Kolya: an open-source package for inclusive semileptonic B decays

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= ArXiv: 2409.15007 & preliminary =

with Matteo Fael, Ilija Milutin, Florian Bernlochner and Markus Prim

Why inclusive decays?

- Set up OPE and heavy quark expansion
- Well established framework
- Extract important CKM parameters $|V_{cb}|, |V_{ub}|$ (and $|V_{cs}?)$
- Extract power corrections from data (inputs to $B \to X_u$, $B \to X_s \ell\ell$ and lifetimes)
- Cross check of exclusive decays

Inclusive $B \to X_c$ decays

Inclusive Decays: Heavy Quark Expansion

- b quark mass is large compared to $\Lambda_{\rm QCD}$
- Setting up the HQE: momentum of b quark: $p_b = m_b v + k$, expand in $k \sim iD$
- Optical Theorem $\to$ (local) Operator Product Expansion (OPE)

$$d\Gamma = d\Gamma_0 + \frac{d\Gamma_1}{m_b} + \frac{d\Gamma_2}{m_b^2} + \dots \qquad d\Gamma_i = \sum_k C_i^{(k)} \left\langle B|O_i^{(k)}|B\right\rangle$$

  - $C_i^{(k)}$ perturbative Wilson coefficients
  - $\langle B|\dots|B\rangle$ non-perturbative matrix elements $\to$ string of iD
  - operators contain chains of covariant derivatives

HQE elements: $\langle B|\mathcal{O}_i^{(n)}|B\rangle = \langle B|\bar{b}_v(iD_\mu)\dots(iD_{\mu_n})b_v|B\rangle$

- Currently extracted from data

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- Currently extracted from data
- Progress on the lattice Juetner et al. [2305.14092]

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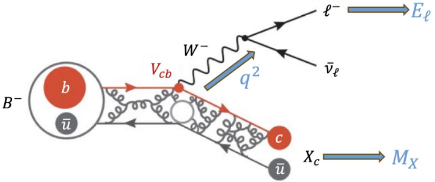
- Γ_2 :μ_π^2 and μ_G^2 at $1/m_b^2$
- Γ_3 :ρ_D^3 and ρ_{LS}^3 at $1/m_b^3$
- Many more at $1/m_b^{4,5}$ Mannel, Turczyk, Uraltsev, JHEP 1010 (2011) 109 (Talk by Ilija)

Moments of the spectrum

BABAR, PRD 68 (2004) 111104; BABAR, PRD 81 (2010) 032003; Belle, PRD 75 (2007) 032005. Pic from M. Fael

Non-perturbative matrix elements obtained from moments of differential rate



$$\langle O^n \rangle_{\rm cut} = \frac{\int_{\rm cut} dO\, O^n \frac{d\Gamma}{dO}}{\int_{\rm cut} dO \frac{d\Gamma}{dO}}$$

$M_X^2 = (p_B - q)^2$, $E_\ell = v_B \cdot p_\ell$ and $q^2 = (p_\nu + p_\ell)^2$

hadronic mass, lepton energy and q^2 moments

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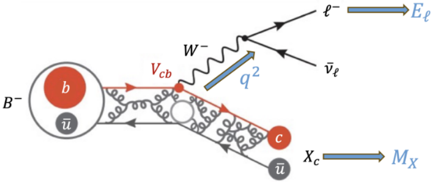

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hadronic mass, lepton energy and q^2 moments

- Different phase space cuts give additional (correlated) observables
- $\mu_\pi^2, \mu_G^2, \rho_D^3 + \cdots$ extracted from data $\to$ total rate $\to |V_{cb}|$

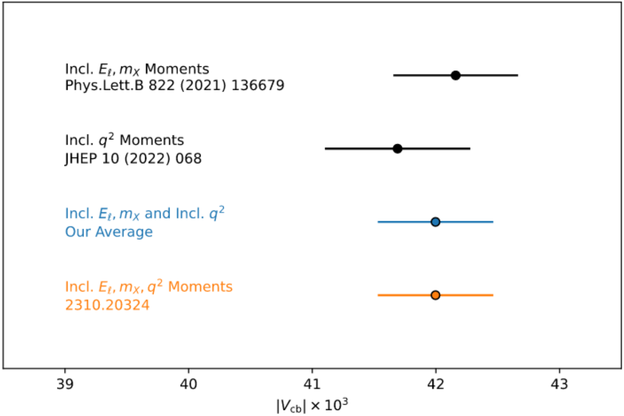
Moments of the spectrum

Moments and total rate are double expansion in α_s and HQE parameters

$$\begin{aligned} L_i =& \frac{1}{\Gamma_0} \int_{E_l \geq E_{\rm cut}} \mathrm{d}E_l \, \mathrm{d}q_0 \, \mathrm{d}q^2 \, (E_l)^i \, \frac{\mathrm{d}^3\Gamma}{\mathrm{d}q^2 \, \mathrm{d}q_0 \, \mathrm{d}E_l} \\ =& (m_b)^i \left[L_i^{(0)} + L_i^{(1)} \frac{\alpha_s(\mu_s)}{\pi} + L_i^{(2)} \left(\frac{\alpha_s(\mu_s)}{\pi} \right)^2 + \frac{\mu_\pi^2}{m_b^2} \left(L_{i,\pi}^{(0)} + L_{i,\pi}^{(1)} \frac{\alpha_s(\mu_s)}{\pi} \right) \right. \\ & + \frac{\mu_G^2(\mu_b)}{m_b^2} \left(L_{i,G}^{(0)} + L_{i,G}^{(1)} \frac{\alpha_s(\mu_s)}{\pi} \right) + \frac{\rho_D^3(\mu_b)}{m_b^3} \left(L_{i,D}^{(0)} + L_{i,D}^{(1)} \frac{\alpha_s(\mu_s)}{\pi} \right) \\ & \left. + \frac{\rho_{LS}^3(\mu_b)}{m_b^3} \left(L_{i,LS}^{(0)} + L_{i,LS}^{(1)} \frac{\alpha_s(\mu_s)}{\pi} \right) + O\left(\frac{1}{m_b^4} \right) \right] , \end{aligned}$$

Summary of $|V_{cb}|$ inclusive (pre Vienna)

Fael, Prim, KKV, Eur. Phys. J. Spec. Top. (2024). https://doi.org/10.1140/epjs/s11734-024-01090-w



- Need new (branching ratio) measurements!

Experimental Inclusive Prospects

Belle II Physics Week https://indico.belle2.org/event/9402/overview

- New hadronic mass, lepton energy and q^2 moments
- Updated branching ratio measurements (with q^2 cut)*
- Unconventional cuts (Lepton energy moments with q^2 cut?)?
- Forward-Backward asymmetry?

*RPI observable = reduced set of parameters

NEW: Inclusive decays: The Kolya package

Kolya package, Fael, Milutin, KKV [2409.15007]

Open source Python package:
https://gitlab.com/vcb-inclusive/kolya

- HQE predictions for several observables:
  - Centralized $\langle E_\ell \rangle$ moments
  - Centralized $\langle q^2 \rangle$ moments
  - Centralized $\langle M_X^2 \rangle$ moments
  - Total rate + branching ratio with kinematic cut

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Features:

- Includes power corrections up to $1/m_b^5$ Mannel, Milutin, KKV [2311.1200]
- Employs kinetic scheme for m_b and $\overline{\mathrm{MS}}$ for m_c
- Interface with CRunDec for automatic RGE evolution Chetyrkin, Kuhn, Steinhauser, Smidth, Herren

QCD corrections in Kolya

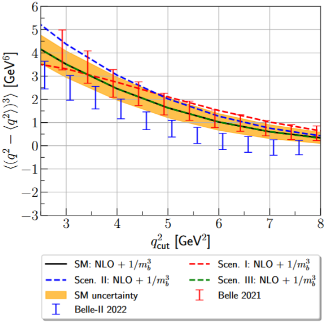
Nir, Pak, Downlin, Egner, Fael, Steinhauser, Gambino, Manohar, Block, Becher, Alberti, Mannel, Turczyk, Dassinger, Schonwald.

$\Gamma_{\rm sl}$	tree	α_s	α_s^2	α_s^3
Partonic				['11]
μ_π^2, μ_G^2		['06, '12,'13,'15]		
ρ_D^3, ρ_{LS}^3		['21]		
$1/m_b^4, 1/m_b^5$	['06,'10,'18,'23]			
$q_n(q^2_{\rm cut})$	tree	α_s	α_s^2	
Partonic			['24]	
μ_G^2, μ_π^2		['12,'13]		
ρ_D^3, ρ_{LS}		['21]		
$1/m_b^4, 1/m_b^5$	['18, '23]			
$\ell_n(E_{\rm cut}), h_n(E_{\rm cut})$	tree	α_s	$\alpha_s^2\beta_0$	α_s^2
Partonic			['05]	['08]*
μ_G^2, μ_π^2		['07,'13]		
ρ_D^3				
$1/m_b^4, 1/m_b^5$	['06, '10,'23]			

- See talk Matteo Fael
- *only known for fixed m_c/m_b and lepton energy cuts

New Physics?

Fael, Rahimi, KKV [2208.04282]



- NP would also influence the moments of the spectrum [Never tested!]
- Requires a simultaneous fit of hadronic parameters and NP

New Physics predictions with Kolya

Fael, Rahimi, KKV [2208.04282]

$P_{L(R)} = 1/2\,(1 \mp \gamma_5)$ and $\sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu]$

$$\mathcal{H}_{\text{eff}} = \frac{4 G_F V_{cb}}{\sqrt{2}} \left[(1 + C_{V_L})\, O_{V_L} + \sum_{i = V_R, S_L, S_R, T} C_i\, O_i \right],$$

$$\begin{aligned} O_{V_{L(R)}} &= (\bar{c}\gamma_\mu P_{L(R)} b)\,(\bar{\ell}\gamma^\mu P_L \nu_\ell)\,, \\ O_{S_{L(R)}} &= (\bar{c} P_{L(R)} b)\,(\bar{\ell} P_L \nu_\ell) \\ O_T &= (\bar{c}\,\sigma_{\mu\nu} P_L b)\,(\bar{\ell}\,\sigma^{\mu\nu} P_L \nu_\ell)\,. \end{aligned}$$

$$\begin{aligned} \langle \mathcal{M} \rangle = \xi_{\text{SM}} &+ |C_{V_R}|^2\, \xi_{\text{NP}}^{\langle V_R, V_R \rangle} + |C_{S_L}|^2\, \xi_{\text{NP}}^{\langle S_L, S_L \rangle} + |C_{S_R}|^2\, \xi_{\text{NP}}^{\langle S_R, S_R \rangle} + |C_T|^2\, \xi_{\text{NP}}^{\langle T, T \rangle} \\ &+ \text{Re}((C_{V_L} - 1)\, C_{V_R}^*)\, \xi_{\text{NP}}^{\langle V_L, V_R \rangle} + \text{Re}(C_{S_L}\, C_{S_R}^*)\, \xi_{\text{NP}}^{\langle S_L, S_R \rangle} + \text{Re}(C_{S_L}\, C_T^*)\, \xi_{\text{NP}}^{\langle S_L, T \rangle} \\ &+ \text{Re}(C_{S_R}\, C_T^*)\, \xi_{\text{NP}}^{\langle S_R, T \rangle}\,, \end{aligned}$$

Expanded moments in terms of C_i

Kolya: Installation and features

Fael, Milutin, KKV [2409.15007], https://gitlab.com/vcb-inclusive/kolya.git

```
$: git clone https://gitlab.com/vcb-inclusive/kolya.git
$: cd kolya
$: pip3 install
```

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- Provide example Jupyter notebooks (see also examples in backup)
- Several cross-checks with literature performed
- Default up to $1/m_b^3$. Higher orders included via flagmb4$=1$ and flagmb5$=1$
- Implemented both "RPI" and "historical" (perp) basis
- LLSA predictions for the HQE elements implemented

Fael, Milutin, KKV [2409.15007], https://gitlab.com/vcb-inclusive/kolya.git

Total Rate

We define the total rate as

$$\Gamma_{\rm sl} = \frac{G_F^2 (m_b^{\rm kin})^5}{192\pi^3} |V_{cb}|^2 X$$

The coefficients X is a function of the quark masses, α_s, the HQE parameters and the Wilson coefficients. It is evaluated by the function `X_Gamma_KIN_MS(par, hqe, wc)`

[5]:
```
hqe = kolya.parameters.HQE_parameters(
    muG = 0.306,
    rhoD = 0.185,
    rhoLS = -0.13,
    mupi = 0.477,
)
wc  = kolya.parameters.WCoefficients()
kolya.TotalRate.X_Gamma_KIN_MS(par,hqe,wc)
```

[5]:
```
0.539225163728085
```

The branching ratio is given by the function `BranchingRatio_KIN_MS(Vcb,par,hqe,wc)`

[6]:
```
Vcb = 42.2e-2
kolya.TotalRate.BranchingRatio_KIN_MS(Vcb,par,hqe,wc)
```

[6]:
```
10.555834162102016
```

Kolya: Installation and features

Fael, Milutin, KKV [2409.15007], https://gitlab.com/vcb-inclusive/kolya.git. Herren, Fael [2403.03976]

```
>>> kolya.Q2moments.moment_1_KIN_MS(8.0, par, hqe, wc, flag_DEBUG=1)
Q2moment n. 1 LO =  9.148659808170105
Q2moment n. 1 NLO = api * -1.319532010835962
Q2moment n. 1 NNLO = api^2 * -9.616956902561078
Q2moment n. 1 NLO pw = api * -0.7873907726673756
Q2moment n. 1 NNLO from NLO pw = api^2 * 8.39048437244325
```

- Includes new NNLO corrections (see talk by Matteo)
- NNLO and NLO to power corrections can be turned off

First extraction of HQE elements from q^2 moments with Kolya

with Markus, Florian, Ilija and Matteo [in progress]

Inclusive fits with Kolya

Bernlochner, Welsch, Fael, Olschewsky, Persson, van Tonder, KKV [2205.10274], Bordone, Capdevila, Gambino [2107.00604], Gambino, Schwanda [2014], Finauri, Gambino [2023] Prim, Milutin, Fael, Bernlochner, KKV [in progress]

New:
Use Kolya + Experimental measurements → $|V_{cb}|$ and HQE parameters

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This talk:
Preliminary results using q^2 moments (with NNLO)

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Challenge: How to deal with theoretical uncertainties?

Theoretical uncertainties

Bernlochner, Welsch, Fael, Olschewsky, Persson, van Tonder, KKV [2205.10274], Bordone, Capdevila, Gambino [2107.00604], Gambino, Schwanda [2014], Finauri, Gambino [2023]

- Vary scale of α_s to account for perturbative corrections
- Vary m_b^{kin} and scale of charm μ_c

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"Known" uncertainties

- Vary scale of α_s to account for perturbative corrections
- Vary $m_b^{\rm kin}$ and scale of charm μ_c

Missing higher order HQE parameters

- Vary ρ_D^3 by 30% to account for missing HQE parameters
- Vary μ_G^2 by 20% to account for missing $\alpha_s\times$ HQE parameters
- Assume theoretical correlations!
  - Strong correlations between different cuts, no correlation between different models see Gambino, Schwanda, Finauri
  - Flexible correlation added as nuisance parameter see Bernlochner, Fael, KKV

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Can we use the $1/m_b^{4,5}$ corrections to better access the theory uncertainty?

Preliminary fit setup

Preliminary! Prim, Milutin, Fael, Bernlochner, KKV [in progress]

- Nuisance parameters: scale α_s, μ_c and m_b^{kin}
- Include (μ_π^2), μ_G^2 and $\rho_D^3, \ldots$ in the likelihood
- Sample nuisance parameter from uniform distribution
- Refit with new sets of nuisance parameters
- Check how strongly the parameters of interest scatter

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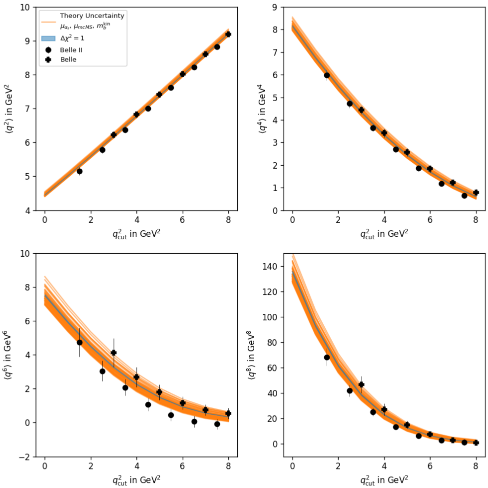
- Sample nuisance parameter from uniform distribution
- Refit with new sets of nuisance parameters
- Check how strongly the parameters of interest scatter

Do not include uncertainty on ρ_D^3 but check order by order if the fit improves

Refitting the data

Preliminary! Refitting at $O3$



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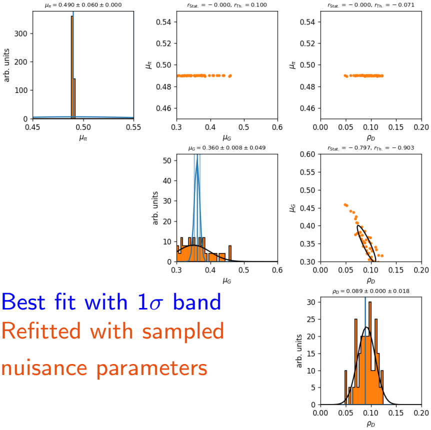
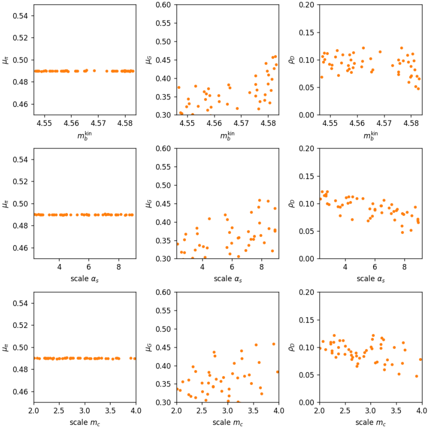




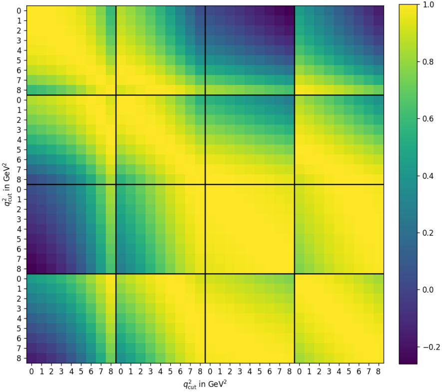
Best fit with 1σ band

Refitted with sampled nuisance parameters

Use central limits theorem to extract Gaussian uncertainty from flat priors

Theoretical correlations

Preliminary! Prim, Milutin, Fael, Bernlochner, KKV [in progress]



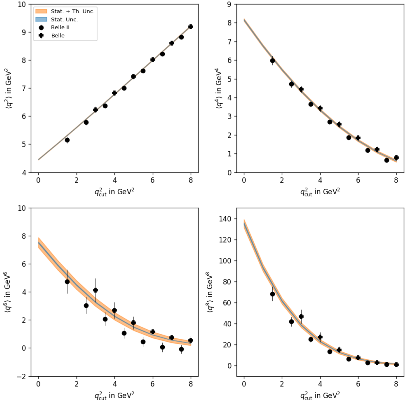
- Large correlations between different cuts
- Correlations between different moments!

Preliminary! Prim, Milutin, Fael, Bernlochner, KKV [in progress]

q^2 moments only, including NNLO

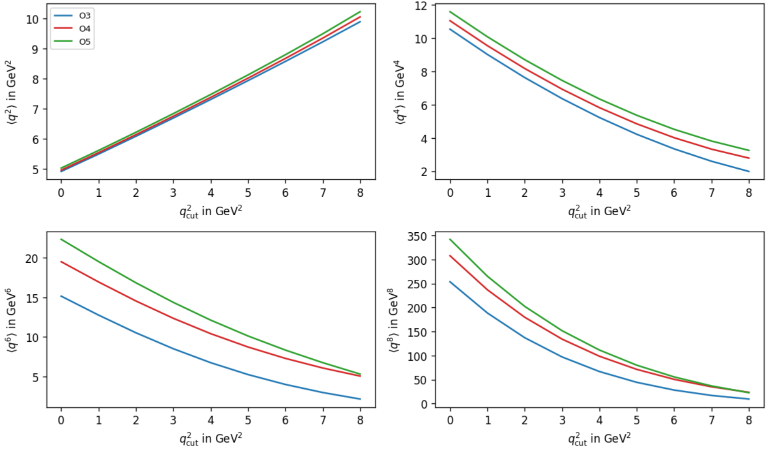
	Central	Fit Unc.	Theory Unc.	Tot Unc.	Precision [%]
μ_π^2	0.490	0.060	0.000	0.060	8.2
μ_G^2	0.360	0.008	0.050	0.050	7.2
ρ_D^3	0.089	0.000	0.018	0.018	5.0

- μ_π^2 does not enter at this order
- No fit uncertainty on ρ_D^3
- External constraint: $\mu_G^2 = 0.36 \pm 0.07$
- Very bad $\chi^2/d.o.f = 800/51$



Effect of higher-order corrections

Kolya



Using LLSA inputs for the HQE parameters and NNLO corrections

Preliminary fit results at $O4$

Preliminary! Prim, Milutin, Fael, Bernlochner, KKV [in progress]

	Central	Fit Unc.	Theory Unc.	Tot Unc.	Precision [%]
μ_π^2	0.492	0.060	0.001	0.060	8.2
μ_G^2	0.579	0.025	0.102	0.105	5.5
ρ_D^3	0.075	0.016	0.021	0.027	2.8
r_E^4	-0.025	0.005	0.003	0.006	3.8
r_G^4	-0.223	0.085	0.056	0.102	2.2
s_E^4	-0.126	0.265	0.163	0.311	0.4
s_B^4	-0.308	0.191	0.064	0.202	1.5
s_{qB}^4	-0.834	0.613	0.215	0.649	1.3

- $(\mu_\pi^2)^2$ and $(\mu_\pi^2)\mu_G^2$ enter!
- Use LLSA ansatz for $1/m_b^4$ with 50% additional uncertainty and 0.05 GeV^4
- Pushes μ_G^2 up by 2σ
- Improves to $\chi^2/d.o.f = 245/51$

Preliminary fit results at $O5$

Preliminary! Prim, Milutin, Fael, Bernlochner, KKV [in progress]

	Central	Fit Unc.	Theory Unc.	Tot Unc.	Precision [%]
μ_π^2	0.463	0.058	0.004	0.059	7.9
μ_G^2	0.547	0.003	0.141	0.141	3.9
ρ_D^3	0.040	0.000	0.059	0.059	0.68
r_E^4	0.015	0.000	0.018	0.018	0.84
r_G^4	0.284	0.002	0.151	0.151	1.9
s_E^4	-0.03	0.005	0.045	0.045	0.7
s_B^4	-0.088	0.007	0.255	0.255	0.4
s_{qB}^4	-1.326	0.022	0.895	0.895	1.5

- $(\mu_G^2)\rho_D^3$ and $(\mu_\pi^2)\rho_D^3$ enter!
- Use LLSA ansatz for $1/m_b^5$ with 50% additional uncertainty and 0.05 GeV^4
- $\chi^2/d.o.f = 248/51$

Hoped convergences does not seem to happen?

What about $|V_{cb}|$?

- Add total rate to extract $|V_{cb}|$
- Use $\mathcal{B} = (10.48 \pm 0.13)\%$ Bernlochner, Fael, KKV

Order	$\lvert V_{cb}\rvert \cdot 10^{-3}$	Moments Unc.	Rate Unc.
O_3	41.64	0.27	0.34
O_3 no NNLO	41.95	0.32	0.35
O_4	42.03	0.34	0.35
O_4 no NNLO	42.24	0.24	0.35
O_5	42.00	0.31	0.36
O_5 no NNLO	42.06	0.35	0.34

Outlook: fits in inclusive decays

- First analysis of q^2 moments with NNLO corrections [preliminary!]
- Relax LLSA Ansatz for higher terms
- Full analysis of all moments ongoing

How to handle missing higher orders and theory uncertainties?

Outlook for Kolya

First version of Kolya available!

Plans to expand Kolya with:

- QED effects Bigi, Bordone, Gambino, Haisch, Piccione [2308.02849]
- Exact results for NNLO corrections to E_ℓ and M_X moments Talk by Matteo
- RGE of HQE paramters
- NLO corrections to HQE parameters for E_ℓ and M_X moments

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And additional observables:

- Forward-backward asymmetry
- $R_X = \Gamma_{B\to X_c \tau \bar{\nu}_\tau} / \Gamma_{B\to X_c l \bar{\nu}_l}$
- Lifetimes
- Predictions for the decay into charmless final states $B \to X_u l \bar{\nu}_l$
- Inclusive D decays Mannel, Fael, KKV [1910.05234] Talk by Alex

Any other suggestions?

Backup

Kolya: Installation and features

Fael, Milutin, KKV [2409.15007], https://gitlab.com/vcb-inclusive/kolya.git

[1]:
```
import kolya
import numpy as np
```

Physical parameters

Physical parameters like quark masses like $m_b^{\rm kin}(\mu_{WC})$, $\overline{m}_c(\mu_c)$ and $\alpha_s(\mu_s)$ are declared in the class `parameters.physical_parameters`. Initialization set default values

[2]:
```
par = kolya.parameters.physical_parameters()
par.show()
```

```
bottom mass:       mbkin( 1.0  GeV)     =  4.563  GeV
charm mass:        mcMS( 3.0  GeV)      =  0.989  GeV
coupling constant: alpha_s( 4.563  GeV) =  0.2182
```

In order to set the quark masses at scales different from the default ones in a consistent way, we include the method FLAG2023 which internally use CRunDec. For instance, we set the quark masses at a scale $\mu_{WC} = \mu_c = 2$ GeV in the following way:

[3]:
```
par = kolya.parameters.physical_parameters()
par.FLAG2023(scale_mcMS=2.0, scale_mbkin=2.0)
par.show()
```

```
bottom mass:       mbkin( 2.0  GeV)     =  4.295730717092438  GeV
charm mass:        mcMS( 2.0  GeV)      =  1.0940623249384822  GeV
coupling constant: alpha_s( 4.563  GeV) =  0.21815198098622618
```

Kolya: Installation and features

Fael, Milutin, KKV [2409.15007], https://gitlab.com/vcb-inclusive/kolya.git

HQE parameters

Non-perturbative matrix elements in the HQE are declared in the class `parameters.HQE_parameters`. This class is defined in the historical basis of hep-ph/1307.4551. By default they are initalized to zero. We can set their values in the following way

```
hqe = kolya.parameters.HQE_parameters(
    muG = 0.306,
    rhoD = 0.185,
    rhoLS = -0.13,
    mupi = 0.477,
)
hqe.show()
```

```
mupi  =  0.477  GeV^2
muG   =  0.306  GeV^2
rhoD  =  0.185  GeV^3
rhoLS =  -0.13  GeV^3
```

```
hqe.show(flagmb4=1)
```

```
mupi  =  0.477  GeV^2
muG   =  0.306  GeV^2
rhoD  =  0.185  GeV^3
rhoLS =  -0.13  GeV^3

m1 =  0  GeV^4
m2 =  0  GeV^4
m3 =  0  GeV^4
m4 =  0  GeV^4
m5 =  0  GeV^4
m6 =  0  GeV^4
m7 =  0  GeV^4
m8 =  0  GeV^4
m9 =  0  GeV^4
```

Kolya: Installation and features

Fael, Milutin, KKV [2409.15007], https://gitlab.com/vcb-inclusive/kolya.git

- Implemented both "RPI" and "historical" (perp) basis
- LLSA predictions for the HQE elements implemented
- Gives "predictions" for moments and total rate

The classes `parameters.LSSA_HQE_parameters` and `parameters.LSSA_HQE_parameters_RPI` store the same HQE parameters as `parameters.HQE_parameters` and `parameters.HQE_parameters_RPI` in the "perp" and RPI basis respectively up to $1/m_b^5$. They are initialized to values predicted by the 'lowest-lying state saturation ansatz' (LSSA).

In [14]:

```
hqe_perp = kolya.parameters.LSSA_HQE_parameters()
hqe_RPI = kolya.parameters.LSSA_HQE_parameters_RPI()

print(r'LSSA prediction for rhoD in the perp basis: ', hqe_perp.rhoD)
print('LSSA prediction for m1 in the perp basis: ', hqe_perp.m1, '\n')

print('LSSA prediction for rhoD in the RPI basis: ', hqe_RPI.rhoD)
print('LSSA prediction for rEtilde in the RPI basis: ', hqe_RPI.rEtilde)
```

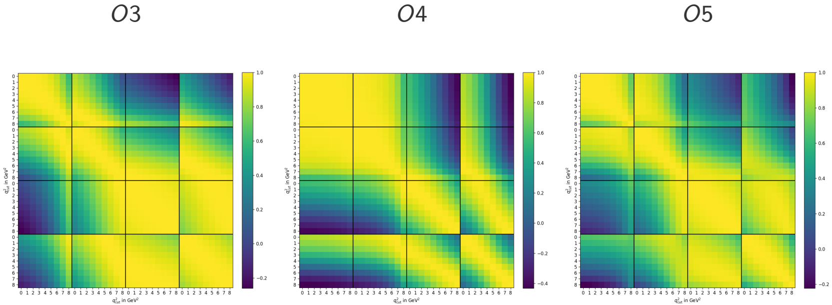
Out [14]:

```
LSSA prediction for rhoD in the perp basis:  0.231
LSSA prediction for m1 in the perp basis:  0.126

LSSA prediction for rhoD in the RPI basis:  0.205
LSSA prediction for rEtilde in the RPI basis:  0.098
```

Theoretical correlations

Preliminary! Prim, Milutin, Fael, Bernlochner, KKV [in progress]



- Order of the analysis changes the correlations
- Large correlations between different cuts
- Correlations between different moments!

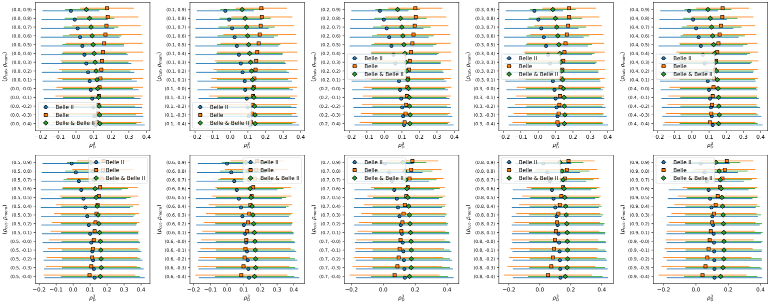
What about theory correlations?

Bernlochner, Welsch, Fael, Olschewsky, Persson, van Tonder, KKV [2205.10274]

- Flexible correlations between moments $\rho_{\rm mom}$ and different cuts $\rho_{\rm cut}$

$$\rho_n[q_n(q_A^2) - q_n(q_B^2)] = \rho_{\rm cut}^x \qquad x = \frac{|q_A^2 - q_B^2|}{0.5\mathrm{GeV}^2}$$

- Included by adding a penalty term to the χ^2
- Scan over large range of values + add as nuisance parameters in fit
- V_{cb} stable w.r.t. theory correlations



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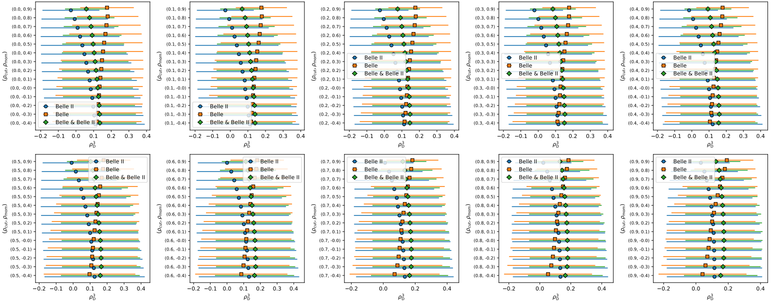
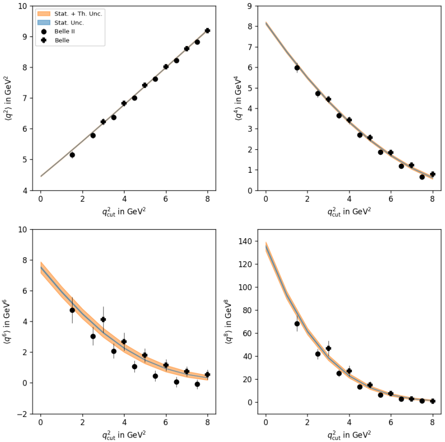
- Included by adding a penalty term to the χ^2
- Scan over large range of values + add as nuisance parameters in fit
- V_{cb} uncertainty includes large range of correlations



Fit details

Preliminary!



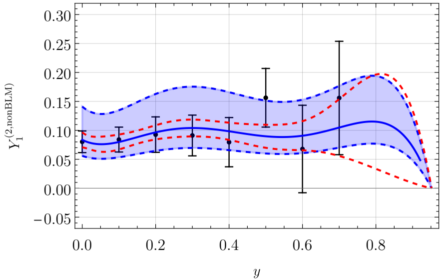
$$q^2_{\rm cut} = 0~{\rm GeV}^2\,, \qquad m_b^{\rm kin} = 4.573~{\rm GeV}\,, \qquad \bar{m}_c(2~{\rm GeV}) = 1.092~{\rm GeV}\,. \qquad ($$

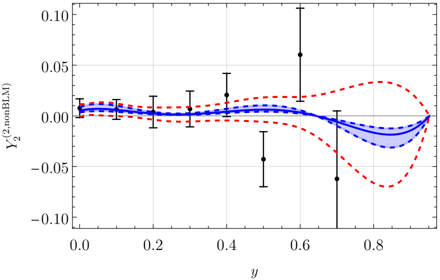
We then find[8]

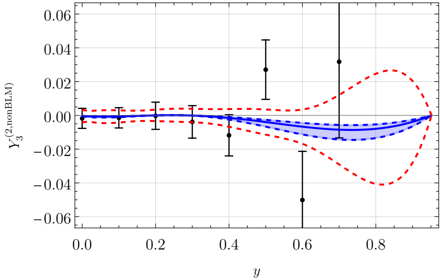
$$\begin{aligned}
q_1 = \frac{m_b^2}{\mu_3}\Big(&0.22\mu_3 - 0.57\frac{\mu_G^2}{m_b^2} - 1.4\frac{(\mu_G^2)^2}{m_b^4\mu_3} - 5.5\frac{\tilde\rho_D^3}{m_b^3} + 16\frac{\tilde r_E^4}{m_b^4} - 5.7\frac{r_G^4}{m_b^4} - 1.7\frac{\tilde s_E^4}{m_b^4}\\
&+ 0.097\frac{s_B^4}{m_b^4} - 0.064\frac{s_{qB}^4}{m_b^4} - 24\frac{\mu_G^2\tilde\rho_D^3}{m_b^5\mu_3} - 19\frac{X_1^5}{m_b^5} + 18\frac{X_2^5}{m_b^5} - 15\frac{X_3^5}{m_b^5} + 2.3\frac{X_4^5}{m_b^5}\\
&+ 6.5\frac{X_5^5}{m_b^5} + 0.91\frac{X_6^5}{m_b^5} - 7.0\frac{X_7^5}{m_b^5} + 8.0\frac{X_8^5}{m_b^5} + 5.2\frac{X_9^5}{m_b^5} - 4.4\frac{X_{10}^5}{m_b^5} + 0.047\frac{X_{\rm IC}^5}{m_b^3 m_c^2}\Big)\,,\\
q_2 = \frac{m_b^4}{\mu_3}\Big(&0.022\mu_3 - 0.12\frac{\mu_G^2}{m_b^2} - 0.61\frac{(\mu_G^2)^2}{m_b^4\mu_3} - 1.6\frac{\tilde\rho_D^3}{m_b^3} + 7.7\frac{\tilde r_E^4}{m_b^4} - 2.1\frac{r_G^4}{m_b^4} - 0.66\frac{\tilde s_E^4}{m_b^4}\\
&+ 0.20\frac{s_B^4}{m_b^4} - 0.082\frac{s_{qB}^4}{m_b^4} - 12\frac{\mu_G^2\tilde\rho_D^3}{m_b^5\mu_3} - 20\frac{X_1^5}{m_b^5} + 15\frac{X_2^5}{m_b^5} - 22\frac{X_3^5}{m_b^5} + 3.2\frac{X_4^5}{m_b^5}\\
&+ 4.2\frac{X_5^5}{m_b^5} - 0.32\frac{X_6^5}{m_b^5} - 4.9\frac{X_7^5}{m_b^5} + 7.6\frac{X_8^5}{m_b^5} + 1.8\frac{X_9^5}{m_b^5} - 2.3\frac{X_{10}^5}{m_b^5} + 0.030\frac{X_{\rm IC}^5}{m_b^3 m_c^2}\Big)\,,\\
q_3 = \frac{m_b^6}{\mu_3}\Big(&0.0012\mu_3 - 0.013\frac{\mu_G^2}{m_b^2} - 0.24\frac{(\mu_G^2)^2}{m_b^4\mu_3} - 0.34\frac{\tilde\rho_D^3}{m_b^3} + 2.9\frac{\tilde r_E^4}{m_b^4} - 0.56\frac{r_G^4}{m_b^4} - 0.19\frac{\tilde s_E^4}{m_b^4}\\
&+ 0.093\frac{s_B^4}{m_b^4} - 0.035\frac{s_{qB}^4}{m_b^4} - 5.8\frac{\mu_G^2\tilde\rho_D^3}{m_b^5\mu_3} - 12\frac{X_1^5}{m_b^5} + 9.3\frac{X_2^5}{m_b^5} - 17\frac{X_3^5}{m_b^5} + 2.5\frac{X_4^5}{m_b^5}\\
&+ 2.0\frac{X_5^5}{m_b^5} - 0.42\frac{X_6^5}{m_b^5} - 2.8\frac{X_7^5}{m_b^5} + 5.1\frac{X_8^5}{m_b^5} + 0.10\frac{X_9^5}{m_b^5} - 1.0\frac{X_{10}^5}{m_b^5} + 0.016\frac{X_{\rm IC}^5}{m_b^3 m_c^2}\Big)\,,\\
q_4 = \frac{m_b^8}{\mu_3}\Big(&0.0010\mu_3 - 0.012\frac{\mu_G^2}{m_b^2} - 0.10\frac{(\mu_G^2)^2}{m_b^4\mu_3} - 0.22\frac{\tilde\rho_D^3}{m_b^3} + 1.6\frac{\tilde r_E^4}{m_b^4} - 0.33\frac{r_G^4}{m_b^4} - 0.11\frac{\tilde s_E^4}{m_b^4}\\
&+ 0.047\frac{s_B^4}{m_b^4} - 0.018\frac{s_{qB}^4}{m_b^4} - 2.6\frac{\mu_G^2\tilde\rho_D^3}{m_b^5\mu_3} - 7.8\frac{X_1^5}{m_b^5} + 7.6\frac{X_2^5}{m_b^5} - 17\frac{X_3^5}{m_b^5} + 2.5\frac{X_4^5}{m_b^5}\\
&+ 1.2\frac{X_5^5}{m_b^5} - 0.26\frac{X_6^5}{m_b^5} - 2.5\frac{X_7^5}{m_b^5} + 4.7\frac{X_8^5}{m_b^5} - 0.40\frac{X_9^5}{m_b^5} - 1.0\frac{X_{10}^5}{m_b^5} + 0.015\frac{X_{\rm IC}^5}{m_b^3 m_c^2}\Big)\,. \qquad (
\end{aligned}$$

NNLO contributions to lepton energy moments

Biswas, Melnikov [0911.4142], Gambino [1107.3100]. Fael, Milutin, KKV [2409.15007]

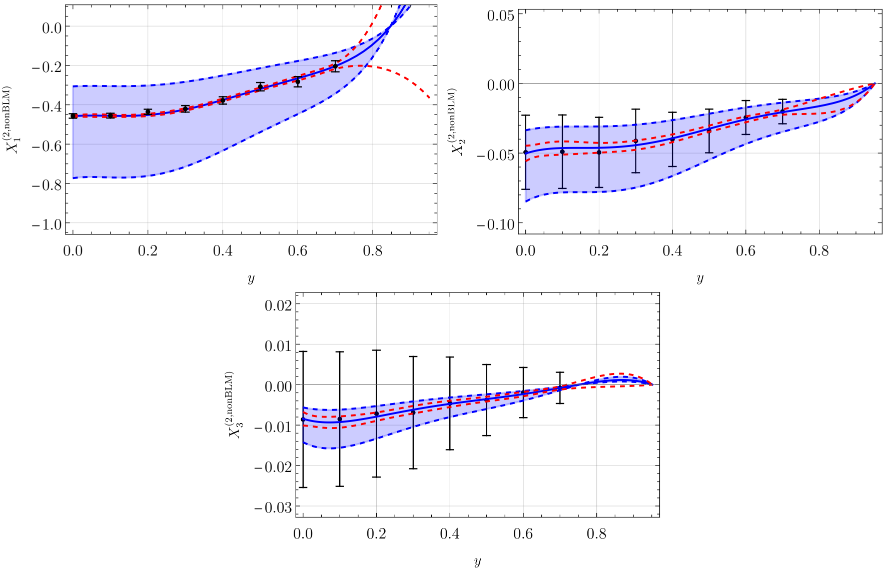






NNLO contributions to M_X moments

Biswas, Melnikov [0911.4142], Gambino [1107.3100]. Fael, Milutin, KKV [2409.15007]



State-of-the-art in inclusive $b \to c$

Jezabek, Kuhn, NPB 314 (1989) 1; Melnikov, PLB 666 (2008) 336; Pak, Czarnecki, PRD 78 (2008) 114015; Becher, Boos, Lunghi, JHEP 0712 (2007) 062; Alberti, Gambino, Nandi, JHEP 1401 (2014) 147; Mannel, Pivovarov, Rosenthal, PLB 741 (2015) 290; Fael, Schonwald, Steinhauser, Phys Rev. D 104 (2021) 016003; Fael, Schonwald, Steinhauser, Phys Rev. Lett. 125 (2020) 052003; Fael, Schonwald, Steinhauser, Phys Rev. D 103 (2021) 014005,

$$\Gamma \propto |V_{cb}|^2 m_b^5 \left[\Gamma_0 + \Gamma_0^{(1)} \frac{\alpha_s}{\pi} + \Gamma_0^{(2)} \left(\frac{\alpha_s}{\pi}\right)^2 + \Gamma_0^{(3)} \left(\frac{\alpha_s}{\pi}\right)^3 + \frac{\mu_\pi^2}{m_b^2} \left(\Gamma^{(\pi,0)} + \frac{\alpha_s}{\pi}\Gamma^{(\pi,1)}\right) \right.$$
$$\left. + \frac{\mu_G^2}{m_b^2} \left(\Gamma^{(G,0)} + \frac{\alpha_s}{\pi}\Gamma^{(G,1)}\right) + \frac{\rho_D^3}{m_b^3}(\Gamma^{(D,0)} + \Gamma_0^{(1)} \left(\frac{\alpha_s}{\pi}\right)) + \mathcal{O}\left(\frac{1}{m_b^4}\right) + \cdots \right)$$

- Include terms up to $1/m_b^4$* see also Gambino, Healey, Turczyk [2016]
- α_s^3 to total rate and kinetic mass Fael, Schonwald, Steinhauser [2020, 2021]
- $\alpha_s \rho_D^3$ for total rate Mannel, Pivovarov [2020]
- Kinetic mass scheme 1411.6560,1107.3100; hep-ph/0401063

E_ℓ, M_X moments:

$$|V_{cb}|_{\rm incl}^{\rm BCG} = (42.16 \pm 0.51) \times 10^{-3}$$

q^2 moments*:

$$|V_{cb}|_{\rm incl}^{q^2} = (41.69 \pm 0.63) \times 10^{-3}$$

Gambino, Schwanda, PRD 89 (2014) 014022;
Alberti, Gambino et al, PRL 114 (2015) 061802;
Bordone, Capdevila, Gambino, Phys.Lett.B 822 (2021) 136679; Bernlochner, Welsch, Fael, Olschewsky, Persson, van Tonder, KKV [2205.10274]