

# Intrinsic Charm and Higher Order $1/m_b$ corrections in inclusive

$$B \rightarrow X_c \ell \bar{\nu}$$

Challenges in Semileptonic B Decays 2024 - Vienna

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in collaboration with  
Thomas Mannel<sup>1</sup> and K. Keri Vos<sup>2</sup>

based on: JHEP 02 (2024) 226, [2311.12002]

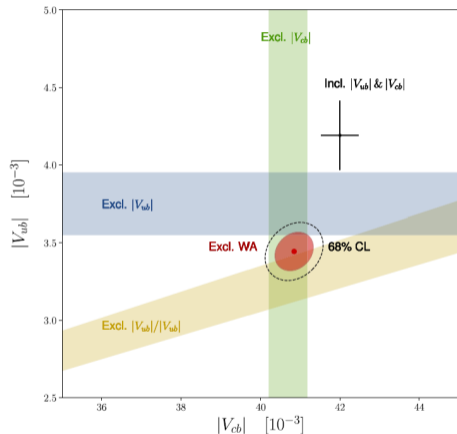
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# Motivation

- $b \rightarrow c l \bar{\nu}$  decays allow for studies of physics beyond the Standard Model and extraction of CKM parameters
- $V_{cb}$  is an important input for many Standard Model predictions
- Long-standing puzzle between different determinations
  - exclusive: known-final state, e.g.  $B \rightarrow D l \bar{\nu}$
  - **inclusive**: summed over all possible final states, e.g.  $B \rightarrow X_c l \bar{\nu}$



Bernlochner, Prim, Vos (*Eur. Phys. J. Spec. Top.* (2024))

# Setting up the Heavy Quark Expansion

- $m_b$  is large compared to  $\Lambda_{\text{QCD}} \rightarrow$  power series in  $1/m_b$

Chay, Georgi, bigi, Shifman, Uraltsev, Vainshtein, Manohar, Wise, Neubert, Mannel,...

$$\begin{aligned}d\Gamma &\propto \sum_X (2\pi)^4 \delta^4(P_B - P_X) |\langle X | \mathcal{H}_{\text{eff}} | B(v) \rangle|^2 \\ &= \int d^4x \langle B(v) | \mathcal{H}_{\text{eff}}^\dagger(x) \mathcal{H}_{\text{eff}}(0) | B(v) \rangle \\ &= 2 \text{Im} \int d^4x \langle B(v) | T \{ \mathcal{H}_{\text{eff}}^\dagger(x) \mathcal{H}_{\text{eff}}(0) \} | B(v) \rangle \\ &= 2 \text{Im} \int d^4x e^{-im_b v \cdot x} \langle B(v) | T \{ \tilde{\mathcal{H}}_{\text{eff}}^\dagger(x) \tilde{\mathcal{H}}_{\text{eff}}(0) \} | B(v) \rangle\end{aligned}$$

- Split the momentum of the heavy quark as  $p_b = m_b v + k$  where  $v^2 = 1$   
 $\rightarrow$  field redefinition of heavy quark field  $b(x) = e^{-im_b v \cdot x} b_v(x)$
- Expand in residual momentum  $k \sim iD$

- Perform Operator Product Expansion (OPE)

Chay, Georgi, bigi, Shifman, Uraltsev, Vainshtein, Manohar, Wise, Neubert, Mannel,...

$$d\Gamma = d\Gamma^{(3)} + \frac{1}{m_b^2} d\Gamma^{(5)} + \frac{1}{m_b^3} d\Gamma^{(6)} + \frac{1}{m_b^4} d\Gamma^{(7)} + \dots, \quad d\Gamma^{(n)} = \sum_i \mathcal{C}_i^{(n)} \langle B | \mathcal{O}_i^{(n)} | B \rangle$$

- $d\Gamma^{(n)}$  are power series in  $\mathcal{O}(\alpha_s)$
- $\mathcal{C}_i^{(n)}$  perturbative Wilson coefficients
- $\mathcal{O}^{(n)}$  local operators of dimension  $n \rightarrow$  chains of covariant derivatives  $iD$
- $\langle B | \mathcal{O}_i^{(n)} | B \rangle$  **non-perturbative matrix elements**  $\rightarrow$  are extracted from data

- The number of matrix elements increases very fast with the dimension:

$$d\Gamma = d\Gamma^{(3)} + \frac{1}{m_b^2}d\Gamma^{(5)} + \frac{1}{m_b^3}d\Gamma^{(6)} + \frac{1}{m_b^4}d\Gamma^{(7)} + \frac{1}{m_b^5}d\Gamma^{(8)} + \dots$$

- $d\Gamma^{(3)}$ : Partonic result ( $d\Gamma^{(4)} = 0$  due to Heavy Quark Symmetries)
- $d\Gamma^{(5)}$ : 2 parameters

$$2m_B\mu_\pi^2 = -\langle B|\bar{b}_v(iD)^2b_v|B\rangle$$

$$2m_B\mu_G^2 = \langle B|\bar{b}_v(-i\sigma^{\mu\nu})(iD_\mu)(iD_\nu)b_v|B\rangle$$

- $d\Gamma^{(6)}$ : 2 parameters

$$2m_B\rho_D^3 = \frac{1}{2}\langle B|\bar{b}_v[iD_\mu, [ivD, iD^\mu]]b_v|B\rangle$$

$$2m_B\rho_{LS}^3 = \frac{1}{2}\langle B|\bar{b}_v\{iD_\mu, [ivD, iD_\nu]\}(-i\sigma^{\mu\nu})b_v|B\rangle$$

- $d\Gamma^{(7)}$ : 9 parameters (at tree level) Mannel, Turczyk, Uraltsev [1009.4622]; Kobach, Pal [1704.00008]
- $d\Gamma^{(8)}$ : 18 parameters (at tree level) Mannel, Turczyk, Uraltsev [1009.4622]; Kobach, Pal [1704.00008]

- In HQE, choice of  $v_\mu$  is not unique Dugan, Golden, Grinstein, Chen, Luke, Manohar, Hill, Solon, Heinonen,...

# Reparametrization Invariance

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- Lorentz invariance of QCD  $\rightarrow$  **Reparametrization Invariance** (RPI) imposed under infinitesimal change  $v_\mu \rightarrow v_\mu + \delta v_\mu$ , leading to

$$\delta_{\text{RP}} v_\mu = \delta v_\mu, \quad \delta_{\text{RP}} iD_\mu = -m_b \delta v_\mu, \quad \delta_{\text{RP}} b_v(x) = im_b(x \cdot \delta v) b_v(x)$$

- Lorentz invariant quantity  $R(v) = \sum_{n=0}^{\infty} C_{\mu_1 \dots \mu_n}^{(n)}(v) \otimes \bar{b}_v(iD^{\mu_1} \dots iD^{\mu_n}) b_v$  Mannel, Vos [1802.09409]

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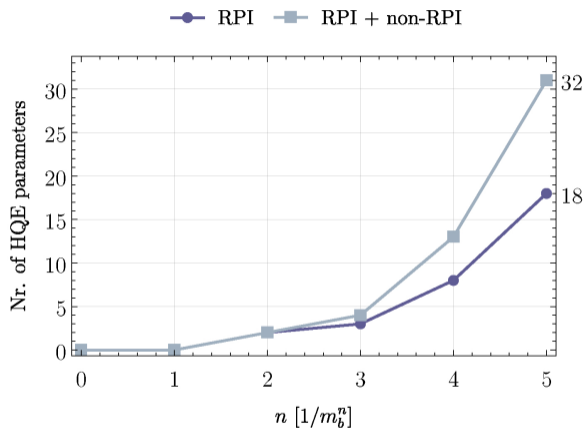
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- **RPI relates different orders**  $n$  in  $1/m_b$  expansion

$$\delta_{\text{RP}} C_{\mu_1 \dots \mu_n}^{(n)}(v) = m_b \delta v^\alpha \left( C_{\alpha \mu_1 \dots \mu_n}^{(n+1)}(v) + C_{\mu_1 \alpha \dots \mu_n}^{(n+1)}(v) + \dots + C_{\mu_1 \dots \mu_n \alpha}^{(n+1)}(v) \right)$$

- This allows us to find combinations of operators which are RPI

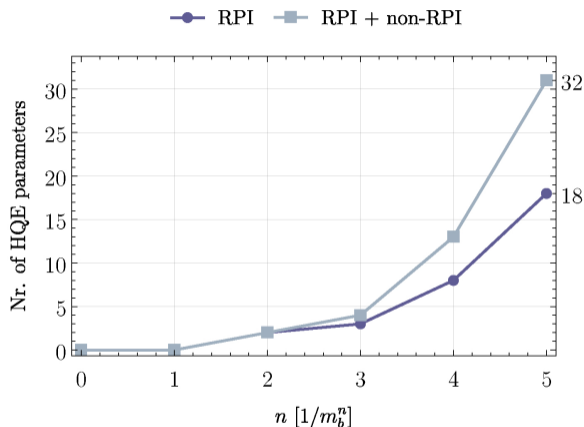


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- Up to  $1/m_b^4$ : total of 8 independent parameters Mannel, Vos [1802.09409]
- At  $1/m_b^5$ , we find 4 spin-indep. and 6 spin-dep. RPI operators:  
**Only 10 RPI operators** at dimension 8, instead of 18 in full basis Mannel, ISM, Vos [2311.12002]

# New inclusive $V_{cb}$ determination

- Need RPI observable: **dilepton invariant mass ( $q^2$ ) moments** used to extract  $|V_{cb}^{\text{incl}}|$   
( $q^2$ -cut needed due to experimental setup)

Bernlochner, Welsch, Fael, Olschewsky, Persson, von Tonder, Vos [2205.10274]

$$\langle (q^2)^n \rangle_{\text{cut}} = \frac{\int_{q^2 > q_{\text{cut}}^2} dq^2 (q^2)^n \frac{d\Gamma}{dq^2}}{\int_{q^2 > q_{\text{cut}}^2} dq^2 \frac{d\Gamma}{dq^2}}$$

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- RPI  $\rightarrow$  only depend on reduced set of RPI operators
- Data  $\rightarrow$  values for reduced set of RPI parameters up to  $1/m_b^4 \rightarrow Br(\bar{B} \rightarrow X_c \ell \bar{\nu}) \rightarrow$

$$|V_{cb}^{\text{incl}}| = (41.69 \pm 0.63) \times 10^{-3} \quad \text{Bernlochner, Vos, et al. [2205.10274]}$$

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- **First determination** of  $V_{cb}$  up to  $\mathcal{O}(1/m_b^4)$  and **first extraction** of  $1/m_b^4$  matrix elements from data
- Agreement at **1 – 2 $\sigma$  level** with previous  $\mathcal{O}(1/m_b^3)$  determinations

Finauri, Gambino [2310.20324]; Bordone, Capdevila, Gambino [2107.00604]; Alberti, Gambino, Healey, Nandi[1411.6560]; Gambino, Schwanda [1307.4551]

# Where do we currently stand?

| $\Gamma$                     | tree | $\alpha_s$ | $\alpha_s^2$ | $\alpha_s^3$ |
|------------------------------|------|------------|--------------|--------------|
| Partonic                     | ✓    | ✓          | ✓            | ✓            |
| $1/m_b^2$                    | ✓    | ✓          |              |              |
| $1/m_b^3$                    | ✓    | ✓          |              |              |
| $1/m_b^4$                    | ✓    |            |              |              |
| $m_b^{\text{kin}}/\bar{m}_c$ |      | ✓          | ✓            | ✓            |

| $\langle (q^2)^n \rangle$ | tree | $\alpha_s$ | $\alpha_s^2$ | $\alpha_s^3$ |
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| Partonic                  | ✓    | ✓          | ✓            |              |
| $1/m_b^2$                 | ✓    | ✓          |              |              |
| $1/m_b^3$                 | ✓    | ✓          |              |              |
| $1/m_b^4$                 | ✓    |            |              |              |

- **Green:** known perturbative corrections Jezabek, Kuhn (1989); Melnikov (2008); Pak, Czarnecki (2008); Becher, Boos, Lunghi (2007); Alberti, Gambino, Nandi (2014); Mannel, Pivovarov, Rosenthal (2015); Gambino, Healey, Turczyk (2016); Mannel, Pivovarov (2020); Fael, Schonwald, Steinhauser (2020, 2021); Fael, Herren (2024) → see talk by Matteo Fael at 14:00
- Next step in HQE is dimension 8:  $1/m_b^5$

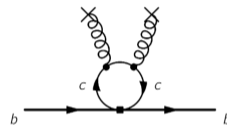
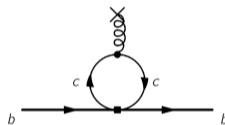
# Going higher in the $1/m_b$ expansion

- Dimension 8 contains: Bigi, Mannel, Turczyk, Uraltsev [0911.3322]

“genuine”  $\frac{1}{m_b^5}$

&

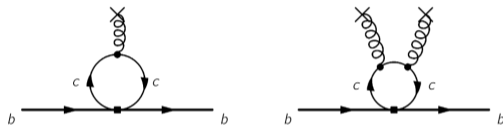
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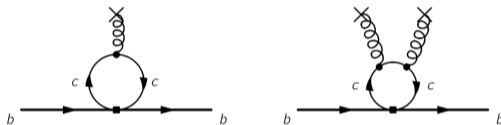
- Numerically:  $m_c^2 \sim m_b \Lambda_{\text{QCD}}$
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- Numerically:  $m_c^2 \sim m_b \Lambda_{\text{QCD}}$
- Dimension 8 contains enhanced terms which contribute like  $1/m_b^4$  terms
- We need  $1/m_b^3 \times 1/m_c^2$  contributions to complete the calculation at  $1/m_b^4$ 
  - We calculate the full dimension 8 contributions
  - We extract the **Intrinsic Charm** contribution
  - We find that only **1 combination of parameters** describes the IC in the  $q^2$ -moments

Mannel, ISM, Vos [2311.12002]

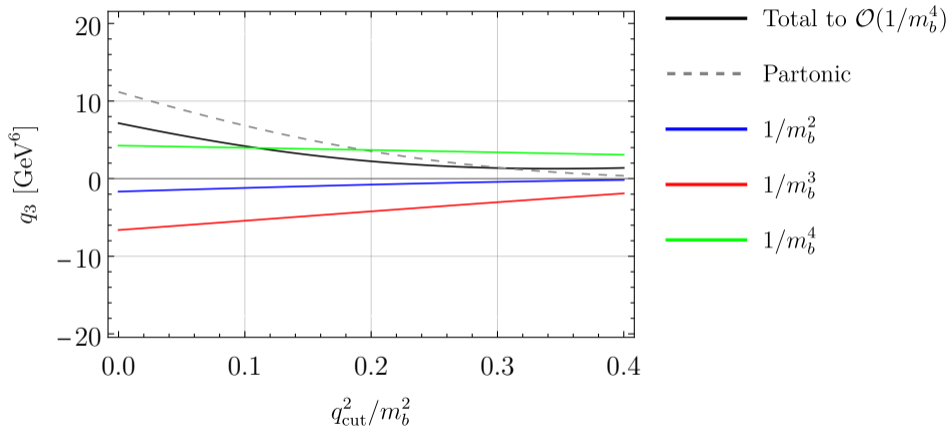
# Lowest-Lying-State Approximation

- We can employ the **LLSA**: Lowest-Lying-State Approximation to estimate the higher order matrix elements by linking them to lower order ones

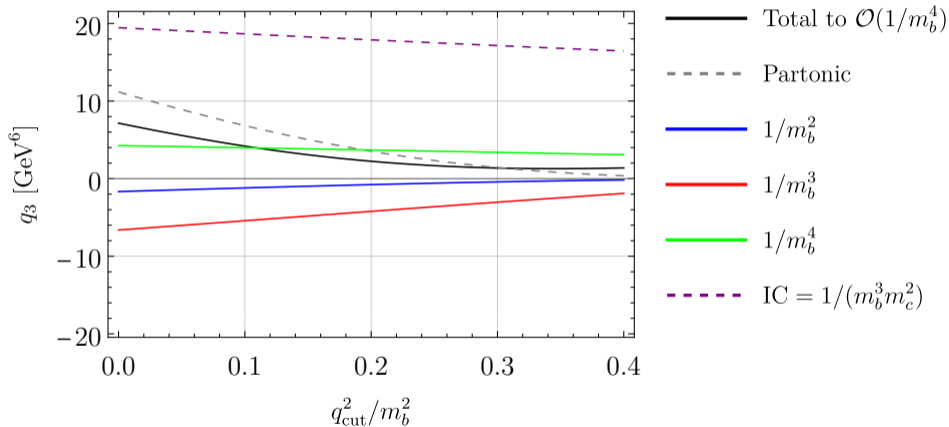
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$$\langle B | \bar{b} A C \Gamma b(0) | B \rangle = \frac{1}{2m_B} \sum_n \langle B | \bar{b} A b(0) | n \rangle \cdot \langle n | \bar{b} C \Gamma b(0) | B \rangle$$

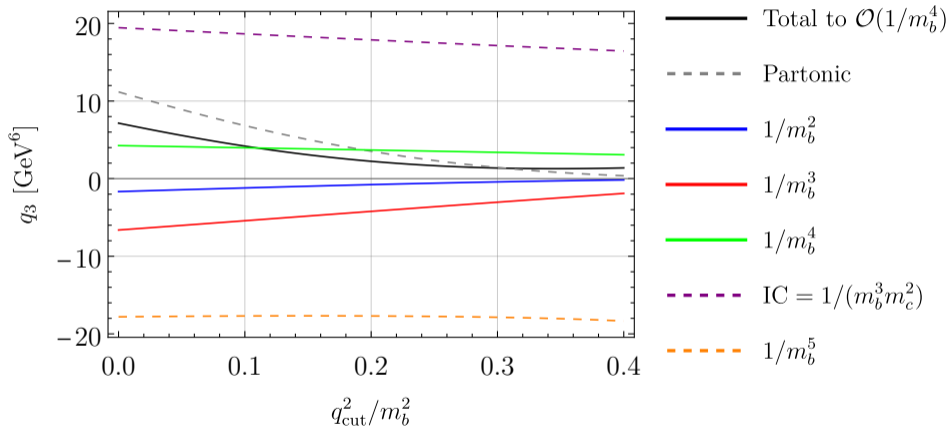
- $A = iD_{\mu_1} \dots iD_{\mu_k}$ ,  $C = iD_{\mu_{k+1}} \dots iD_{\mu_n}$



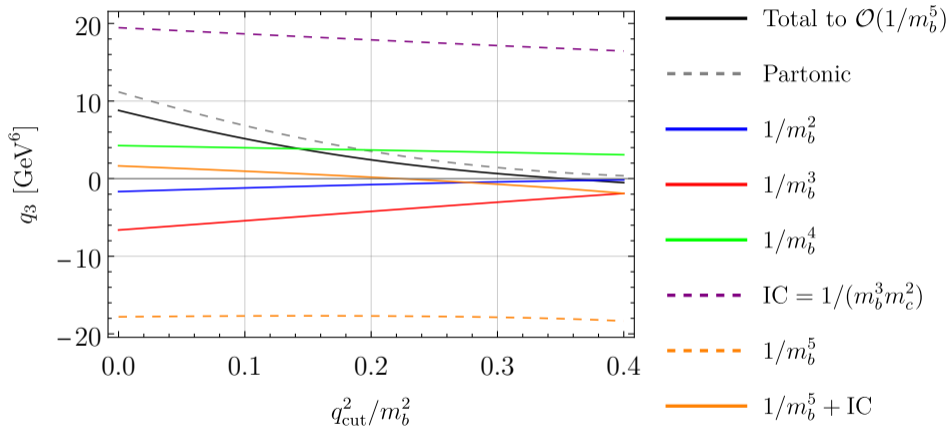
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- We find an **unexpectedly small** overall contribution of the dimension-8 operators

# Outlook

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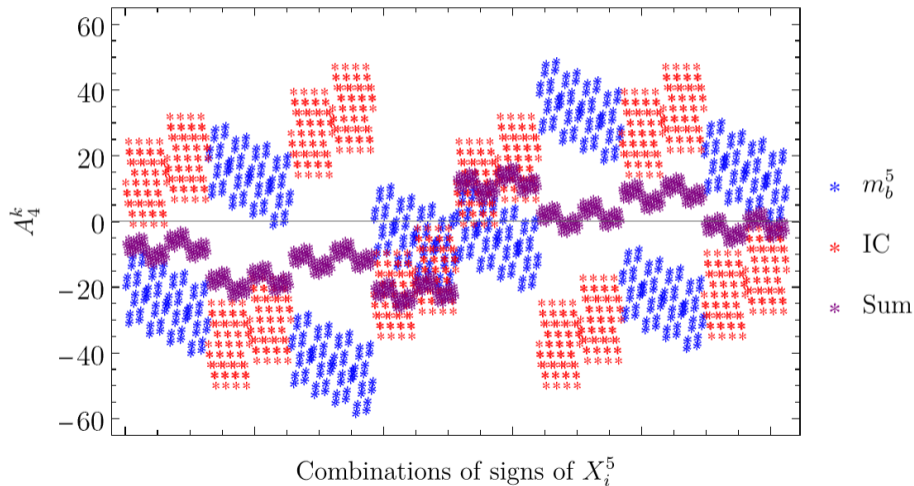
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Thank you for your attention

# Back-up: Cancellation without LLSA

- Assume for all dimension 8 operators that  $X_i^5 \sim \Lambda_{\text{QCD}}^5$  and vary signs:





- **Step 1:** expand charm propagator ( $Q^\mu = m_b v^\mu$ )

$$\begin{aligned} -iS_{\text{BGF}} &= \frac{1}{Q + iD - m_c} \\ &= \frac{1}{Q - m_c} - \frac{1}{Q - m_c} (iD) \frac{1}{Q - m_c} \\ &\quad + \frac{1}{Q - m_c} (iD) \frac{1}{Q - m_c} (iD) \frac{1}{Q - m_c} + \dots \end{aligned}$$

- **Step 2:** insert in forward matrix element

$$\begin{aligned} T = & \left[ \Gamma \frac{1}{Q - m_c} \Gamma^\dagger \right]_{\alpha\beta} \langle \bar{b}_\alpha b_\beta \rangle \\ & - \left[ \Gamma \frac{1}{Q - m_c} \gamma^\mu \frac{1}{Q - m_c} \Gamma^\dagger \right]_{\alpha\beta} \langle \bar{b}_\alpha (iD_\mu) b_\beta \rangle \\ & + \left[ \Gamma \frac{1}{Q - m_c} \gamma^\mu \frac{1}{Q - m_c} \gamma^\nu \frac{1}{Q - m_c} \Gamma^\dagger \right]_{\alpha\beta} \langle \bar{b}_\alpha (iD_\mu) (iD_\nu) b_\beta \rangle \\ & + \dots \end{aligned}$$

# Back-up: Calculation of forward matrix element

- **Step 3:** Determine **Trace formula**

- Start at dimension 8  $\rightarrow$  lengthy, but systemically calculable
- Compute dim-7, including  $1/m_b$  correction through e.o.m.

$$(ivD)b_v = -\frac{1}{2m_b}(iD)(iD)b_v$$

- ...
- Compute dim-3, including corrections up to  $1/m_b^5$

$$\langle \bar{b}_\alpha b_\beta \rangle = 2m_B \left( \frac{1+v}{4} + \frac{1}{8m_b^2}(\mu_G^2 - \mu_\pi^2) + \mathcal{O}(1/m_b^6) \right)_{\beta\alpha}$$

- Need full (non-RPI) set of **basic parameters** up to  $1/m_b^5$
- **Step 4:** Compute the trace with the geometric series