The q^2 Moments in Inclusive Semileptonic B Decays

Gael Finauri

Challenges in Semileptonic B Decays

Vienna - 25 September 2024

based on GF, Paolo Gambino 2310.20324, + new results

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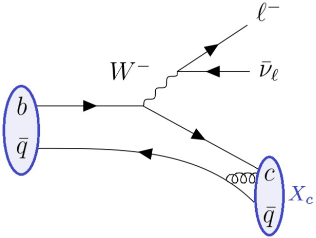

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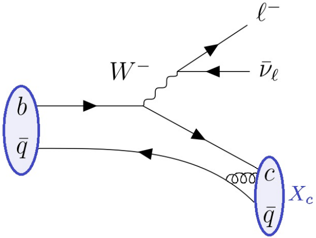

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is computed through a power expansion in $\Lambda_{\text{QCD}}/m_b \sim 0.1$

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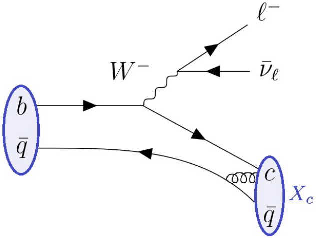

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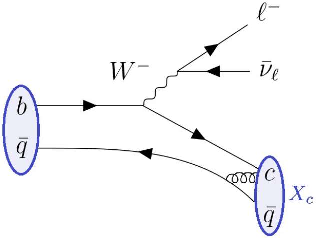

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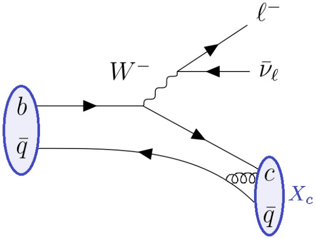

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They can also be extracted from **DATA**!

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So far only existed fits to (E_ℓ, m_X^2) moments [Bordone et al. '21] or q^2 separately [Bernlochner et al. '22].
First combined (E_ℓ, m_X^2, q^2) fit in [GF, Gambino '23]. Now updated results!

Theory State of the Art in $\bar{B} \to X_c \ell \bar{\nu}$

	dE_ℓ	dm_X^2	dq^2	Γ
1	α_s^2 [Melnikov 2008]; [Pak, Czarnecki 2008]	α_s^2 [Pak, Czarnecki 2008]	α_s^2 [Fael, Herren 2024]	α_s^3 [Fael, Schönwald, Steinhauser 2020]
$1/m_b^2$	α_s [Alberti, Ewerth, Gambino, Nandi 2012, 2013]	α_s [Alberti, Ewerth, Gambino, Nandi 2012, 2013]	α_s [Alberti, Ewerth, Gambino, Nandi 2012, 2013]	α_s
$1/m_b^3$	1 [Gremm, Kapustin 1997]	1 [Gremm, Kapustin 1997]	α_s [Mannel, Moreno Pivovarov 2021]	α_s [Mannel, Pivovarov 2019]
$1/m_b^{4,5}$, $1/(m_b^3 m_c^2)$	1 [Mannel, Turczyk, Uraltsev 2010]	1 [Mannel, Turczyk, Uraltsev 2010]	1 [Mannel, Milutin, Vos 2023]	1 [Mannel, Turczyk, Uraltsev 2010]

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Proliferation of non-perturbative parameters

RPI can reduce them, but restricted to dq^2

[Fael, Mannel, Vos 2018]

q^2 Moments

In practice measured at several lower cuts in q^2

$$M_n(q^2_{\text{cut}}) = \int_{q^2_{\text{cut}}}^{q^2_{\text{max}}} dq^2 \, \frac{d\Gamma}{dq^2} (q^2)^n \,, \qquad \langle q^{2n} \rangle = \frac{M_n(q^2_{\text{cut}})}{M_0(q^2_{\text{cut}})}$$

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coefficient functions have perturbative expansion (NNLO only for leading power $S^{(0)}$)

$$S^{(i)}(q^2) = S^{(i,0)}(q^2) + \frac{\alpha_s}{\pi} S^{(i,1)}(q^2) + \frac{\alpha_s^2}{\pi^2} S^{(i,2)}(q^2) + \mathcal{O}(\alpha_s^3)$$

full two-loop [Fael, Herren, '24] new implementation to the fit, before only BLM $\mathcal{O}(\alpha_s^2 \beta_0)$

Kinetic Scheme

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on-shell $\rightarrow$ kinetic scheme ($\mu_k = 1$ GeV, $\alpha_s^{(4)}(m_b) = 0.2185$)

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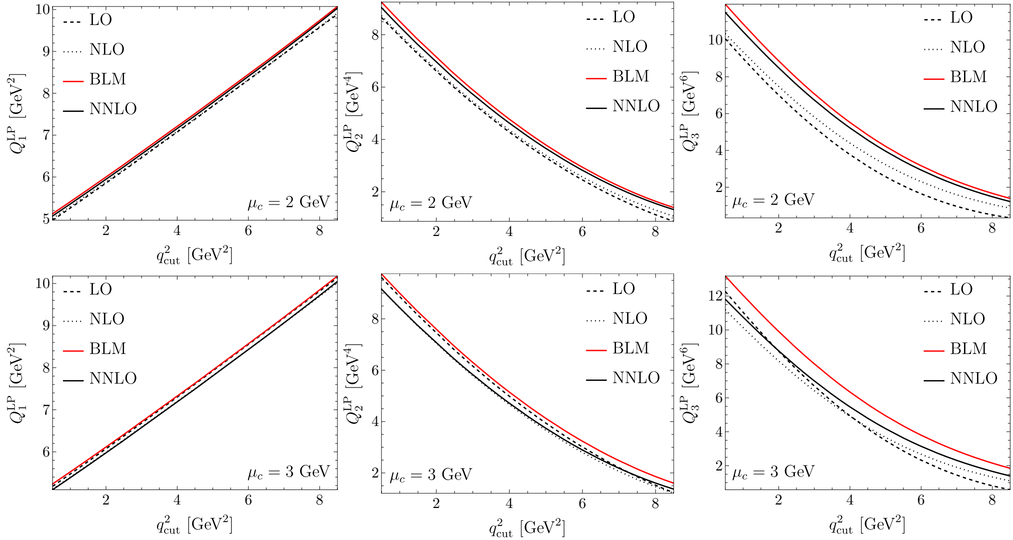
- m_c: on-shell $\to$ $\overline{\text{MS}}$ at μ_c

  $$m_c^{\text{OS}} = m_c(2\text{ GeV})\left(1 + 0.18_{\alpha_s} + 0.14_{\alpha_s^2}\right) = m_c(3\text{ GeV})\left(1 + 0.25_{\alpha_s} + 0.18_{\alpha_s^2}\right)$$

BLM vs Full NNLO



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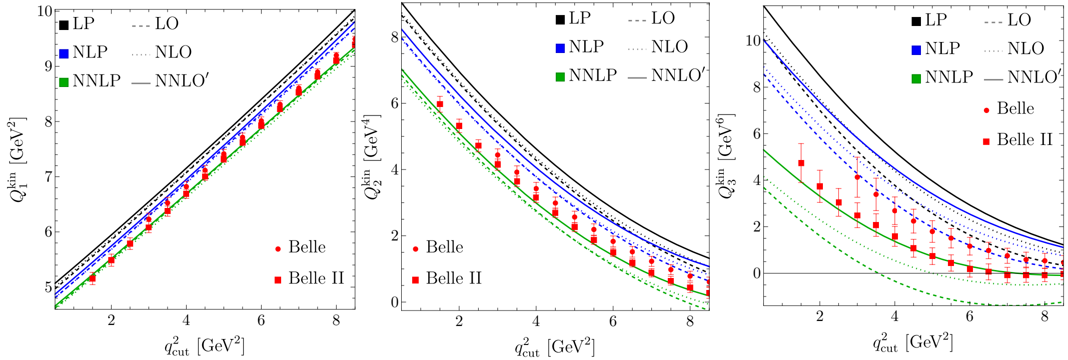
with HQE parameters from new **global** fit (preliminary!)



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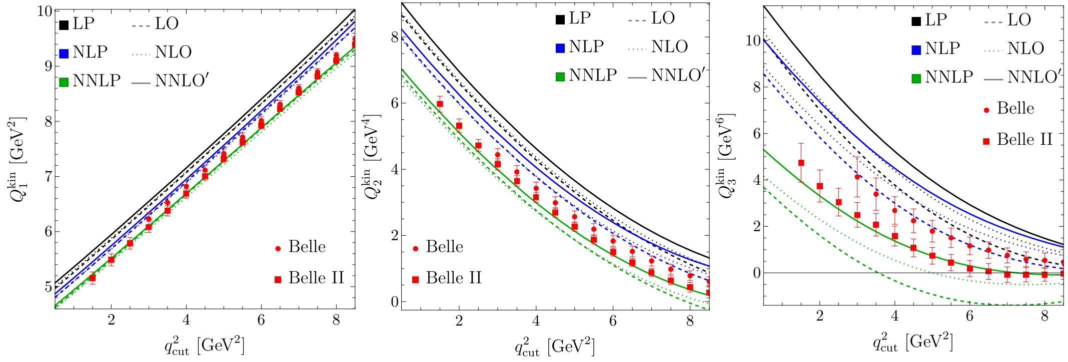

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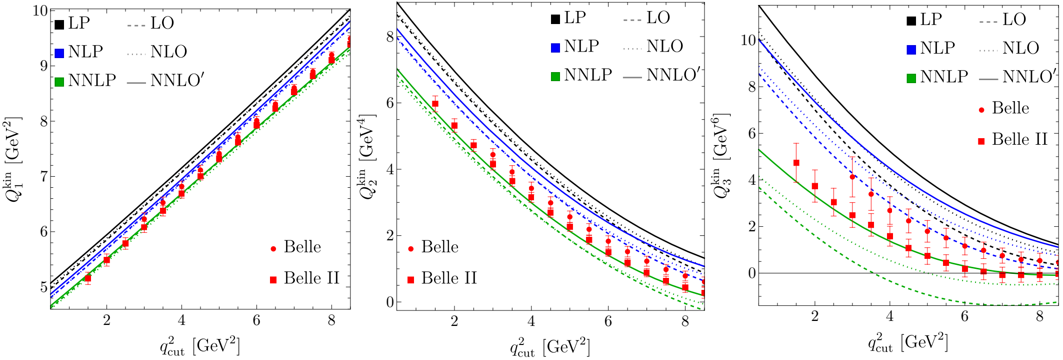

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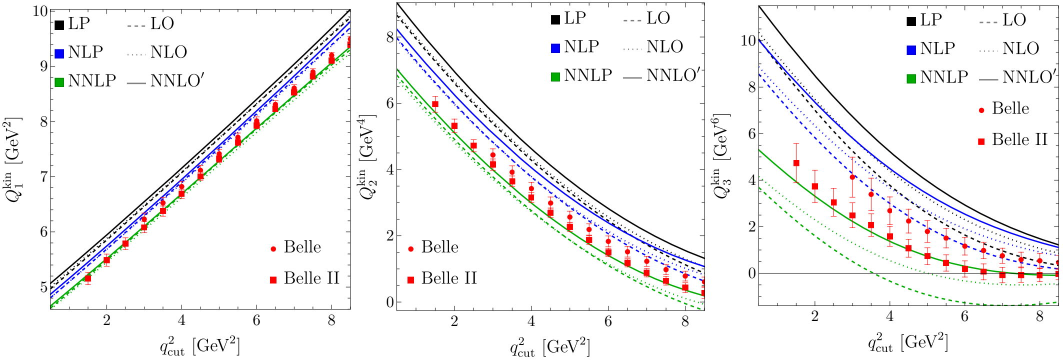

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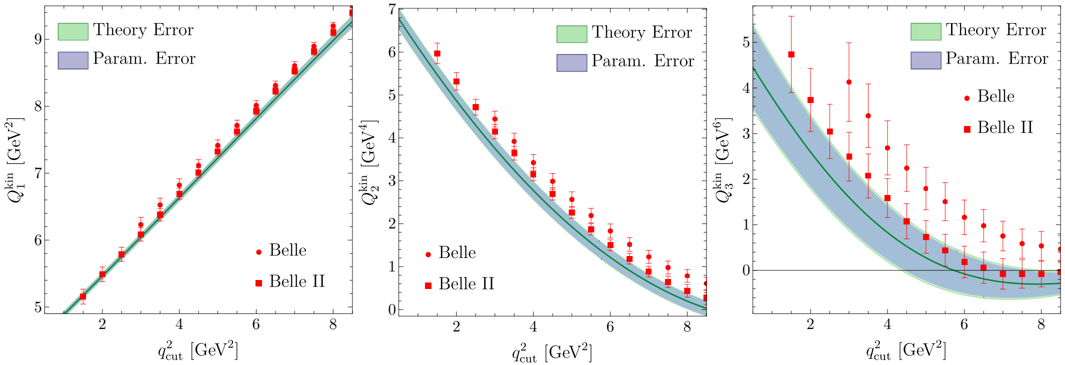
Inclusion of NLO and NNLO terms can have big impact on HQE parameters!

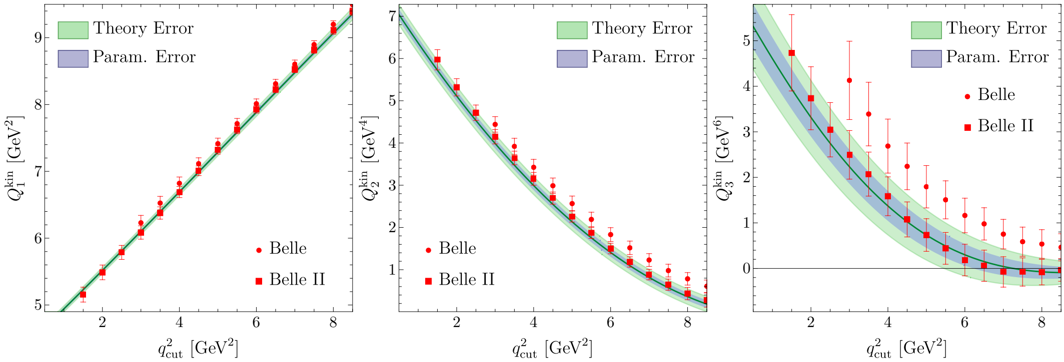
Fit Results

Fit to (E_ℓ, m_X^2) moments [Bordone, Capdevila, Gambino 2021]



q^2 moms probe different direction in parameter space
$\Rightarrow$ reduce parametric uncertainty!

Fit to (E_ℓ, m_X^2, q^2) moments (new, preliminary!)



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Fit Results (PRELIMINARY)

m_b^{kin}	$\overline{m}_c(2\,\text{GeV})$	μ_π^2	μ_G^2	ρ_D^3	ρ_{LS}^3	$\text{BR}_{c\ell\nu}$	$10^3\lvert V_{cb}\rvert$
4.572	1.090	0.430	0.282	0.161	−0.091	10.61	41.83
0.012	0.010	0.040	0.048	0.018	0.089	0.15	0.47
1	0.389	−0.229	0.561	−0.025	−0.181	−0.062	−0.422
	1	0.019	−0.238	−0.030	0.083	0.033	0.076
		1	−0.097	0.536	0.262	0.142	0.334
			1	−0.261	0.006	0.006	−0.260
				1	−0.019	0.022	0.139
					1	−0.011	0.067
						1	0.697
							1

reached a precision of 1.1% on $|V_{cb}|$ ($\chi^2/\text{dof} = 0.59$)

Big improvement in $\sigma_{\mu_\pi^2}$ (0.056 → 0.040) and $\sigma_{\rho_D^3}$ (0.031 → 0.018) w.r.t. (E_ℓ, m_X^2) fit

Impact of NNLO q^2: ρ_D^3 : $0.176 \to 0.161$ and $10^3|V_{cb}|$: $41.97 \to 41.83$

*other small improvements to the fit: inclusion of QED effects [Bigi et al. '23], ..., see [2310.20324] for details.

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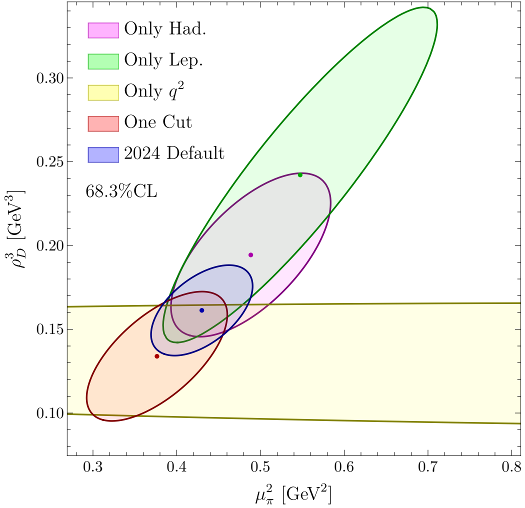

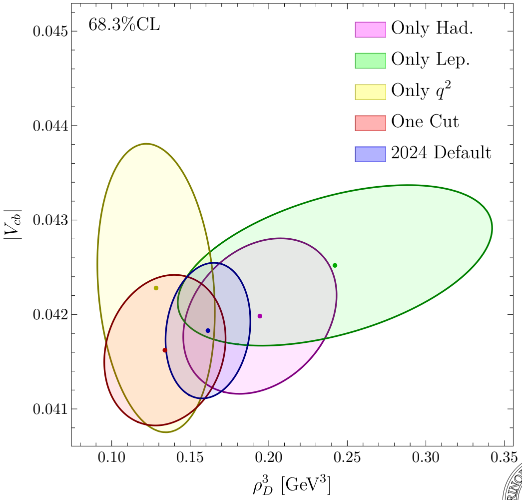

1σ regions. q^2 moments independent on μ_π^2

Summary & Outlook

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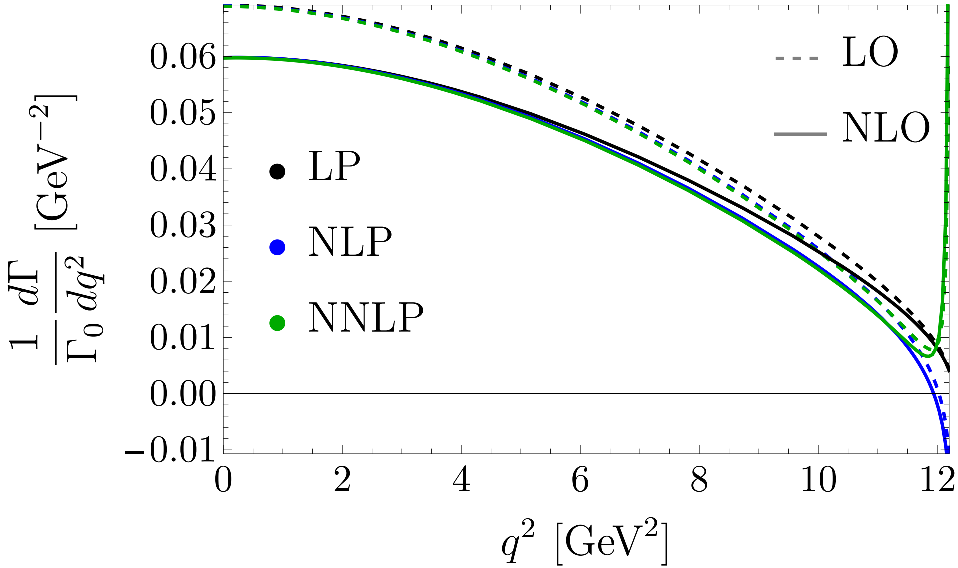
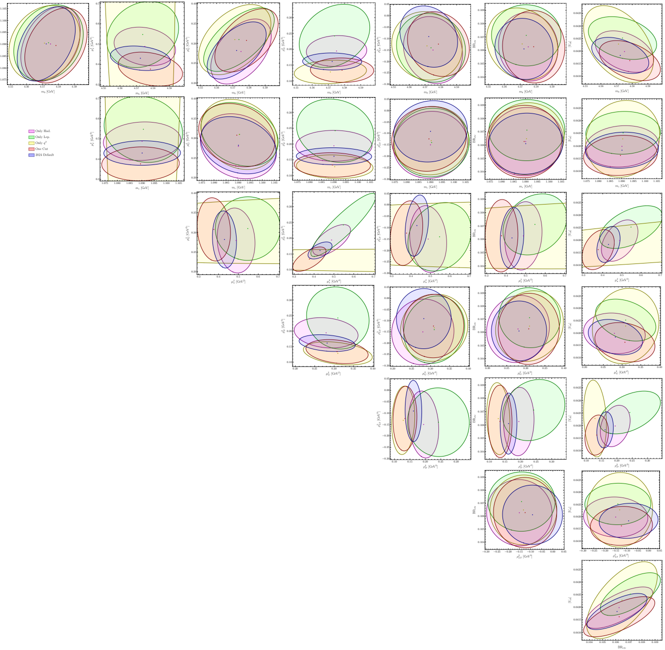
Thank You!

Backup Slides

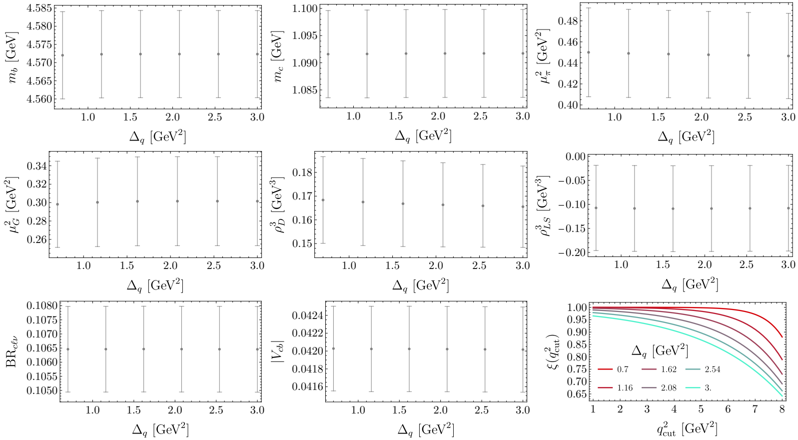


$\frac{1}{\Gamma_0}\frac{d\Gamma}{dq^2}$ [GeV^{-2}]
0.06
0.05
0.04
0.03
0.02
0.01
0.00
−0.01
0
2
4
6
8
10
12
q^2 [GeV^2]
LO
NLO
LP
NLP
NNLP



Only Had.
Only Lep.
Only q^2
One Cut
2024 Default

Theoretical Correlations



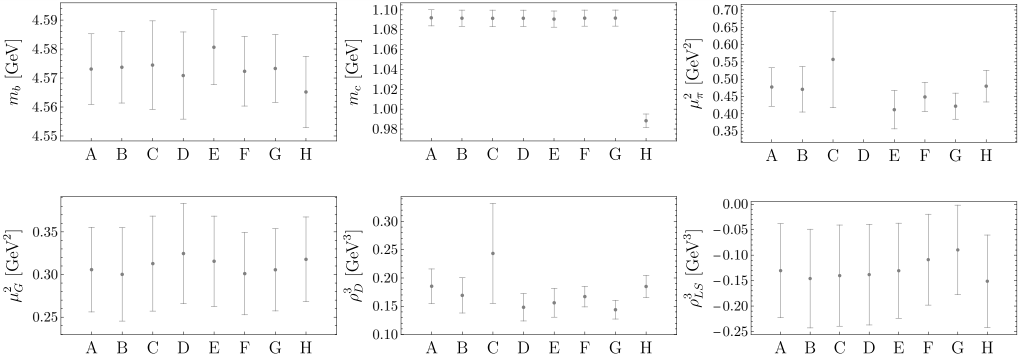
Correlations between different central moments set to 0

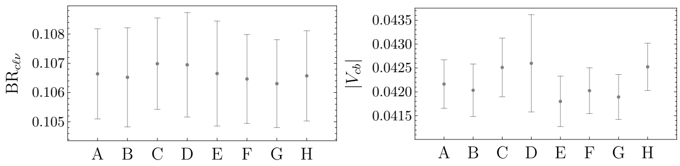
Correlations between same moments at 0.5 GeV^2 distance in q^2_{cut}:

$$\xi(q^2_{\text{cut}}) = 1 - \frac{1}{2} e^{-\frac{9\text{GeV}^2 - q^2_{\text{cut}}}{\Delta_q}}$$

q^2_{cut} dependent to take into account spectrum endpoint

Fit Variations





A: 2021 Default

B: Only Hadronic

C: Only Leptonic

D: Only q^2

E: One Cut

F: All Data

G: $\mu_s = m_b$

H: $\mu_c = 3$ GeV