
Piwinski angle $(\theta_x \sigma_{z,\mathrm{BS}})/\sigma^*_x$		26.6	3.6	6.6	0.94
Lifetime (q + BS + lattice)	[sec]	11800	4500	6000	7700
Lifetime (lum) [b]	[sec]	1330	960	600	670
Luminosity / IP	$[10^{34}/\mathrm{cm}^2\mathrm{s}]$	143	20	7.5	1.38

[a] incl. hourglass.
[b] only the energy acceptance is taken into account for the cross section, no beam size effect.

beamstrahlung dominated

Tune scan (lattice + beam-beam + beamstrahlung with SAD/BBWS)

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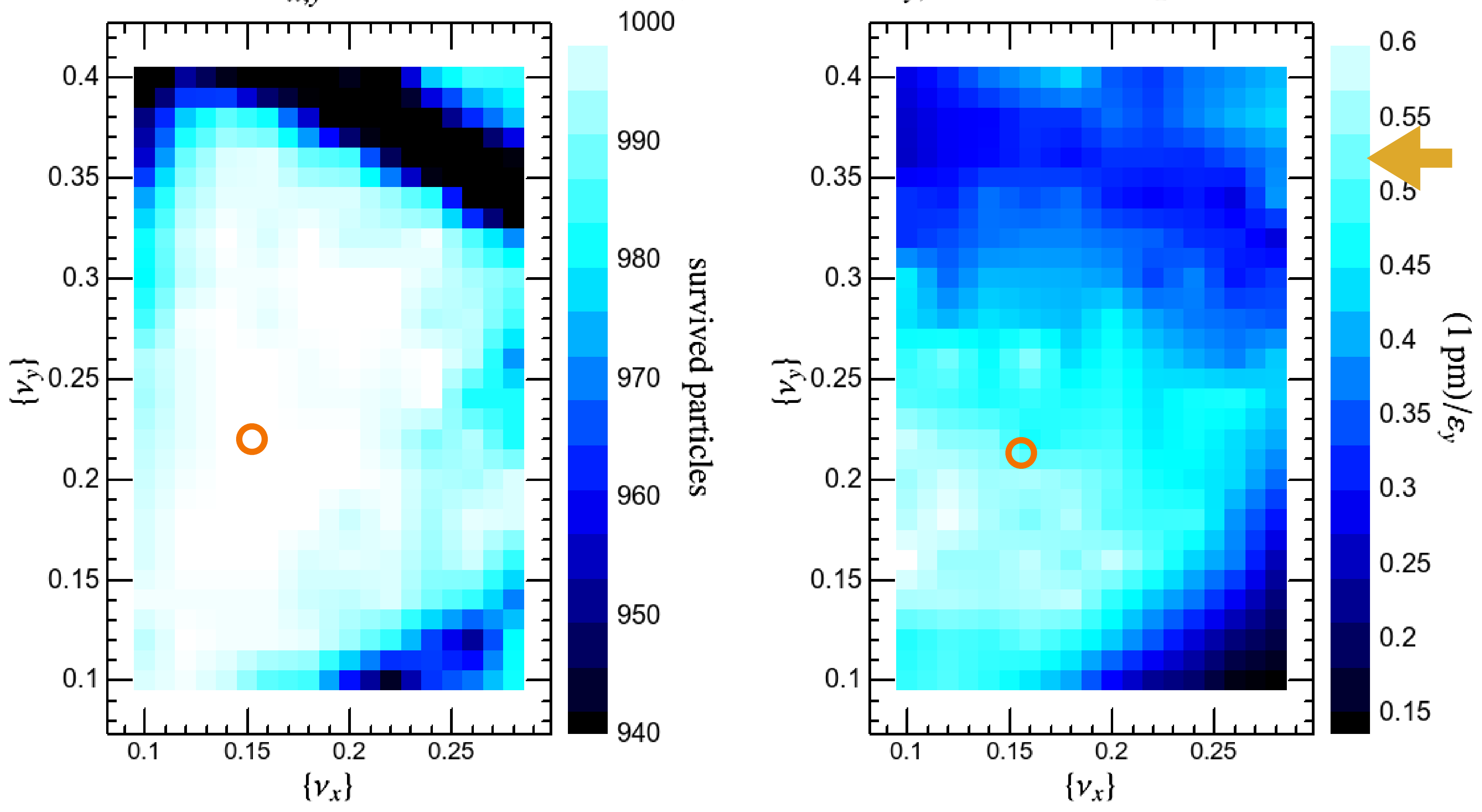
Z

FCCee_z_605_nosol_2_bb_ts.sad

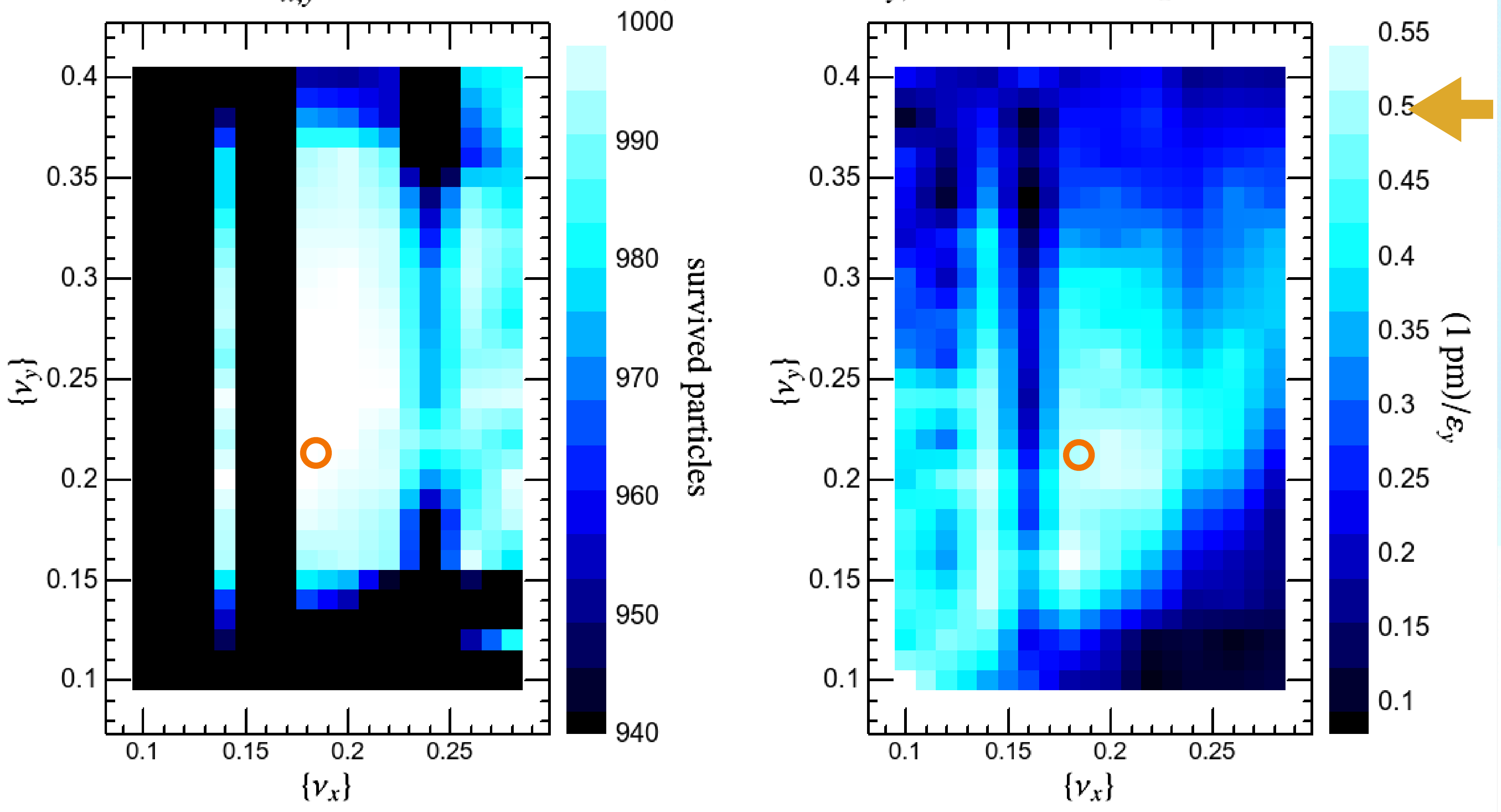
$N = 2.16 \times 10^{11}$, Crab waist = 70%, turns = 4000, particles = 1000, $\beta^*_{x,y} = \{.11\,\mathrm{m},\ .7\,\mathrm{mm}\}$, $\nu_z = -.0288$, $\varepsilon_{y,\mathrm{lattice}} = .8712\,\mathrm{pm}$



$W^\pm$

FCCee_w_605_nosol_9_bb_ts.sad

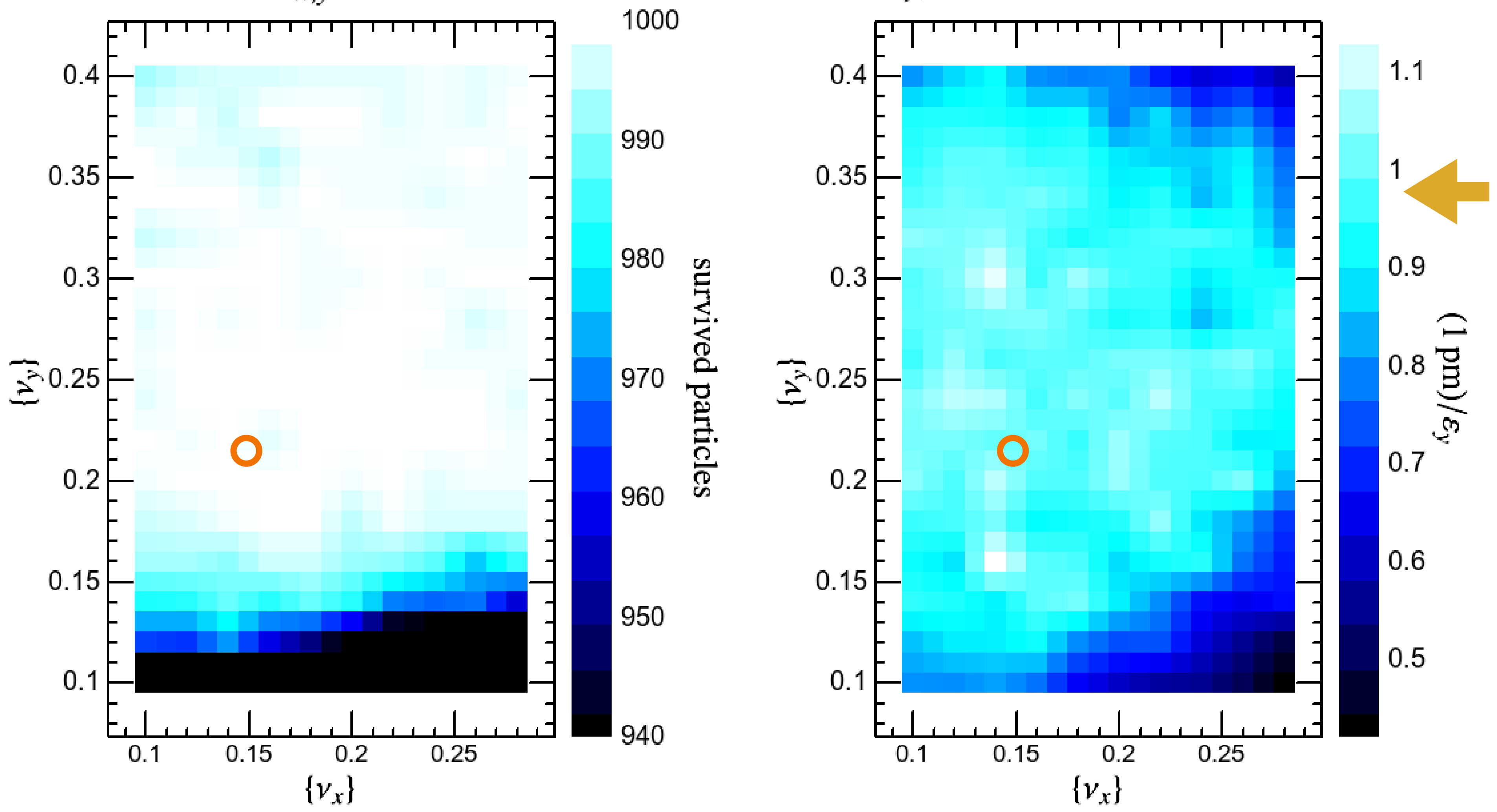
$N = 1.38 \times 10^{11}$, Crab waist = 55%, turns = 4000, particles = 1000, $\beta^*_{x,y} = \{.22\,\mathrm{m},\ 1\,\mathrm{mm}\}$, $\nu_z = -.0808$, $\varepsilon_{y,\mathrm{lattice}} = 1.2019\,\mathrm{pm}$



Zh

FCCee_h_605_nosol_2_bb1_ts.sad

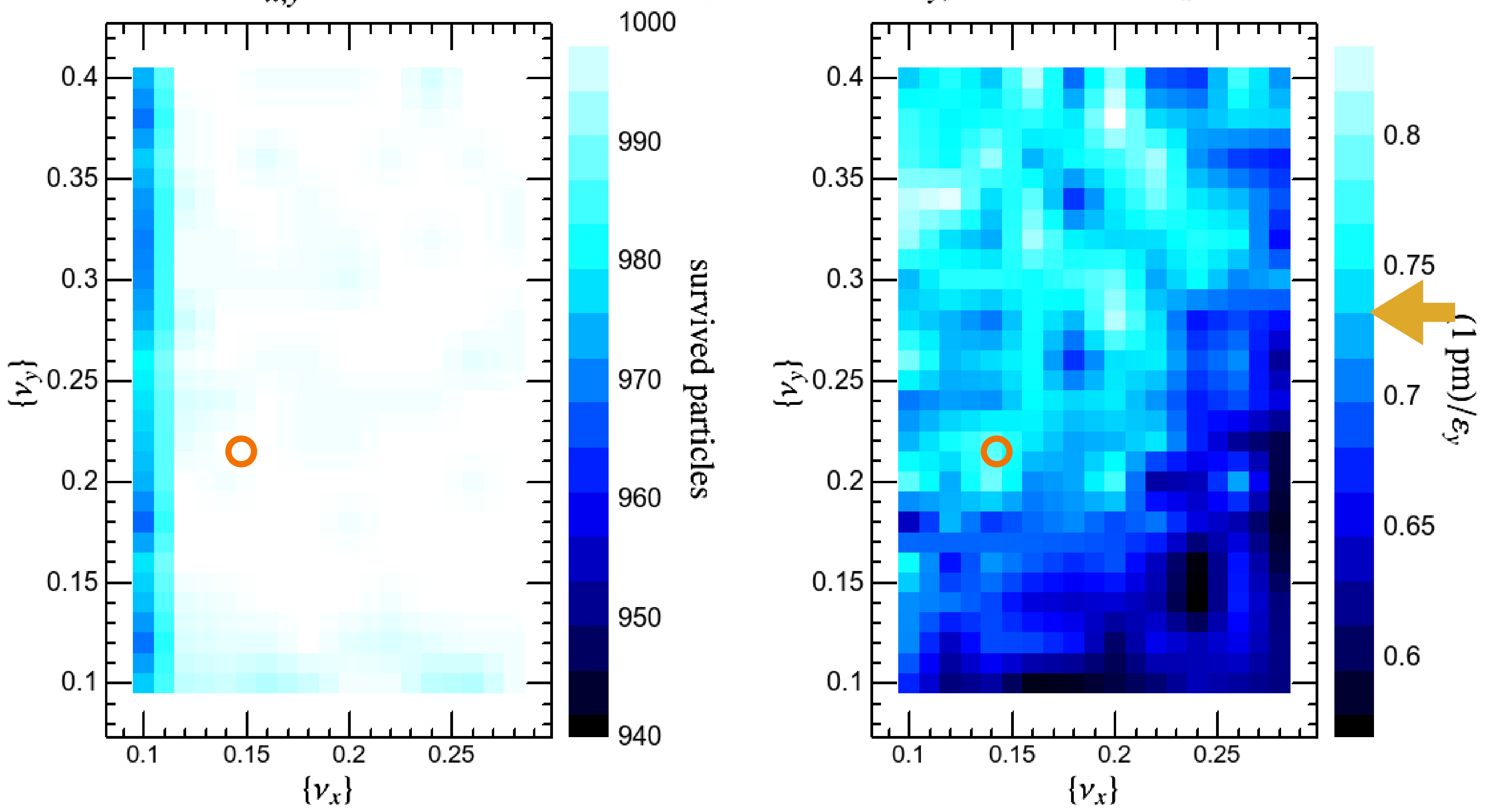
$N = 1.69 \times 10^{11}$, Crab waist = 50%, turns = 4000, particles = 1000, $\beta^*_{x,y} = \{.24\,\mathrm{m},\ 1\,\mathrm{mm}\}$, $\nu_z = -.0327$, $\varepsilon_{y,\mathrm{lattice}} = .5718\,\mathrm{pm}$



$t\bar{t}$

FCCee_t_605_nosol_13_bb_ts.sad

$N = 1.48 \times 10^{11}$, Crab waist = 40%, turns = 4000, particles = 1000, $\beta^*_{x,y} = \{.88\,\mathrm{m},\ 1.36\,\mathrm{mm}\}$, $\nu_z = -.0872$, $\varepsilon_{y,\mathrm{lattice}} = .9494\,\mathrm{pm}$



- Each plot shows the particle loss (left) and vert. emittance after collision (right) with each lattice. The circles show the current working point.
  - The gradation of the design emittance is shown by the yellow arrow.
- Very strong synchrotron sidebands $\nu_x + n\nu_z = N$, $(n = 1,2,3)$ are seen at $W^\pm$.
- A strong "chromatic-crab" resonance line $\nu_x + 2\nu_y - \nu_z = N$ is observed with Z & W lattices. **This resonance strongly depends on the magnitude of the crab waist (see slides later).**
  - At higher energies (Zh, $t\bar{t}$), the chromatic-crab resonance seems weaker or invisible.

Dynamic aperture (z-x)

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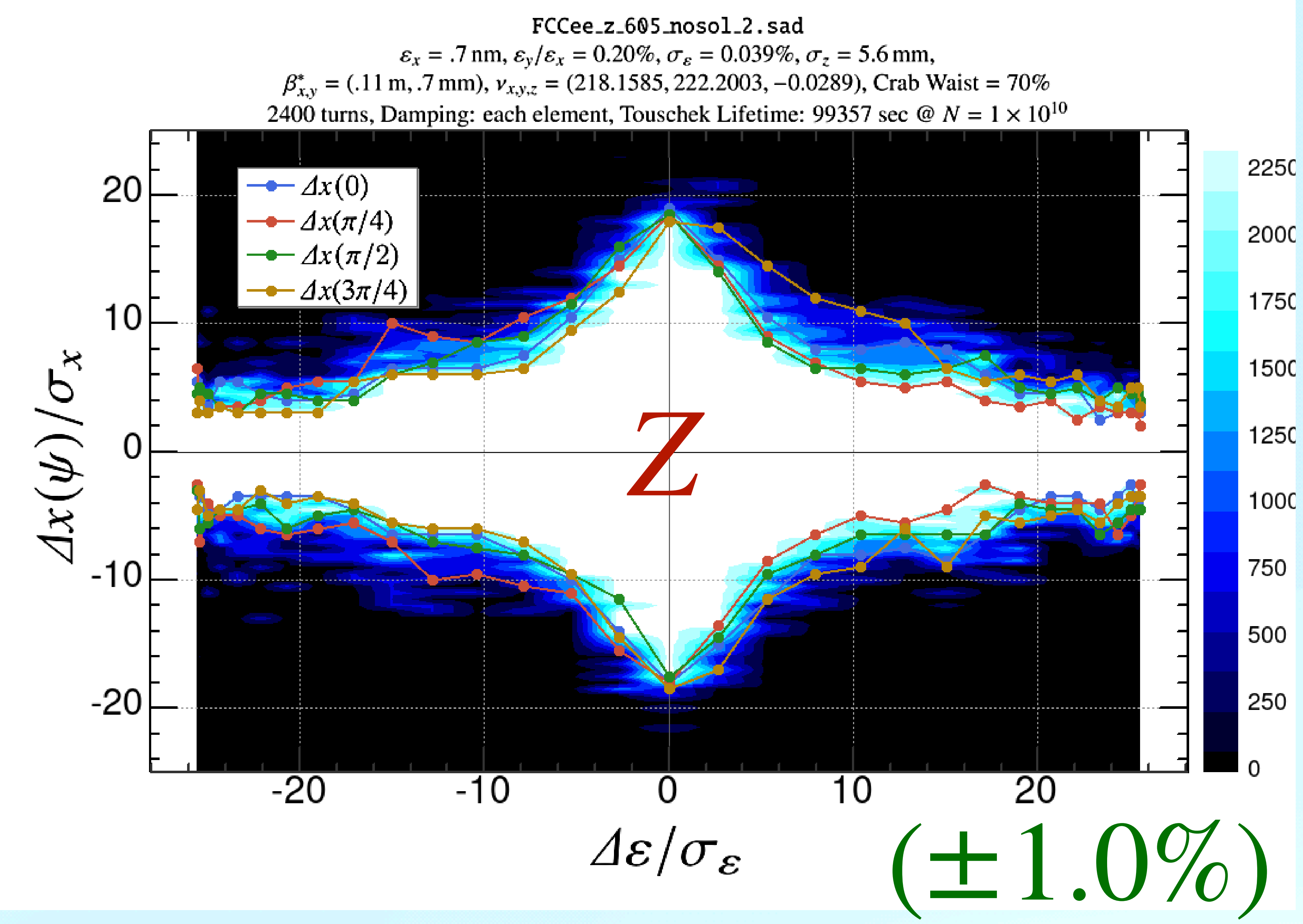
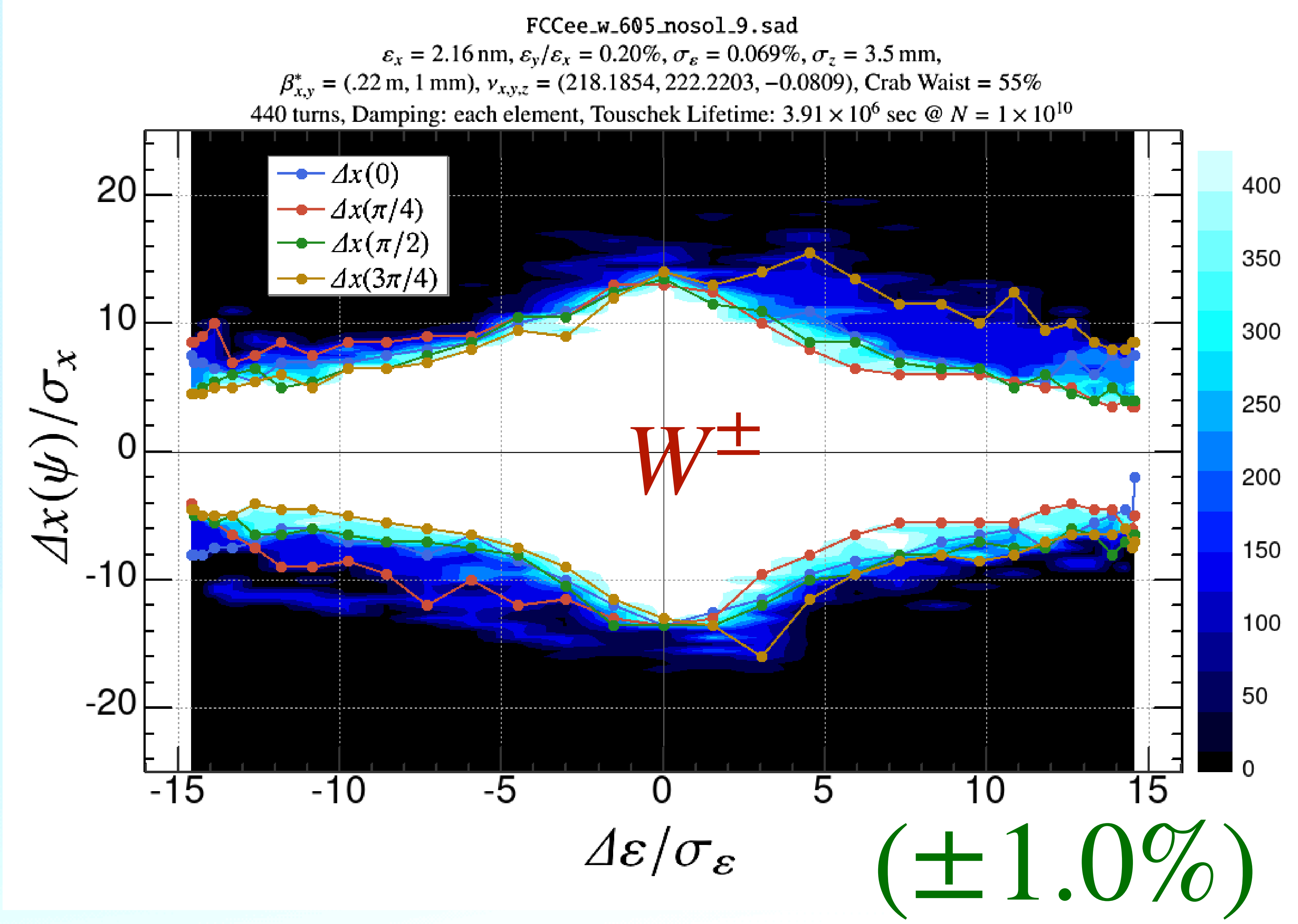
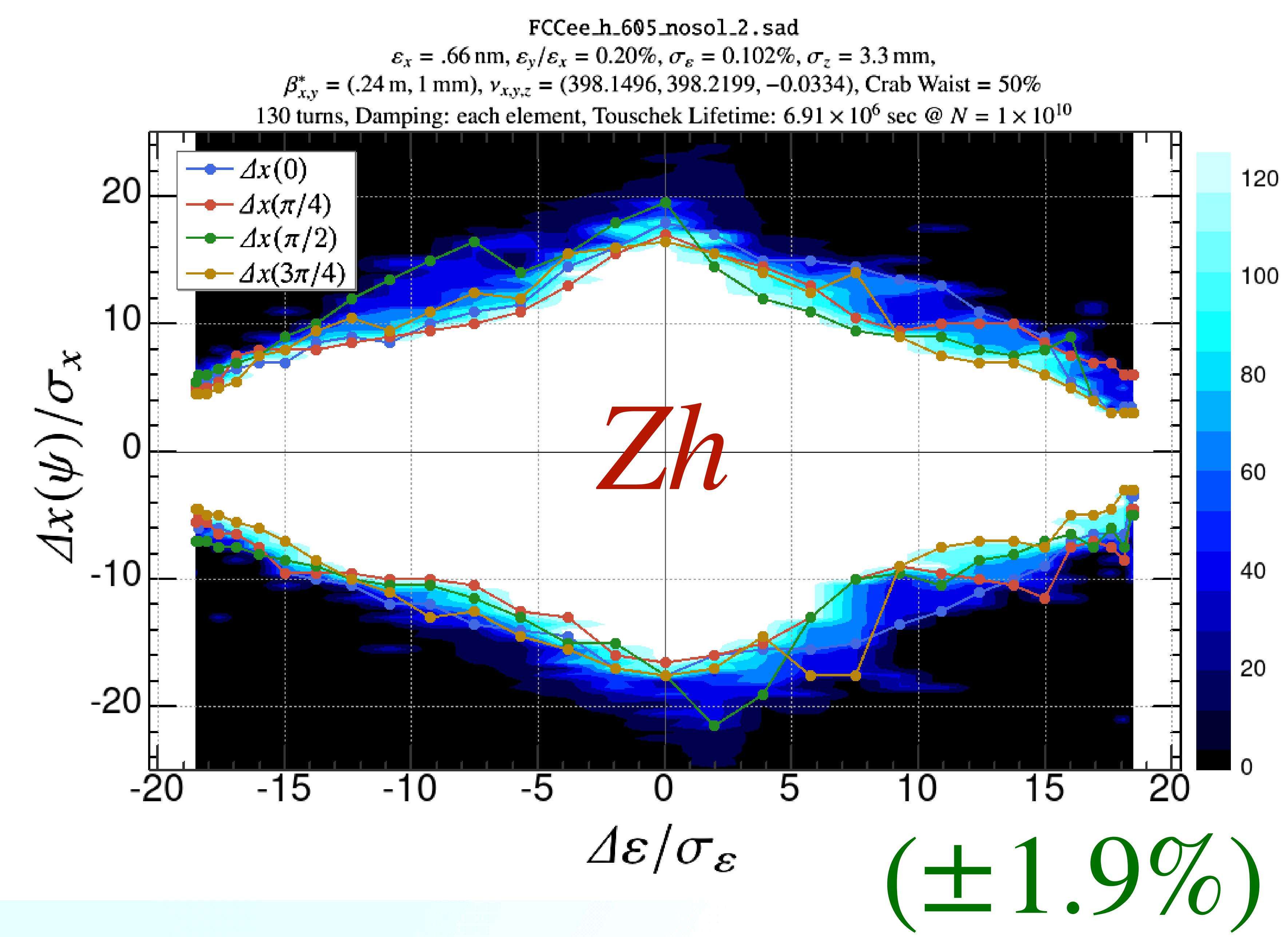
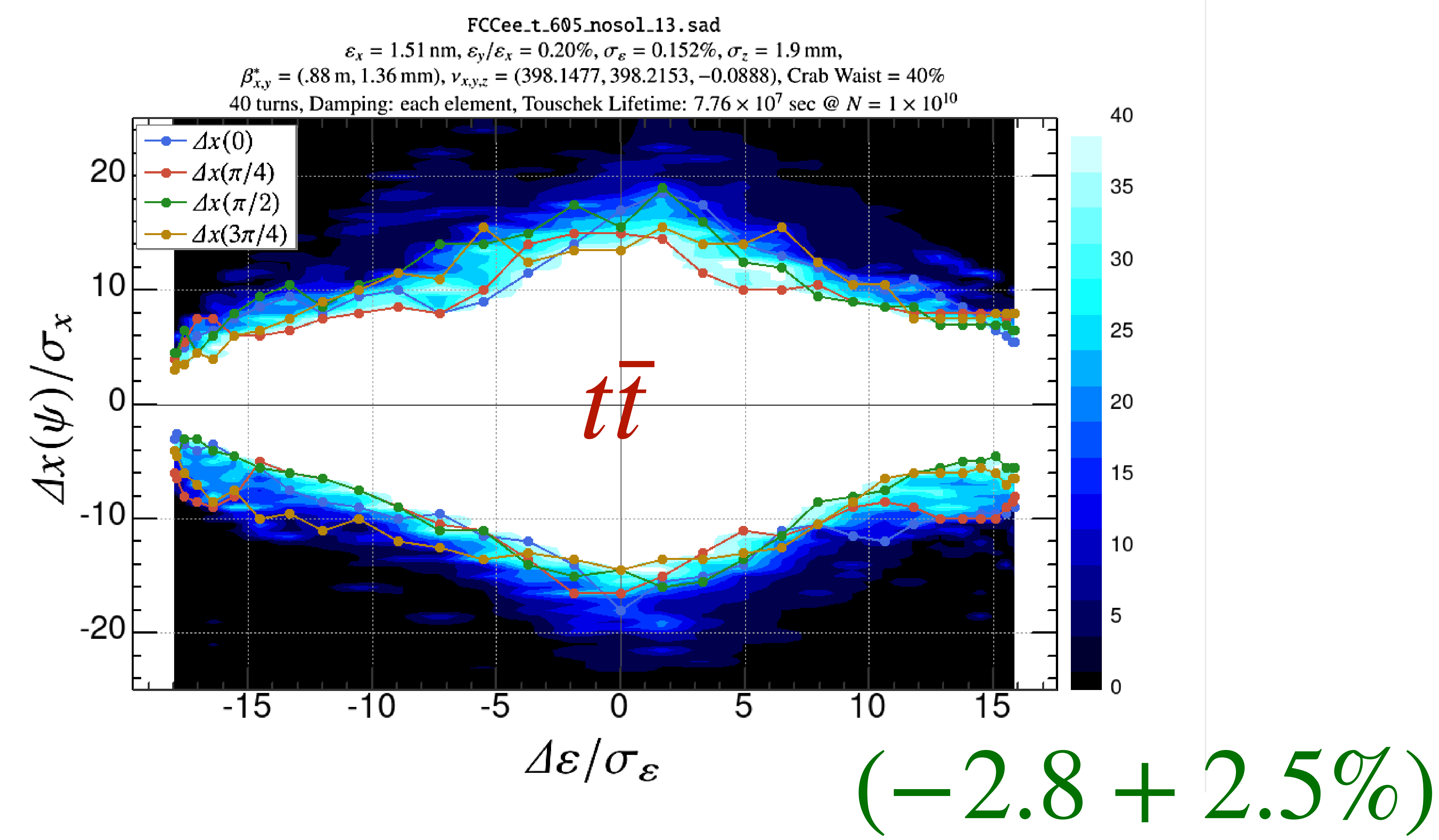




- Some DA(MA)s still seem immature, for instance at Z. However, the beam-beam lifetime looks OK.
- The beam-beam effect does not affect the DA, as the beam-beam force becomes smaller for large amplitudes.

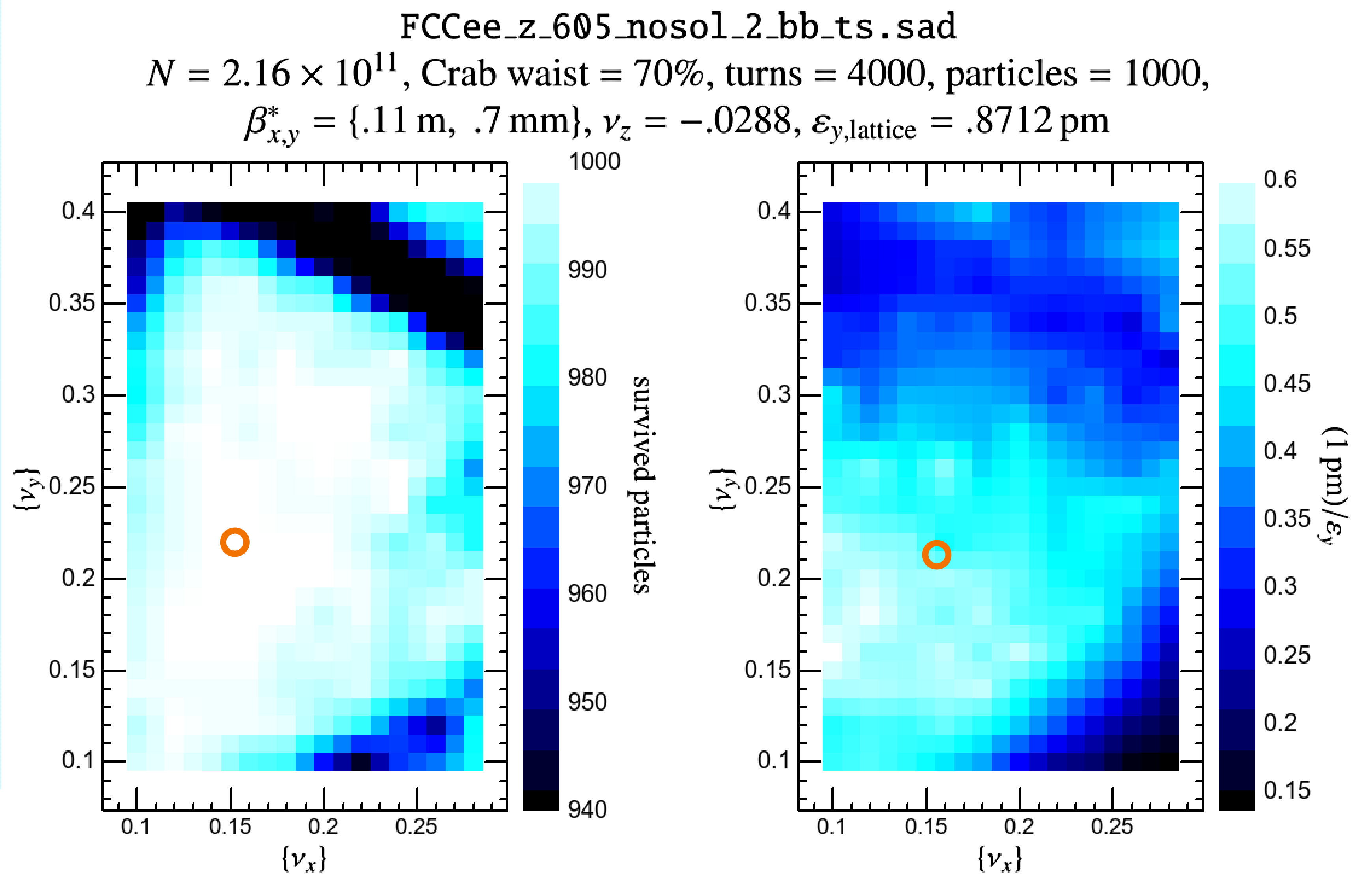




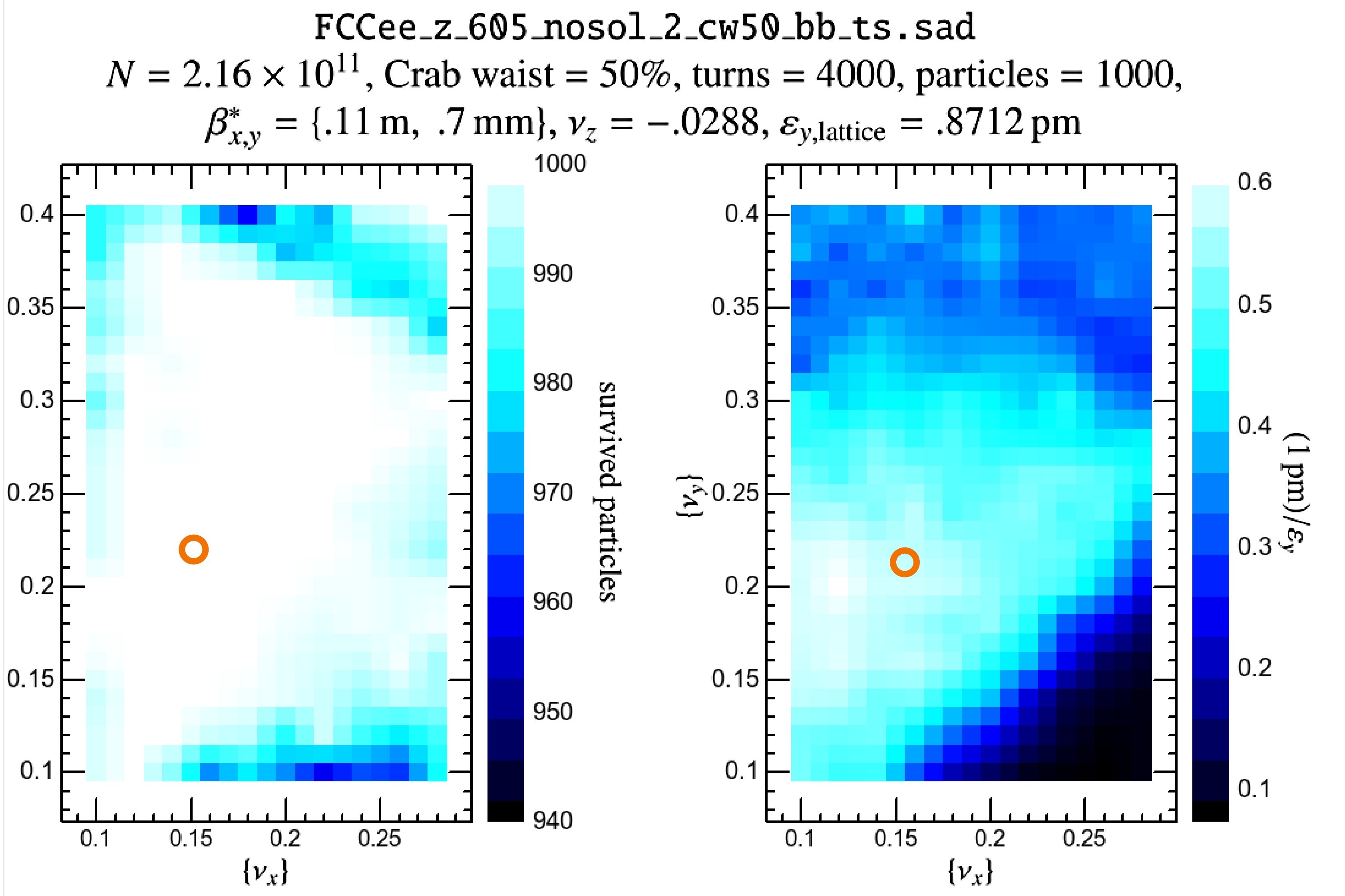
Resonance due to crab waist ratio @Z

Z

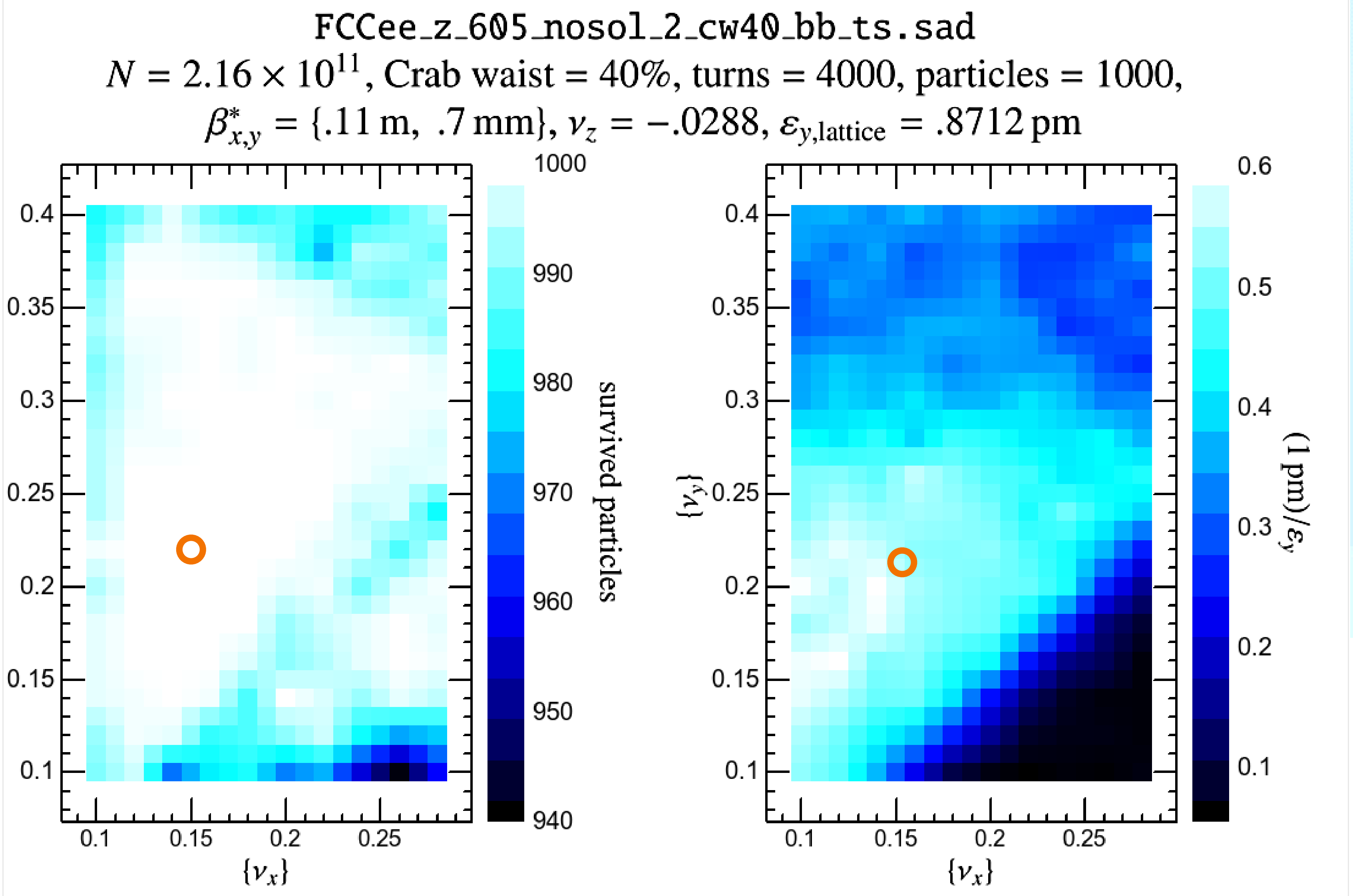
CW = 70%



CW = 50%



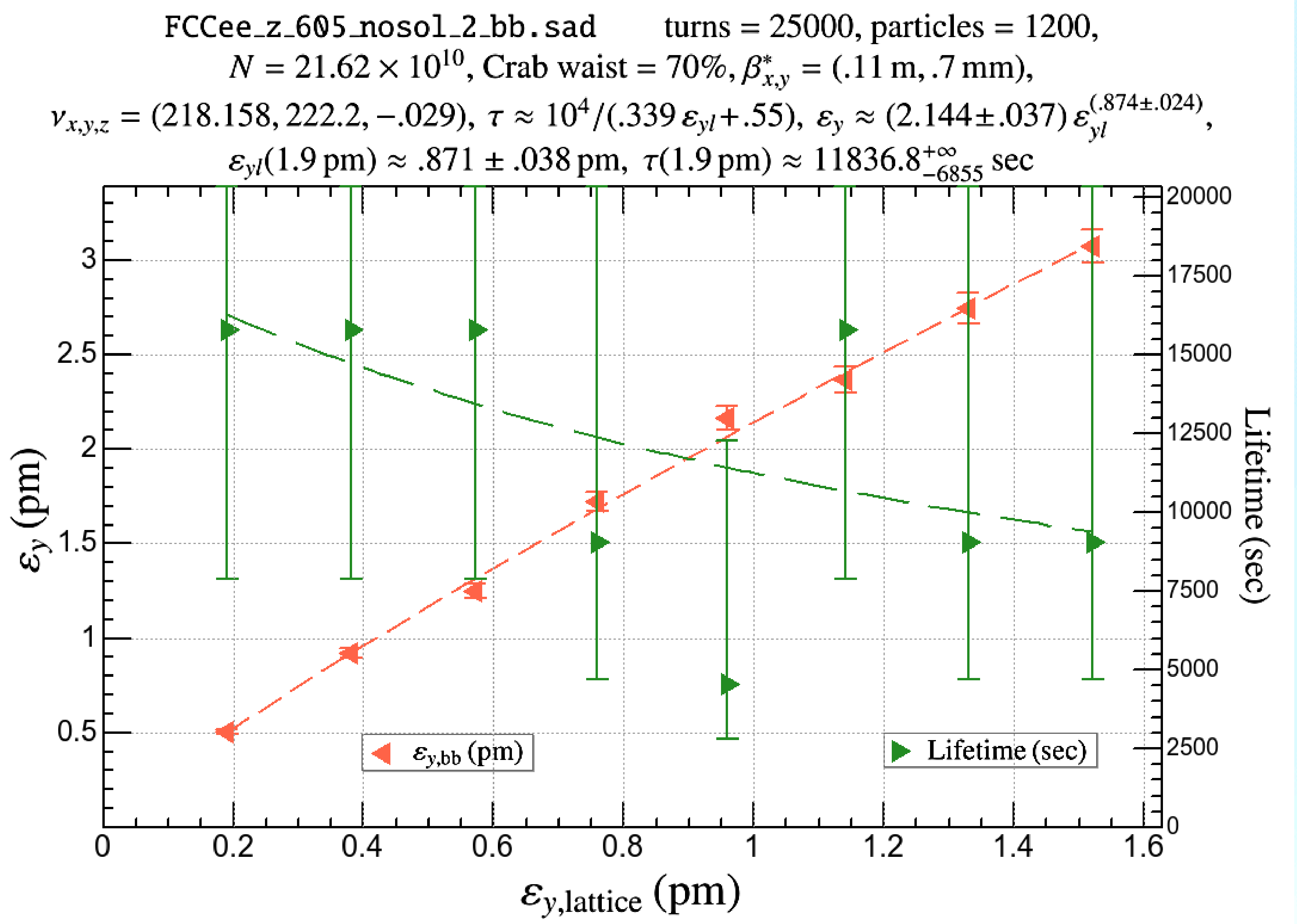
CW = 40%



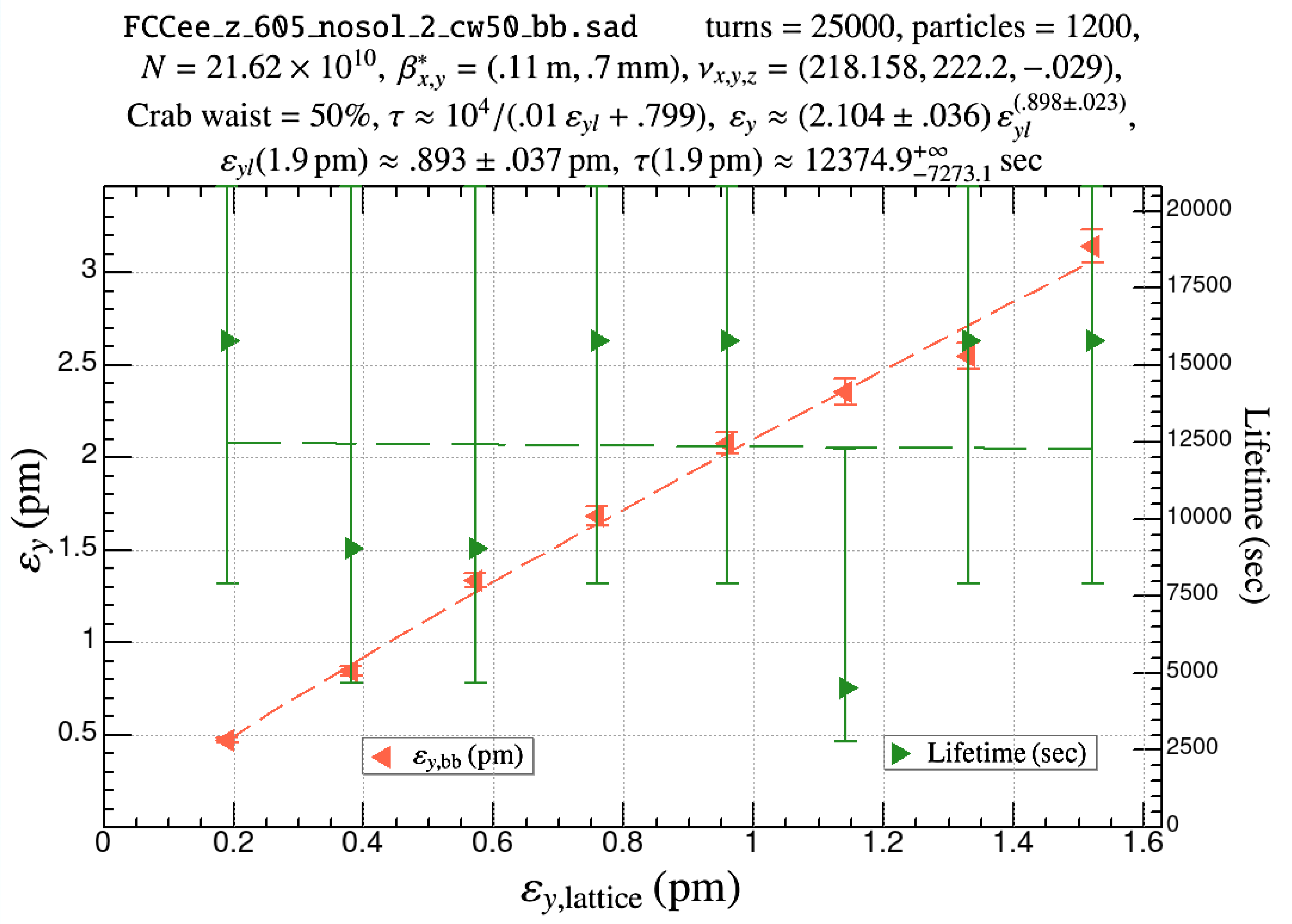
Emittance blowup & lifetime dependence on crab waist @Z

Z

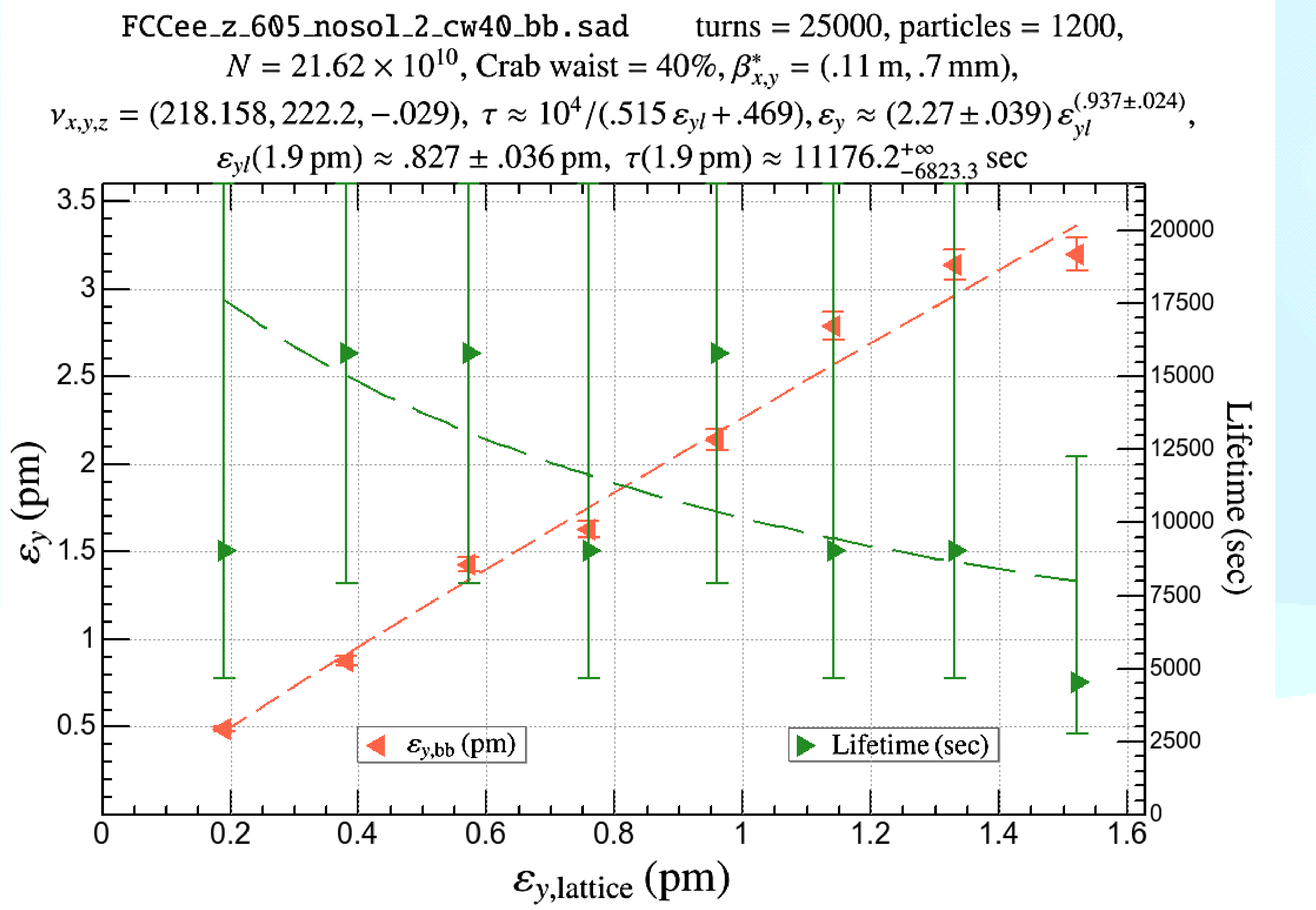
CW = 70%



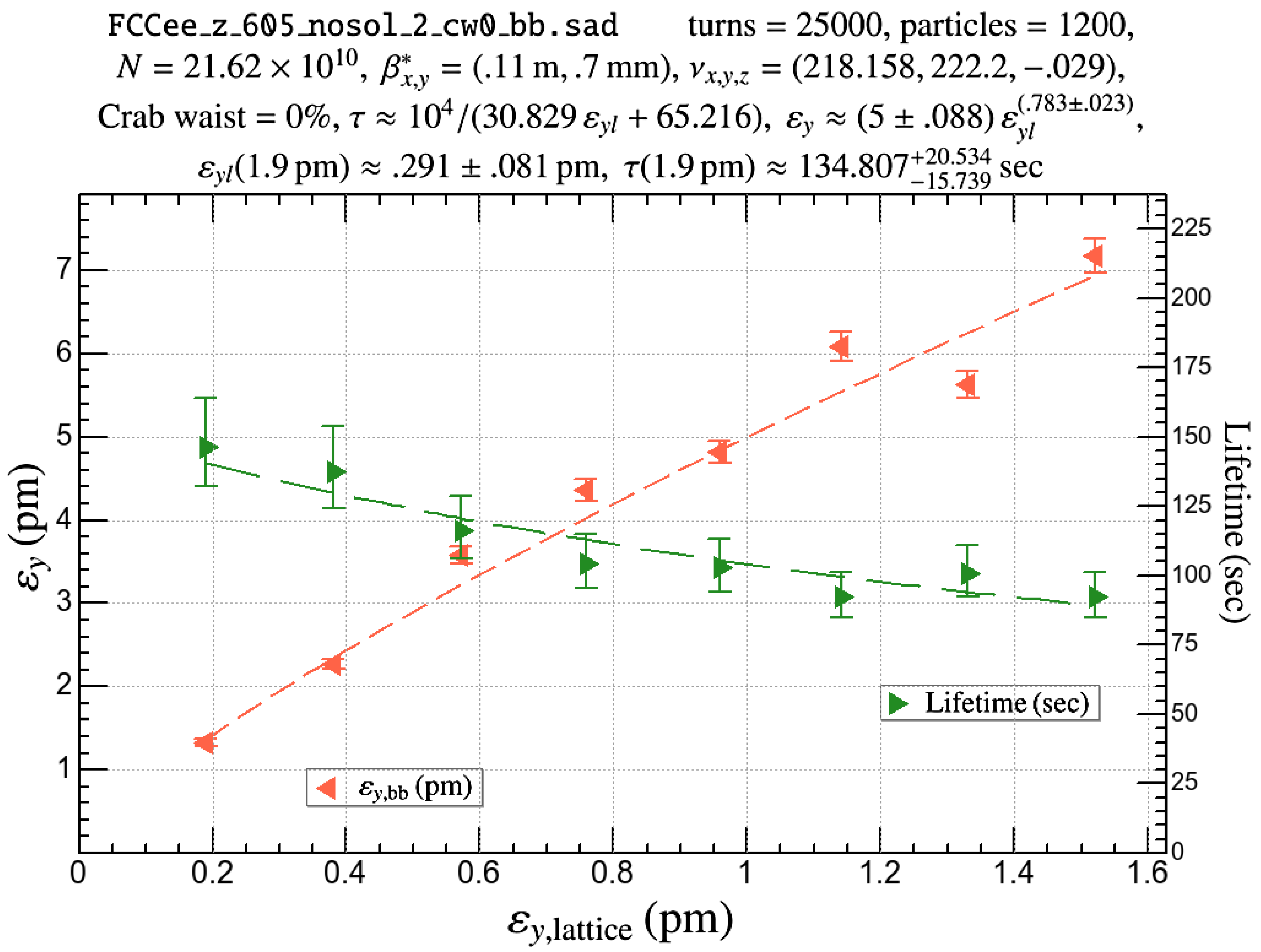
CW = 50%



CW = 40%



CW = 0%



Crab waist (%)	$\varepsilon_{y,\mathrm{lattice}}$ (pm)	τ (sec)
0	0.29 ± 0.08	135^{+20}_{-16}
40	0.83 ± 0.04	$11200^{+\infty}_{-6800}$
50	0.89 ± 0.04	$12400^{+\infty}_{-7300}$
70	0.87 ± 0.04	$11800^{+\infty}_{-6900}$

Optimum crab waist ratio

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- According to the simulations above, the optimum crab waist ratio for Z seems to be around 50%.
  - higher CW induces additional resonances due to beam-beam and crab.
  - simulations by the CDR did not take the lattice into account, gave the optimum at nearly 100% for Z.
- … but what is the reason?
  - Is this lattice-specific?
  - currently the strong beam in BBWS does not take the CW on the beam shape.
  - needs independent simulations, incl. strong-strong model.

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The source of emittance blowup

- There has been seen $\sim \times 2$ blowup of the vertical emittance *universally* for all energies or lattices, unless $CW = 0$.
  - Even for the "LCCS" lattice by P. Raimondi.
- A naïve speculation is the *dynamic emittance* due to the beam-beam focusing:
  - If a focusing element is inserted into a ring, it changes the equilibrium emittance.
- The equilibrium emittance satisfies the equation of beam-matrix for one turn:

K. Ohmi, K. Hirata, K. Oide, *Phys.Rev.E* 49 (1994) 751-765

$$\left\langle \boldsymbol{x}\boldsymbol{x}^T \right\rangle = \mathrm{M} \left\langle \boldsymbol{x}\boldsymbol{x}^T \right\rangle \mathrm{M}^T + \mathrm{b}\ , \tag{1}$$

where $\boldsymbol{x}$, M, and b are the 6D coordinates around the closed orbit, the one-turn transfer matrix including the radiation damping, and the expected excitation $\left\langle \boldsymbol{\Delta x}\boldsymbol{\Delta x}^T \right\rangle$ due to the radiation fluctuation, respectively.

- For a simplicity, let us consider a 1D case below. Then Eq. (1) is rewritten using

$$\left\langle \boldsymbol{x}\boldsymbol{x}^T \right\rangle = \begin{pmatrix} \sigma_y^2 & \sigma_{ypy} \\ \sigma_{ypy} & \sigma_{py}^2 \end{pmatrix}\ , \tag{2}$$

$$\mathrm{M} \approx e^{-\delta} \begin{pmatrix} \cos\mu & \sin\mu \\ -\sin\mu & \cos\mu \end{pmatrix}\ , \tag{3}$$

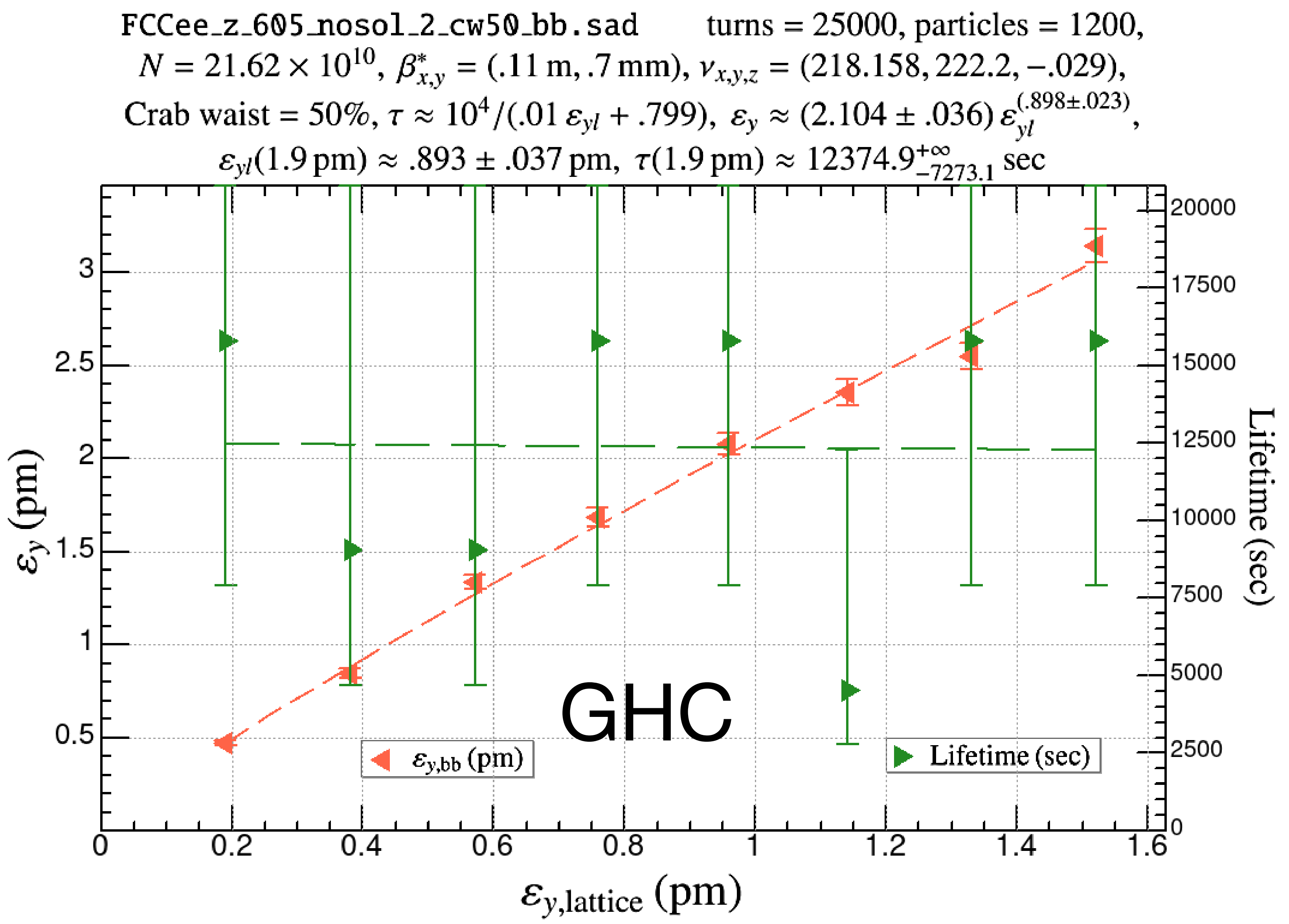
$$\mathrm{b} = \begin{pmatrix} a & c \\ c & b \end{pmatrix}\ , \tag{4}$$

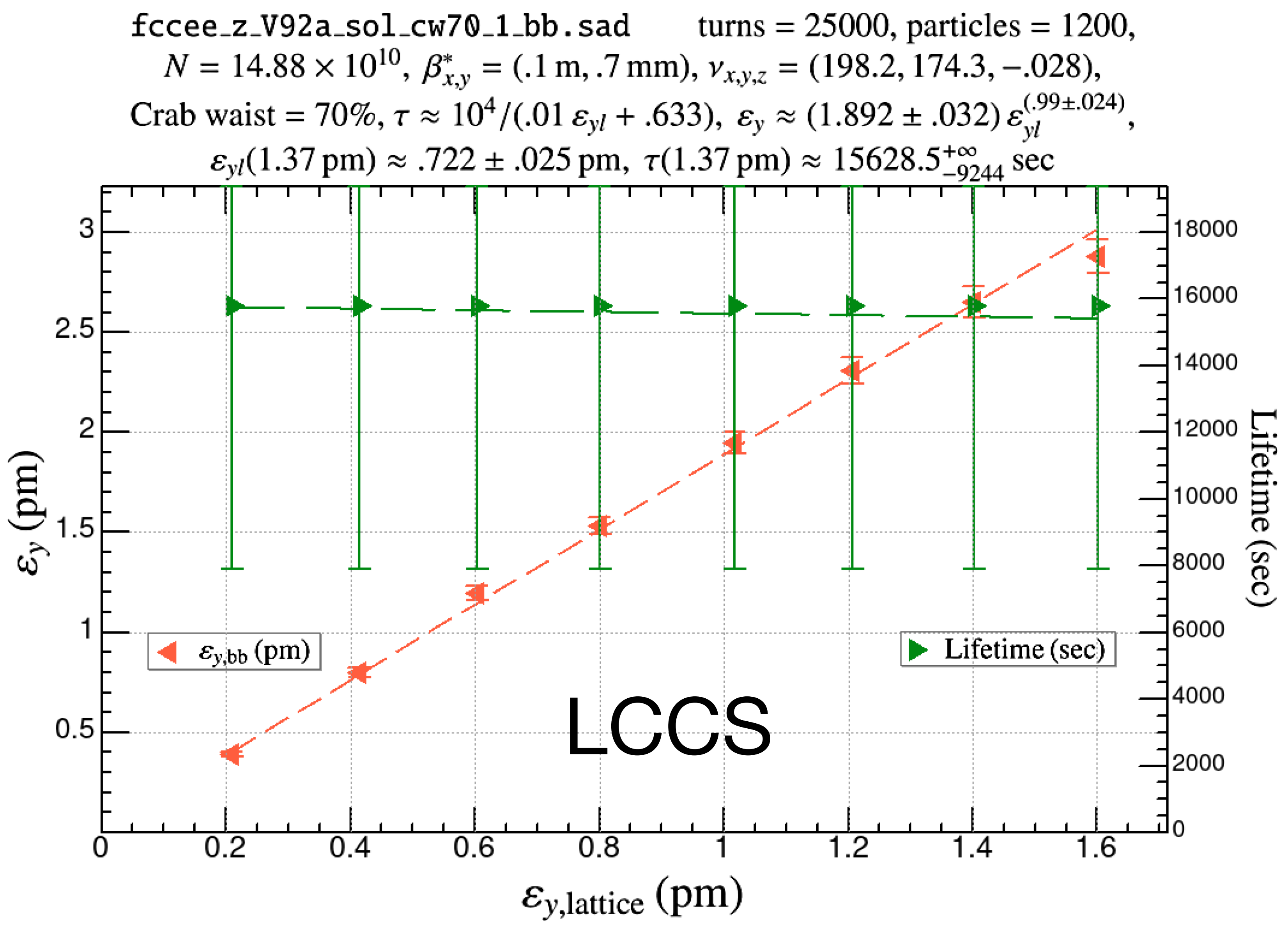
where μ and δ are the ring tune ($\times 2\pi$) and the damping fraction, and we have expressed in the normalized coordinate,

- If there is a focusing element inserted at the end of the beam line (in this case at the IP), The equilibrium can be calculated by simply replacing M in Eq. (1) by:

$$\mathrm{M}' = \begin{pmatrix} 1 & 0 \\ -k & 1 \end{pmatrix} \mathrm{M}\ , \tag{5}$$

where k is the focusing strength. In the case of beam-beam, $k = 4\pi\xi$.





The source of emittance blowup (2)

- Then it is not difficult to obtain the equilibrium beam-matrix:

$$\begin{pmatrix} \sigma_y^2 \\ \sigma_{py}^2 \\ \sigma_{ypy} \end{pmatrix} = \frac{1}{\delta} \frac{ak\cos\mu + (a+b+ck)\sin\mu}{4k\cos\mu + (4-k^2)\sin\mu} \begin{pmatrix} 1 \\ 1+k\cot\mu \\ k/2 \end{pmatrix} , \tag{6}$$

and the equilibrium emittance:

$$\varepsilon_y \equiv \sqrt{\sigma_y^2 \sigma_{py}^2 - \sigma_{ypy}^2} \tag{7}$$

$$= \frac{a+b+ck+ak\cot\mu}{2\delta\sqrt{4-k^2+4k\cot\mu}} . \tag{8}$$

- Once we know the components a, b, c of the radiation excitation matrix b without beam-beam, the equilibrium emittance (or *dymanic emittance*) can be easily calculated, without doing the calculation over the ring again.
- Actually this calculation is performed in the 6D phase space.
- The issue is that the excitation matrix in the vertical plane is basically given by the machine errors such as misalignments. So we have to know where the vertical excitation occurs and how much. The models shown here depends on the particular radiation source.
- Anyway the dynamic emittance effect is unavoidable. The actual emittance blowup must occur at least for the linear focusing regime ($y \lesssim \sigma_y$).

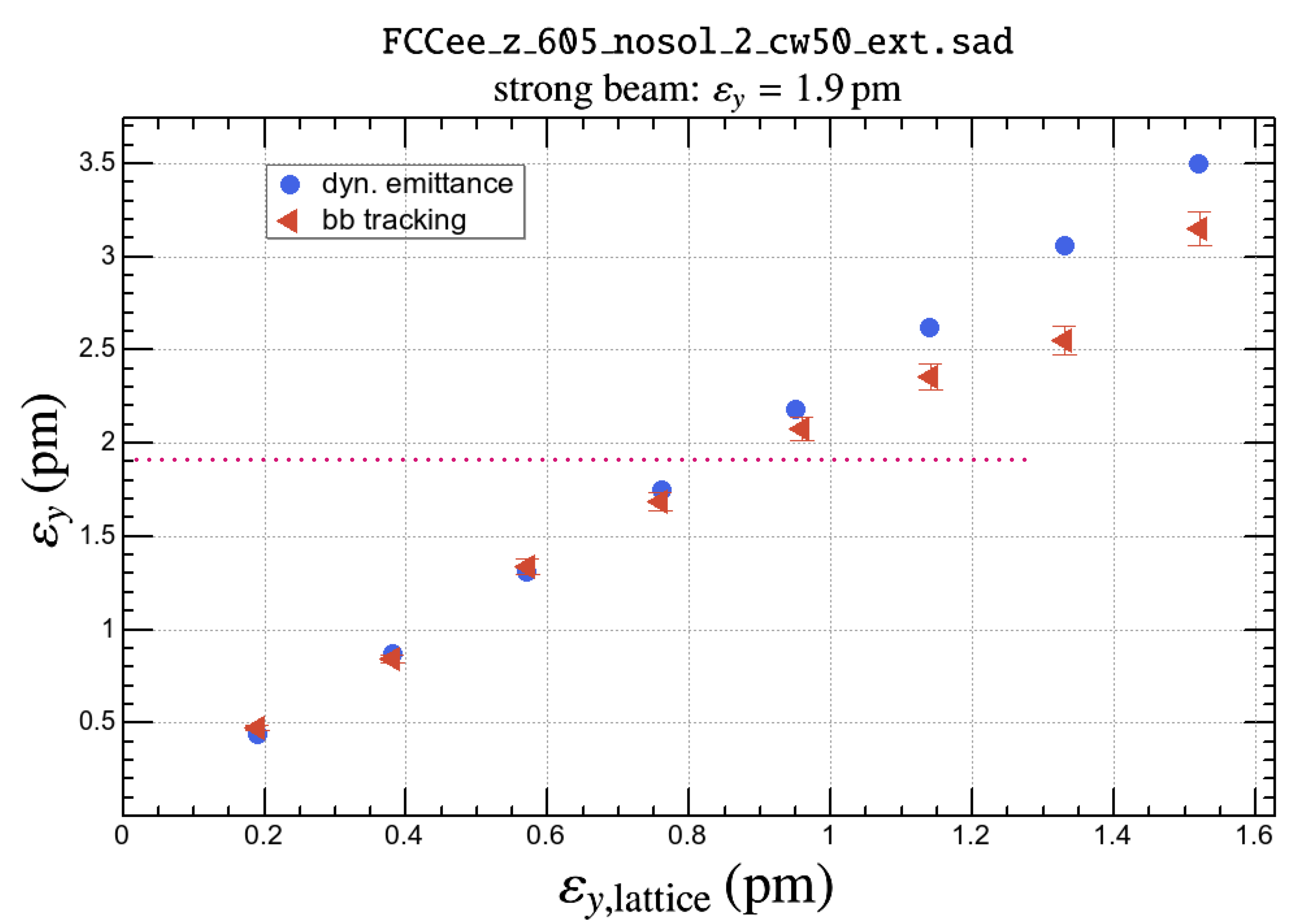
a	0.364 fm
b	0.028 fm
c	−0.102 fm
δ	1.09×10^{-4}
k_y	1.23
μ_y	$2\pi \times 0.550$
ε_y	1.80 pm

Eq. (8) gives a good estimation of ε_y.

6D: 2.0 pm

Tracking: 1.9 pm

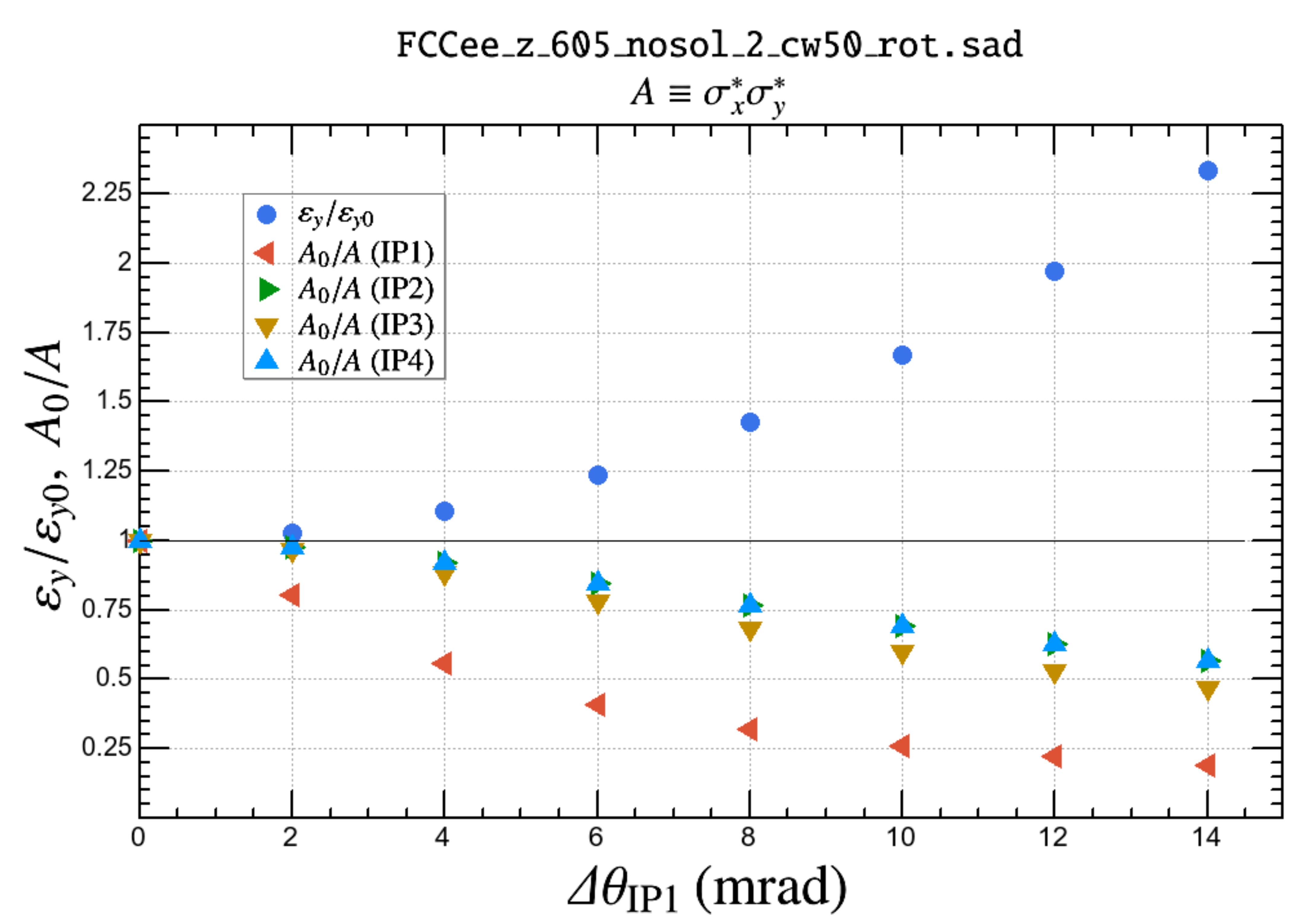
The source of emittance blowup (3)

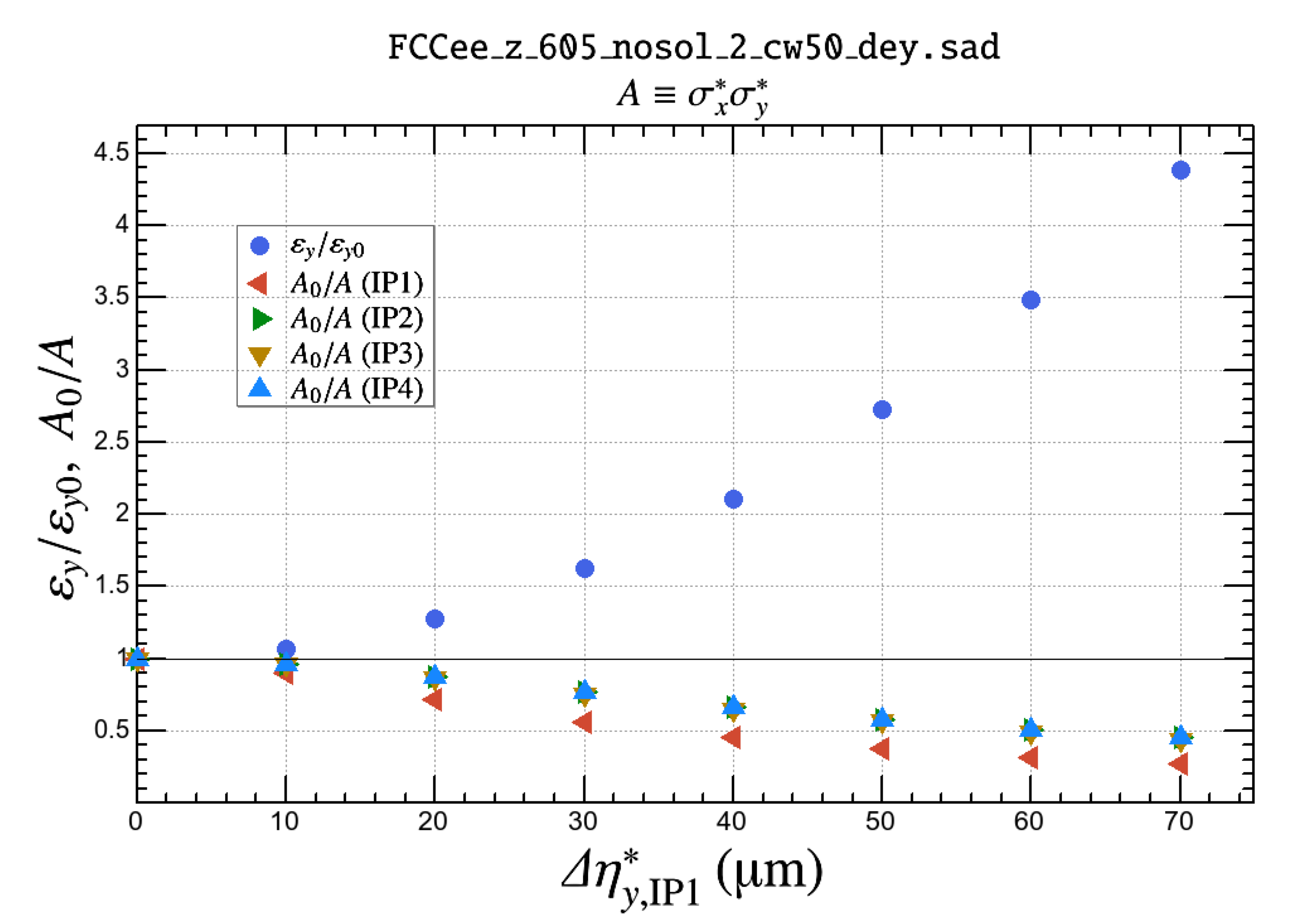


* Here the beam-beam focusing is applied as a thin lens at each IP.

- The plot above is the vertical emittance blowup between the tracking (red) and dynamic emittance calculation (blue).
- The agreement looks quite good, up to the emittance of the strong beam (dashed line).
  - It is reasonable, as the blowup exceeds the size of the strong beam, the focusing will be more nonlinear.
- If this method works, it is very convenient, as the computation of the dynamic emittance is 1 / 10000 of the beam-beam simulation, or even less.

Application of dynamic emittance calculation





- If the dynamic emittance method works, it is very easy to calculate the effect of errors in the collision condition.
- Plots above show effects of roll (left) and vertical dispersion (right) at one of the IPs in the ring.
  - Resulting vertical emittance increase (circle) and $1/\sigma_x^*\sigma_y^*$ at each IP (triangle) are plotted.
  - The luminosity of other IPs are affected through the emittance increase as well as coupling/dispersion propagation.
  - Machine tuning just looking at the luminosity of each IP interferes to each other.

Summary

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- The weak-strong(WS) beam-beam simulation with lattice & beamstrahlung has been the key method in the design of the ring for FCC-ee.
  - Looking at the beam-beam is mandatory to complete the design. The usual plots of the dynamic aperture is insufficient, for instance, for choosing the ring tune.
  - Strong-strong simulations with lattice will (or will not) confirm these results.
    - Preliminary results by K. Ohmi more or less agrees with the WS so far (https://indico.cern.ch/event/1398060/).
- The primitive beam envelope matrix calculation agrees with the emittance blowup obtained by the tracking.
- Any error in the collision at one IP may interfere the luminosities of other IPs through the emittance growth and propagation of optics errors.