

# $\bar{B} \rightarrow D^{(*)} \ell^{-} \bar{\nu}_{\ell}$ with hadronic tagging at *BaBar*

[PRL123,091801(2019) + PRD110,032018(2024)]

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(on behalf of the BaBar Collaboration)

Challenges in SL decays, Vienna 2024



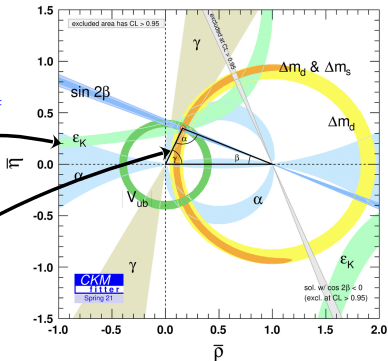
# $|V_{ub}|$ AND $|V_{cb}|$ : FLAGSHIP SM VARIABLES

- $V_{xb}$  play critical roles in the unitarity test of  $V_{\text{CKM}}$ .

$$\begin{bmatrix} 1 - \frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{bmatrix} \begin{matrix} V_{ub} \\ V_{cb} \sim 0.04 \\ 1 \end{matrix}$$

Unitarity  
Triangle

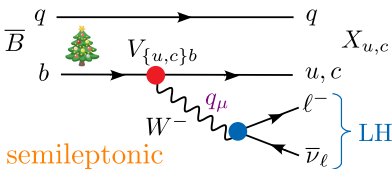
$$\left\{ \begin{array}{l} \varepsilon_K \propto |V_{cb}|^4 \\ \frac{|V_{ub}|}{|V_{cb}|} \sim (10 \pm 0.4)\% \end{array} \right.$$



- $\sin 2\beta$  (loop) known to better than **2%**. Side opposite is  $|V_{ub}|/|V_{cb}|$ .
- Rare FCNC processes  $\propto |V_{cb}|^2 \left[ \frac{|V_{tb}^* V_{ts}|^2}{|V_{cb}|^2} \right] \Rightarrow$  theory **uncertainty**.

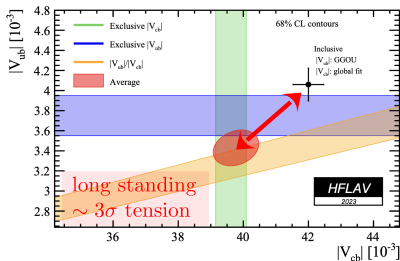
# $|V_{ub}|$ AND $|V_{cb}|$ : PROCEDURES AND TENSIONS

- Leptonic  $B_{u,c}^+ \rightarrow \tau^+ \nu_\tau$ : theoretically clean, experimentally hard.



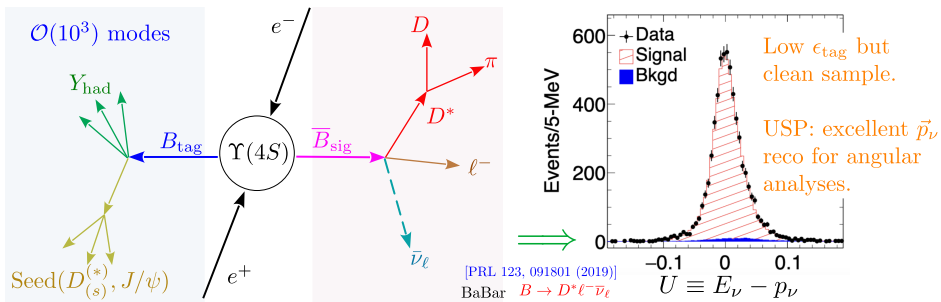
- $X_{u,c}$  is **exclusive**  $\{\pi, \rho, \omega, D^{(*)} \dots\}$ . Or **inclusive** sum of states.
- Different theory inputs: **OPE** (inclusive) and **FFs** (exclusive)

- Exclusive** systematically **lower** than **inclusive** for both  $|V_{ub}|$  and  $|V_{cb}|$  by  $\sim 5-10\%$ .
- QCD effects, experimental issues with the normalizations, or NP?



## TO TAG OR NOT TO TAG

- $\mathcal{B}(B \rightarrow X_c \ell^- \bar{\nu}_\ell) \sim 10\%$ . Dominant statistics, but at least one **missing neutrino**  $\Rightarrow$  hadronic tagging at  $e^+e^-$  machines.



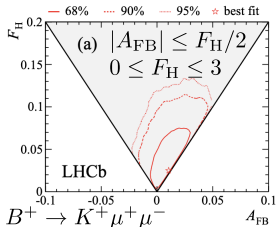
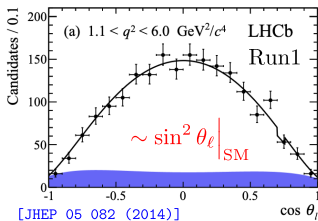
- BF measurements: **calibration** for ensuring  $\frac{\epsilon_{\text{tag}}^{\text{MC}}|_{\text{signal}}}{\epsilon_{\text{tag}}^{\text{MC}}|_{\text{control}}} \rightarrow 1$  is **hard**.

$$B \rightarrow D\ell^-\bar{\nu}_\ell$$

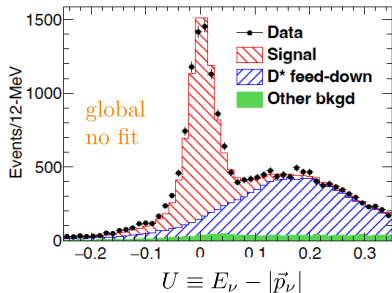
[PRD110,032018 (2024)]

$\bar{B} \rightarrow D\ell^{-}\bar{\nu}_\ell$ : ORIGINAL MOTIVATION

- $\theta_\ell$ : lepton helicity angle in  $W^*$  rest frame.
- In SM, (almost) pure  $\sin^2 \theta_\ell$ . Look for non-zero  $F_H$  and  $A_{FB}$  in rate  $\propto \frac{3}{4}(1 - F_H) \sin^2 \theta_\ell + \frac{1}{2}F_H + A_{FB} \cos \theta_\ell \Rightarrow$  SM null test.
- Done at LHCb for  $B^+ \rightarrow K^+ \mu^+ \mu^-$ .
- Unfortunately, BABAR MC generated as  $\sin^2 \theta_\ell$ . Needed flat.



- $\bar{B} \rightarrow D\ell^{-}\bar{\nu}_\ell$  at BABAR complicated by the fact it's a sum of 10 different modes with independent backgrounds, angular efficiencies.

$B \rightarrow D\ell^{-}\bar{\nu}_\ell$ : ANALYSIS SETUP

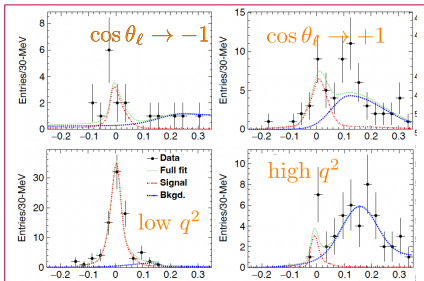
- Full *BABAR* dataset w/ hadronic tagging. **2d problem** in  $\{q^2, \cos\theta_\ell\}$ .

$$\frac{d\Gamma}{dq^2 d\cos\theta_\ell} = \frac{G_F^2 |V_{cb}|^2 \eta_{EW}^2}{32\pi^3} k^3 |f_+(q^2)|^2 \sin^2\theta_\ell$$

- Rate factorizes but efficiency and background **correlated**.

- $U$ -fit in “**local**” phase-space for given event. Signal/background *shapes* from sim.  $\Rightarrow$  data-driven.

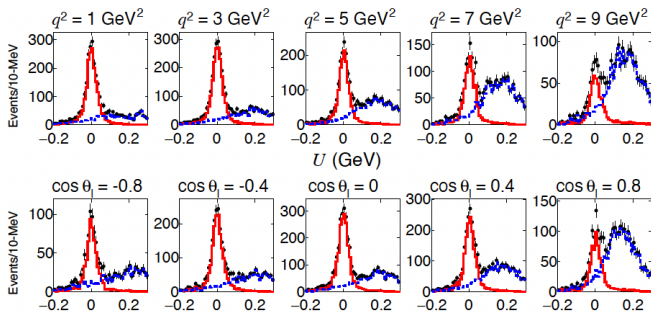
- **Event-wise weight** for unbinned angular fits:  $Q_i = \frac{\mathcal{S}(U_i)}{\mathcal{S}(U_i) + \mathcal{B}(U_i)}$



## BKGD SUBTRACTION VALIDATIONS

$\ell^-$	$D$	decay mode	mode	$N_{\text{sig}}$	$N_{\text{bkgd}}$
$e^-$	$D^0$	$K^-\pi^+\pi^0$	0	539	63
			1	813	196
			2	550	82
$e^-$	$D^+$	$K^-\pi^+\pi^+\pi^0$	3	721	41
			4	204	120
			5	433	64
$\mu^-$	$D^0$	$K^-\pi^+\pi^0$	6	798	221
			7	608	84
			8	665	55
$\mu^-$	$D^+$	$K^-\pi^+\pi^+\pi^0$	9	233	134
			Total	5563	1061

- Lineshapes also depend on the reconstruction mode. **10 modes** are treated independently.
- Unlike sWeights, tracks **correlations** between sig/bkgd lineshapes and phase-space.



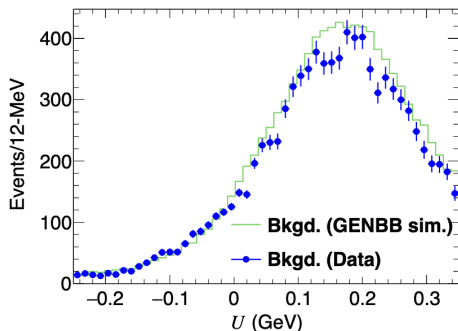
- 1d projections, integrated over all 10 modes.

signal  $Q_i$  background  $(1 - Q_i)$   $U$  (GeV)



## BKGD SUBTRACTION VALIDATIONS (CONTD.)

- Final  $Q$ -value fits done with signal/bkgd. shapes fixed from MC. Within statistics, seen to work quite well. Variations as systematic uncertainties.
- Comparison between final data and generic  $B\bar{B}$  MC bkgd.:



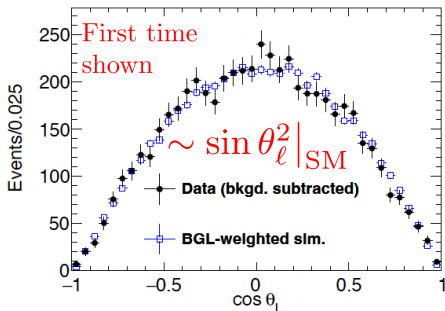
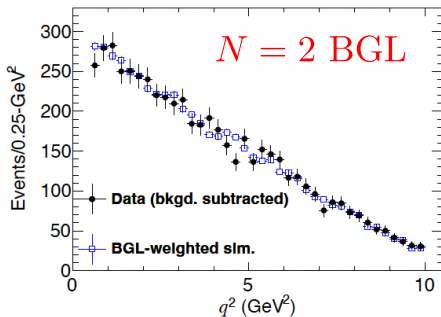
- Fortunately the MC does a pretty good job in getting the shapes right.
- Detailed mode-by-mode fit validations in fine 1-d bins in  $\{q^2, \cos\theta_\ell\}$  performed.

# BGL FIT RESULTS

- **BGL**  $z$ -expansion fit to form-factors  $f_+$  and  $f_0$  ( $\rightarrow$  lattice, time-like)

$$f_i(z) = \frac{1}{P_i(z)\phi_i(z)} \sum_{n=0}^N a_n^i z^n$$

- **Unbinned** non-extended ML **2d fits** to **BABAR** + **FNAL/MILC** (Gaussian constraints).  $d\Gamma/dq^2$  from **Belle-16** optionally included.
- Independent  $[-2\ln\mathcal{L}]_k$  from 10 modes summed at the end.

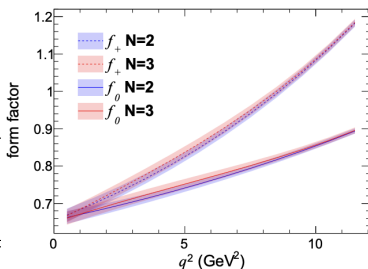


EFFECT OF  $N = 3$  BGL EXPANSION

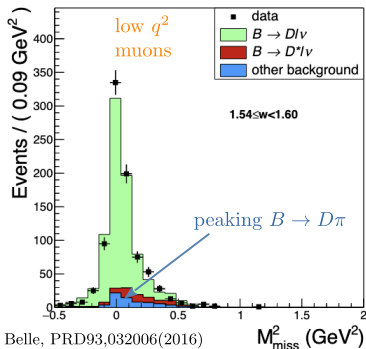
- From FF fits,  $\Gamma' = \frac{\Gamma_{\text{tot}}^{\text{FF}}}{|V_{cb}|^2} \cdot |V_{cb}| = \sqrt{\frac{\mathcal{B}}{\Gamma' \tau_B}}$  from updated HFLAV  $\mathcal{B}$ .

- BGL fits using BABAR, FNAL/MILC, Belle-16 ( $d\Gamma/dq^2$  shape):

$\mathcal{B}$ measurement (had-tagged)	$ V_{cb}  \times 10^3$	
	$N = 2$	$N = 3$
BaBar-10, $B^0$	$40.02 \pm 1.76$	$39.60 \pm 1.76$
BaBar-10, $B^+$	$38.67 \pm 1.41$	$38.25 \pm 1.42$
Belle-16, $B^0$	$41.66 \pm 1.21$	$41.22 \pm 1.24$
Belle-16, $B^+$	$41.27 \pm 1.22$	$40.82 \pm 1.25$
	$R(D)$ SM prediction	
	$0.2983 \pm 0.0047$	$0.2995 \pm 0.0038$



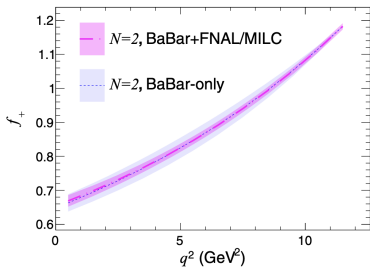
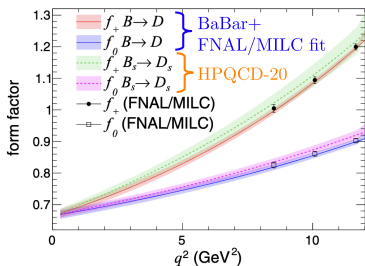
- For  $B \rightarrow D$ , mild effect of going from  $N = 2$  to  $N = 3$  BGL.
- Summer-23 tension in  $R(D) \sim 2\sigma$ .

COMMENT: LOW  $q^2$  DIFFICULTY

- Clean, but very hard to get rid of the small remnant background in a data-driven fashion (no sidebands).
- In particular, for muon mode,  $B \rightarrow D\pi$  shows up as peaking.
- Possible (Belle II) to PID-substitute  $\mu \rightarrow \pi$  and kin. fit to nothing missing.

- Current *BABAR* analysis, we trim at  $q^2 > 0.5 \text{ GeV}^2$  to avoid this peaking component.

## COMMENT: COMPARISONS W/ LATTICE



- (Qualitatively) good agreement with **flavor SU(3)** in comparison with HPQCD.
- Also good agreement in the FF shape for  $f_+$ , between *BABAR*-only and *BABAR* +FNAL/MILC fit.
- **Full covariance matrices** of BGL fits included in the PRD as supplementary material.

COMMENT: EXCLUSIVE  $|V_{cb}|$  FROM  $B \rightarrow D$ 

Measurement	$\mathcal{B}(\bar{B} \rightarrow D\ell^-\bar{\nu}_\ell) \times 10^2$	$ V_{cb}  \times 10^3$
<i>BABAR</i> -10 [14]	$\mathcal{B}_{B^0} = (2.15 \pm 0.11 \pm 0.14)$	$40.02 \pm 1.76$
<i>BABAR</i> -10 [14]	$\mathcal{B}_{B^+} = (2.16 \pm 0.08 \pm 0.13)$	$38.67 \pm 1.41$
Belle-16 [15]	$\mathcal{B}_{B^0} = (2.33 \pm 0.04 \pm 0.11)$	$41.66 \pm 1.22$
Belle-16 [15]	$\mathcal{B}_{B^+} = (2.46 \pm 0.04 \pm 0.12)$	$41.27 \pm 1.23$

- This analysis relies on external BF input for the overall  $|V_{cb}|$  normalization.
- Belle-16 systematically higher than *BABAR*-10

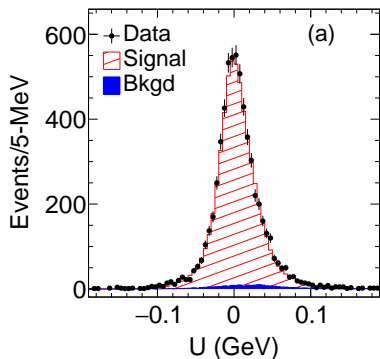
- Naively,  $B \rightarrow D |V_{cb}|$  is closer to inclusive+in **tension with  $B \rightarrow D^*$** , but this being driven by the Belle-16 input.
- Note: prelim. Belle II untagged  $B \rightarrow D |V_{cb}| = (38.3 \pm 1.2) \times 10^{-3}$  (FPCP'23).

$$B \rightarrow D^* \ell^- \bar{\nu}_\ell$$

[PRL123,091801(2019) + 24XX.XXXX]

# RECAP OF BABAR-19 $B \rightarrow D^*$ PAPER [PRL123, 091801 (2019)]

- First full 4-d  $\bar{B} \rightarrow D^* \ell^- \bar{\nu}_\ell$  angular analysis with **hadronic tagging**.
- As for  $B \rightarrow D$ , overall normalization from external BF.



- Extremely clean. Background subtraction from generic  $B\bar{B}$  MC + systematic.
- $\sim 6000$  signal events.  $N = 2$  (linear) BGL fit adequate.  
 $|V_{cb}| = (38.36 \pm 0.90) \times 10^{-3}$ .
- Negligible effect on extracted  $|V_{cb}|$  between **BGL** and **CLN** FF parameterisations.



# HQET FF'S AND THE RATIO OBSERVABLES

- $H_\lambda$  amplitudes are written in terms of **four form-factors**.
- HQET: FF's only depend on  $w$ , the gamma-factor between  $B$  and recoiling  $D^*$ .

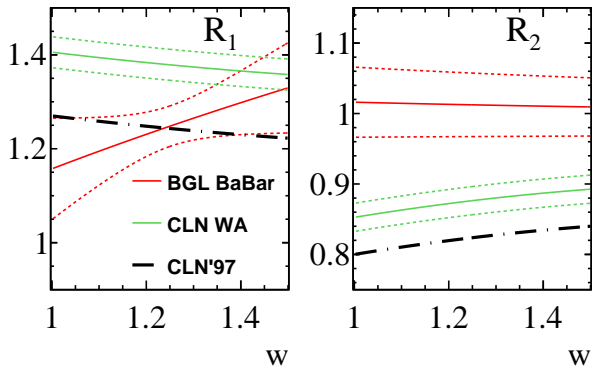
$$\frac{\langle D^*(v', \varepsilon) | V^\mu | \bar{B}(v) \rangle}{\sqrt{m_B m_{D^*}}} = i h_V(w) \varepsilon^{\mu\nu\alpha\beta} \varepsilon_\nu^* v'_\alpha v_\beta \quad A_1 = \frac{w+1}{2} r' h_{A_1}$$

$$\frac{\langle D^*(v', \varepsilon) | A^\mu | \bar{B}(v) \rangle}{\sqrt{m_B m_{D^*}}} = h_{A_1}(w)(w+1)\varepsilon^{*\mu} - h_{A_2}(w)(\varepsilon^* \cdot v)v^\mu - h_{A_3}(w)(\varepsilon^* \cdot v)v'^\mu \quad A_2 = \frac{r h_{A_2} + h_{A_3}}{r'} \equiv \frac{R_2 h_{A_1}}{r'}$$

$$V = \frac{h_V}{r'} \equiv \frac{R_1 h_{A_1}}{r'}$$

- HQS limit:  $\{h_V, h_{A_1}, h_{A_3}\} \rightarrow \zeta(w)$  and  $h_{A_2} \rightarrow 0$ .
- The two ratio observables  $R_{1,2}$  have reduced hadronic uncertainties.
- BGL basis  $\{f_0, F_1, g, F_2\}$ : rewrites  $h_{V, A_1, A_2, A_3}$ .
- Small **RH admixture**  $\epsilon_R$ : add  $(1 \pm \epsilon_R)$  factor for the vector ( $g$ ) and axial-vector ( $f, F_1$ ) FFs, respectively.

# BABAR-19: CONUNDRUMS IN $R_{1,2}$



- Figure as is, from the *BABAR-19* paper using BGL fits.
- “CLN-WA” used HFLAV16 numbers.
- CLN’97: original paper w/o uncertainties.

- $R_1(1)$  moved from  $1.404 \pm 0.032$  (HFLAV16) to  $1.269 \pm 0.026$  (HFLAV21, *BABAR-19* not included). Almost  $3.3\sigma$  change! Latest number is close to *BABAR-19*.
- HFLAV21 (excluding *BABAR-19*) quotes  $R_2(1) \sim 0.85$ .
- These are *BABAR-only* BGL. *BABAR + lattice* fits important.

## ONGOING DEVELOPMENT AT BABAR

- New  $w > 1$  lattice data: FNAL/MILC and JLQCD ( $B \rightarrow D^*$ ) and HPQCD ( $B_{(s)} \rightarrow D_{(s)}^*$ ).
- Preliminary joint BABAR +lattice  $B \rightarrow D^*$  fits were shown at ICHEP'22. Being finalized.
- Available unbinned  $B \rightarrow D$  data being added to joint  $B \rightarrow D^{(*)}$  BABAR +lattice HQET fits.
- Angular moments analysis of  $\bar{B} \rightarrow \{D^*, \rho^0\} \ell^- \bar{\nu}_\ell$  in  $q^2$  bins.

# SUMMARY

- Hadronic tagged *BABAR* SL still a unique dataset. Excellent for unbinned full angular analyses.
- For  $B \rightarrow D$ : FF's seem quite stable and getting more precise. **Normalization** remains a concern.
- For  $B \rightarrow D^*$ : still much to understand about the **FFs**. Apparent  $|V_{cb}|$  exclusive discrepancies among  $B_{(s)} \rightarrow D_{(s)}^{(*)}$  concerning.