

Crab waist history and prospects

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- Crab waist collisions rationale
- Implementation on Daphne and SuperKEKB
- Lessons learned
- Projects based on CW
- Challenges of CW based colliders
- Design of CW based colliders
- Conclusions

Factories		Design Luminosity	Achieved Luminosity
KEKB	B-Factory KEK, Japan	1.0×10^{34}	2.1×10^{34}
PEP-II	B-Factory SLAC, USA	3.0×10^{33}	1.2×10^{34}
DAΦNE phase I	Φ-Factory Frascati, Italy	1.0×10^{32}	1.6×10^{32}
DAΦNE upgrade	Φ-Factory Frascati, Italy	5.0×10^{32}	4.5×10^{32}
BEPCII	C-Tau-Factory Beijing, China	1.0×10^{33}	3.3×10^{32}

Beam Current Records at Factories

Parameters	PEP-II		KEKB		DAΦNE	
	LER	HER	LER	HER	e+	e-
Circumference, m	2200	2200	3016	3016	97.69	97.69
Energy, GeV	3.1	9.0	3.5	8.0	0.51	0.51
Damping time, turns	8.000	5.000	4.000	4.000	110.000	110.000
Beam Currents, A	3.21	2.07	1.70*	1.25*	1.40	2.45

Maximum positron beam current

Maximum currents with SC cavities

Maximum electron beam current

* 2.00 A and 1.40 A without crab cavities

Conventional Strategy

$$L = N_b f_0 \frac{N^2}{4\pi\sigma_x^* \sigma_y^*} = N_b f_0 \frac{\pi\gamma^2 \xi_x \xi_y \varepsilon_x}{r_e^2 \beta_y^*} \left(1 + \frac{\sigma_y^*}{\sigma_x^*}\right)^2$$

$$\xi_{x,y} = \frac{Nr_e}{2\pi\gamma} \frac{\beta_{x,y}^*}{\sigma_{x,y}^* (\sigma_x^* + \sigma_y^*)}$$

- Small beta function at the IP β_y^*
- Higher number of particles per bunch N
- More colliding bunches N_b
- Larger beam emittance ε_x
- Round beams $\sigma_x^* = \sigma_y^*$
- Higher tune shift parameters $\xi_{x,y}$
- Small crossing angle $\theta \ll 1$
- Small Piwinski angle $\Phi = \sigma_z \text{tg}(\theta/2) / \sigma_x < 1$

Flat beams $\sigma_x^* \gg \sigma_y^*$
for DA requirements

To avoid parasitic
crossings (PC)

To reduce strength of
SB resonances

Standard Collision Scheme Limitations

1. Hour-glass effect limits minimum beta function at IP $\beta_y^* \geq \sigma_z$
2. Drastic bunch length reduction is impossible:
bunch lengthening, microwave instability, CSR
3. Further multibunch current increase would result in:
coupled bunch instabilities, HOM heating, higher wall plug power
4. Higher emittances conflict with
stay-clear and dynamics aperture limitations
5. Tune shifts saturate, beam lifetime drops due to
beam-beam interaction

Against Standard Logic?

1. Small emittance ϵ_x
2. Large Piwinski angle $\Phi \gg 1$
3. Larger crossing angle θ
4. Longer bunch length σ_z
5. Strong nonlinear elements (sextupoles)



Crabbed Waist Advantages

1. Large Piwinski's angle

$$\Phi = \text{tg}(\theta/2)\sigma_z/\sigma_x$$

- a) Luminosity gain with N
- b) Very low horizontal tune shift
- c) Vertical tune shift decreases with oscillation amplitude

2. Vertical beta comparable with overlap area

$$\beta_y \approx 2\sigma_x/\theta$$

- a) Geometric luminosity gain
- b) Lower vertical tune shift
- c) Suppression of vertical synchro-betatron resonances

3. Crabbed waist transformation

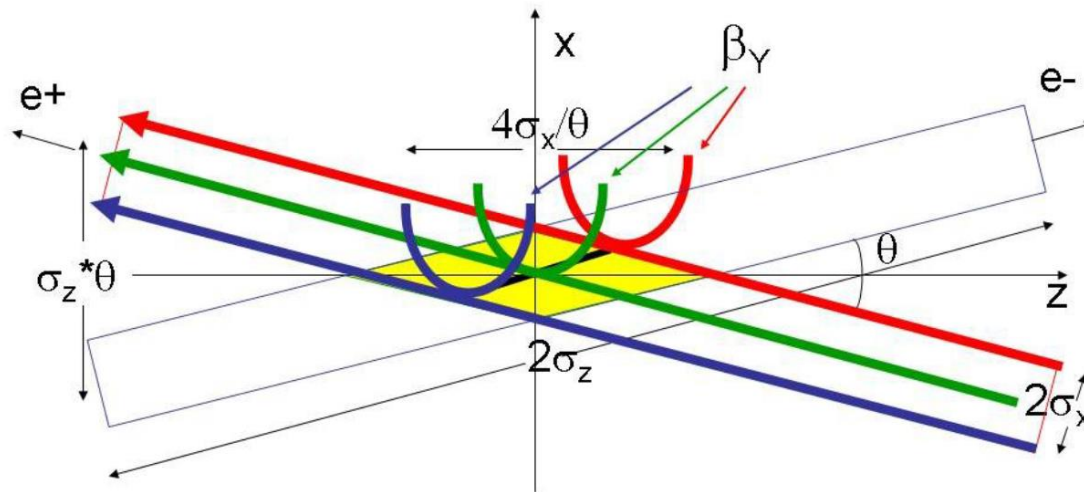
$$y = xy'/\theta$$

- a) Geometric luminosity gain
- b) **Suppression of X-Y betatron and synchro-betatron resonances**

Parameters		BEPCII	SuperC-Tau
Energy	E , GeV	1.89	2
Circumference	C , m	238	767
Damping time	$\tau_x/\tau_y/\tau_z$, ms	25/25/12.5	30/30/30
Beam current	I , A	0.91	1.68
Bunches	n_b	93	384
Energy spread	σ_E	5.16×10^{-4}	7.1×10^{-4}
Bunch length	σ_z , cm	1.5	0.9
Beta functions	β_x^*/β_y^* , m	1/0.015	0.04/0.0008
Emittances	$\varepsilon_x/\varepsilon_y$, nm-rad	144/2.2	8/0.04
Beam sizes (IP)	σ_x/σ_y , μm	380/5.7	17.9/0.179
Crossing angle	θ , mrad	11x2	30x2
Powinski angle	Φ	0.435	15.1
Tune shifts	ξ_y/ξ_x	0.04/0.04	0.13/0.0044
Luminosity	L, $\text{cm}^{-2}\text{s}^{-1}$	1.0×10^{33}	1.1×10^{35}

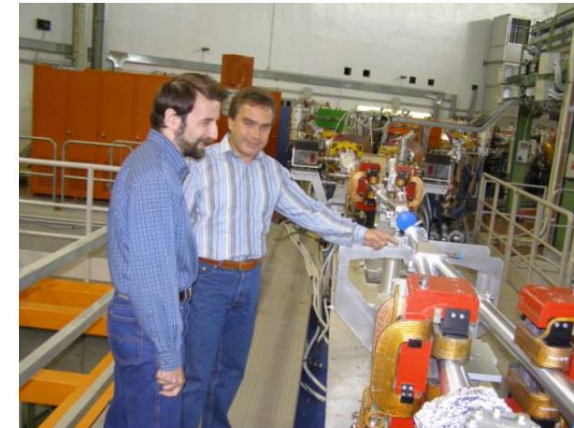
Crab Waist in 3 Steps

1. Large Piwinski's angle $\Phi = \text{tg}(\theta/2)\sigma_z/\sigma_x$
2. Vertical beta comparable with overlap area $\beta_y \approx 2\sigma_x/\theta$
3. Crab waist transformation $y = xy'/\theta$

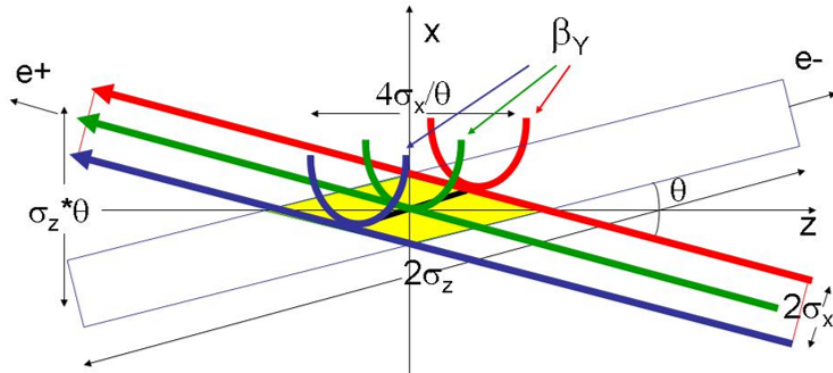
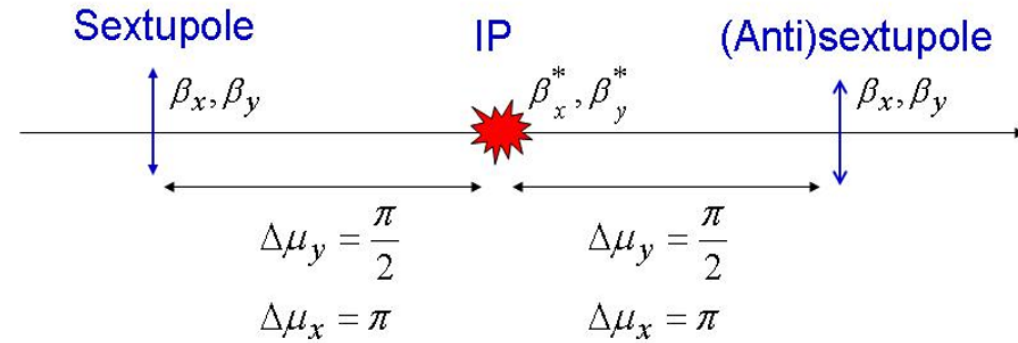
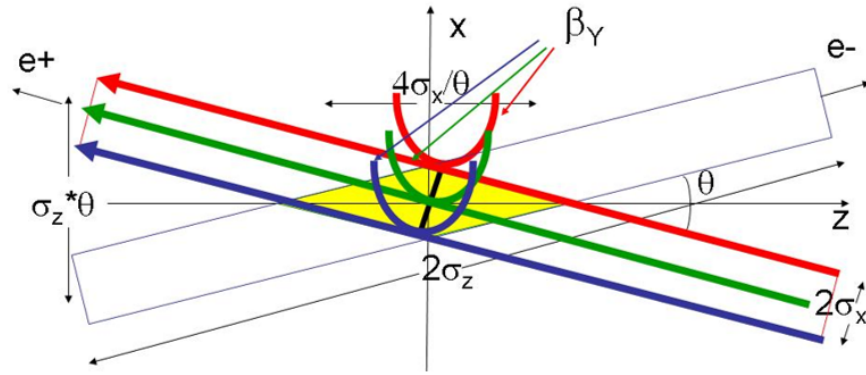


1. P.Raimondi, 2° SuperB Workshop, March 2006

2. P.Raimondi, D.Shatilov, M.Zobov, physics/0702033



Crab Waist collision scheme



- Large Piwinski Angle Φ (smaller emittance, large crossing angle, lower horizontal beta)
- Small vertical beta function at IP
- Suppression of beam-beam resonances using sextupoles in the interaction region

1. P.Raimondi, 2^o SuperB Workshop, March 2006
2. P.Raimondi, D.Shatilov, M.Zobov, physics/0702033
3. M.Zobov et al., Phys.Rev.Lett. 104 (2010) 174801

$$\Phi = \frac{\sigma_z}{\sigma_x} \tan\left(\frac{\theta}{2}\right); \quad l_{\text{int}} \approx \frac{\sigma_z}{\Phi}; \quad L \cong n_b f_0 \frac{1}{4\pi\gamma\sigma_x\sigma_y} \left[\frac{N^2}{\sqrt{1+\Phi^2}} \right]$$

$$\xi_y \cong \frac{r_e \beta_y}{2\pi\gamma\sigma_x\sigma_y} \left[\frac{N}{\sqrt{1+\Phi^2}} \right]; \quad \xi_x \cong \frac{r_e \beta_x}{2\pi\gamma\sigma_x^2} \left[\frac{N}{1+\Phi^2} \right]$$

LPW & CW inception

In the quest for the Next Linear Collider A Seryi and me had developed a new scheme for efficient demagnification of the beams at the IP.

A Seryi demonstrated that it can be successfully applied to low energies as well (at ATF)

Myself I've got intrigued with the possibility of using this capability for circular colliders.

Using Guinea-Pig (D Shultze code) it did seem that a making making a few passes in a ring, this beam can produce a significant luminosity before blowing up.

To mitigate the hour-glass effect, two RF-quadrupoles at the side of the IP are used to to generate a traveling focusing:

the head of the bunch is focused earlier (in Z), e.g. at the center of its colliding region
This greatly reduce the beam blowup, and improve somewhat the geometric overlap.

Subsequently I realized that the beam components with large betas do not generate much luminosity but a lot of beam blowup => a crossing angle is very beneficial to remove this blowup with little loss of luminosity

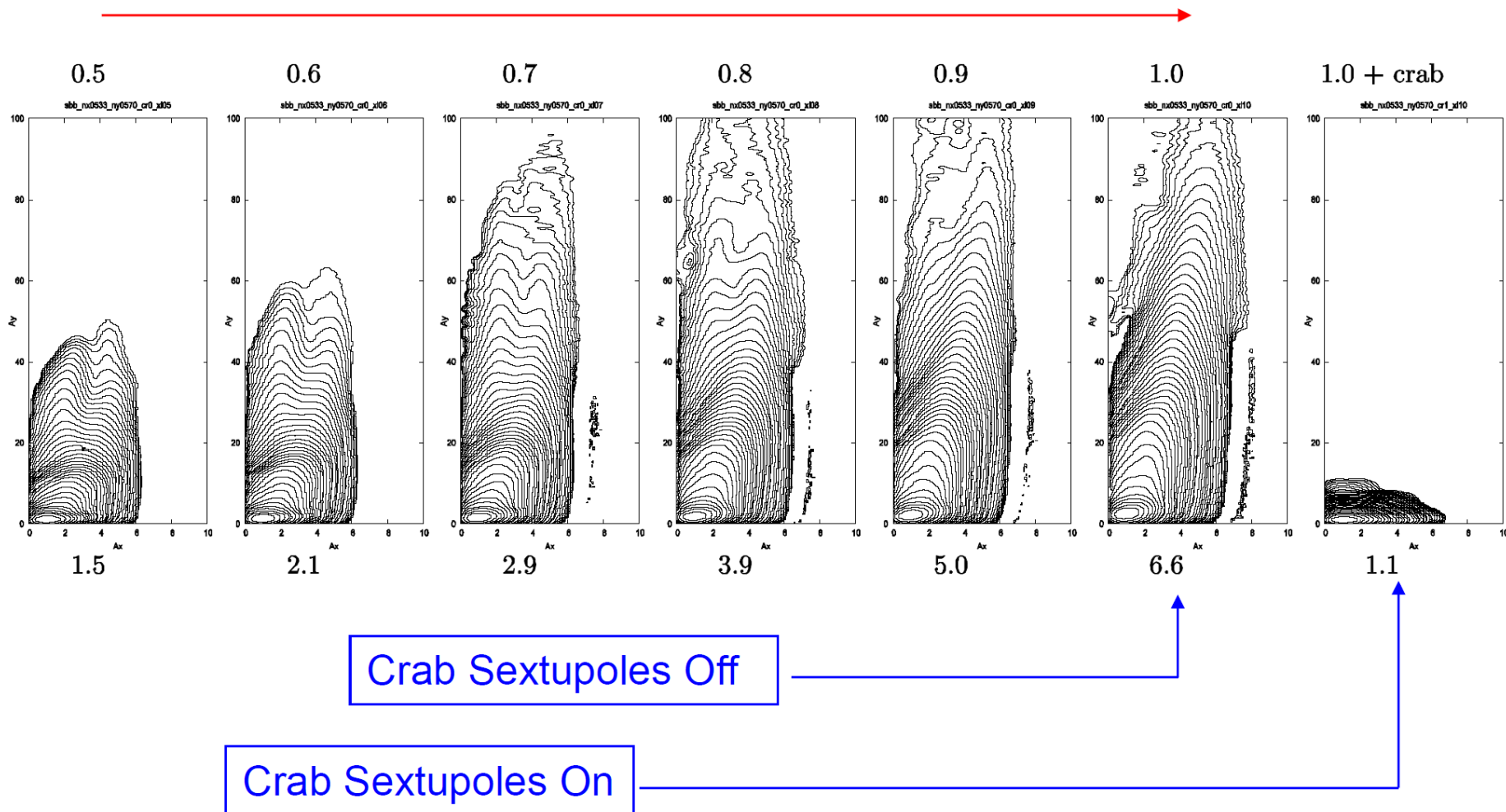
But with the crossing angle the traveling focus becomes ineffective!

After a lot of thinking I realized that since the crossing angle swaps the Z coordinate with the X coordinate, the "traveling focus" had to be done in the horizontal plane => **The Crab Waist**

It took one single pass with Guinea-Pig to see that the emittance dilution with this set-up is negligible

Beam Blowup and Tails in SuperB

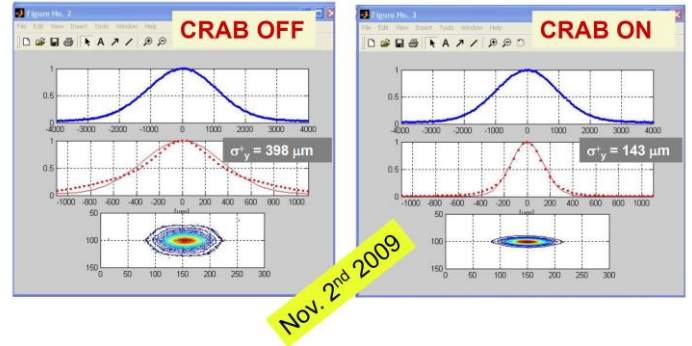
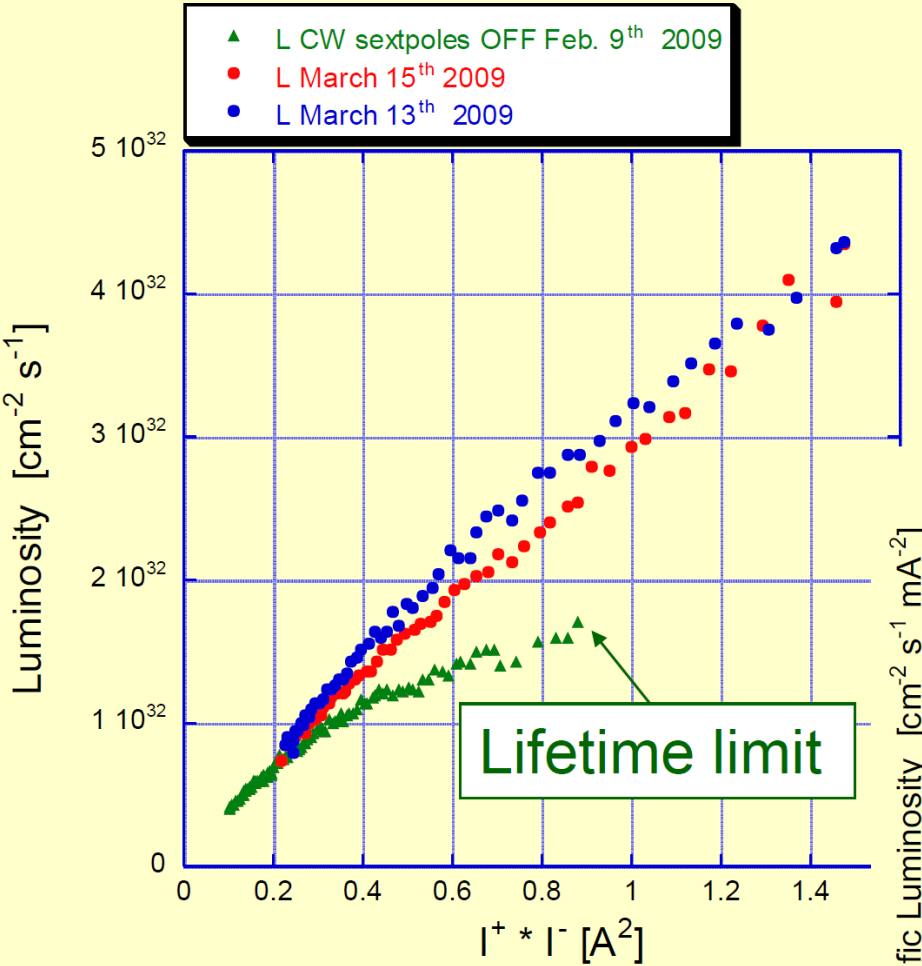
Bunch Current



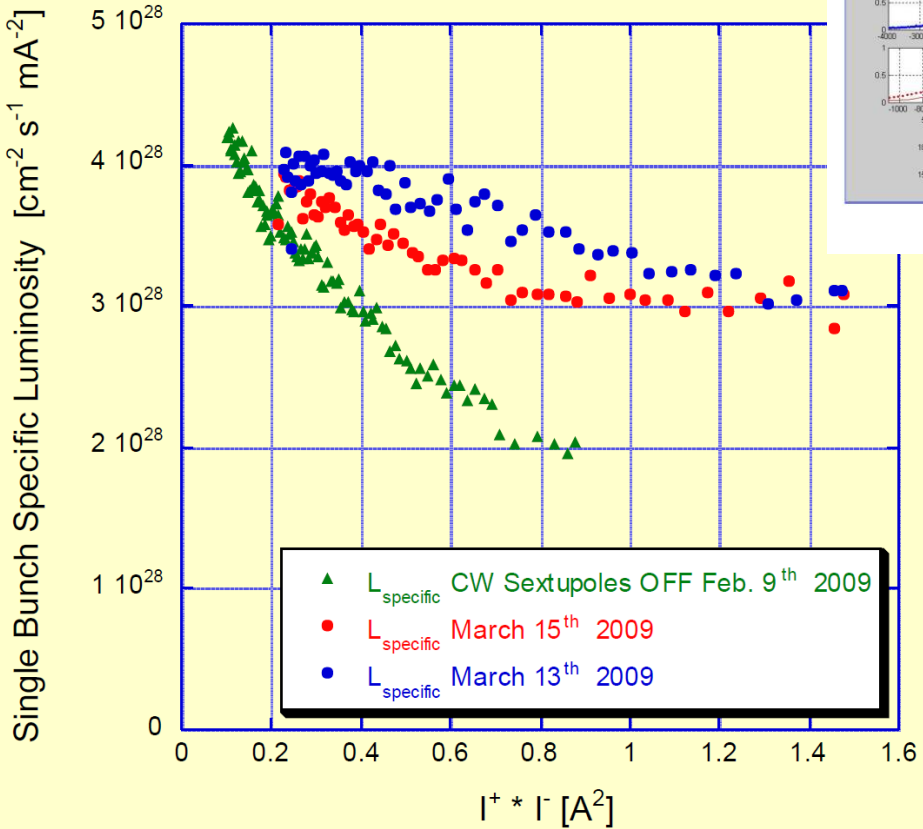
Crab waist collisions at Daphne

Crab on/off Specific Luminosity vs Current Product

Transverse Beam Profile Measurements



Crab on/off Luminosity vs Current Product

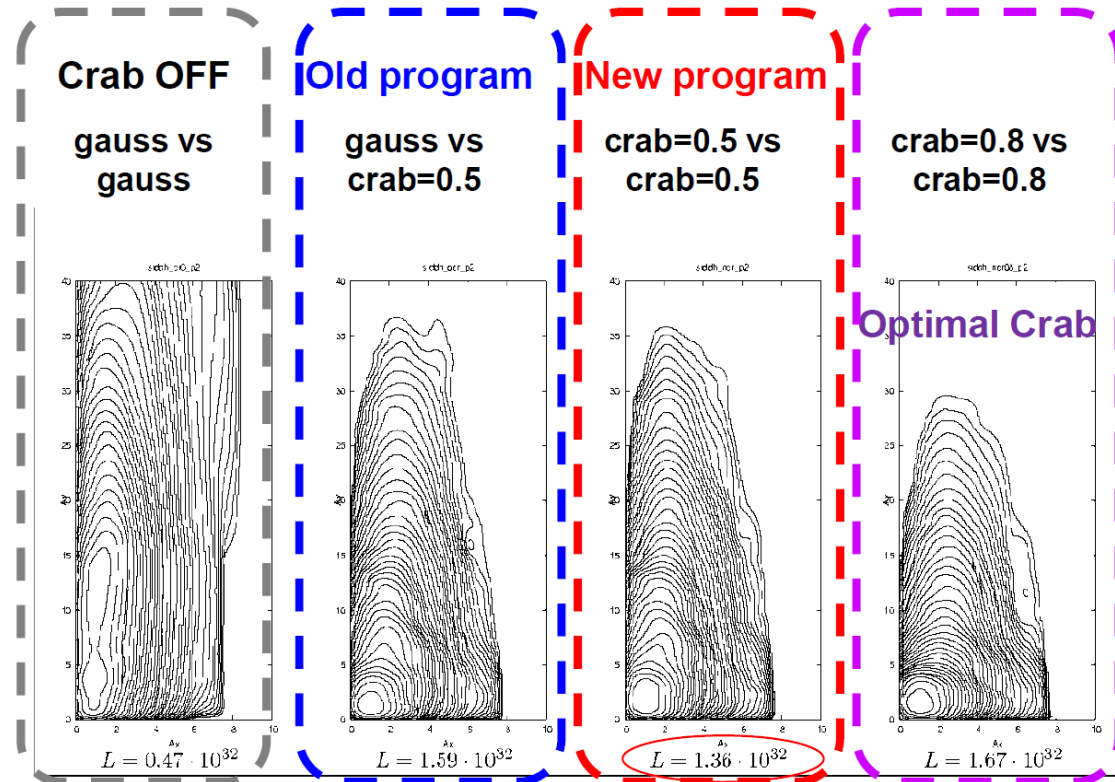


Specific luminosity drop consistent with single beam collective effects

Weak-Strong Simulations

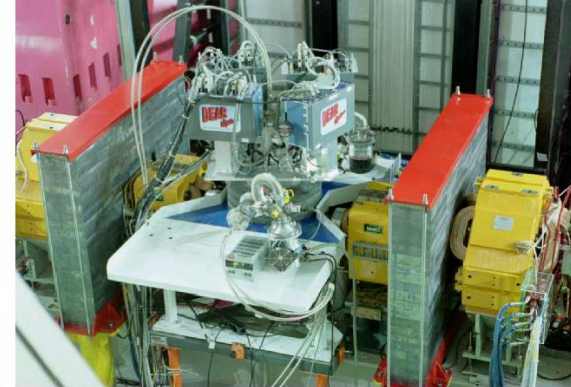
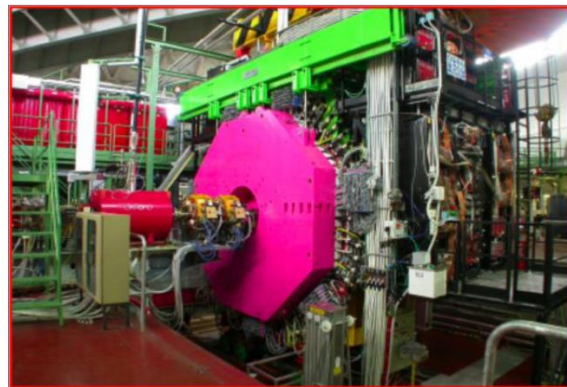
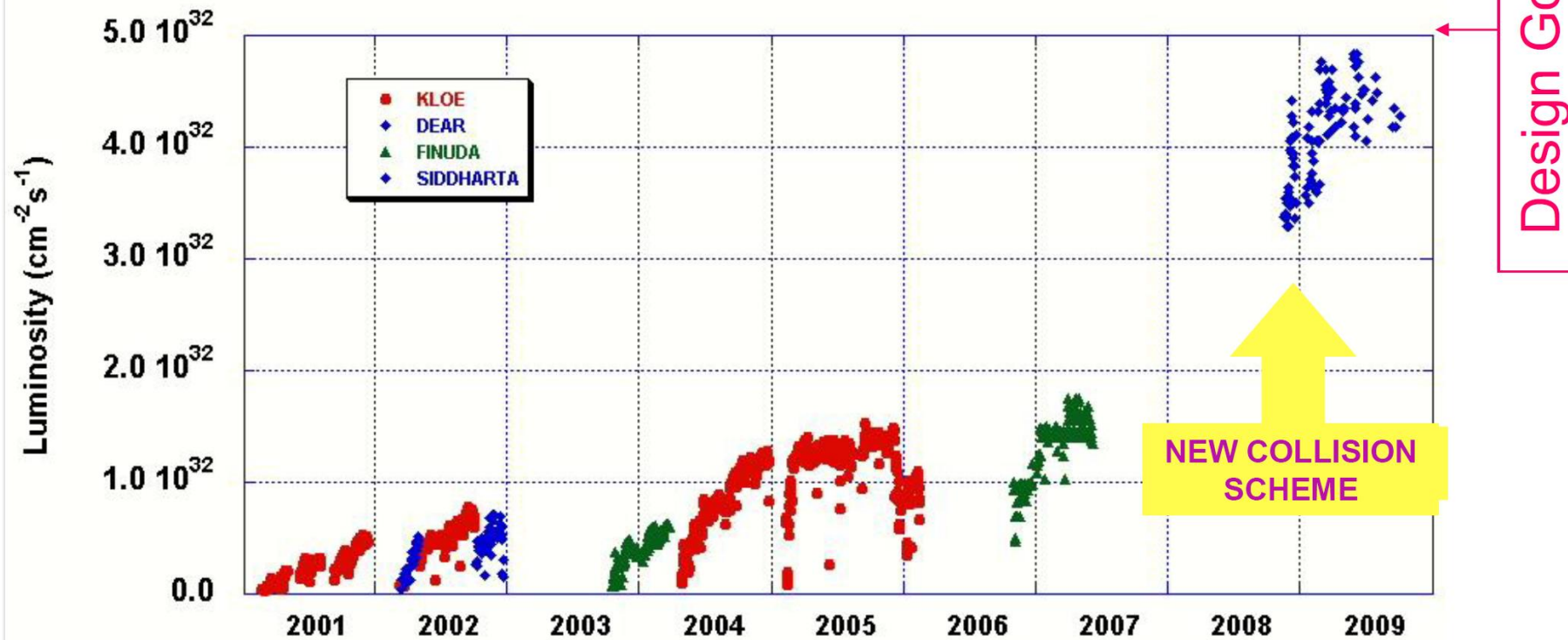
(Crabbed Strong Beam)

$$\Delta v_y = 0.0894$$



L = 1.36E+32

DAΦNE Peak Luminosity

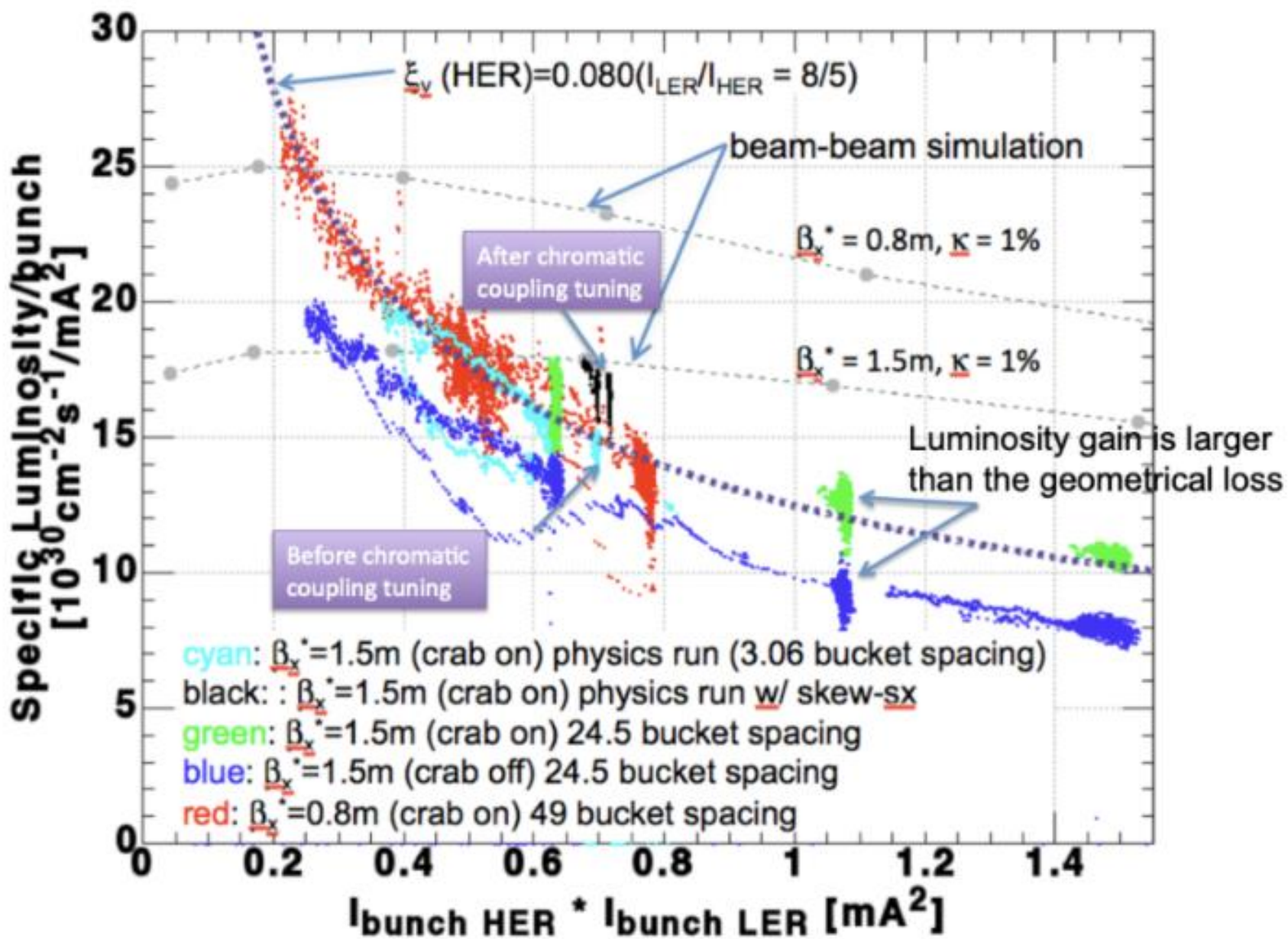


Implementation of LPW and CW at SuperKEKB

Table 1. Comparison of KEKB and SuperKEKB machine parameters.

	KEKB		SuperKEKB		SuperKEKB		SuperKEKB	
	Achieved		2020 May 1st		2022 June 22nd		Design	
	LER	HER	LER	HER	LER	HER	LER	HER
$I_{\text{beam}} [\text{A}]$	1.637	1.188	0.438	0.517	1.363	1.118	3.6	2.6
# of bunches	1585		783		2249		2500	
$I_{\text{bunch}} [\text{mA}]$	1.033	0.7495	0.5593	0.6603	0.606	0.497	1.440	1.040
$\beta_y^* [\text{mm}]$	5.9	5.9	1.0	1.0	1.0	1.0	0.27	0.30
ξ_y	0.129 ^{a)}	0.090 ^{a)}	0.0236 ^{b)}	0.0219 ^{b)}	0.0398 ^{b)}	0.0278 ^{b)}	0.0881 ^{c)}	0.0807 ^{c)}
	0.10 ^{b)}	0.060 ^{b)}			0.0565 ^{d)}	0.0434 ^{d)}	0.069 ^{b)}	0.061 ^{b)}
$\mathcal{L} [10^{34} \text{cm}^{-2} \text{s}^{-1}]$	2.11		1.57		4.71		80	
$\int \mathcal{L} dt [\text{ab}^{-1}]$	1.04		0.03		0.424		50	

^{a)} Values of ξ_y^{ih} calculated by Eq. (2.11); ^{b)} Values of ξ_y^L calculated by Eq. (2.8); ^{c)} Values of ξ_y^{ih} calculated by Eq. (2.9); ^{d)} Values in high bunch current study with 393 bunches by Eq. (2.8).



Lessons learned from CW operations

- Dafne performances ultimately limited by:
 - poor lifetime due to Touschek effect, small Momentum Acceptance ($\sim \pm 1.5\%$)
 - beam sizes enlargement due to collective effects:
 - ion trapping, ecloud, microwave instability, instabilities etc.
 - strong e⁺ horizontal instability due to ecloud limiting the maximum e⁺ current
- SuperKEKB performances affected by:
 - High detector background
 - poor lifetime
 - low injection efficiency => small transverse Dynamic Aperture (DA)
 - poor lifetime in collision => small MA (<1%)
 - fast instabilities ~one turn
 - All of the above worsen by lowering Betay*

Colliders based on Crab Waist concept

Colliders	Location	Status
DAΦNE	Φ-Factory Frascati, Italy	In operation (SIDDHARTA, KLOE-2, SIDDHARTA-2)
SuperKEKB	B-Factory Tsukuba, Japan	In operation, the world record luminosity has been achieved
SuperC-Tau	C-Tau-Factory Sarov, Russia	Russian mega-science project
FCC-ee	Z,W,H,tt-Factory CERN, Switzerland	100 km, CDR released in December 2018
CEPC	Z,W,H,tt-Factory China	100 km, CDR released in September 2018
HIERA	2-7 GeV China	Considered base line option

Challenges of CW based colliders

- Luminosity in LPA&CW collision roughly scales with the Piwinsky Angle
- Luminosity scales with $1/\epsilon_{mix}^{1.5}$ if the optic & ring support:
 - scaling of betax with $\sqrt{\epsilon_{mix}}$
 - scaling of betay with ϵ_{mix}
 - maintaining a constant $k_{coupling} = \epsilon_{mix} / \epsilon_{mix}$
 - increase the current with $1/\epsilon_{mix}^{0.5}$
 - maintaining the DA&MA at a level that affects minimally:
 - lifetime and lifetime in collision
 - Injection efficiency
 - topup operations
 - Detector Background and radiation budget
- ARC lattice should provide the smallest possible emittance while ensuring the largest possible DA&MA
- Extremely low beta insertion(s) should:
 - preserve DA&MA
 - allow “transparent” rescale of betas*
 - include all necessary tuning knobs, knobs should be orthogonal
 - be Crab Sextupoles transparent
 - etc.

$$Brilliance = \frac{\text{Photons}}{\text{sec} \cdot \text{mrad}^2 \cdot \text{mm}^2 \cdot 0.1\% \text{ BW}} \propto \frac{I}{\varepsilon_x \varepsilon_y}$$

Parameters	Unit	MAX-IV	ESRF-EBS	SIRIUS	SuperKEKB (LER/HER)		FCC-ee (at Z)	CEPC (at Z)
Energy	GeV	3.0	6.0	3.0	4.0	7.0	45.6	45.5
Circumference	km	0.528	0.844	0.518	3.02	3.02	90.66	100
Beam current	A	0.50	0.20	0.35	3.6	2.6	1.27	0.80
Horizontal emittance	nm rad	0.328	0.133	0.25	3.2	4.6	0.71	0.27

Design values

Operating 4th generation light sources

Biggest future lepton circular colliders

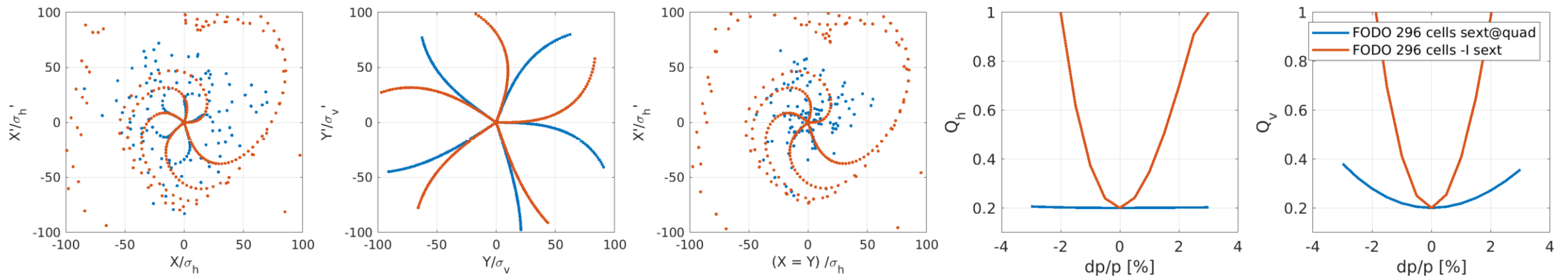
1. Both modern light sources and future lepton colliders based on the crab waist collision concept require smaller emittances
2. The future colliders beam currents should be close to the best values achieved in the factory-class lepton colliders

The quest for ARC low emittance lattice for colliders

In general, multiplying the number of cells of a FODO lattice by a factor 2:

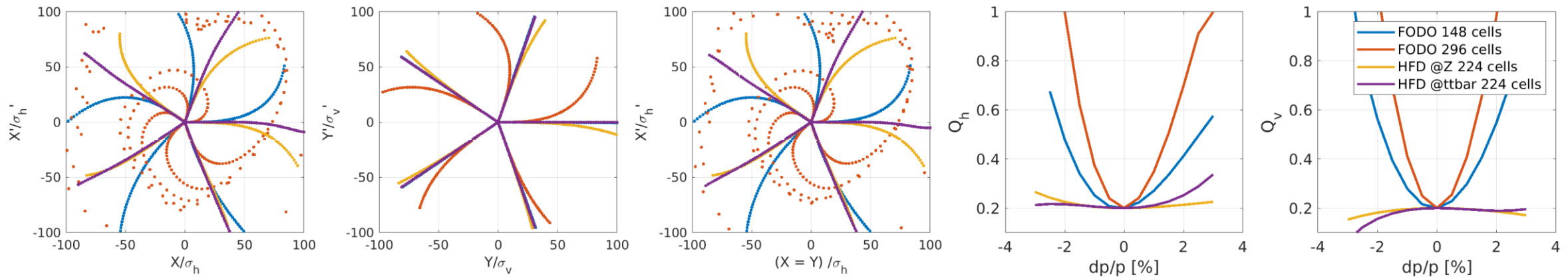
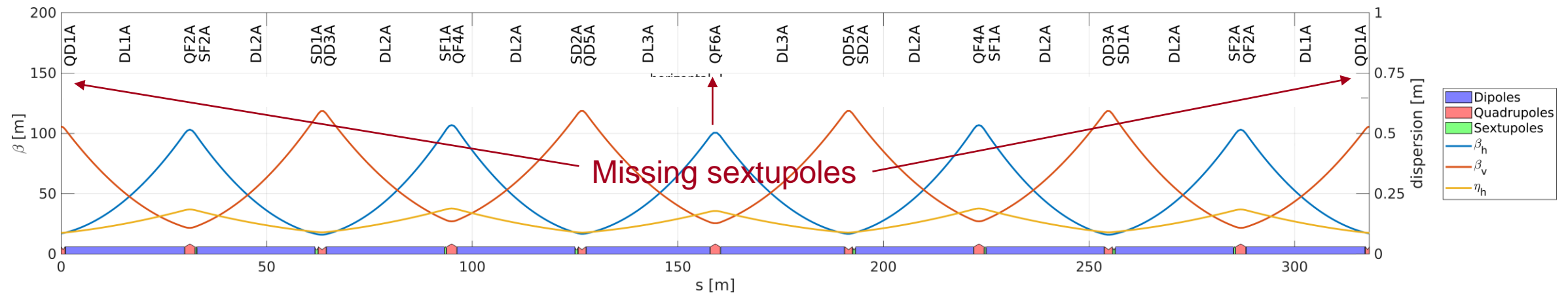
- Emittance decreases by 2^3 (8)
- Quadrupoles gradient and number doubles
- Sextupole gradients increase by 2^3 and number of sextupoles doubles
- MA decreases by a factor 2 and DA decreases by a factor $> 2^3$

Anharmonicity and achromaticity become more severe with more cells



Detuning with amplitude and momentum for two version of a FODO ARC lattice with sextupoles at -I locations or at all quadrupoles

It is possible to obtain detuning-free FODOs by proper placing and tuning of the sextupoles in the sequence



Coordinates at each of five turns tracking particles starting on the horizontal axis (first plot on the left), vertical axis (second plot from the left) and for particle with identical horizontal and vertical coordinates (third plot from left). The fourth and fifth plot are horizontal and vertical detuning with momentum. The curves refer to four ARC lattice options: GHC 148 cells (Z), GHC 296 cells (ttbar), HFD with low phase advance (Z) and large phase advance (ttbar). HFD optics are the most anharmonic and achromatic options

It is possible to design highly achromatic and anharmonic low beta insertions => LCC optic

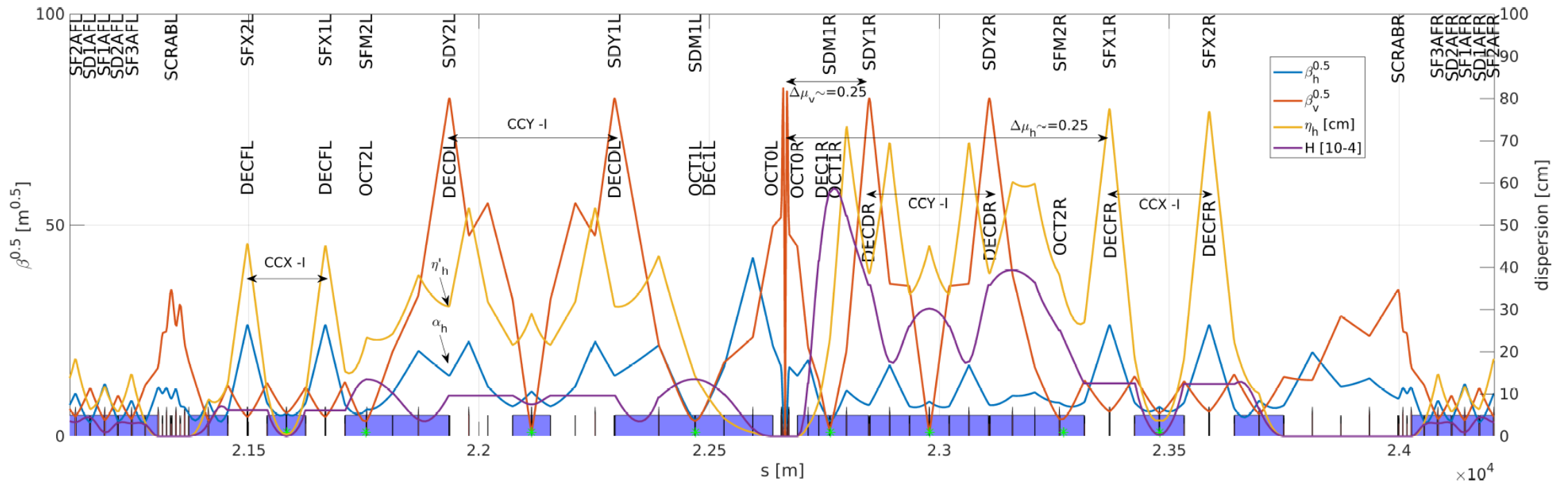


FIG. 4. One of the four final focus systems. The interaction point is located in the center of the displayed region. Sextupole names are reported in correspondence of their location. Crab sextupoles are named SCRAB.

Final Focus chromaticity compensation

8

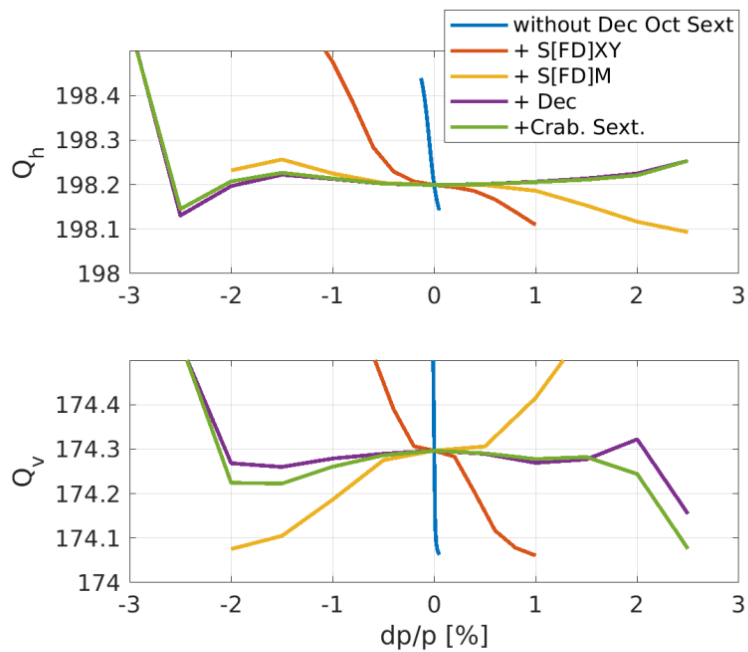
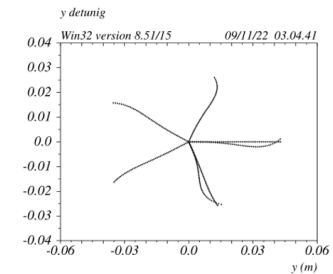
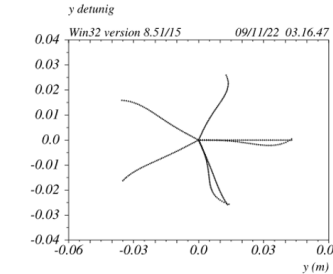


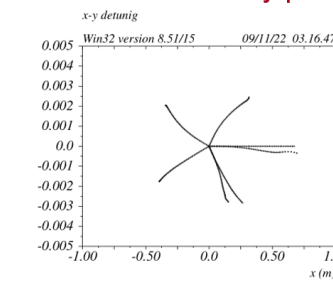
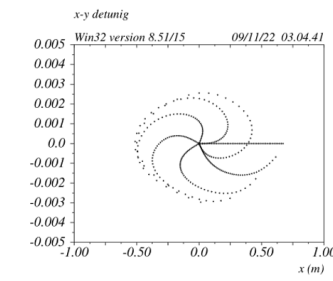
FIG. 14. Tunes vs momentum introducing one by one the non-linear multipoles in the Final Focus optics. Results are obtained by 6D particle tracking without synchrotron radiation.



R12 $\sim -0.3 \cdot L_{\text{sext}}^{10^{**}(-3)}$ between the SDy pair



R34 $\sim -0.3 \cdot L_{\text{sext}}^{10^{**}(-3)}$ between the SDy pair



Higher order effects are optimized by proper dimensioning of dipoles and beta functions

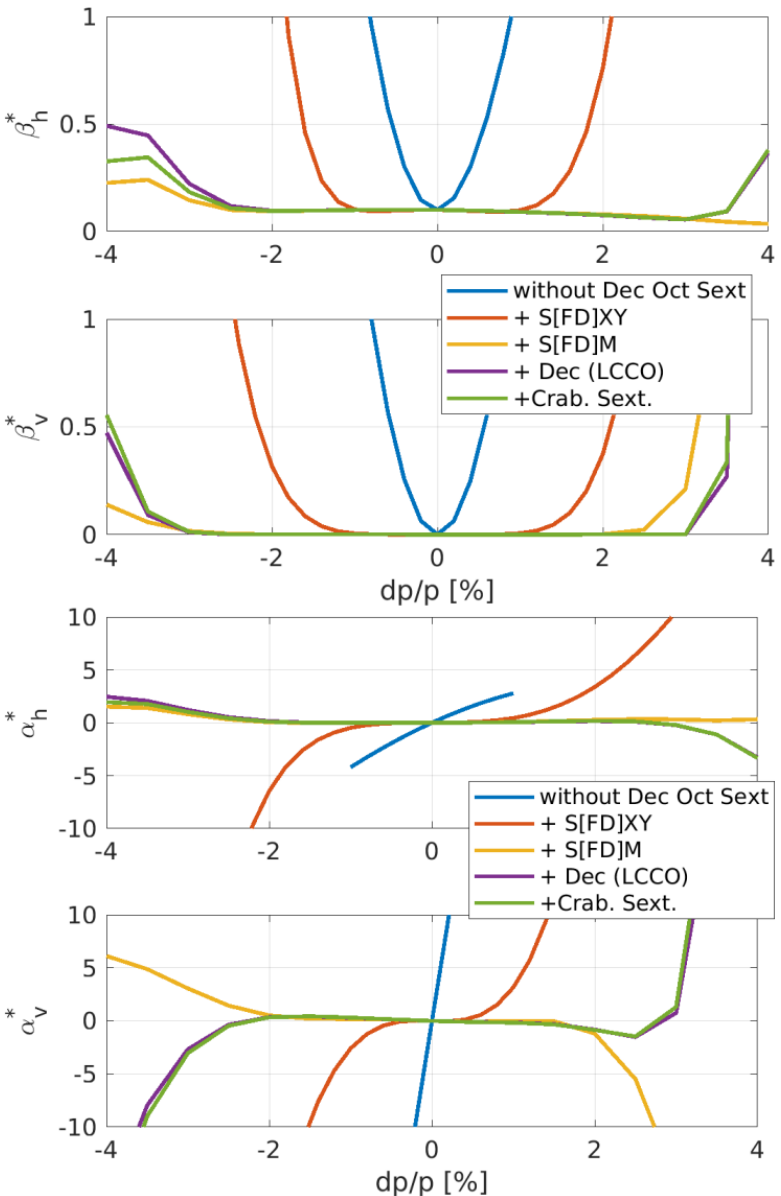


FIG. 15. β^* at IP (top) and α^* at IP as a function of the energy deviation considering only the FF optics turning on progressively the multipoles for non-linear corrections.

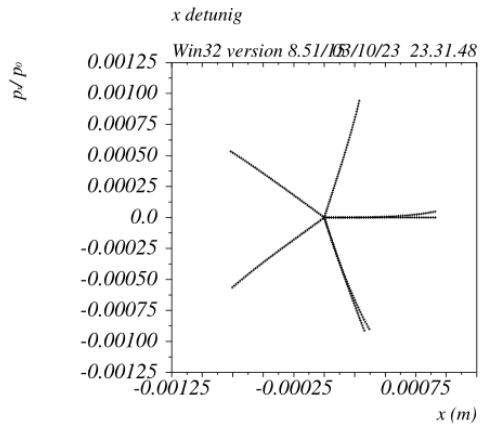


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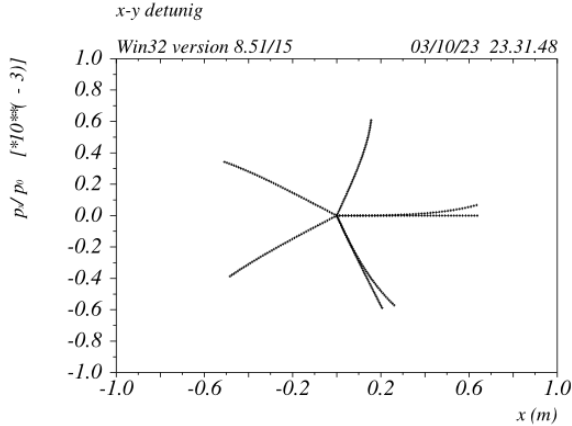


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[$\times 10^{**(-3)}$]

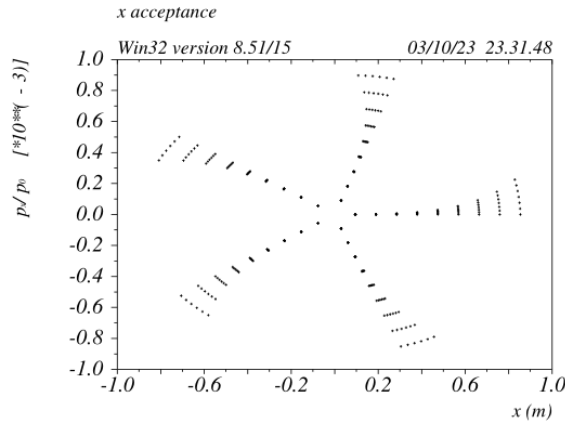


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[$\times 10^{**(-3)}$]

On energy dynamic is linear.
“Resonances” are virtually not existing.
Extremely favourable dynamics to minimize BeamBeam degradation (DS)

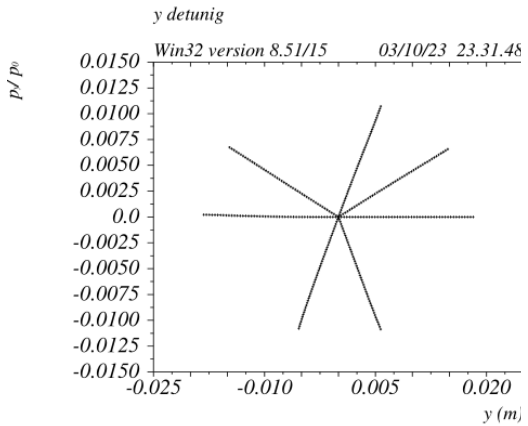


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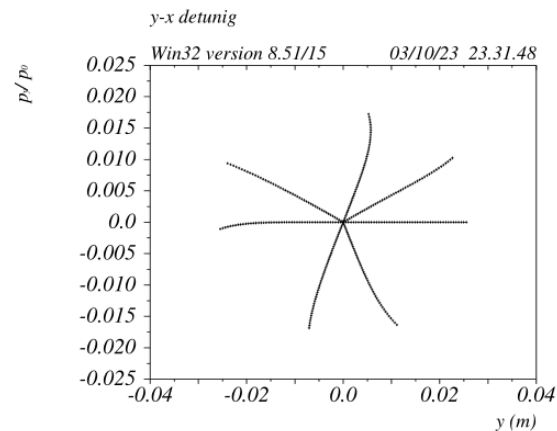


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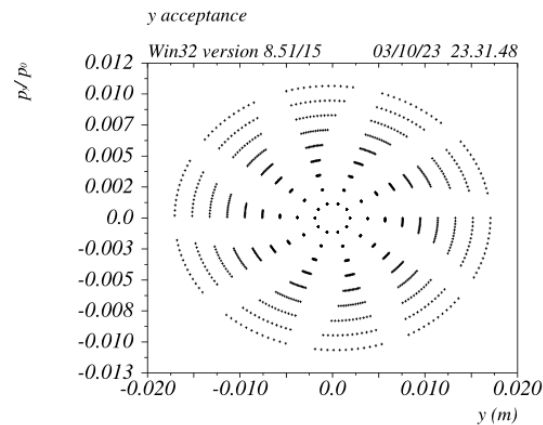
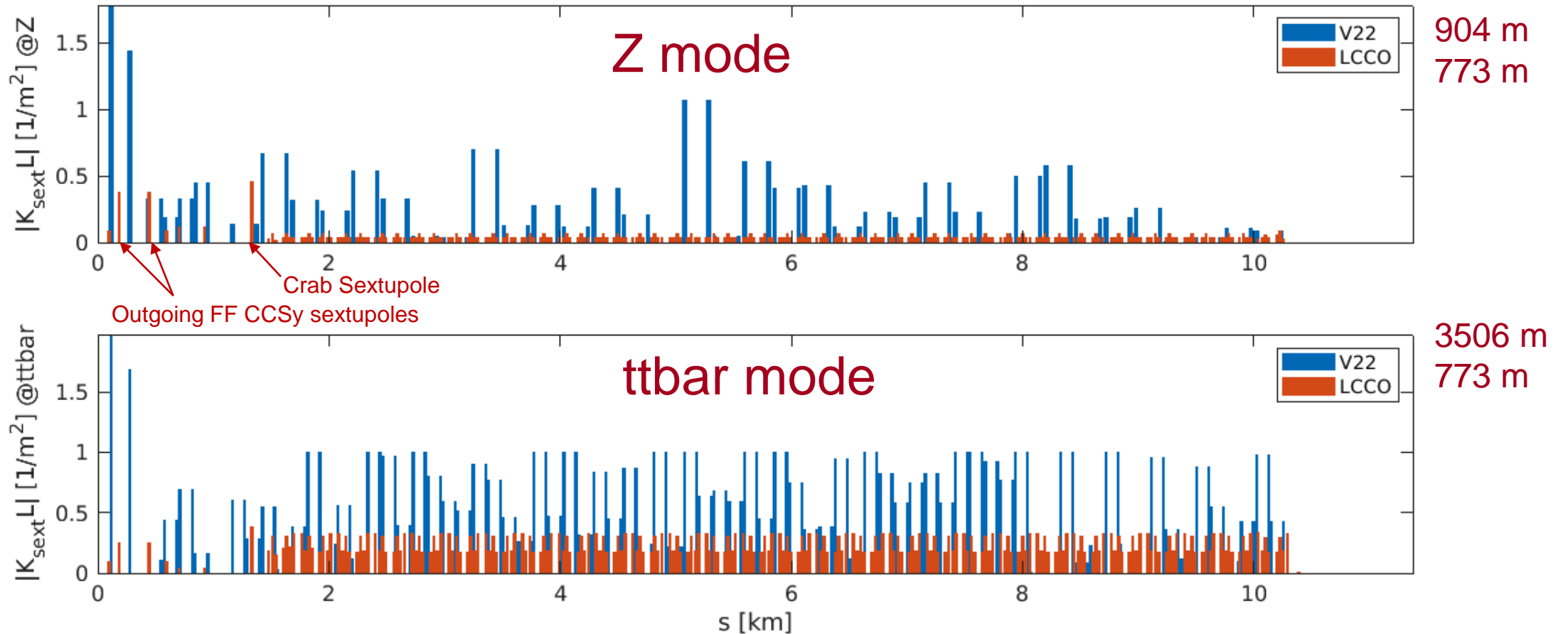


Table name = TRAC

[$\times 10^{**(-3)}$]

The quest/dream for a “quasi” time-independent trajectory is at reach!

Sextupoles gradients (1 octant)



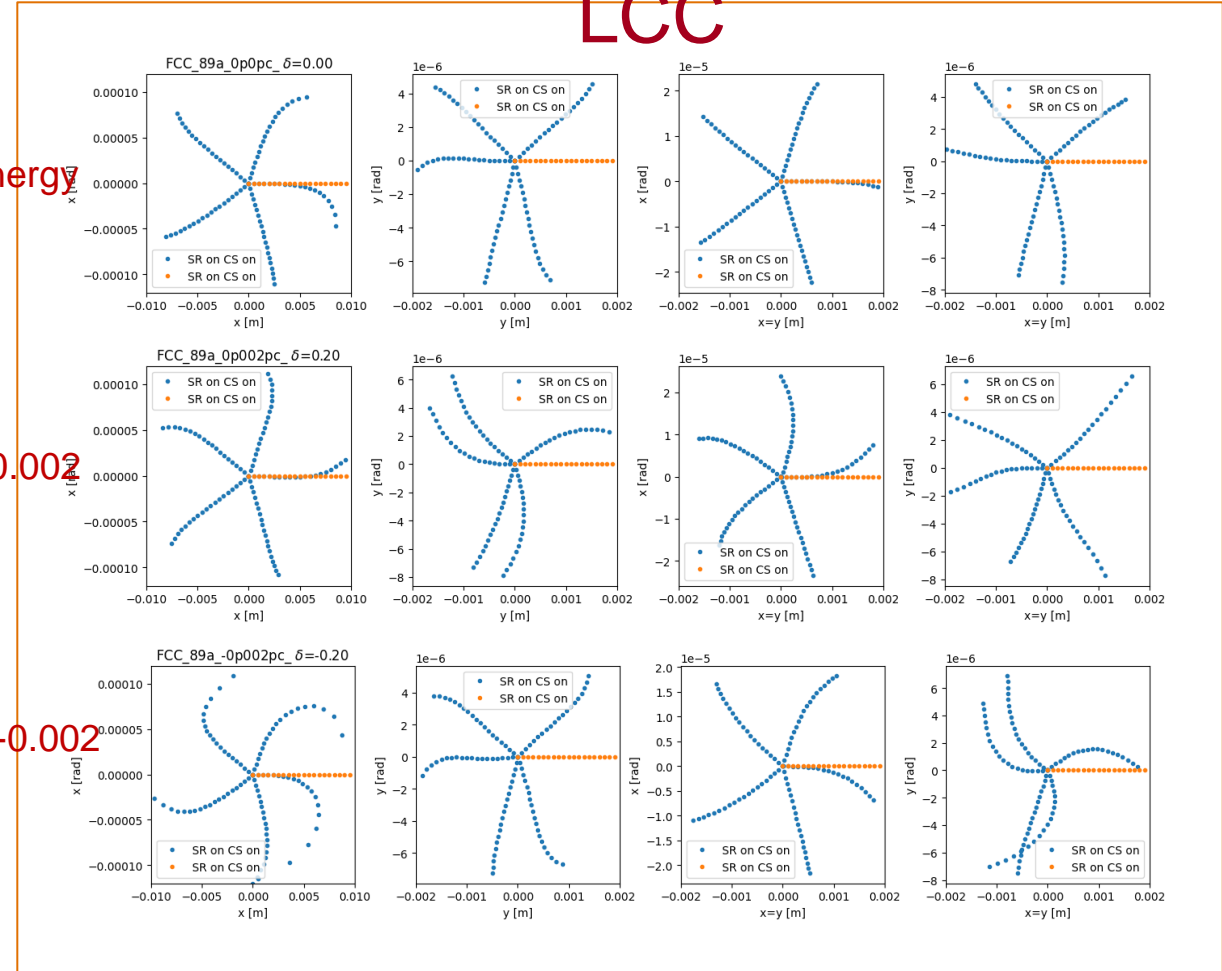
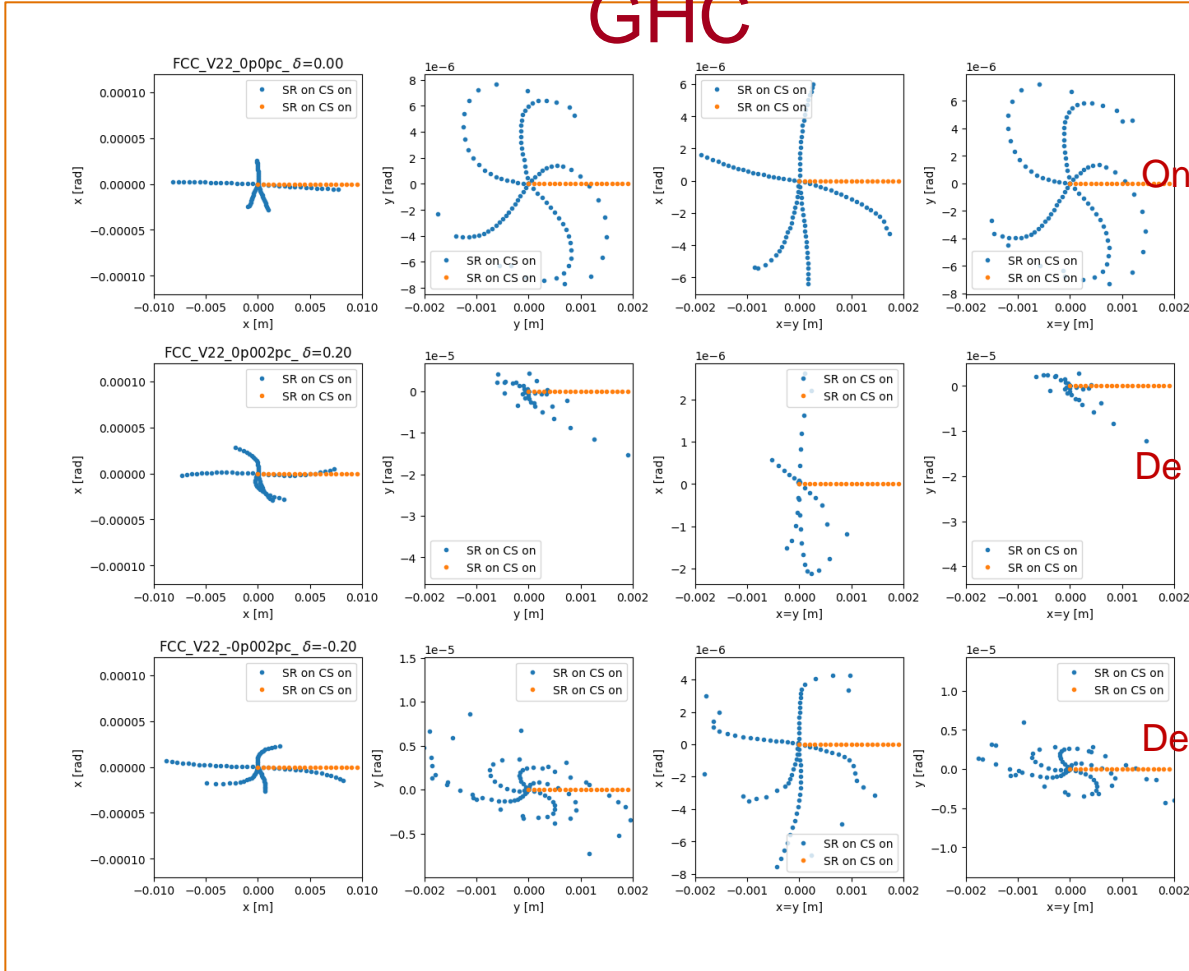
Smaller sextupole gradients → Relaxed requirements and tolerances.

Phase space evolution over 5 turns on and off energy

No SR in dipoles (no effect)

GHC

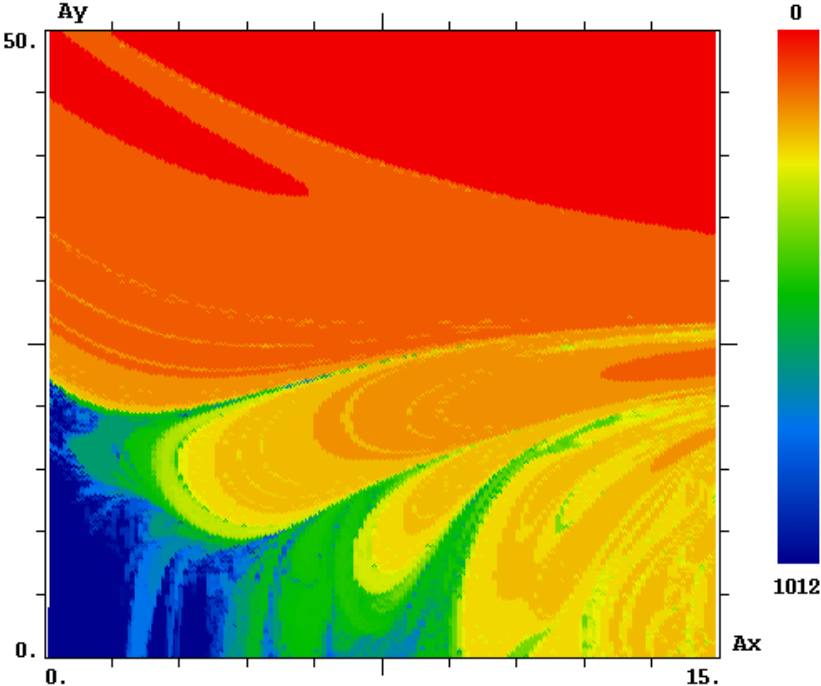
LCC



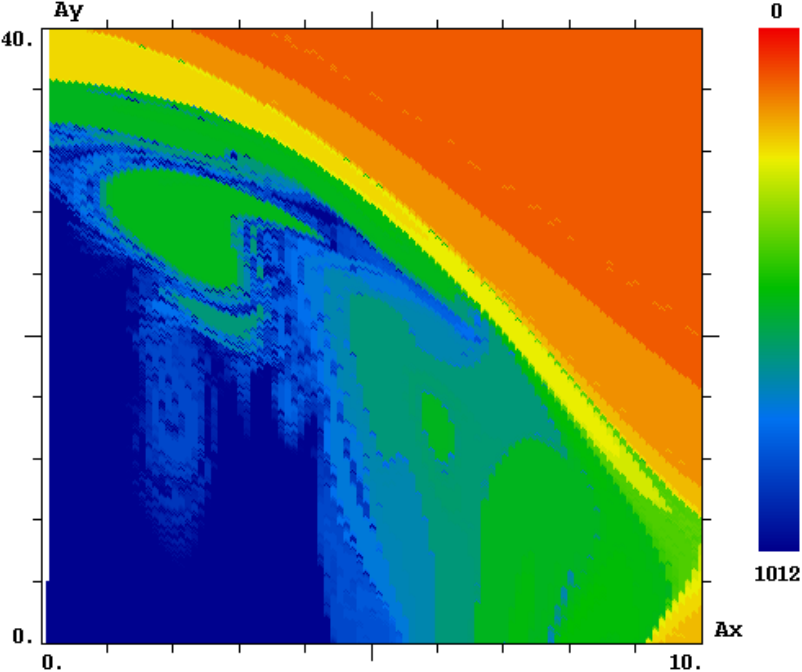
SYNCHROTRON RADIATION AND CRAB SEXTUPOLES ON

Starfish plots provide a “quick” overview of the combined effect of all the resonant driving terms

DA with BeamBeam for GHC and LCC optics



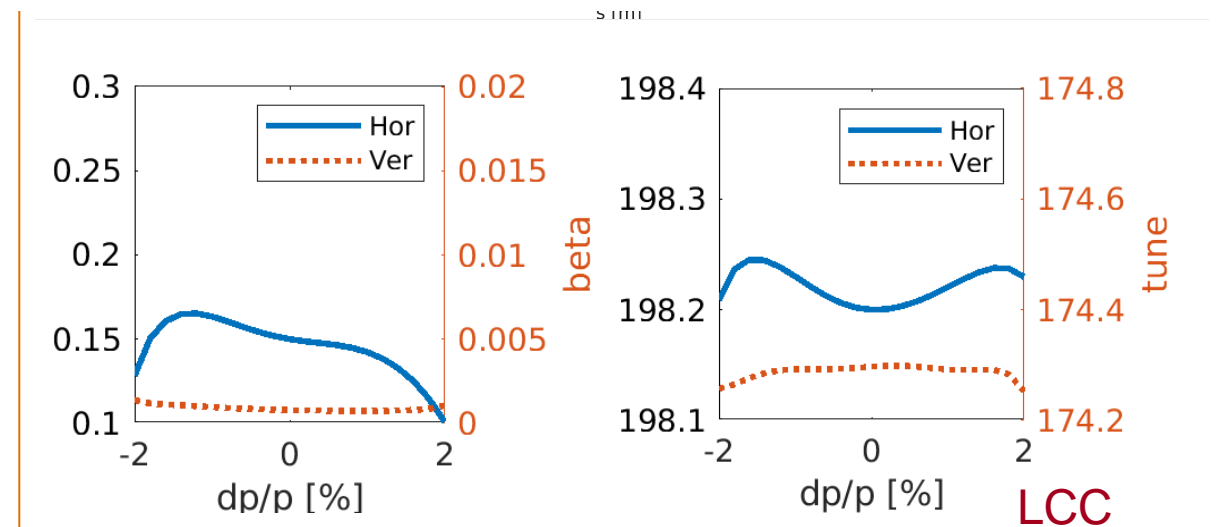
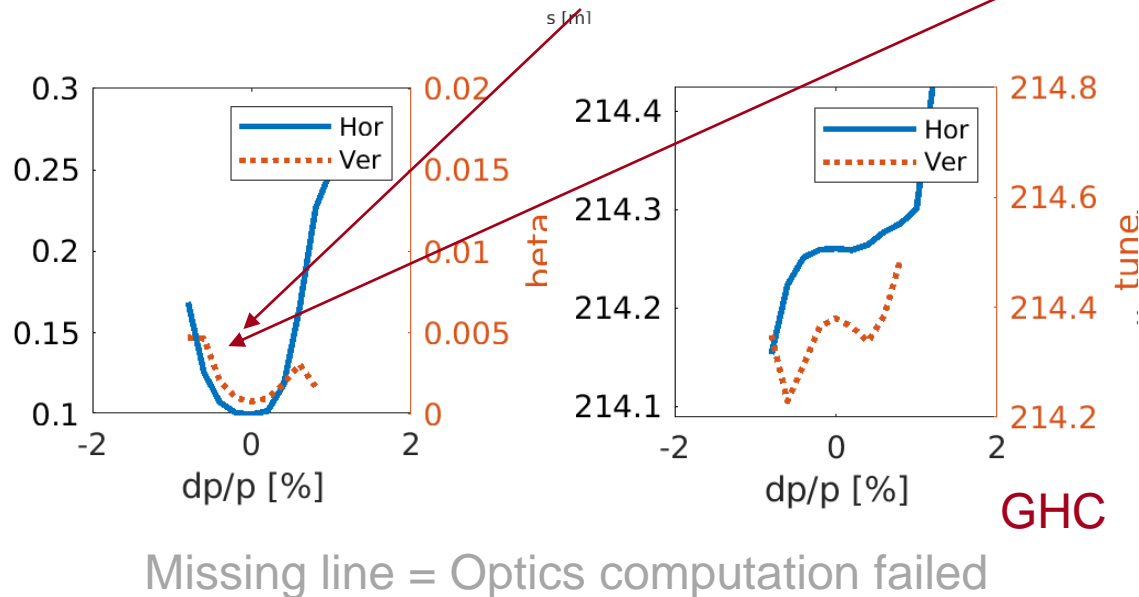
GHC



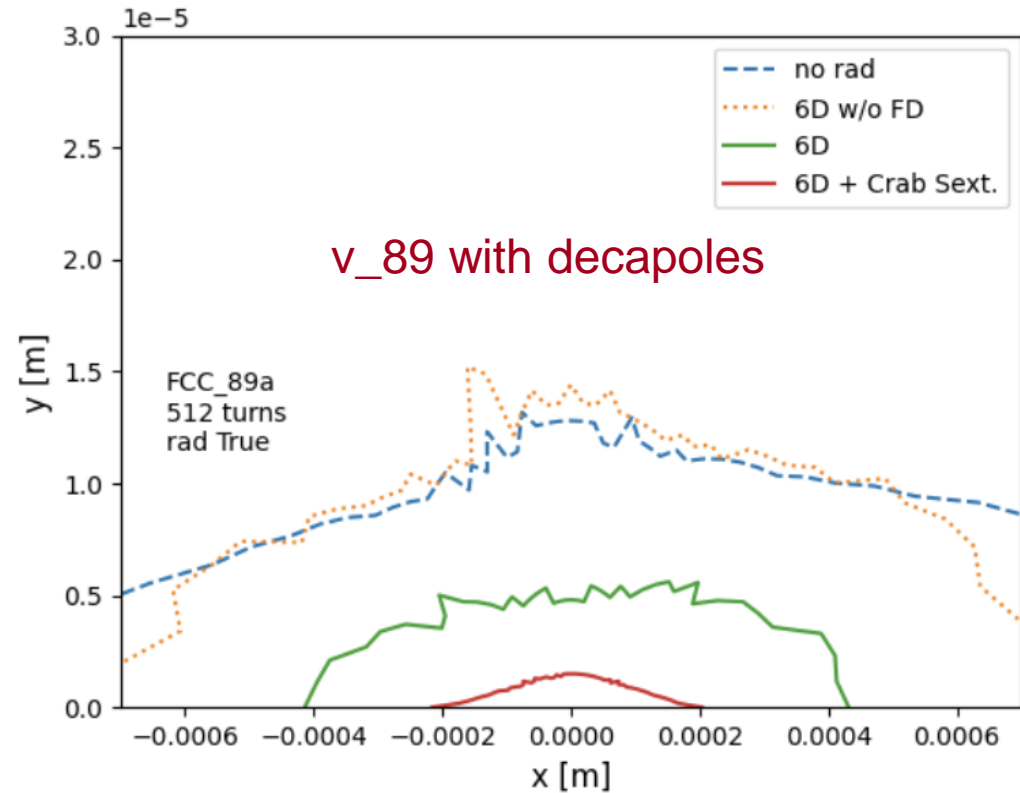
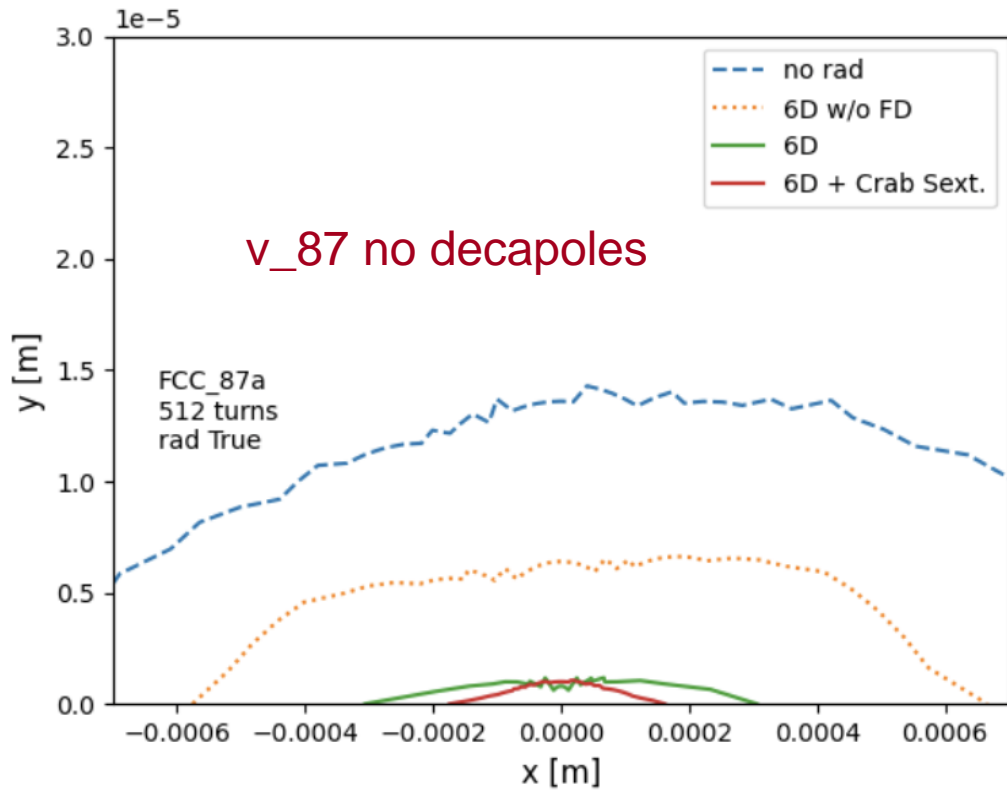
LCC (preliminary optic)

Crab waist concept applied to 6D

- Beambeam is minimized when all colliding particles have the same betas independently from their position
- Beambeam could improve when all colliding particles have the same betas independently from their energy
- Due to beamstrahlung, particles lose energy at the IP \Rightarrow a large DA and MA at the IP is desirable moreover if the BW is not flat, they will also exercise betabeating further reducing the effective IP MA



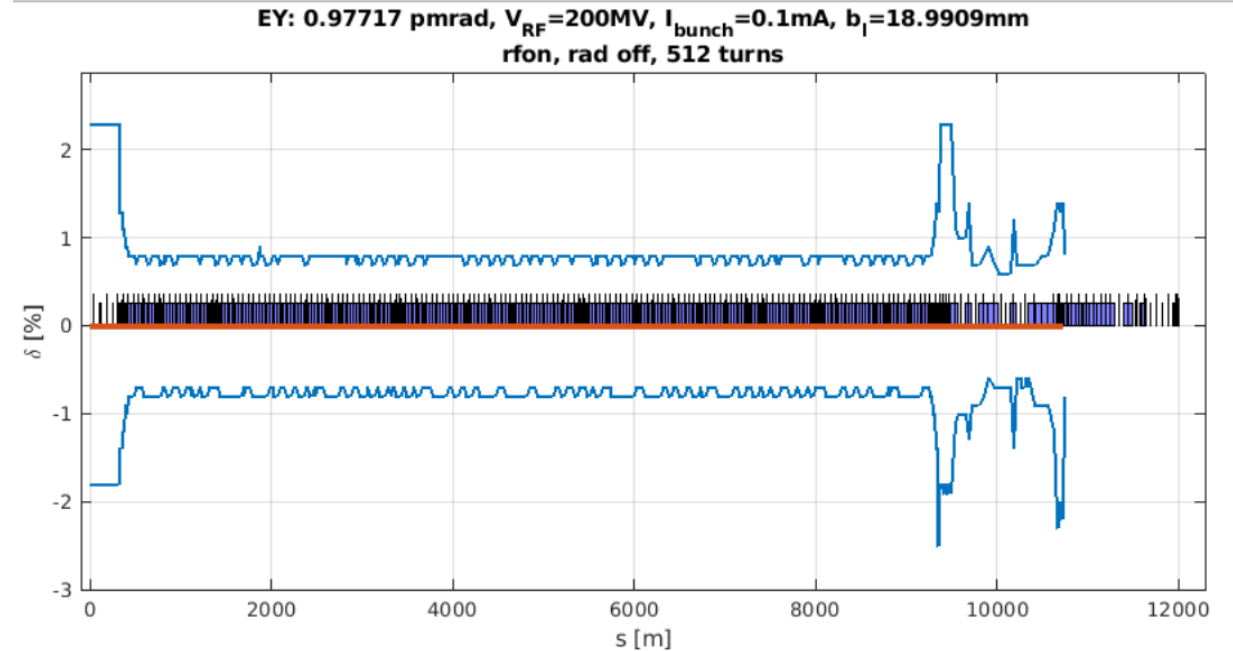
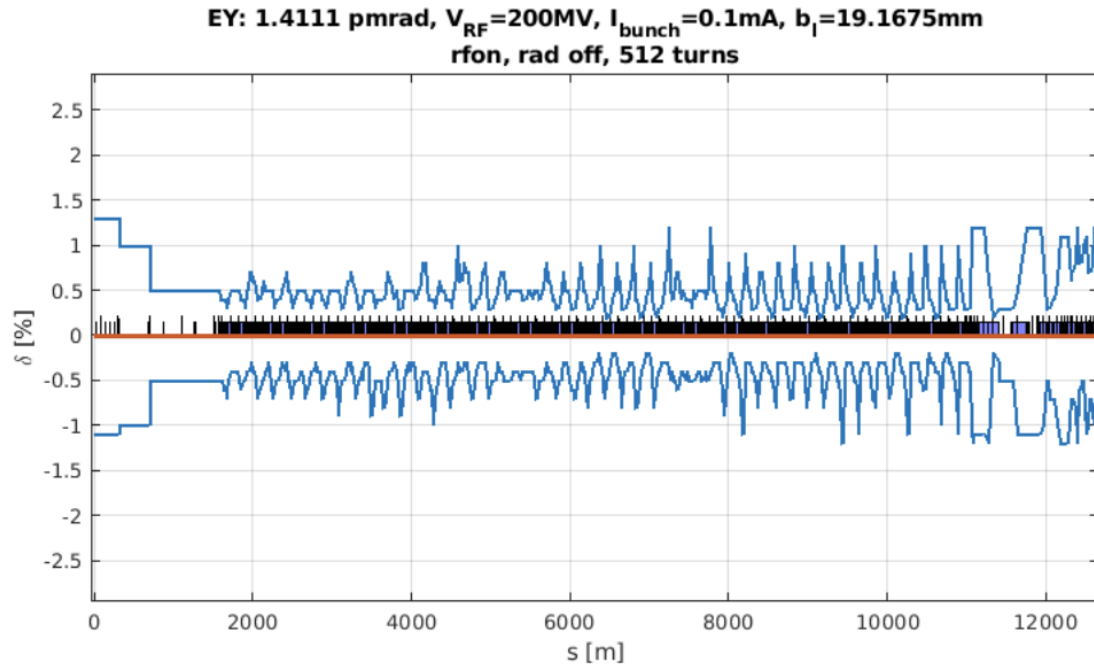
Dynamic aperture without and with Synchrotron Radiation for FCC Z-mode



Vertical DA and MA are the ultimate limit (optically) to the achievable betay* and luminosity

Local Momentum Acceptance no synchrotron Radiation (OPTIMISTIC)

LCCO 76



Small momentum acceptance locations have large impact on final Vacuum and Touschek Lifetime, LMA with errors further shrinks

It is also worrisome the local angular acceptance (not shown) that due to non linearities and errors can be extremely small, enhancing sensitivity to instabilities

Conclusions

- LPA&CW are very powerful tools to minimize and linearize BB forces
- LPA&CW have the potential of delivering high luminosity, but very challenging optics-related beam parameters are required, in particular low emittances and small IP-betas
- These parameters **should not come with a reintroduction of nonlinearities** that might diminish or negate the LPA&CW benefits
- We can build optics up to the challenge. We have tools and knowhow to develop solutions up to the task.
- LPA&CW has moved the optimum cross-over point between Circular Colliders and Linear Colliders by about 50-100GeV/CM. It cannot be excluded that novel solutions developed for the LCs can shift the balance again.

Conclusions: personal remarks

- LPA&CW concept has been developed based on analytical considerations
- A very small emittance, anharmonic and achromatic FODO has been developed by building the minimum periodic sequence with enough degrees of freedom to cancel the transverse detunings and second order chromaticity
- Final focus systems can be developed from concepts applied for design of SLC/NLC/SUPERB/etc. The methods to mitigate high order aberrations are determined and optimized analytically
- Optimizations based on computer algorithms are very effective for fine optimization of DA&MA, they are less effective in cases where they are used to optimize systems that have very large aberrations as starting with