Imperfections in the crab-waist transform

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Outline

- Introduction
- Theory of beam-beam resonances for CW colliders
- Imperfections in the crab-waist transform
- Summary

Introduction

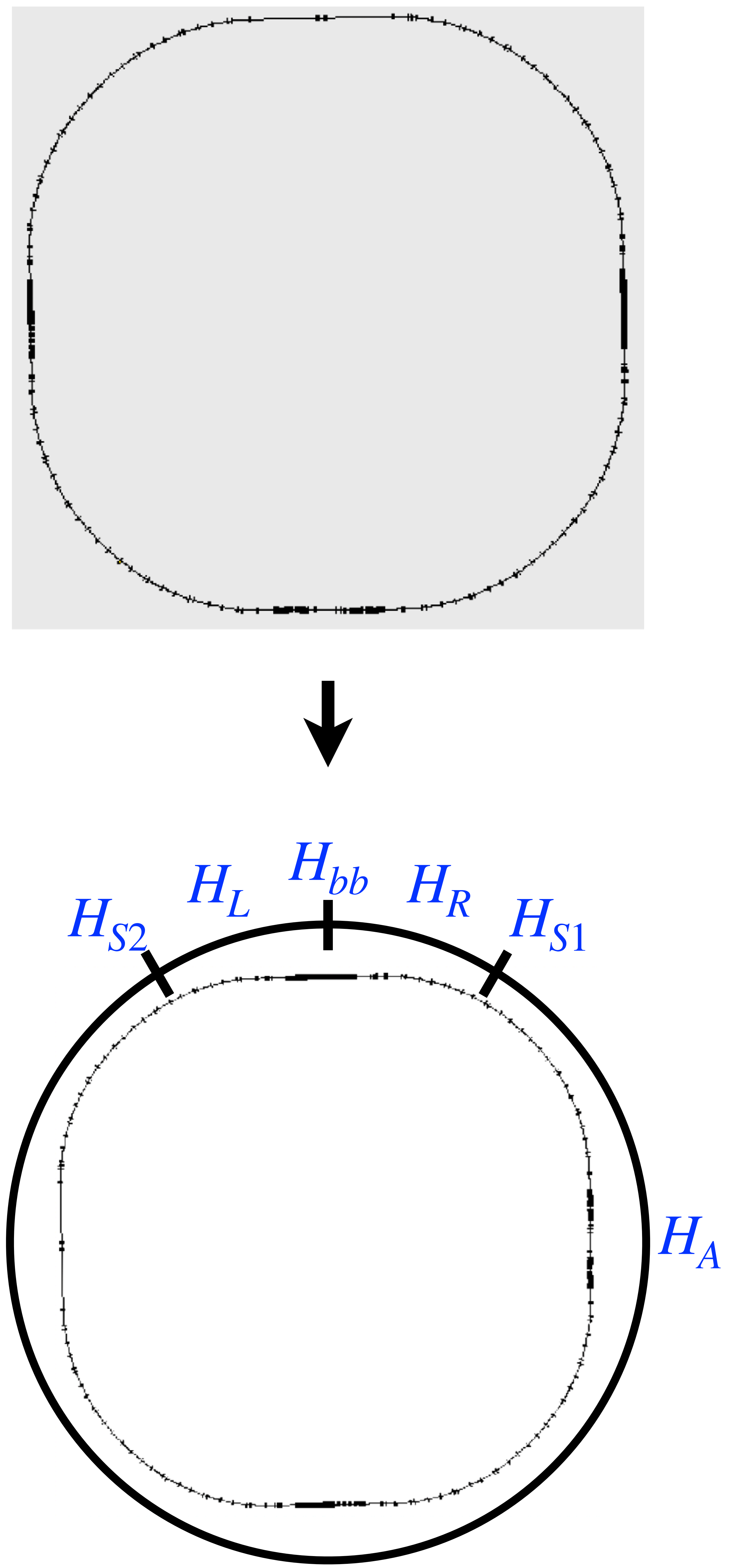
- Model of a CW collider ring
  - In terms of Lie maps, the one-turn map is

$$M = e^{-:H_R:}e^{-:H_{S1}:}e^{-:H_A:}e^{-:H_{S2}:}e^{-:H_L:}e^{-:H_{bb}:}$$

  - Sequence of elements:
    - H_R, H_L: right and left side of IR
    - H_{S1}, H_{S2}: first and second CW sextupole
    - H_A: arc and straight sections
    - H_{bb}: beam-beam kick at IP
  - The one-turn map of an ideal CW collider ring is

$$M_i = e^{-:H_0:}e^{-:H_{cw}:}e^{-:H_{bb}:}e^{:H_{cw}:} \qquad H_{cw} = \frac{\chi}{2\tan(2\theta_c)}xp_y^2$$

    - χ=1 for full CW strength
    - H_0 is determined only by $\beta^*_{x,y,z}$ and $\nu_{x,y,z}$
- The imperfections in the CW transform are embedded within the Hamiltonians being analyzed.

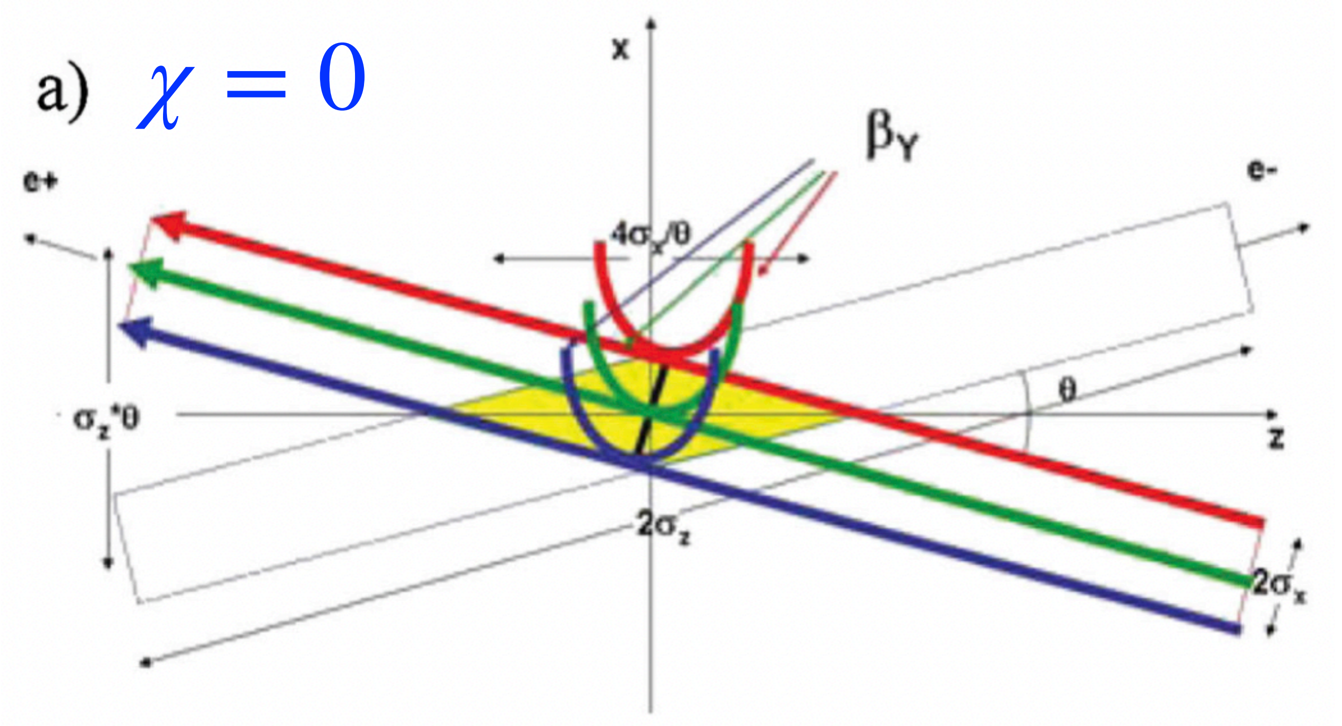


Introduction

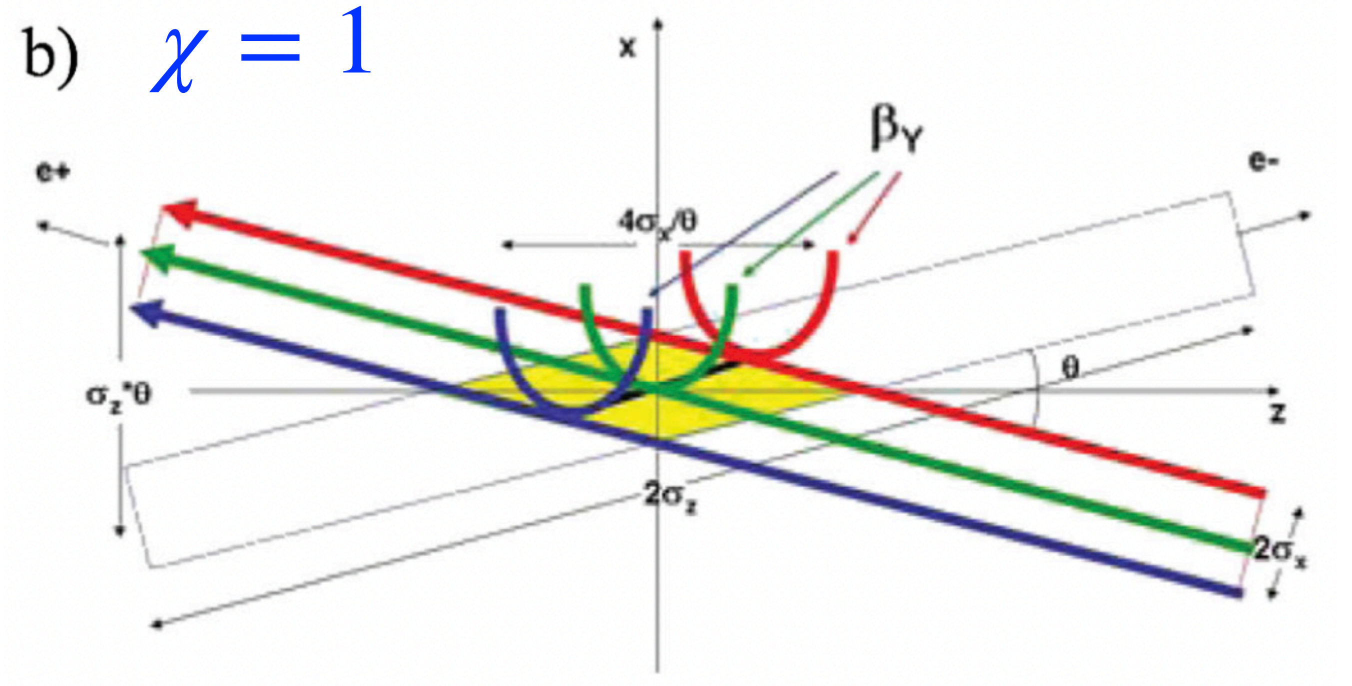
- An ideal CW collider
  - Beam distribution around the IP

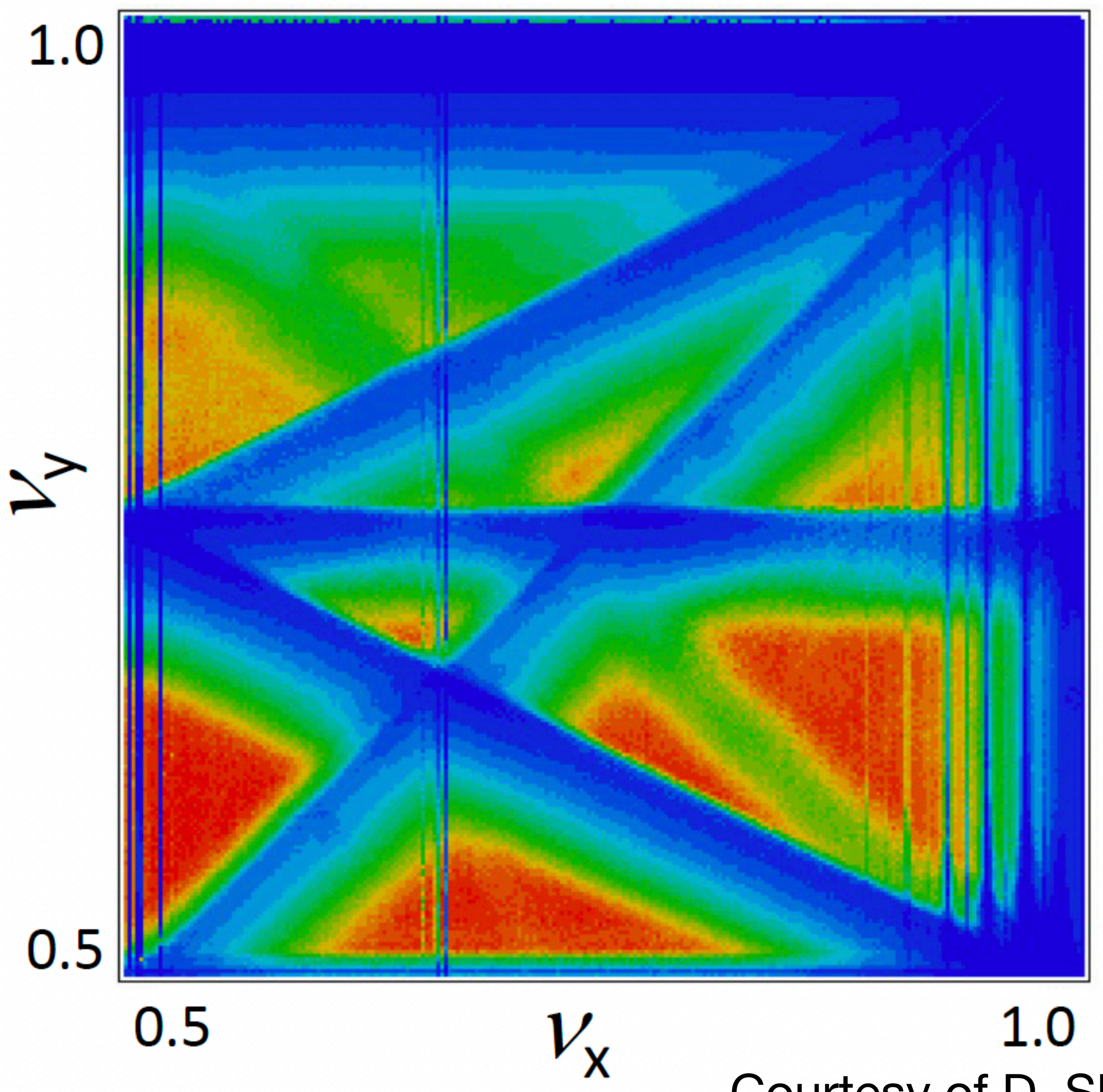
$$\rho(\vec{r};s)=\frac{e^{-\frac{x^2}{2\sigma_x^{*2}}-\frac{y^2}{2\sigma_y^2(s)}-\frac{z^2}{2\sigma_z^2}}}{(2\pi)^{3/2}\sigma_x^*\sigma_y(s)\sigma_z} \qquad \sigma_y(s)=\sigma^*\sqrt{1+\frac{1}{\beta_y^{*2}}\left(z+s+\frac{\chi x}{\tan(2\theta_c)}\right)^2}$$

  - Features:
    - Flat beam: $R_0=\sigma_y^*/\sigma_x^*\ll 1$
    - Large Piwinski angle: $\phi_0=\sigma_z\tan\theta_c/\sigma_x^*\gg 1$
    - Small β_y^*: $\zeta_x=\sigma_x^*/(\beta_y^*\tan(2\theta_c))\lesssim 0.5$
    - Working point per IP: fractional (ν_x,ν_y) around (0.5, 0.5)
  - Important beam-beam resonances
    - Horizontal synchrobetatron resonances: $2\nu_x-2k\nu_z=N$, $k=1,2,3,\ldots$
    - Betatron resonance: $\nu_x+2n\nu_y=N$, $n=\pm 1,\pm 2,\ldots$
    - Vertical synchrobetatron resonances: $2\nu_y-2k\nu_z=N$, $k=1,2,3,\ldots$
    - 3D synchrobetatron resonances: $m_x\nu_x+m_y\nu_y+m_z\nu_z=N$



M. Zobov et al., PRL 104, 174801 (2010).





Courtesy of D. Shatilov

Theory of beam-beam resonances for ideal CW colliders

- N.S. Dikansky and D.V. Pestrikov, NIM-A 600 (2009) 538-544
  - Beam-beam potential

$$V_{bb} = -\frac{N_0 r_e R_0}{\pi\gamma}\iiint_{-\infty}^{\infty} d\tau dt_x dt_y \frac{\lambda(\tau)}{R_0^2 t_x^2 + t_y^2} e^{it_x(\tau+q_x+\phi_0 q_z)-it_y q_y} e^{-\frac{t_x^2}{2}-\frac{t_y^2}{2}\left(1+\zeta_{x0}^2(\tau+\phi_0 q_z)^2\right)}$$

$$\lambda(\tau) = \frac{1}{\sqrt{2\pi}\phi_0} e^{-\frac{\tau^2}{2\phi_0^2}}$$

$$\zeta_{x0} = \sigma_{x0}^*/(\beta_{y0}^* \tan(2\theta_c))$$

  - Particle motion in the weak beam

$$x = \sqrt{2\beta_x^* J_x}\cos\psi_x, \quad y(s') = \sqrt{2\beta_y(s')J_y}\cos\phi_y(s'), \quad z = \sqrt{2\beta_z J_z}\cos\psi_z$$

  - Hourglass effect and CW transform

$$\beta_y(s') = \beta_y^*\left(1 + \frac{1}{\beta_y^{*2}}\left(s' + \frac{\chi x}{\tan(2\theta_c)}\right)^2\right), \quad \phi_y(s') = \psi_y + \arctan\left(\frac{s' + \frac{\chi x}{\tan(2\theta_c)}}{\beta_y^*}\right)$$

  - Beam-beam resonances

$$V_{bb}\delta(\theta) = \sum_{\vec{m},n} V_{m_x m_y m_z} e^{i(m_x\psi_x + m_y\psi_y + m_z\psi_z - n\theta)}, \quad V_{m_x m_y m_z} = \frac{1}{(2\pi)^4}\iiint_0^{2\pi} d\psi_x d\psi_y d\psi_z V_{bb} e^{-i(m_x\psi_x + m_y\psi_y + m_z\psi_z)}$$

Theory of beam-beam resonances for ideal CW colliders

- Horizontal synchrobetatron resonances
  - Amplitude

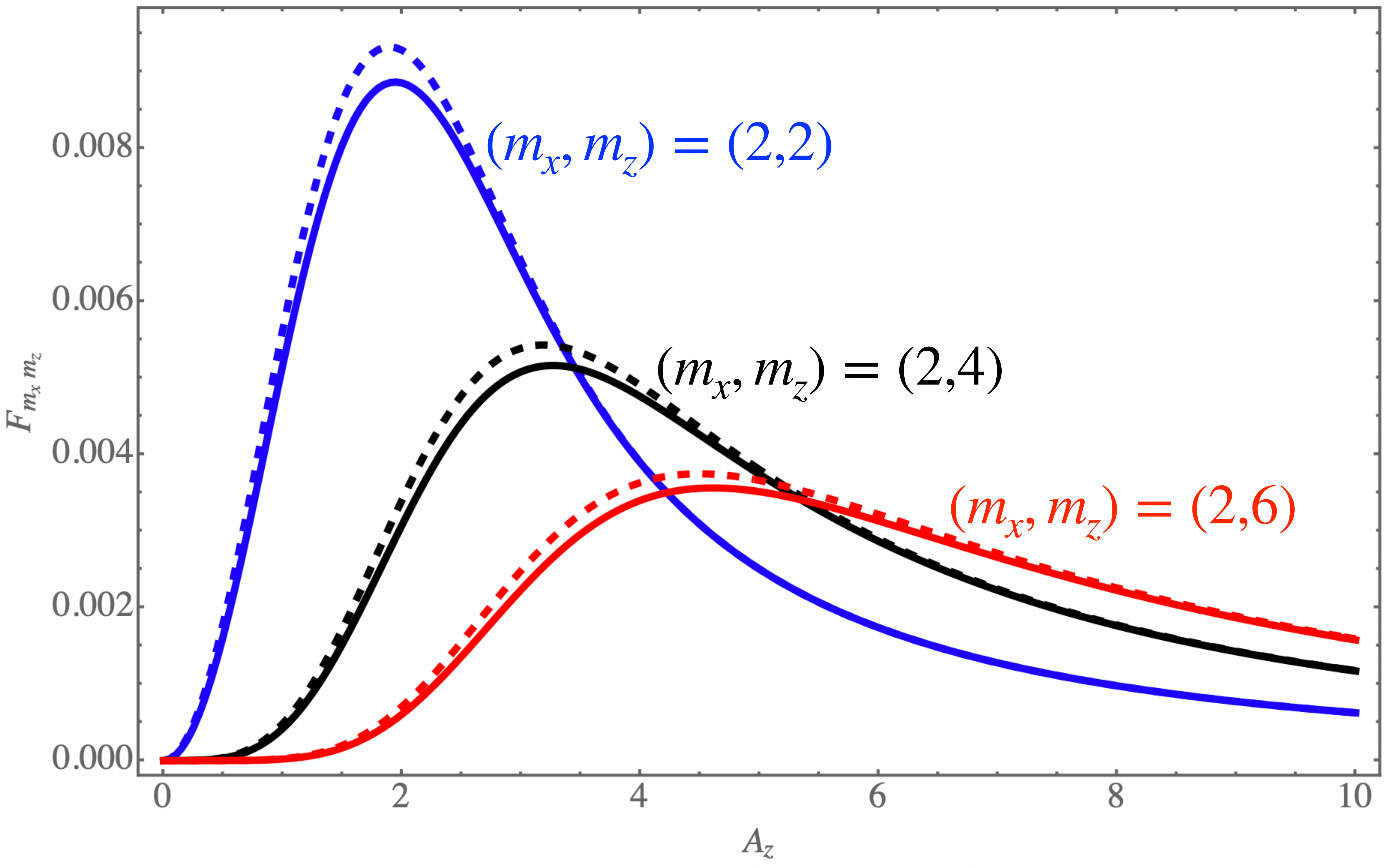
$$V_{m_x 0 m_z} \approx -\frac{N_0 r_e}{\pi\gamma} i^{m_x+m_z} F_{m_x m_z}(A_x, A_z)$$

$$F_{m_x m_z}(A_x, A_z) = \int_0^\infty \frac{dk}{k} e^{-\frac{k^2}{2}} J_{m_x}\left(\frac{kA_x}{\sqrt{\phi_0^2+1}}\right) J_{m_z}\left(\frac{k\phi_0 A_z}{\sqrt{\phi_0^2+1}}\right)$$

  - Approximation for $\phi_0 \gg 1$

$$F^a_{2m_z} \approx \frac{\sqrt{2\pi}}{32\phi_0^2} A_x^2 A_z e^{-\frac{A_z^2}{4}} \left[I_{\frac{m_z-1}{2}}\left(\frac{A_z^2}{4}\right) - I_{\frac{m_z+1}{2}}\left(\frac{A_z^2}{4}\right) \right]$$

  - Findings
    - Excited resonances: $m_x + m_z =$ even
    - **CW does not suppress horizontal resonances**
    - There is only one way to reduce the strength: Increasing ϕ_0



Conditions for the plot: $A_x = 5$, $\phi_0 = 10$
Solid lines: F_{2m_y}
Dashed lines: $F^a_{2m_z}$

Theory of beam-beam resonances for ideal CW colliders

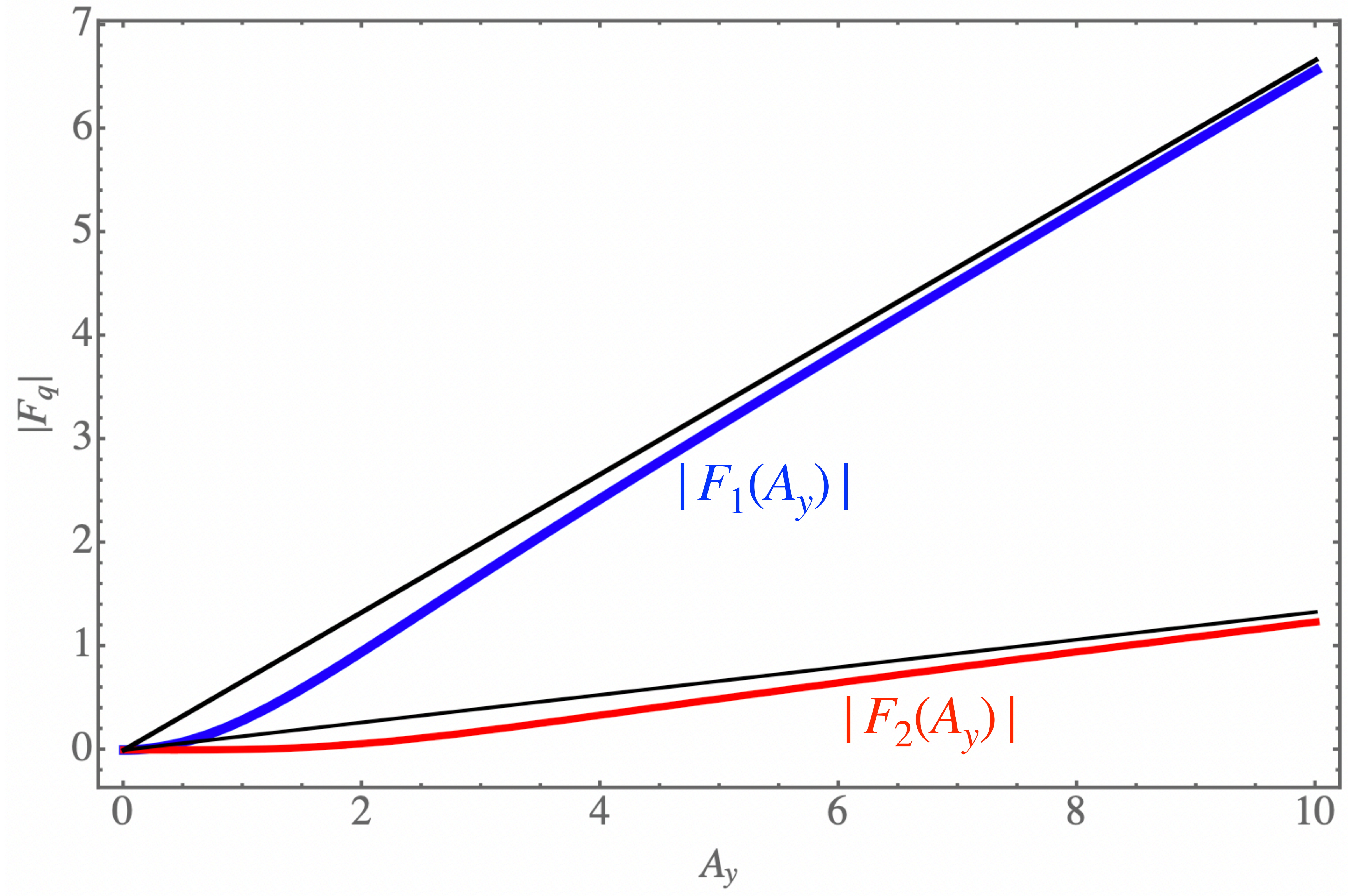
- Resonances with $m_y = 2q > 0$
  - Amplitude

$$V_{m_x m_y m_z} = -\frac{N_0 r_e R_0}{\pi\gamma(2\pi)^3}\iint_0^{2\pi} d\psi_x d\psi_z e^{-i(m_x\psi_x + m_z\psi_z)}\sqrt{2\pi}\int_{-\infty}^{\infty} dr\sqrt{1+\zeta_x^2(r+(\chi-1)q_x)^2}\lambda(r-q_x-\phi_0 q_z)e^{-\frac{r^2}{2}}\left(\frac{1+i\zeta_x(r+(\chi-1)q_x)}{1-i\zeta_x(r+(\chi-1)q_x)}\right)^q F_q(A_y)$$

$$F_q(A_y) \approx \frac{(-1)^q}{4q^2-1}\sqrt{\frac{\pi}{2}}e^{-\frac{A_y^2}{4}}\left[(2+4q+A_y^2)I_q\left(\frac{A_y^2}{4}\right)+A_y^2 I_{q+1}\left(\frac{A_y^2}{4}\right)\right]$$

$$F_q(A_y) \approx (-1)^q\frac{2A_y}{4q^2-1} \text{ for } A_y \gg 1$$

$$V_{m_x m_y m_z} \approx -\frac{N_0 r_e R_0}{\pi\gamma\phi_0}F_q(A_y)\overline{G}_{m_x m_y m_z}(A_x, A_z)$$

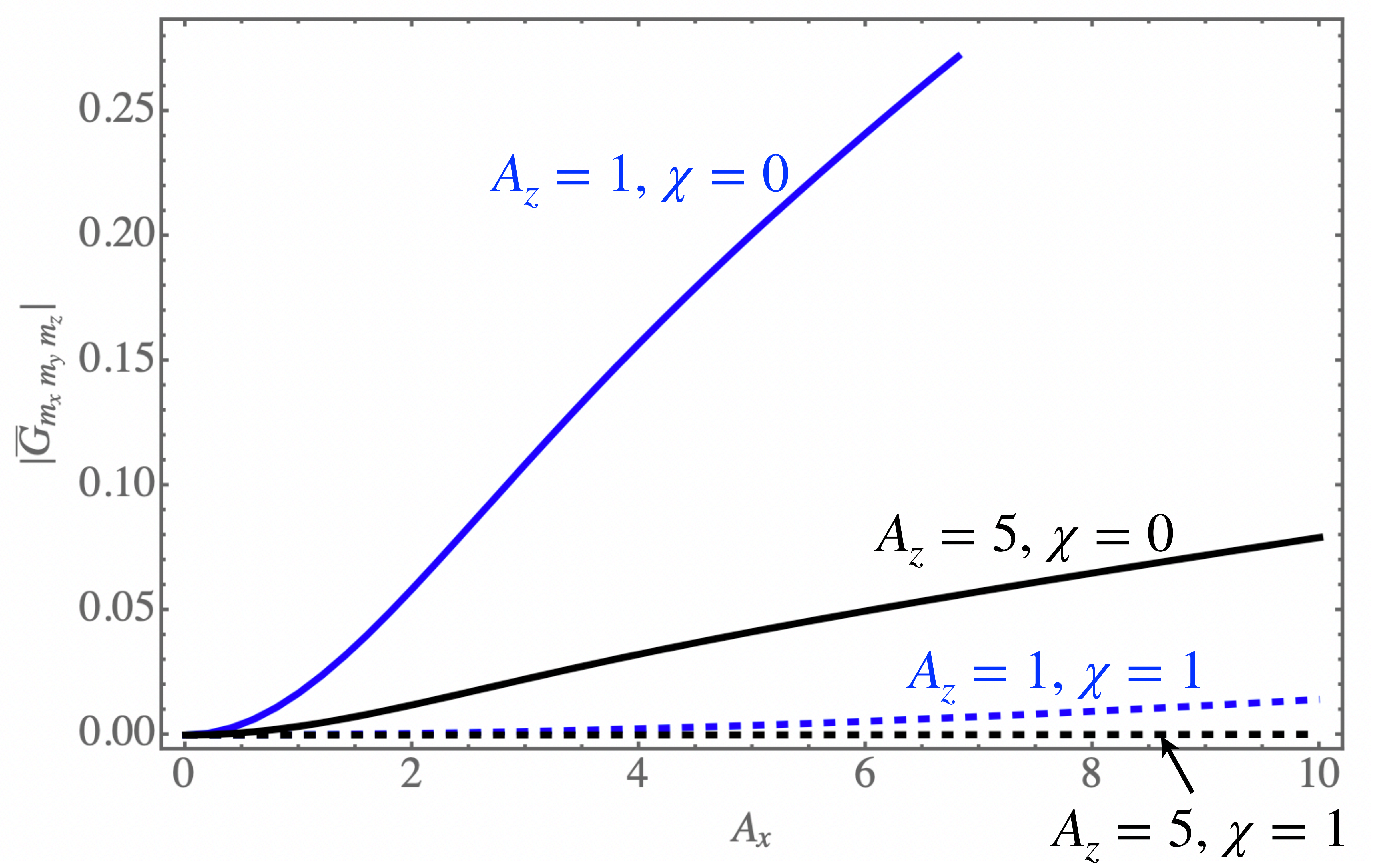


  - Findings
    - $V_{m_x m_y m_z}$ is linearly proportional to A_y when $A_y \gg 1$

Theory of beam-beam resonances for ideal CW colliders

- Betatron resonances $V_{m_x m_y 0}$ with $m_y = 2q > 0$
  - Findings
    - With CW, resonances with $m_x = 1,3,5,\ldots$ will not be excited.
    - Resonances with $m_x = 2,4,6,\ldots$ will be significantly suppressed, but have a finite amplitude
    - **The power of CW is to suppress betatron resonances** $V_{m_x m_y 0}$

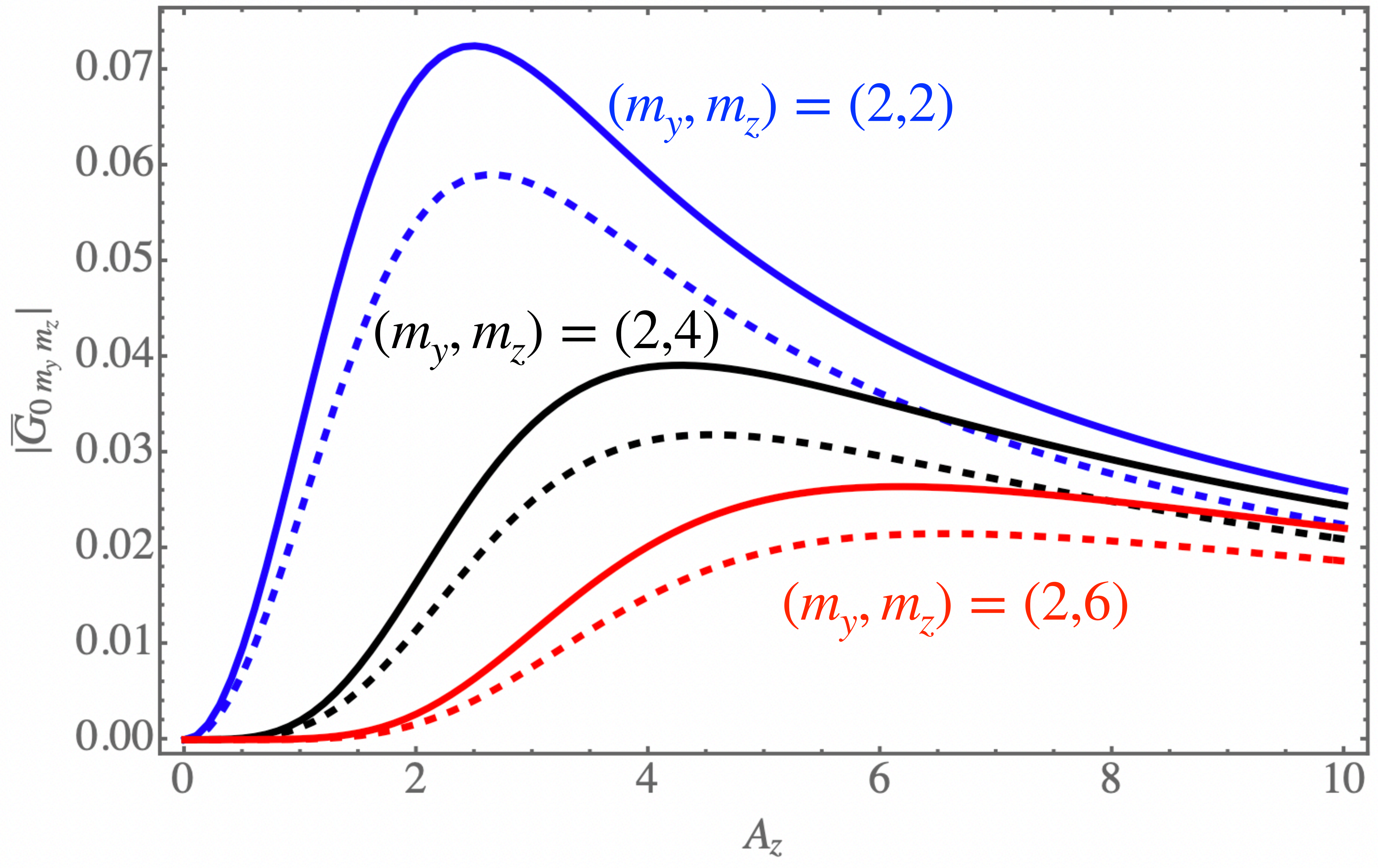
$$H_{cw} = \frac{\chi}{2\tan(2\theta_c)} x p_y^2$$



Conditions for the plot: $\zeta_x = 0.5$, $\phi_0 = 10$
Solid lines: without CW
Dashed lines: with CW

Theory of beam-beam resonances for ideal CW colliders

- Vertical synchrobetatron resonances $V_{0m_ym_z}$ with $m_y = 2q > 0$
  - Findings
    - Resonances with $m_y = 2,4,6,\ldots$ and $m_z = 2,4,6,\ldots$ can be excited.
    - CW has some suppressive effect on particles with large horizontal amplitudes, but it is not effective in fully suppressing these resonances.

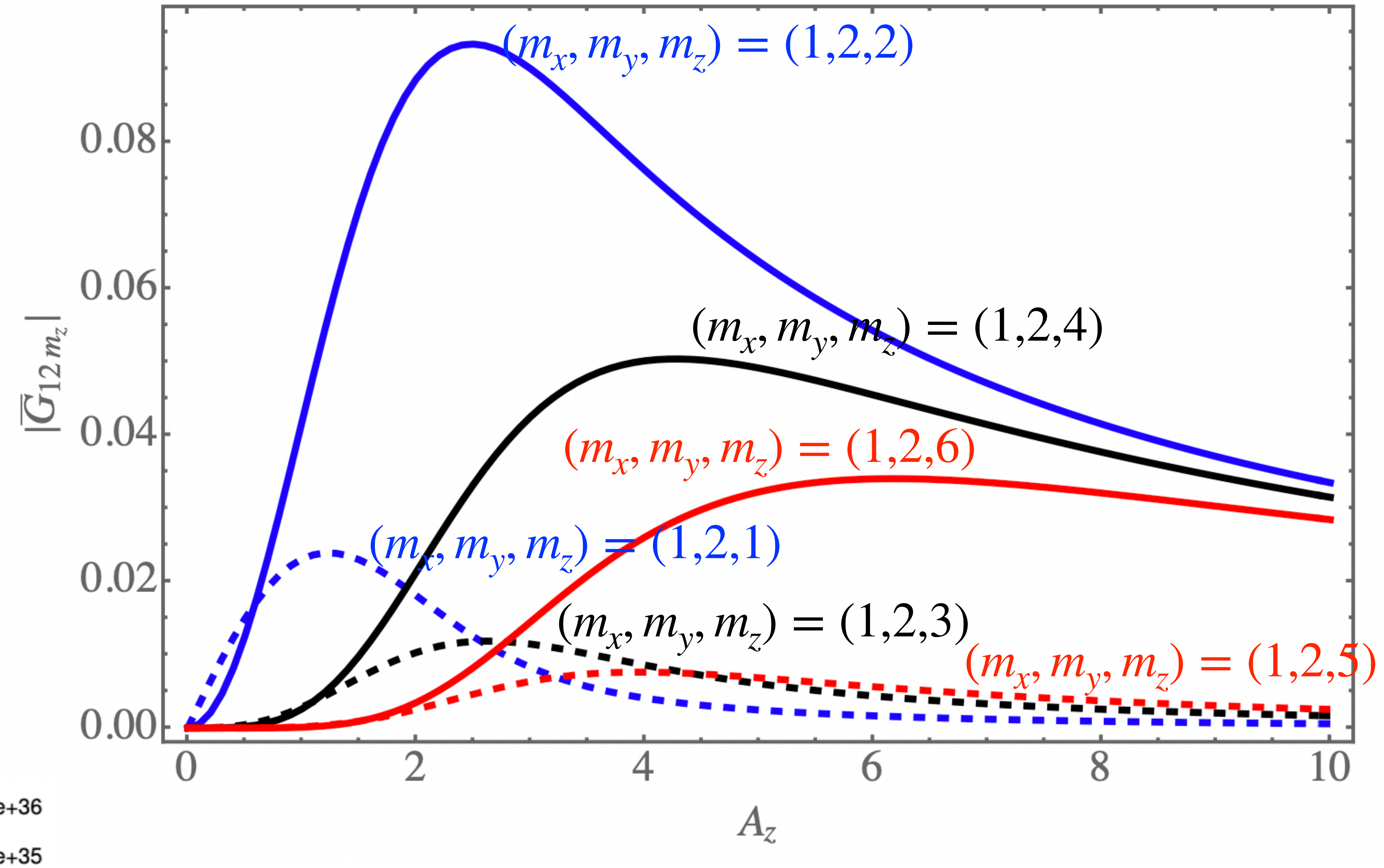


Conditions for the plot: $A_x = 5$, $\zeta_x = 0.5$, $\phi_0 = 10$
Solid lines: without CW
Dashed lines: with CW

Theory of beam-beam resonances for ideal CW colliders

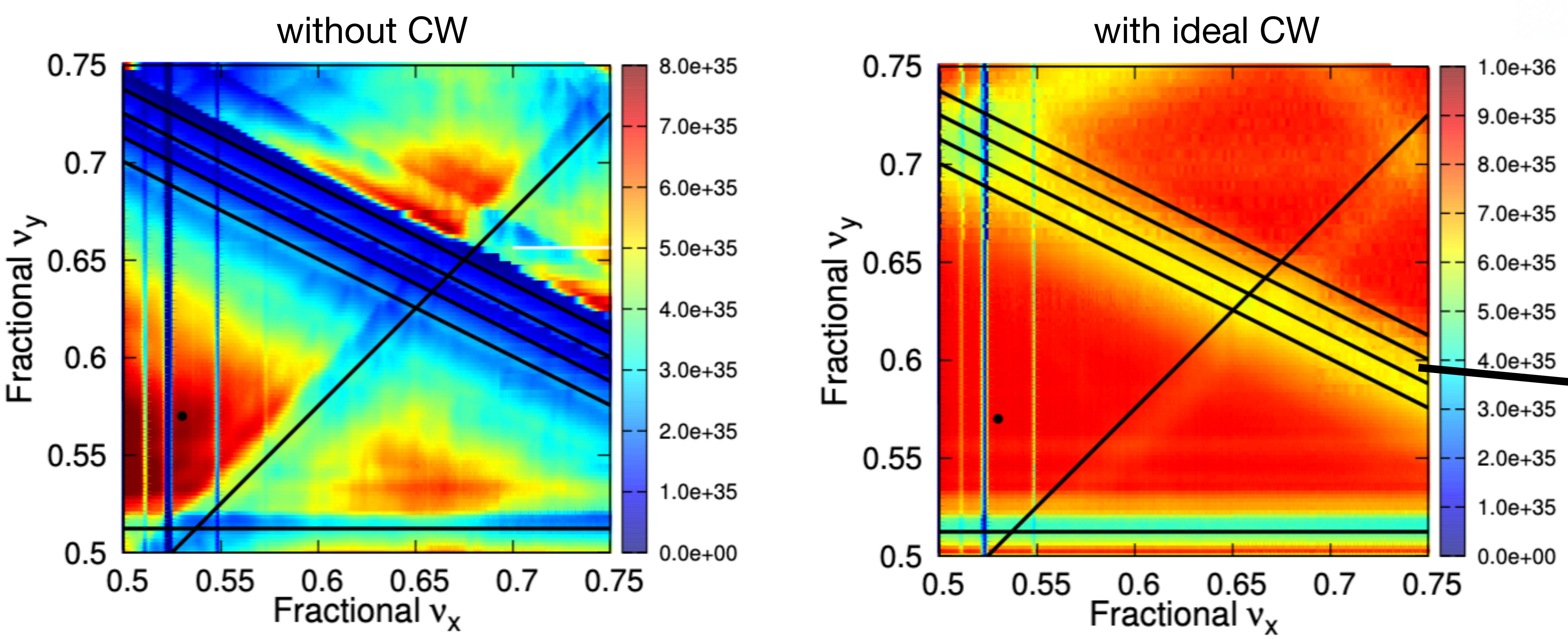
- 3D synchrobetatron resonances $V_{m_x m_y m_z}$ with

  $m_y = 2q > 0$
  - Findings
    - Betatron resonance satellites can be excited
    - Without CW, only satellites with m_z = even can be excited
    - With full CW, only satellites with $m_x + m_z$ = even are excited



Conditions for the plot: $A_x = 5$, $\zeta_x = 0.5$, $\phi_0 = 10$
Solid lines: without CW
Dashed lines: with CW

Tune scan for SuperKEKB final design ($\beta_y^* = 0.3/0.27$ mm)

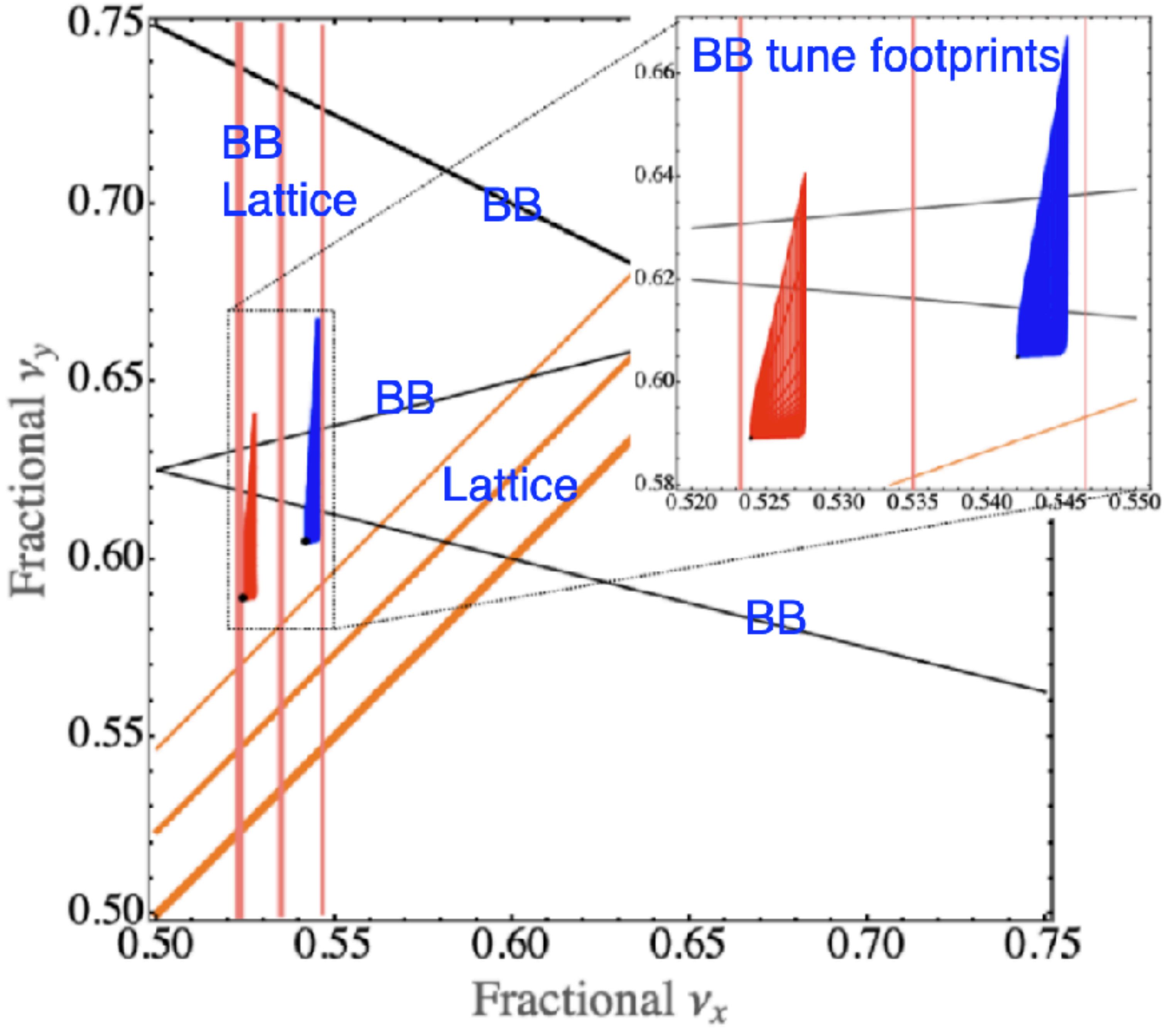


Are these synchrobetatron satellites?

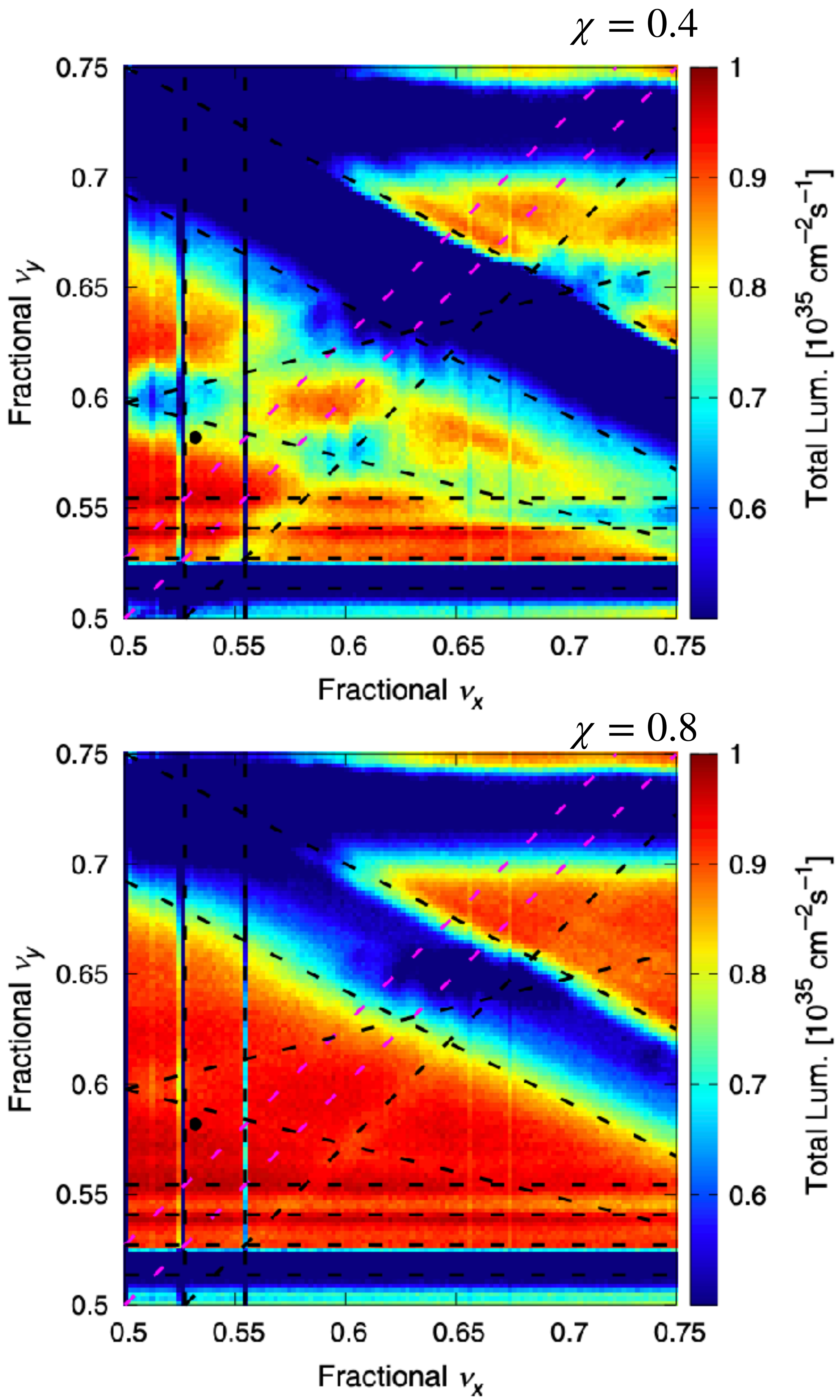
It can be identified by careful analysis of simulation data

Theory of beam-beam resonances for ideal CW colliders

- Theory applied to interpret weak-strong beam-beam simulations
  - Luminosity and beam sizes can be correlated with beam-beam resonances $m_x\nu_x + m_y\nu_y + m_z\nu_z = N$
  - Consider the tunes as functions of many variables $\nu_{x\pm,y\pm,z\pm}(I_{b\pm}, I_{b\mp}, J_{x\pm,y\pm,z\pm}, \beta^*_{x\pm,y\pm}, \beta^*_{x\mp,y\mp}, \epsilon_{x\mp,y\mp}, \ldots)$ due to multiple beam physics aspects.



Tune diagram for SuperKEKB LER [1]



SuperKEKB HER [1]

[1] D. Zhou et al., PRAB 26, 071001 (2023).

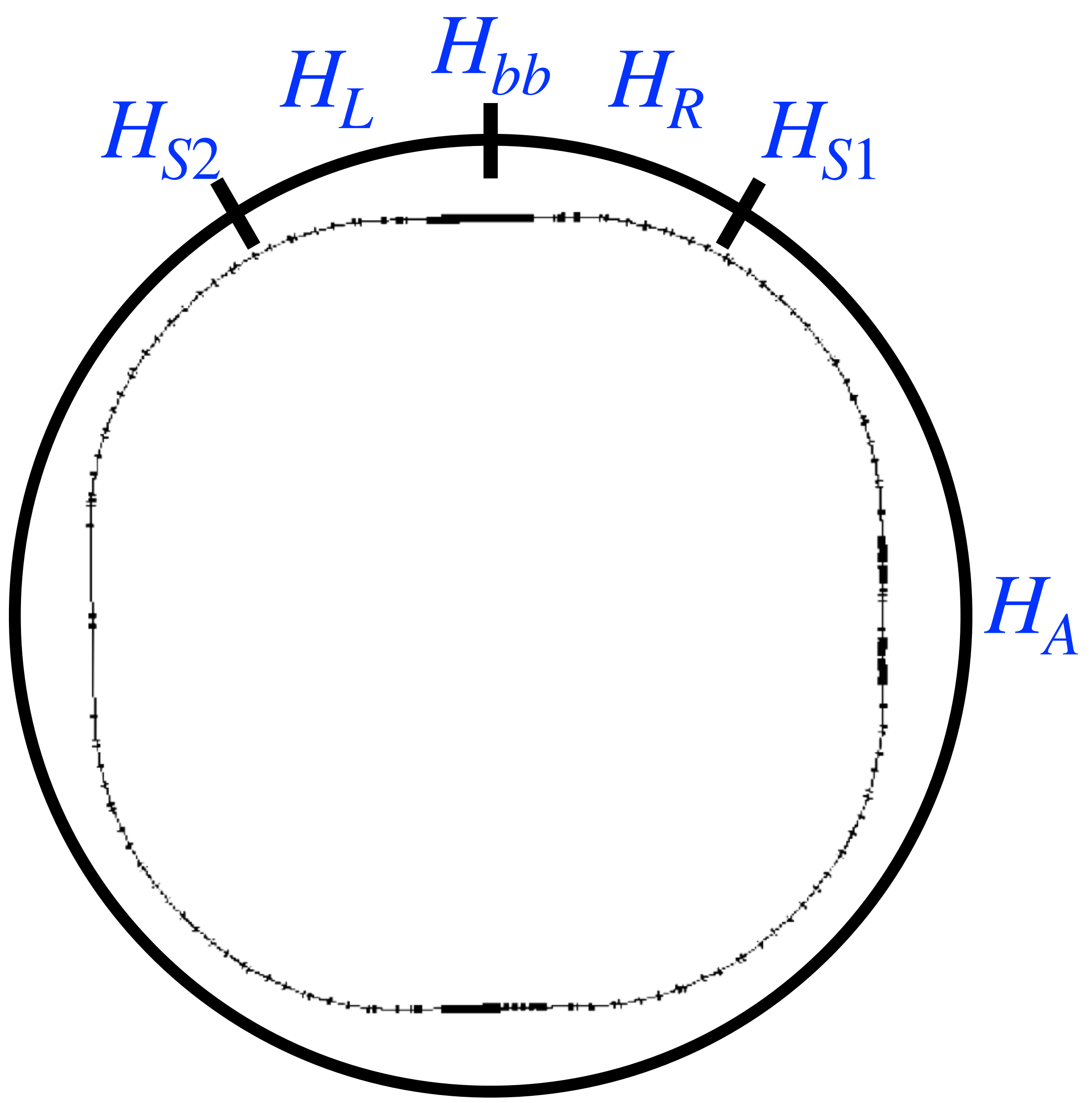
Imperfections in the crab-waist transform

- Categorization of CW imperfections
  - H_{bb}
    - Dynamic beta and emittance
    - Synchrobetatron resonances
  - H_R, H_L
    - Phase advances between IP and CW sextupoles
    - IR nonlinearities
  - H_{S1}, H_{S2}
    - Residual nonlinear terms in CW transform
    - Orbit offset at CW sextupoles
    - Dispersions at CW sextupoles
  - H_0
    - Linear IP aberrations (beta-beat, alpha, dispersion, couplings, etc.)
    - Chromatic IP aberrations
    - Impedance effects
- The impact of various CW imperfections can be analyzed through **theoretical approaches, simulations, and experimental studies.**

$$M = e^{-:H_R:}e^{-:H_{S1}:}e^{-:H_A:}e^{-:H_{S2}:}e^{-:H_L:}e^{-:H_{bb}:}$$

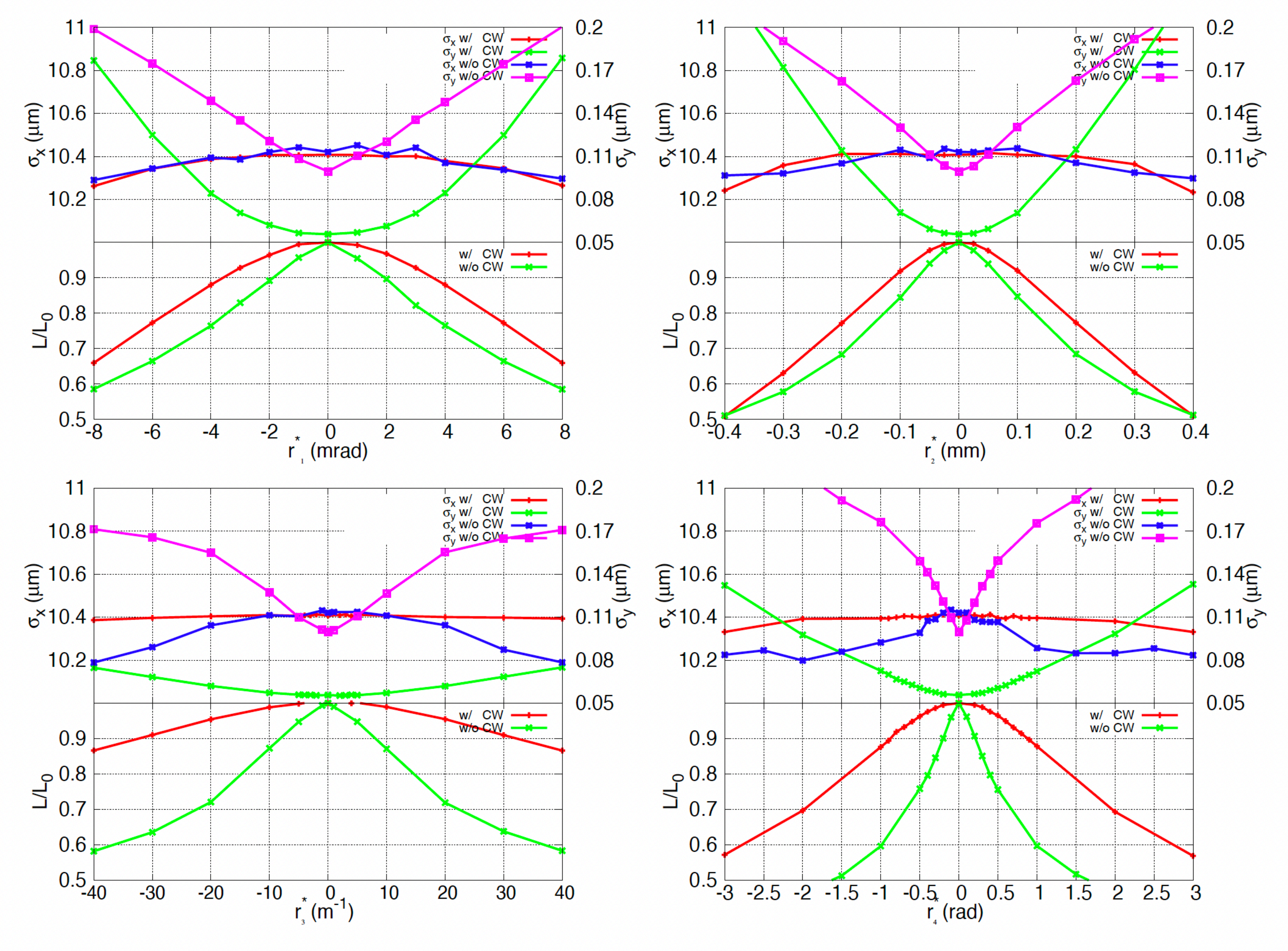
$$M_i = e^{-:H_0:}e^{-:H_{cw}:}e^{-:H_{bb}:}e^{:H_{cw}:} \qquad H_{cw} = \frac{\chi}{2\tan(2\theta_c)}xp_y^2$$

$$K_2 = \pm\frac{1}{\tan(2\theta_c)}\frac{1}{\beta_{y,crab}\beta_y^*}\sqrt{\frac{\beta_x^*}{\beta_{x,crab}}}$$



Imperfections in the crab-waist transform

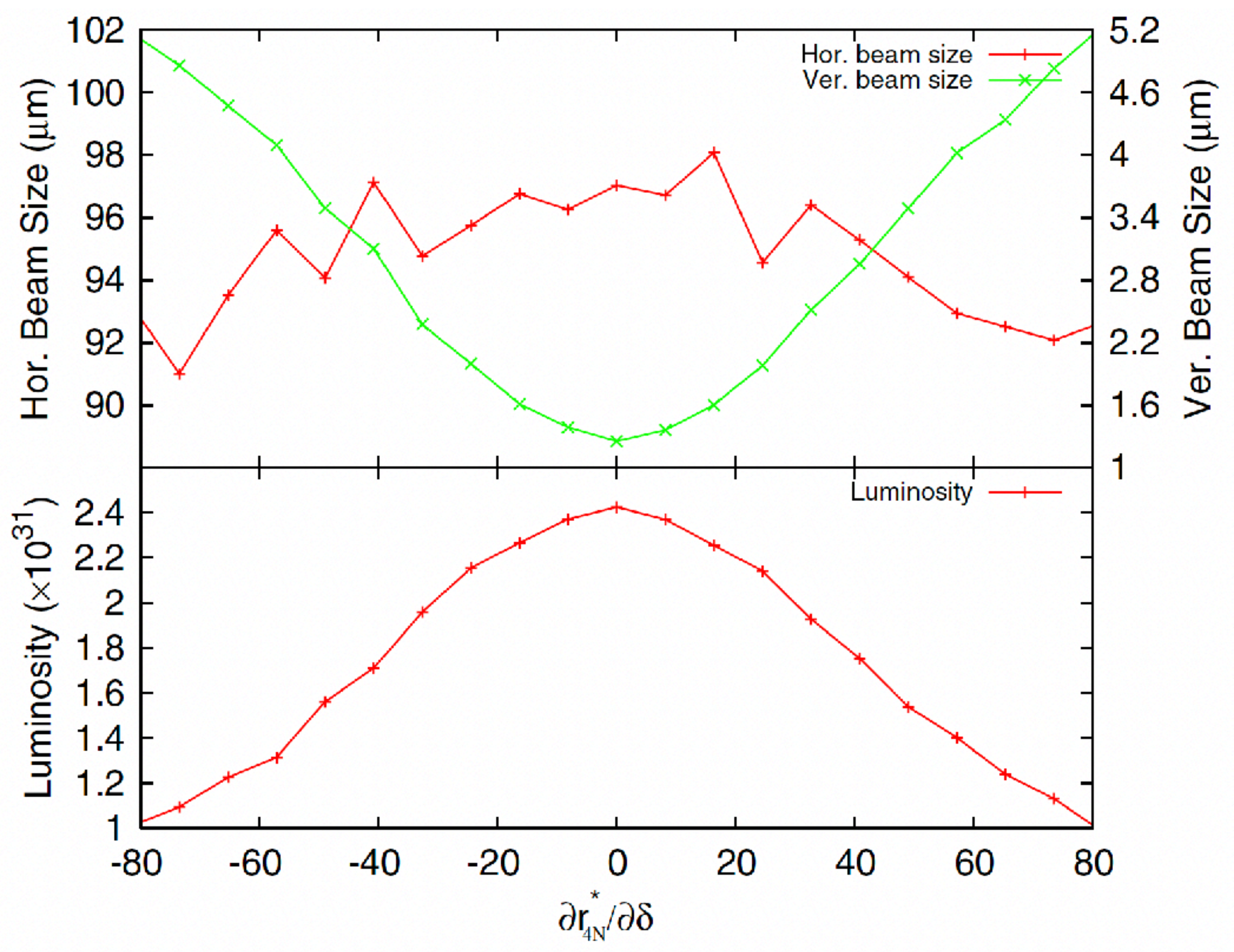
- Linear IP aberrations
  - Recognized to be important in KEKB and SuperKEKB (K. Ohmi, eeFACT'18)
  - IP knobs are necessary to suppress these IP aberrations
  - Through simulations, tolerances can be defined in design stages of CW colliders



Scan of IP couplings for SuperKEKB w/ and w/o CW (D. Zhou et al., IPAC'10)

Imperfections in the crab-waist transform

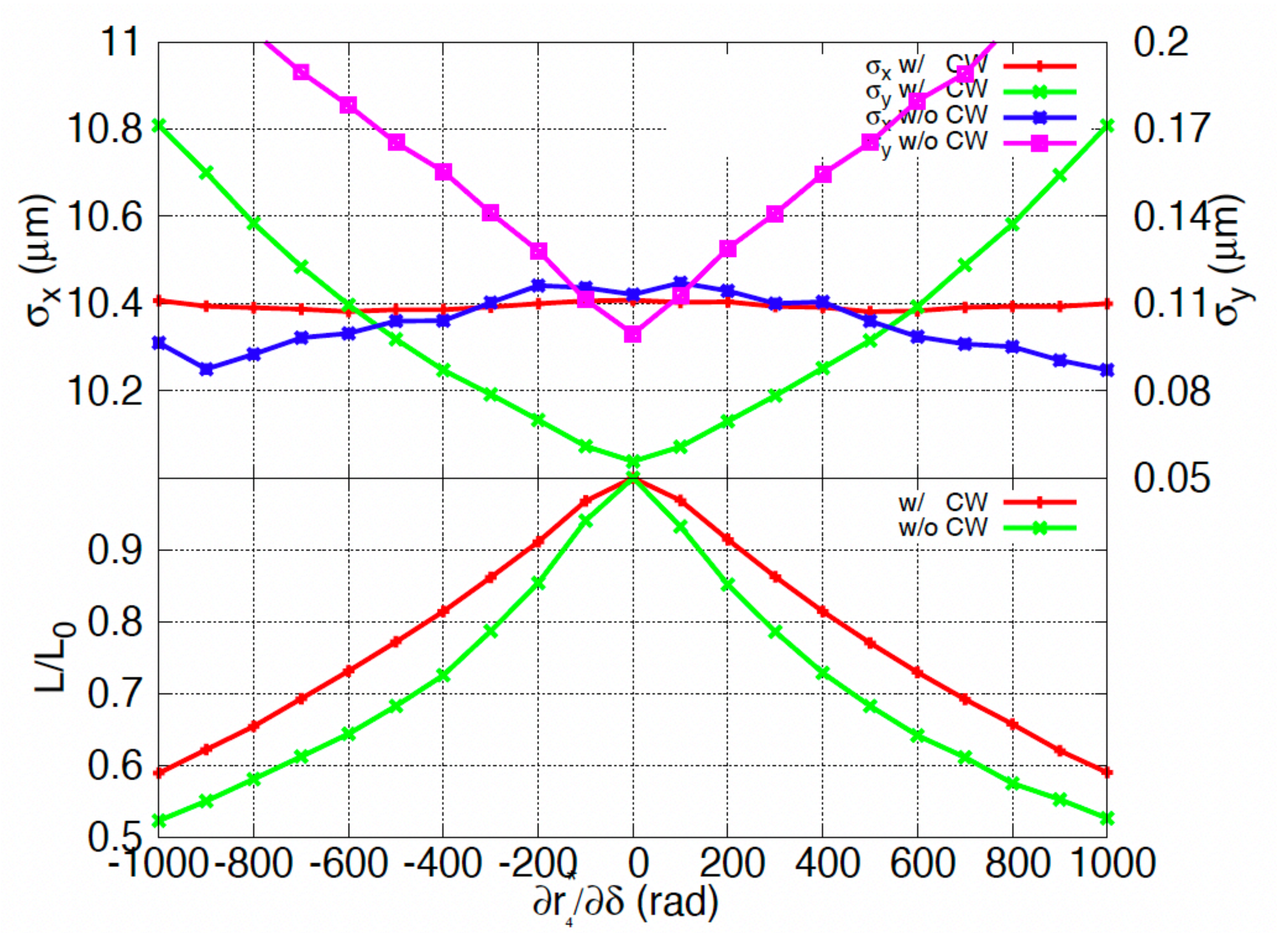
- Nonlinear IP aberrations
  - Chromatic couplings were found important in KEKB (D. Zhou et al., PRST-AB 13, 021001 (2010)) and SuperKEKB
  - IP knobs are necessary to suppress these chromatic IP aberrations
  - Through simulations, tolerances can be defined in design stages of CW colliders



Scan of $dR_4^*/d\delta$ for KEKB (D. Zhou et al., PRAST-AB (2010))

Table 3: Tolerances for the linear and chromatic X-Y couplings at the IP of the SuperKEKB LER, assuming a rate of 20% luminosity degradation.

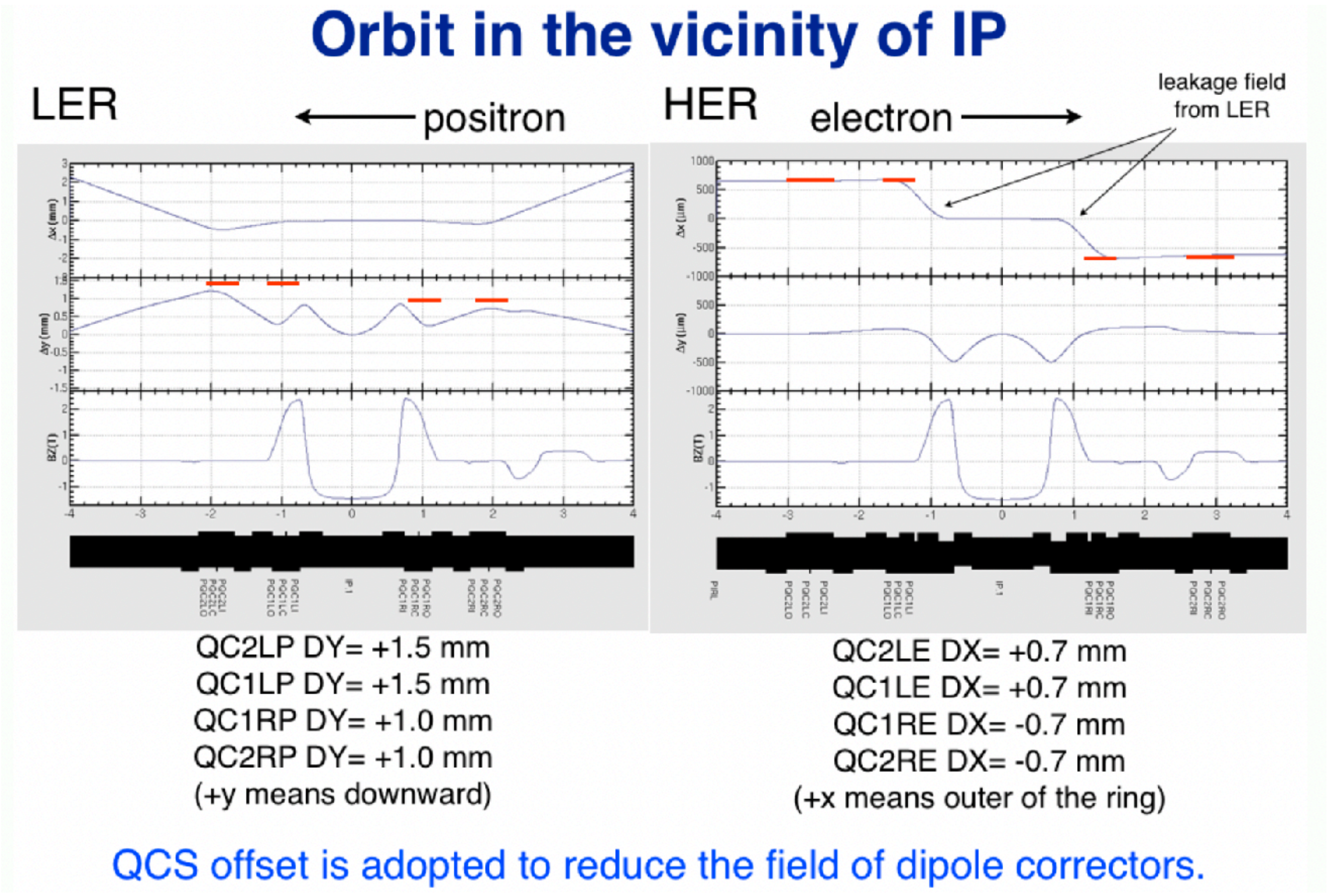
Parameter	w/ crab waist	w/o crab waist
$r_1^*\,(mrad)$	± 5.3	± 3.5
$r_2^*\,(mm)$	± 0.18	± 0.13
$r_3^*\,(m^{-1})$	± 55	± 15
$r_4^*\,(rad)$	± 1.4	± 0.4
$r_{11}\,(rad)$	± 2.3	± 2.0
$r_{21}\,(m)$	± 0.09	± 0.07
$r_{31}\,(m^{-1})$	± 11000	± 9400
$r_{41}\,(rad)$	± 430	± 280

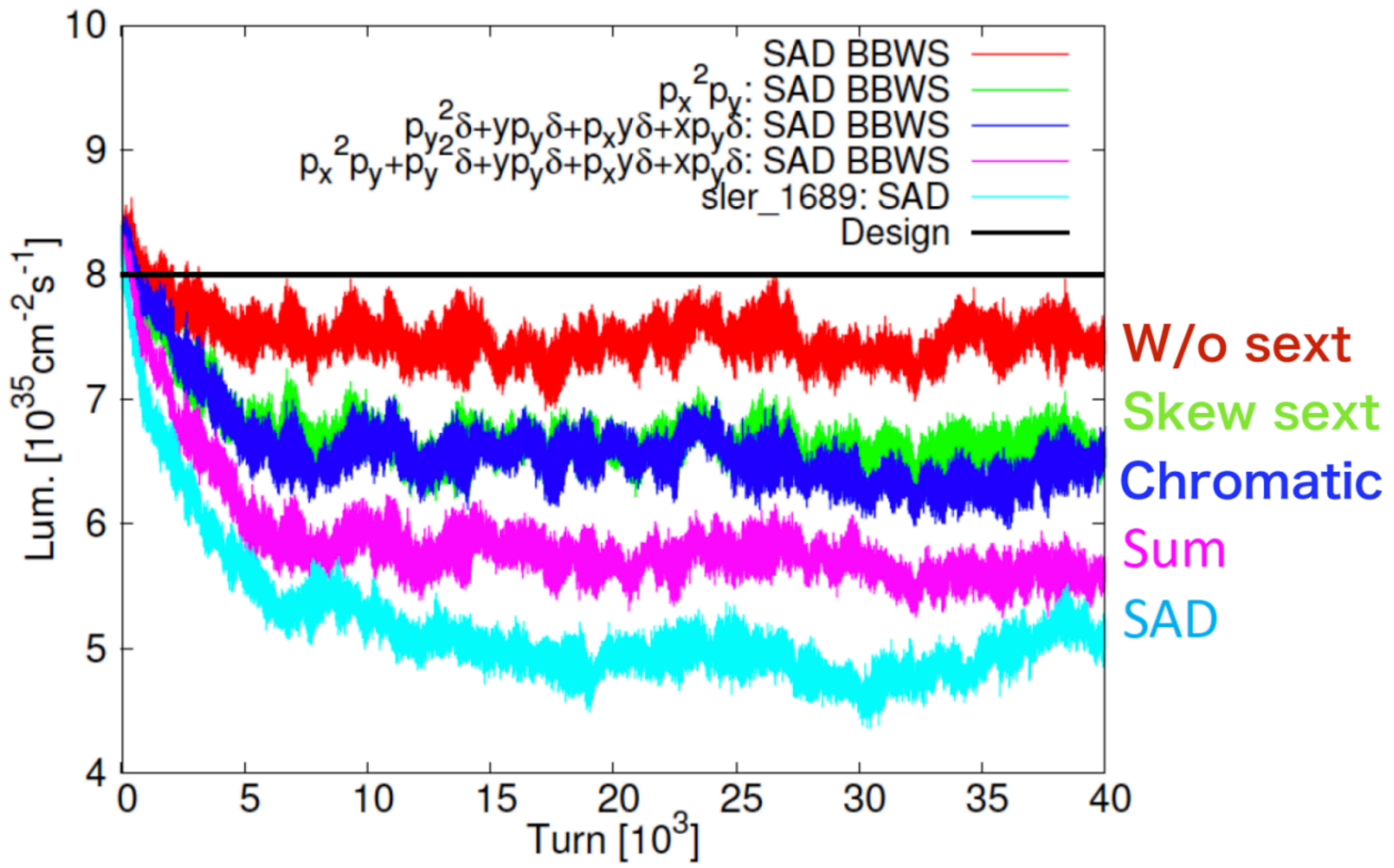


Scan of $dR_4^*/d\delta$ for SuperKEKB (D. Zhou et al., IPAC'10)

Imperfections in the crab-waist transform

- Nonlinear IR at SuperKEKB
  - Extremely small β_y^* → Nonlinear effects from kinematic term of IP drift and fringe fields of final focus (FF) quadrupoles [1] → Fundamental limit on dynamic aperture and lifetime [1,2,3] → Poor injection efficiency [4] and high detector background [5].
  - Overlap of solenoid and FF quadrupoles, offsets of FF quadrupoles, etc. → Vertical emittance growth (single-beam) due to local linear and chromatic couplings [6] → Vertical emittance growth (two-beam) from interplay of beam-beam and lattice nonlinearity [7,8] → Imperfect CW due to nontransparent IR [2].





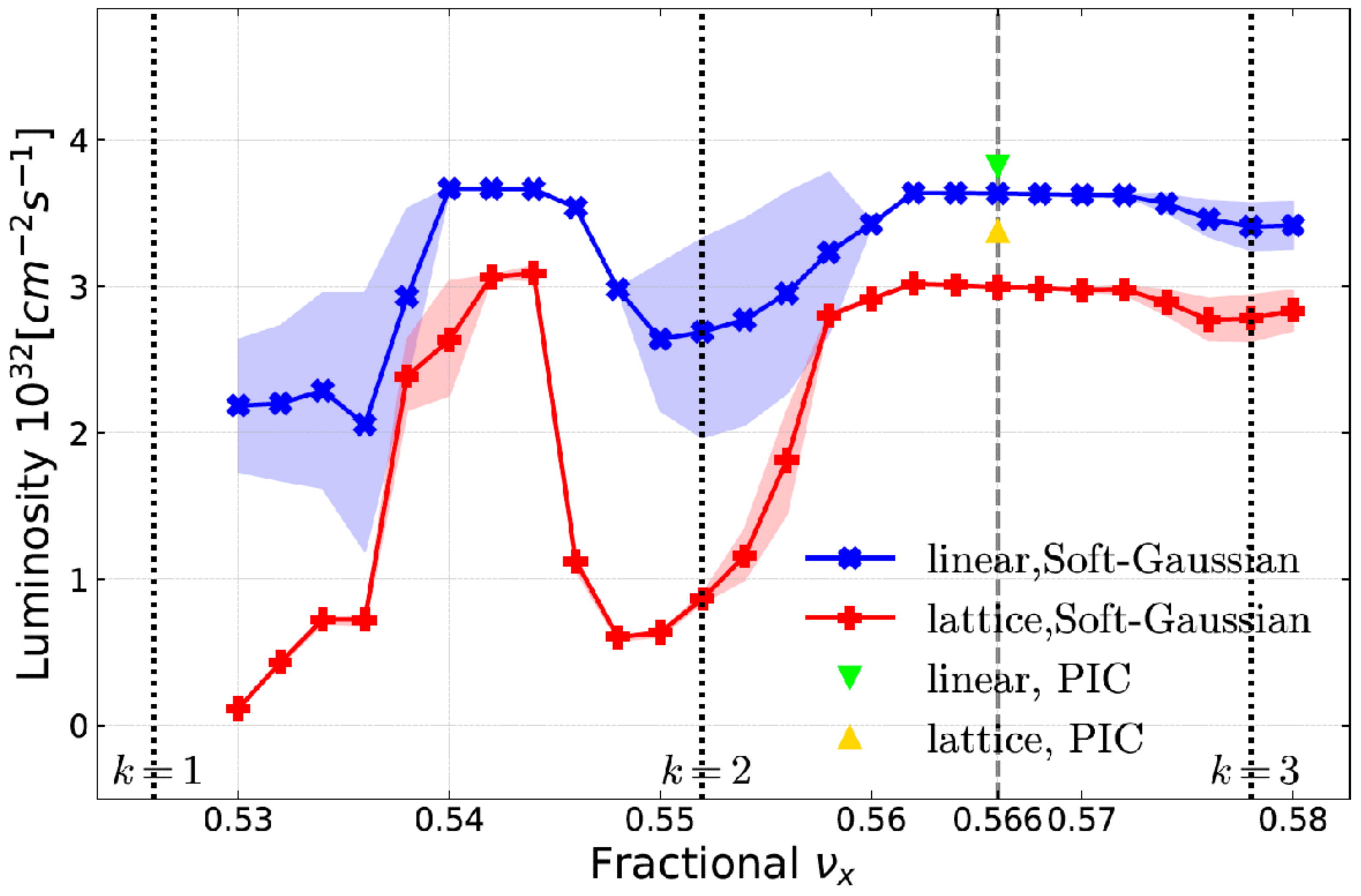
[1] K. Oide and H. Koiso, Phys. Rev. E 47, 2010 (1993). [2] SuperKEKB TDR. [3] Y. Suetsugu, et al., PRAB 26, 013201 (2023). [5] A. Natochii, et al., "Beam background expectations for Belle II at SuperKEKB".
[6] M. Masuzawa, IPAC'22. [7] D. Zhou et al., "Beam Dynamics Issues in the SuperKEKB". [8] K. Hirosawa et al., J. Phys.: Conf. Ser. 1067 062004 (2018).

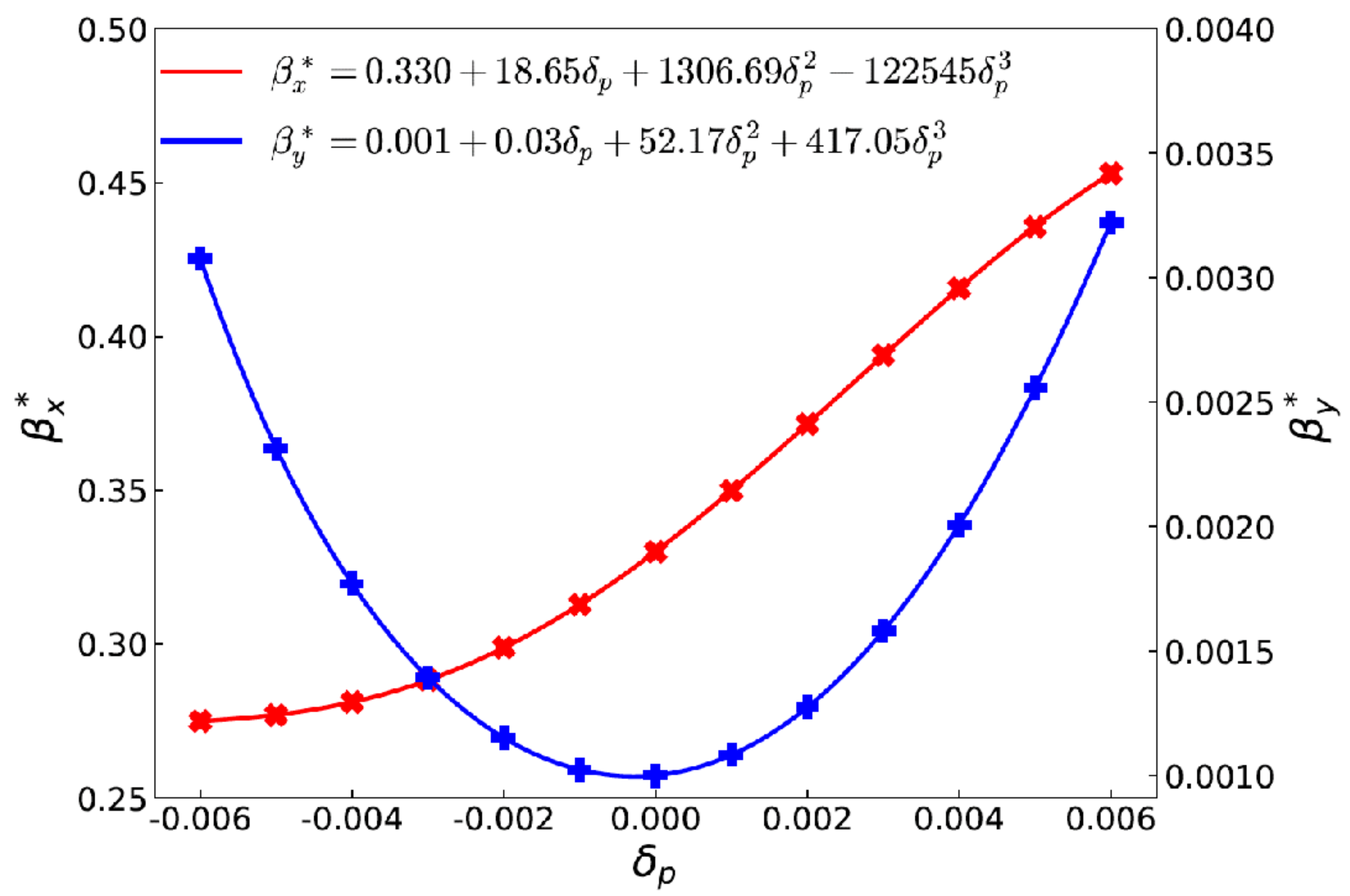
Imperfections in the crab-waist transform

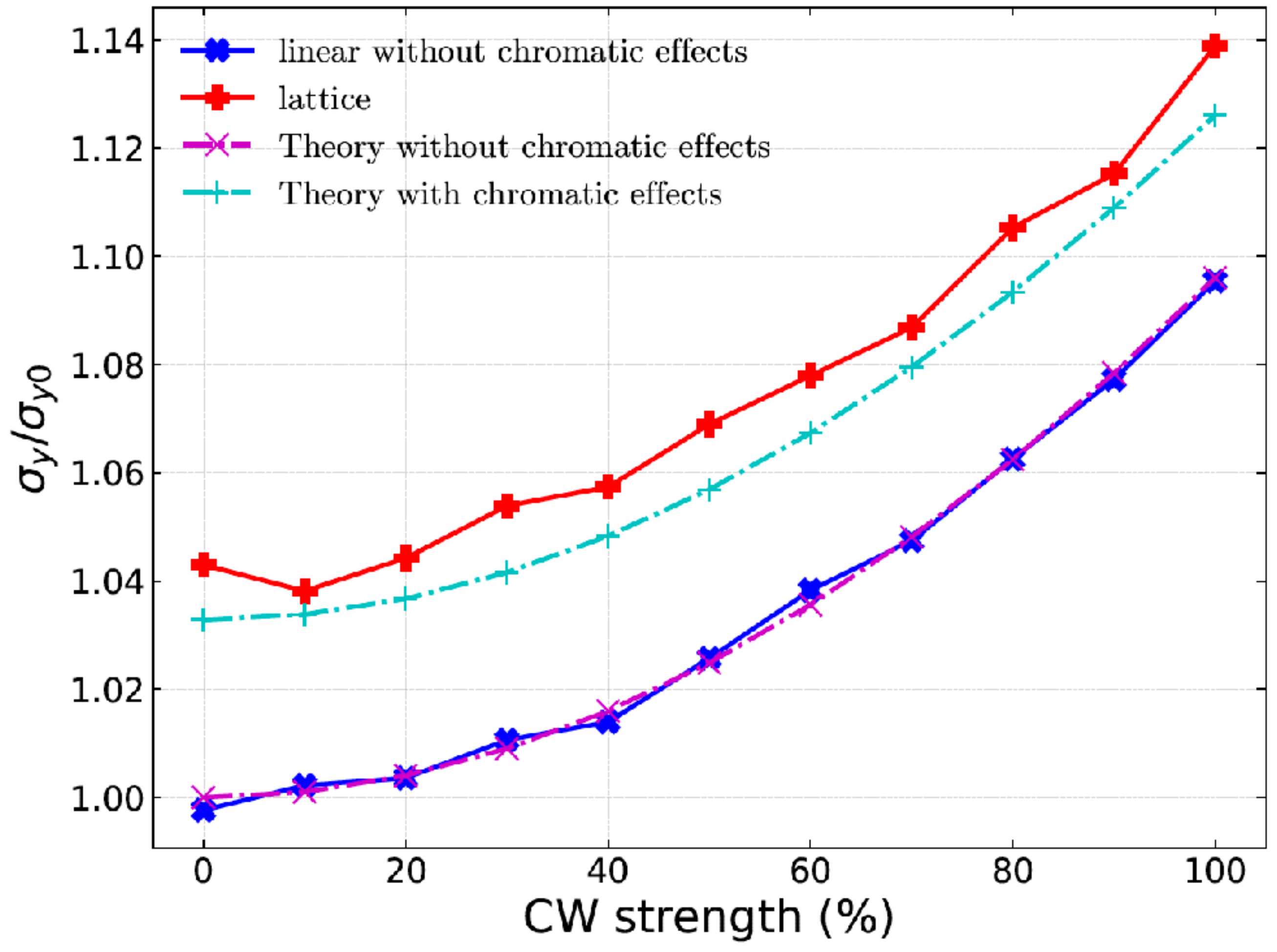
- Chromatic effects at CEPC [1]
  - Macroparticle tracking by **APES-T** with full lattice and beam-beam
  - Chromatic effects were identified as the main sources of luminosity loss

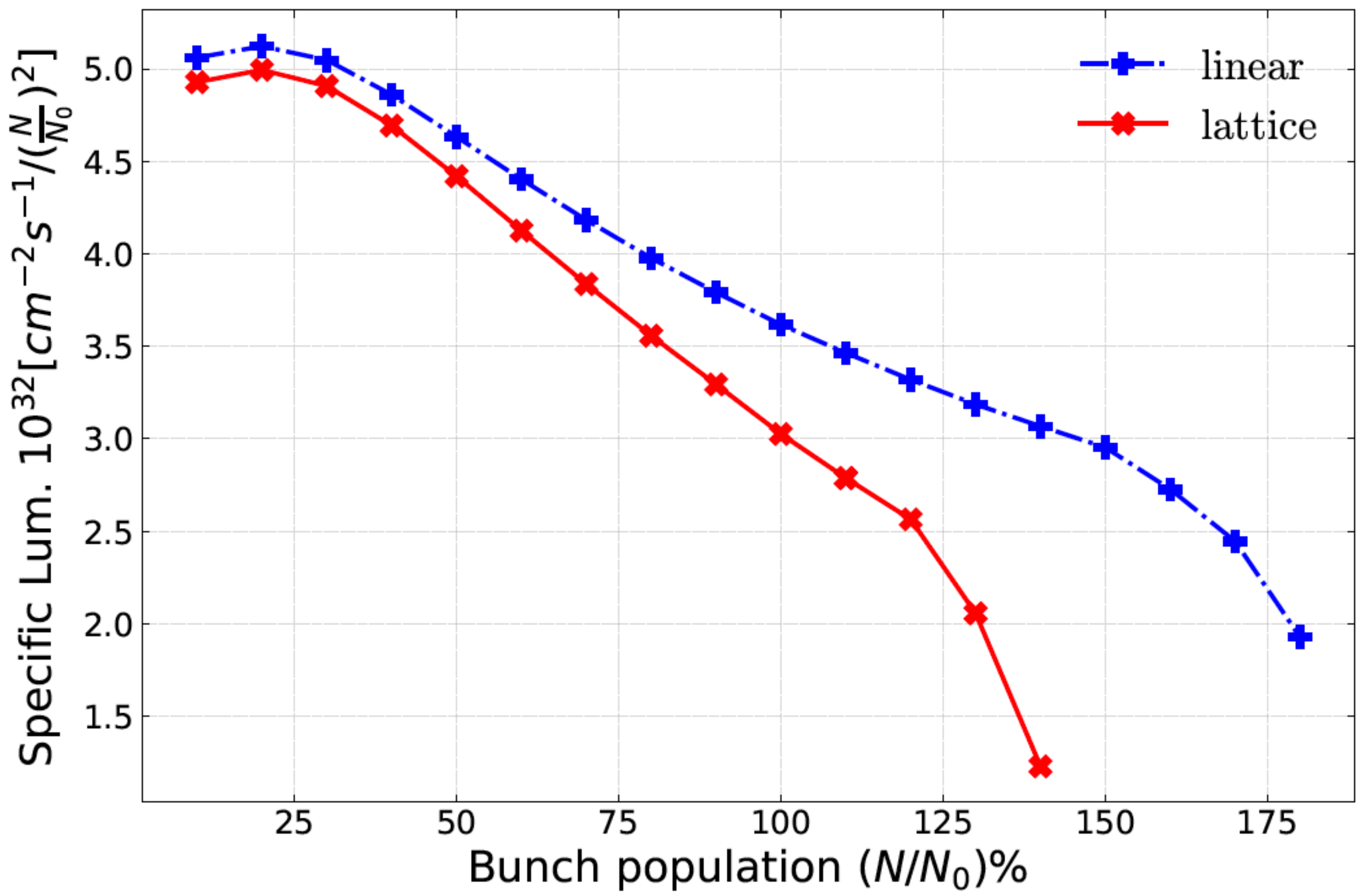
$$\sigma_y^* \approx \sigma_{y0}^* \sqrt{1 + \frac{\chi^2 \epsilon_x}{\beta_{y0}^* \tan^2(2\theta_c)} + (b_2 + a_1^2)\sigma_p^2}$$

$$\beta_y^*(p_z) = \beta_{y0}^*(1 + b_1 p_z + b_2 p_z^2) \qquad \alpha_y^*(p_z) = a_0 + a_1 p_z + a_2 p_z^2$$









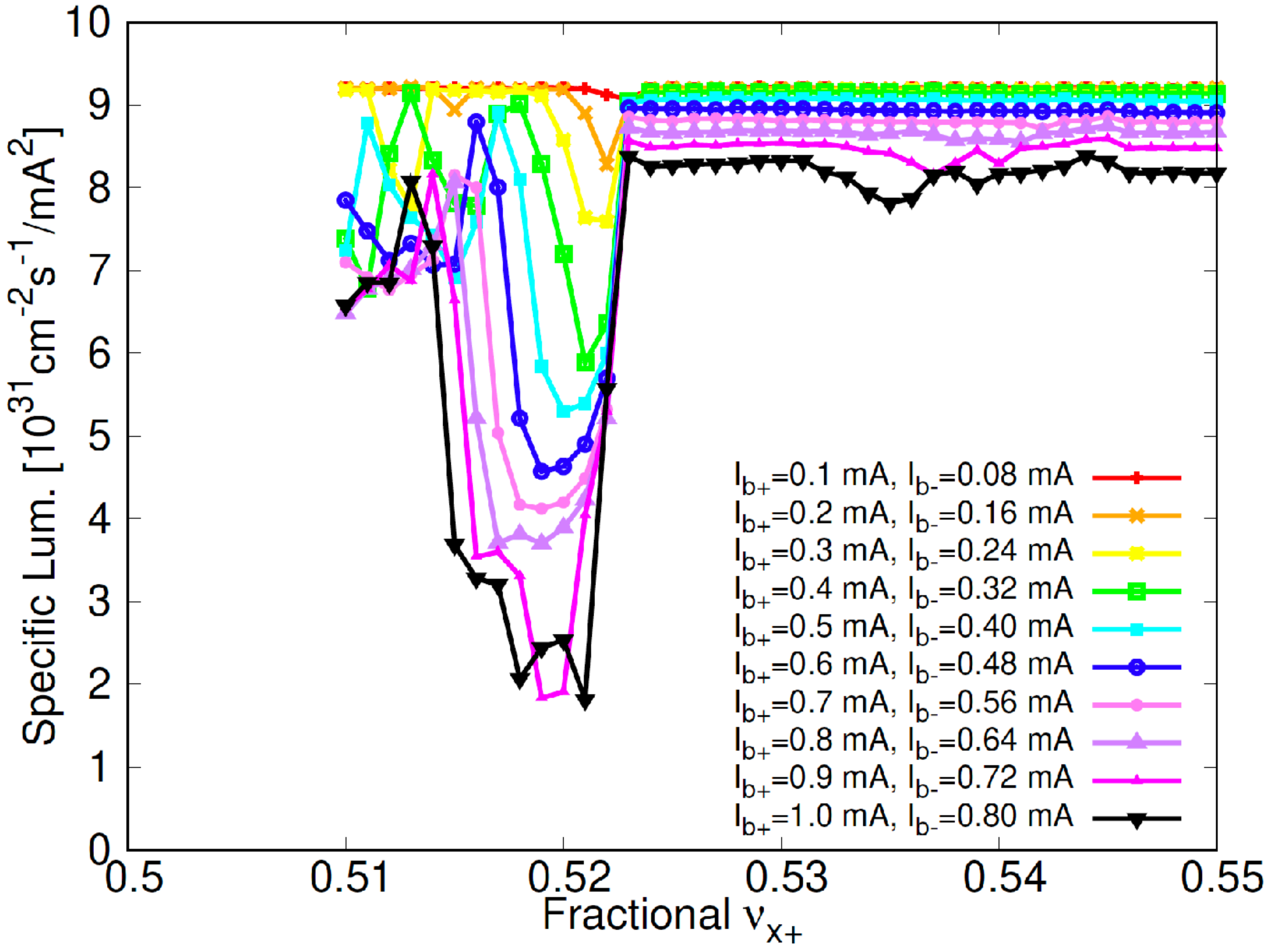
Z. Li, this workshop

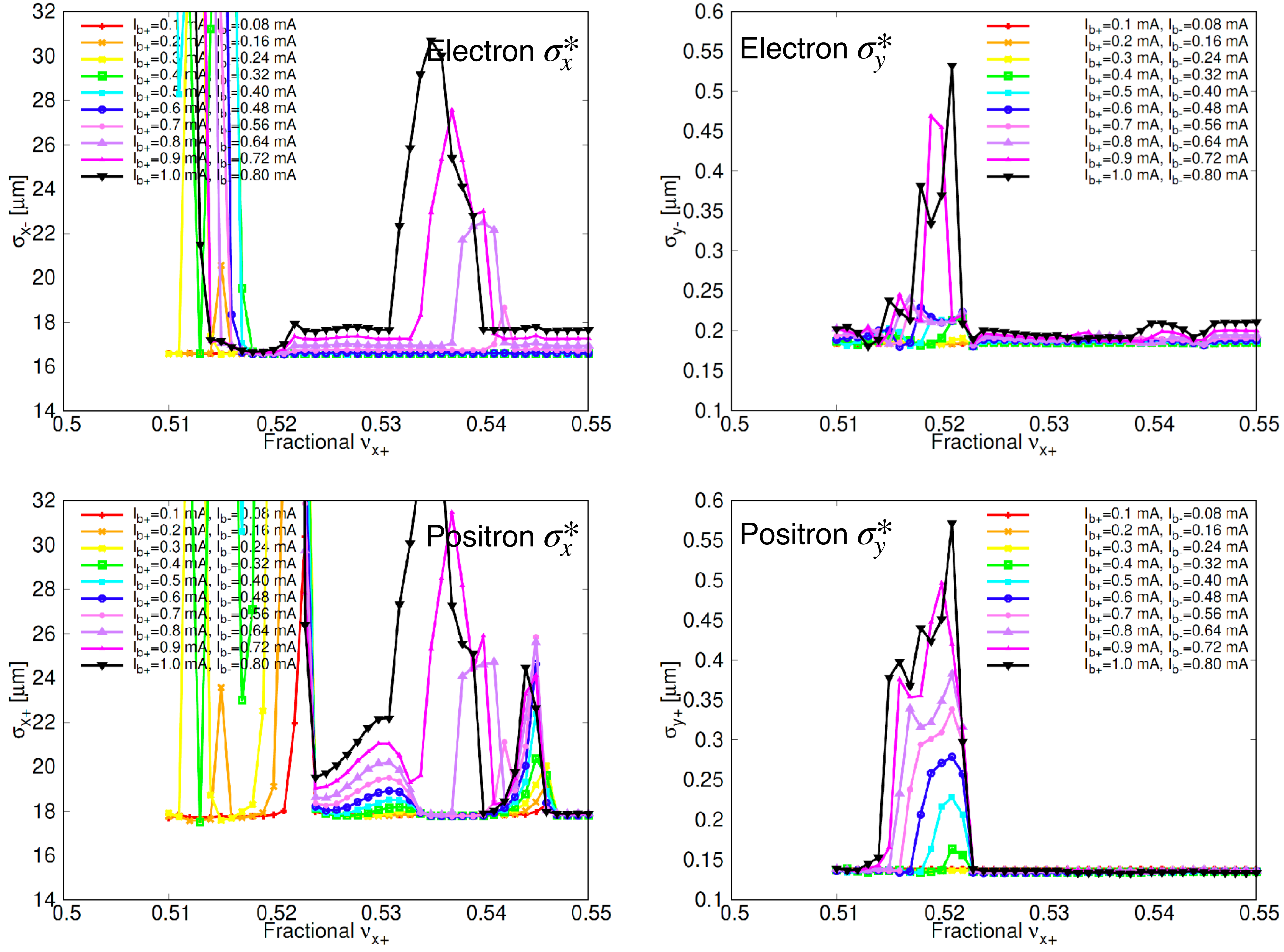
[1] Z. Li et al., NIM-A 1064 (2024): 169386.

Imperfections in the crab-waist transform

- Impedance effects
  - Strong **coherent X-Z instability** around resonance $2\nu_x - 2k\nu_s(J_z) = N$ and weak blowup due to **synchrobetatron (SB) resonances** [1]
  - Mitigation: **Squeezing** β_x^* [2]

Strong-strong simulations of beam-beam + impedance effects for SuperKEKB





[1] D. Zhou et al., PRAB 26, 071001 (2023). [2] K. Ohmi et al., PRL 119, 134801.

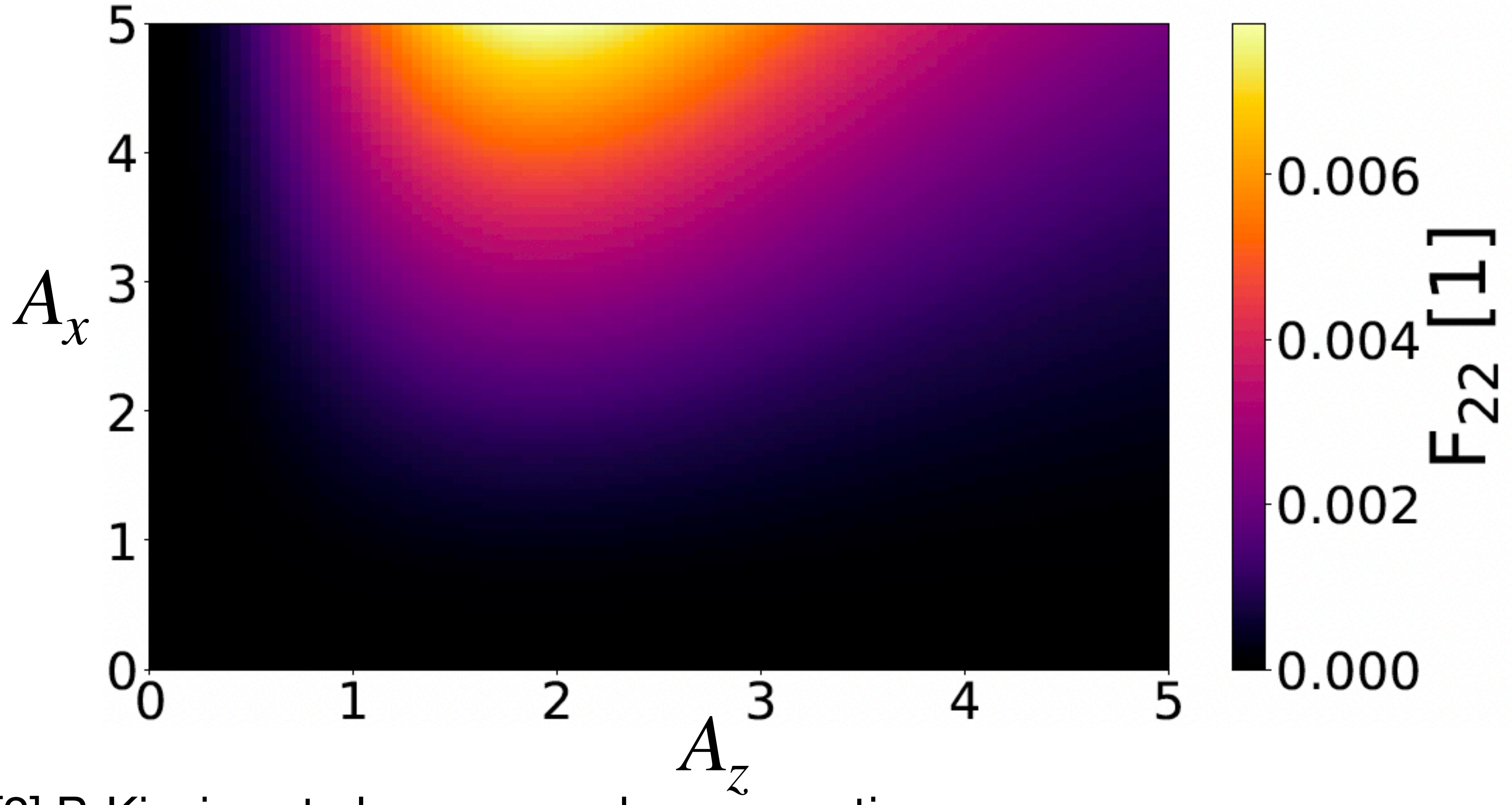
Imperfections in the crab-waist transform

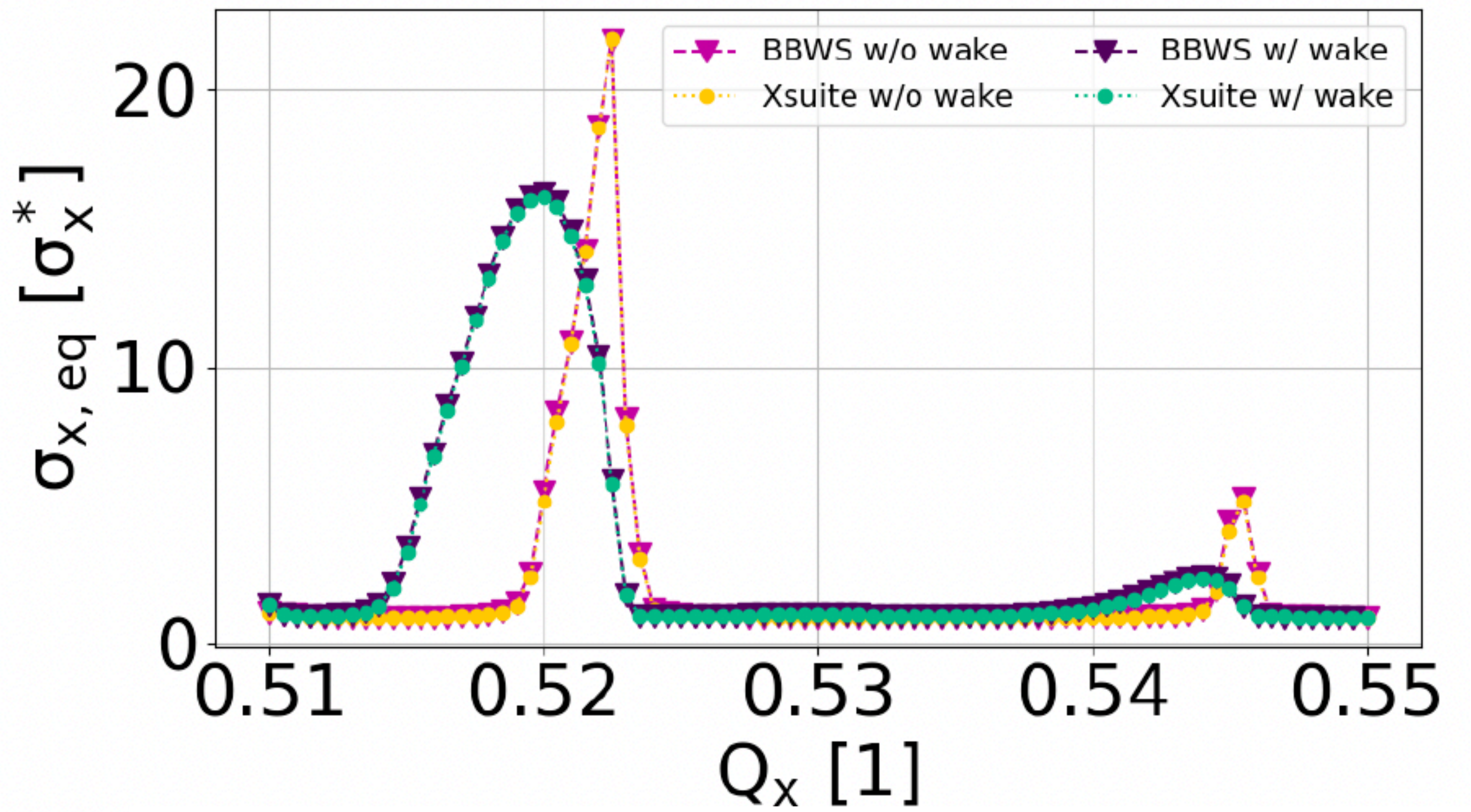
- Impedance effects
  - Horizontal SBRs cause blowup in ϵ_x, altered by impedance effects [1, 2].
  - Resonance condition:

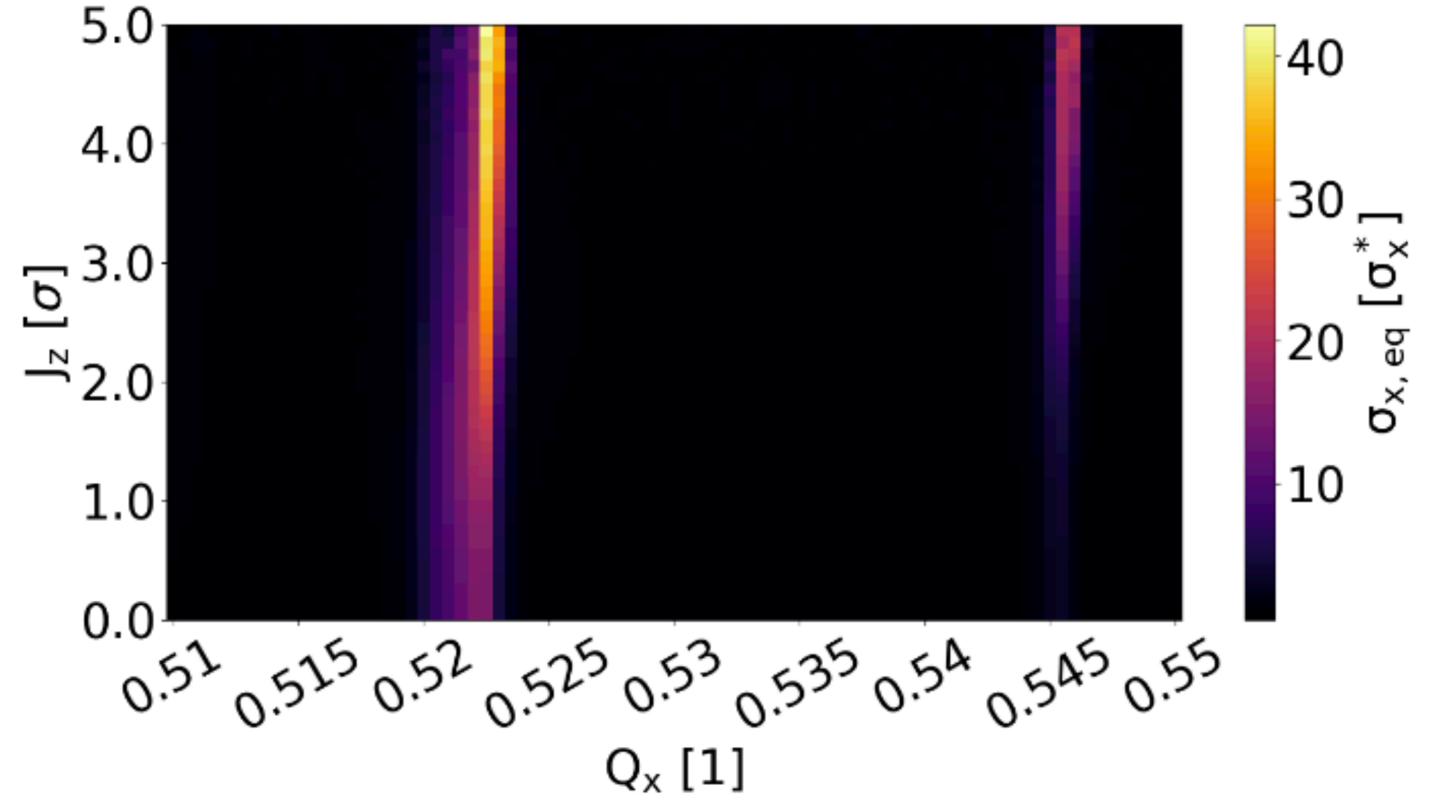
$$m_x \nu_x + m_z \nu_z = N \qquad \nu_x = \nu_{x0} + \Delta\nu_z^{bb}(J_x, 0, J_z)$$

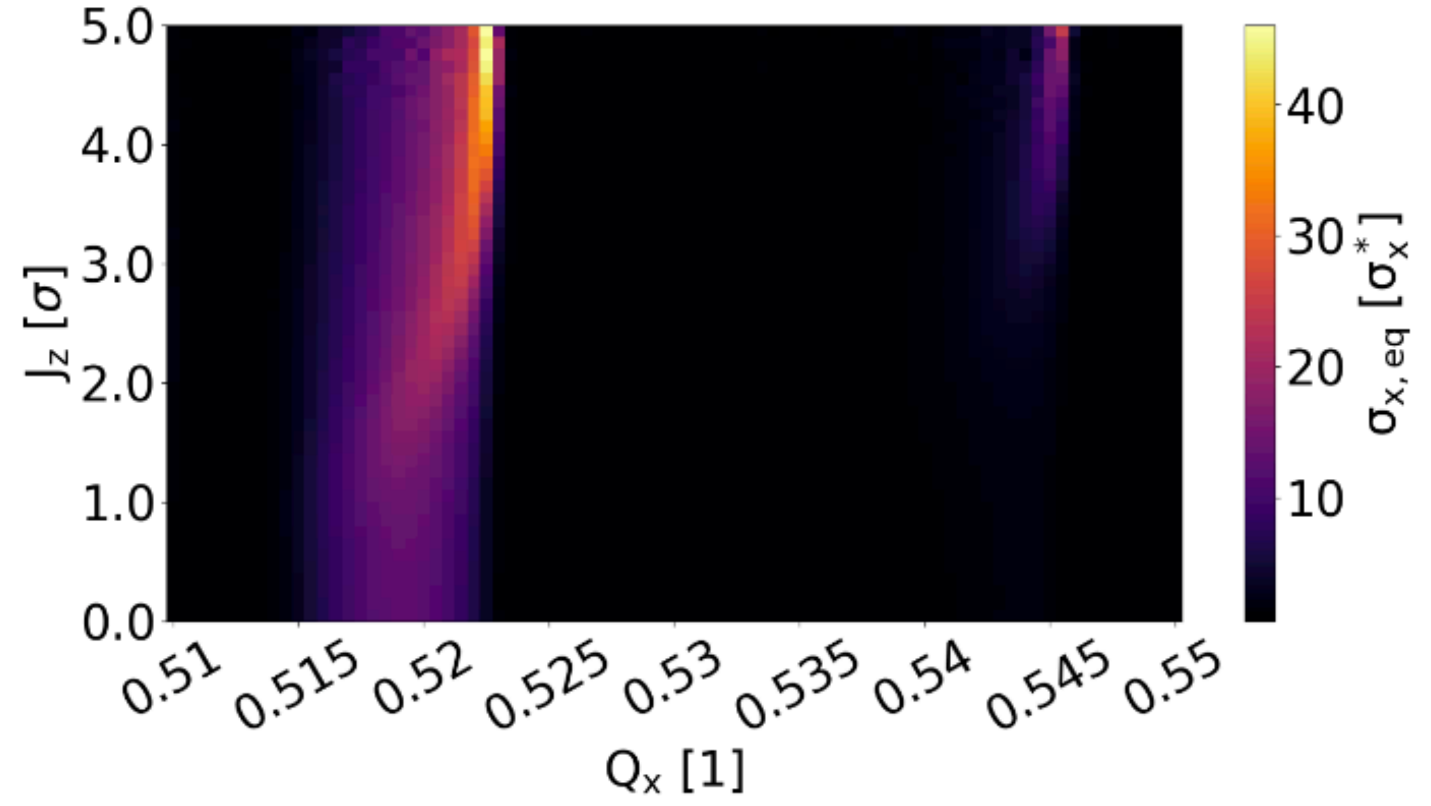
$$\nu_z = \nu_{z0} + \Delta\nu_z^{bb}(J_x, 0, J_z) + \Delta\nu_z^{wake}(J_z)$$

$$F_{m_x m_z}(A_x, A_z) = \int_0^\infty \frac{dk}{k} e^{-\frac{k^2}{2}} J_{m_x}\left(\frac{kA_x}{\sqrt{\phi_0^2+1}}\right) J_{m_z}\left(\frac{k\phi_0 A_z}{\sqrt{\phi_0^2+1}}\right)$$









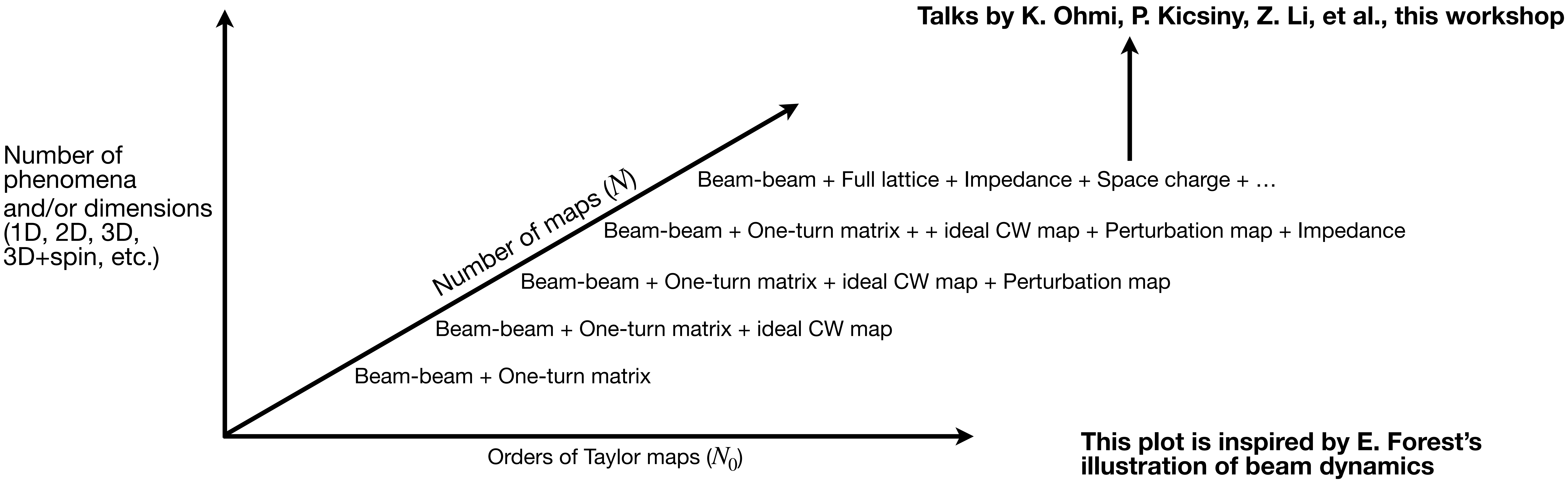
P. Kicsiny, this workshop

[1] D. Zhou et al., PRAB 26, 071001 (2023). [2] P. Kicsiny et al., paper under preparation.

Summary

- The effectiveness of CW in colliders may be diminished by various imperfections.
- These CW imperfections can be systematically studied using a framework that includes **theoretical analysis, simulations, and experimental investigations**, allowing for controlled complexity at each stage.

$$M = e^{-:H_1:}e^{-:H_2:}e^{-:H_3:}\ldots e^{-:H_N:} \qquad M = e^{:f_2:}e^{:f_3:}e^{:f_4:}\ldots e^{:f_{N_0}:}$$



This plot is inspired by E. Forest's illustration of beam dynamics

Acknowledgements

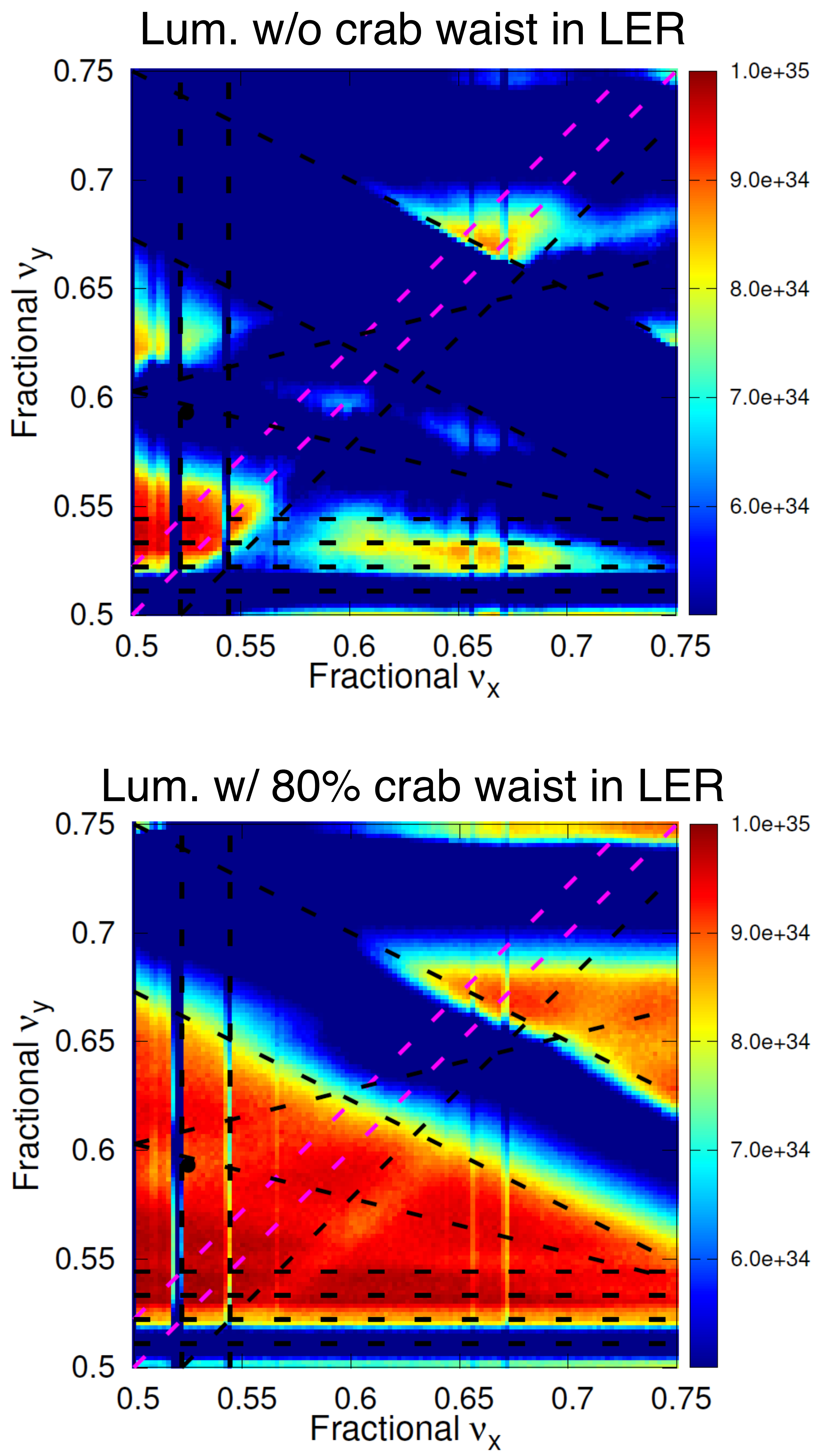
- I thank K. Ohmi, Z. Li, P. Kicsiny, and X. Buffat for inspiring discussions on beam-beam issues in crab-waist colliders.

Backup

Crab waist applied to SuperKEKB

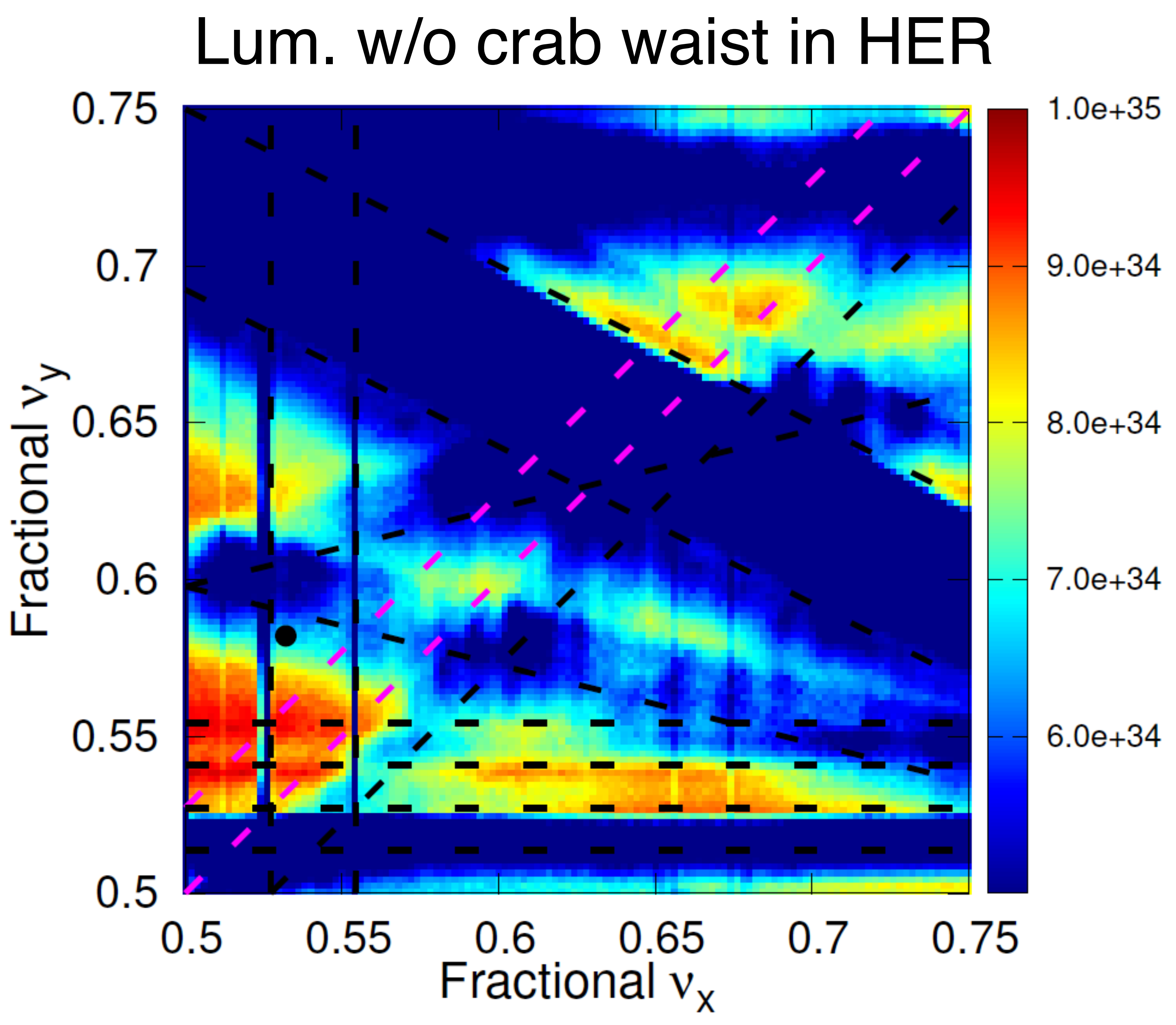
- SuperKEKB 2021b run ($\beta_y^* = 1$ mm) with ideal crab waist
  - Tune scan using BBWS showed that 80% crab waist ratio in LER is effective in suppressing vertical blowup caused by beam-beam resonances (mainly $\nu_x \pm 4\nu_y + \alpha = N$).

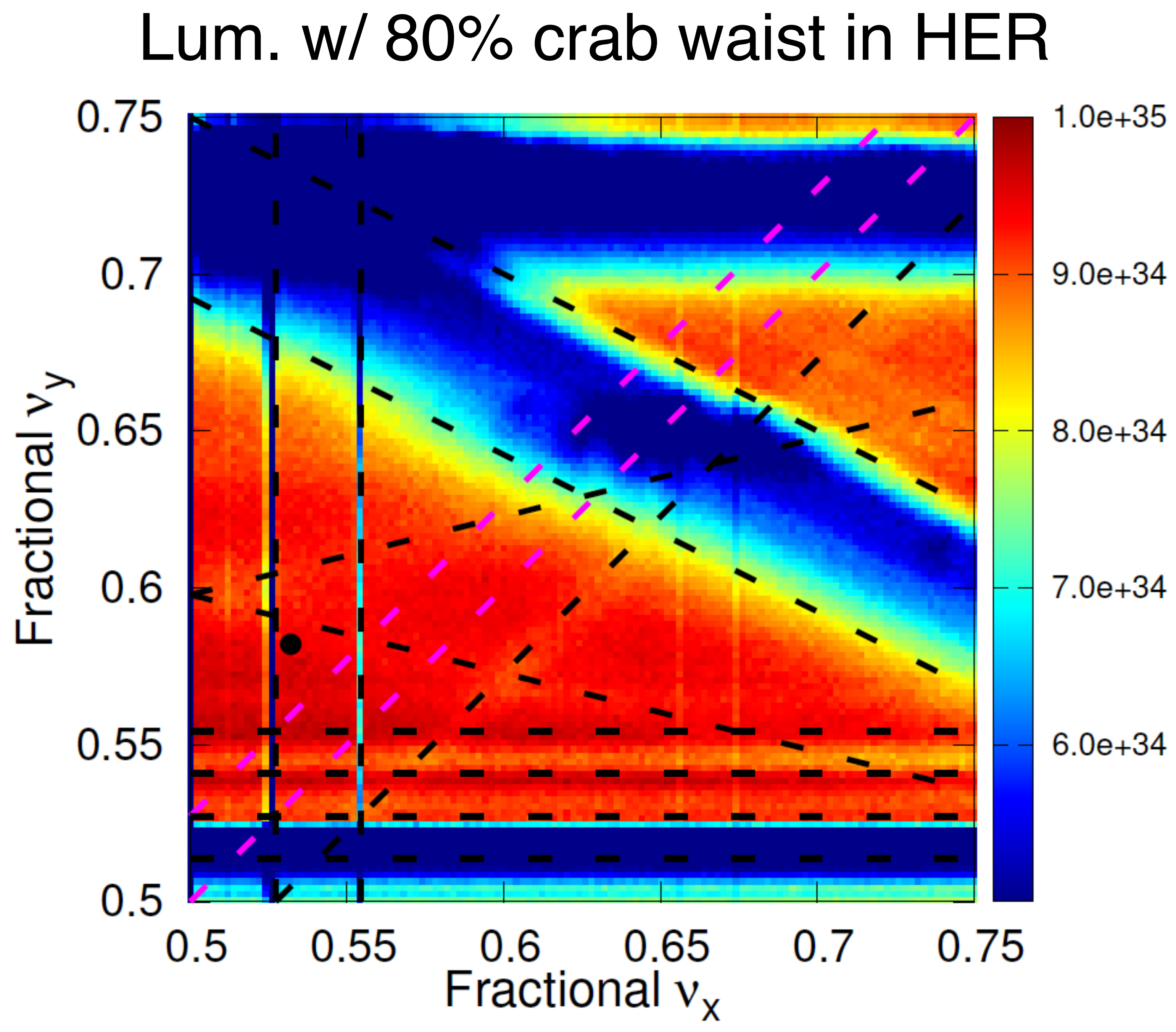
	2021.07.01		Comments
	HER	LER	
I_{bunch} (mA)	0.80	1.0	
# bunch	1174		Assumed value
ε_x (nm)	4.6	4.0	w/ IBS
ε_y (pm)	23	23	Estimated from XRM data
β_x (mm)	60	80	Calculated from lattice
β_y (mm)	1	1	Calculated from lattice
σ_{z0} (mm)	5.05	4.84	Natural bunch length (w/o MWI)
ν_x	45.532	44.525	Measured tune of pilot bunch
ν_y	43.582	46.593	Measured tune of pilot bunch
ν_s	0.0272	0.0221	Calculated from lattice
Crab waist	40%	80%	Lattice design

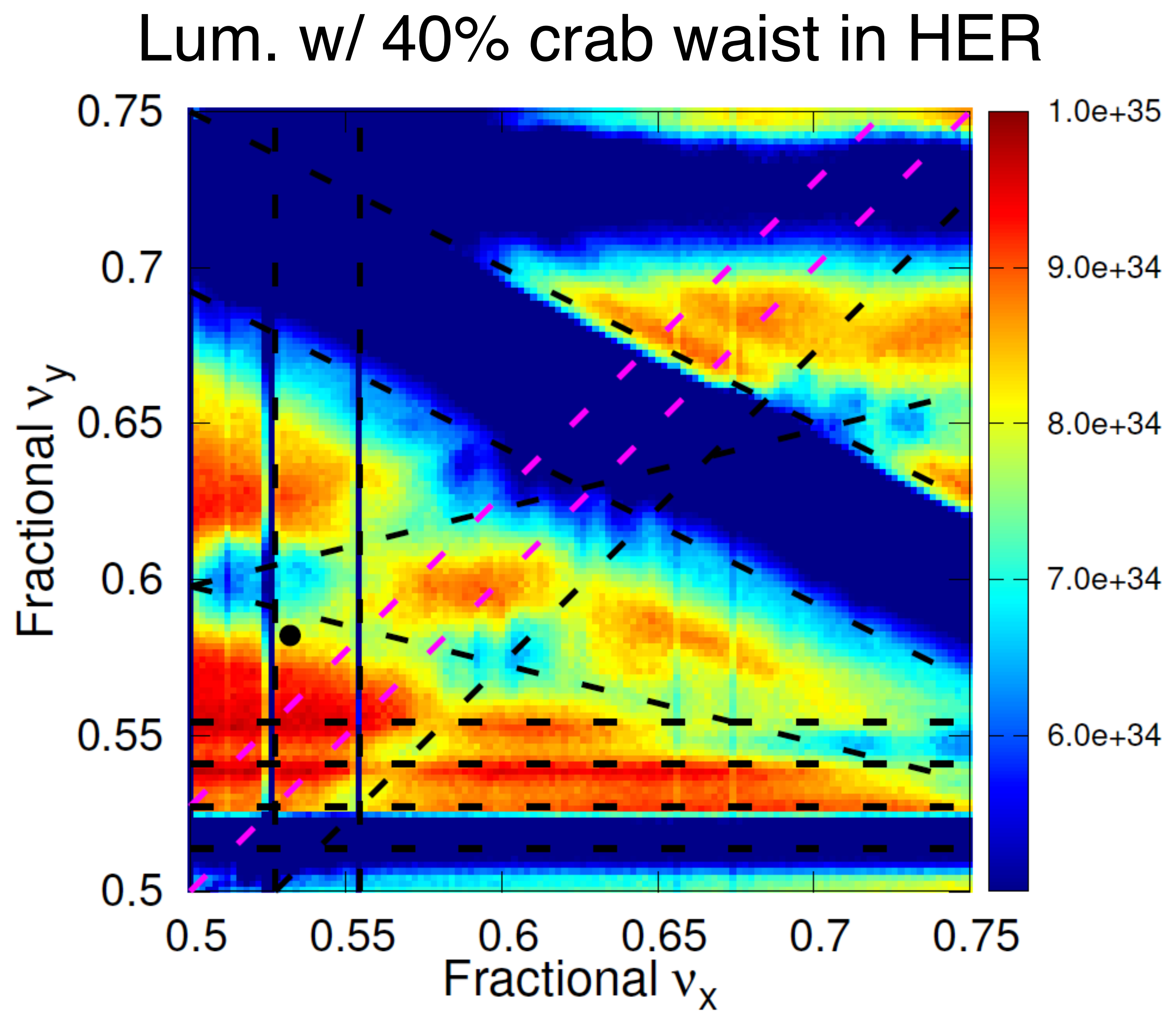


Crab waist applied to SuperKEKB

- SuperKEKB 2021b run ($\beta_y^* = 1$ mm) with ideal crab waist
  - Tune scan using BBWS showed that 40% crab waist ratio (current operation condition) in HER is not enough for suppressing vertical blowup caused by beam-beam resonances (mainly $\nu_x \pm 4\nu_y + \alpha = N$).







Theory of beam-beam resonances for ideal CW colliders

- Beam-beam parameter
  - Findings
    - ζ_x should be less than 0.5

$$\xi_y = \frac{N_0 r_e \beta_y^*}{2\pi\gamma\sigma_{y0}^*(\bar{\sigma}_{x0} + \sigma_{y0}^*)}\Theta_y(\zeta_{x0}, \zeta_x)$$

$$\Theta_y = \frac{e^{u_0}}{2\sqrt{2\pi}\zeta_{x0}^3}\left[(2\zeta_{x0}^2 - \zeta_x^2)K_0\left(u_0\right) + \zeta_x^2 K_1\left(u_0\right)\right]$$

$$u_0 = 1/(4\zeta_{x0}^2) \qquad \zeta_{x0} = \sigma_{x0}^*/(\beta_{y0}^* \tan(2\theta_c))$$

$$\zeta_x = \sigma_{x0}^*/(\beta_y^* \tan(2\theta_c))$$