Interplay Between Beam-Beam Interaction and Noise in EIC

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Sept. 2 – 5, 2024, Lausanne, Switzerland
ICFA mini workshop: BB24

Outline

1. Introduction
2. Electron orbit/size ripple
3. IBS diffusion
4. Other noise

Introduction — EIC beam-beam features

The EIC is designed to achieve a peak luminosity of up to 10^{34} cm^{-2}s^{-1}, incorporating a crossing angle of 25 mrad

- Large Beam-beam parameters: proton ~ 0.015, electron ~ 0.1, **combination never demonstrated before**
- Crab crossing collision with local crab cavities: crab cavities demonstrated at KEKB and SPS, **not used in hadron collider operation**
- Flat hadron beam with large transverse emittance ratio: experimentally demonstrated at RHIC, **not tested with large beam-beam parameters**

Compared with HERA, the EIC seeks to increase in luminosity by **two to three orders of magnitude**, accompanied by a **fourfold** increase in both proton and electron beam-beam parameters

Introduction — EIC beam-beam studies

Objective: To understand and explain the long-term emittance growth of the proton beam observed in the EIC beam-beam simulation:

- The linear beam optics imperfection: available knobs to restore beam-beam performance
- Dynamic fluctuations including electron orbit/size ripple, and proton random diffusion (such as IBS)
- Self-consistent simulation: limited by PIC solver

We have been using weak-strong simulation to study the interplay between beam-beam interactions and physical fluctuations.

Current Focus:

- For each kind of fluctuation, determine the **maximum permissible tolerance** using the weak-strong simulation method.
- Identify the pattern of vertical emittance growth under different fluctuations and attempt to understand the **underlying mechanism**.

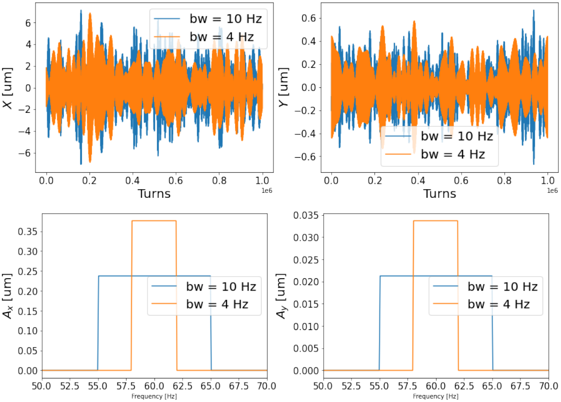
Electron orbit/size ripple — Modeling

Two methods to introduce the electron orbit jitter

- Applying a band-pass filter to the white noise
- **Generating the signal in the frequency domain, then transforming it into the time domain by FFT**

2% ripple in time domain

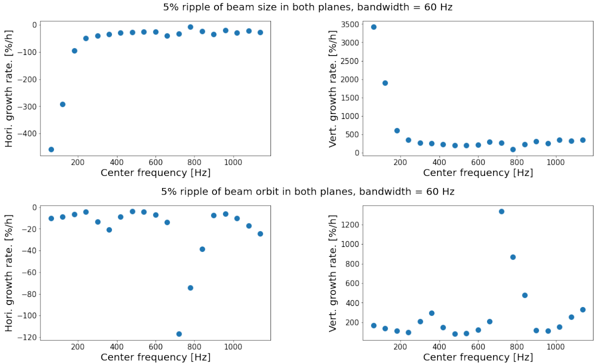
2% ripple in frequency domain



Either applying ripple to electron beam orbit or size

Electron orbit/size ripple — Scan ripple frequency

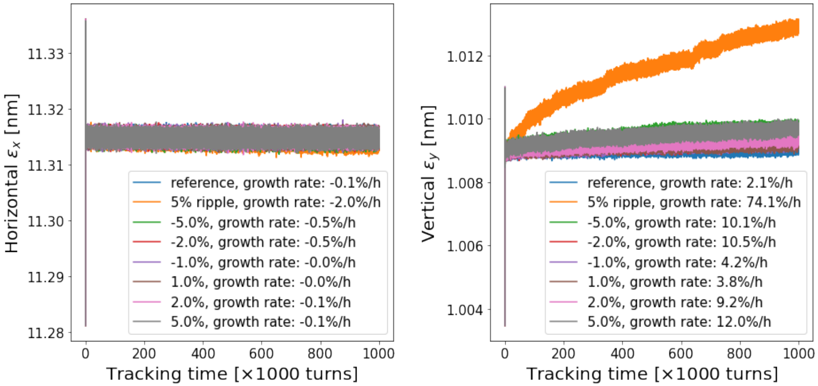
Top: size ripple, bottom: orbit ripple



- The emittance transfer is observed in both size and orbit ripples
- Different frequency responses: (1) for orbit ripple, the local maxima appears at $\nu_z/2$ and ν_z, (2) for size ripple, the emittance transfer is mitigated at larger frequency
- Around 60 Hz, the emittance transfer is much more severe in electron size ripple

Electron orbit/size ripple — Fixed electron orbit offset

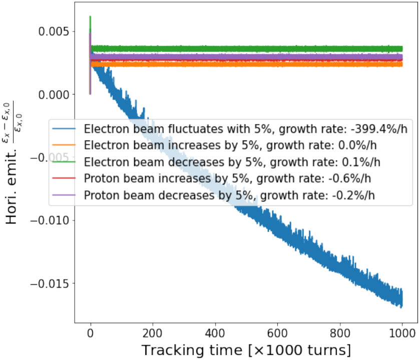
Emittance evolution by weak-strong simulation, suggested by @V. Ptitsyn

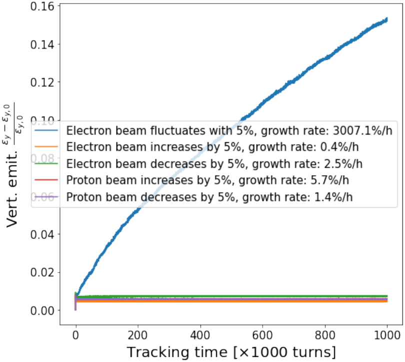


- No difference between positive and negative offset
- With same RMS magnitude, fixed offset causes much less vertical emittance growth than dynamic fluctuation

Electron orbit/size ripple — Fixed beam size change

Relative emittance evolution by weak-strong simulation, suggested by @D. Shatilov

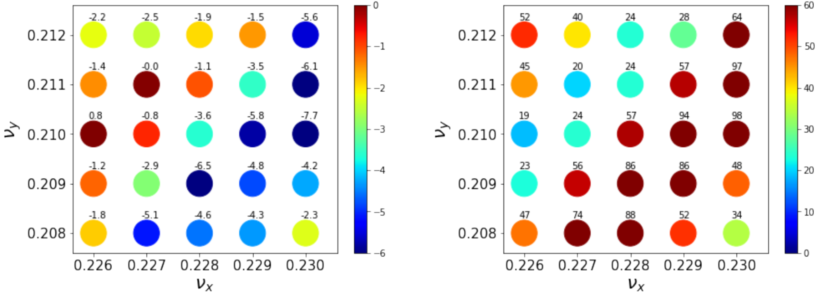




The constant beam size change, involving neither the strong electron beam nor the weak proton beam, has a small impact on proton emittance growth. However, the dynamic ripple introduces significant emittance transfer.

Electron orbit/size ripple — Transverse tune scan

Emittance growth rate for 5% **orbit ripple**, left: horizontal, right: vertical

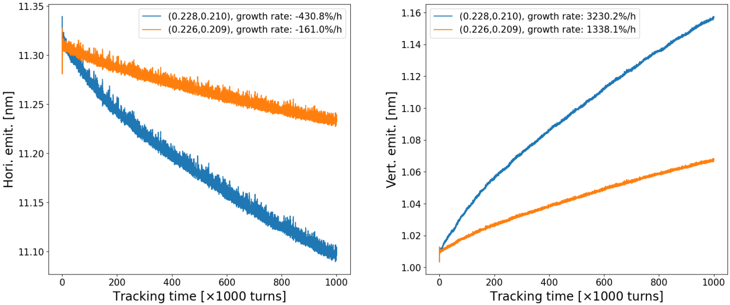


where the color indicates the emittance growth rate in unit of %/h

- The diagonal line has similar growth rate which indicates that the vertical emittance growth depends on the synchro-betatron resonance $2\nu_x - 2\nu_y + p\nu_z = 0$
- Taking 20%/h as the threshold, if the working point is optimized, such as $(0.226, 0.209, -0.010)$, the 5% ripple is acceptable

Electron orbit/size ripple — $(0.226, 0.209)$

Emittance evolution with 5% ripple of electron beam size



$(0.226, 0.209)$ is also better for electron size ripple, which is similar to electron orbit ripple.

Electron orbit/size ripple — Other findings

The proton emittance transfer depends on **synchro-betatron resonance**, specifically $2\nu_x - 2\nu_y \pm 4\nu_z = 0$ and **dynamic fluctuations**.

- The emittance transfer is not observed in round beam collision. Even with a significant emittance transfer, $\epsilon_x + \epsilon_y$ is still a good invariant.
- Minimizing synchro-betatron coupling helps control emittance transfer. Effective strategies for this include: shortening the bunch length to lessen the hourglass effect, decreasing chromaticity, and freezing longitudinal dynamics by setting longitudinal tune $\nu_z = 0$.
- The emittance transfer is reduced at negative longitudinal tune compared to positive one when both have the same absolute value.
- At high frequencies close to betatron tune, the threshold of allowable ripple is much smaller. However, the ripple amplitude is significantly attenuated due to magnet inductance as well as eddy currents in the vacuum chamber @B. Podobedov.

Additional simulation results are available in the backup slides.

IBS diffusion — Modeling

- The IBS diffusion and cooling are simulated by a lumped element at IP, similar to radiation damping and excitation

$$u_{n+1} = \lambda_u u_n + R_u \sigma_u \sqrt{1-\lambda_u^2}, \qquad p_{n+1} = \lambda_u p_n + R_p \sigma_p \sqrt{1-\lambda_u^2}$$

  where $u = x, y, z$, $\lambda_u = \exp\left(-T_{\mathrm{rev}}/\tau_u\right)$, τ_u IBS growth time or cooling time in u plane, and $R_{u,p}$ the random number following a normal distribution.
- The linear normal form is applied to ensure that diffusion and cooling processes occur within the eigenplanes.
- Turn off the cooling, the diffusion map is

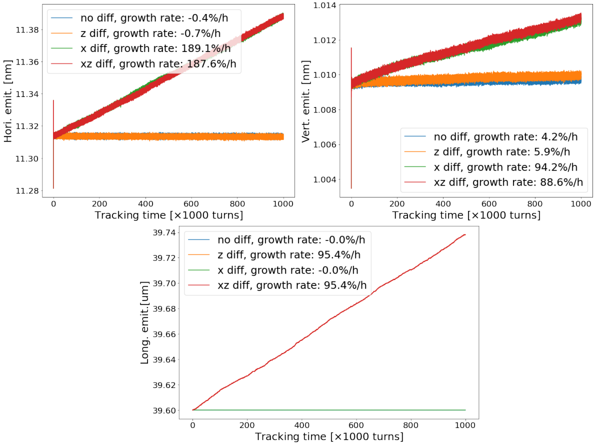
$$u_{n+1} = u_n + R_u \sigma_u \sqrt{1-\lambda_u^2} \approx u_n + \frac{T_{\mathrm{rev}}}{\tau_u} \sigma_u R_u$$

$$p_{n+1} = p_n + R_p \sigma_u \sqrt{1-\lambda_u^2} \approx p_n + \frac{T_{\mathrm{rev}}}{\tau_u} \sigma_p R_p$$

- More precise IBS modeling will be considered in the future.

IBS diffusion — With beam-beam

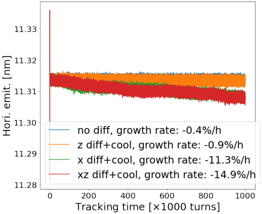
Set $\tau_x = 1$ h or ∞, $\tau_y = \infty$, and $\tau_z = 2$ h or ∞, where $\tau_{x,y,z}$ are IBS time in terms of beam size growth

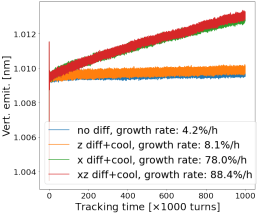


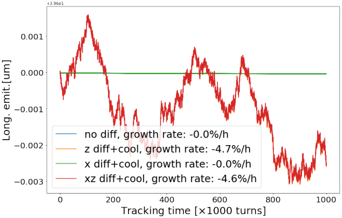
Significant vertical growth is observed when $\tau_x < \infty$. The required vertical cooling time may be as large as 1 h

IBS diffusion — Turn on cooling

Set $\tau_x = 1$ h or ∞, $\tau_y = \infty$, and $\tau_z = 2$ h or ∞, where $\tau_{x,y,z}$ are IBS time (or cooling time) in terms of beam size growth



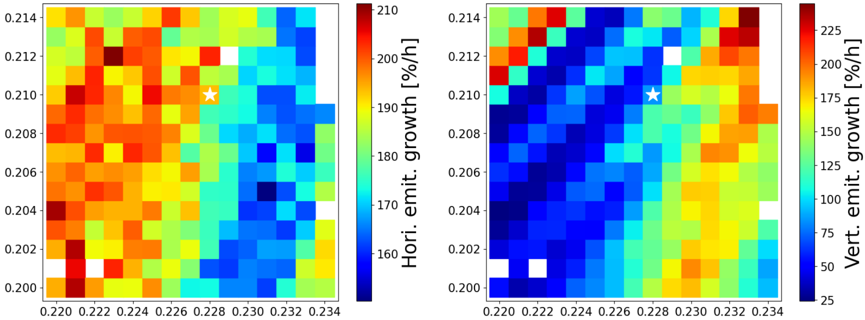




Unfortunately, the large cooling rate in hori. and long. planes can't cool the vert. plane

IBS diffusion — Transverse tune scan

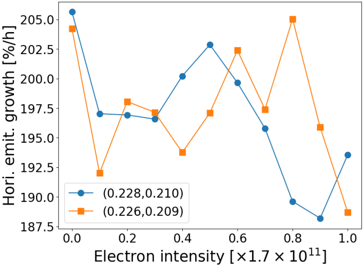
Scan $\nu_{x,y}$ while $\nu_z = -0.01$

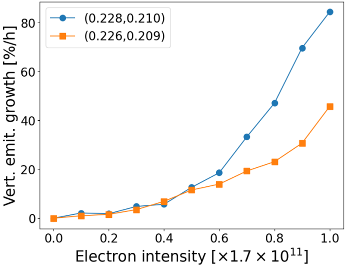


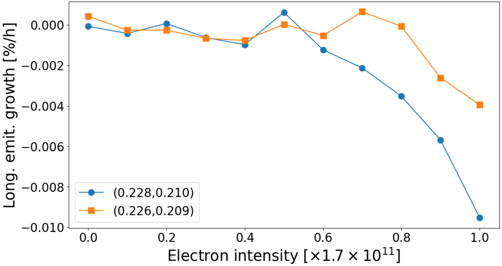
- The vertical emittance growth has a clear pattern
- The minimum growth appears around

$$\nu_x - \nu_y = 0.011 \qquad \text{or} \qquad 0.012$$
$$\nu_x - \nu_y = 0.016 \qquad \text{or} \qquad 0.017$$

IBS diffusion — $(0.228, 0.210)$ vs. $(0.226, 0.209)$







$(0.226, 0.209)$ is better which is similar to the electron orbit ripple study

IBS diffusion — Other findings

- No big difference between crab crossing and head-on which aligns with strong-strong simulation: the vertical growth has no difference in strong-strong simulation between different crossing schemes.
- The longitudinal plane becomes unstable as ν_z nears 0, due to a non-zero energy kick in Hirata's map. When ν_z is zero and there is a DC component in the white noise of IBS diffusion, an integer resonance occurs in the longitudinal plane.
- Similar to electron orbit/size ripple, minimizing synchro-betatron coupling (hour-glass effect, chromaticity etc.) and reducing longitudinal emittance help mitigate the vertical emittance growth.

More simulation results are available in the backup slides.

Other noise — Crab cavity phase

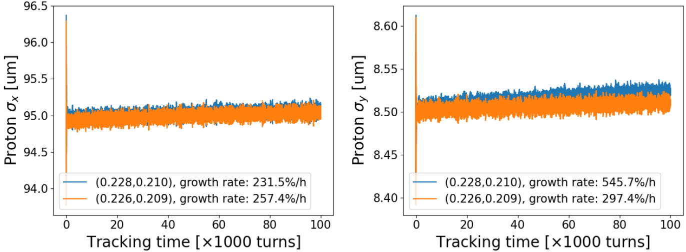
- The horizontal emittance growth is understandable and the simulation aligns well with the theoretical formula P. Baudrenghien and T. Mastoridis, PhysRevSTAB.18.101001
- However, when the beam-beam interaction is included, the vertical emittance growth is observed, and the RMS amplitude of the phase noise should be less than 1 μrad (@R. Huang, @V. Morozov, @Y. Hao, @Y. Luo, see IPAC21-24 proceedings)
- **The vertical emittance growth is only observed when the spectrum of phase noise overlaps with the beam-beam footprint** @Y. Luo
- The proton centroid due to a single crab cavity kick:

$$X = -\theta_c \left[\frac{\sin(k_c z + \phi)}{k_c} - z \right], \quad \frac{(X)_{\text{rms}}}{\sigma_x} \approx \frac{\sigma_\phi \Theta_{\text{PW}}}{k_c \sigma_z}$$

1 μrad RMS phase noise corresponds to 3.3×10^{-5} orbit ripple, which is consistent with orbit ripple tolerance at high frequency

Other noise — PIC noise in strong-strong simulation

- Physical noise is indistinguishable from numerical noise in our beam-beam simulation
- Previous studies have been demonstrated that the emittance growth in strong-strong simulation is dominated by PIC noise
- The optimized working point $(0.226, 0.209, -0.010)$ behaves better than the nominal one $(0.228, 0.210, -0.010)$ in all scenarios, even in the strong-strong simulation



Not only the vertical emittance growth rate becomes smaller, the discrepancy between horizontal and vertical plane is also reduced

Summary

Based on our weak-strong simulation: Taking 10%/h of vertical emittance growth rate as the limit, the noise threshold is summarized below

Noise	working point	RMS threshold	unit
Electron orbit ripple	$(0.226, 0.209)$	2.5%	$\sigma^*_{x,y}$
Electron size ripple	$(0.228, 0.210)$	0.1%	$\sigma^*_{x,y}$
Proton crab cavity phase	$(0.228, 0.210)$	1	µrad
IBS horizontal diffusion	$(0.219, 0.202)$	2	hours

- We observed emittance transfer in the simulations, where horizontal ripple may lead to vertical emittance growth, potentially compromising the flat beam profile.
- A diagonal pattern emerged during the tune scan, and emittance transfer ceases when the transverse-longitudinal coupling is reduced or eliminated. Theoretical analysis confirms that **this emittance transfer is driven by streaming around the synchro-betatron resonance**, specifically $2\nu_x - 2\nu_y \pm 4\nu_z = 0$.

Thank you!

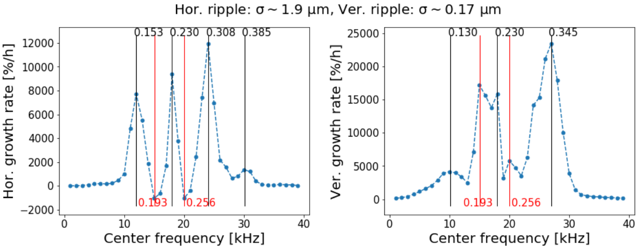
Backup slides

Backup slides for orbit ripple — High frequency scan

Scan for 2% ripple

- The center frequency is scanned from 1 kHz to 39 kHz($0.5 f_{\text{rev}}$)
- Bandwidth is chosen as 1 kHz $> 0.012 f_{\text{rev}} = 938$ Hz

Horizontal (left) and vertical (right) growth rate

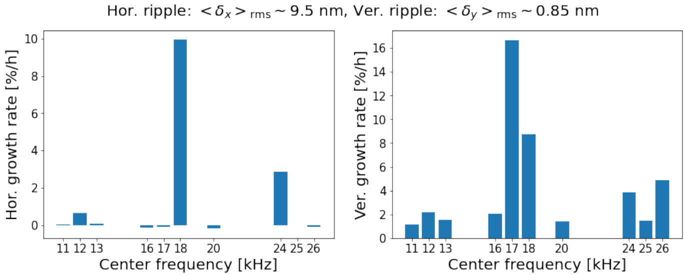


$\nu_x = 0.228$ is the proton horizontal tune, and ν_{e} is the center frequency of external ripple (normalized by the revolution frequency)

- Horizontal high peaks: $2\nu_x/3, \nu_x, 4\nu_x/3 \cdots$
- Vertical peaks are more complicated: (1) at ν_y, (2) at ν_x via coupling

Backup slides for orbit ripple — High frequency scan

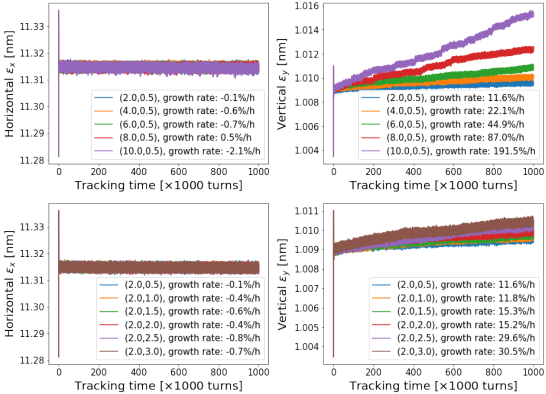
Electron orbit ripple scan with 10^{-4} ripple



- The largest growth in the horizontal (vertical) plane happens at 18 kHz (17 kHz) which is close to ν_x (ν_y)
- At the betatron frequencies, the ripple should be reduced less than 10^{-4}
- At these high frequencies, the ripple amplitude is significantly attenuated due to magnet inductance as well as eddy currents in the vacuum chamber @B. Podobedov

Backup slides for orbit ripple — Around 60 Hz

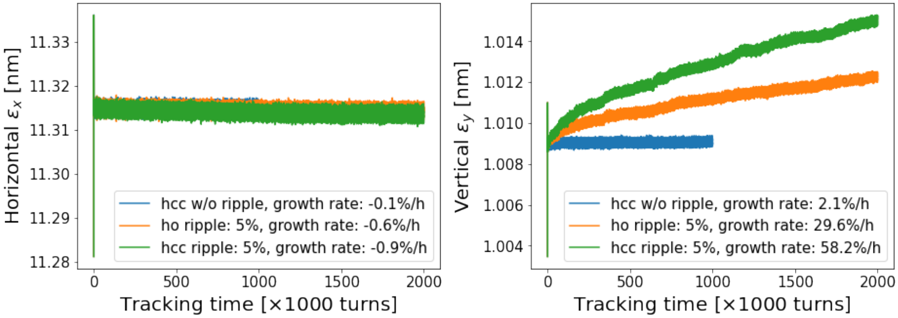
Scan horizontal or vertical ripple: top (horizontal), bottom (vertical)



- When the horizontal ripple increases linearly, the vertical growth increases exponentially
- When the vertical ripple increases linearlly, the vertical growth increases in distinct steps

Backup slides for orbit ripple — HO vs. CC

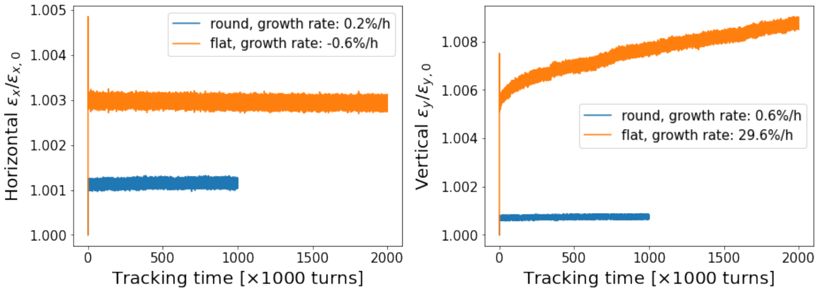
Emittance tracking by weak-strong simulation, HO: head-on, CC: crab crossing, HCC: crab crossing with higher harmonic crab cavity



- No significant growth in horizontal plane
- Vertical growth is reduced by half when switching to head-on collision
- Negative correlation between horizontal and vertical growth

Backup slides for orbit ripple — Flat vs. round

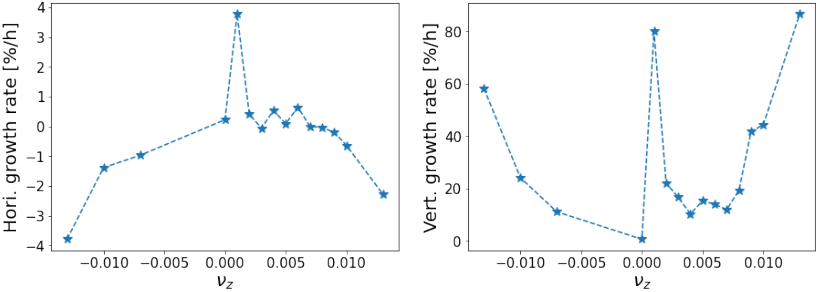
Relative emittance evolution in head-on collision with 5% ripple for flat (EIC) and round (RHIC) beam, beam-beam parameters are similar



- The round beam parameter is similar to RHIC; the flat is same as EIC
- The working point is same $(0.228, 0.210, 0.010)$
- The ripple is 5% but of different $\sigma_{x,y}$ in both round and flat

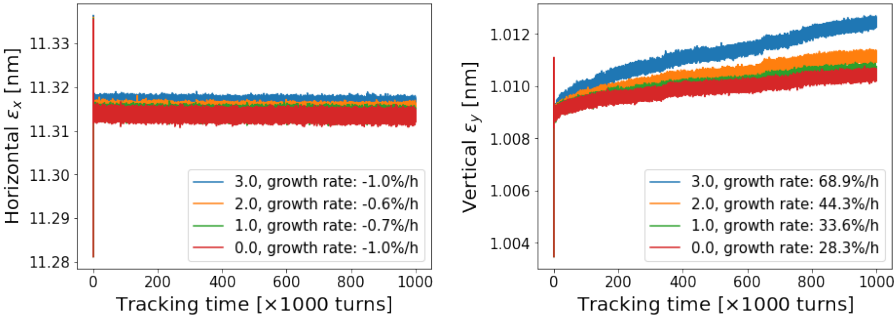
Backup slides for orbit ripple — Longitudinal tune scan

Emittance evolution in head-on collision for 5% ripple



- When $\nu_z = 0.0$, no growth in vertical emittance is observered provides further evidence that synch-betatron resonances are responsible for the vertical emittance growth
- When $\nu_z = 0.001$, the longitudinal frequency matches the external modulation frequency, leading to a substantial increase in vertical emittance
- Negative ν_z causes less vertical growth

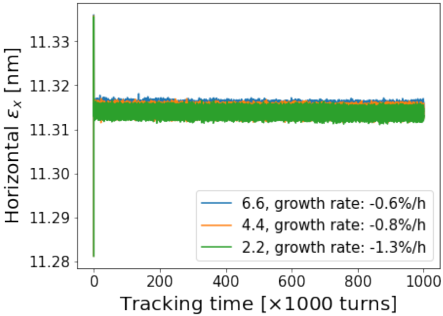
Emittance evolution in head-on collision with 5% ripple and different chromaticities, woring point: $(0.228, 0.210, 0.010)$

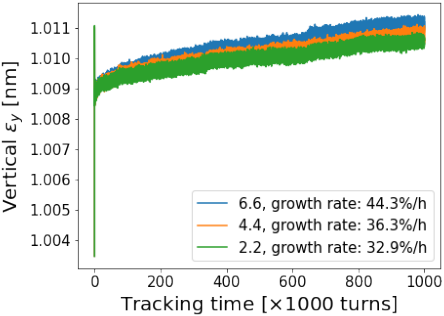


Reducing the chromaticity helps to mitigate the vertical growth

Backup slides for orbit ripple — Momentum spread

Emittance evolution in head-on collision with 5% ripple and different momentum spread ($\times 10^{-4}$), woring point: $(0.228, 0.210, 0.010)$

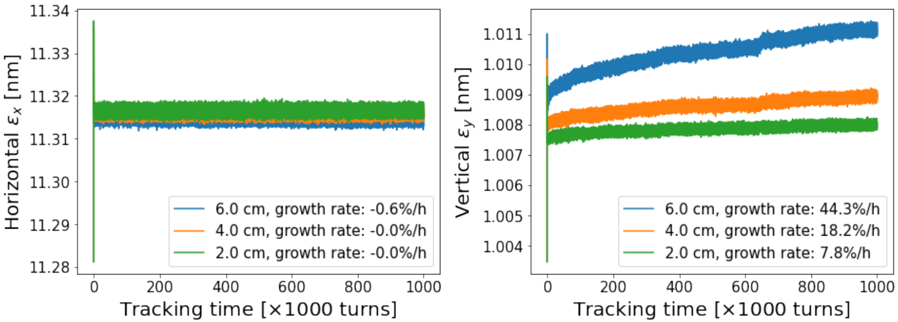




Reducing momentum spread helps to mitigate the vertical growth

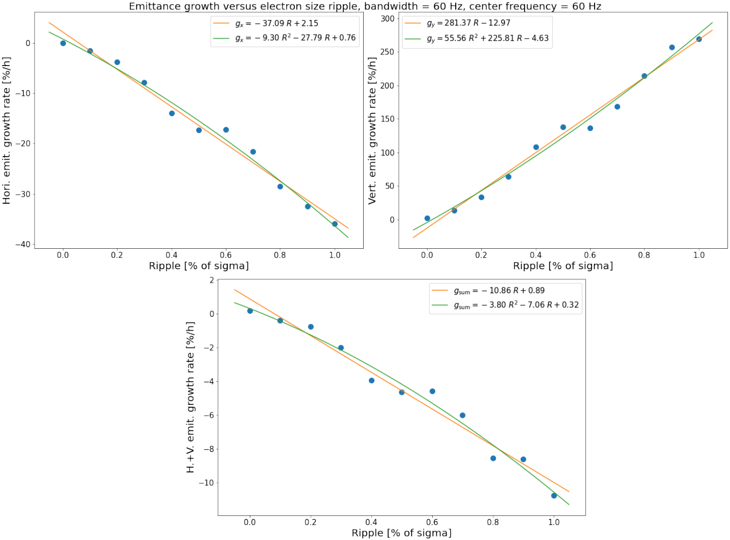
Backup slides for orbit ripple — Bunch length

Emittance evolution in head-on collision with 5% ripple and different bunch length, woring point: $(0.228, 0.210, 0.010)$



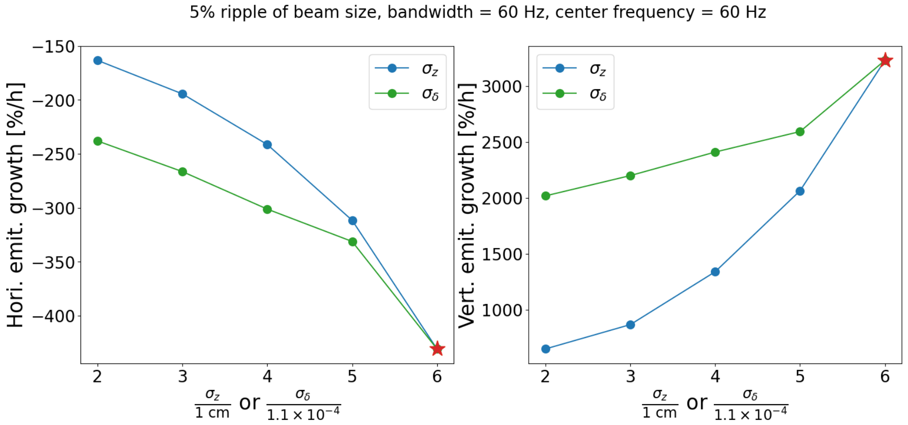
Small bunch length not only means small longitudinal emittance, but also mitigate the hour-glass effect. Therefore, reducing proton bunch length significantly mitigate the vertical emittance growth.

Backup slides for size ripple — Scan ripple amplitude



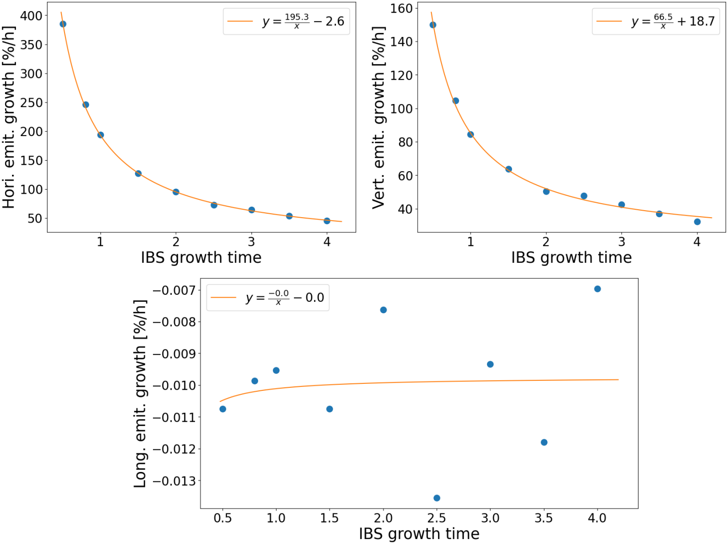
Taking 10%/h as the threshold, size ripple should be within $0.1\%\sigma_{x,y}$

Backup slides for size ripple — Scan proton σ_z and σ_δ



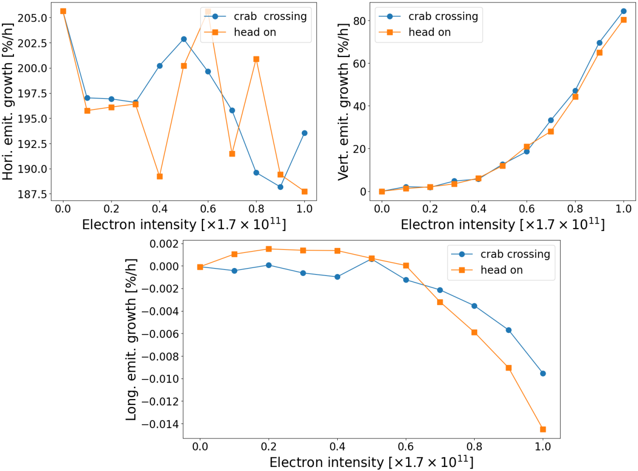
Similar to orbit ripple, reducing longitudinal emittance helps suppress vertical growth, and reducing σ_z is more effective as it mitigates hour-glass effect as well

Backup slides for IBS — Horizontal diffusion amplitude



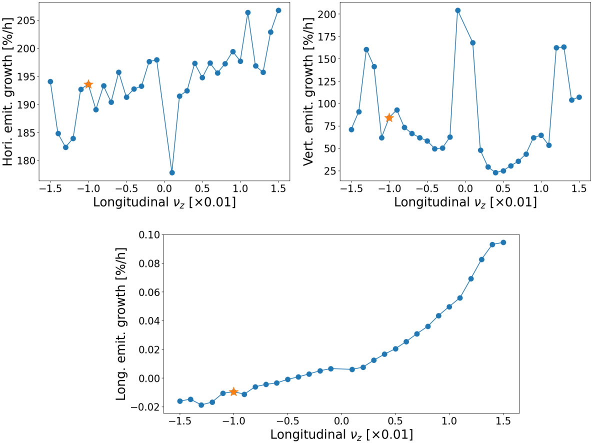
Both horizontal and vertical growth fit $1/A$ scaling law

Backup slides for IBS — Crab crossing vs. head on



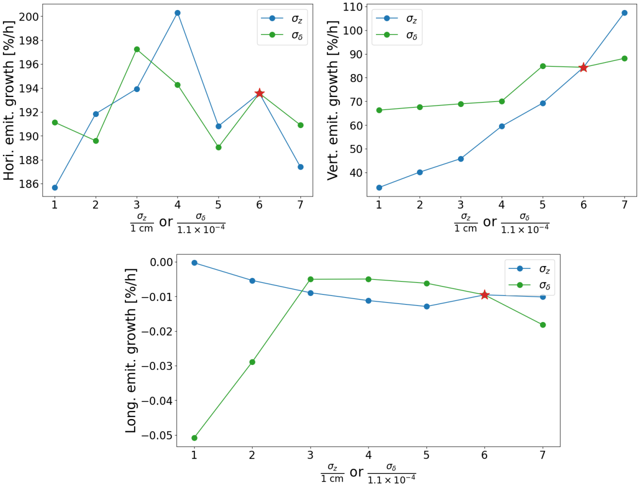
No big difference between crab crossing and head-on which aligns with previous study: the vertical growth has no difference in strong-strong simulation between different crossing schemes

Backup slides for IBS — Longitudinal tune scan



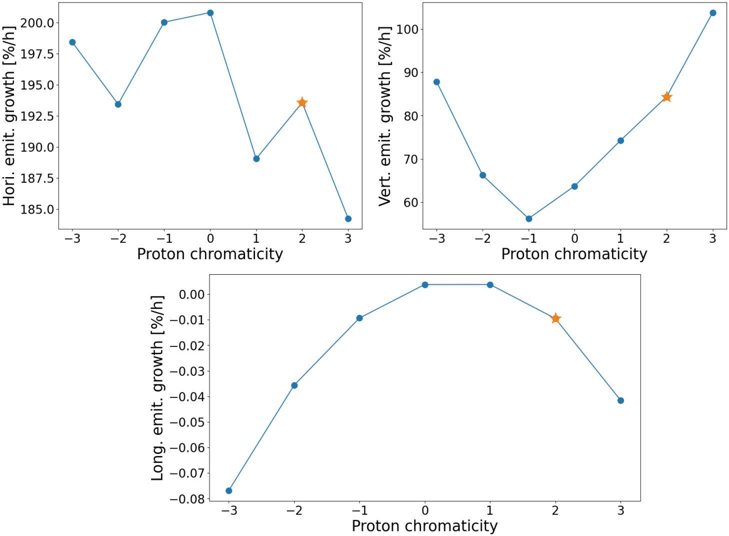
Longitudinal plane tends to unstable when ν_z approaches 0, which leads to large vertical emittance growth.

Backup slides for IBS — Proton σ_z and σ_δ



Smaller longitudinal emittance can reduce the vertical growth a little; a small σ_z is more effective as it mitigates hour-glass effect as well

Backup slides for IBS — Chromaticity



Since the chromaticity also introduces synchro-betatron coupling, it is not surprising that the chromaticity can change vertical growth