PRECISION QCD CORRECTIONS TO SEMILEPTONIC B DECAYS

Matteo Fael (CERN)

Challenges in Semileptonic B Decays - Vienna - Sept. 25th 2024

in collaboration with F. Herren, J. Usovitsch

Funded by the European Union

OUTLINE

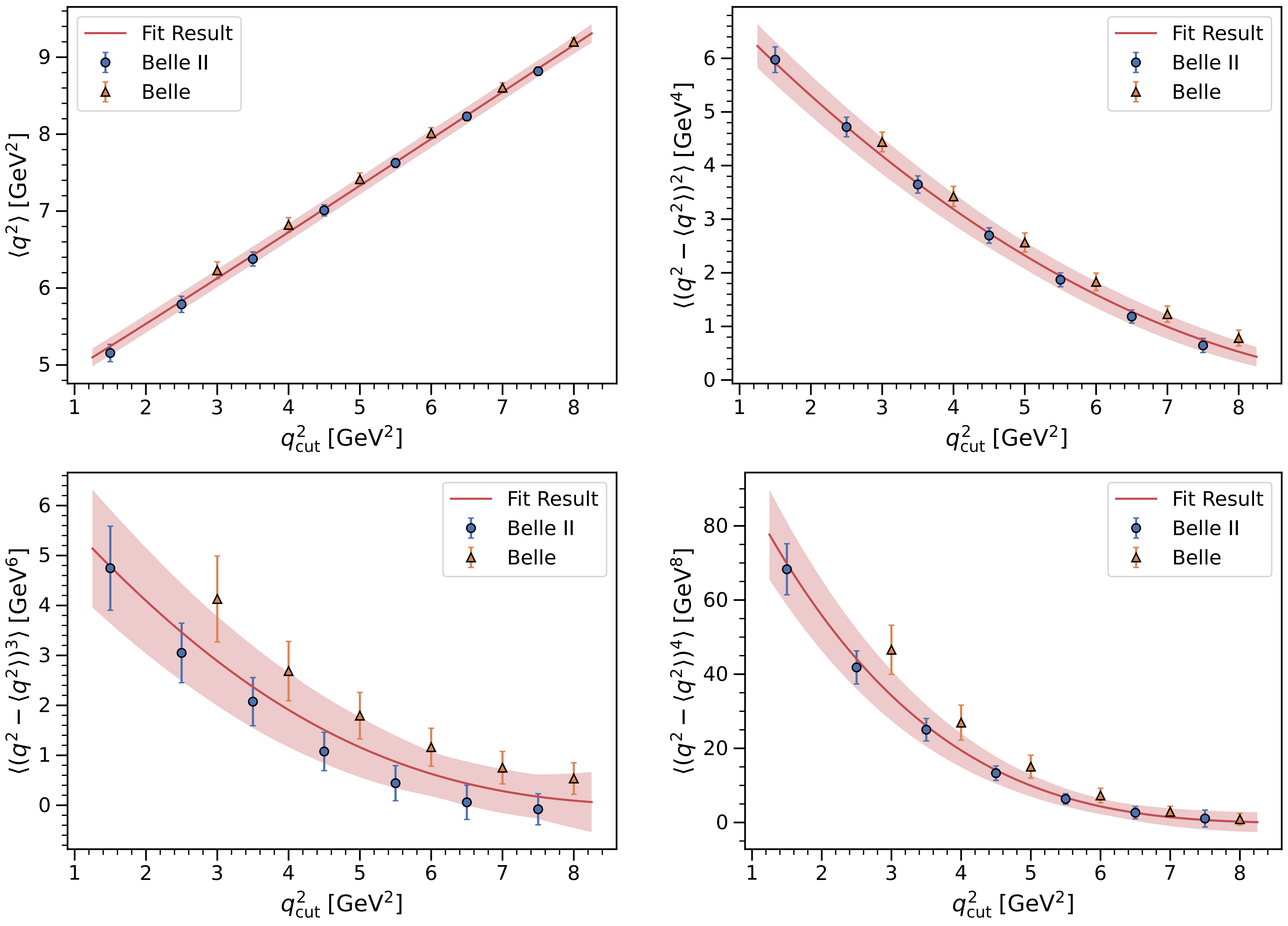
- NNLO QCD corrections to the q^2 spectrum of $B \to X_c l \bar{\nu}_l$
- Third order corrections to $\Gamma(B \to X_u l \bar{\nu}_l)$

NNLO QCD corrections to the q^2 spectrum of $B \to X_c l \bar{\nu}_l$

MF, Herren, JHEP 05 (2024) 287

See also talk by K. Vos

$|V_{cb}|$ FROM q^2 MOMENTS



$$|V_{cb}| = (41.69 \pm 0.59_{\mathrm{fit}} \pm 0.23_{\mathrm{h.o.}}) \times 10^{-3}$$
$$= (41.69 \pm 0.63) \times 10^{-3}$$

Bernlochner, **MF**, Olschwesky, Person, van Tonder, Vos, Welsch, JHEP 10 (2022) 068

Γ	tree	α_s	α_s^2	α_s^3
Partonic	✓	✓	✓	✓
μ_G^2	✓	✓		
ρ_D^3	✓	✓		
$1/m_b^4$	✓			
$m_b^{\mathrm{kin}}/\overline{m}_c$		✓	✓	✓

$\langle (q^2)^n \rangle$	tree	α_s	α_s^2	α_s^3
Partonic	✓	✓		
μ_G^2	✓	✓		
ρ_D^3	✓	✓		
$1/m_b^4$	✓			

NNLO corrections missing!

N3LO corrections to the total rate!
MF, Schönwald, Steinhauser, Phys.Rev.Lett. 125 (2020) 5, 052003
Phys.Rev.D 103 (2021) 1, 014005, Phys.Rev.D 104 (2021) 1, 016003

See also talk by G. Finauri

COMBINED FIT: q^2, E_l AND M_X^2 MOMENTS

Finauri, Gambino, JHEP 02 (2024) 206

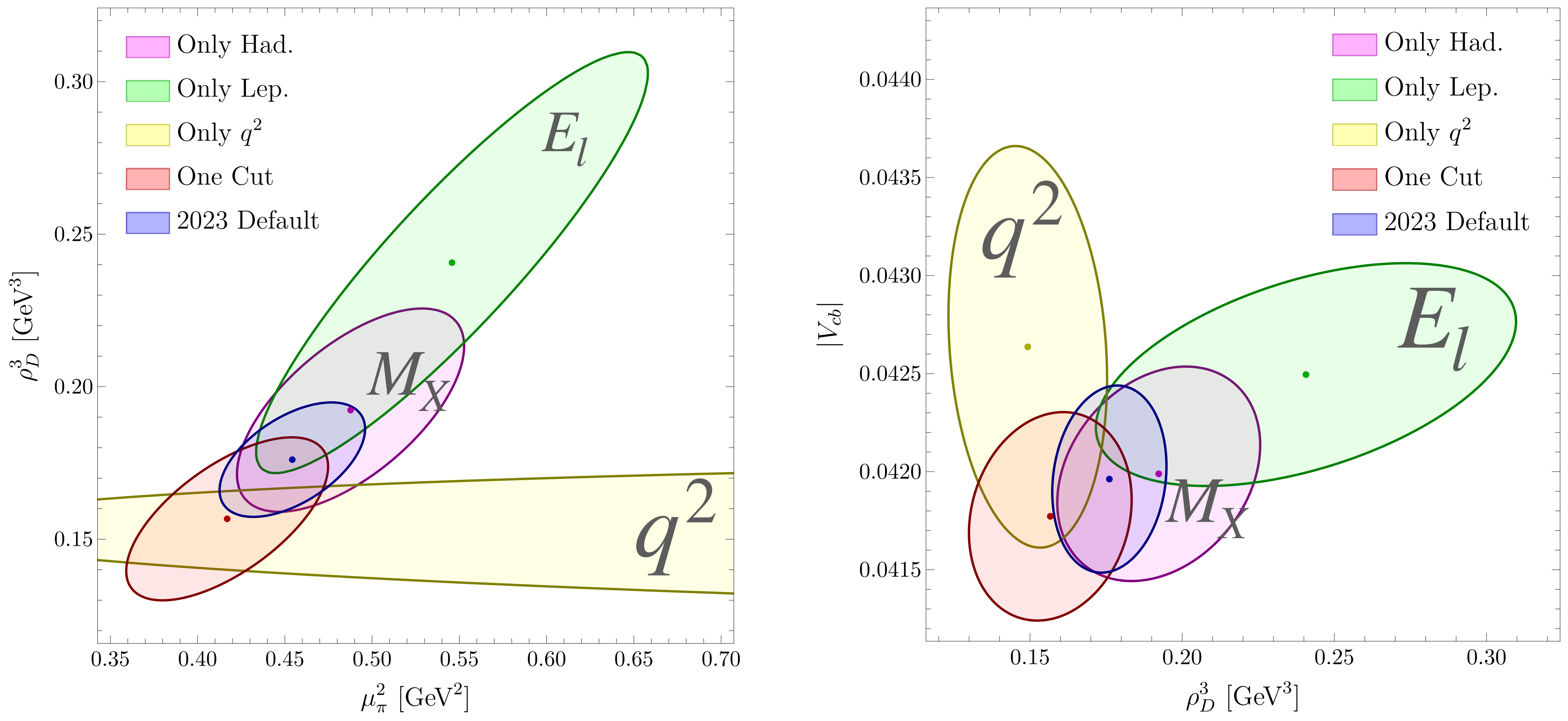
- Old DELPHI, CDF, BaBar, Belle data:
  $\langle E_l \rangle_{E_{\rm cut}}$, $\langle M_X^2 \rangle_{E_{\rm cut}}$, $\Delta {\rm Br}_{E_{\rm cut}}$
- New Belle & Belle II: $\langle q^2 \rangle_{q^2_{\rm cut}}$

$$|V_{cb}| = (41.97 \pm 0.27_{\rm exp} \pm 0.31_{\rm th} \pm 0.25_{\Gamma}) \times 10^{-3}$$
$$= (41.97 \pm 0.48) \times 10^{-3}$$

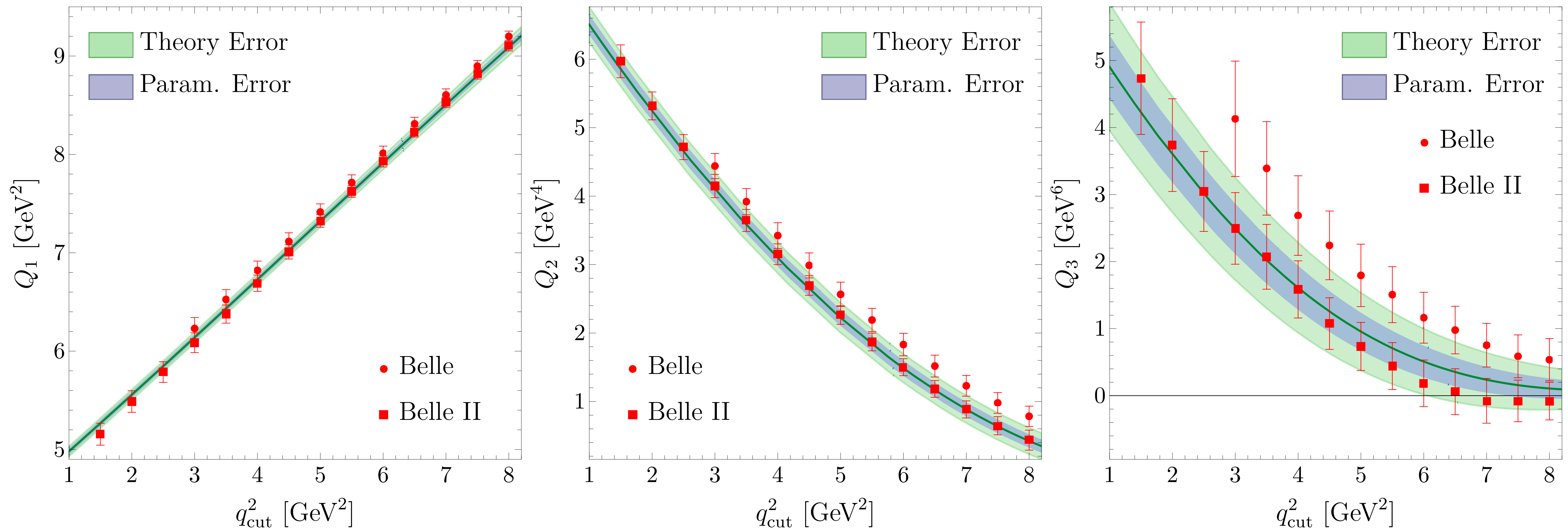
Compared with 2021 fit: 0.51 → 0.48 reduction

0.031 → 0.019 reduction

$m_b^{\rm kin}$	$\overline{m}_c(2\,{\rm GeV})$	μ_π^2	$\mu_G^2(m_b)$	$\rho_D^3(m_b)$	ρ_{LS}^3	${\rm BR}_{c\ell\nu}$	$10^3\lvert V_{cb}\rvert$
4.573	1.090	0.454	0.288	0.176	−0.113	10.63	41.97
0.012	0.010	0.043	0.049	0.019	0.090	0.15	0.48
1	0.380	-0.219	0.557	-0.013	-0.172	-0.063	-0.428
	1	0.005	-0.235	-0.051	0.083	0.030	0.071
		1	-0.083	0.537	0.241	0.140	0.335
			1	-0.247	0.010	0.007	-0.253
				1	-0.023	0.023	0.140
					1	-0.011	0.060
						1	0.696
							1



Only $\alpha_s^2\beta_0$ corrections included for $\langle q^2 \rangle$



hep-ph/0911.4142

Second order QCD corrections to inclusive semileptonic $b \longrightarrow X_c l \bar{\nu}_l$ decays with massless and massive lepton

Sandip Biswas[1] **and Kirill Melnikov**

Department of Physics and Astronomy, Johns Hopkins University, Baltimore, MD 21218, U.S.A.

E-mail: sbiswas@pha.jhu.edu, melnikov@pha.jhu.edu

NNLO corrections for E_l and M_X moments at specific values of ρ and $E_{\rm cut}$

Sandip Biswas 26. January 2023 at 14:13

Re: Hep-ph/0911.4142

To: Matteo Fael

Hi Mateo,
Nice to e-meet you. Yes, I am one of the authors but unfortunately no, I lost access to that MC code a while back.

SANDIP
BISWAS

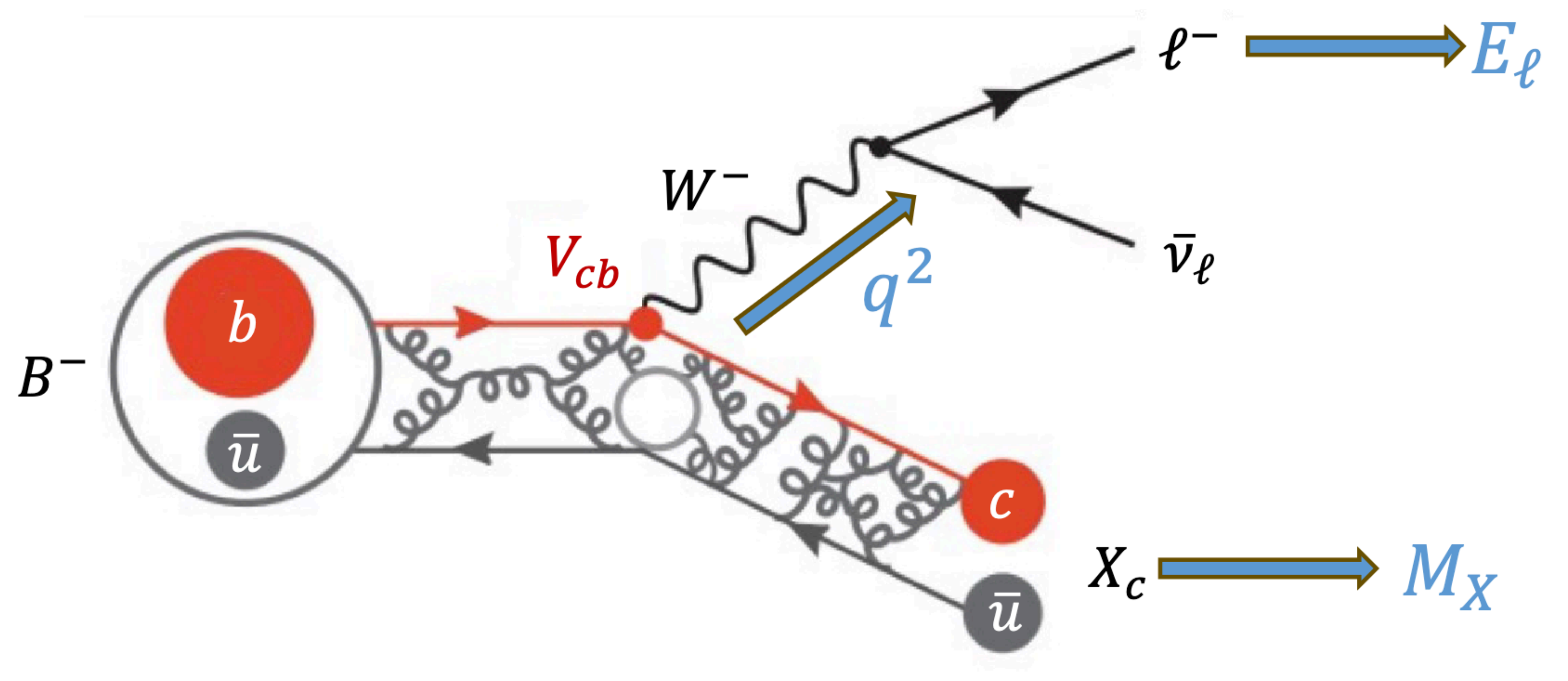
sbiswas@resultant.com
410-802-2245

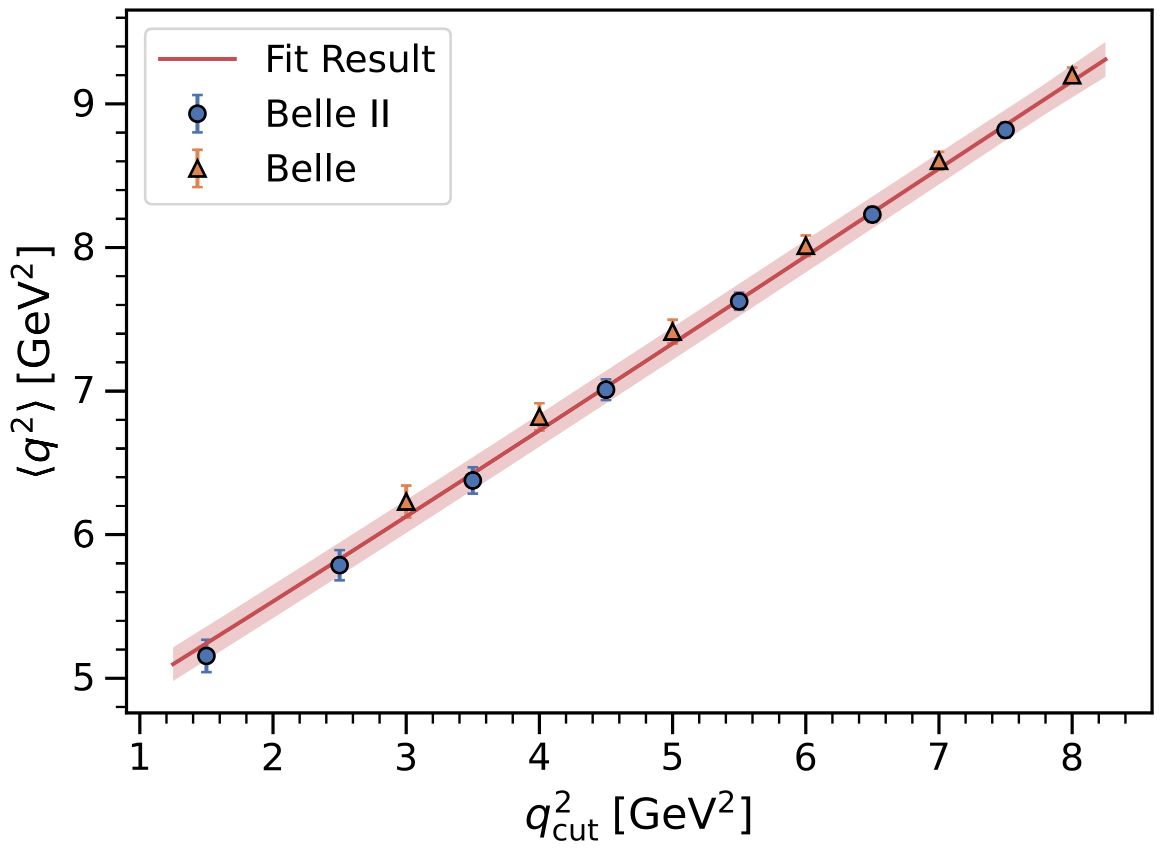
RESULTANT.COM

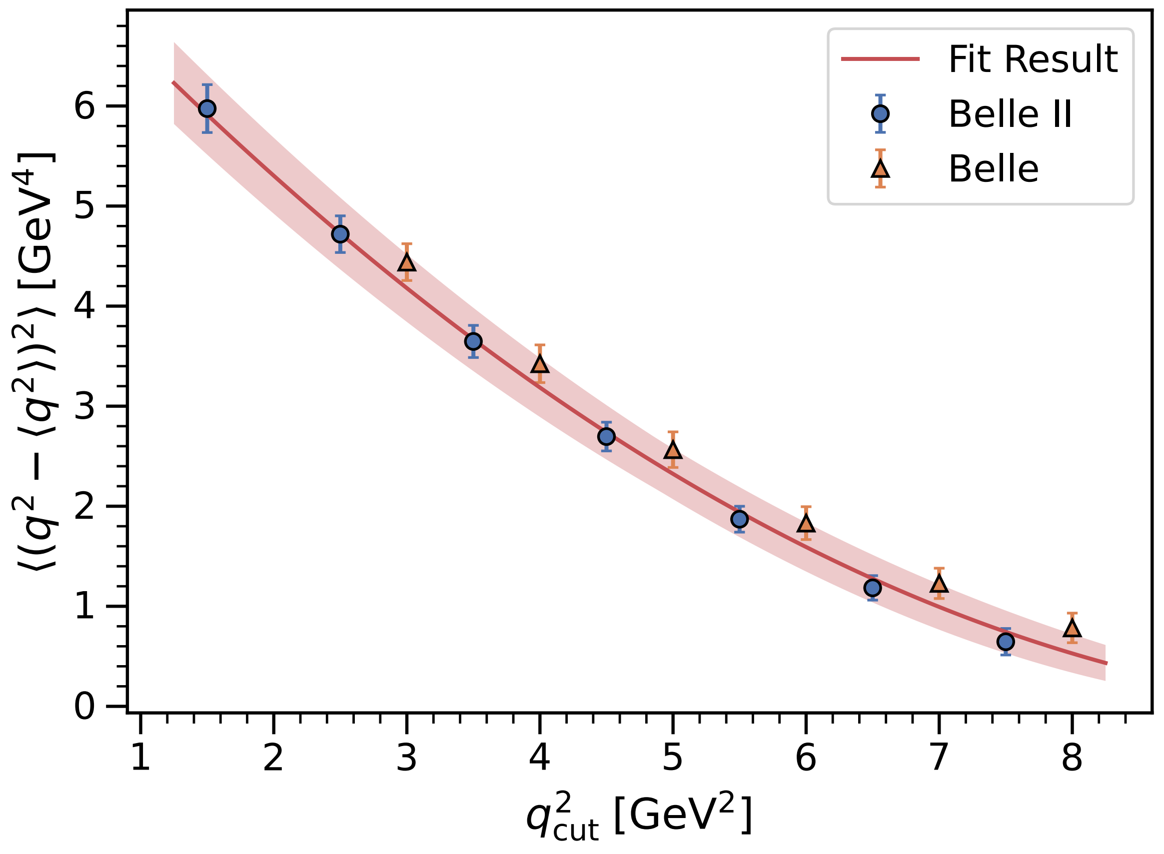
NNLO CORRECTIONS TO q^2 SPECTRUM

$$\rho = m_c/m_b \quad \hat{q}^2 = q^2/m_b^2$$

MF, Herren,, hep-ph/2403.03976







Bernlochner, MF, et al, 2205.10274 [hep-ph]
see also: MF, Mannel, Vos, JHEP 02 (2019) 177

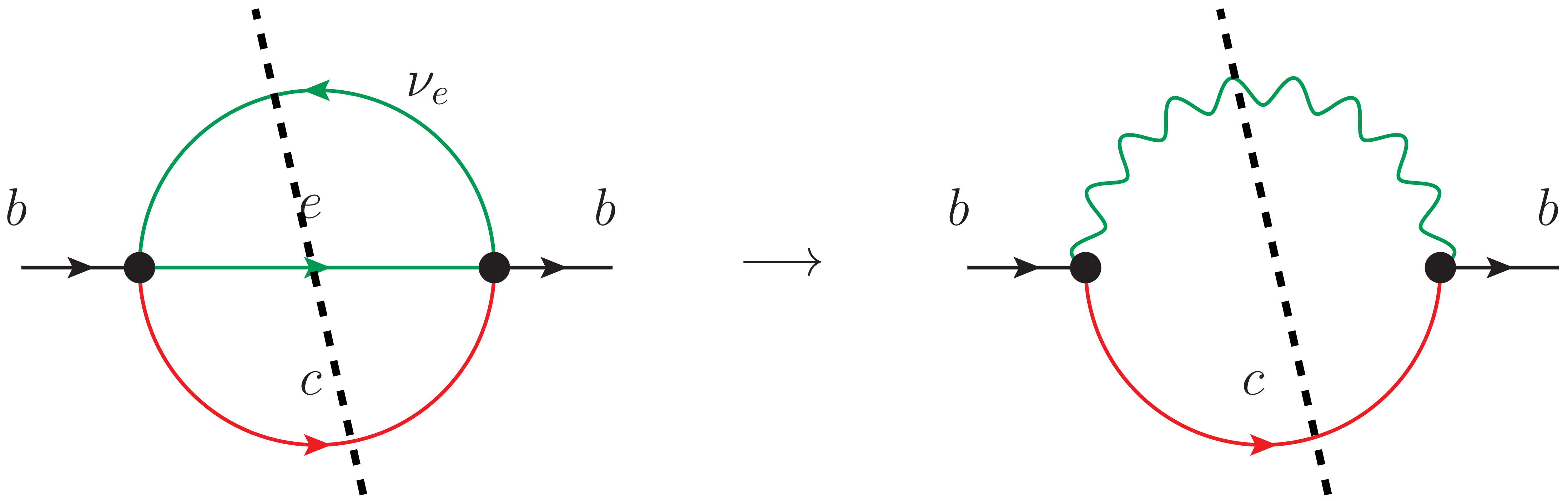
$$\frac{d\Gamma}{d\hat{q}^2} = \frac{G_F^2 m_b^5}{192\pi^3} |V_{cb}|^2 \left[F_0(\rho, \hat{q}^2) + \frac{\alpha_s}{\pi} F_1(\rho, \hat{q}^2) + \left(\frac{\alpha_s}{\pi}\right)^2 F_2(\rho, \hat{q}^2) \right] + O\left(\frac{1}{m_b^2}\right)$$

See also talk by M. Czaja

Jezabeck, Kühn, Nucl. Phys. B 314 (1989) 1
Moreno, Mannel, Pivovarov, *Phys.Rev.D 105 (2022) 5, 054033*

Normalised moments $\langle (q^2)^n \rangle_{q^2_{\rm cut}} = \int_{q^2 > q^2_{\rm cut}} (q^2)^n \frac{d\Gamma}{dq^2} dq^2 \Big/ \int_{q^2 > q^2_{\rm cut}} \frac{d\Gamma}{dq^2} dq^2$

$\delta(p_L^2 - q^2) \times$



Jezabeck, Kühn, Nucl. Phys. B 314 (1989) 1
Moreno, Mannel, Pivovarov, *Phys.Rev.D 105 (2022) 5, 054033*

Integration w.r.t. neutrino-electron phase space

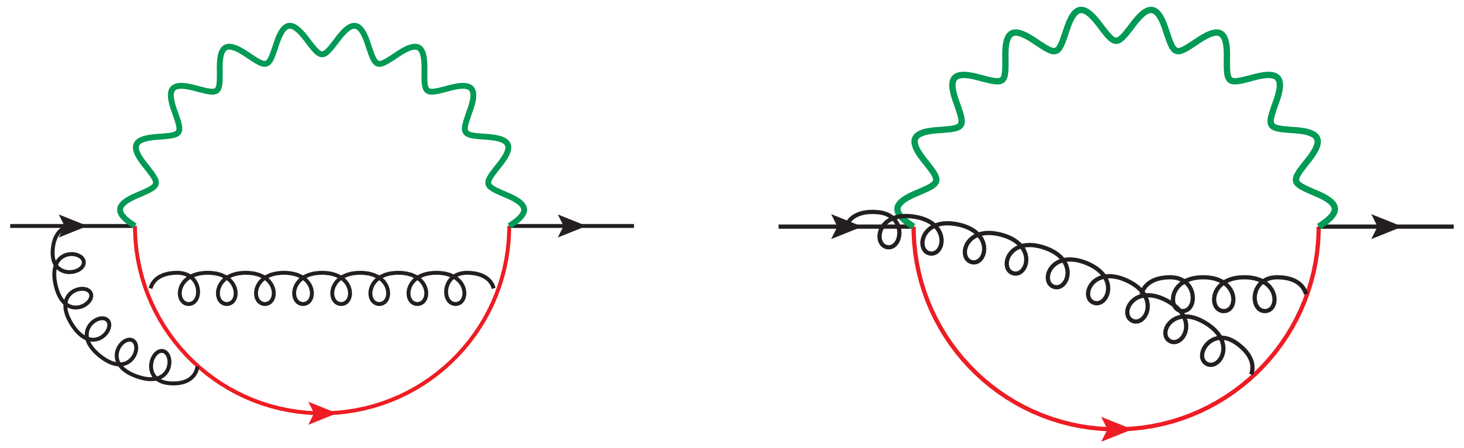
$$\mathscr{L}^{\mu\nu}(p_L) = \int L^{\mu\nu} d\Phi_2(p_L; p_l, p_\nu) = \frac{1}{384\pi^5}\left(1 - \frac{m_\ell^2}{p_L^2}\right)^2 \left[\left(1 + \frac{2m_\ell^2}{p_L^2}\right) p_L^\mu p_L^\nu - g^{\mu\nu} p_L^2 \left(1 + \frac{m_\ell^2}{2p_L^2}\right)\right]$$

Inverse unitarity

$$\delta(p_L^2 - q^2) \to \frac{1}{2\pi i}\left[\frac{1}{p_L^2 - q^2 - i0} - \frac{1}{p_L^2 - q^2 + i0}\right]$$

NNLO calculation

- Three-loop diagrams
- Three different masses: m_b^2, m_c^2, q^2

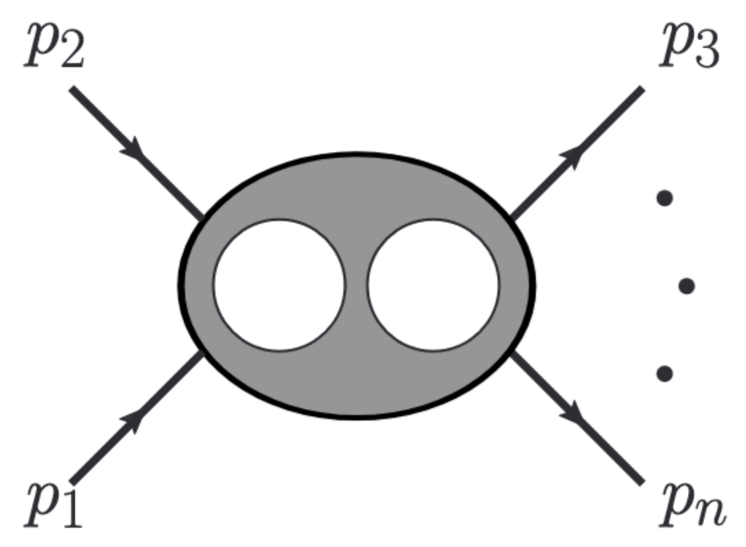


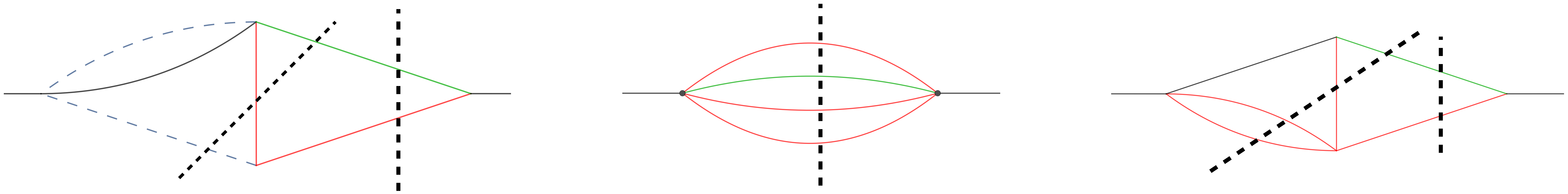
MASTER INTEGRALS

- 98 master integrals with cuts
- Ignore cuts through 3 charm quarks

see also: Egner, **MF**, Schönwald, Steinhauser, HEP 09 (2023) 112

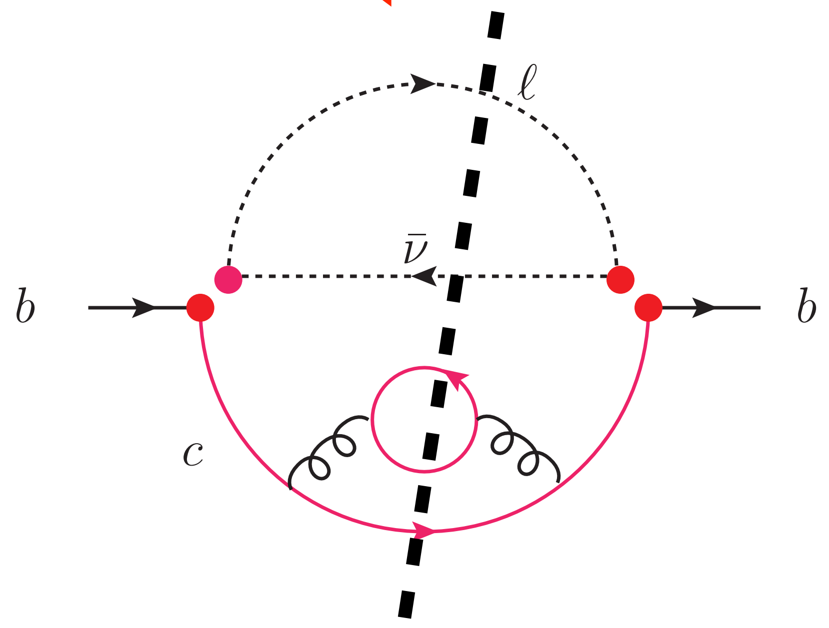
Rational functions Master integrals

$\mathscr{M} =$  $= \sum_i c_i I_i$



ONE CHARM CUTS THREE CHARMS THREE CHARMS ONE CHARM CUTS

Elliptic integrals



$$\mathrm{Br}(b \to c c \bar{c} l \bar{\nu}_l) \simeq 10^{-7}$$

CANONICAL FORM

Henn, Rev. Lett. 110 (2013) 251601

Find rational transformation $\mathbb{T}(u, \rho; \epsilon)$

Libra, R.N. Lee, Comput. Phys. Commun. 267 (2021) 108058

$$\frac{\partial \vec{I}}{\partial \rho} = \hat{M}_\rho(\hat{q}^2, \rho, \epsilon)\, \vec{I}(\hat{q}^2, \rho, \epsilon)$$

$$\frac{\partial \vec{I}}{\partial \hat{q}^2} = \hat{M}_{q^2}(\hat{q}^2, \rho, \epsilon)\, \vec{I}(\hat{q}^2, \rho, \epsilon)$$

$$\vec{I} = \mathbb{T}\vec{I}' \longrightarrow$$

$$\frac{\partial \vec{I}'}{\partial \rho} = \epsilon \hat{M}_\rho(u, \rho)\, \vec{I}'(u, \rho, \epsilon)$$

$$\frac{\partial \vec{I}'}{\partial u} = \epsilon \hat{M}_{q^2}(u, \rho)\, \vec{I}'(u, \rho, \epsilon)$$

Analytic solution expressed via Generalised Polylogarithms

$$G(a_1, \ldots, a_n; z) = \int_0^z \frac{dt}{t - a_1} G(a_2, \ldots, a_n; t)$$

$$G(a_1, \ldots, a_n; z) = \int_0^z \frac{dt}{t - a_1} G(a_2, \ldots, a_n; t)$$

Examples

$$G(0; z) = \log(z) \qquad G(x, z) = \log\left(1 - \frac{z}{x}\right)$$

$$G(\underbrace{0,\ldots,0}_{n}; z) = \frac{\log^n(z)}{n!} \qquad G(\underbrace{0,\ldots,0}_{n}, x, z) = -\,\mathrm{Li}_n\left(\frac{z}{x}\right)$$

Fast numerical evaluation: GiNaC+PolyLogTools

http://www.ginac.de
Duhr, Dulat, JHEP 08 (2019) 135

$$G\left(x, \frac{1+x^2}{x}, x, \frac{1}{x}; z\right)\Bigg|_{x=1/2,\, z=1/3} = 0.00151860208899279\ldots$$

BOUNDARY CONDITIONS

AMFLOW with ~ 250 digits

GPLs with ~ 250 digits

PolyLogTools + GiNaC: Duhr, Dulat, JHEP 08 (2019) 135; Bauer, Frink, Kreckel, , J. Symb. Comput. 33 (2002) 1

$$(\hat{q}^2, \rho^2) \in \{(1/5, 1/5), (4/25, 4/25)\}$$

PSLQ constant reconstruction

Ferguson, Bailey, Arno, Mathematics of Computation 68 (1999) 351.

Analytic boundary

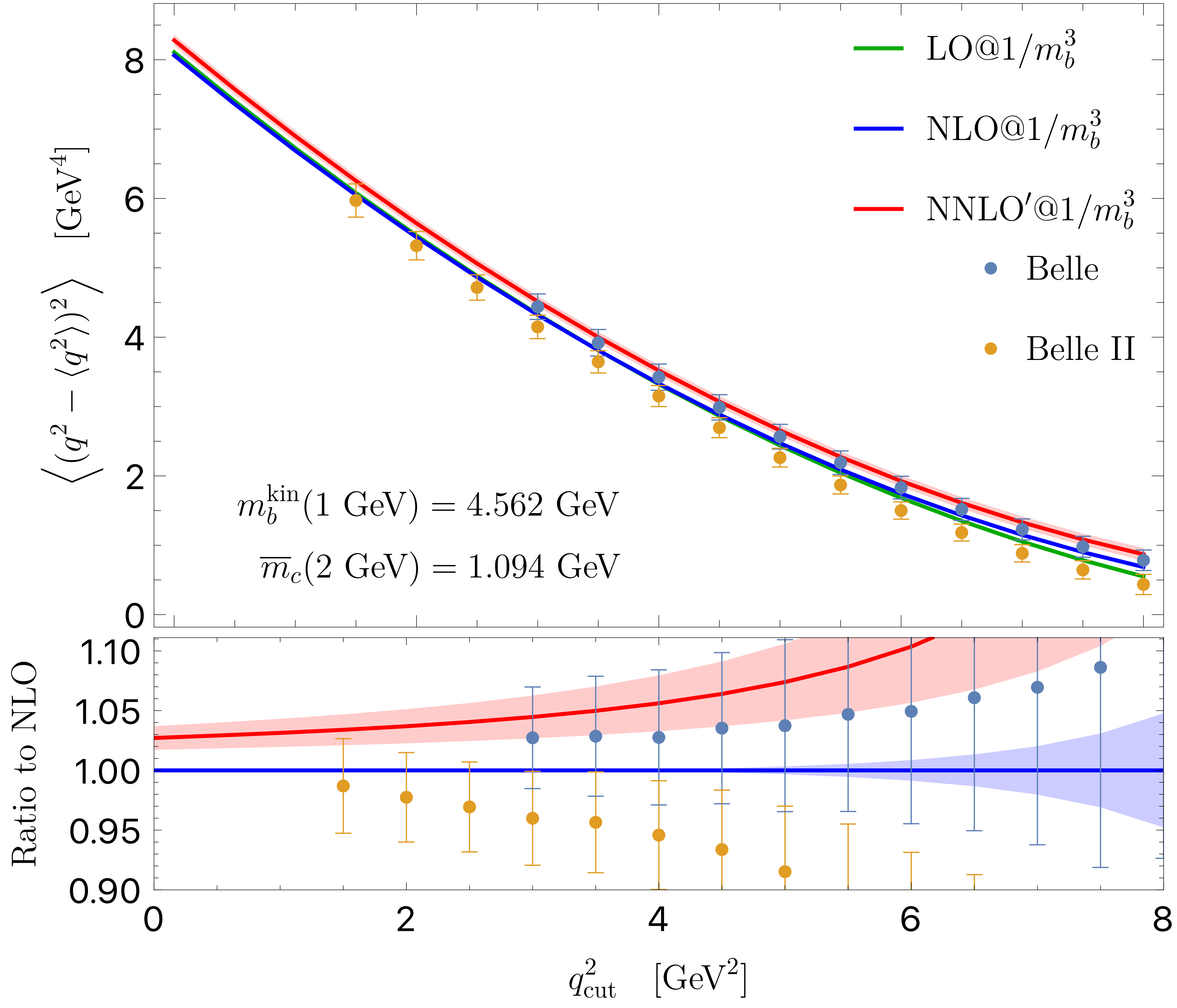
$$2.18269754013877673 46\ldots = \frac{13\pi^2}{72} + \frac{\zeta_3}{3}$$

NEW: NNLO CORRECTIONS Q2 SPECTRUM

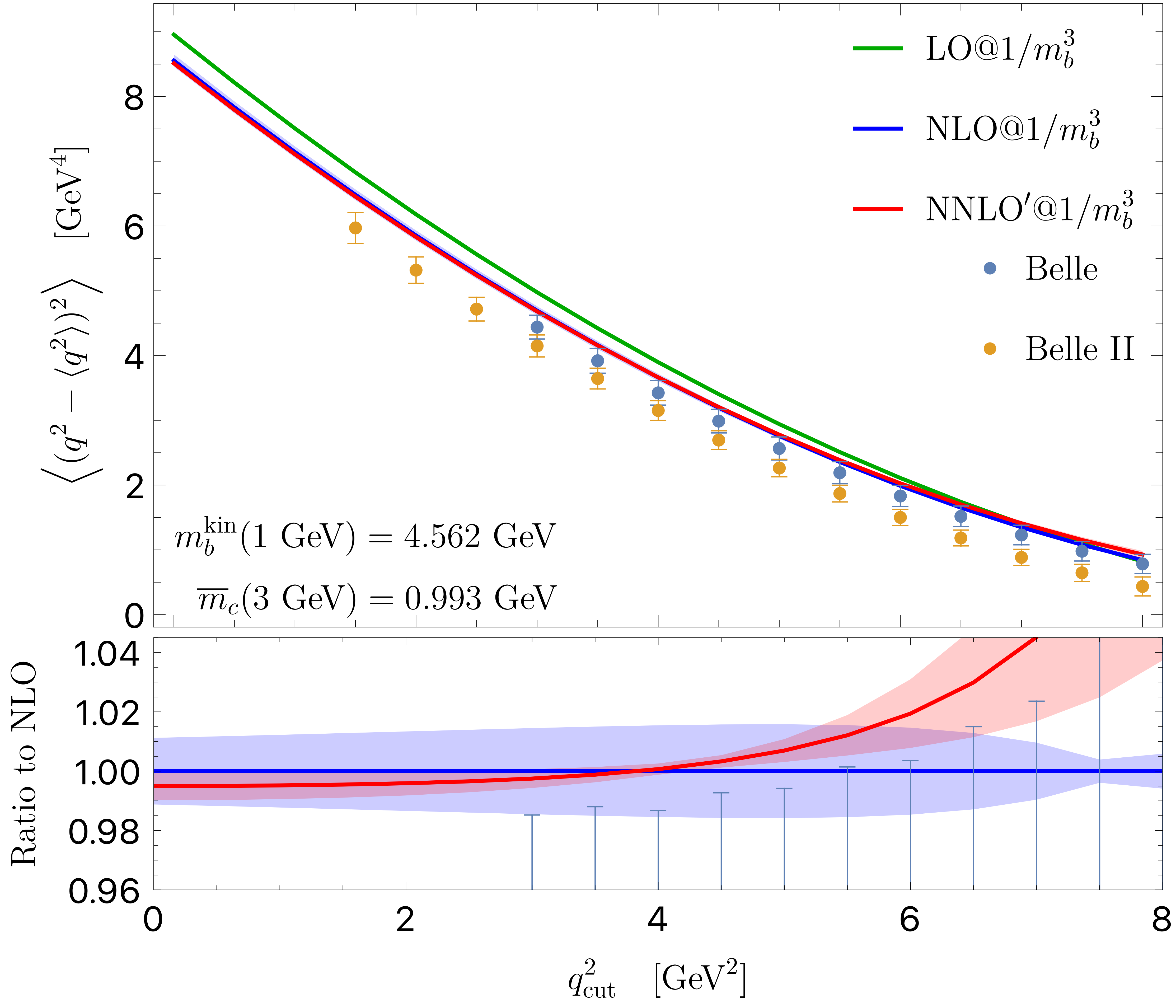
MF, Herren, JHEP 05 (2024) 287

NNLO effects mainly re-absorbed in the fit into a shift of ρ_D, r_E and r_G with reduced uncertainty. No major shift in $|V_{cb}|$.

setup from: Bernlochner, **MF**, et al, JHEP 10 (2022) 068



Unfortunate choice of $\overline{m}_c(2\,{\rm GeV})$



Much better $\overline{m}_c(3\,{\rm GeV})$

NNLO CORRECTIONS TO TAUONIC MODE AND R(X)

$$R(X_{\ell_1/\ell_2}) = \frac{\Gamma_{B\to X\ell_1\bar{\nu}_1}}{\Gamma_{B\to X\ell_2\bar{\nu}_{l_2}}}$$

$$R^{\exp}(X_{e/\mu}) = 1.007 \pm 0.009(\text{stat}) \pm 0.019(\text{syst})$$

Belle II, Phys.Rev.Lett. 131 (2023) 5, 051804

$$R^{\exp}(X_{\tau/l}) = 0.228 \pm 0.016(\text{stat}) \pm 0.036(\text{syst})$$

Belle II, hep-ex/2311.07248

$$R^{\text{SM}}(X_{\tau/l}) = 0.225 \pm 0.005$$

Rahimi, Vos, JHEP 11 (2022) 007
Ligeti, Luke, Tackmann, Phys. Rev. D 105, 073009 (2022)

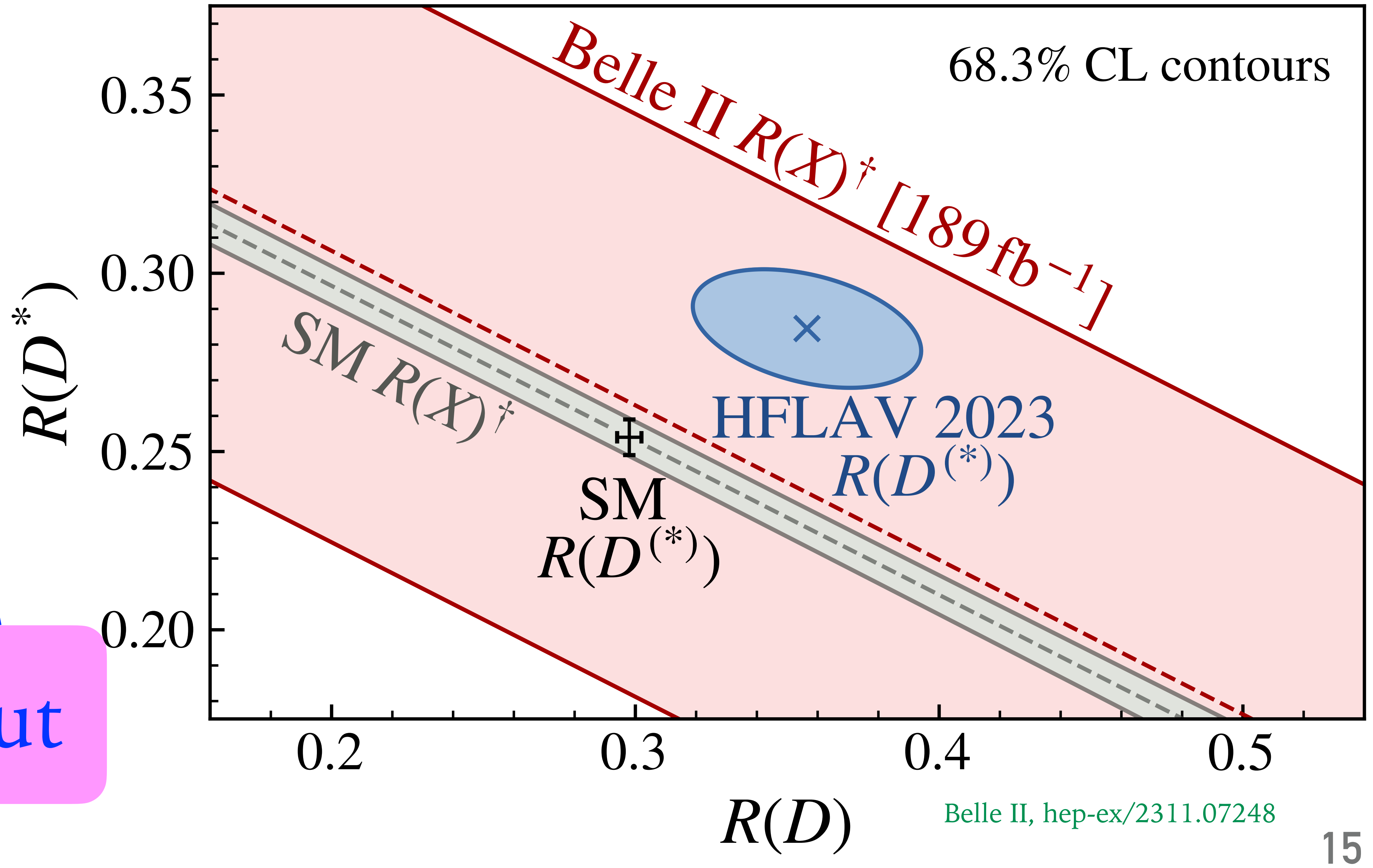
$$R(X_c) = 0.241\left[1 - 0.156\frac{\alpha_s}{\pi} - 1.766\left(\frac{\alpha_s}{\pi}\right)^2\right]$$

$$R(X_c)\Big|_{q^2>6\,\text{GeV}^2} = 0.350\left[1 - 0.782\frac{\alpha_s}{\pi} - 8.355\left(\frac{\alpha_s}{\pi}\right)^2\right]$$

MF, Herren, JHEP 05 (2024) 287

Enrichment with q^2 selection cut

† = with expected SM contributions of $D^{**}_{(\text{gap})}, X_u$ removed



Belle II, hep-ex/2311.07248

COMMENTS ON THE IMPLEMENTATION IN KOLYA

$\rho = m_c/m_b \quad \hat{q}^2 = q^2/m_b^2$

$$\frac{d\Gamma}{d\hat{q}^2} = \frac{G_F^2 m_b^5}{192\pi^3} |V_{cb}|^2 \left[F_0(\rho, \hat{q}^2) + \frac{\alpha_s}{\pi} F_1(\rho, \hat{q}^2) + \left(\frac{\alpha_s}{\pi}\right)^2 F_2(\rho, \hat{q}^2) \right]$$

- Fast numerical implementation, but not that fast…
- Needs to integrate the differential rate

$$G(a_1, \ldots, a_n; z) = \int_0^z \frac{dt}{t - a_1} G(a_2, \ldots, a_n; t)$$

$$Q_n^{(2)}(\hat{q}^2_{\mathrm{cut}}) = \int_{\hat{q}^2 > \hat{q}^2_{\mathrm{cut}}} (\hat{q}^2)^n F_2(\rho, \hat{q}^2)\, d\hat{q}^2$$

- Cannot be performed on-the-fly, e.g. during a fit

IMPLEMENTATION

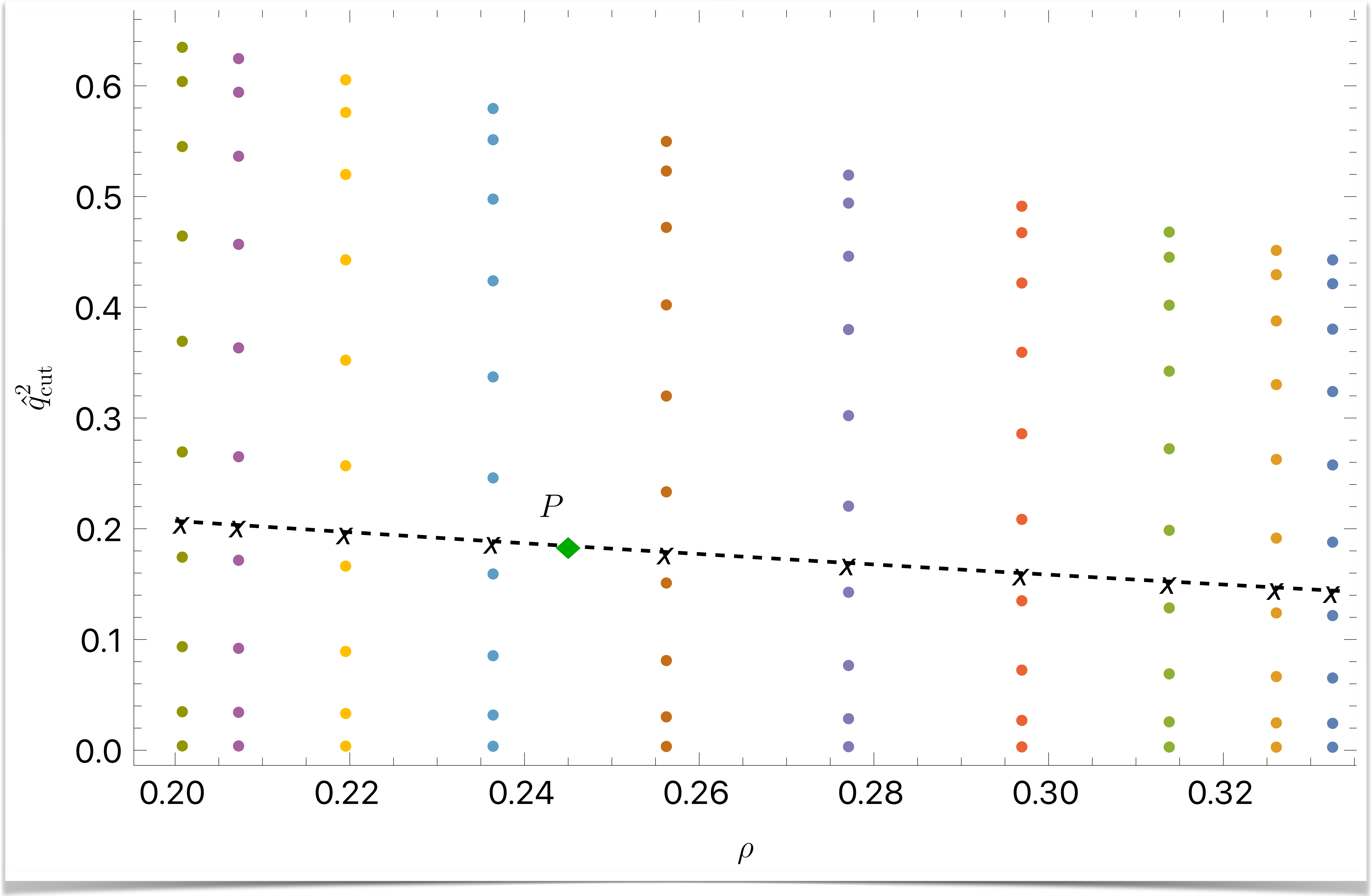
$\rho = m_c/m_b \quad \hat{q}^2 = q^2/m_b^2$

- Chebyshev interpolation grids for QCD corrections to the moments

$$Q_n^{(2)}(\hat{q}^2_{\rm cut}) = \int_{\hat{q}^2>\hat{q}^2_{\rm cut}} (\hat{q}^2)^n F_2(\rho, \hat{q}^2)\, d\hat{q}^2$$

- Use Numba for fast numerical evaluation https://numba.pydata.org

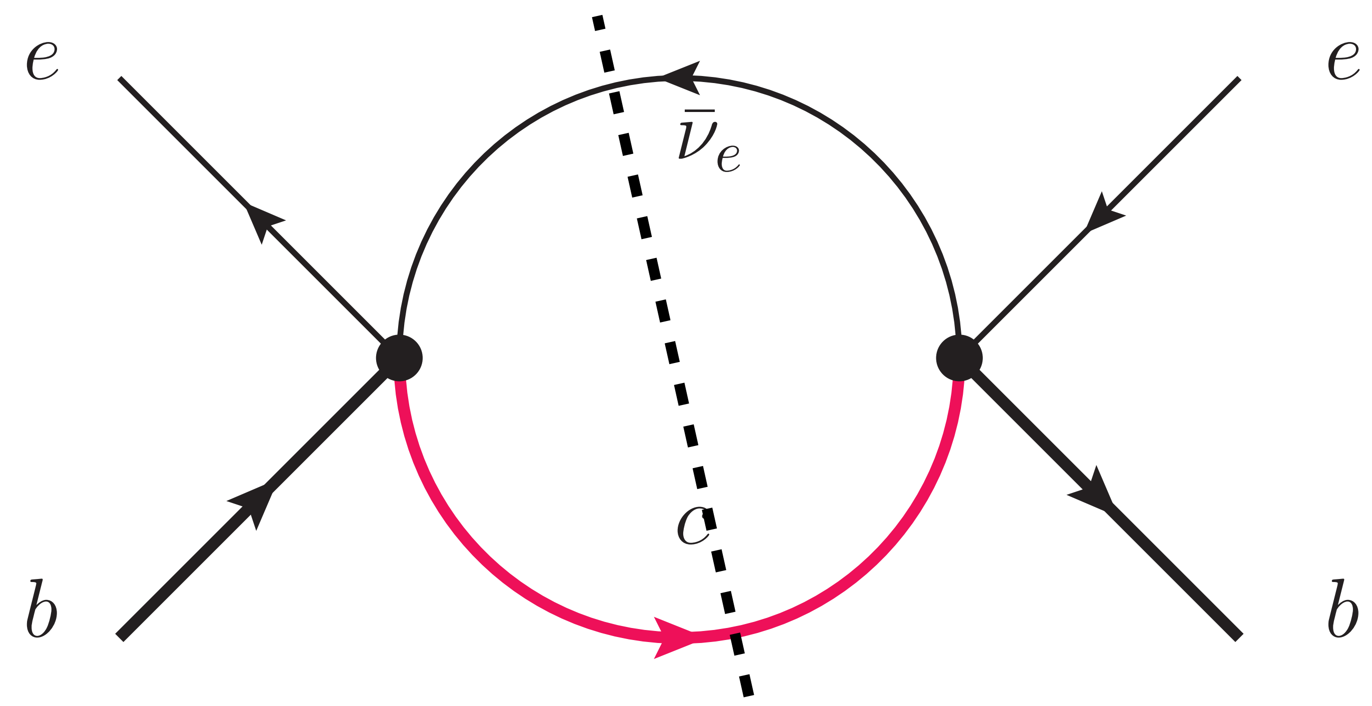
- Checks in 100 random points: agreement better than 10^{-5}



NNLO CORRECTIONS TO E_l AND M_X MOMENTS

Follow the strategy in Gambino, JHEP 09 (2011) 055

REVISITING NNLO CORRECTIONS TO E_l MOMENTS



$$\simeq \quad \frac{d\Gamma}{dE_l} \quad \rightarrow \quad \langle E_l^n \rangle_{E_{\text{cut}}} = \int_{E_l > E_{\text{cut}}} (E_l)^n \frac{d\Gamma}{dE_l} \, dE_l$$

- Feynman integrals depend on two scales: $\rho = m_c/m_b$ and E_l.
- At NLO there are 9 master integrals.
- Perfect numerical agreement with integration of differential rate.

Aquila, Gambino, Ridolfi, Uraltsev, Nucl.Phys.B 719 (2005) 77

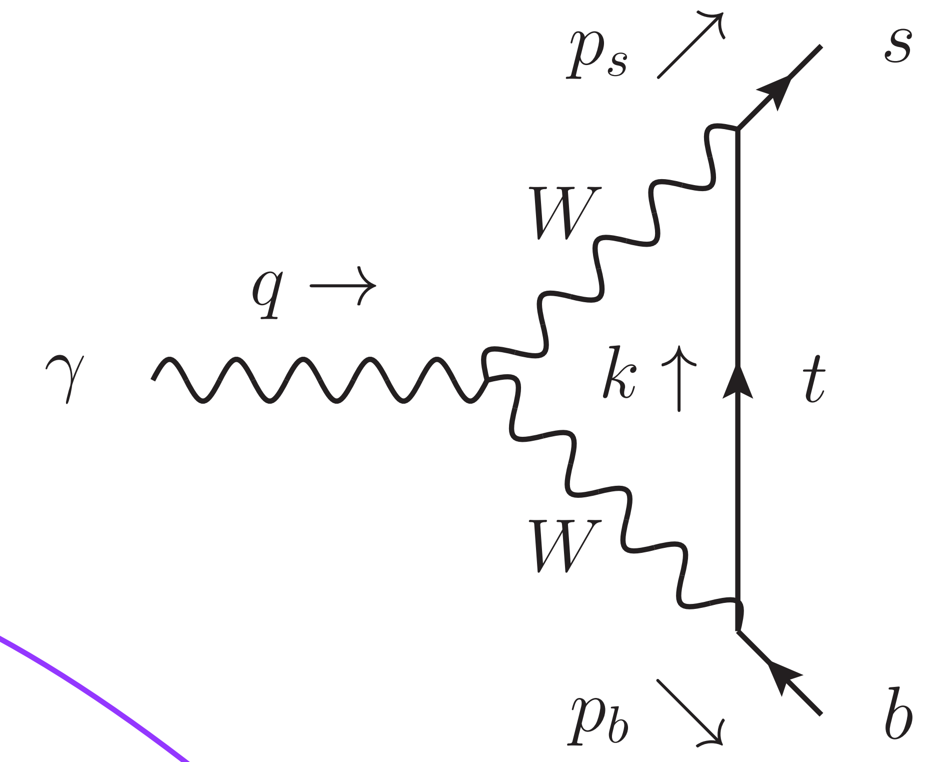
- Possibility to extend the calculation at NNLO under study.

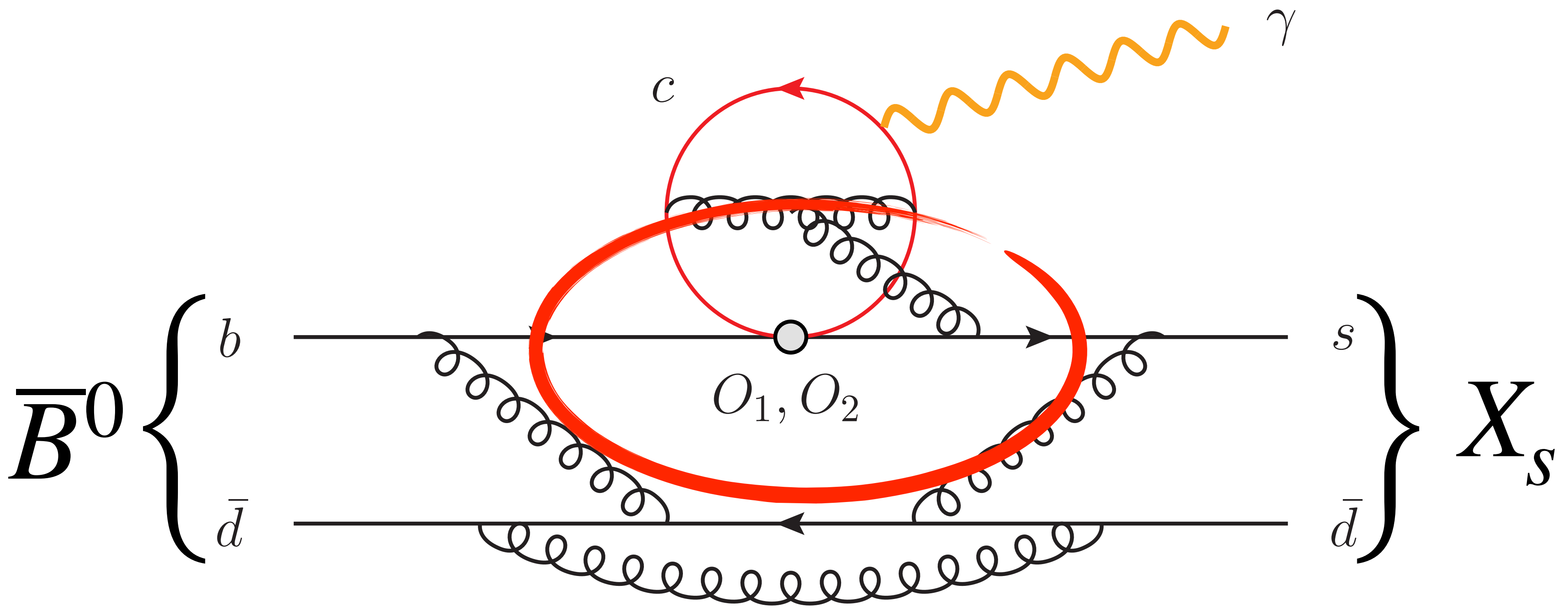
MF, Herren, Schönwald, work in progress

Third order corrections to $\Gamma(B \to X_u l \bar{\nu}_l)$

MF, Usovitsch, Phys.Rev.D 108 (2023) 11, 11

MOTIVATION





Belle II: 5% → 2.5%

$$\mathscr{B}^{\exp}(B \to X_s\gamma)\Big|_{E_\gamma > 1.6\,\mathrm{GeV}} = (3.49 \pm 0.19) \times 10^{-4}$$

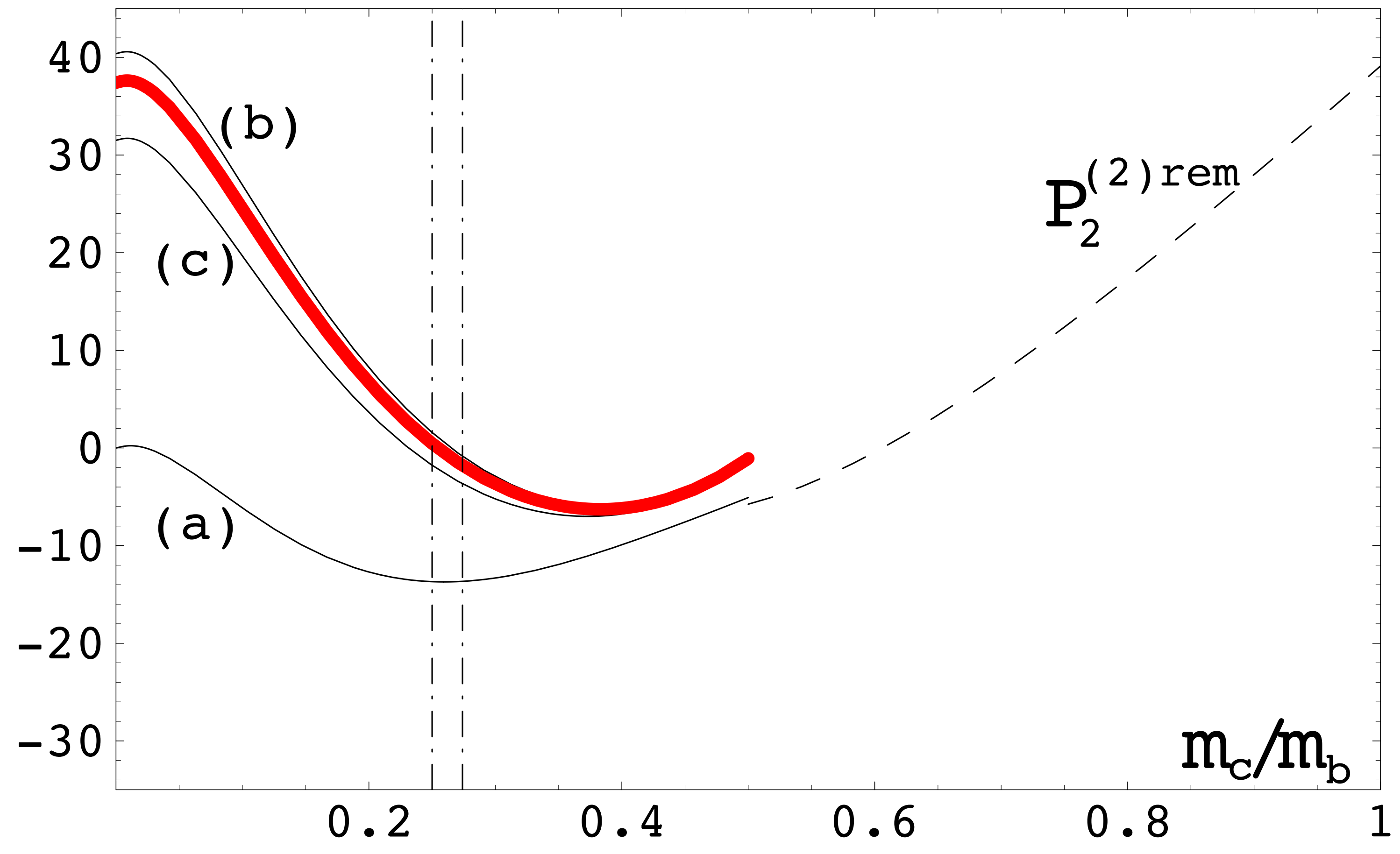
HFLAV - 2023

$$\mathscr{B}^{\mathrm{SM}}(B \to X_s\gamma)\Big|_{E_\gamma > 1.6\,\mathrm{GeV}} = (3.40 \pm 0.17) \times 10^{-4}$$

Misiak et al. Phys.Rev.Lett. 114 (2015) 22, 221801
Misiak, Rehman, Steinhauser, JHEP 06 (2020) 175

Rare decay $B \to X_s\gamma$

m_c interpolation: 3% | QCD higher orders: 3% | parametric: 2.5%



Czakon, Fiedler, Huber, Misiak, Schutzmeier, Steinhauser, JHEP 04 (2015) 168

Ongoing: NNLO QCD corrections without m_c interpolation

Misiak, Rehman, Steinhauser, JHEP 06 (2020) 175
MF, Lange, Schönwald, Steinhauser, JHEP 11 (2023) 166
Czaja, Czakon, Huber, Misiak, Niggetiedt, Rehman, Schönwald, Steinhauser, Eur.Phys.J.C 83 (2023) 12, 1108

PHASE SPACE RATIO C

$$C = \left|\frac{V_{ub}}{V_{cb}}\right|^2 \frac{\Gamma(B \to X_c l \bar{\nu}_l)}{\Gamma(B \to X_u l \bar{\nu}_l)}$$

$$= 0.568 \pm 0.007_{\text{par}} \pm 0.010_{\text{h.o.}} \; (2.1\%)$$

Gambino, Misiak, hep-ph/0104034,
Gambino, Giordano, hep-ph/0805.0271,
Alberti, et al, hep-ph/1411.6560

$$\text{Br}(B \to X_s \gamma)_{E_\gamma > E_0} = \tau_B \, \Gamma(B \to X_s \gamma)_{E_\gamma > E_0}$$

Significant source of uncertainty

- $B \to X_s \gamma$
- $B \to X_s l \bar{l}$

PHASE SPACE RATIO C

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- $B \to X_s \gamma$
- $B \to X_s l \bar{l}$

$$\frac{|V_{ts}^{\star} V_{tb}|^2}{|V_{cb}|^2} \simeq \frac{[1 + \lambda^2(2\bar{\rho} - 1) + O(\lambda^4)]\,|V_{cb}|^2}{|V_{cb}|^2} = (0.965 \pm 0.001)$$

$$\text{Br}(B \to X_s \gamma)_{E_\gamma > E_0} = \tau_B \Gamma(B \to X_c l \bar{\nu}_l) \frac{\Gamma(B \to X_s \gamma)_{E_\gamma > E_0}}{\Gamma(B \to X_c l \bar{\nu}_l)}$$

PHASE SPACE RATIO C

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Significant source of uncertainty

- $B \to X_s \gamma$
- $B \to X_s l \bar{l}$

$$\text{Br}(B \to X_s \gamma)_{E_\gamma > E_0} = \tau_B \Gamma(B \to X_c l \bar{\nu}_l) \left(\frac{|V_{cb}|^2 \Gamma(B \to X_u l \bar{\nu}_l)}{|V_{ub}|^2 \Gamma(B \to X_c l \bar{\nu}_l)} \right) \frac{\Gamma(B \to X_s \gamma)_{E_\gamma > E_0}}{|V_{cb}|^2 / |V_{ub}|^2 \Gamma(B \to X_u l \bar{\nu}_l)}$$

PHASE SPACE RATIO C

$$C = \left|\frac{V_{ub}}{V_{cb}}\right|^2 \frac{\Gamma(B \to X_c l \bar{\nu}_l)}{\Gamma(B \to X_u l \bar{\nu}_l)}$$

$$= 0.568 \pm 0.007_{\text{par}} \pm 0.010_{\text{h.o.}} \; (2.1\%)$$

Gambino, Misiak, hep-ph/0104034,
Gambino, Giordano, hep-ph/0805.0271,
Alberti, et al, hep-ph/1411.6560

Significant source of uncertainty

- $B \to X_s \gamma$
- $B \to X_s l \bar{l}$

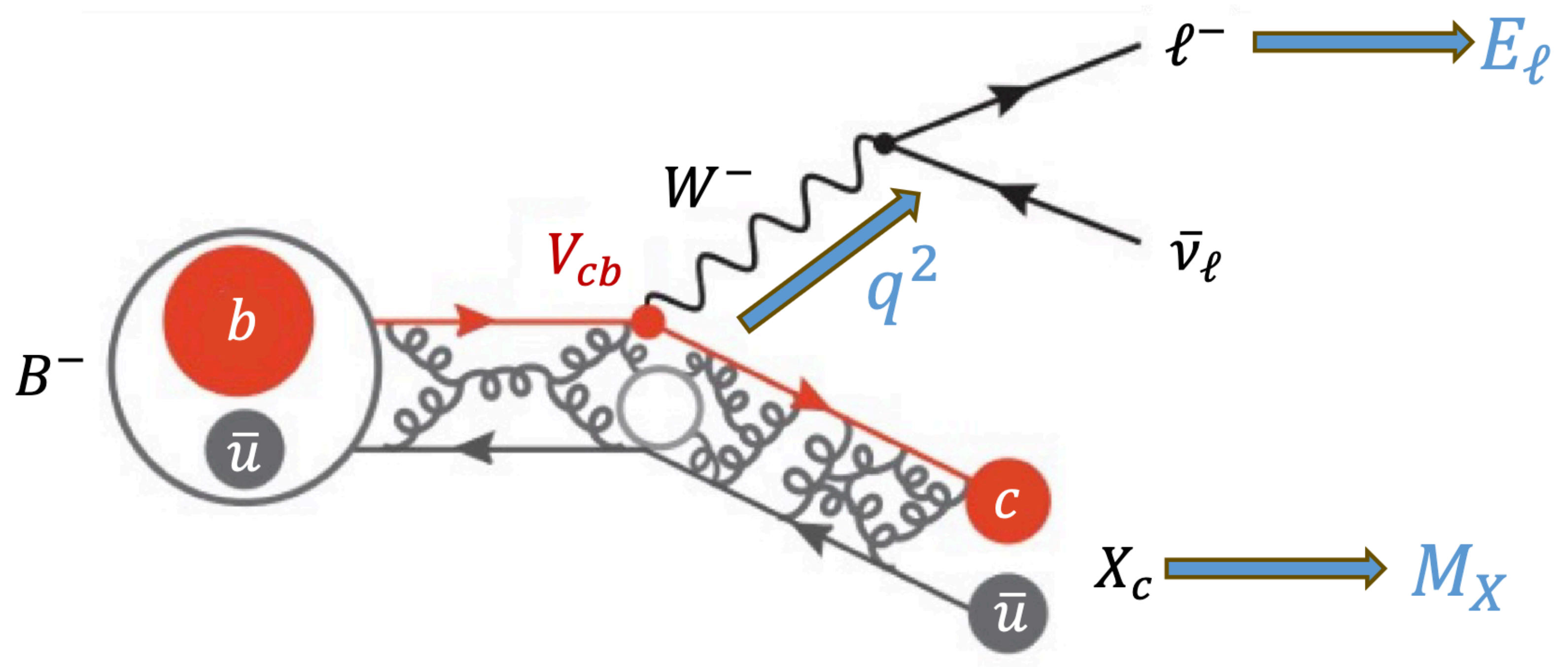
$$|V_{ts}^{\star} V_{tb}|^2 \simeq [1 + \lambda^2(2\bar{\rho} - 1) + O(\lambda^4)]\,|V_{cb}|^2$$
$$= (0.965 \pm 0.001)\,|V_{cb}|^2$$

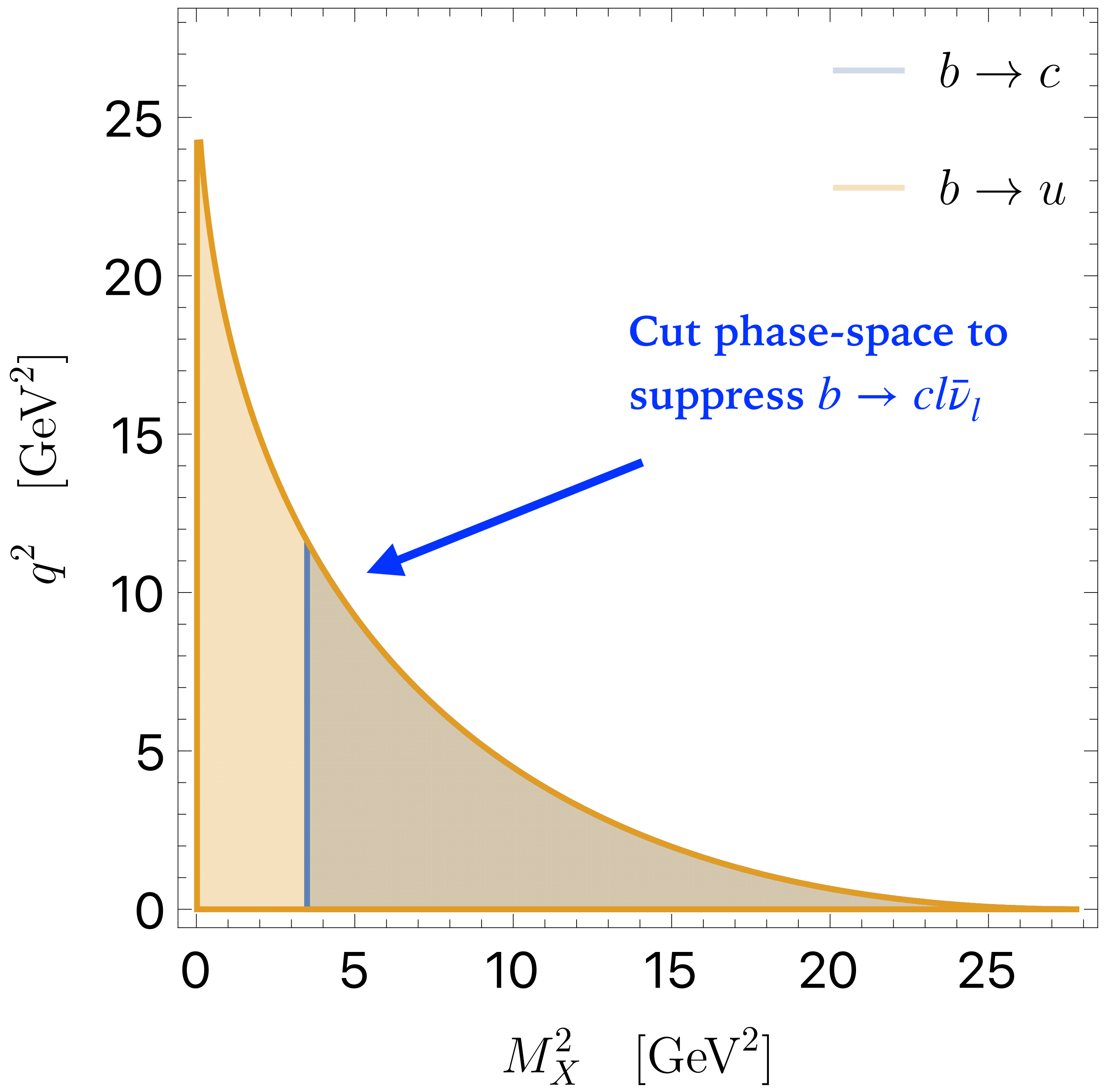
NNLO QCD corrections

$$\text{Br}(B \to X_s \gamma)_{E_\gamma > E_0} = \frac{\text{Br}^{\text{exp}}(B \to X_c l \bar{\nu}_l)}{C} \left|\frac{V_{ts}^{\star} V_{tb}}{V_{cb}}\right|^2 \frac{6\alpha_{\text{em}}}{\pi} \left[1 + \delta_{NP}\right] P(E_0)$$

Normalisation factor: up to N3LO?

V_{ub}/V_{cb} EXTRACTION





- $|V_{cb}/V_{ub}|^2 \simeq 100$
- Strong experimental cuts to suppress $b \to c$ contamination
- First Belle extraction of $|V_{ub}|/|V_{cb}|$

Belle collaboration: 2311.00458 [hep-ex]

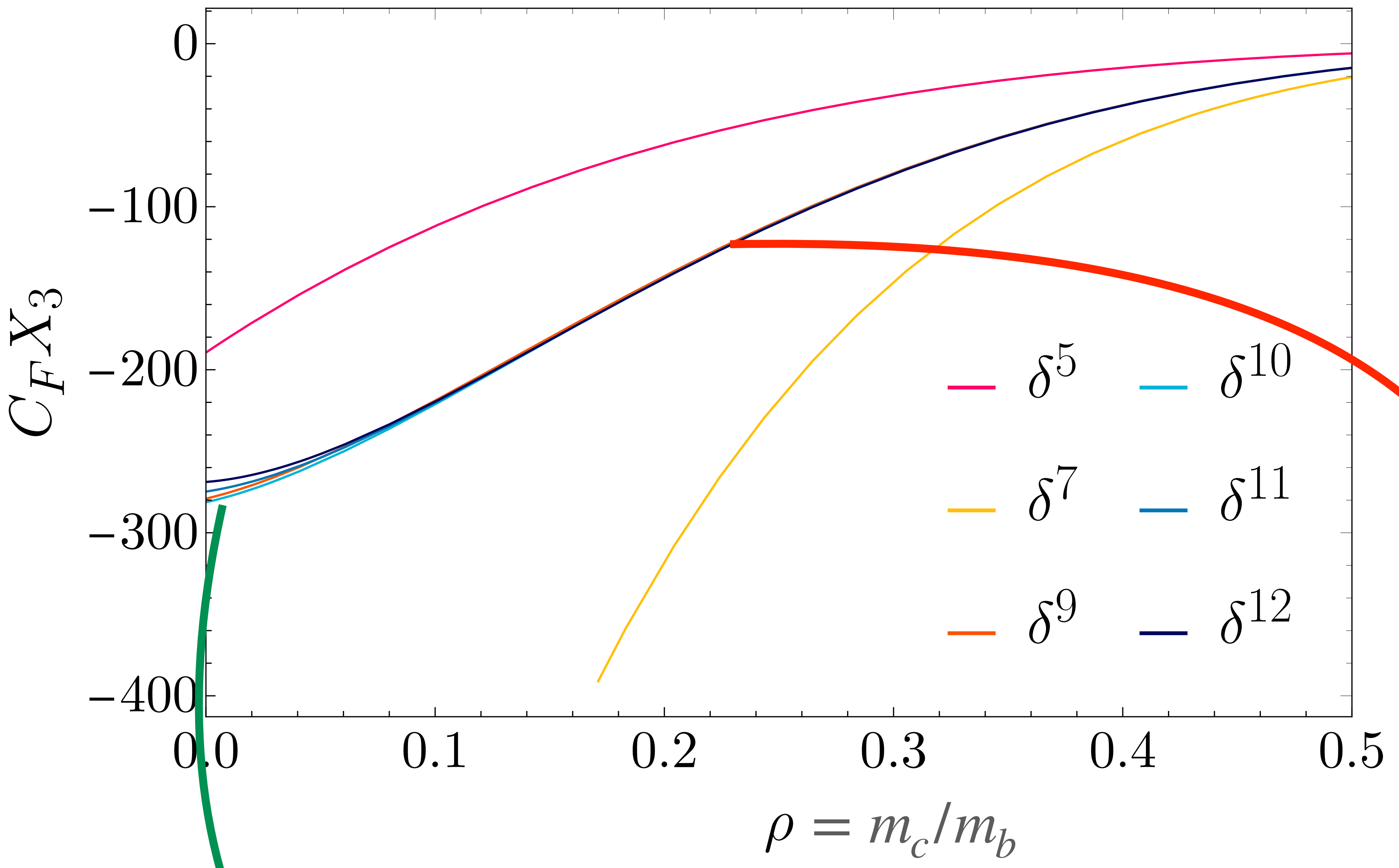
$$\frac{\Delta\mathscr{B}(B \to X_u l \bar{\nu}_l)}{\Delta\mathscr{B}(B \to X_c l \bar{\nu}_l)} = 0.0196\,(1 \pm 8.4\%_{\text{stat}} \pm 7.9\%_{\text{syst}})$$

$$\frac{|V_{ub}|}{|V_{cb}|}^{\text{GGOU}} = 0.0996\left(1 \pm 4.2\%_{\text{stat}} \pm 3.9\,\%_{\text{syst}} \pm 2.3\%_{\Delta\Gamma(B\to X_u l\nu)} \pm 2.0\,\%_{\Delta\Gamma(B\to X_c l\nu)}\right)$$

about 80% of the phase space

THEORETICAL UNCERTAINTY OF THE TOTAL RATE

Equal mass expansion $\delta = 1 - m_c/m_b \ll 1$



$$\Gamma_{\rm sl} = \frac{G_F^2 m_b^5 A_{\rm ew}}{192\pi^3} |V_{qb}|^2 \left(X_0(\rho) + C_F \sum_n \left(\frac{\alpha_s}{\pi}\right)^n X_n(\rho) \right)$$

with $\rho = m_q/m_b$

$$C_F X_3(\rho = 0.28) = -91.2 \pm 0.4\,(0.4\%)$$

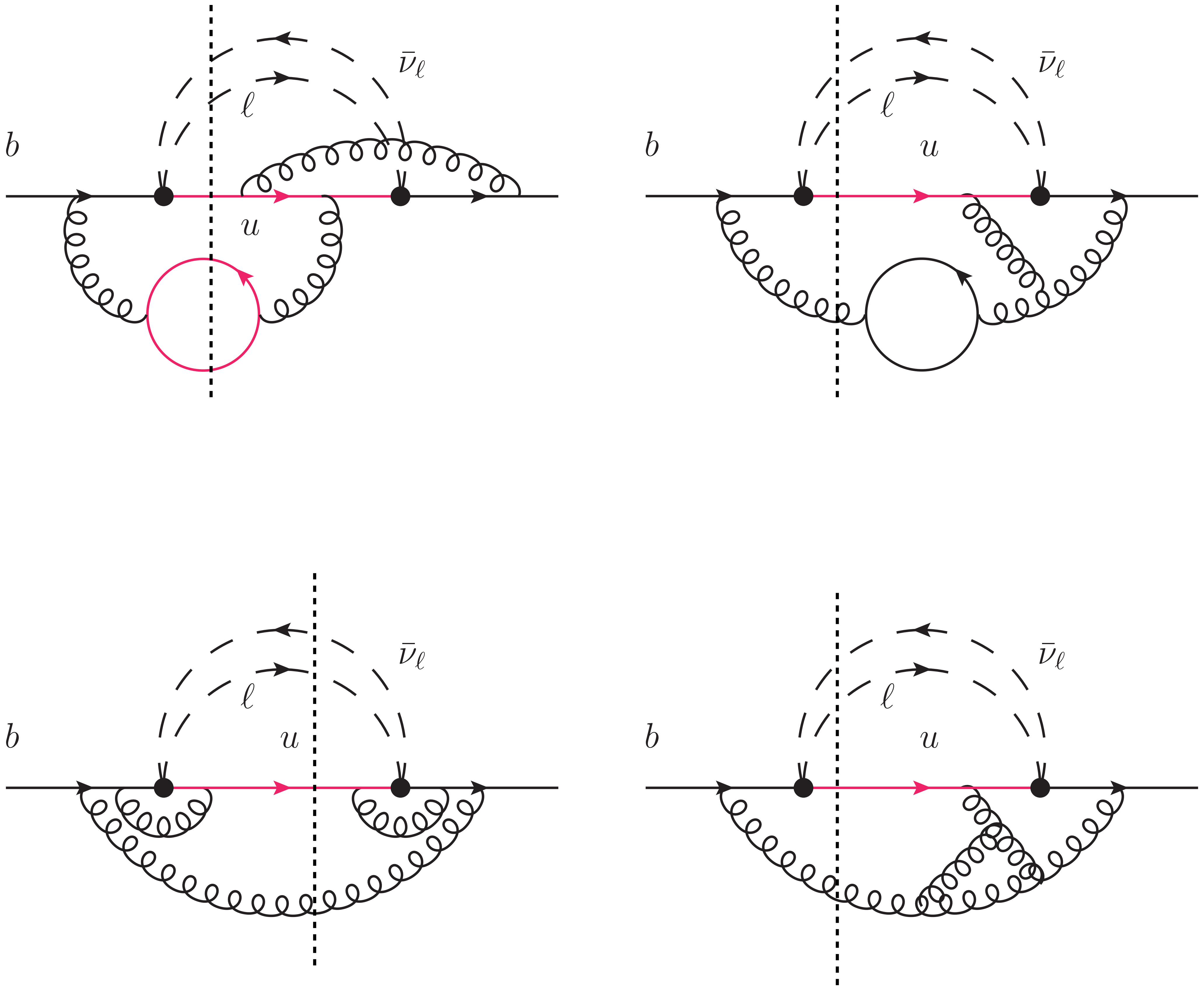
MF, Schönwald, Steinhauser, *Phys.Rev.D* 104 (2021) 016003

$$\delta\Gamma(B \to X_c l \bar{\nu}_l) = 1.2\,\%$$

Bordone, Capdevila, Gambino, Phys.Lett.B 822 (2021) 136679

$$C_F X_3(\rho = 0) = -269 \pm 27\,(10\%)$$

$$27 \times \left(\frac{\alpha_s(m_b)}{\pi}\right)^3 = 1\,\%$$



Fermionic corrections

$$X_3 = N_L^2 T_F^2 X_{N_L^2} + N_H^2 T_F^2 X_{N_H^2} + N_H N_L T_F^2 X_{N_H N_L}$$
$$+ N_L T_F (C_F X_{N_L C_F} + C_A X_{N_L C_A})$$
$$+ N_H T_F (C_F X_{N_H C_F} + C_A X_{N_H C_A})$$

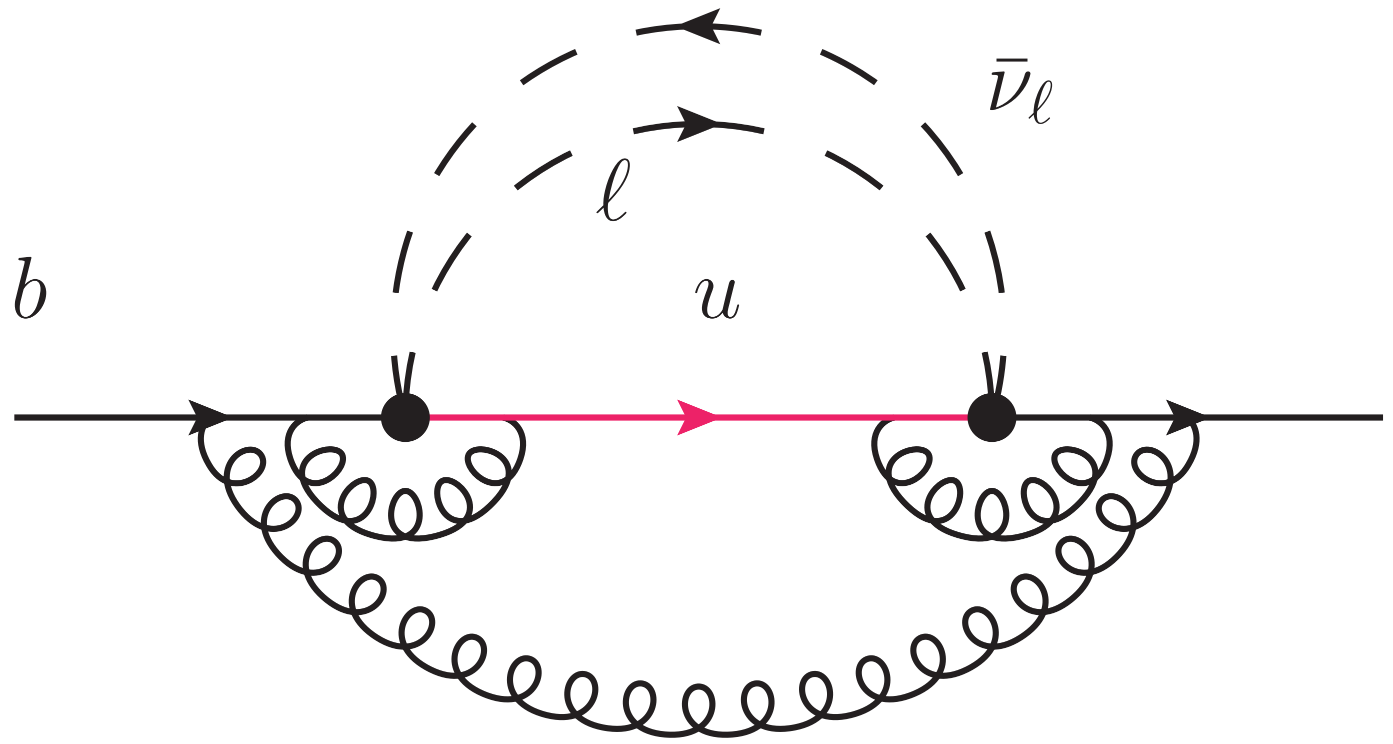
$$+ C_F^2 X_{C_F^2} + C_F C_A X_{C_F C_A} + C_A^2 X_{C_A^2}$$

Bosonic corrections

IBP REDUCTION AT 5 LOOPS

Challenging 5loop families:

12 propagators + 8 numerators



Trade electron-neutrino loop for a denominator raised to a symbolic power

$$\int d^d p \frac{p^{\mu_1}\ldots p^{\mu_N}}{(-p^2)[-(p-q)^2]} = \frac{i\pi^{2-\epsilon}}{(-q^2)^\epsilon} \sum_{i=0}^{[N/2]} f(\epsilon, i, N) \left(\frac{q^2}{2}\right)^i \{[g]^i [q]^{N-2i}\}^{\mu_1\ldots\mu_N}$$

Map 5-loop families into 4-loop ones

$$I_5(n_1, n_2, \ldots, n_{20}) \leftrightarrow \sum_{\vec{m}\in M} f_{\vec{m}}(\epsilon)\, J_{4\epsilon}(m_1, m_2, \ldots, m_{14})$$

Use Kira with: **`symbolic_ibp: [1]`**

Klappert, Lange, Maierhöfer, Usovitsch, Comput. Phys. Commun. 266 (2021) 108024
Klappert, Lange, Comput.Phys.Commun. 247 (2020) 106951

ELIMINATE SECTORS WITHOUT CUTS

We eliminate up to 70% of the non-trivial sectors

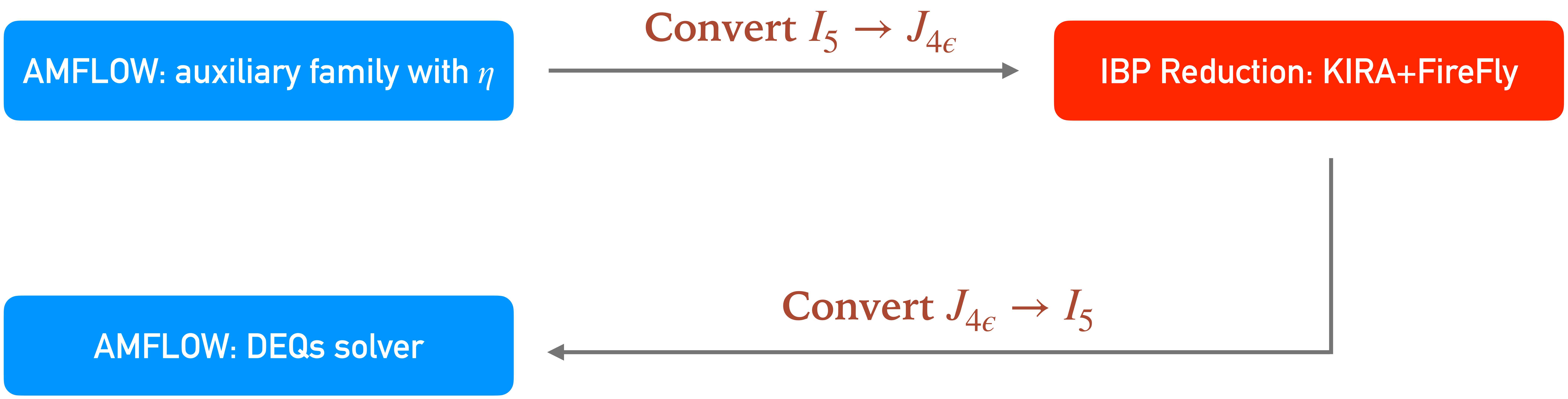
- Identify non-trivial sectors
- For each family, identify the sectors with a physics cut

- Set to zero sectors without cuts: **zero_sectors: [1,2, ...]**
- Full reduction (up to 5 scalar products) with Kira+FireFly

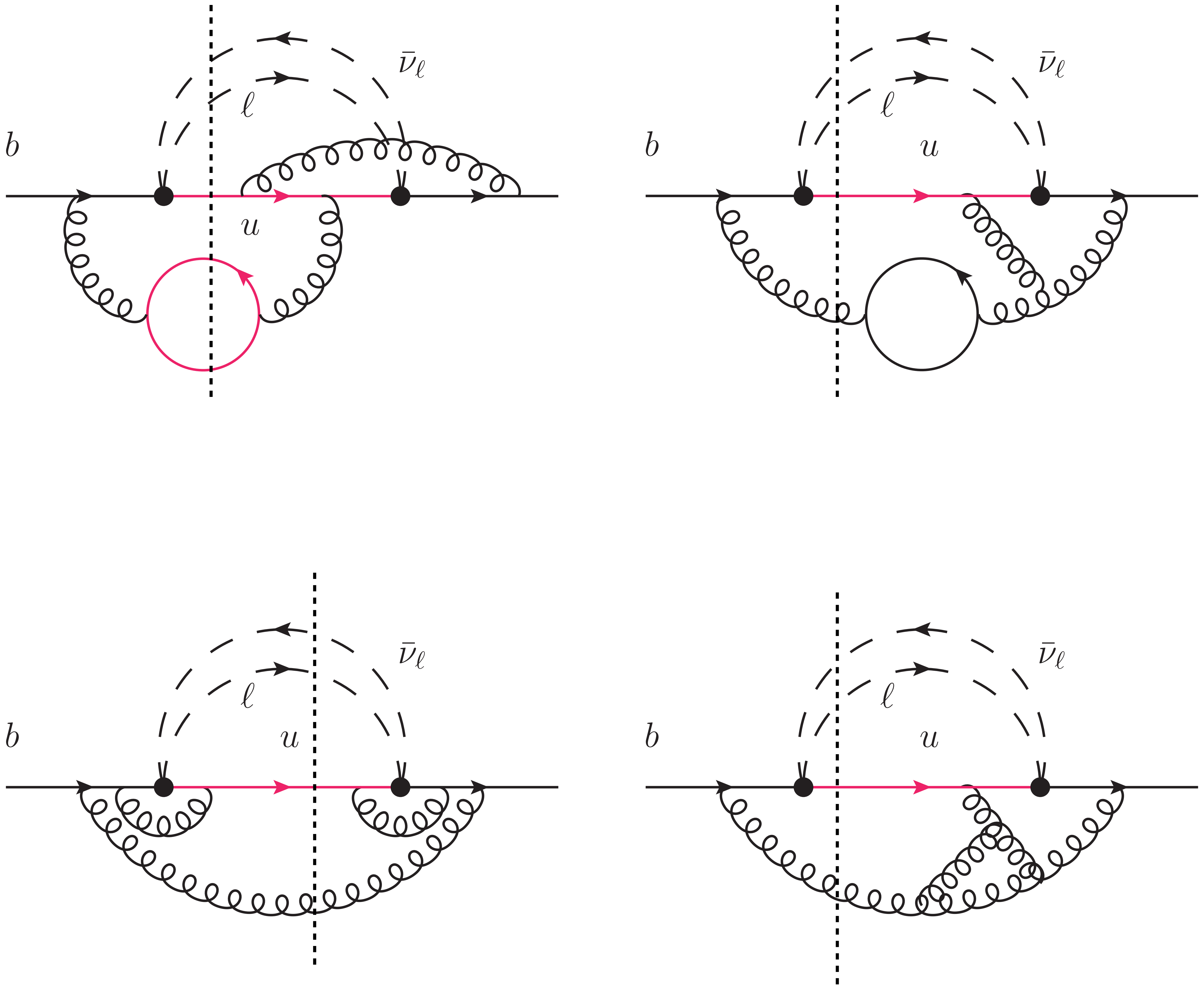
$$I_5(n_1, n_2, \ldots, n_{20}) \leftrightarrow \sum_{\vec{m} \in M} f_{\vec{m}}(\epsilon)\, J_{4\epsilon}(m_1, m_2, \ldots, m_{14})$$

NUMERICAL EVALUATION WITH AMFLOW

- 48 families - 1369 master integrals



- All non-trivial sectors must be included
- Requires 40 digits of precision



Fermionic corrections

$$X_3 = N_L^2 T_F^2 X_{N_L^2} + N_H^2 T_F^2 X_{N_H^2} + N_H N_L T_F^2 X_{N_H N_L}$$
$$+ N_L T_F (C_F X_{N_L C_F} + C_A X_{N_L C_A})$$
$$+ N_H T_F (C_F X_{N_H C_F} + C_A X_{N_H C_A})$$
$$+ C_F^2 X_{C_F^2} + C_F C_A X_{C_F C_A} + C_A^2 X_{C_A^2}$$

Bosonic corrections

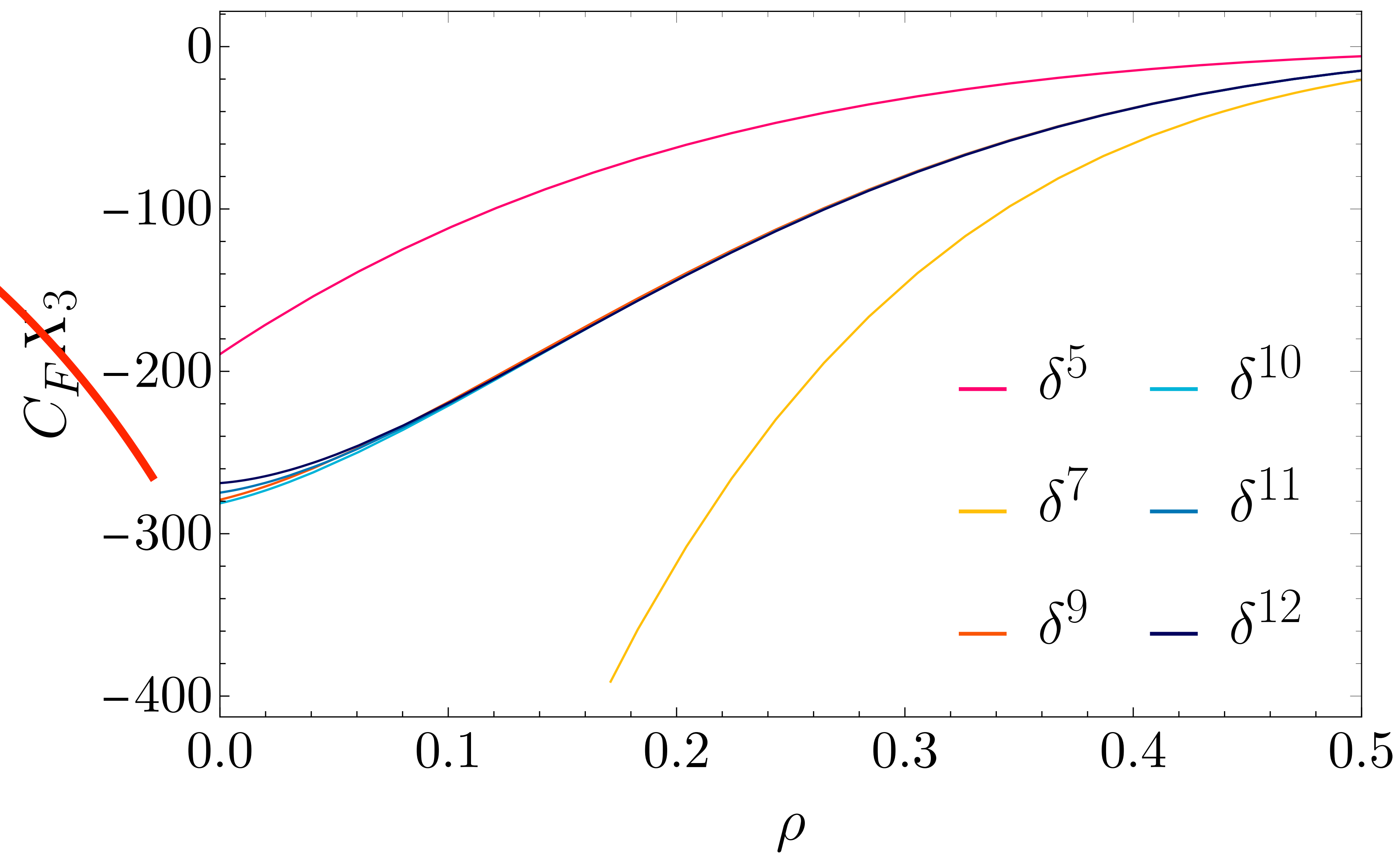
RESULTS

- Poles $\epsilon^{-3}, \epsilon^{-2}, \epsilon^{-1}$ cancel with 37, 35 and 33 digits

	This work	Ref. [28]	Difference
$T_F^2 N_L^2$	-6.9195	-6.34 (42)	8.3%
$T_F^2 N_H^2$	-1.8768×10^{-2}	$-1.97\,(42) \times 10^{-2}$	5.0%
$T_F^2 N_H N_L$	-1.2881×10^{-2}	$-1.1\,(1.1) \times 10^{-2}$	12%
$C_F T_F N_L$	-7.1876	$-5.65\,(55)$	22%
$C_A T_F N_L$	42.717	39.7 (2.1)	7%
$C_F T_F N_H$	2.1098	2.056 (64)	2.5%
$C_A T_F N_H$	-0.45059	$-0.449\,(18)$	0.4%

MF, Usovitsch, Phys.Rev.D 108 (2023) 11, 11

$$
\begin{aligned}
C_F X_3 &= 280.2 && \text{fermionic} \\
&\quad -536.4 && \text{bosonic, large } N_c \\
&\quad -11.6\,(2.7) && \text{bosonic, subleading } N_c \\
&= -267.8\,(2.7)
\end{aligned}
$$



MF, Schönwald, Steinhauser, *Phys.Rev.D* 104 (2021) 016003

- Compatible with previous estimate $C_F X_3(\rho = 0) = -269 \pm 27\,(10\%)$
- Parallel calculation large-N_c limit
  Chen, Li, Li, Wang, Wand, Wu, Phys.Rev.D 109 (2024) 7, L071503
- Agree with unpublished results for N_L and N_L^2
  Long Chen, Xiang Chen, Xin Guan, Yan-Qing Ma, hep-ph/2309.01937

SHORT DISTANCE MASS (PRELIMINARY)

$$\Gamma_0 = \frac{m_b^5 G_F^2}{192\pi^3} |V_{ub}|^2$$

$$\alpha_s \equiv \alpha_s^{(4)}(m_b)$$

$$\Gamma(B \to X_u \ell \bar{\nu}_\ell) = \Gamma_0^{\text{pole}} \left[1 - 2.413 \frac{\alpha_s}{\pi} - 21.3 \left(\frac{\alpha_s}{\pi}\right)^2 - 267.8\,(2.7) \left(\frac{\alpha_s}{\pi}\right)^3 \right]$$

Scheme conversion

$$m^{\text{pole}} \to m^X(\mu) + \mu \sum_{n=1}^{\infty} a_n^X(\mu_s/\mu) \left(\frac{\alpha_s(\mu_s)}{\pi}\right)^n$$

SHORT DISTANCE MASS (PRELIMINARY)

assuming $m_c = 0$

$$m_b^{\text{MSR}}(2\,\text{GeV}): \quad \Gamma(B \to X_u \ell \bar{\nu}_\ell) = \Gamma_0^{\text{MSR}} \left[1 + 0.039\,|_{\alpha_s} + 0.019\,|_{\alpha_s^2} + 0.010\,|_{\alpha_s^3} \right]$$

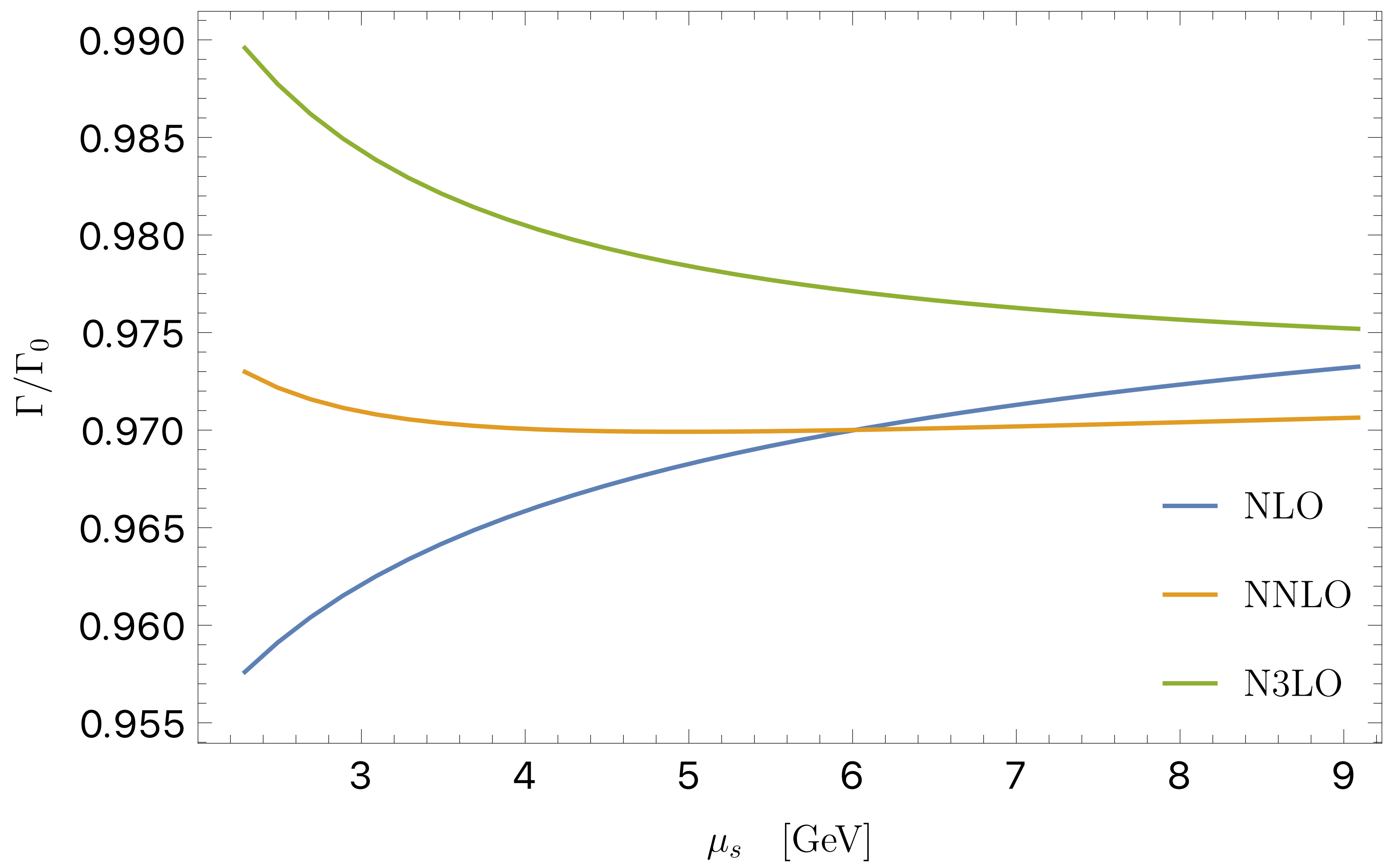
$$m_b^{1\text{S}}: \quad \Gamma(B \to X_u \ell \bar{\nu}_\ell) = \Gamma_0^{1\text{S}} \left[1 - 0.114\epsilon - 0.031\epsilon^2 + 0.002\epsilon^3 \right]$$

$$m_b^{\text{kin}}(1\,\text{GeV}): \quad \Gamma(B \to X_u \ell \bar{\nu}_\ell) = \Gamma_0^{\text{kin}} \left[1 - 0.020\,|_{\alpha_s} - 0.012\,|_{\alpha_s^2} + 0.017\,|_{\alpha_s^3} \right]$$

$$m_b^{\text{kin}}(1\,\text{GeV}): \quad \Gamma(B \to X_u \ell \bar{\nu}_\ell) = \Gamma_0^{\text{kin}} \left[1 - 0.033\,|_{\alpha_s} - 0.0026\,|_{\alpha_s^2} + 0.0095\,|_{\alpha_s^3} \right]$$

Simplified kinetic mass

$$m^{\text{pole}} \to m^{\text{kin}}(\mu) + [\overline{\Lambda}(\mu)]_{\text{pert}}$$



CONCLUSIONS

- In the last years, the theory of inclusive decays has greatly profited from developments in computational methods for multi-loop integrals.
- Complete NNLO corrections to the q^2 spectrum!
- Work in progress for the E_l spectrum
- N3LO corrections to $b \to u l \bar{\nu}_l$ must be scrutinised
  - Resummation of $\alpha_s^{n+1} \beta_0^n$ terms
  - Understand how subleading renormalons cancel in the kinetic scheme
  - We have to keep into account larger theoretical uncertainties in $b \to u l \bar{\nu}_l$

MARIE CURIE ACTIONS

Funded by
the European Union

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