Beam-Beam Simulation Models and Numerical Noises for Flat Beam Collisions in the EIC

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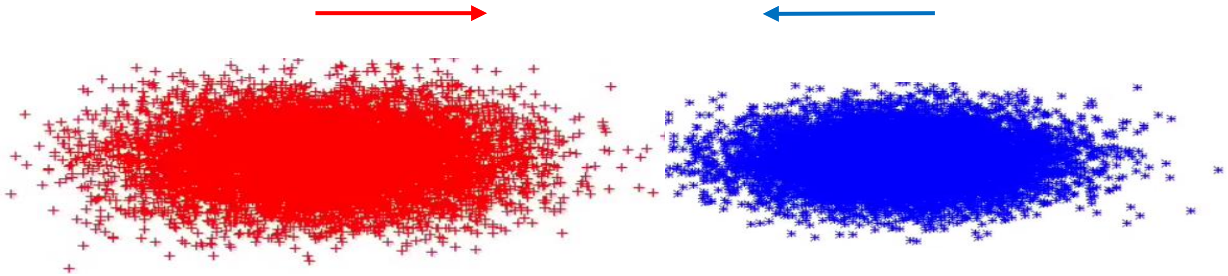
Electron-Ion Collider

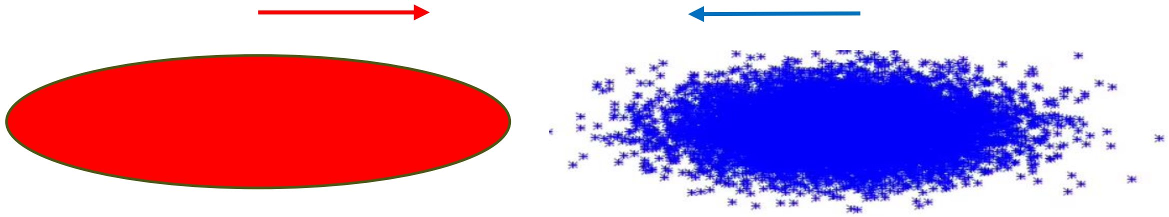
Numerical Simulation Models for the EIC

- **Strong-strong simulation model**



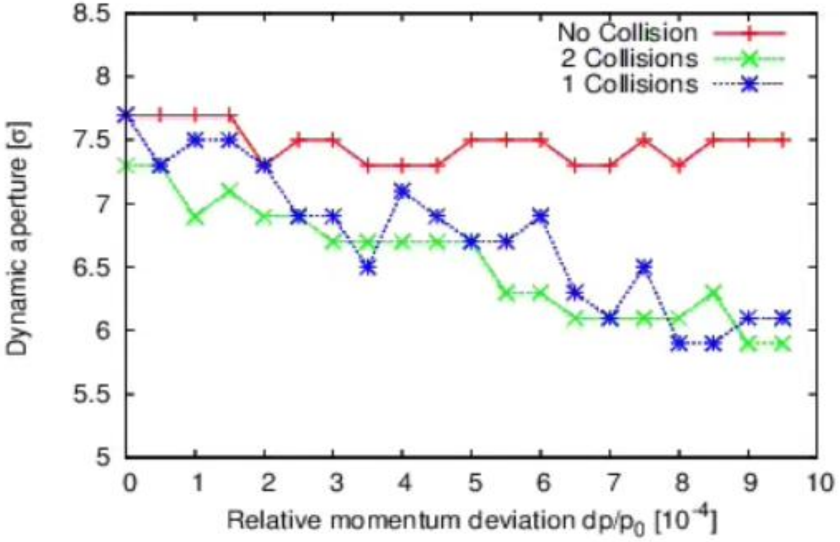
Both bunches are represented by ~ a million macro-particles. Subject to numerical noises.

- **Weak-Strong simulation model**



Strong bunch is rigid while weak bunch is represented by macro-particles. Not consistent.

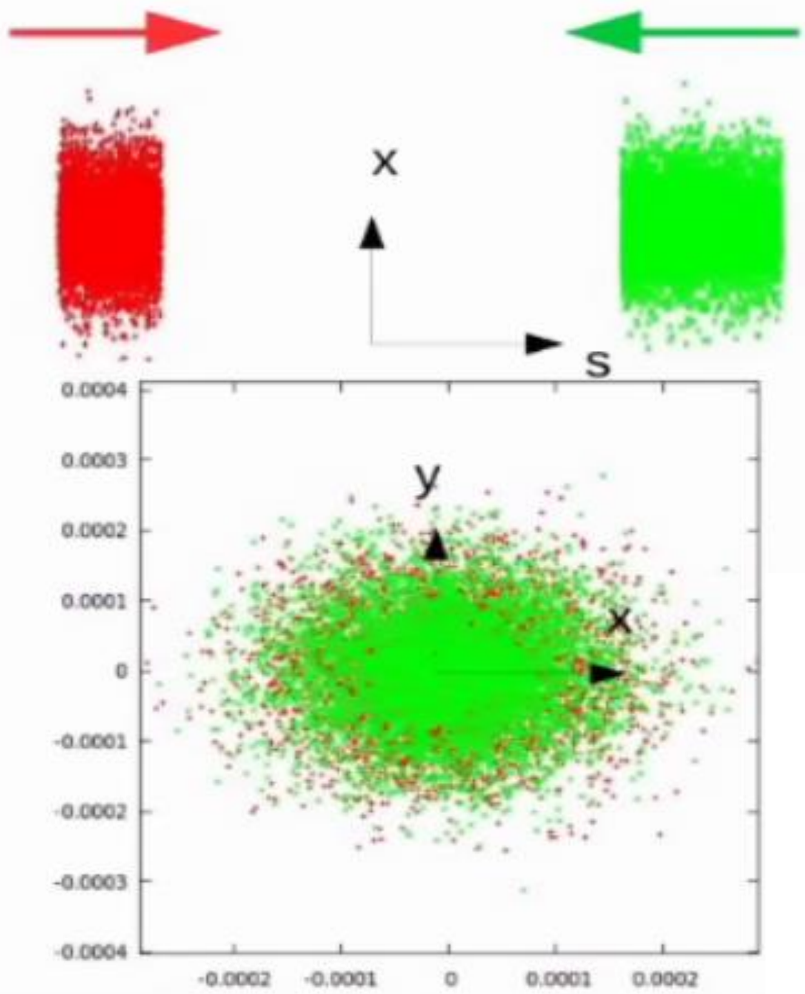
- **Dynamic aperture calculation**

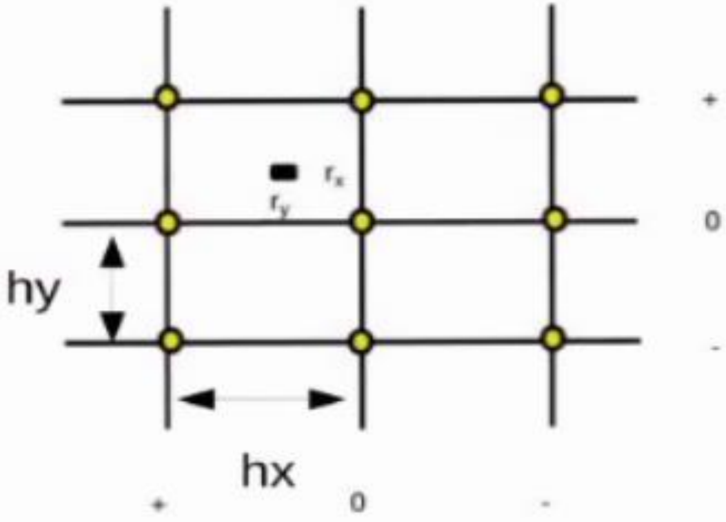


- Importance of long-term proton stability is crucial for the EIC beam-beam interaction simulation stuudies.

Strong-Strong Model (Particle-In-Cell, Poisson Solver)

Head-on frame





Triangular-Shaped Cloud (TSA) technique used to distribute particle charge onto 256 * 256 mesh points.

- **Poisson's Equation** (FFT method)

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\phi(x,y) = -2\pi\rho_c(x,y)$$

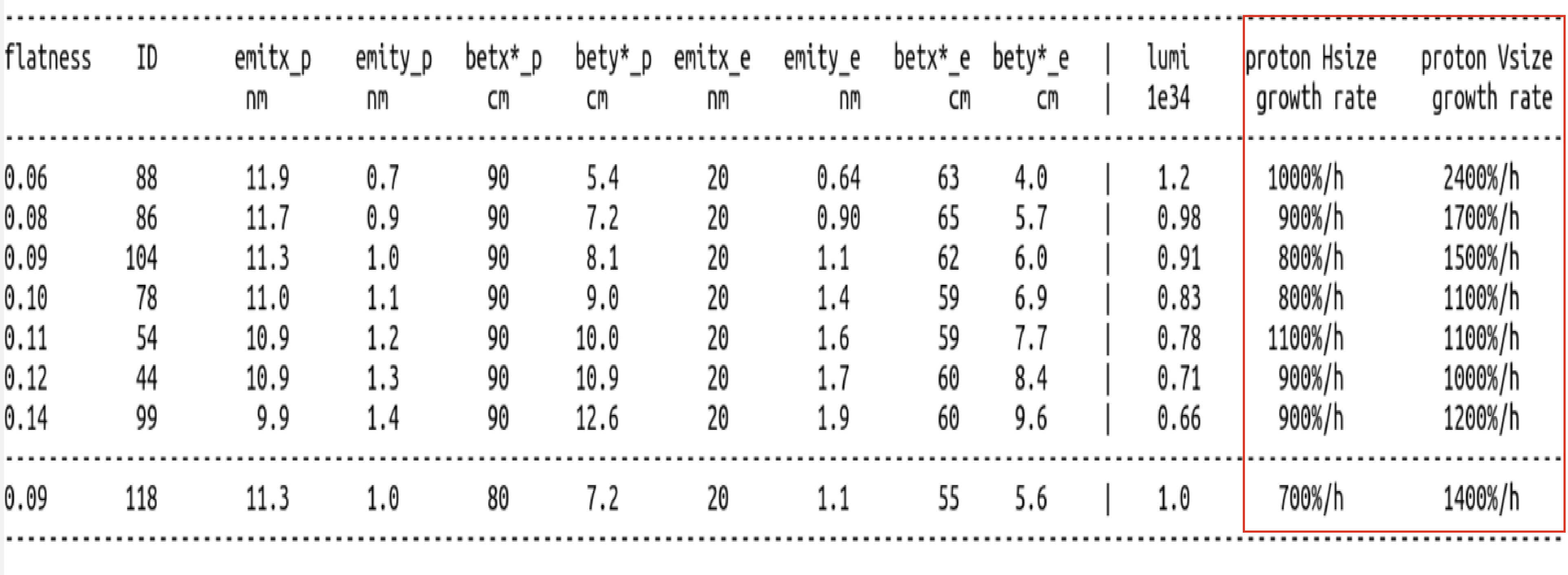
$$\vec{E} = -\nabla\phi \qquad \mathbf{B} = \vec{\beta}\times\mathbf{E}/c$$

We can see that PIC Poisson solver is exact if charges are really on grids.
Numerical errors happen in charge deposition and BB EM filed calculation (interpolation).

Flatness Scan Study: Strong-Strong Model

Flatness is defined as σ^*_y / σ^*_x at IP

Calculated with BBSS

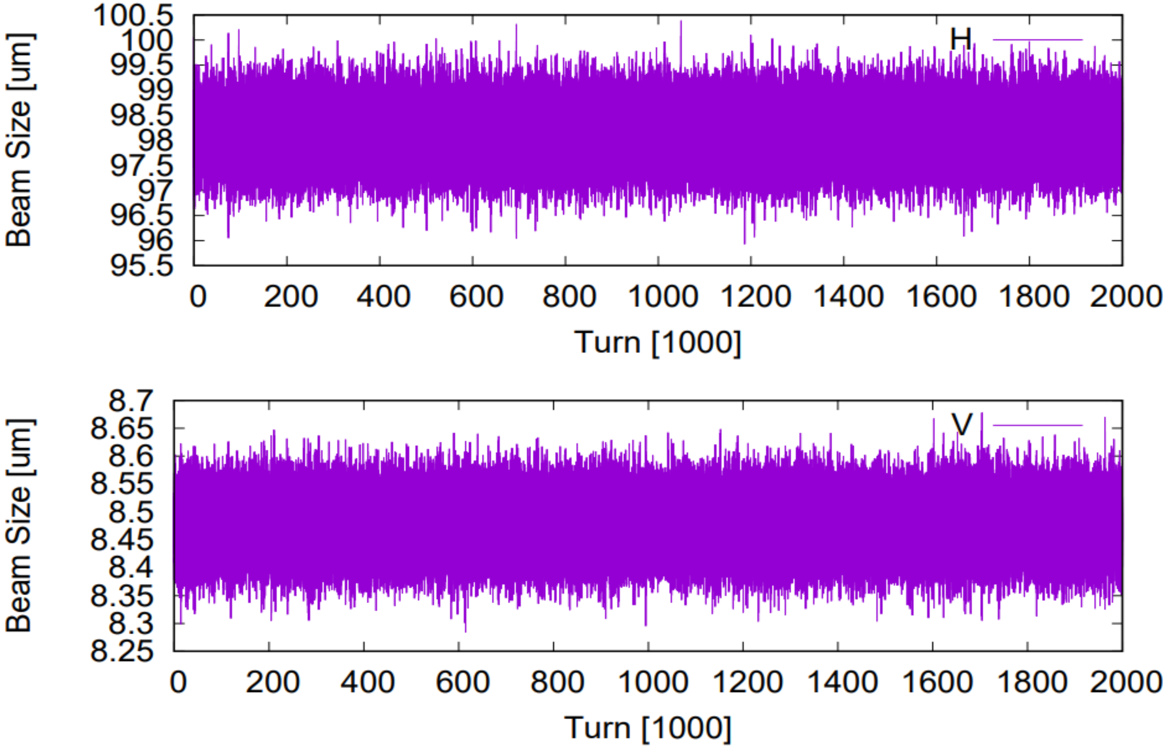


```
---------------------------------------------------------------------------------------------------------------------------
flatness    ID      emitx_p   emity_p   betx*_p   bety*_p  emitx_e   emity_e   betx*_e  bety*_e  |   lumi    proton Hsize    proton Vsize
                      nm        nm        cm        cm        nm        nm        cm       cm    |   1e34     growth rate     growth rate
---------------------------------------------------------------------------------------------------------------------------
0.06        88       11.9      0.7       90        5.4       20       0.64       63      4.0    |   1.2      1000%/h         2400%/h
0.08        86       11.7      0.9       90        7.2       20       0.90       65      5.7    |   0.98      900%/h         1700%/h
0.09       104       11.3      1.0       90        8.1       20       1.1        62      6.0    |   0.91      800%/h         1500%/h
0.10        78       11.0      1.1       90        9.0       20       1.4        59      6.9    |   0.83      800%/h         1100%/h
0.11        54       10.9      1.2       90       10.0       20       1.6        59      7.7    |   0.78     1100%/h         1100%/h
0.12        44       10.9      1.3       90       10.9       20       1.7        60      8.4    |   0.71      900%/h         1000%/h
0.14        99        9.9      1.4       90       12.6       20       1.9        60      9.6    |   0.66      900%/h         1200%/h
---------------------------------------------------------------------------------------------------------------------------
0.09       118       11.3      1.0       80        7.2       20       1.1        55      5.6    |   1.0       700%/h         1400%/h
---------------------------------------------------------------------------------------------------------------------------
```

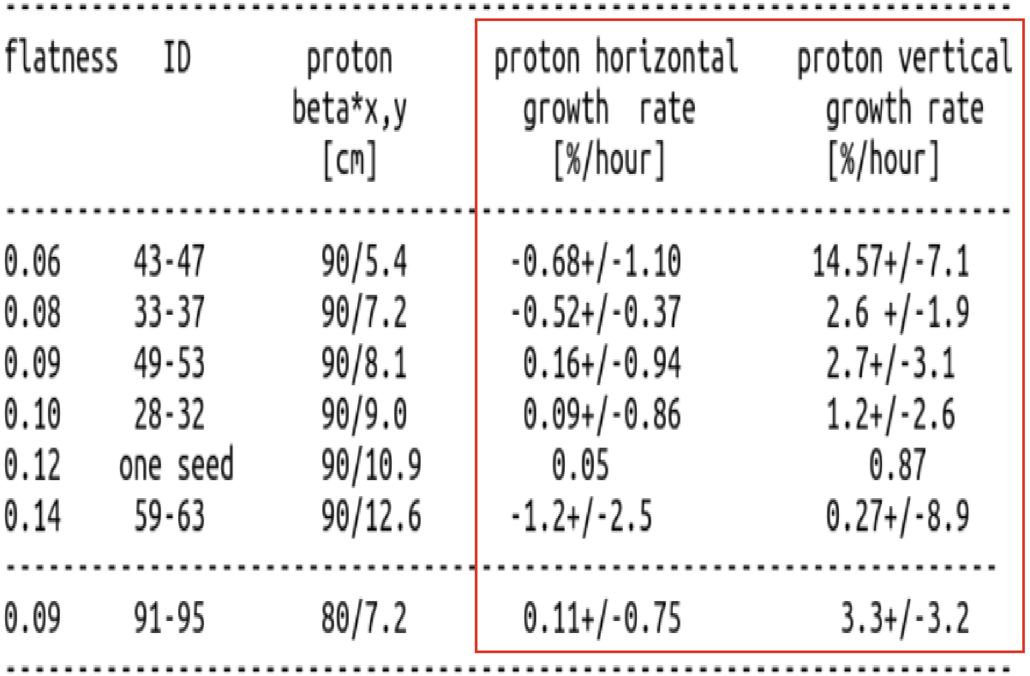
In strong-strong simulation, half million macro-particles used and tracked to 50k turns. We linearly fit beam sizes between 25k to 50k turns (5-10 electron SR damping periods) to get a rough growth rate in unit of %/hour (extrapolated from 25k turns).

Growth Rates vs Flatness: Weak-Strong Model

An example with flatness 0.09



Calculated with SimTrack

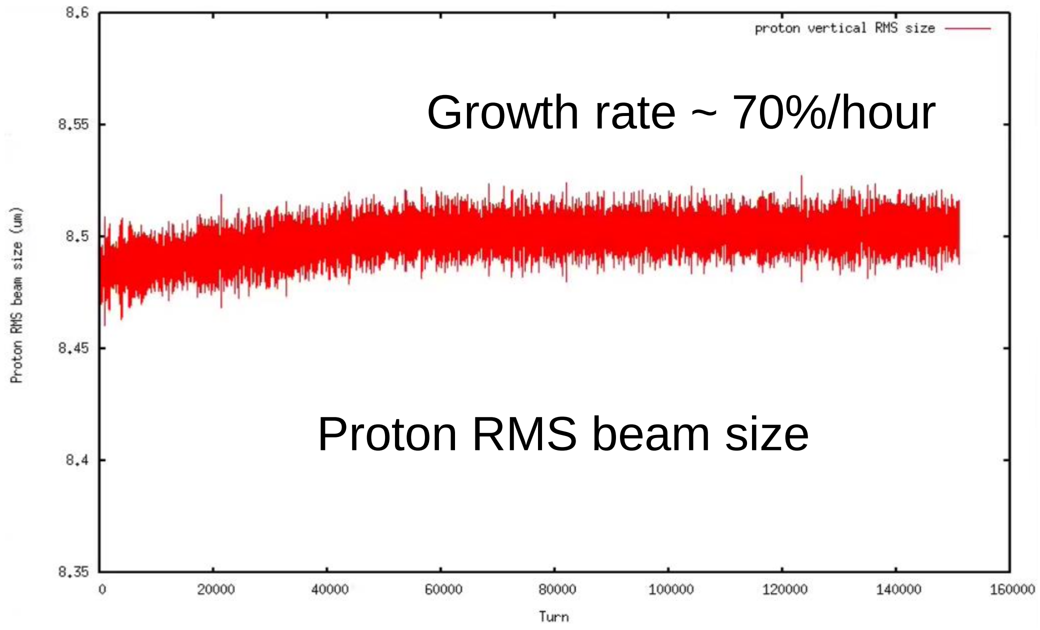


flatness	ID	proton beta*x,y [cm]	proton horizontal growth rate [%/hour]	proton vertical growth rate [%/hour]
0.06	43-47	90/5.4	-0.68+/-1.10	14.57+/-7.1
0.08	33-37	90/7.2	-0.52+/-0.37	2.6 +/-1.9
0.09	49-53	90/8.1	0.16+/-0.94	2.7+/-3.1
0.10	28-32	90/9.0	0.09+/-0.86	1.2+/-2.6
0.12	one seed	90/10.9	0.05	0.87
0.14	59-63	90/12.6	-1.2+/-2.5	0.27+/-8.9
0.09	91-95	80/7.2	0.11+/-0.75	3.3+/-3.2

In weak-strong simulation, electron bunch is rigid with design bunch sizes. 5 electron slices were used. Proton bunch represented by 10k macro-particles and tracked to 2 million turns. Several seeds of initial distribution are used to get a RMS error bar in calculated growth rates.

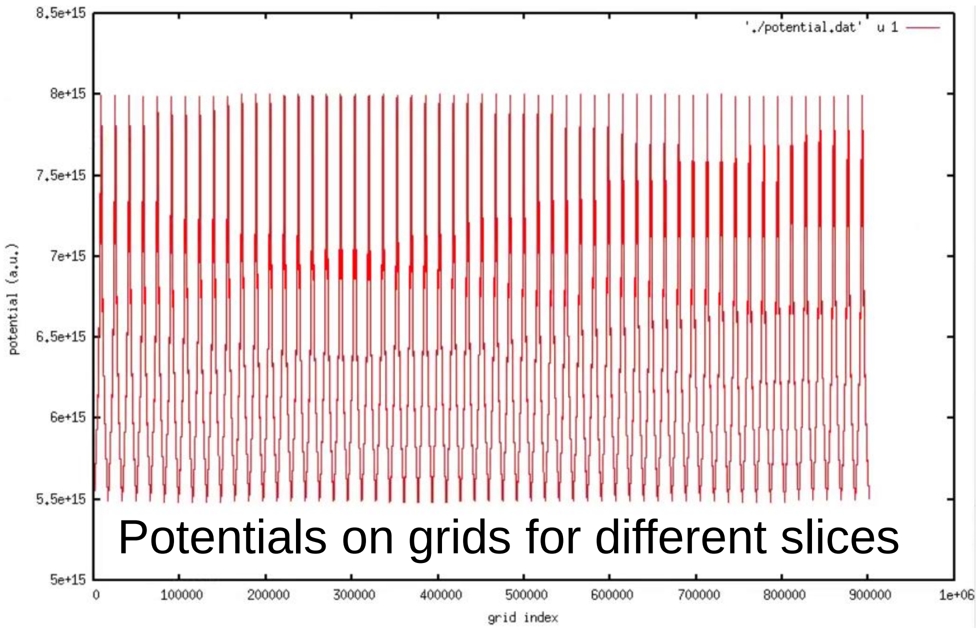
Frozen Model

- After the electron bunch reaches its quasi-equilibrium in a strong-strong simulation, we freeze the electron particle distribution and its space charge potential. Then we do weak-strong simulation with the electron potentials. In the code, potentials are averaged in 1000 turns.
- The proton vertical beam size growth rate from frozen model is between the strong-strong and weak-strong models. Frozen model gets ride of TBT variation but still carries errors in PIC Poisson solver.



Growth rate ~ 70%/hour

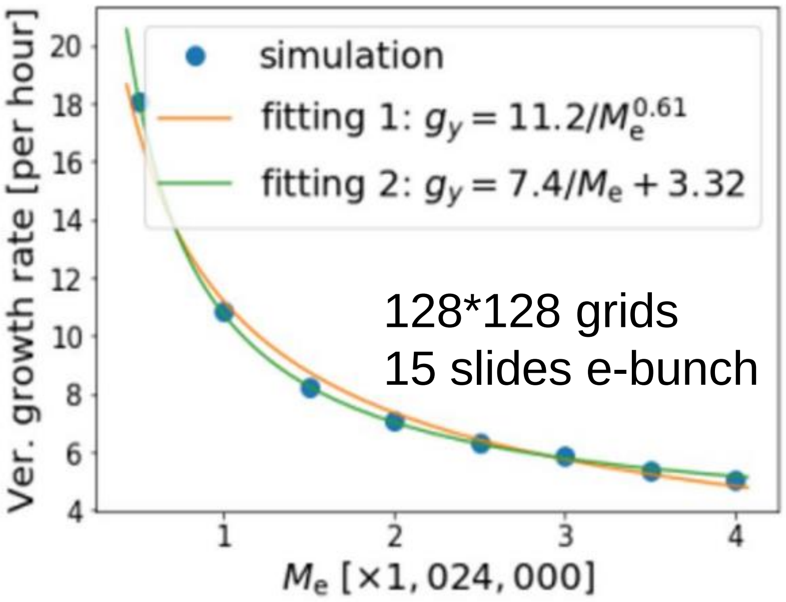
Proton RMS beam size



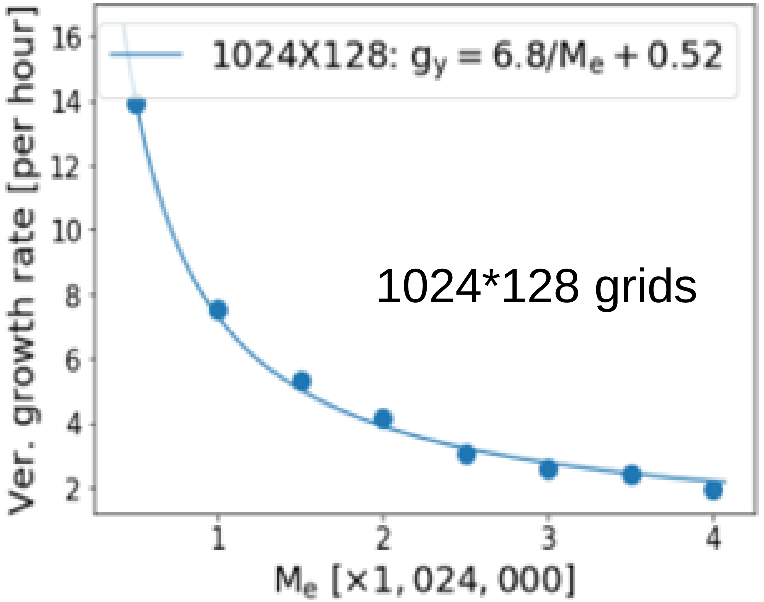
Potentials on grids for different slices

Converging Studies

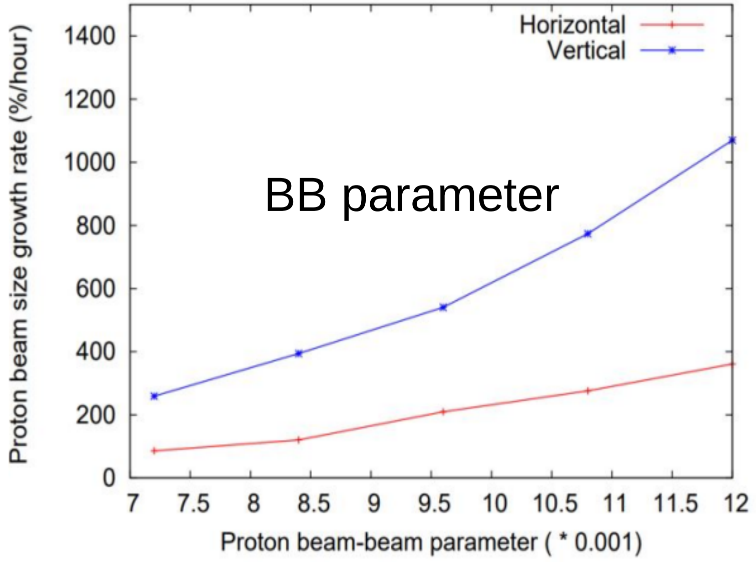
- Analytically, the numerical noise introduced artificial emittance growth rate in the strong-strong beam-beam simulation **is inversely proportional to the number of macroparticles** and **proportional to the square of the beam-beam parameter**, if beam-beam nonlinear effect is weak enough.
- In EIC strong-strong simulation, the growth rate goes down with increased macro-particles, transverse grids, and longitudinal slices. It also decreases with a smaller BB parameter and with a rounder transverse beam.



128*128 grids
15 slides e-bunch



1024*128 grids

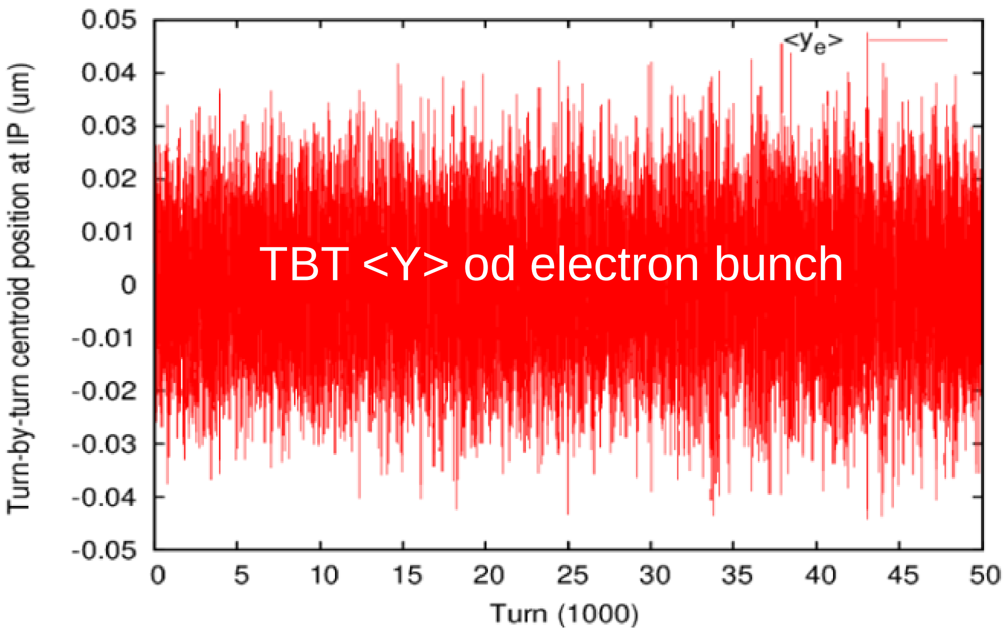


BB parameter

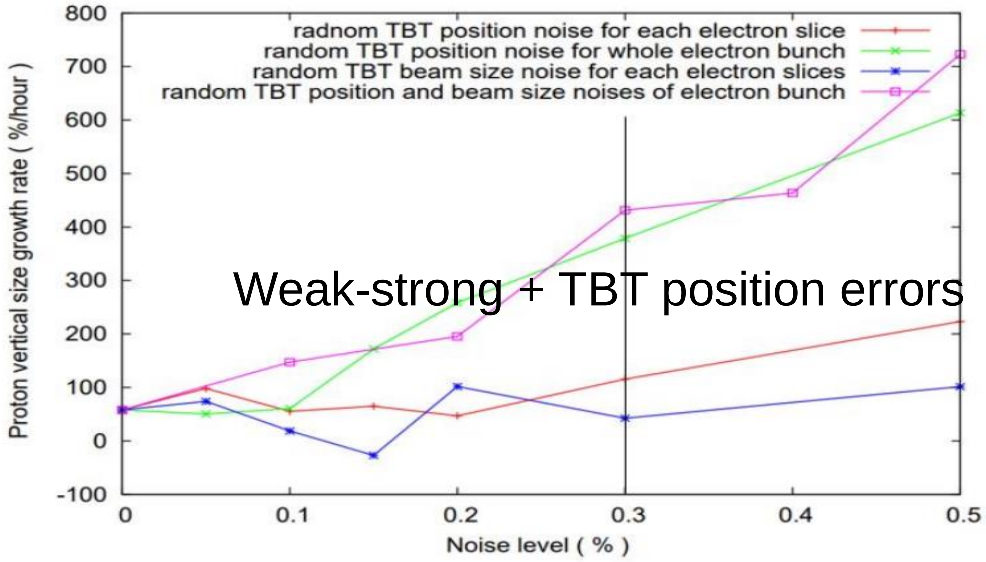
D. Xu, et al, NAPAC2022, MOYD4.
Y. Luo, et al, IPAC2022, WEPOPT038.

Turn-by-Turn Position and Beam Size Variation

- With typically 0.5 to 1 million macro-particles per bunch, the **RMS variations of electron bunch center and transverse beam sizes are about 0.2-0.3% of their RMS beam sizes at IP**.
- The impact of dipole moment of electron bunch can be eliminated by introducing virtual symmetric macro-particles in the PIC Poisson solver but growth rate doesn't go down much with this trick. **Dipole motion is not the main source for emittance growth**.
- TBT Beam size variation in simulation code is less than 0.2% of 1 sigma, which is not the main source for observed emittance growth either.



TBT <Y> od electron bunch

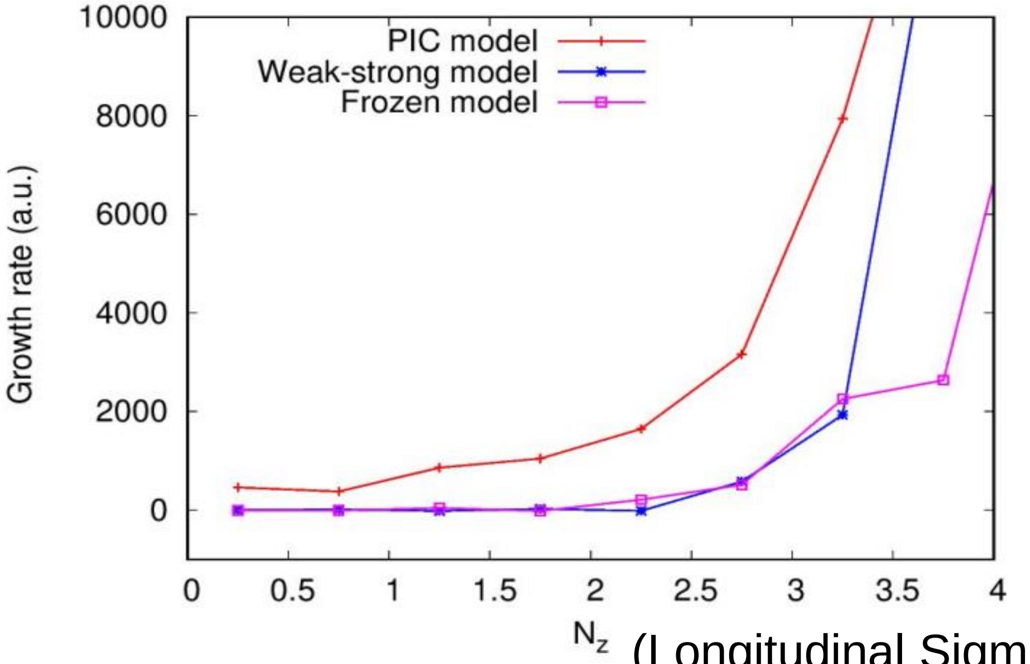


Weak-strong + TBT position errors

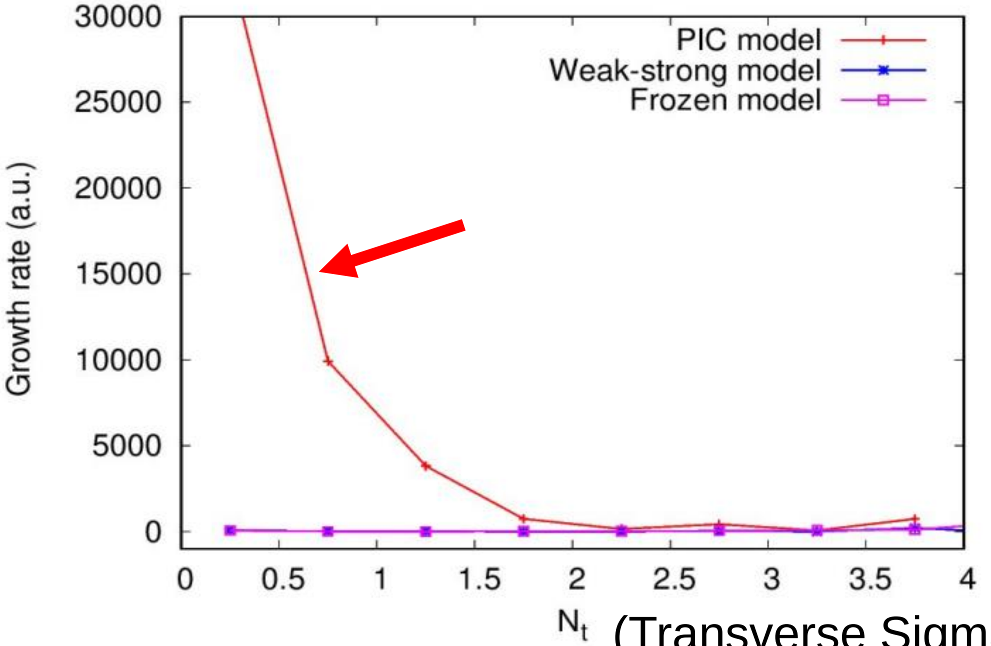
Macro-particles Contribute Most Emittance Growth

- To identify which macro-particles contribute most of proton emittance growth rates, we calculate RMS beam size growth rate of macro-particles as function of their initial longitudinal and transverse actions.

- **We found that macro-particles in the transverse bunch core contribute most of the artificial emittance growth rate in the strong-strong beam-beam simulation.**



(Longitudinal Sigma)



(Transverse Sigma)

Y. Luo, et al, IPAC2022, WEPOPT036

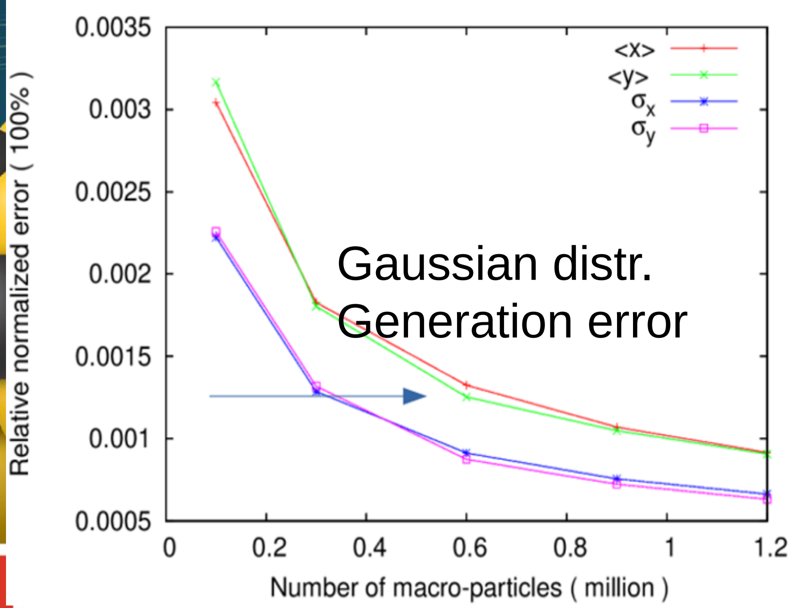
Tests of PIC Based Poisson Solver

- We estimate RMS variation of beam-beam kick from PIC Poisson solver for a given 4-D Gaussian distribution (with same RMS beam sizes) of 1000 particle distributions.

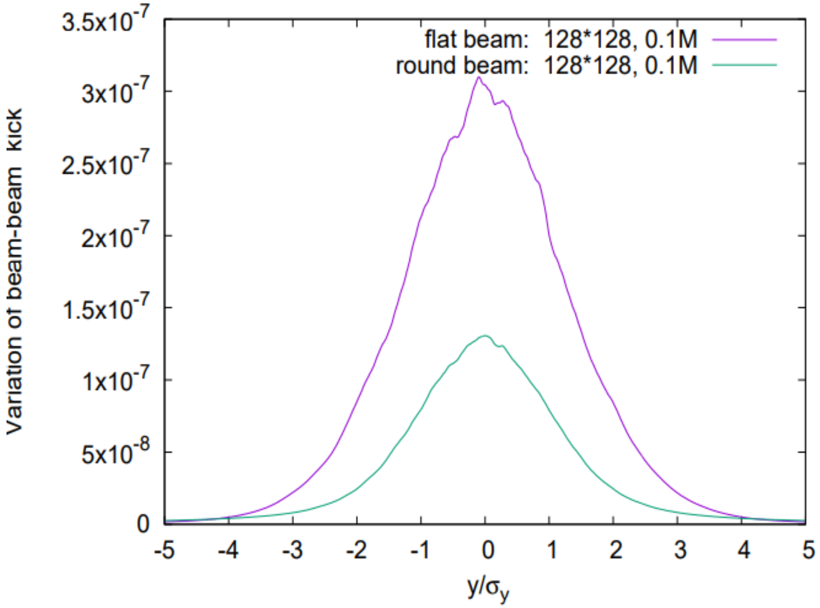
- **We found that PIC based Poisson solver tends to have a larger variation (noise) in BB kick for particles in the bunch core, especially in the vertical plane for a flat beam.**

This explained why we observed much faster beam size growth rates for macro-particles in the bunch core in the strong-strong simulation.

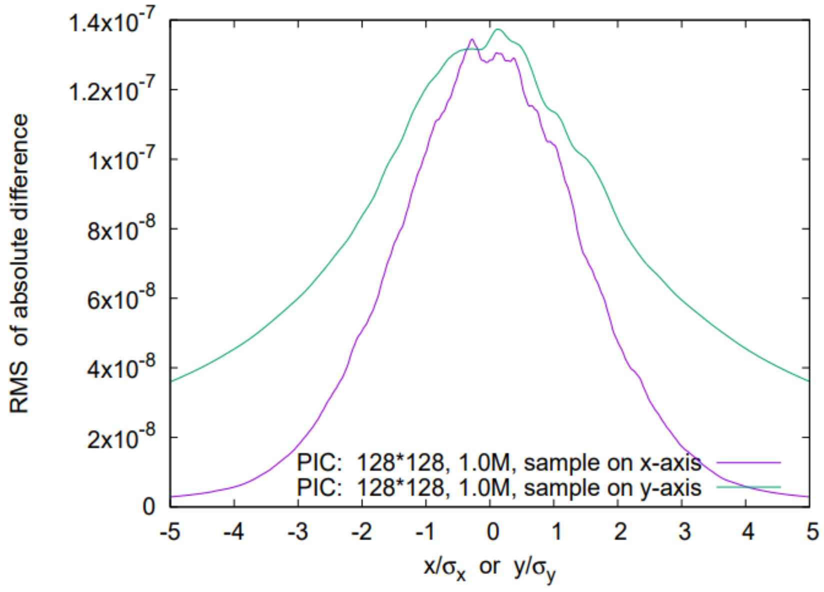
Y. Luo, et al, IPAC2022, WEPOPT040.



Gaussian distr.
Generation error



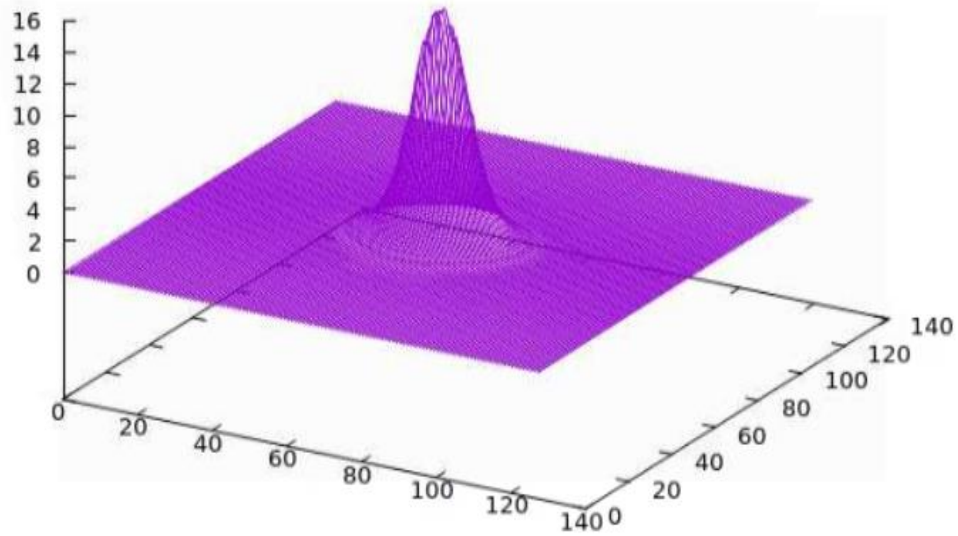
Flat beam vs round beam



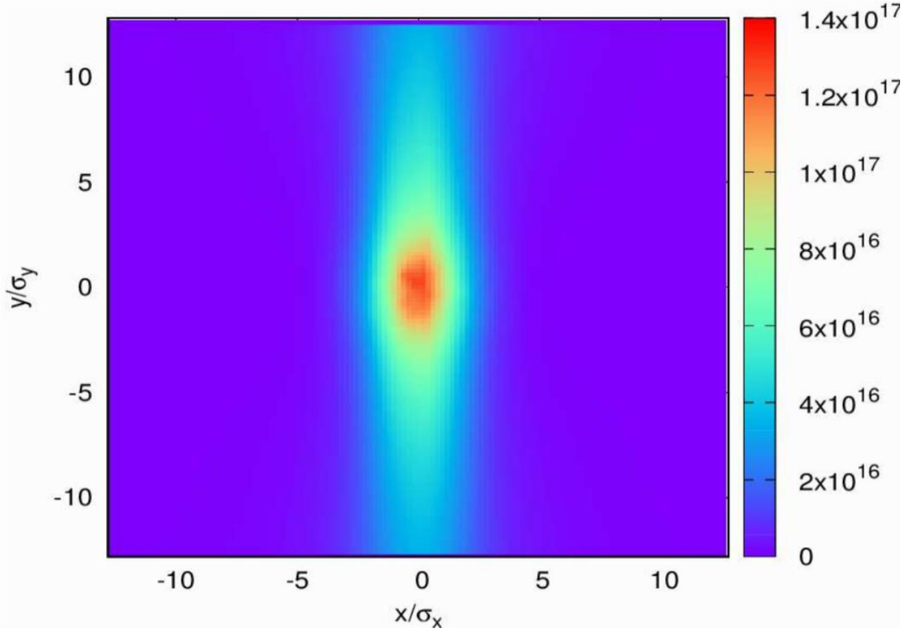
Flat beam: Horizontal vs vertical

Turn-by-Turn Variation in BB Kicks

We dug into a real strong-strong simulation with 0.5M * 0.5M macro-particles each bunch and PIC Poisson solver with128 *128 grids. **1000 turns after equilibrium are used for variation calculation.**



RMS variation of charge density on 128*128 grids.

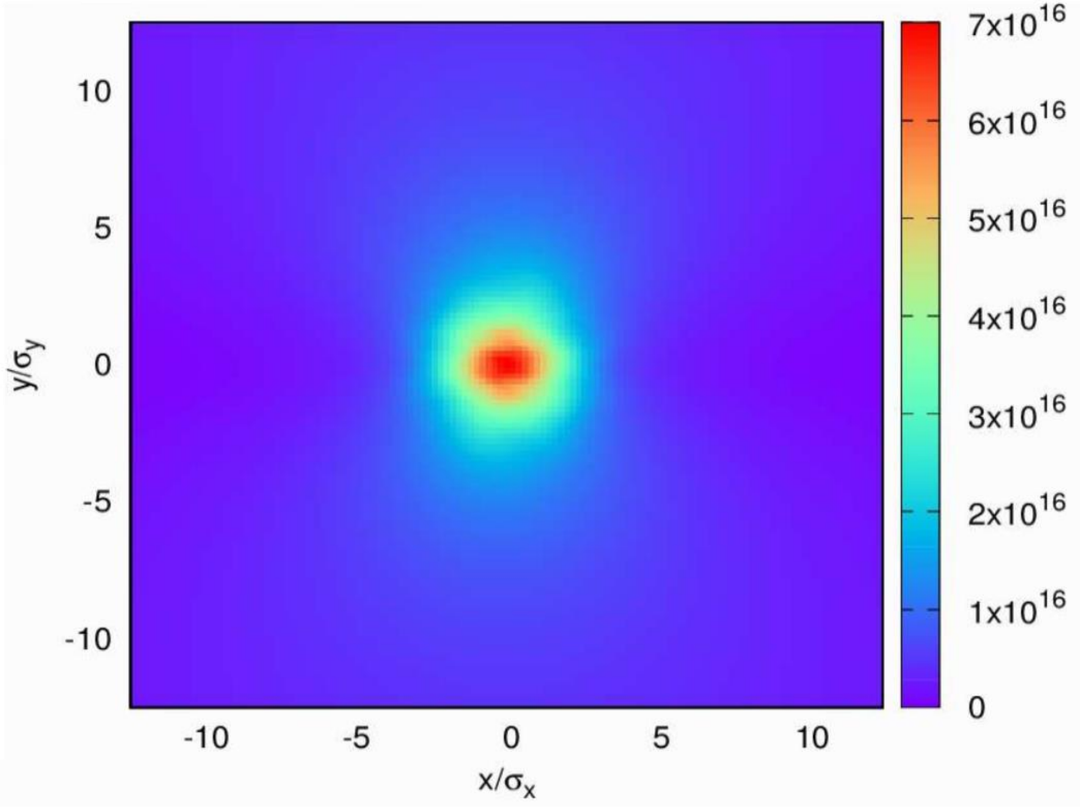


RMS variation of vertical BB kick on grids.

- **We observed larger variations in charge density and BB kick for central area of e-bunch. Also, variation of BB kick goes down slowly in the vertical direction for a flat bunch.**

As a comparison: round beams in RHIC

Vertical BB kick variation



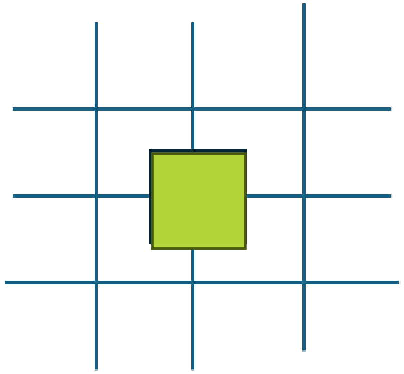
Extracted from a strong-strong BB simulation for RHIC.

- For a round beam, beam-beam variation has no much difference in horizontal and vertical beam-beam kicks.
- BB kick variation is overall smaller than flat beam.
- We also observed a larger BB force variation for bunch core particles.
- From physics, particles in bunch core is more stable than tail particles. We don’t observe emittance growth rates from strong-strong simulation.

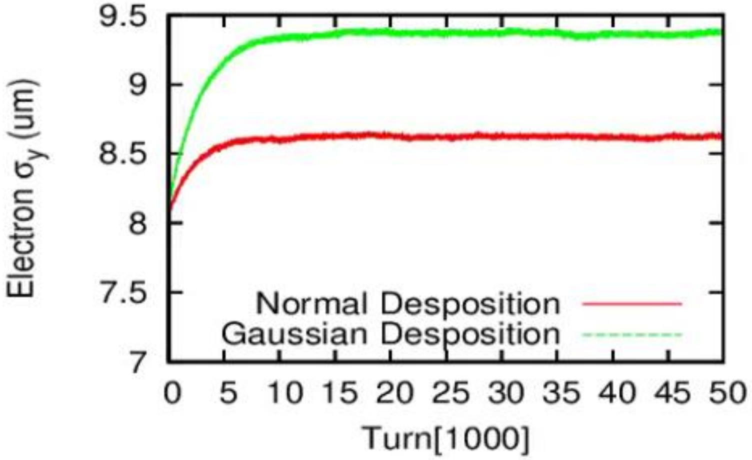
Recent Test with Analytical Density Deposition

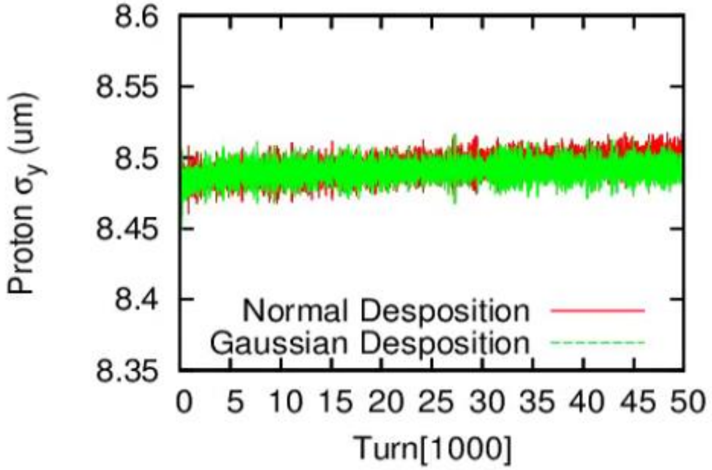
- At each collision point, we first calculate centers and RMS size of macro-particles in one slice. Then we assign charge onto each grid analytically assuming Gaussian charge distribution. PIC solver and Ex, Ey calculation remain the same as normal strong-strong codes.

- By doing so, we notice that the proton emittance growth rate goes down to ~200%.hour, still far away from the weak-strong growth rat.   This method doesn't exclude the TBT variations and electrical field interpolation.



Gaussian charge in the square +/-1/2 grid sizes assigned to the grid





Summary

- Numerical noise can be reduced by greatly increasing macro-particles, transverse grids, and longitudinal slices in PIC solver.
- Turn-by-turn variation of bunch motion can increase proton emittance growth rate but not the main contributor to the artificial emittance growth in strong-strong simulation.
- PIC solver generates a larger numerical variation in beam-beam force calculation for macro-particles in the bunch core.
- It is found with S-S simulation codes that macro-particles near bunch core contribute most of artificial emittance growth in strong-strong simulation.
- Strong-strong beam-beam simulation based on PIC is subject to large numerical noises, therefore it is not suitable for quantitative calculation for emittance growth rates.
- However, strong-strong BB simulation is suitable for design parameter optimization, tune scan, coherent BB motion, fast beam instabilities with wake fields, etc.