



Quantum-limited amplification and photon counting based on Josephson photonics

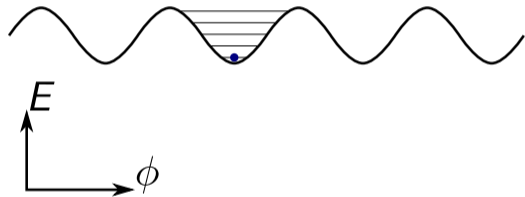
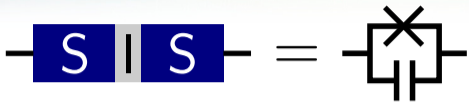
Max Hofheinz

Institut Quantique and GEG
Université de Sherbrooke

QMUL SNOLAB Quantum Workshop

SNOLAB, Sudbury, Jan 15-18, 2024

The Josephson junction in quantum circuits

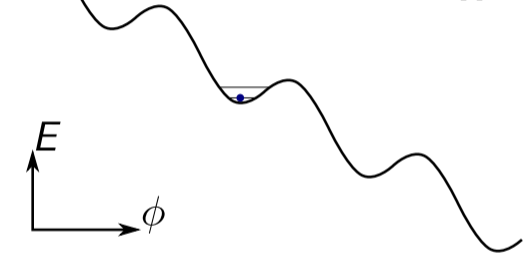


- Josephson junction forms anharmonic oscillator → qubit

$$H = -E_J \cos(\phi)$$

$$V = \frac{\hbar}{2e} \frac{d\phi}{dt}$$

The Josephson junction in quantum circuits

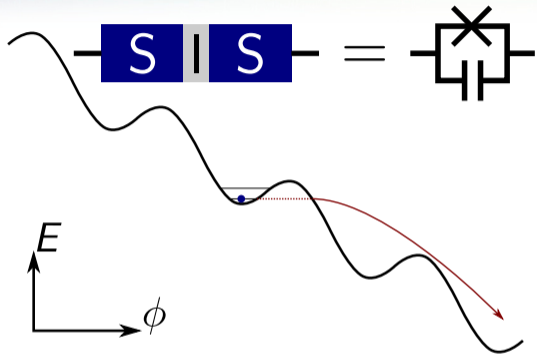


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- DC current tilts potential

$$H = -E_J \cos(\phi)$$

$$V = \frac{\hbar}{2e} \frac{d\phi}{dt}$$

The Josephson junction in quantum circuits



- Josephson junction forms anharmonic oscillator \rightarrow qubit
- DC current tilts potential
- current too high \rightarrow phase runs down potential
 - $V > 0$
 - energy gets dissipated somewhere
 - qubit is gone

$$H = -E_J \cos(\phi)$$

$$V = \frac{\hbar}{2e} \frac{d\phi}{dt}$$

Stay below the critical current!

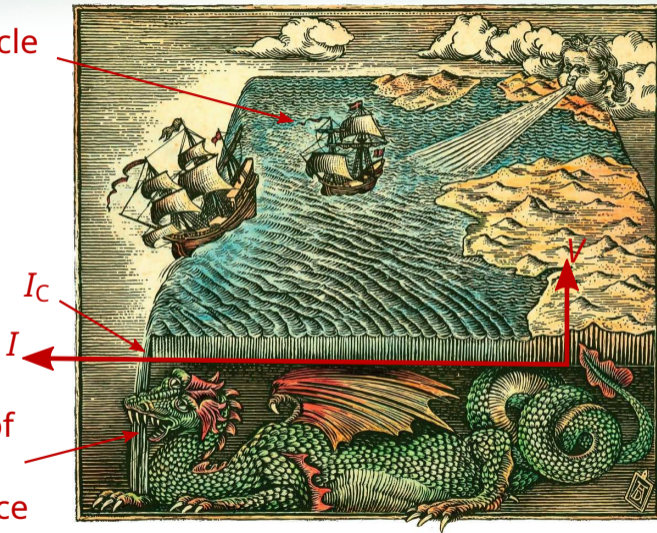
The Josephson junction in quantum circuits



Artist: Antar Dayal

The Josephson junction in quantum circuits

phase particle



monster of
instant
decoherence

Artist: Antar Dayal

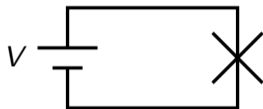
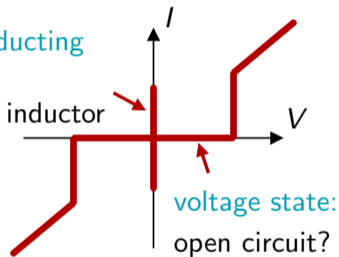
Voltage state of the Josephson junction: Semi-classical view



Josephson junction in the voltage state is also dissipationless!

superconducting
state:

nonlinear inductor



$$I = I_C \sin(\omega_J t)$$

$$\omega_J = \frac{2eV}{\hbar}, \quad I_C = \frac{2eE_J}{\hbar}$$

AC current but no DC
current

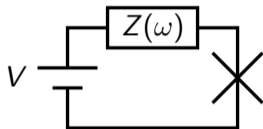
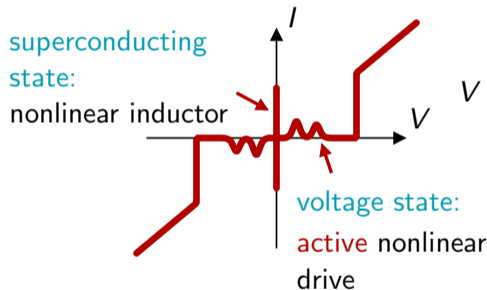
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Ingold, Nazarov, in Single Charge Tunnelling, cond-mat/0508728 (1992)

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$$I = I_C \sin(\omega_J t)$$

$$\omega_J = \frac{2eV}{\hbar}, \quad I_C = \frac{2eE_J}{\hbar}$$

Dissipated power

$$P = \text{Re } Z(\omega_J) \frac{I_C^2}{2}$$

Power is drawn from bias:

$$I = \frac{P}{V} = \frac{2e}{\hbar} \frac{\text{Re } Z(\omega_J)}{\omega_J} \frac{I_C^2}{2}$$

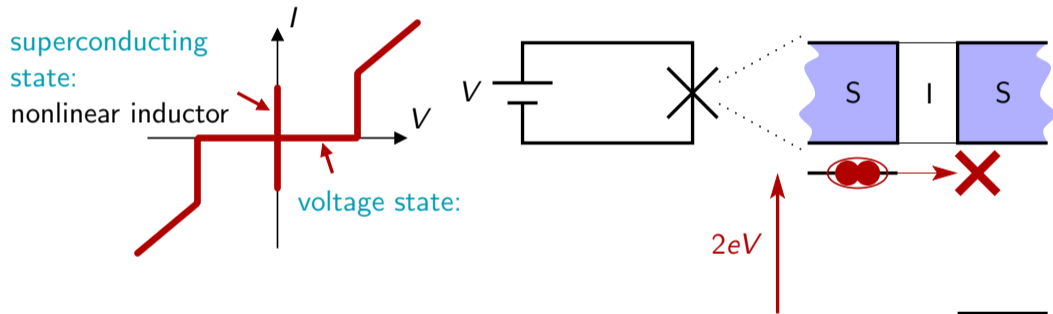
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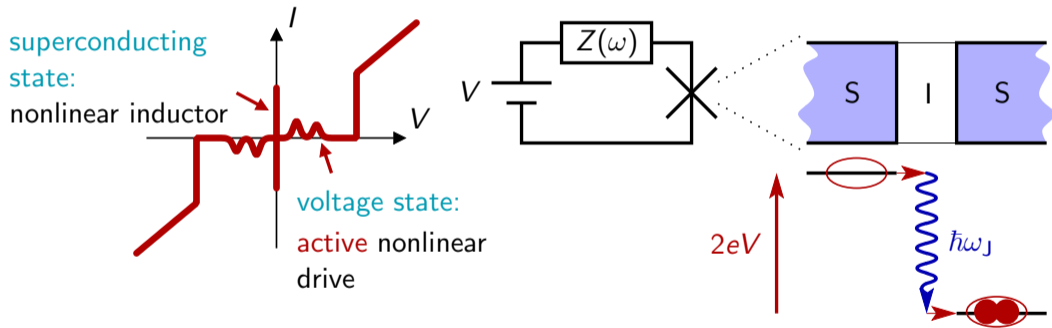


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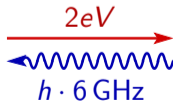
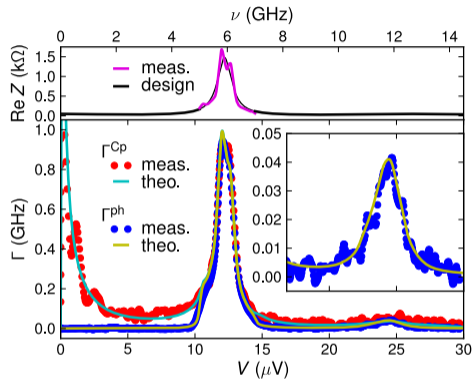
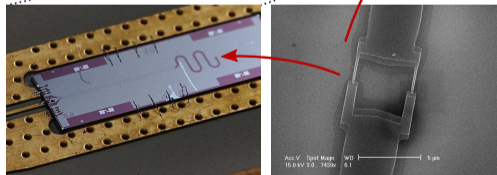
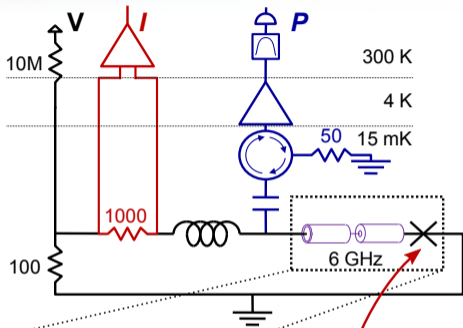
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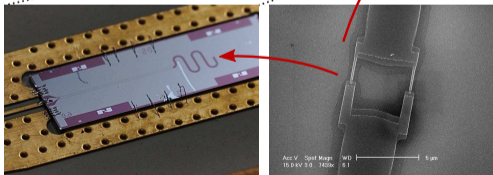
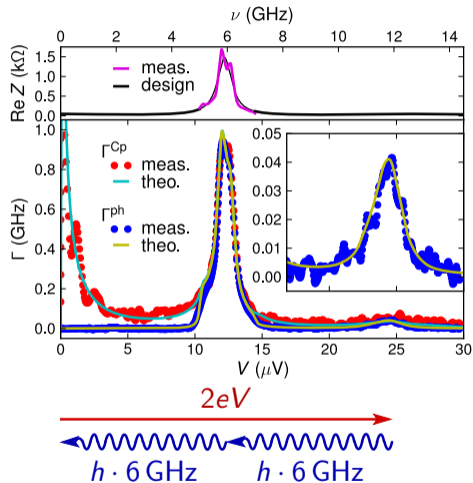
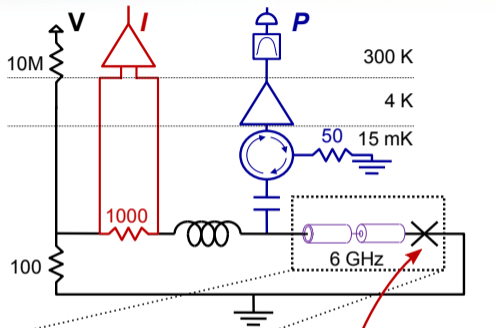
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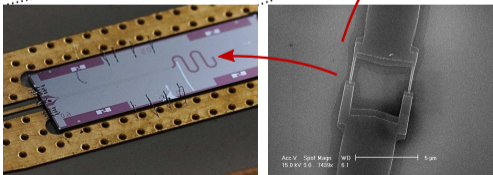
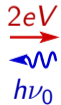
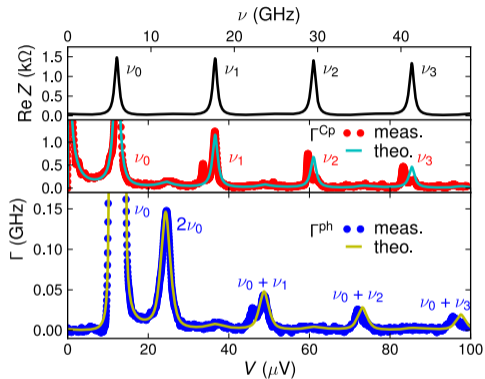
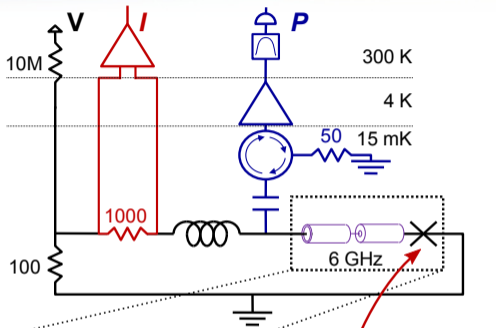
Bright side of inelastic Cooper-pair tunnelling



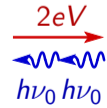
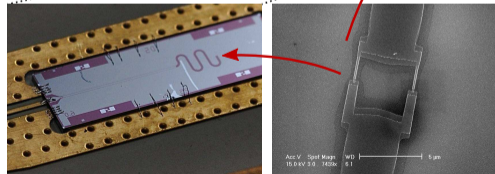
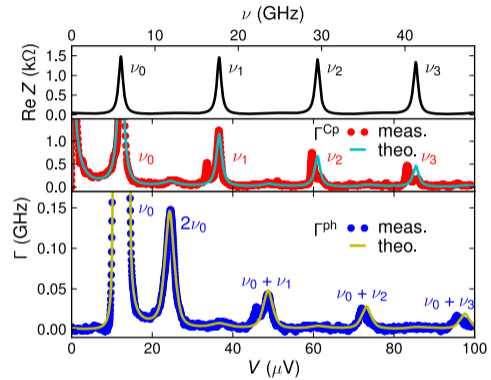
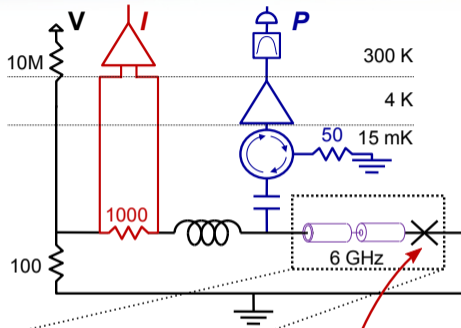
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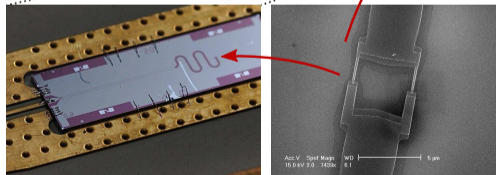
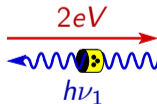
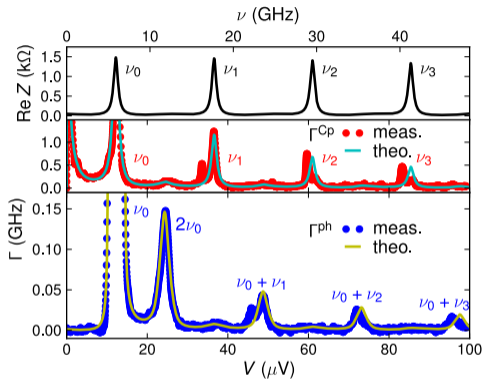
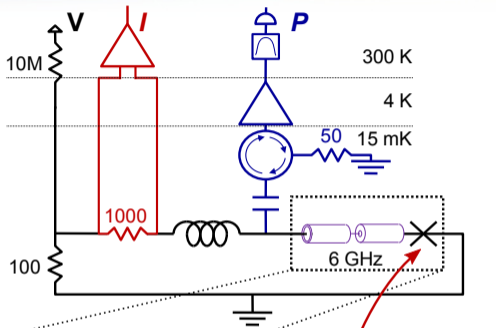


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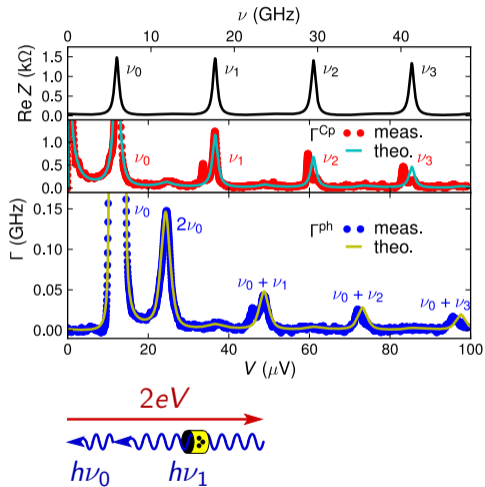
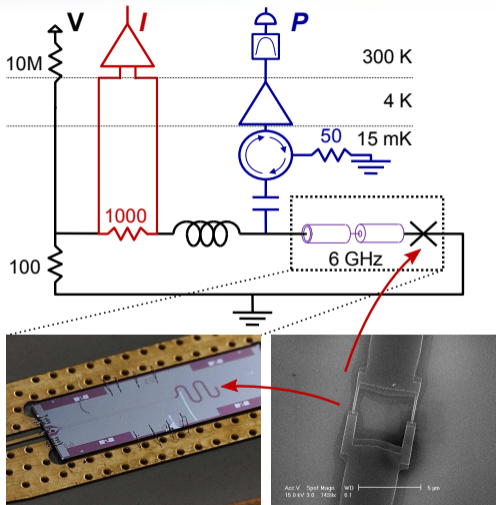


Hofheinz *et al.*, Phys. Rev. Lett. **106**, 217005 (2011)

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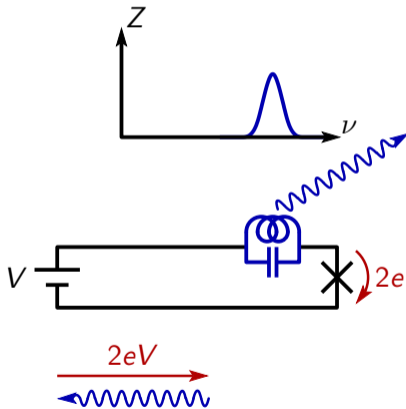


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Engineering $Z(\nu)$ \longrightarrow Full toolbox for wideband quantum microwave devices

Sources

- Coherent
- Single photons
- Entangled photons



Measurement

- Amplifiers
- Frequency shifters
- Photomultipliers

Holst *et al.*,
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Gramich *et al.*,
Phys. Rev. Lett. **111** 247002 (2013)

Chen *et al.*,
Phys. Rev. B **90**, 020506(R) (2014)

Cassidy *et al.*,
Science **355** 939 (2017)

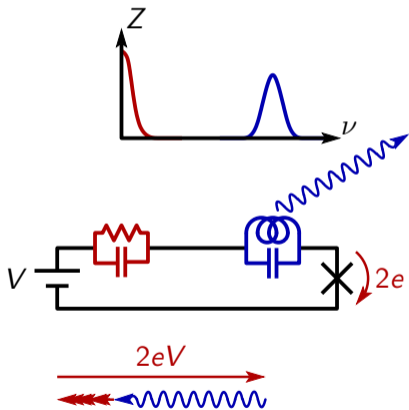
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Armour *et al.*,
Phys. Rev. B **91** 184508 (2015)

Dambach *et al.*,
Phys. Rev. B **92** 054508 (2015)

Souquet *et al.*,
Phys. Rev. A **93** 060301 (2016)

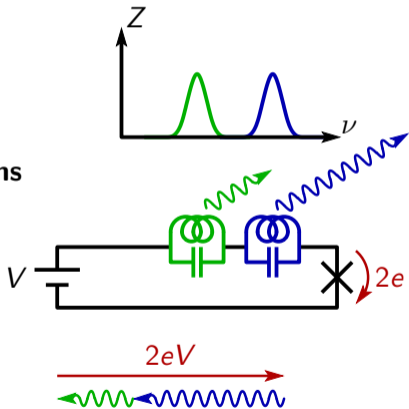
Grimm *et al.*,
Phys. Rev. X **9** 021016 (2019)

Rolland *et al.*,
Phys. Rev. Lett. **122** 186804 (2019)

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Phys. Rev. Lett. **110** 267004 (2013)

Trif *et al.*,
Phys. Rev. B **92** 014503 (2015)

Westig *et al.*,
Phys. Rev. Lett. **119** 137001 (2017)

Wood *et al.*,
prb **104** 155424 (2021)

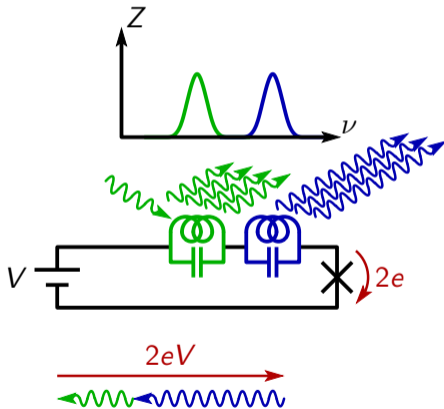
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Safi *et al.*,
Phys. Rev. B **84** 205129 (2011)

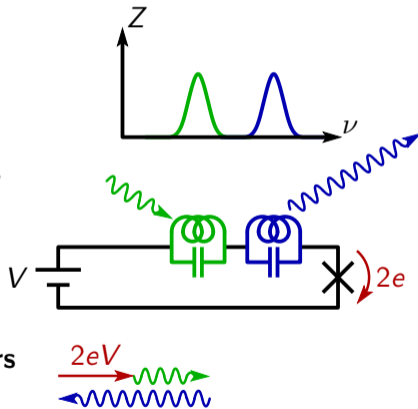
Lähteenmäki *et al.*,
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Jebari *et al.*,
Nat. Electron. **1** 223 (2018)

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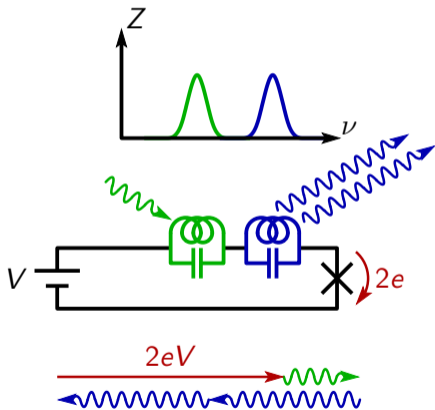
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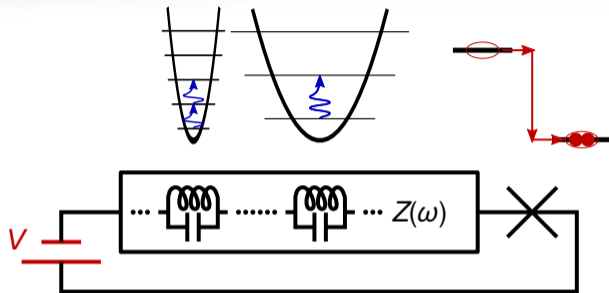
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Leppäkangas *et al.*,
Phys. Rev. A **97** 013855 (2018)

Albert *et al.*,
arxiv:2303.03173 (2023)

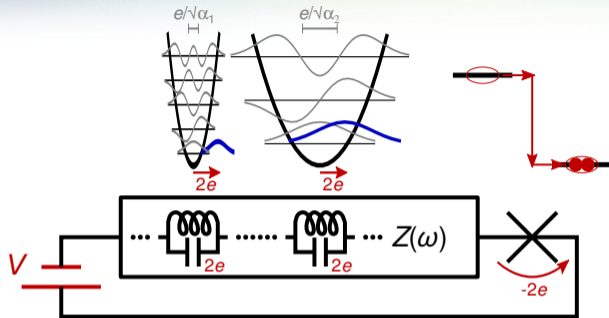
Inelastic Cooper pair tunnelling: Nonlinearity depends on impedance



$$\Gamma \propto E_J^2 \delta(2eV - n_1 \hbar \omega_1 - n_2 \hbar \omega_2 \dots)$$

- One or several modes can absorb $2eV$ as photons

Inelastic Cooper pair tunnelling: Nonlinearity depends on impedance



$$M_n^{(k)} = \left| \langle n | e^{i\sqrt{\alpha_k}(a+a^\dagger)} | 0 \rangle \right|^2 = \frac{\alpha_k^n e^{-\alpha_k}}{n!}$$

$$\alpha_k = \pi \frac{4e^2}{h} Z_k$$

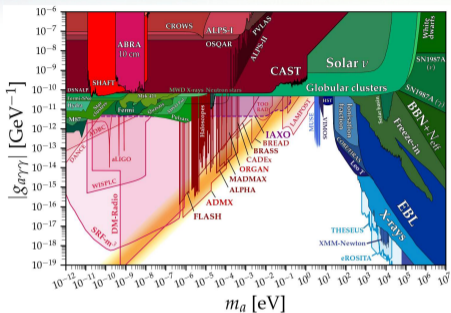
$\sqrt{\alpha_k}$: 0-point phase fluctuations

$$\Gamma \propto E_J^2 M_{n_1}^{(1)} M_{n_2}^{(2)} \dots \delta(2eV - n_1 \hbar \omega_1 - n_2 \hbar \omega_2 \dots)$$

- One or several modes can absorb $2eV$ as photons
- Z_k determine **how** $2eV$ is split up into photons

$Z(\omega)$ can be engineered, V controlled \rightarrow very versatile

Quantum measurement devices for THz blind spot



Ciaran O'Hare, cajohare.github.io/AxionLimits

Gap frequencies $2\Delta/h$

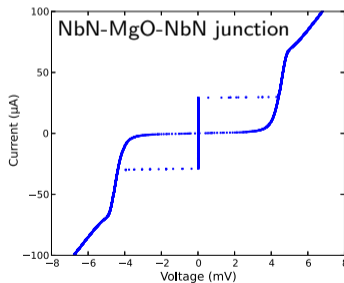
Al: 90 GHz

Nb: 700 GHz

NbN: 1.2 THz

Josephson photonics at high frequency

- no microwave pump needed
- Josephson inductance cancels
- Frequency only limited by gap



Grimm *et al.*, *Supercond. Sci. Technol.* **30** 105002 (2017)

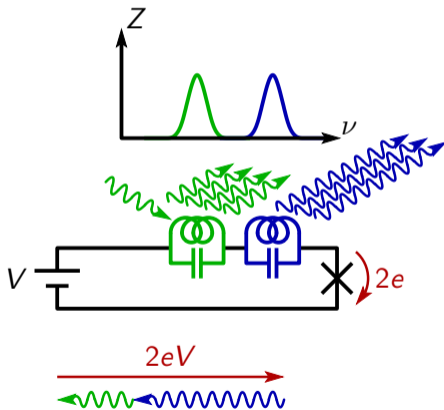
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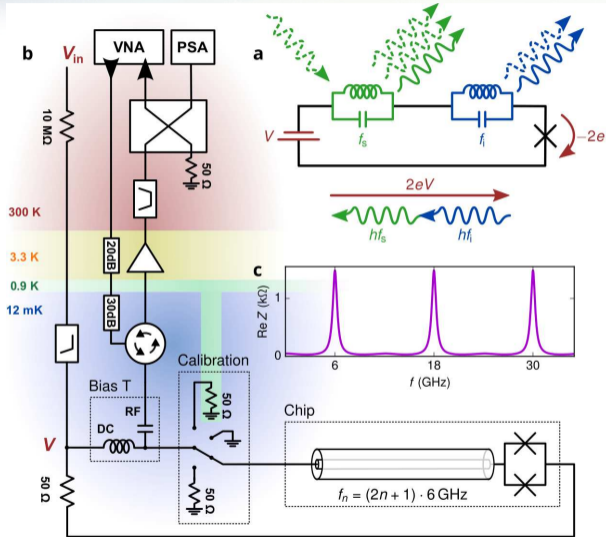


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Weak nonlinearity: Amplification

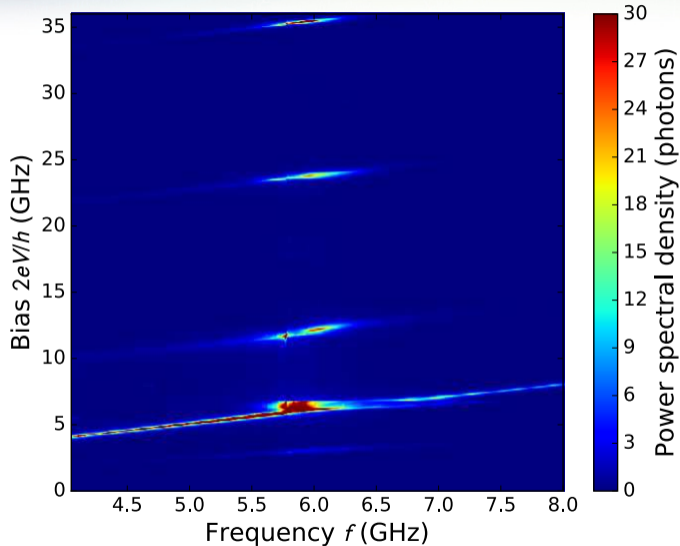


- send signal to one of the modes
- chose any mode as idler
- bias at the sum of the two modes
- Adjust E_J for large gain:

$$\frac{E_J}{\hbar \sqrt{\Gamma_a \Gamma_b}} \pi \frac{4e^2}{h} \sqrt{Z_a Z_b} \rightarrow 1$$

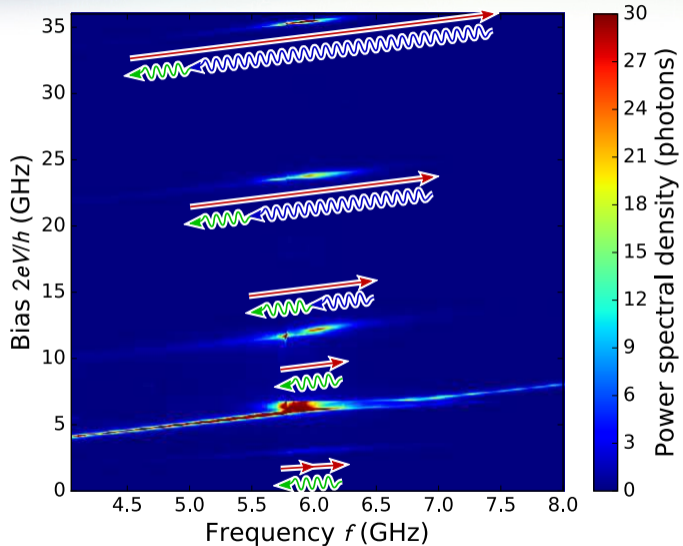
➔ quantum limited amplification?

Power spectral density



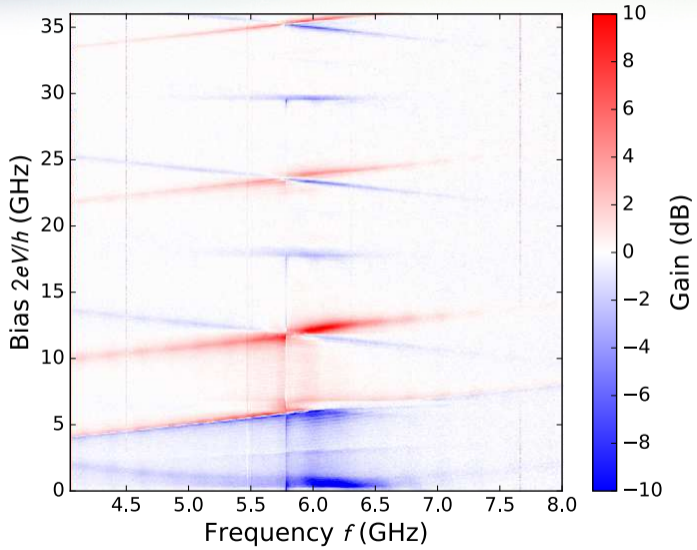
- Same sample as in the beginning
- Resolve photon emission rate in frequency

Power spectral density

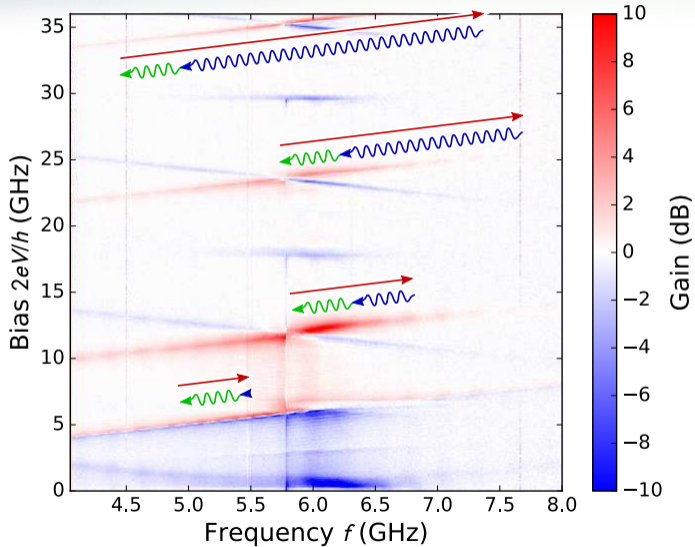


- Same sample as in the beginning
- Resolve photon emission rate in frequency
- ➔ Amplifier noise

Gain

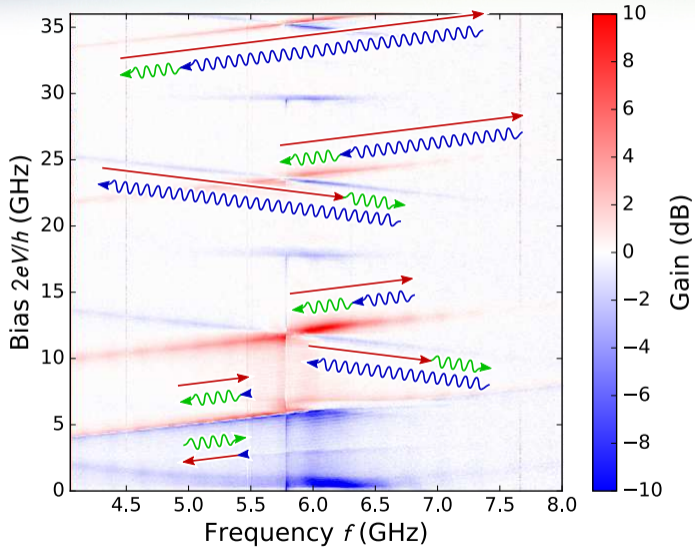


Gain



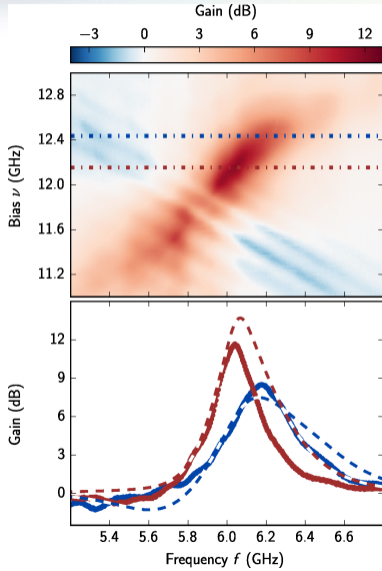
- Down conversion
- ➔ Gain

Gain



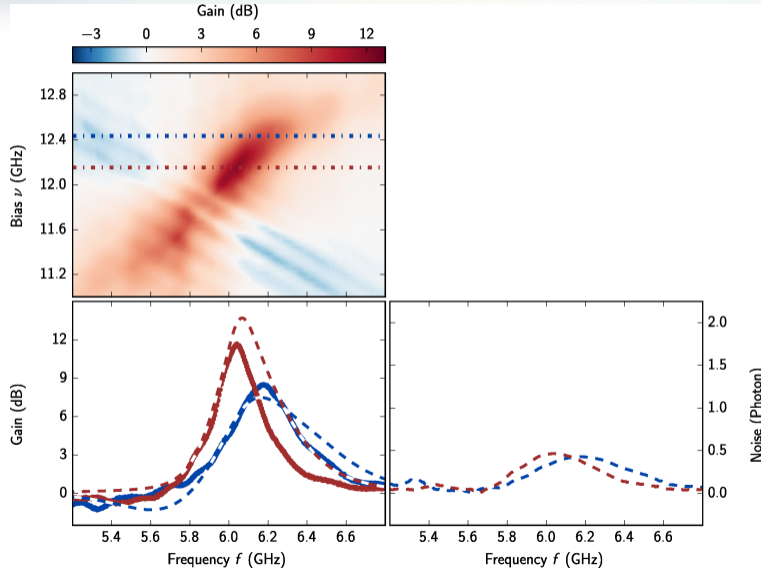
- Down conversion
- ➔ Gain
- Frequency conversion
- ➔ Loss

Amplification close to the quantum limit



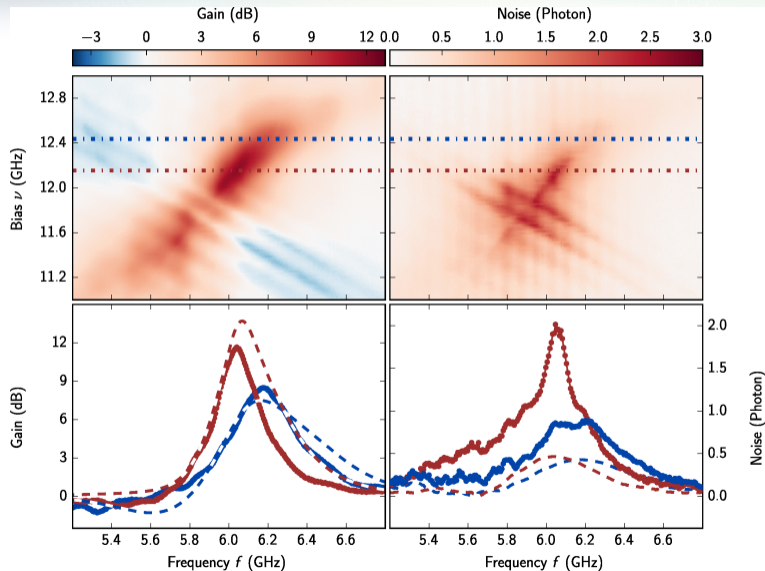
- Gain > 10 dB for sample not designed as amplifier
- Qualitatively explained by $P(E)$ theory

Amplification close to the quantum limit



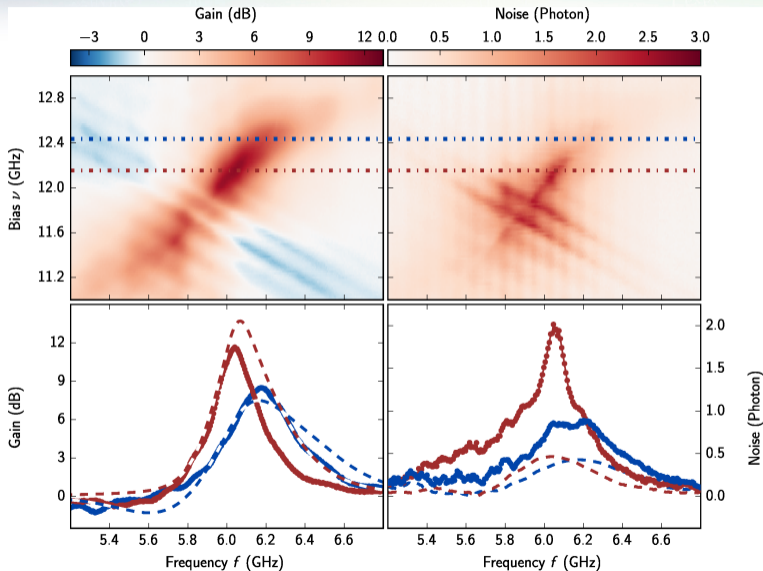
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Amplification close to the quantum limit



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- Qualitatively explained by $P(E)$ theory
- quantum limit $\frac{1}{2}|1 - G^{-1}|$
- Best noise $\sim 2 \times \text{QL}$

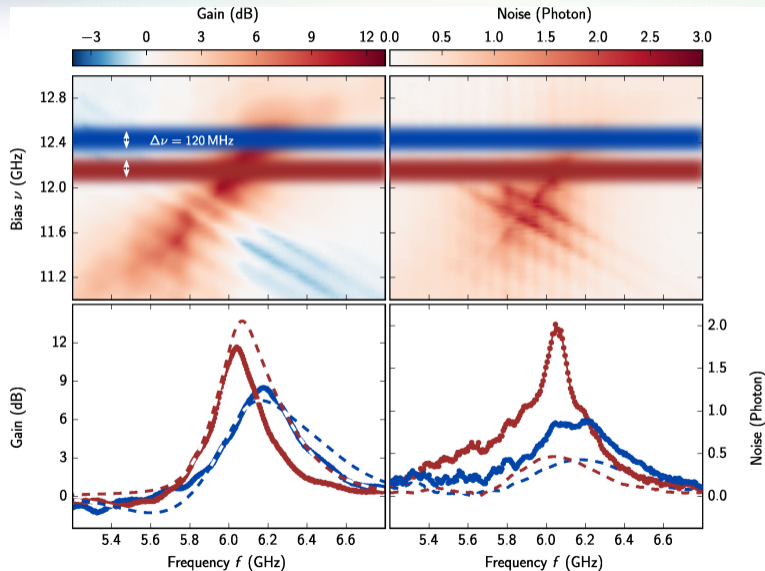
Amplification close to the quantum limit



Limited performance

- Gain limited to ~ 10 dB
- Best noise $\sim 2 \times$ QL

Amplification close to the quantum limit



Limited performance

- Gain limited to ~ 10 dB
- Best noise $\sim 2 \times$ QL

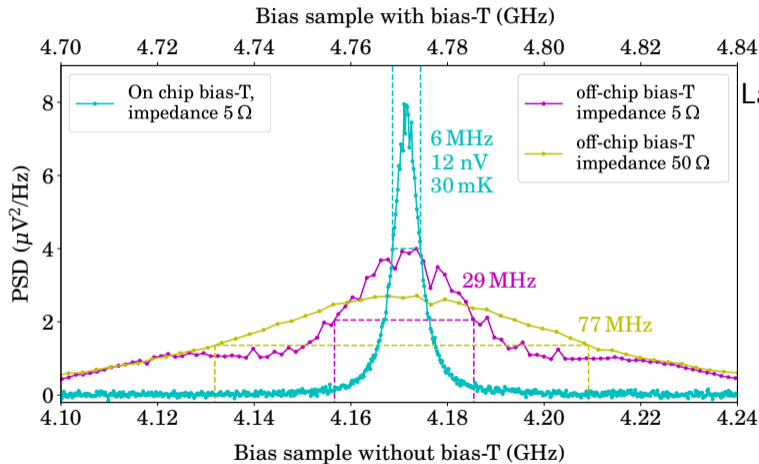
Reason: Pump fluctuations $\Delta\nu$

- JPA: $\sim 1 \mu\text{Hz}$
- ICTA: ~ 100 MHz

Optimize:

- reduce voltage noise
- increase bandwidth

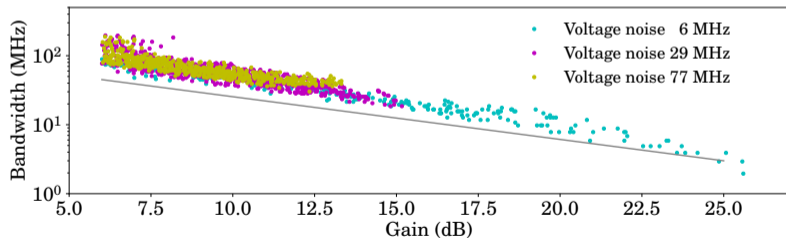
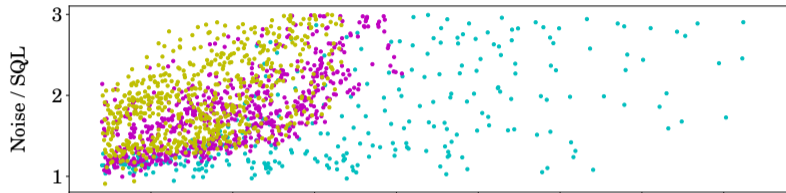
Reducing voltage-bias noise



Last filter generation:

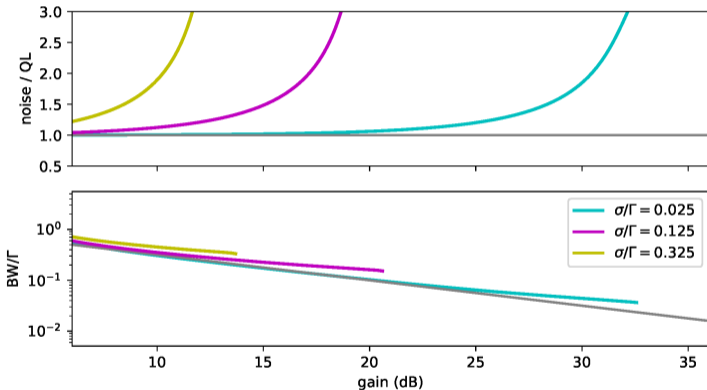
- 3 discrete *LCR* stages + silver epoxy
- 700 Hz cutoff
- flat 5Ω output impedance up to 1 GHz
- effective temperature of 5Ω : 30 mK

Gain is limited by voltage noise



- Various operating points
- BW reduces with gain:
 $\sqrt{G} \cdot BW = \Gamma$
- Need bandwidth $>$ voltage noise
- Lower bias noise allows for larger gain

JPA theory with adiabatic pump-frequency fluctuations



- JPA theory on resonance
- average over Gaussian distribution of bias frequency
- qualitative agreement with measurement

Inelastic Cooper-pair Tunneling Amplifier



- Near quantum limited noise
- GBWP ~ 100 MHz
- $P_{\text{in}}^{-1\text{dB}} \sim -130$ dBm @ $G = 20$ dB
- Less hardware overhead
- No pump tone required
- Should work to gap of superconductor
- Expect easy scaling to mm-wave to THz frequencies

Jebari *et al.*, Nat. Electron. **1** 223 (2018)



Salha Jebari



Florian Blanchet



Ulrich Martel



Naveen Nehra

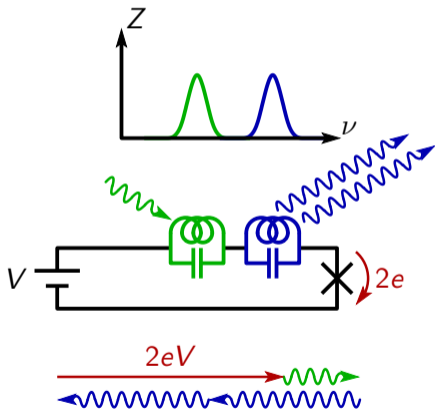
Engineering $Z(\nu)$ \longrightarrow Full toolbox for wideband quantum microwave devices

Sources

- Coherent
- Single photons
- Entangled photons

Measurement

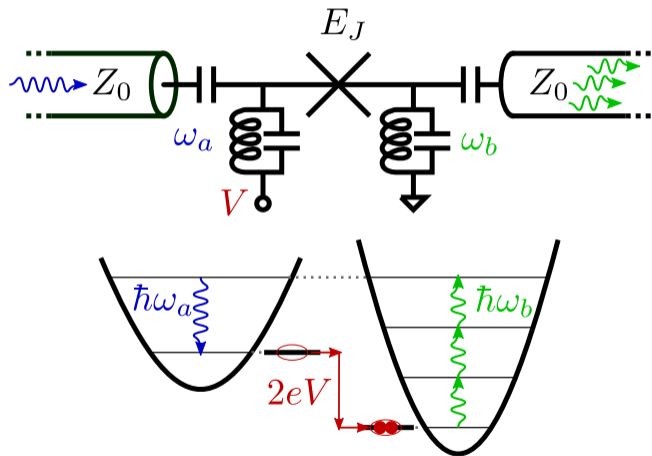
- Amplifiers
- Frequency shifters
- **Photomultipliers**



Leppäkangas *et al.*,
Phys. Rev. A **97** 013855 (2018)

Albert *et al.*,
arxiv:2303.03173 (2023)

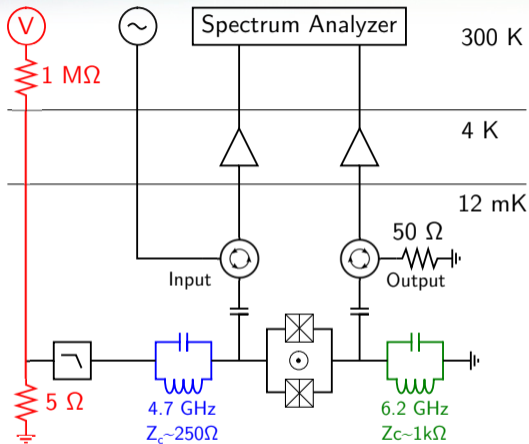
Strong nonlinearity: Photomultiplication



- spontaneous tunneling forbidden
- incident photon provides energy complement
- ➔ tunneling creates several photons in other mode
- process involving ≥ 3 photons
- need $Z_{\text{out}} \sim 2 \text{ k}\Omega$
- adjust E_J to cancel reflection

Leppäkangas et al., Phys. Rev. A **97** 013855 (2018)

Device



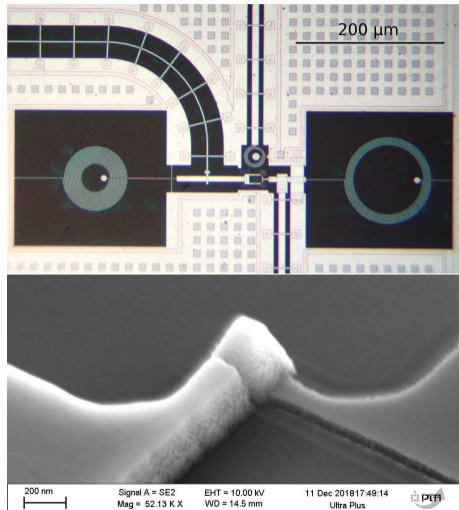
300 K

4 K

12 mK

Josephson junctions: Nb/Al/AIO_x/Nb

Albert *et al.*, arxiv:2303.03173



200 nm

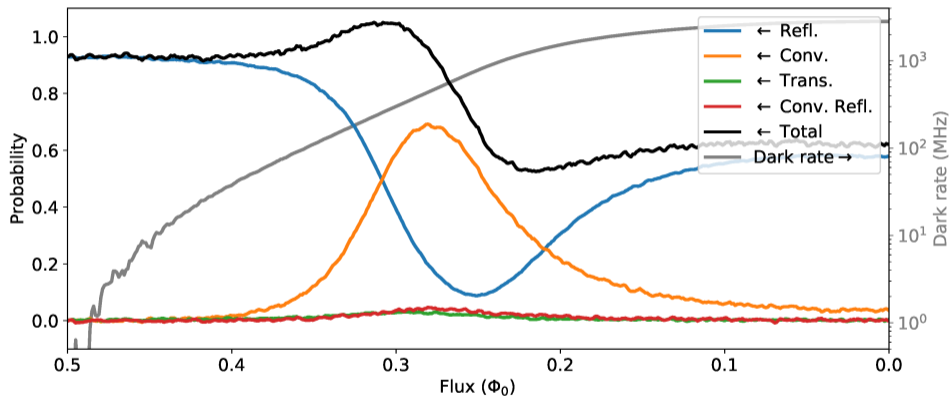
Signal A = SE2
Mag = 52.13 K X

EHT = 10.00 kV
WD = 14.5 mm

11 Dec 2018 17:49:14
Ultra Plus

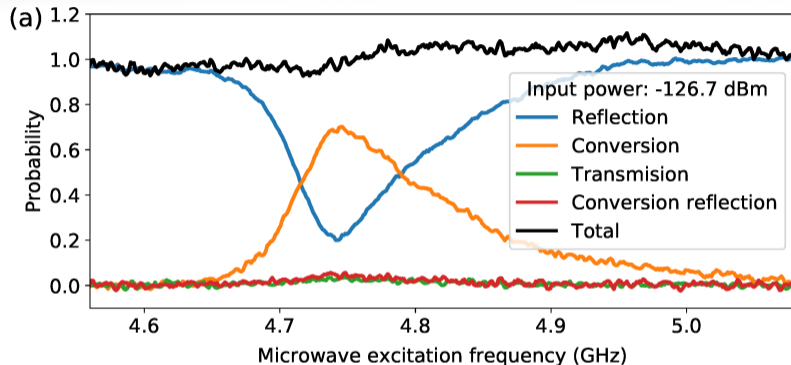


Conversion 1 → 3



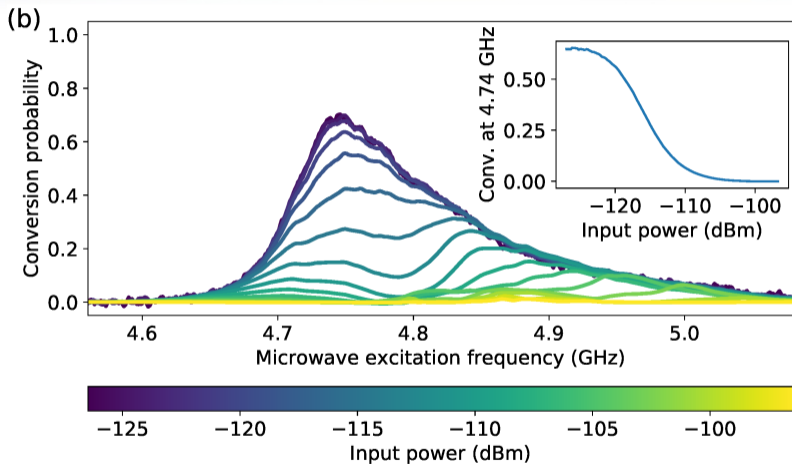
Input frequency	Input power	Bias $2eV/h$
4.74 GHz	-127 dBm	13.37 GHz

Bandwidth



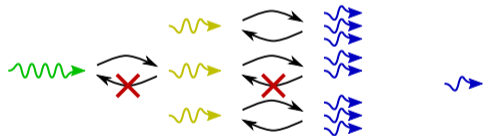
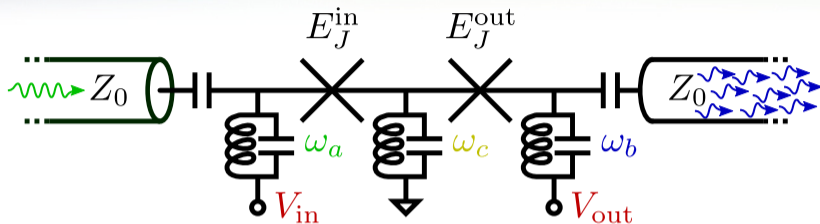
Input power: -127 dBm
Bias voltage: 13.37 GHz
SQUID flux : $0.28 \phi_0$
Efficiency : 73%

Power handling



Bias voltage: 13.37 GHz
SQUID flux : $0.28 \phi_0$
Efficiency : 73 %
Saturation : 6 photons

Cascaded photomultipliers \rightarrow single photon detector



- photon is either fully converted or reflected
- impedance matching by tuning one Josephson energy
- need 2 to 3 stages followed by quantum limited amplifier
- number resolving, no dead time

Leppäkangas *et al.*, Phys. Rev. A **97** 013855 (2018)

Photomultiplier

Where we are at:

- ✓ linear to a few photons
- ✓ 0.7 quantum efficiency
- ✗ high dark rate

Reasons for high dark rate:

- high $E_J \rightarrow$ non RWA processes
- understood with $P(E)$

Much lower dark rate with:

- lower BW
- $2eV < \hbar\omega_{\text{output}}$
- preliminary result: < 1 MHz



Juha Leppäkangas



Romain Albert



Joël Griesmar



Nicolas Bourlet

Leppäkangas *et al.*, Phys. Rev. A **97** 013855 (2018)

Albert *et al.*, arxiv:2303.03173, to appear in Phys. Rev. X

Quantum measurement devices based on Josephson photonics

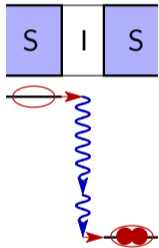
10

New physics

- Nonlinearity in drive, not modes
- very open systems

Useful devices

- no microwave pump needed
- very strong nonlinearities
- not limited by plasma frequency
- good candidate for covering THz blind spot in quantum measurement



Jebari *et al.*, Nat. Electron. **1** 223 (2018)

Albert *et al.*, arxiv:2303.03173, to appear in Phys. Rev. X

