Determination of V_{cb} from Exclusive Decays

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Challenges in Semileptonic B Decays

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based on work in collaboration with Paolo Gambino and Martin Jung

A lot of recent activity

Theory

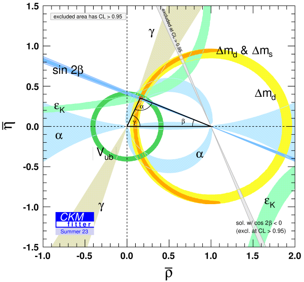
- Fermilab Lattice/MILC 2105.14019, HPQCD 2304.03137, JLQCD 2306.05657
- Bernlochner Ligeti Papucci Prim Robinson Xiong 2206.11281
- Bobeth Bordone Gubernari Jung van Dyk 2104.02094
- Bordone Jüttner 2406.10074, Flynn Jüttner Tsang 2303.11285
- Martinelli Simula Vittorio 2109.15248, 2310.03680
- Fedele Blanke Crivellin Iguro Nierste Simula Vittorio 2305.15457
- Gambino Jung Schacht 24xx.soon

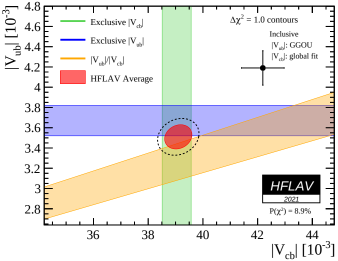
Experiment

- Babar 2311.15071
- Belle 2301.07529, 2310.20286.
- Belle II 2210.13143, 2308.02023, 2310.01170, 2401.02840.
- LHCb 2302.02886, 2305.01463, 2406.03387

Why is V_{cb} so important?

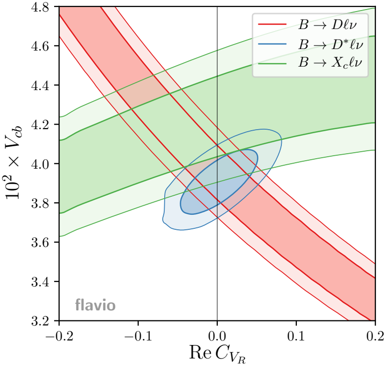
- Fundamental parameter of the Standard Model.
- Important role in Unitarity Triangle.
- Important for prediction of $\varepsilon_K \propto x\,|V_{cb}|^4 + \ldots$.
- Important for predictions of FCNCs.
- Ratio $|V_{ub}/V_{cb}|$ directly constrains one side of the Unitarity Triangle.





Is BSM the answer?

- Requires sizable New Physics incompatible with kinematic distributions [Straub Jung 1801.01112].
- Conflict with $Z \to b\bar{b}$, connected through SMEFT [Crivellin Pokorski 1407.1320].



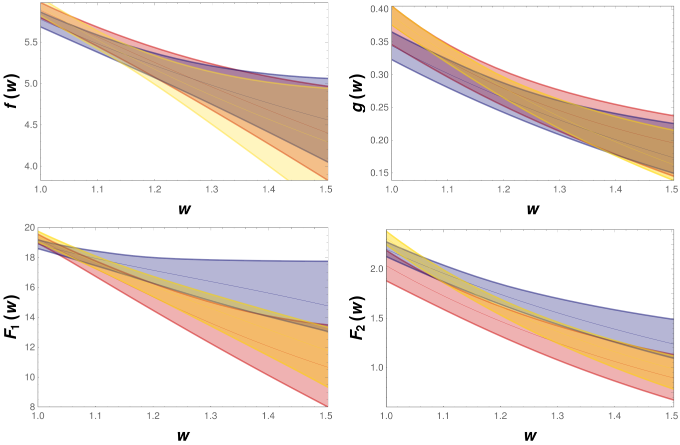
Constraints on RH currents [Straub Jung 1801.01112]

It is a puzzle.

$B \to D^*$ Form Factors from Lattice: $N = 3$ BGL fits

[Gambino Jung Schacht 24xx.soon, preliminary]

- FNAL/MILC 2105.14019
- HPQCD 2304.03137
- JLQCD 2306.05657



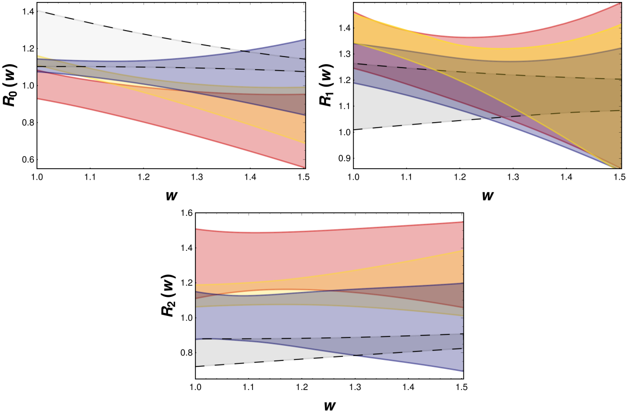
- No major discrepancy, but …

Comparison of ratios to HQET results

[Gambino Jung Schacht 24xx.soon, preliminary]

- …differences may get enhanced in combinations.

- FNAL/MILC 2105.14019
- HPQCD 2304.03137
- JLQCD 2306.05657

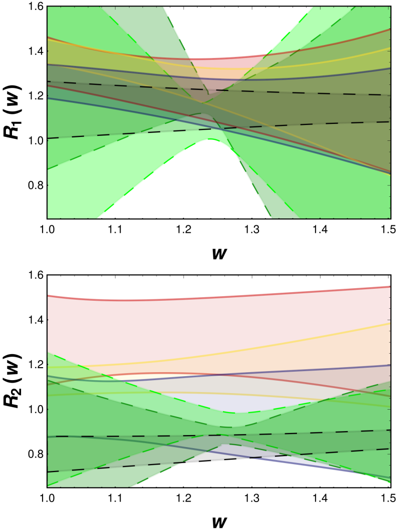


- HQET (black) [Bordone, Jung, van Dyk: 1908.09398]

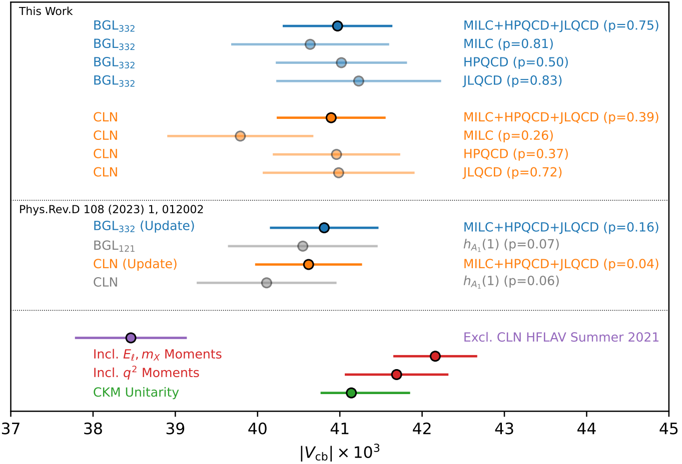
Comparison to experimental data

[Gambino Jung Schacht 24xx.soon, preliminary]

- FNAL/MILC 2105.14019
- HPQCD 2304.03137
- JLQCD 2306.05657
- HQET (black) [Bordone, Jung, van Dyk: 1908.09398]
- Experiment
  Light: Belle-II 2308.02023, 2310.01170
  Dark: Belle 1809.03290.



At ∼1% precision things become complicated



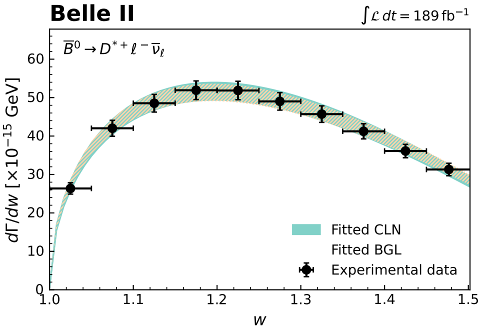
[overview from Belle 2310.20286]

Outline

- Convergence of the z-expansion
- Impact of different statistical methods
- Angular observables in Experiment and Theory

Convergence of the z-expansion

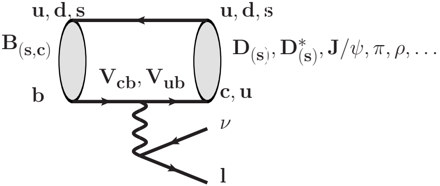
Kinematics



[Belle II, 2310.01170]

- Invariant lepton mass squared $q^2 = (p_B - p_{D^*})^2 = (p_l + p_\nu)^2$.
- Dimensionless quantity $w = \frac{m_B^2 + m_{D^*}^2 - q^2}{2 m_B m_{D^*}}$.
- High q^2 ⇔ low w. Low q^2 ⇔ high w.

Form Factors for Exclusive Decays



# of FFs	$B \to Pl\nu_l$	$B \to Vl\nu_l$
$l = e, \mu$	1	3
$l = \tau$	2	4

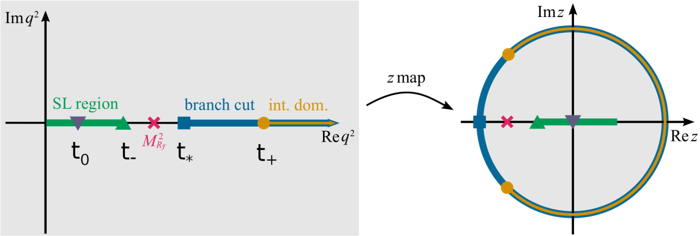
- Example:

$$\langle D(k)|\, \bar{c}\gamma^\mu b \left|\bar{B}(p)\right\rangle =$$

$$\left((p+k)^\mu - \frac{m_B^2 - m_D^2}{q^2} q^\mu\right) f_+^{B\to D}(q^2) + \left(\frac{m_B^2 - m_D^2}{q^2} q^\mu\right) f_0^{B\to D}(q^2).$$

- Information on form factors:
  - Lattice QCD.
  - Light Cone Sum Rules (LCSR, at low q^2 $\Leftrightarrow$ high w).
  - Heavy Quark Expansion.
  - Experiment.

The $q^2 \mapsto z$ mapping



[figure adapted from Gubernari, Reboud, van Dyk, Virto: 2305.06301]

$$z(t, t_*, t_0) = \frac{\sqrt{t_* - t} - \sqrt{t_* - t_0}}{\sqrt{t_* - t} + \sqrt{t_* - t_0}}$$

- $t \geq t_*$: branch cut: multi-particle resonances.
- $t_\pm = (m_B \pm m_{D^*})^2$. t_0: free expansion parameter.
- Red cross: B_c^* resonances below branch cut.

Result: Model-ind. form factor parametrization

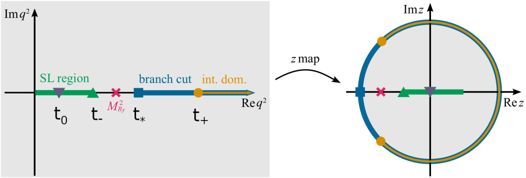
Boyd Grinstein Lebed parametrization

$$f_i(z) = \frac{1}{B_i(z)\phi_i(z)} \sum_{n=0}^{\infty} a_n^i z^n , \qquad z = \text{function of } w(q^2) ,$$

- $0 < z < 0.06$ for $B \to D^* l\nu \Rightarrow z^3 \sim 10^{-4}$.
- $B_i(z)$: “Blaschke factor”: removes poles below branch cut threshold.
- $\phi_i(z)$: phase space factors.

[Boyd Grinstein Lebed: hep-ph/9412324, hep-ph/9504235, hep-ph/9508211, hep-ph/9705252]

Unitarity Constraints



[figure adapted from Gubernari, Reboud, van Dyk, Virto: 2305.06301]

- Use dispersion relations to relate semileptonic region $m_l^2 \leq t \leq t_-$ to pair-production region beyond threshold $t \geq t_+$.
- Constrain form factors in pair-production region with pert. QCD.
- Translate constraint to semileptonic region using analyticity.
- Original BGL: $t_* = t_+$ (“yellow = blue”), resulting in

$$\sum_i^{\infty} a_i^2 \leq 1\,.$$

Generalized Unitarity Constraints

- Original unitarity bounds based on approximation:
  Multi-particle branch cut actually begins at $t = (m_{B_c^*} + 2m_\pi)^2$
  (two pions due to isospin) $\Rightarrow t_* < t_+$.
- For $b \to s$ transition, effect has been accounted for by introduction of "generalized unitarity constraints". [Gubernari, van Dyk Virto: 2011.09813, Blake, Meinel, Rahimi, van Dyk: 2205.06041, Flynn Jüttner Tsang: 2303.11285]

$$\sum_{j,k=0}^{\infty} a_j^{(i)} a_k^{(i)} \langle z^j | z^k \rangle < 1\,,$$

$$\langle z^i | z^j \rangle = \begin{cases} \frac{\sin(\alpha(i-j))}{\pi(i-j)} & i \neq j \\ \frac{\alpha}{\pi} & i = j. \end{cases}\,,$$

$$\alpha = \arg[z(t_+, t_\star, t_0)]\,.$$

- Model-dep. estimate for $b \to c$ finds small effect [BGL, hep-ph/9508211].

Questions

- Are subthreshold effects in $b \to c$ still negligible with today's data?
  Does the z-expansion converge with generalized unitarity?

- Where should we truncate the series?

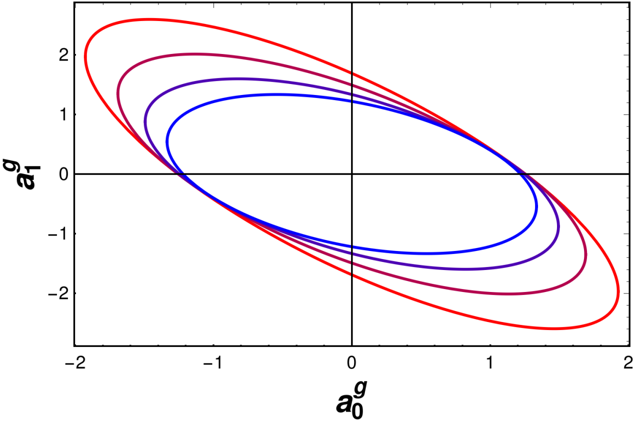
- How shall we include the unitarity constraints?

Questions intertwined with each other and also related to the used statistics.

Answers determine methodology.

Constraints from Generalized Unitarity Only

[Gambino Jung Schacht 24xx.soon, preliminary]

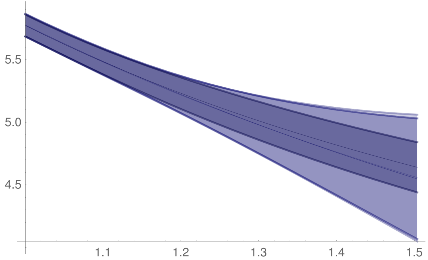


- Constraints in the $a_0^g - a_1^g$ plane.
- In (Blue) ⇒ Out (Red): $N = 1, \ldots, 4$.

Convergence for Lattice Only Fit: $f(w)$ (JLQCD)

[Gambino Jung Schacht 24xx.soon, preliminary]

- $N = 1, 2, 3, t_* = t_+$.
- Fast convergence.
- Higher orders can hardly be distinguished.

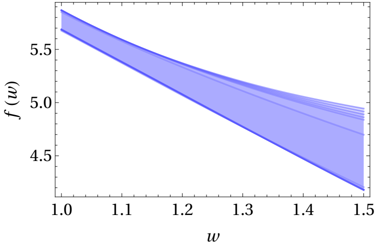


BGL order	χ^2
$N = 1$	1.42
$N = 2$	1.07
$N = 3$	0.97

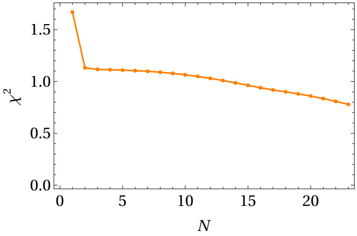
Convergence of $f(w)$ for $t_* = (M_{B_c} + 2m_\pi)^2$

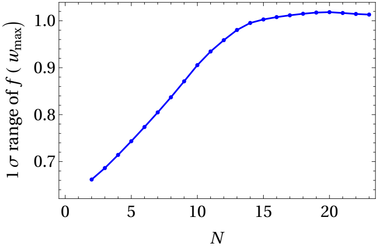
[Gambino Jung Schacht 24xx.soon, preliminary]

- Signs of convergence only at very high order.
- May look different with data, may not be relevant for V_{cb}.
  Requires more investigation.
- BGL3, $t_* = t_+ \colon \chi^2_{min} = 0.97$,
  $4.048 < f(w_{\text{max}}) < 5.061$.



$N = 1, \ldots, 6.$





Impact of different statistical methods

"Standard" (frequentist)

- χ^2 fit with hard unitarity constraints as side condition.
- Increase BGL order until χ^2 stable. [Gambino Jung Schacht 1905.08209]

Feldman Cousins (frequentist)

- Well-defined CL including hard unitarity constraints, using toy Monte Carlo data. [Gambino Jung Schacht 24xx.soon]

Nested Hypothesis Test to determine truncation order. (frequentist)

- Go to order $N + 1$ if $\Delta\chi^2 = \chi^2_N - \chi^2_{N+1} \geq 1$. Check unitarity a posteriori. [Bernlochner Ligeti Robinson 1902.09553]
- Alternative: Akaike Information Criterion. [Persson Bernlochner Ligeti Prim Robinson 2024]

Bayesian inference

- BGL unitarity constraints as prior [Flynn Jüttner Tsang 2303.11285, Bordone Jüttner 2406.10074]
- Dispersive Matrix approach [Di Carlo, Martinelli, Naviglio, Sanfilippo, Simula, Vittorio 2105.02497]

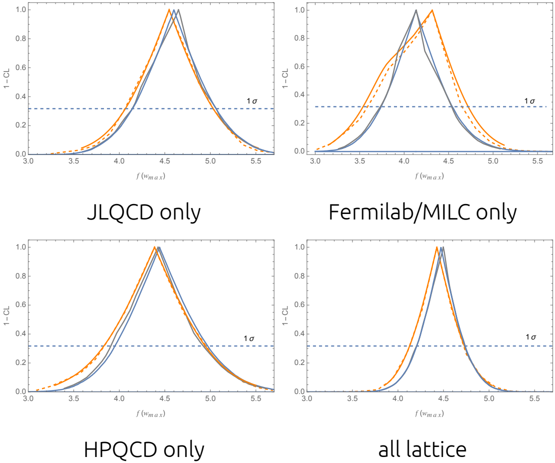
$f(w_{max})$: Lattice Only BGL[2] Fits, standard unitarity

[Gambino Jung Schacht 24xx.soon, preliminary]

- Standard: orange solid
- Feldman Cousins: orange dashed

$$1 - \mathrm{CL} \equiv p \equiv \int_{\Delta\chi^2_{\mathrm{data}}}^{\infty} f(t)dt = \frac{N(\Delta\chi^2_{\mathrm{toy}} > \Delta\chi^2_{\mathrm{data}})}{N_{\mathrm{toys}}},$$

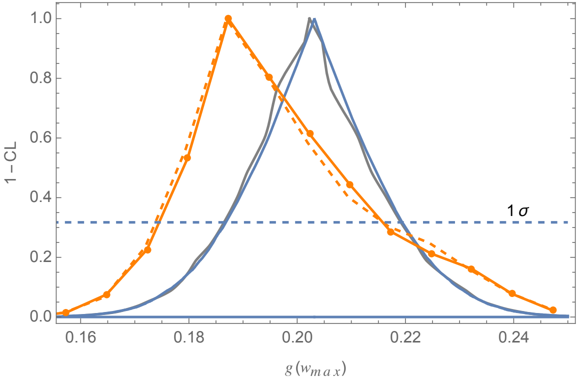
- Bayesian: blue solid



$g(w_{max})$: Lattice Only BGL^2 Fits, standard unitarity

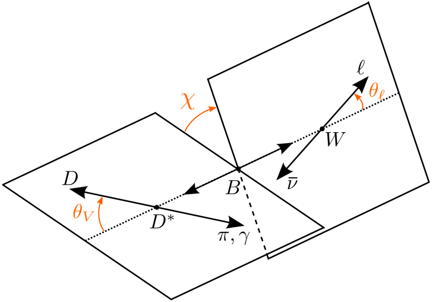
[Gambino Jung Schacht 24xx.soon, preliminary]

- Standard: orange solid
- Feldman Cousins: orange dashed
- Bayesian: blue solid



all lattice

Angular observables: Experiments and Theory

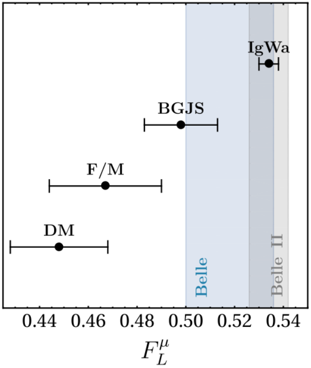


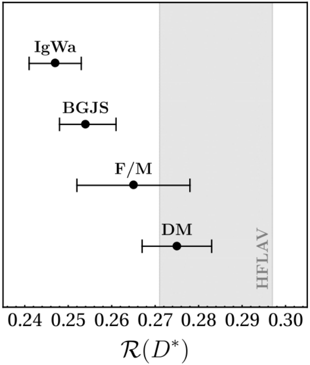
[Belle 2301.07529]

Motivation also beyond V_{cb}: Correlation with $R(D^*)$

[Fedele Blanke Crivellin Iguro Nierste Simula Vittorio 2305.15457]

- Ang. obs. A_{FB} and F_L^μ are correlated with $R(D^*)$.
- Lattice only FNAL/MILC results agree with $R(D^*)$ but disagree with F_L^μ.





Total Differential Rate

- We use the notation of [Bobeth, Bordone, Gubernari, Jung, van Dyk: 2104.02094].

$$\frac{8\pi}{3}\frac{d^4\Gamma^{(l)}}{dq^2 d\cos\theta_l d\cos\theta_D d\chi} = \left(J_{1s}^{(l)} + J_{2s}^{(l)}\cos 2\theta_l + J_{6s}^{(l)}\cos\theta_l\right)\sin^2\theta_D$$
$$+\left(J_{1c}^{(l)} + J_{2c}^{(l)}\cos 2\theta_l + J_{6c}^{(l)}\cos\theta_l\right)\cos^2\theta_D$$
$$+\left(J_{3}^{(l)}\cos 2\chi + J_{9}^{(l)}\sin 2\chi\right)\sin^2\theta_D\sin^2\theta_l$$
$$+\left(J_{4}^{(l)}\cos\chi + J_{8}^{(l)}\sin\chi\right)\sin 2\theta_D\sin 2\theta_l$$
$$+\left(J_{5}^{(l)}\cos\chi + J_{7}^{(l)}\sin\chi\right)\sin 2\theta_D\sin\theta_l$$

- Take the CP average. CP-averaged, normalized coefficients:

$$S_i^{(l)} = \frac{J_i^{(l)} + \bar{J}_i^{(l)}}{\Gamma^{(l)} + \bar{\Gamma}^{(l)}}\,.$$

CP-averaged differential rate

$$\frac{8\pi}{3\hat{\Gamma}^{(l)}} \frac{d^4\hat{\Gamma}^{(l)}}{dq^2 d\cos\theta_l d\cos\theta_D d\chi} = \left(S_{1s}^{(l)} + S_{2s}^{(l)} \cos 2\theta_l + S_{6s}^{(l)} \cos\theta_l\right) \sin^2\theta_D$$
$$+ \left(S_{1c}^{(l)} + S_{2c}^{(l)} \cos 2\theta_l + S_{6c}^{(l)} \cos\theta_l\right) \cos^2\theta_D$$
$$+ S_3^{(l)} \cos 2\chi \sin^2\theta_D \sin^2\theta_l$$
$$+ S_4^{(l)} \cos\chi \sin 2\theta_D \sin 2\theta_l$$
$$+ S_5^{(l)} \cos\chi \sin 2\theta_D \sin\theta_l \,.$$

CP-averaged, normalized coefficients: $S_i^{(l)} = \dfrac{J_i^{(l)} + \bar{J}_i^{(l)}}{\Gamma^{(l)} + \bar{\Gamma}^{(l)}}$,

$$J_i^{(l)} + \bar{J}_i^{(l)} = \begin{cases} 2J_i^{(l)} & (i = 1(s,c) \ldots 6(s,c) \quad \text{(CP-even)} \\ 0 & i = 7,8,9 \quad \text{(CP-odd)} \end{cases} ,$$

Corrections only from dim-8 operators or QED corrections $\Rightarrow$ applies to BSM, too.

Non-trivial relation between coefficients

$$q^2 \text{ integration:} \quad \langle X \rangle \equiv \int dq^2 X(q^2)\,,$$

Total decay rate:

$$\Gamma^{(l)} = 2\langle J_{1s}^{(l)} \rangle + \langle J_{1c}^{(l)} \rangle - \frac{1}{3}\left(2\langle J_{2s}^{(l)} \rangle + \langle J_{2c}^{(l)} \rangle\right)\,.$$

Implies for normalized coefficients (model-independent):

$$2\langle S_{1s}^{(l)} \rangle + \langle S_{1c}^{(l)} \rangle - \frac{1}{3}\left(2\langle S_{2s}^{(l)} \rangle + \langle S_{2c}^{(l)} \rangle\right) = 1\,.$$

This may look more familiar: 1D coefficients

$$\frac{1}{\Gamma}\frac{d\Gamma^{(l)}}{d\cos\theta_l} = \frac{1}{2} + \langle A_{\rm FB}^{(l)}\rangle \cos\theta_l + \frac{1}{4}\left(1 - 3\langle \widetilde{F}_L^{(l)}\rangle\right)\frac{3\cos^2\theta_l - 1}{2}$$

$$\frac{1}{\Gamma}\frac{d\Gamma^{(l)}}{d\cos\theta_D} = \frac{3}{4}\left(1 - \langle F_L^{(l)}\rangle\right)\sin^2\theta_D + \frac{3}{2}\langle F_L^{(l)}\rangle\cos^2\theta_D$$

$$\frac{1}{\Gamma}\frac{d\Gamma^{(l)}}{d\chi} = \frac{1}{2\pi} + \frac{2}{3\pi}\langle S_3^{(l)}\rangle\cos 2\chi + \frac{2}{3\pi}\langle S_9^{(l)}\rangle\sin 2\chi$$

Example: Forward-Backward Asymmetry:

$$A_{\rm FB}^{(l)}(q^2) \equiv \left(\frac{d\Gamma^{(l)}}{dq^2}\right)^{-1}\left(\int_0^1 - \int_{-1}^0\right) d\cos\theta_l \frac{d^2\Gamma^{(l)}}{d\cos\theta_l dq^2},$$

$$\langle A_{FB}^{(l)}\rangle = \langle S_{6s}^{(l)}\rangle + \frac{1}{2}\langle S_{6c}^{(l)}\rangle\,.$$

Model-Ind. Extraction of Angular Coefficients

Model-independent Assumptions (no form factor input)

- Form of CP-averaged differential decay rate.
- Non-trivial relation between CP-averaged coefficients.
- $S_{7,8,9} = 0.$

Complication: Matching literature to common notation

$$A_{FB}^{(l)} = A_{FB}^{(l),\text{Belle'23}} = A_{FB}^{(l),\text{Belle-II'23}}, \qquad F_L^{(l)}(D^*) = F_L^{(l)}(D^*)^{(l),\text{Belle'23}}$$

$$\langle S_3^{(l)} \rangle = \frac{3\pi}{4} S_3^{(l),\text{Belle'23}} = \frac{3\pi}{8} S_3^{(l),\text{Belle-II'23}}$$

$$\langle S_5^{(l)} \rangle = S_5^{(l),\text{Belle'23}} = S_5^{(l),\text{Belle-II'23}}$$

$$\langle S_7^{(l)} \rangle = S_7^{(l),\text{Belle'23}} = S_7^{(l),\text{Belle-II'23}}$$

$$\langle S_9^{(l)} \rangle = \frac{3\pi}{4} S_9^{(l),\text{Belle'23}} = \frac{3\pi}{8} S_9^{(l)\text{Belle-II'23}}$$

Belle'23: 2310.20286

Belle-II'23: 2308.02023

Model-Ind. Extraction of Angular Coefficients

[Gambino Jung Schacht 24xx.soon, preliminary]

$$\delta\mathcal{A}_x = \mathcal{A}_{x,\text{hi}} - \mathcal{A}_{x,\text{lo}}$$
$$\Sigma\mathcal{A}_x = (\mathcal{A}_x^\mu + \mathcal{A}_x^e)/2$$
$$\Delta\mathcal{A}_x = \Delta\mathcal{A}_x - \mathcal{A}_x^e$$

- Belle II'23: different data sets. (2308.02023, 2310.01170)
- Belle'23: same data set (both tagged) (2301.07529, 2310.20286)
- Belle'18: untagged (1809.03290)

Measurement	Belle-II 23a [3]	Belle-II 23b [4]	Belle 23a [1]	Belle 23b [2]	Belle 18 [5]
χ^2/dof	7.5/16	53/42	113/118	118/118	48/52
Observable					
$\Sigma A_{\text{FB,tot}}$	0.171(23)	0.189(19)	0.238(11)	0.244(14)	0.212(5)
$\Sigma S_{3,\text{tot}}$	-0.130(29)	-0.141(8)	-0.126(22)	-0.126(24)	-0.139(6)
$\Sigma S_{5,\text{tot}}$	0.173(25)	—	—	0.177(20)	—
$\Sigma F_{\text{L,tot}}$	—	0.524(8)	0.500(13)	0.530(18)	0.5302(35)
$\Sigma \tilde{F}_{\text{L,tot}}$	—	0.515(20)	0.523(20)	0.514(23)	0.543(7)
$\Sigma\delta A_{\text{FB}}$	-0.050(29)	—	—	−0.041(29)§	—
$\Sigma\delta S_3$	0.062(28)	—	—	0.15(5)§	—
$\Sigma\delta S_5$	-0.03(5)	—	—	0.09(4)§	—
$\Sigma\delta F_{\text{L}}$	—	—	—	0.27(4)§	—
$\Sigma\delta \tilde{F}_{\text{L}}$	—	—	—	0.24(5)§	—
$\Delta A_{\text{FB,tot}}$	-0.03(5)	-0.020(22)	-0.002(22)	0.020(27)	0.035(9)
$\Delta S_{3,\text{tot}}$	-0.08(6)	-0.023(17)	-0.04(4)	-0.04(5)	-0.013(11)
$\Delta S_{5,\text{tot}}$	-0.03(5)	—	—	0.04(4)	—
$\Delta F_{\text{L,tot}}$	—	0.007(9)	0.027(25)	0.02(4)	-0.006(6)
$\Delta \tilde{F}_{\text{L,tot}}$	—	-0.015(28)	0.001(38)	-0.02(5)	-0.011(14)
$\Delta\delta A_{\text{FB}}$	-0.14(6)	—	—	0.04(6)§	—
$\Delta\delta S_3$	-0.03(6)	—	—	−0.11(10)§	—
$\Delta\delta S_5$	-0.02(6)	—	—	−0.08(8)§	—
$\Delta\delta F_{\text{L}}$	—	—	—	0.00(7)§	—
$\Delta\delta \tilde{F}_{\text{L}}$	—	—	—	0.28(10)§	—

Model-Ind. Extraction of Angular Coefficients

[Gambino Jung Schacht 24xx.soon, preliminary]

- '23 $\Delta A_{\text{FB,tot}}$ analyses not conclusive yet.
- $\Delta\delta A_{FB} \neq 0$ at 2.3σ and $\Delta\delta \widetilde{F}_L \neq 0$ at 2.8σ. Coefficients of same distribution ($\cos\theta_l$).
- $\Sigma A_{FB,\text{tot}}$ Belle II 23a vs. Belle 23b 2.7σ.
- $\Sigma F_{L,\text{tot}}$ Belle 23a vs. Belle 18 2.2σ.

Measurement	Belle-II 23a [3]	Belle-II 23b [4]	Belle 23a [1]	Belle 23b [2]	Belle 18 [5]
χ^2/dof	7.5/16	53/42	113/118	118/118	48/52
Observable					
$\Sigma A_{\text{FB,tot}}$	0.171(23)	0.189(19)	0.238(11)	0.244(14)	0.212(5)
$\Sigma S_{3,\text{tot}}$	-0.130(29)	-0.141(8)	-0.126(22)	-0.126(24)	-0.139(6)
$\Sigma S_{5,\text{tot}}$	0.173(25)	—	—	0.177(20)	—
$\Sigma F_{\text{L,tot}}$	—	0.524(8)	0.500(13)	0.530(18)	0.5302(35)
$\Sigma \tilde{F}_{\text{L,tot}}$	—	0.515(20)	0.523(20)	0.514(23)	0.543(7)
$\Sigma\delta A_{\text{FB}}$	-0.050(29)	—	—	$-0.041(29)^{\S}$	—
$\Sigma\delta S_3$	0.062(28)	—	—	$0.15(5)^{\S}$	—
$\Sigma\delta S_5$	-0.03(5)	—	—	$0.09(4)^{\S}$	—
$\Sigma\delta F_{\text{L}}$	—	—	—	$0.27(4)^{\S}$	—
$\Sigma\delta \tilde{F}_{\text{L}}$	—	—	—	$0.24(5)^{\S}$	—
$\Delta A_{\text{FB,tot}}$	-0.03(5)	-0.020(22)	-0.002(22)	0.020(27)	0.035(9)
$\Delta S_{3,\text{tot}}$	-0.08(6)	-0.023(17)	-0.04(4)	-0.04(5)	-0.013(11)
$\Delta S_{5,\text{tot}}$	-0.03(5)	—	—	0.04(4)	—
$\Delta F_{\text{L,tot}}$	—	0.007(9)	0.027(25)	0.02(4)	-0.006(6)
$\Delta \tilde{F}_{\text{L,tot}}$	—	-0.015(28)	0.001(38)	-0.02(5)	-0.011(14)
$\Delta\delta A_{\text{FB}}$	-0.14(6)	—	—	$0.04(6)^{\S}$	—
$\Delta\delta S_3$	-0.03(6)	—	—	$-0.11(10)^{\S}$	—
$\Delta\delta S_5$	-0.02(6)	—	—	$-0.08(8)^{\S}$	—
$\Delta\delta F_{\text{L}}$	—	—	—	$0.00(7)^{\S}$	—
$\Delta\delta \tilde{F}_{\text{L}}$	—	—	—	$0.28(10)^{\S}$	—

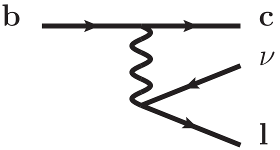
Conclusions

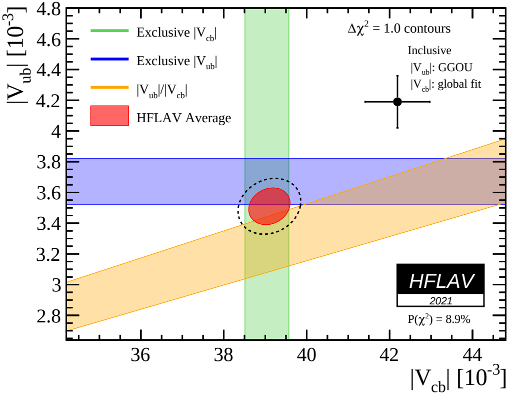
- It is an exciting time for V_{cb}.
- In the precision era of V_{cb}, parametrizations, their convergence and corresponding uncertainties require great care.
- As a community, we should fix a convention for angular observables.
- Stay tuned for our paper.

BACK-UP

B Anomalies: I and II

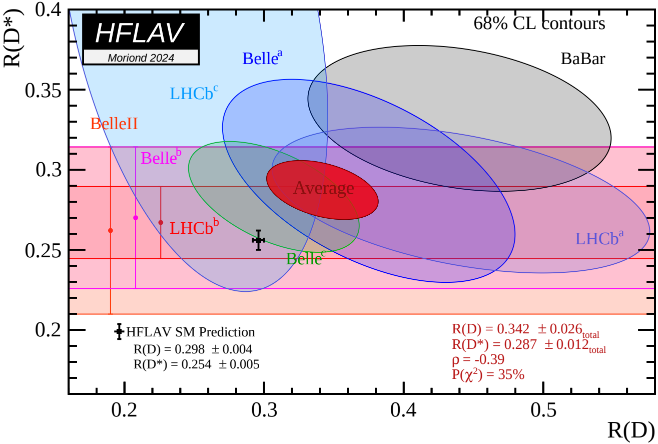
[HFLAV 2206.07501]





- Exclusive vs. Inclusive V_{cb}, V_{ub}
- Recent years: Enter era of precision measurements.
  New results from Belle, new lattice form factor results.

B Anomalies: III and IV 3.31σ [HFLAV 2206.07501]



Need to understand V_{cb} puzzle for complete picture.