# On the potential of Light-Cone Sum Rules without semi-global Quark-Hadron Duality

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Challenges in semileptonic $B$ decays
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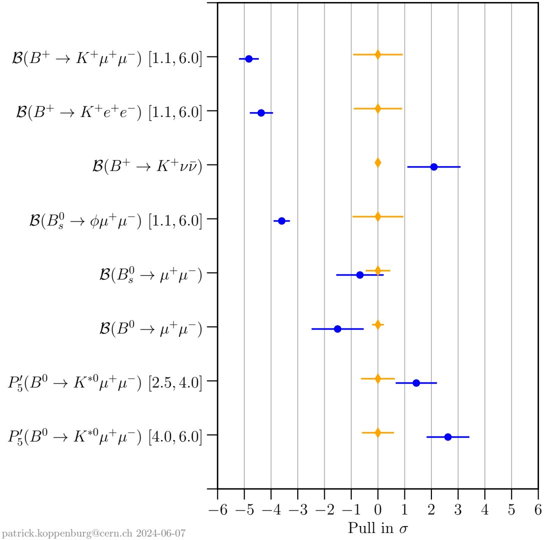
# Status of $B$-anomalies in $b \to s\ell\ell$

- Anomalies in the clean observables $R_{K^{(*)}}, BR(B_{(s)} \to \mu\mu)$ are gone.
- Discrepancies are remaining in theoretically challenging observables: $P_5'(B \to K^*\mu\mu)$, $\mathrm{BR}(B \to K^+\mu\mu)$, ...
- *Are $B$-anomalies due to New Physics or misunderstood SM effect?*



# Theory of $B \to M\ell\ell$ $(b \to s\ell\ell)$

$$A(B \to M\ell^+\ell^-) = -\frac{G_F\alpha}{\sqrt{2}\pi} V_{tb} V_{ts}^* \left[(A_\mu + T_\mu)\bar{u}_\ell\gamma^\mu v_\ell + B_\mu \bar{u}_\ell \gamma^\mu\gamma_5 v_\ell\right], \quad (1)$$

where

$$\begin{aligned} A_\mu &= C_9\langle \bar{M}|\bar{s}\gamma_\mu P_L b|\bar{B}\rangle - C_7\frac{2im_b}{q^2}q^\nu\langle \bar{M}|\bar{s}\sigma_{\mu\nu}P_R b|\bar{B}\rangle, \\ B_\mu &= C_{10}\langle \bar{M}|\bar{s}\gamma_\mu P_L b|\bar{B}\rangle, \end{aligned} \quad (2)$$

the $B \to M$ matrix elements are expressed in terms of (local) form factors. The remaining term is the non-local contribution

$$T_\mu = -\frac{4\pi^2}{q^2} i \int d^4x\, e^{iq\cdot x}\langle \bar{M}|T j_\mu(x) H^c(0)|\bar{B}\rangle. \quad (3)$$

Non-local contributions do not seem to explain the anomalies (still debated):

- Unitarity bounds (Bobeth, Chrzaszcz, van Dyk, Virto 2017 [1], Gubernari, van Dyk, Virto 2020 [2]).
- $q^2$-independence of the fitted NP Wilson coefficients (e.g. Bordone, Isidori, Mächler, Tinari 2024 [3]).

# Local form factors for $B \to$ light at low $q^2$

**The status of the $B$-anomalies seems to rely essentially on the calculation of the local form factors in the SM.** In this talk I focus on the low-$q^2$ range where most of the anomalies are. There are three ways to compute the form factors in this region:

- LCSR with light-meson distribution amplitude (DA) (see e.g. Khodjamirian Russov 2017 [4])
- LCSR with $B$-meson DA (**this talk**) (e.g. Cui, Huang, Shen, Wang, Wang 2022 [5] in SCET, Gubernari, Kokulu, Van Dyk 2018 [6] in HQET). GKvD used e.g. in LHCb 2024 [7].
- Lattice QCD, at $q^2 = 0$ only $f_{+,T}^{B\to K}$ (HPQCD 2022 [8])

# Establishing the sum rule

Khodjamirian, Mannel, Offen 2005 [9]

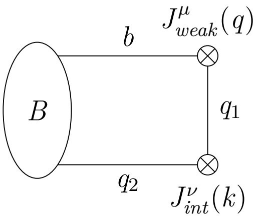
$$\Pi^{\mu\nu}(q,k) = i\int d^4x e^{ik.x} \langle 0|\, T\left\{J^{\nu}_{int}(x) J^{\mu}_{weak}(0)\right\} |\bar{B}(p_B = q+k)\rangle\,, \quad (4)$$

From analyticity (taking $k^2 < 0$) and unitarity

$$\Pi^{\mu\nu}(q,k) = \frac{\langle 0|J^{\nu}_{int}|M(k)\rangle\langle M(k)|J^{\mu}_{weak}|\bar{B}(q+k)\rangle}{m_M^2 - k^2} + \int_{s_{cont}}^{+\infty} ds \frac{\rho^{\mu\nu}(q,s)}{s-k^2}, \quad (5)$$

From the identification of Lorentz structures in the definition of the form factors

$$\Pi_F(q^2,k^2) = Y_F \frac{F(q^2)}{m_M^2 - k^2} + \int_{s_{cont}}^{\infty} \frac{\rho_F(q^2,s)}{s-k^2}, \quad (6)$$



# Establishing the sum rule

We reproduce the results of Gubernari, Kokulu, van Dyk 2018 [6], with 2-particles up to twist-5 and and 3-particles up to twist-3. For 2-particles e.g.:

$$\Pi_F^{LCOPE} = \int_0^{+\infty} d\sigma \sum_{n=1}^{+\infty} \frac{I_n^F(\sigma)}{(s(\sigma)-k^2)^n}, \qquad s(\sigma) \equiv \sigma m_B^2 + \frac{m_1^2 - \sigma q^2}{1-\sigma} \tag{7}$$

$$I_n^{F,(2\mathrm{p})}(\sigma, q^2) = \frac{f_B m_B}{(1-\sigma)^n} \sum_{\psi_{2\mathrm{p}}} C_n^{F,\psi_{2\mathrm{p}}}(\sigma, q^2)\, \psi_{2\mathrm{p}}(\sigma m_B) \tag{8}$$

where $\psi_{2p} = \phi_+, \phi_-, g_+, g_-$ are the distributions amplitudes. B-LCDAs from Braun, Ji, Manoshov 2017 [10] using the exponential model (using the alternative models yield a $0-10\%$ variation). *We derive the full expression for the twist-5 2-particle LCDA $g_-$. (Wandzura-Wilczek approximation in GKvD 2018)*

# Borel Transform vs Moments method

***Global* quark-hadron duality**

$$\Pi_F = Y_F \frac{F(q^2)}{m_M^2 - k^2} + \int_{s_{cont}}^{\infty} \frac{\rho_F(q^2, s)}{s - k^2} \simeq \Pi_F^{LCOPE} = \int_0^{+\infty} d\sigma \sum_{n=1}^{+\infty} \frac{I_n^F(\sigma)}{(s(\sigma) - k^2)^n} \quad (9)$$

**Borel transform:**

$$\frac{e^{m_M^2/M^2}}{Y_F} \mathcal{B}_{M^2} \Pi_F(q^2, k^2)$$

$$\equiv \frac{e^{m_M^2/M^2}}{Y_F} \lim_{\substack{p, -k^2 \to +\infty \\ -k^2/p = M^2}} \frac{(-k^2)^{(p+1)}}{p!} \left(\frac{\partial}{\partial k^2}\right)^p \Pi_F(q^2, k^2)$$

$$= F(q^2) + \int_{s_{cont}}^{+\infty} \frac{\rho_F(s)}{Y_F} e^{(m_M^2 - s)/M^2}$$

$$= \int_0^{+\infty} d\sigma \sum_{n=1}^{+\infty} \frac{I_n^F(\sigma)}{M^{2(n-1)}(n-1)!} e^{(m_M^2 - s(\sigma))/M^2} \quad (10)$$

**Moments method** (used in this work)

$$\frac{(m_M^2 - k^2)^{p+1}}{p!\, Y_F} \left(\frac{\partial}{\partial k^2}\right)^p \Pi_F(q^2, k^2)$$

$$= F(q^2) + \int_{s_{cont}}^{+\infty} \frac{\rho_F(s)}{Y_F} \left(\frac{m_M^2 - k^2}{s - k^2}\right)^{p+1}$$

$$= \int_0^{\infty} d\sigma \sum_{n=1}^{\infty} \frac{(n+p-1)!}{(n-1)!} \times \frac{I_n^{(F)}(\sigma)}{(s(\sigma) - k^2))^{n+p}} \quad (11)$$

$$\boxed{M^2 \sim \frac{-k^2}{p}}$$

# Semi-global Quark Hadron Duality

**_Semi-Global_ Quark-Hadron Duality approximation**

$$\int_{s_{cont}}^{+\infty} \rho(s) e^{-s/M^2} = \frac{1}{\pi}\int_{s_0}^{+\infty} \mathrm{Im}\Pi^{LCOPE}(s) e^{-s/M^2} \tag{12}$$

Thus

$$Y_F F(q^2) e^{-m_M^2/M^2} = \mathcal{B}_{M^2}\Pi_F^{LCOPE} - \frac{1}{\pi}\int_{s_0}^{+\infty} \mathrm{Im}\Pi^{LCOPE}(s) e^{-s/M^2} \tag{13}$$

where the effective threshold is set to verify e.g.

$$m_M^2 = \frac{\frac{d}{d(-1/M^2)}\left[\mathcal{B}_{M^2}\Pi_F^{LCOPE} - \frac{1}{\pi}\int_{s_0}^{+\infty} \mathrm{Im}\Pi^{LCOPE}(s) e^{-s/M^2}\right]}{\mathcal{B}_{M^2}\Pi_F^{LCOPE} - \frac{1}{\pi}\int_{s_0}^{+\infty} \mathrm{Im}\Pi^{LCOPE}(s) e^{-s/M^2}} \tag{14}$$

# Dependence on $\lambda_B^{-1}$ and effective threshold determination strategy in $B$-meson LCSR

| form factor | $B \to K^*$ | | | $B \to \rho$ | | |
|---|---|---|---|---|---|---|
| | GKvD [6] | (i) | (ii) | GKvD [6] | (i) | (ii) |
| $V$ | $0.33 \pm 0.11$ | $0.31^{+0.19}_{-0.15}$ | $0.48^{+0.24}_{-0.20}$ | $0.27 \pm 0.14$ | $0.16^{+0.12}_{-0.09}$ | $0.27^{+0.16}_{-0.13}$ |
| $A_1$ | $0.26 \pm 0.08$ | $0.25^{+0.14}_{-0.12}$ | $0.36^{+0.18}_{-0.15}$ | $0.22 \pm 0.10$ | $0.14^{+0.09}_{-0.07}$ | $0.21^{+0.11}_{-0.10}$ |
| $A_2$ | $0.24 \pm 0.09$ | $0.22^{+0.16}_{-0.12}$ | $0.36^{+0.20}_{-0.17}$ | $0.19 \pm 0.11$ | $0.11^{+0.10}_{-0.07}$ | $0.20^{+0.14}_{-0.10}$ |
| $T_1$ | $0.29 \pm 0.10$ | $0.27^{+0.17}_{-0.13}$ | $0.41^{+0.20}_{-0.17}$ | $0.24 \pm 0.12$ | $0.15^{+0.10}_{-0.08}$ | $0.24^{+0.13}_{-0.11}$ |
| $T_{23}$ | $0.58 \pm 0.13$ | $0.58^{+0.19}_{-0.20}$ | $0.73^{+0.16}_{-0.21}$ | $0.56 \pm 0.15$ | $0.43^{+0.16}_{-0.15}$ | $0.56^{+0.16}_{-0.16}$ |
| $s_0$ (GeV$^2$) | $[1.4, 1.7]$ | $1.53^{+0.35}_{-0.09}$ | $1.54^{+0.34}_{-0.10}$ | $1.6 \pm 0.032$ | $1.03^{+0.08}_{-0.04}$ | $1.05^{+0.09}_{-0.04}$ |

Table 1: Prediction of $B \to \rho, K^*$ form factors at $q^2 = 0$ following the calculation of GKvD 2018 using a different method for the determination of the effective threshold. $\lambda_B^{-1} = 2.2 \pm 0.6$ GeV$^{-1}$ (GKvD 2018 [6]) (i) and $\lambda_B^{-1} = 2.72 \pm 0.66$ GeV$^{-1}$ (Khodjamirian et al. 2020 [11]) (ii).

# Challenges of $B$-meson LCSR

- **Effective threshold**: For $B \to \pi, \rho, K$, the effective threshold $s_0$ cannot be determined using the meson mass sum rule. Thresholds from QCD sum rules are used in GKvD 2018.
- **Borel window**: In the literature $M^2 \in [0.5, 1.5]\,\mathrm{GeV}^2$ (KMO '06, GKvD '18) or alternatively $M^2 \in [1, 1.5]$ GeV$^2$ (CHSWW '22, DKVV '23). *Up to* $20\%$ *variations of the form factor between* $M^2 = 0.5\,GeV^2$ *and* $M^2 = 1.5\,GeV^2$.
- $B$-**LCDA** are only known asymptotically at low $\omega$, full DAs are model dependent. Plagued by end-point divergences which can break the twist hierarchy.
- $B$-**LCDA parameters** are poorly known. We use our own average of the results of $\lambda_B^{-1}$ Khodjamirian, Mandal, Mannel 2020 [11] $\lambda_B^{-1} = 2.72 \pm 0.66$ GeV$^{-1}$ (GKvD: $\lambda_B^{-1} = 2.2 \pm 0.6$ GeV$^{-1}$)

## Definitions

We define

$$\widetilde{\Pi}_F^{(p)}(q^2,k^2) \equiv \frac{(m_M^2-k^2)^{p+1}}{p!\;Y_F}\Pi_F^{(p)}(q^2,k^2), \qquad R_F(p,q^2,k^2) \equiv \int_{s_{cont}}^{\infty} \frac{\rho_F(s)}{Y_F}\left(\frac{m_M^2-k^2}{s-k^2}\right)^{p+1},$$

and

$$\widetilde{m}_M^2(p,\ell,k^2) \equiv \left[\frac{p!}{(p-\ell)!}\frac{\Pi_F^{(p-\ell)}}{\Pi_F^{(p)}}\right]^{1/\ell} + k^2,$$

The sum rule becomes

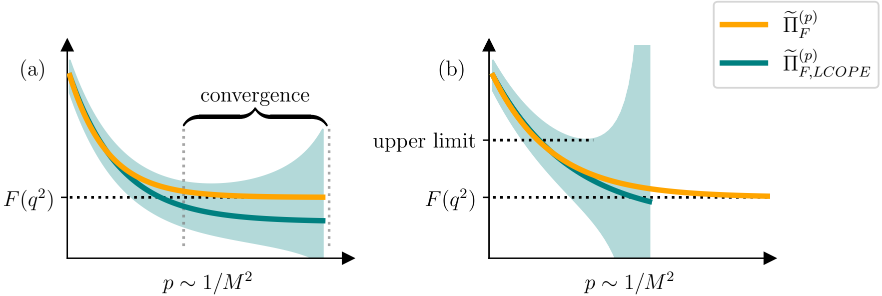
$$\widetilde{\Pi}_F^{(p)}(q^2,k^2) = F(q^2) + R_F(p), \tag{15}$$

and

$$\boxed{\widetilde{\Pi}_F^{(p)}(q^2,k^2) \xrightarrow[p\to\infty]{} F(q^2), \quad R_F(p,q^2,k^2) \xrightarrow[p\to\infty]{} 0, \quad \widetilde{m}_M^2(p,\ell,k^2) \xrightarrow[p\to\infty]{} m_M^2.} \tag{16}$$

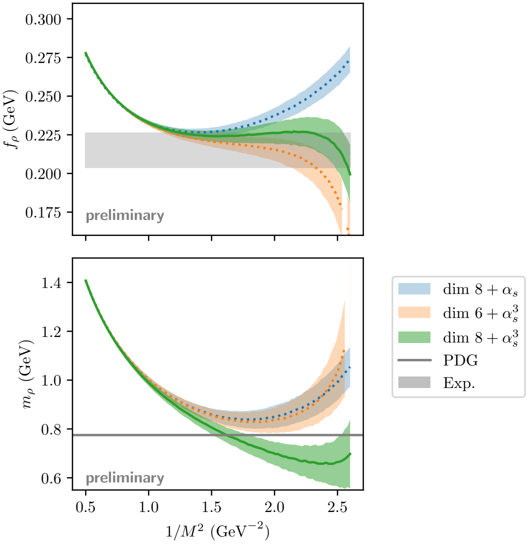
# (LC) Sum Rule without semi-global QHD

Accounting for the truncation error, $\Pi_F^{(p)} = \Pi_{F,LCOPE}^{(p)}$ within uncertainties. For $p$ ($M^2$) large (low) enough, $R_F \ll F(q^2)$ and $R_F \ll \mathrm{std}(\Pi_{F,LCOPE}^{(p)})$, thus $F(q^2) = \Pi_{F,LCOPE}^{(p)}(q^2)$ within uncertainties.




Same behaviour for $\widetilde{m}_M^2$, $\widetilde{m}_{M,pert}^2$ and $m_M^2$, thus the daughter sum rule can tell us when (if) we reach the convergence regime. $p \to \infty \leftrightarrow 1/M^2 \to \infty$.

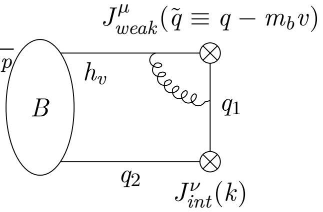
# Check with two-point QCD sum rule for the $\rho$ meson - WIP




- Shifman-Vainshtein-Zakharov [12] sum rule successfully predicts e.g. the $\rho$ meson decay constant
- $\Pi_{SVZ} = \sum_d C_d(q^2)\langle O_d \rangle$ where $\langle O_d \rangle$ are quark-gluon vacuum condensates
- $\langle \bar{q}q \rangle$ from FLAG 2021, $\langle \bar{s}s \rangle$ from HPQCD [13], $\langle \alpha_s GG \rangle = 0.012(4)$ (SVZ)
- Dimension 8 contribution for the $\rho$ sum rule extracted from $\frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$ [14]
- dim 0: $\alpha_s^3$, dim 3: $\alpha_s^2$, dim 6: $\alpha_s$
- Caveat: This figure only includes parametric errors
- **We obtain $f_\rho$ from SVZ sum rules without semi-global QHD**

# QCD perturbativity

$$\Pi^{(p)}_{F,LCOPE}(q^2,k^2) = \int_0^{\sigma_{\max}} d\sigma \sum_{n=1}^{\infty} \frac{(n+p-1)!}{(n-1)!} \frac{I_n^{(F)}(\sigma)}{(s(\sigma)-k^2))^{n+p}}$$
$$\equiv \int_0^{\sigma_{\max}} d\sigma I^{(F,p)}_{\text{tot}}(\sigma,k^2)$$

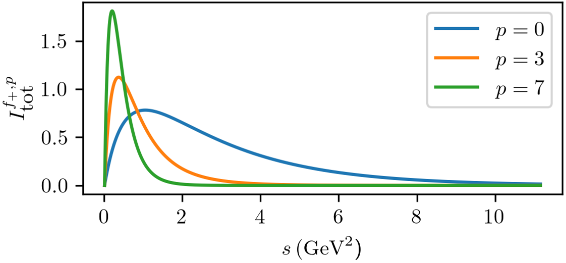



We define the average virtuality

$$\langle s \rangle = \frac{\int_0^{\sigma_{\max}} d\sigma |I^{(F,p)}_{\text{tot}}(\sigma,k^2)| s(\sigma)}{\int_0^{\sigma_{\max}} d\sigma |I^{(F,p)}_{\text{tot}}(\sigma,k^2)|}$$

To keep the radiative corrections under control we require

$$|\langle s \rangle - m_1^2| \gg \Lambda^2_{QCD}, \quad |k^2|, |\tilde{q}^2| \gg \Lambda^2_{QCD}.$$



# QCD error model

For the characteristic QCD scale we take

$$\mu_{\rm QCD} \equiv \text{min}(\sqrt{\langle s\rangle - m_1^2}, \sqrt{|k^2|}, \sqrt{|\tilde{q}^2|}). \tag{17}$$

Interestingly, $\langle s\rangle > -k^2/p \approx M^2$ when $p$ becomes large.

$$\Pi_F^{(p)} = \Pi_{F,LO}^{(p)} \left[1 + w_{\alpha_s}(\mu) \sum_{n=1} \left(\frac{\alpha_s(\mu)}{\pi}\right)^n\right] = \Pi_{F,LO}^{(p)} \left[1 + w_{\alpha_s}(\mu) \frac{\alpha_s(\mu)/\pi}{1-\alpha_s(\mu)/\pi}\right], \tag{18}$$

where $w_{\alpha_s}(\mu \sim m_B) \sim 1$ and $w_{\alpha_s}(\mu \to \Lambda_{QCD}) = 0$. We take $\mu = \mu_{QCD} > 0.8$ GeV and

$$w_{\alpha_s} \in [-1.5, 1.5] \tag{19}$$

# LCOPE truncation error

$$\Pi_{F,LCOPE}(q^2,k^2) = \underbrace{\Pi_{2p}^{twist-2,3} + \Pi_{3p}^{twist-3,4}}_{\propto (x^2)^0:\ \text{LT}} + \underbrace{\Pi_{2p}^{twist-4,5} + \Pi_{3p}^{twist-5,6}}_{\propto x^2:\ \text{NLT}} + \ldots \tag{20}$$

$$\begin{aligned}\Pi_F^{(p)} &= \sum_{t\geq 2} \Pi^{(p)}_{twist=t} = \Pi^{(p)}_{\text{LT}} + \Pi^{(p)}_{\text{NLT}} + \sum_{t\geq 6} \Pi^{(p)}_{twist=t} \\ &\equiv \Pi^{(p)}_{\text{LT}} + \Pi^{(p)}_{\text{NLT}} + w_{LCOPE} \times \frac{(\Pi^{(p)}_{\text{NLT}})^2}{|\Pi^{(p)}_{\text{LT}}| - |\Pi^{(p)}_{\text{NLT}}|}.\end{aligned} \tag{21}$$

To be conservative we take a uniformly distributed $w_{LCOPE}$

$$w_{LCOPE} \in [-2, 2] \tag{22}$$

and keep $\Pi^{(p)}_{NLT}/\Pi^{(p)}_{LT} < 20\%$

# Strategy summary

- **Requirements:**
  - Large average virtuality $\langle s \rangle - m_1^2 \gg \Lambda_{QCD}^2$ (pQCD)
  - $\Pi_{NLT}/\Pi_{LT} \ll 1$ (LCOPE)
- **Checks:**
  - $k^2$- and $p$- independence (one extra check provided by the moments method)
  - Convergence towards the physical mass of the final meson $\tilde{m}_M^2 \to m_M^2$

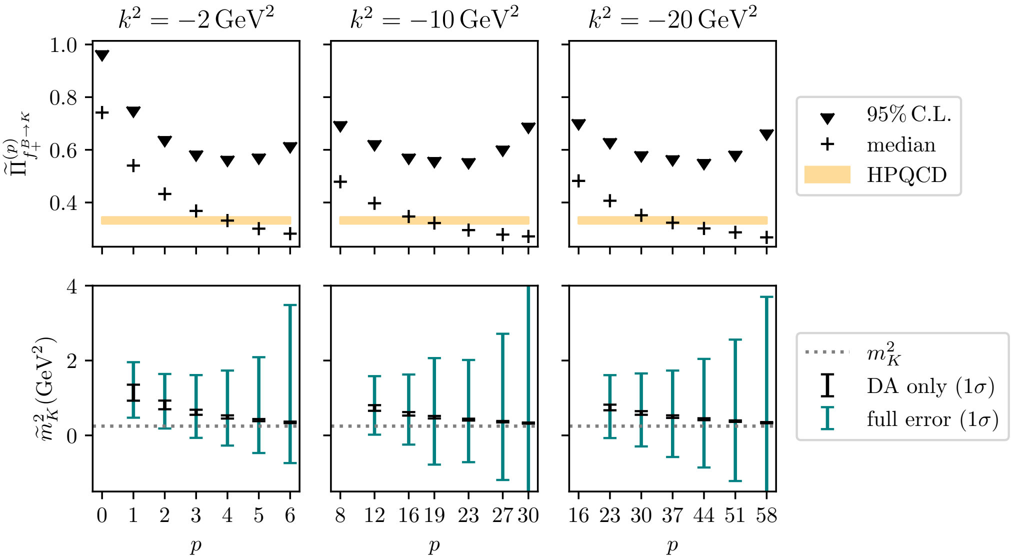
## $-k^2/p$-dependence of $V^{B\to K^*}$

| $-k^2/p$ (GeV$^2$) | 20/13 | 20/16 | 20/20 | 20/25 | 20/30 | 20/35 |
|---|---|---|---|---|---|---|
| $R_{V^{B\to K^*}}$ | $0.50^{+0.15}_{-0.13}$ | $0.37^{+0.11}_{-0.11}$ | $0.25^{+0.08}_{-0.08}$ | $0.15^{+0.05}_{-0.04}$ | $0.08^{+0.03}_{-0.02}$ | $0.03^{+0.01}_{-0.01}$ |
| $V^{B\to K^*}$ | $0.43^{+0.24}_{-0.19}$ | $0.45^{+0.25}_{-0.20}$ | $0.47^{+0.28}_{-0.22}$ | $0.49^{+0.30}_{-0.21}$ | $0.52^{+0.31}_{-0.23}$ | $0.53^{+0.35}_{-0.24}$ |
| $\widetilde{\Pi}^{(p)}_{V^{B\to K^*}}$ | $0.94^{+0.37}_{-0.33}$ | $0.84^{+0.36}_{-0.31}$ | $0.72^{+0.35}_{-0.30}$ | $0.64^{+0.34}_{-0.26}$ | $0.60^{+0.32}_{-0.25}$ | $0.55^{+0.35}_{-0.25}$ |
| $s_0$(GeV$^2$) | $1.43^{+0.026}_{-0.012}$ | $1.48^{+0.03}_{-0.01}$ | $1.56^{+0.04}_{-0.02}$ | $1.68^{+0.06}_{-0.03}$ | $1.87^{+0.1}_{-0.05}$ | $2.21^{+0.25}_{-0.10}$ |

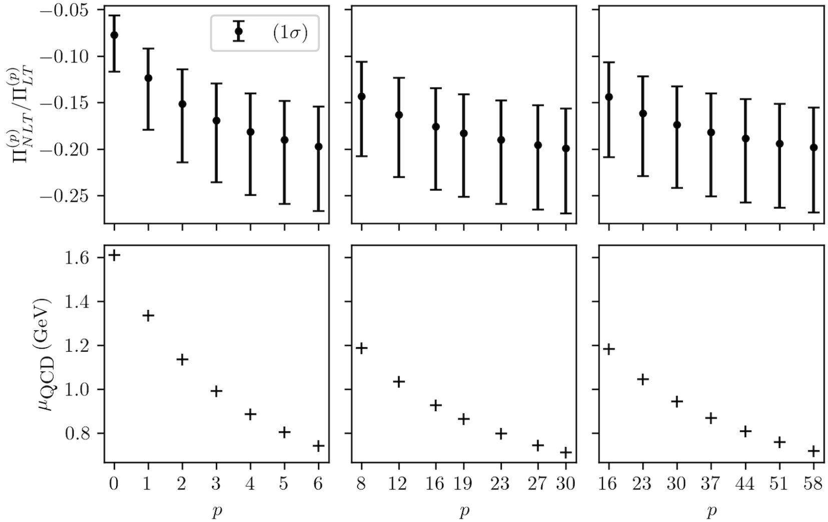
Table 2: Central values and $68\%$ C.L. intervals of $\widetilde{\Pi}^{(p)}_{V^{B\to K^*}}$ $(q^2=0)$. The corresponding $R_{V^{B\to K^*}}$, $V^{B\to K^*}$ and $s_0$ are estimated using DSR and semi-global QHD.

At $-k^2/p$ $M^2 \sim 1.53\text{GeV}^2$ $R_F/\widetilde{\Pi}^{(p)}_F = 53\%$! We advocate for $-k^2/p$ $M^2 < 1$ GeV$^2$.
Variation of $V^{B\to K^*}$ of $20\%$ between $M^2 = 0.5$ GeV$^2$ and $M^2 = 1.5$ GeV$^2$.

# Results - $f_+^{B\to K}$



# Results - $f_+^{B\to K}$

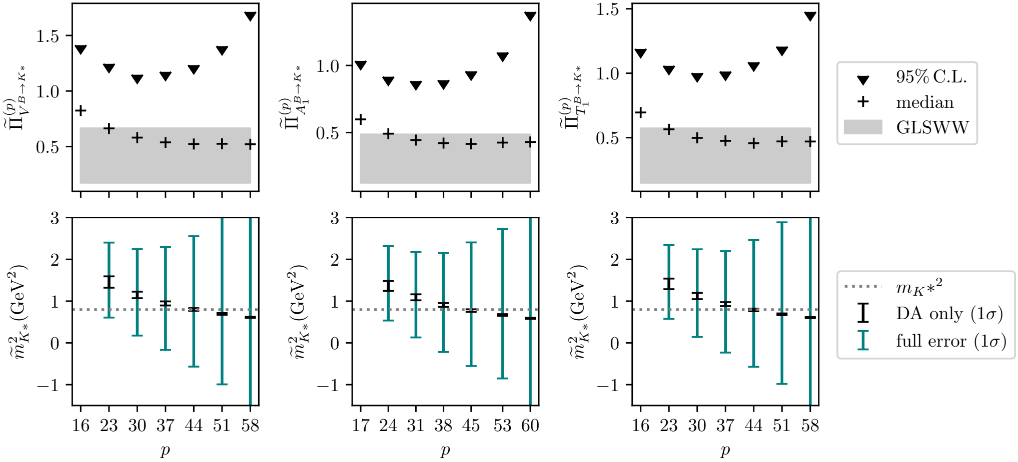


# Results - $B \to K$

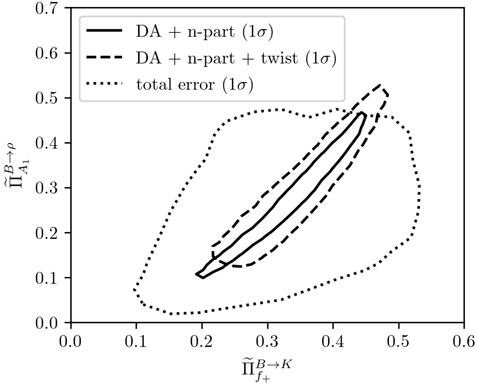
| form factor | $-k^2/p$ | $R_F(p,k^2)$ | upper limit @ 95% C.L. | $\widetilde{\Pi}_F^{(p)}$ $(1\sigma)$ | literature | Ref. |
|---|---|---|---|---|---|---|
| $f_+^{B\to K}$ | $10/19$ | $0.02^{+0.05}_{-0.04}$ | $0.57$ | $0.32^{+0.15}_{-0.12}$ | $0.332(12)$<br>$0.27(8)$<br>$0.325(85)$<br>$0.395(33)$ | HPQCD [8]<br>GvDK [6]†<br>CHSWW [5]<br>KR [4] |
| $f_T^{B\to K}$ | $10/8$ | $0.03^{+0.06}_{-0.11}$ | $0.46$ | $0.34^{+0.08}_{-0.07}$ | $0.332(21)$<br>$0.25(7)$<br>$0.381(27)$<br>$0.381(97)$ | HPQCD [8]<br>GvDK [6]†<br>KR [4]<br>CHSWW [5] |

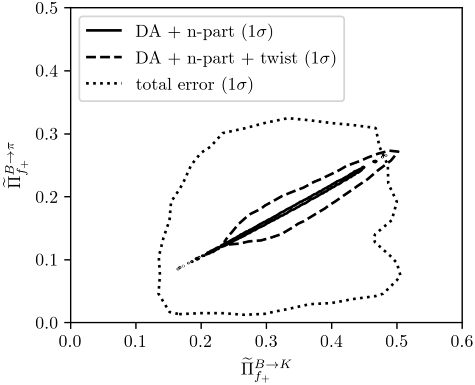
Table 3: Upper limits at the $95\%$ confidence level and central value of $\widetilde{\Pi}_F^{(p)}$ for $B \to K$. We include the corresponding values of $-k^2$ and $p$ as well as an estimate of $R_F(p,k^2)$ using quark-hadron duality. $R_F$ is calculated using $s_0$ from GKvD 18.

# $B \to K^*$ - $k^2 = -20\text{GeV}^2$



# Correlation



## Conclusion and prospects

- We propose to push the LCSR method to low Borel parameters (or $-k^2/p$) to suppress (virtually) entirely the spectral integral and avoid the semi-global QHD.
- In this limit the predicted FFs are less sensitive to the choice of the $B$-DA model.
- We find that the spectral density integral is negligible in the region $0.4\ \mathrm{GeV}^2 < M^2 < 0.5\ \mathrm{GeV}^2$ and the LCOPE is under control for $B \to$ light channels. Radiative corrections in this range are potentially large but calculable.
- Avoiding the determination of the effective threshold yields higher correlations between form factors with different final mesons.
- This approach works well for LCSR with e.g. $K$-LCDAs for which the radiative corrections are known and higher twists have been calculated. 24xx.soon
- We have started computing the radiative corrections for $B$-LCSR in HQET.

Thank you!

# B-meson Light-Cone Distribution Amplitude

We take the B-LCDAs from Braun, Ji, Manoshov 2017 [10] using the exponential model (using the alternative models yield a $0 - 10\%$ variation). For 2-particles the DA's are defined as

$$\langle 0 \,|\bar{q}(x)\Gamma[x,0]h_v(0)|\, \bar{B}(v)\rangle = -\frac{i}{2}F_B\,\mathrm{Tr}\left[\gamma_5\Gamma P_+\right]\int_0^\infty d\omega\, e^{-i\omega(vx)}\left\{\phi_+(\omega) + x^2 g_+(\omega)\right\}$$
$$+\frac{i}{4}F_B\,\mathrm{Tr}\left[\gamma_5\Gamma P_+ \not{x}\right]\frac{1}{vx}\int_0^\infty d\omega\, e^{-i\omega(vx)}\left\{\left[\phi_+ - \phi_-\right](\omega) + x^2\left[g_+ - g_-\right](\omega)\right\} \tag{23}$$

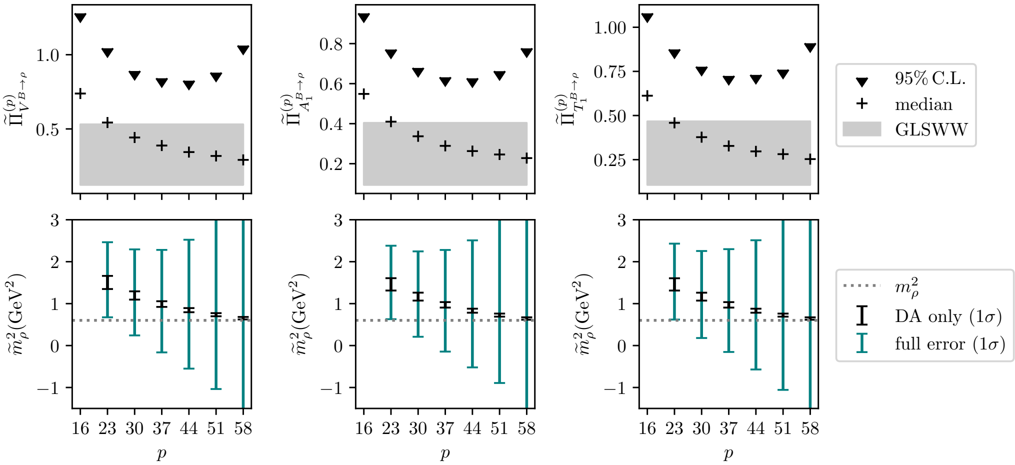
We derive the full expression for the twist-5 2-particle LCDA $g_-$. (Only the Wandzura-Wilczek approximation was used for $g_-$ in GKvD [6])

# Dependence on $\lambda_B^{-1}$ and effective threshold determination strategy in $B$-meson LCSR

| form factor | $B \to \pi$ | | | $B \to K$ | | |
|---|---|---|---|---|---|---|
| | GKvD [6] | (iii) | (iv) | GKvD [6] | (iii) | (iv) |
| $f_+$ | $0.21(7)$ | $0.023(7)$ | $0.26^{+0.08}_{-0.08}$ | $0.27(8)$ | $0.24(7)$ | $0.34^{+0.09}_{-0.09}$ |
| $f_T$ | $0.19(7)$ | $0.024(7)$ | $0.24^{+0.06}_{-0.06}$ | $0.25(7)$ | $0.24(7)$ | $0.31^{+0.06}_{-0.08}$ |
| $s_0$ (GeV$^2$) | $0.7 \pm 0.014$ | $0.0393(1)$ | $0.7 \pm 0.014$ | $1.05 \pm 0.021$ | $0.54^{+0.03}_{-0.02}$ | $1.05 \pm 0.021$ |

Table 4: Prediction of $B \to \pi, K$ form factors at $q^2 = 0$ following the calculation of GKvD 2018 [6]. Our results are obtained using $\lambda_B^{-1} = 2.72 \pm 0.66$ GeV$^{-1}$ and $s_0$ obtained from a daughter sum rule (iii) and using the same threshold $s_0$ as in [6] (iv).

# Results - $k^2 = -20\text{GeV}^2$



# Mass prediction

- The mass sum rule has huge error bars because we decorrelated the radiative corrections of the successive derivatives.
- The parametric uncertainties in the DA parameters cancel out when $p$ increases

$$\widetilde{m}_M^2 = m_M^2 + (m_M^2 - k^2)\left[\frac{1}{\ell}\cdot\frac{R_F(p-\ell)-R_F(p)}{F(q^2)} + \mathcal{O}\left(\frac{R_F}{F}\right)^2\right], \qquad (24)$$

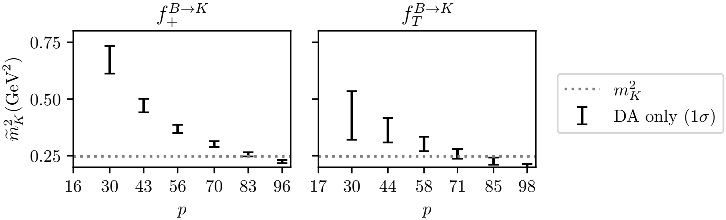
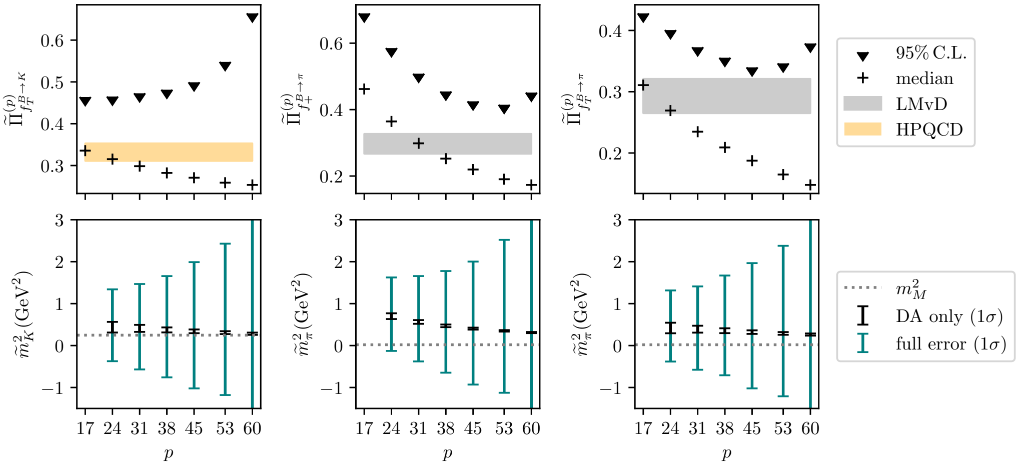



Figure 1: Mass predictions accounting for DA parametric error only $k^2 = -20$ GeV$^2$

# Results - $k^2 = -20\text{GeV}^2$



5 Backup Slides

6 References

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