$\bar{B}_{(s)} \to D_{(s)}^{(*)}$ form factors at $\mathcal{O}(1/m_c^2)$

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Based on
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Challenges in Semileptonic B Decays
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Introduction

Hadronic matrix elements

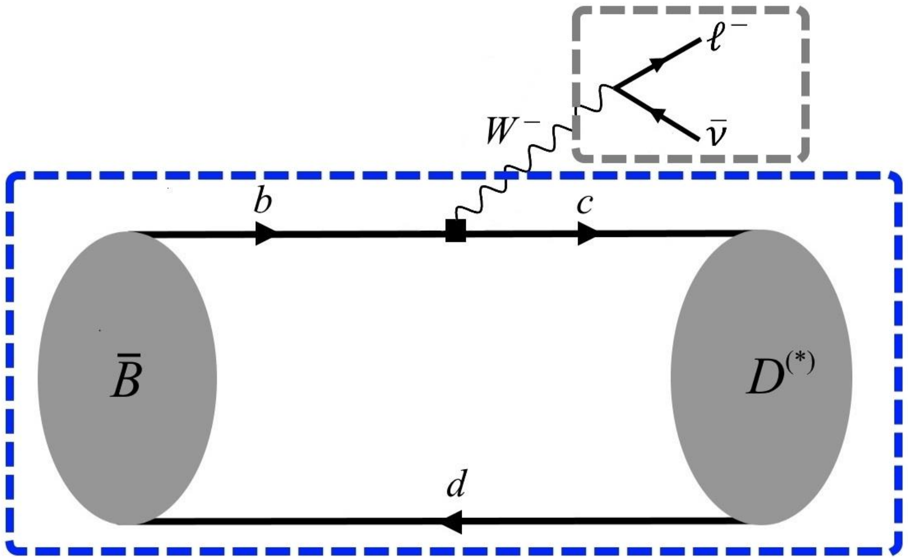
study $\boldsymbol{B}$-meson decays to test the $b \to c\ell\nu$ transitions
factorise decay amplitude (neglecting QED corrections)

$$\langle \bar{D}^{(*)}\ell\nu_\ell|\mathcal{O}_{eff}|B\rangle = \langle \ell\nu_\ell|\mathcal{O}_{lep}|0\rangle\langle D^{(*)}|\mathcal{O}_{had}|B\rangle$$

leptonic matrix elements: perturbative objects, high accuracy
QED corrections mostly unknown but small (~1%)

hadronic matrix elements: non-perturbative QCD effects,
usually large uncertainties (~10%)
$\Longrightarrow$ biggest challenge for percent precision

calculate them using lattice QCD or light-cone sum rule



Definition of the form factors

decompose matrix elements in terms of form factors (FFs) (assume only Lorentz invariance)

$$\langle D(k)|\bar{c}\,\gamma_\mu b|B(q+k)\rangle = 2\,k_\mu f_+(q^2) + q_\mu\big(f_+(q^2) + f_-(q^2)\big)$$

$$\langle D(k)|\bar{c}\,\sigma_{\mu\nu} q^\nu b|B(q+k)\rangle = \frac{i f_T(q^2)}{m_B + m_P}\big(q^2(2k+q)_\mu - (m_B^2 - m_P^2)q_\mu\big)$$

FFs are functions of the momentum transfer squared q^2

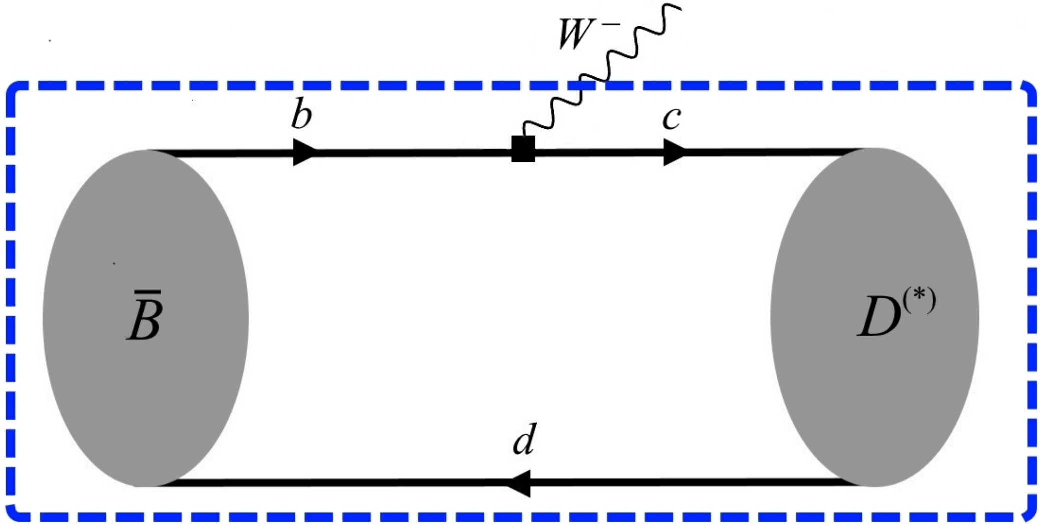
number of independent vector (or tensor) FFs

2(+1) for $B \to D$

4(+3) for $B \to D^*$

4(+?) for $B^* \to D$

10(+?) for $B^* \to D^*$



Heavy Quark Expansion (HQE)

use HQE: perform a power series expansion in Λ_{QCD}/m_c and Λ_{QCD}/m_b of FFs

$$\mathcal{F}^i_{\text{HQE}}(q^2) = \xi(q^2)\left(c_0^i + c_1^i \frac{\alpha_s}{\pi}\right) + c_2^i \frac{1}{m_b} L_k(q^2) + c_3^i \frac{1}{m_c} L_k(q^2) + c_4^i \frac{1}{m_c^2} l_k(q^2)$$

all the $B^{(*)} \to D^{(*)}$ FFs can be expressed in terms of 10 Isgur-Wise functions
(1 leading, 3 subleading, 6 subsubleading)

essential to include $\Lambda_{\text{QCD}}^2/m_c^2$ corrections (CLN not sufficient) [Bordone/Jung/van Dyk 2019]

$\Longrightarrow$ relations between $B^{(*)} \to D^{(*)}$ FFs

alternative method to include $1/m_c^2$ corrections proposed in Bernlochner F. et al. (2022)
$\Longrightarrow$ fewer parameters but model dependence introduced

The importance of HQE

several (new) precise lattice QCD calculations available

- $B \to D$ at high q^2
  [FNAL/MILC 2015] [HPQCD 2015]
- $B \to D^*$ at high q^2
  [FNAL/MILC 2021] [JLQCD 2023]
  at any q^2 (in the physical region)
  [HPQCD 2023]
- $B_s \to D_s$ at any q^2
  [HPQCD 2019]
- $B_s \to D_s^*$ at any q^2
  [HPQCD 2021] [HPQCD 2023]

LCSRs available for the four processes at low q^2

using HQE is important because

- theoretical calculations must fulfil **HQE relationships** (within uncertainties)
- extract information about **lesser-known form FFs** (vector FFs ⟺ tensor FFs)
- apply **strong unitarity bounds** (not possible with a la BGL or similar parametrizations)

Our goal: a comprehensive HQE analysis

combine theoretical constraints in a HQE analysis of all $B^{(*)} \to D^{(*)}$ and $B_s^{(*)} \to D_s^{(*)}$ FFs
include tensor FFs and corresponding strong bounds for the first time

steps to perform this analysis:

1. define the $B^* \to D^{(*)}$ HQE FFs
2. expand them in terms of Isgur-Wise functions at $\Lambda^2_{\mathrm{QCD}}/m_c^2$
3. define the $B^* \to D^{(*)}$ helicity FFs to write the unitarity bounds
4. relate HQE and helicity $B^* \to D^{(*)}$ FFs
5. fit the Isgur-Wise functions to the theoretical calculations and impose unitarity bounds

work in progress: all steps completed for $B^* \to D$, only step 1-3 completed for $B^* \to D^*$

Form factor definitions

HQE-like form factors definitions

$B^{(*)} \to D^{(*)}$ matrix elements are functions of the momenta (or velocities) and polarizations vectors appearing in the process, e.g.

$$\langle D^*(v',\eta')|\bar{c}\,\Gamma_\mu b|B^*(v,\eta)\rangle = g(v,v',\eta,\eta')$$

factorize Lorentz structures and scalar functions of $w = v \cdot v' \propto q^2$

$$\langle D^*(v',\eta')|\bar{c}\,\Gamma_\mu b|B^*(v,\eta)\rangle = \sum_i \mathcal{S}^i_\mu(v,v',\eta,\eta')\,\mathcal{F}^i_{\mathrm{HQE}}(w)$$

number of **independent Lorentz structures** $\Longrightarrow$ number of **independent form factors**

for $B^* \to D$ FFs similar definitions to $B \to D^*$ (use crossing symmetry) $\Longrightarrow$ 3 tensor FFs

for $B^* \to D^*$ 7 tensor FFs

HQE form factors expansion

expand $B^{(*)} \to D^{(*)}$ FFs in terms of Isgur-Wise functions

$$\mathcal{F}^i_{\mathrm{HQE}}(w) = \xi(w)\left(c^i_0 + c^i_1 \frac{\alpha_s}{\pi}\right) + c^i_2 \frac{1}{m_b} L_k(w) + c^i_3 \frac{1}{m_c} L_k(w) + c^i_4 \frac{1}{m_c^2} l_k(w)$$

coefficients $c_i^{\mathcal{F}}$ depend on the form factor considered

(axial-)vector and tensor FFs depend on the same **10 Isgur-Wise functions**

$B_s^{(*)} \to D_s^{(*)}$ FFs have 10 different Isgur-Wise functions
assume $SU(3)_F$ for subsubleading Isgur-Wise functions $\Longrightarrow$ **14 independent functions**

$$\mathcal{F}^i_{\mathrm{HQE}}(w) = \xi^s(w)\left(c^i_0 + c^i_1 \frac{\alpha_s}{\pi}\right) + c^i_2 \frac{1}{m_b} L^s_k(w) + c^i_3 \frac{1}{m_c} L^s_k(w) + c^i_4 \frac{1}{m_c^2} l_k(w)$$

Helicity form factors

calculate helicity amplitudes

(polzation vectors for virtual W boson ϵ_{λ_q}, D^*meson η'_{λ_k}, B^*meson η_{λ_p} , with $\lambda = 0, +, -, (t)$

$$\mathcal{A}^{B\to D}_{\lambda_q} \propto \epsilon^{\mu *}_{\lambda_q} \langle D(k) | \bar{c}\, \sigma_{\mu\nu} q^\nu b | B(p) \rangle$$

...

$$\mathcal{A}^{B^*\to D^*}_{\lambda_q,\lambda_k,\lambda_p} \propto \epsilon^{\mu *}_{\lambda_q} \left\langle D^*(k, \eta'_{\lambda_k}) \middle| \bar{c}\, \sigma_{\mu\nu} q^\nu b \middle| B^*(p, \eta'_{\lambda_p}) \right\rangle$$

in general, one finds, e.g.,

$$\mathcal{A}^{B^*\to D^*}_{\lambda_q,\lambda_k,\lambda_p} \propto \sum_i h^i(m_{D^*}, m_{B^*}, q^2)\, \mathcal{F}^i(q^2)$$

helicity FFs are defined so that each helicity amplitude depends on only one FF (or vanish)

$$\boxed{\mathcal{A}^{B^*\to D^*}_{\lambda_q,\lambda_k,\lambda_p} \propto g^i(m_{D^*}, m_{B^*}, q^2) \mathcal{F}^i_{\text{hel}}(q^2)}$$

Properties of helicity FFs

unitarity bounds can only be applied (directly) to helicity FFs since

$$\left|\langle D^{(*)}|\bar{c}\,\sigma_{\mu\nu}q^{\nu}b|B^{(*)}\rangle\right|^2 \propto \sum_i \left|\mathcal{F}^i_{\mathbf{hel}}\right|^2$$

however the definition of helicity FFs is not unique as $g^i(m_{D^*}, m_{B^*}, q^2)$ can be chosen arbitrarily

$$\mathcal{A}^{B^*\to D^*}_{\lambda_q,\lambda_k,\lambda_p} \propto g^i(m_{D^*}, m_{B^*}, q^2)\mathcal{F}^i_{\mathbf{hel}}(q^2)$$

how to fix $g^i(m_{D^*}, m_{B^*}, q^2)$, a few tips

- keep $\mathcal{F}^i_{\mathbf{hel}}$ dimensionless
- avoid zeros and poles in the relations between $\mathcal{F}^i_{\mathbf{hel}}$ and $\mathcal{F}^i_{\mathbf{HQE}}$
- keep $\mathcal{F}^i_{\mathbf{hel}}$ positive

derive relations between $\mathcal{F}^i_{\mathbf{hel}}$ and $\mathcal{F}^i_{\mathbf{HQE}}$ for $B^{(*)} \to D^{(*)}$

HQE parametrization

BGL(-like) parametrization

define the map

$$z(q^2) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}$$

for $B \to D^{(*)}$ it has been shown that a parametrization in the form

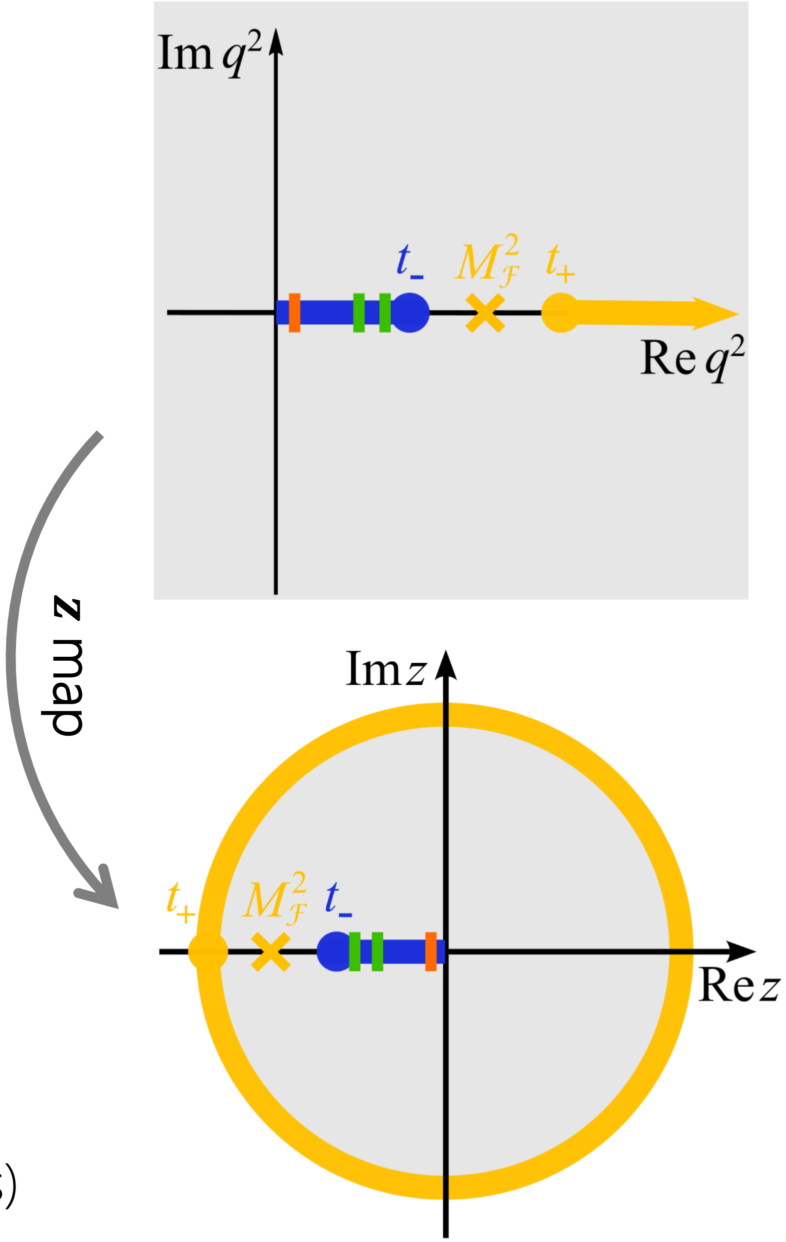
$$\mathcal{F}^i_{\mathrm{hel}}(z) = \frac{1}{\mathcal{P}(z)\phi(z)} \sum_{n=0}^{\infty} a^i_n\, z^n$$

($\mathcal{P}(z)\phi(z)$ are known functions)
fulfils the unitarity bound [Boyd/Grinstein/Lebed 1997]

$$\sum_{n=0}^{\infty} \left|a^i_n\right|^2 < 1$$

extend approach to tensor $B^* \to D^{(*)}$ FFs

$\Longrightarrow$ calculate outer functions for the new FFs (depend on the g^i functions)



HQE parametrization

expand Isgur-Wise functions around $\boldsymbol{w = 1}$ (max recoil)

$$\xi(w) = \sum_{m=0}^{N} \frac{\xi^{(m)}}{m!}(w-1)^m \qquad L_k(w) = \sum_{n=0}^{M} \frac{L_k^{(m)}}{m!}(w-1)^m \qquad l_k(w) = \sum_{n=0}^{N} \frac{l_k^{(m)}}{m!}(w-1)^m$$

the derivatives $\xi^{(m)}, L_k^{(m)}, l_k^{(m)}$ are the parameters of our parametrization

$\boldsymbol{N/M/K}$ parametrization

3/2/1 parametrization is the minimal order to achieve a good description [Bordone/Jung/van Dyk 2019]

apply unitarity bounds

- change variable $\boldsymbol{w \to z(w)}$ (use $\boldsymbol{t_0 = t_-}$)
- write $\mathcal{F}^i_{\mathrm{HQE}}$ in terms of $\mathcal{F}^i_{\mathrm{hel}}$
- choose truncation order in $\boldsymbol{z}$

$$z(w) = \frac{\sqrt{w+1} - \sqrt{2}}{\sqrt{w+1} + \sqrt{2}}$$

Weak and strong unitarity bounds

write the unitarity bound in terms of **Isgur-Wise parameters**

for one FF the bound reads

$$\sum_{n=0}^{\infty} |a_n^i|^2 \Rightarrow \sum_{n=0}^{\infty} \left|a_n^i(\xi^{(m)}, L_k^{(m)}, l_k^{(m)})\right|^2 < 1$$

sum contributions in the same channel (e.g. $B \to D^*$) with same J^P $\Longrightarrow$ **weak bound**

sum contribution of all channels related by heavy quark symmetry ($B^{(*)} \to D^{(*)}$)

$$\Downarrow$$

strong unitarity bound

$$\boxed{\sum_{\mathcal{F}^i} \sum_{n=0}^{\infty} \left|a_n^i(\xi^{(m)}, L_k^{(m)}, l_k^{(m)})\right|^2 < 1}$$

More on unitarity bounds

the HQE strong unitarity bound have some crucial advantages

- include the contribution of $B^* \to D^{(*)}$ decays $\Longrightarrow$ increase bound saturation
  not possible at the moment for BGL, as there are no $B^* \to D^{(*)}$ FFs predictions

- relate (axial-)vector and tensor FFs (constrain each other)
  not possible in BGL

- keep the number of fit parameters relatively low

other channels not related through HQE symmetry can be added $\Longrightarrow$ *ultra* strong unitarity bound

$$\underbrace{\sum_{\mathcal{F}^i}\sum_{n=0}^{\infty}\left|a_n^i(\xi^{(m)}, L_k^{(m)}, l_k^{(m)})\right|^2}_{B^{(*)}\to D^{(*)}} + \underbrace{\sum_{\mathcal{F}^i}\sum_{n=0}^{\infty}\left|a_n^i(\xi^{s,(m)}, L_k^{s,(m)}, l_k^{s,(m)})\right|^2}_{B_s^{(*)}\to D_s^{(*)}} + \underbrace{\sum_{\mathcal{F}^i}\sum_{n=0}^{\infty}\left|a_n^i(\xi^{\Lambda,(m)}, L_k^{\Lambda,(m)}, l_k^{\Lambda,(m)})\right|^2}_{\Lambda_b\to\Lambda_c} + \cdots < 1$$

Some (very) preliminary results

Preliminary HQE fit

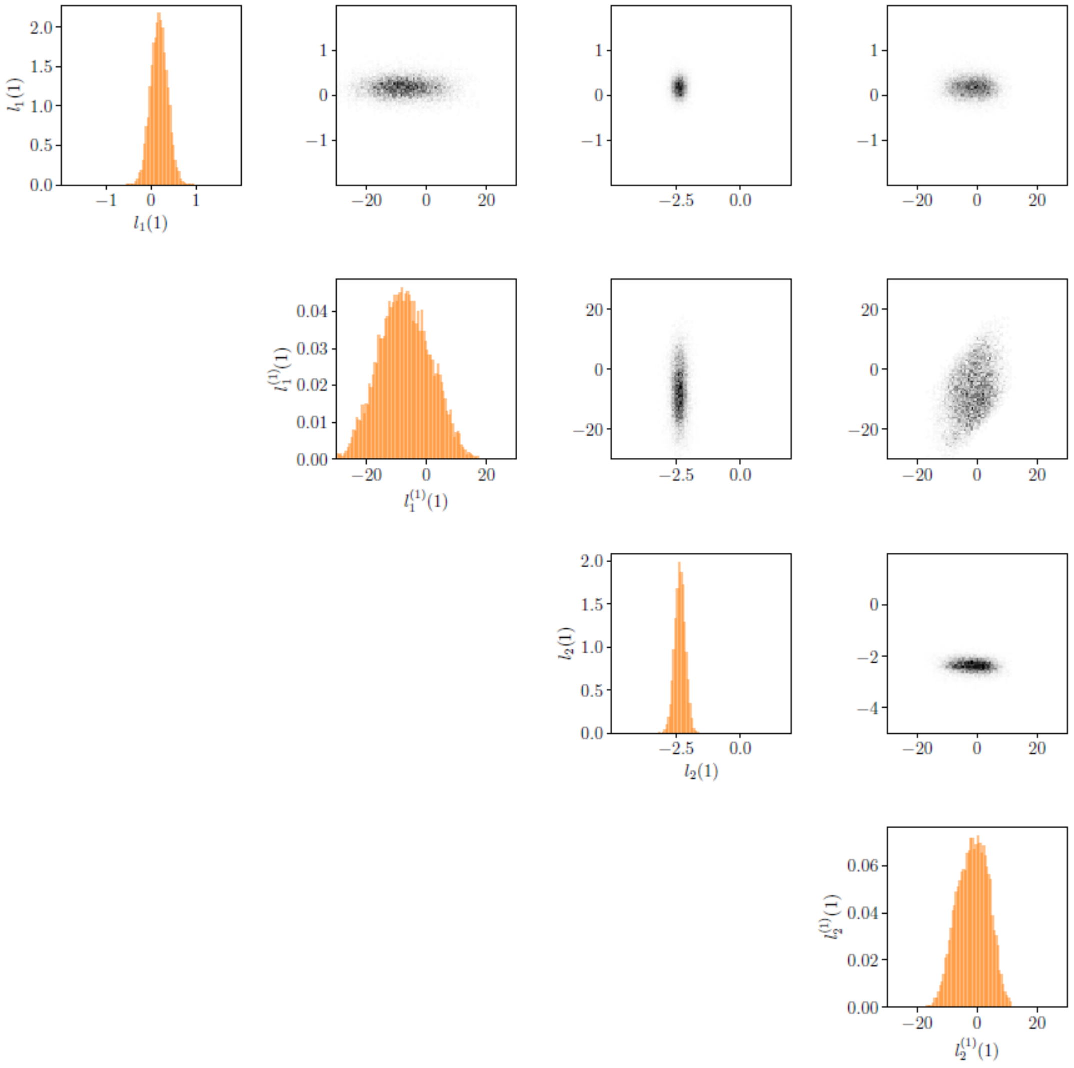
perform fits using EOS software (nested sampling)

use HQE 3/2/1 model (include to $1/m_c^2$ corrections)

31 free parameters for $B_{(s)}^{(*)} \to D_{(s)}^{(*)}$ FFs

available theory constraints

- lattice QCD
- light-cone sum rules
- QCD sum rules for Isgur-Wise functions
- unitarity bounds



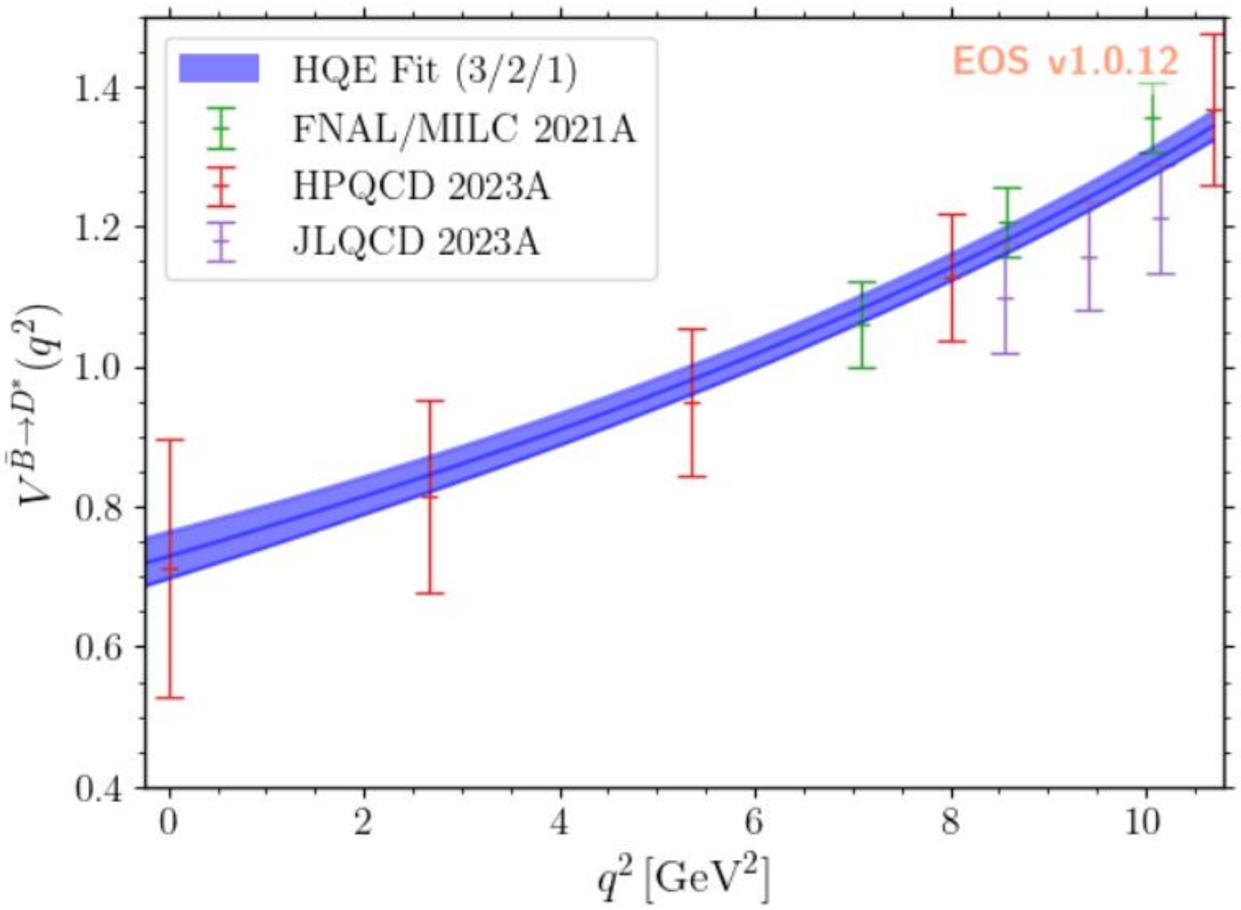
Preliminary HQE fit (FFs)

first attempt:
fit all available $B \to D^{(*)}$ FFs from LQCD
$B \to D$: [FNAL/MILC 2015] [HPQCD 2015]
$B \to D^*$: [FNAL/MILC 2021] [JLQCD 2023] [HPQCD 2023]

use unitarity bounds
(tensor contribution not fully implemented)

obtain a good fit p-value ~ 50%

obtain $B \to D^{(*)}$ FFs
predict physical observable

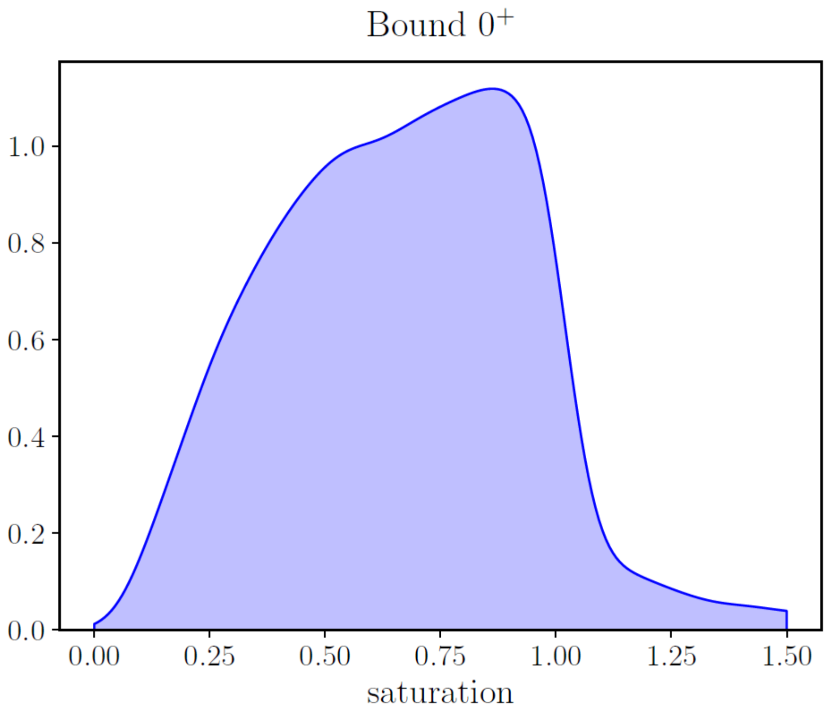


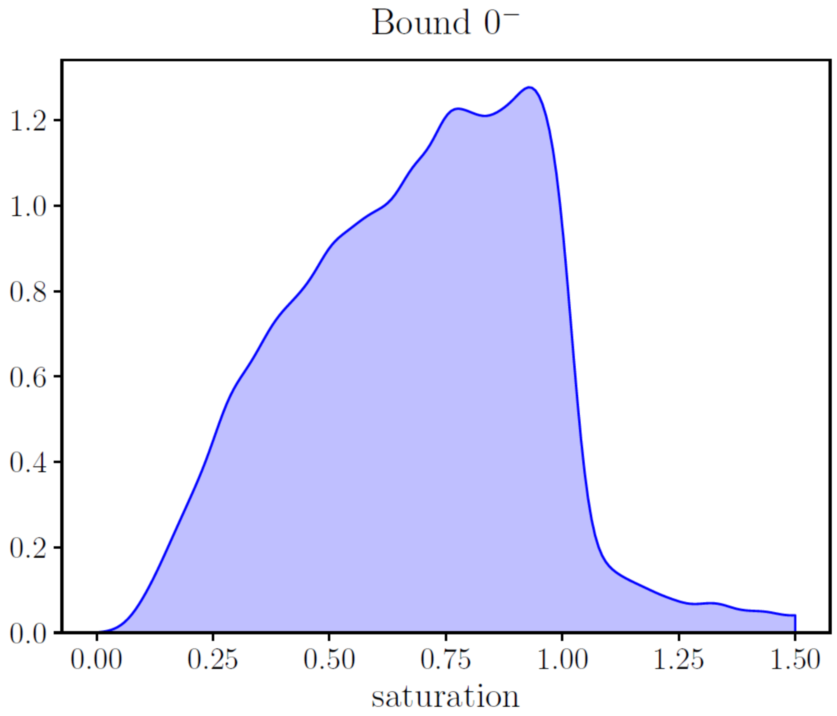
Preliminary HQE fit (bounds)

strong unitarity bound is (suspiciously) almost saturated by LQCD results. $\Longrightarrow$ essential for the analysis

also observed by Martinelli et al.

unitarity bounds not only control the truncation error but also **check the consistency of LQCD results**





Summary and outlook

Summary and outlook

HQE pheno analyses are important for the interpretation of FFs calculations and measurements
$\Longrightarrow$ obtain complementary information

strong unitarity bounds fully exploit the theoretical constraints
obtain relations and information on lesser-known FFs

$\Longrightarrow$ new bounds on the $B^* \to D^{(*)}$ tensor FFs will improve our knowledge of other FFs

define the $B^* \to D^*$ helicity FFs and relate them to the HQE FFs

perform fits using different sets of theoretical constraints

Thank you!