

Implications of SMEFT for semileptonic processes and effects beyond SMEFT

Based on : arXiv:2404.10061, arXiv: 2408.13069 and arXiv: 2305.16007

In collaboration with Prof. Amol Dighe, Susobhan Chattopadhyay, and Dr. Rick S. Gupta.

Challenges in semileptonic B decays, Vienna, Sep 23-27, 2024

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Standard Model Effective Field Theory (SMEFT) :

$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}} + \frac{1}{\Lambda} C^{(5)} O^{(5)} + \frac{1}{\Lambda^2} \sum_i C_i^{(6)} O_i^{(6)} + \mathcal{O}\left(\frac{1}{\Lambda^3}\right).$$

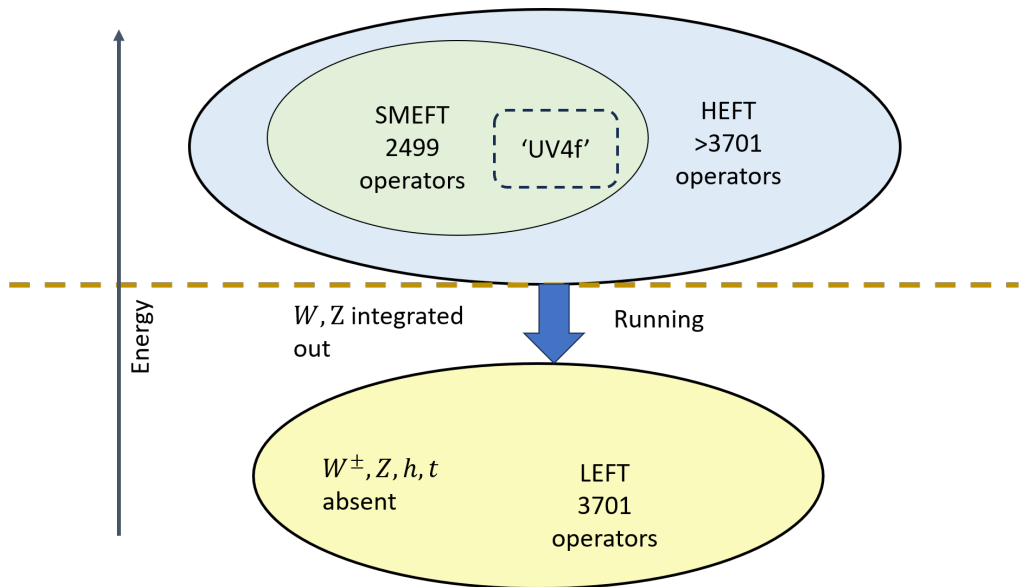
- Includes SM fields only.
- Follows $SU(3)_C \times SU(2)_L \times U(1)_Y$.
- Electroweak (EW) symmetry linearly realized.

Current uncertainties in Higgs coupling measurements allow more generalized EFTs e.g. **Higgs Effective Field Theory (HEFT)**. In HEFT:

- $SU(2)_L \times U(1)_Y$ non-linearly realized.
- Higgs boson is not embedded in a $SU(2)_L$ -doublet: \rightarrow More general coupling of Higgs.
- HEFT \supset SMEFT \supset SM

- In the energy scale much below the EW symmetry breaking, the relevant EFT is **Low Energy Effective Field Theory (LEFT)**
- LEFT can be derived from HEFT by integrating out the heavier particles – W^\pm , Z , Higgs and top quark.

HEFT, SMEFT and LEFT



- More number of operator in HEFT/LEFT than in SMEFT \implies relations among HEFT/LEFT WCs
- Relations among HEFT/LEFT WCs \implies indirect bounds
- Violation of these relations \implies physics beyond SMEFT

- SMEFT-predicted relations among LEFT/HEFT Wilson coefficients

- SMEFT-predicted constraints on LEFT Wilson coefficients

- Violations of SMEFT-predicted relation.
 - Effects beyond SMEFT in neutral-current semileptonic processes.
 - Effects beyond SMEFT in charged-current semileptonic processes.

SMEFT predictions for semileptonic processes: Operators and matching

An example derivation of relations among $U(1)_{em}$ invariant operators:

Vector operators $LLLL$ (HEFT)		
	NC	Count
$[\mathbf{c}_{eLdL}^V]^{\alpha\beta ij}$	$(\bar{e}_L^\alpha \gamma_\mu e_L^\beta)(\bar{d}_L^i \gamma^\mu d_L^j)$	81 (45)
$[\mathbf{c}_{euLL}^V]^{\alpha\beta ij}$	$(\bar{e}_L^\alpha \gamma_\mu e_L^\beta)(\bar{u}_L^i \gamma^\mu u_L^j)$	81 (45)
$[\mathbf{c}_{\nu dLL}^V]^{\alpha\beta ij}$	$(\bar{\nu}_L^\alpha \gamma_\mu \nu_L^\beta)(\bar{d}_L^i \gamma^\mu d_L^j)$	81 (45)
$[\mathbf{c}_{\nu uLL}^V]^{\alpha\beta ij}$	$(\bar{\nu}_L^\alpha \gamma_\mu \nu_L^\beta)(\bar{u}_L^i \gamma^\mu u_L^j)$	81 (45)
CC		
$[\mathbf{c}_{LL}^V]^{\alpha\beta ij}$	$(\bar{e}_L^\alpha \gamma_\mu \nu_L^\beta)(\bar{u}_L^i \gamma^\mu d_L^j)$	162 (81)

Vector operators $LLLL$ (SMEFT)		
	Operator	Count
$[\mathcal{C}_{lq}^{(1)}]^{\alpha\beta ij}$	$(\bar{l}^\alpha \gamma_\mu l^\beta)(\bar{q}^i \gamma^\mu q^j)$	81 (45)
$[\mathcal{C}_{lq}^{(3)}]^{\alpha\beta ij}$	$(\bar{l}^\alpha \gamma_\mu \tau^I l^\beta)(\bar{q}^i \gamma^\mu \tau^I q^j)$	81 (45)

$$\begin{aligned}
 & C_{lq}^{(1)\alpha\beta ij} O_{lq}^{(1)\alpha\beta ij} \\
 &= C_{lq}^{(1)\alpha\beta ij} (\bar{l}^\alpha \gamma_\mu l^\beta) (\bar{u}_L^i \gamma^\mu u_L^j + \bar{d}_L^i \gamma^\mu d_L^j)
 \end{aligned}$$

Matching among SMEFT and HEFT:

$$[\mathbf{c}_{\nu uLL}^V]^{\alpha\beta ij} = ([\mathcal{C}_{lq}^{(1)}]^{\alpha\beta ij} + [\mathcal{C}_{lq}^{(3)}]^{\alpha\beta ij}), \quad [\mathbf{c}_{euLL}^V]^{\alpha\beta ij} = ([\mathcal{C}_{lq}^{(1)}]^{\alpha\beta ij} - [\mathcal{C}_{lq}^{(3)}]^{\alpha\beta ij}),$$

$$[\mathbf{c}_{\nu dLL}^V]^{\alpha\beta ij} = ([\mathcal{C}_{lq}^{(1)}]^{\alpha\beta ij} - [\mathcal{C}_{lq}^{(3)}]^{\alpha\beta ij}), \quad [\mathbf{c}_{edLL}^V]^{\alpha\beta ij} = ([\mathcal{C}_{lq}^{(1)}]^{\alpha\beta ij} + [\mathcal{C}_{lq}^{(3)}]^{\alpha\beta ij}),$$

$$[\mathbf{c}_{LL}^V]^{\alpha\beta ij} = 2[\mathcal{C}_{lq}^{(3)}]^{\alpha\beta ij}.$$

[E.E. Jenkins, A.V. Manohar and P. Stoffer, JHEP03(2018)016]

[R. Bause, H. Gisbert, M. Golz and G. Hiller, Eur.Phys.J.C 82(2022)164]

$$\begin{aligned}
 u_L^i &\rightarrow S_{L\ ij}^u u_L^j, & u_R^i &\rightarrow S_{R\ ij}^u u_R^j, \\
 d_L^i &\rightarrow S_{L\ ij}^d d_L^j, & d_R^i &\rightarrow S_{R\ ij}^d d_R^j, \\
 V_{\text{CKM}} &= (S_L^u)^\dagger S_L^d.
 \end{aligned}$$

Resulting relations among HEFT/LEFT $LLLL$ Wilson Coefficients

Category	Analytic relations	Count
$LLLL$	$V_{ik}^\dagger [\hat{\mathbf{c}}_{euLL}^V]^{\alpha\beta kl} V_{lj} = U_{\alpha\rho}^\dagger [\hat{\mathbf{c}}_{\nu dLL}^V]^{\rho\sigma ij} U_{\sigma\beta}$	81 (45)
	$V_{ik} [\hat{\mathbf{c}}_{edLL}^V]^{\alpha\beta kl} V_{lj}^\dagger = U_{\alpha\rho}^\dagger [\hat{\mathbf{c}}_{\nu uLL}^V]^{\rho\sigma ij} U_{\sigma\beta}$	81 (45)
	$V_{ik}^\dagger [\hat{\mathbf{c}}_{LL}^V]^{\alpha\beta kj} = [\hat{\mathbf{c}}_{edLL}^V]^{\alpha\rho ij} U_{\rho\beta}^\dagger - U_{\alpha\sigma}^\dagger [\mathbf{c}_{\nu dLL}^V]^{\sigma\beta ij}$	162 (81)

[S. Karmakar, A. Dighe, R. S. Gupta, *arXiv:2404.10061*]

- These relations are independent of any assumptions for the flavor structure in NP.
- We derive 17 classes of such relations (2223 relations with explicit flavor indices).
- In the scenario when SMEFT only contains four-fermionic operators i.e. the ‘UV4f’ scenario, the above relations will be applicable for WCs in LEFT as well.

SMEFT predictions for semileptonic processes: Relations among LEFT WCs

$$\begin{aligned}
 u_L^i &\rightarrow S_{Lij}^u u_L^j, & u_R^i &\rightarrow S_{Rij}^u u_R^j, \\
 d_L^i &\rightarrow S_{Lij}^d d_L^j, & d_R^i &\rightarrow S_{Rij}^d d_R^j, \\
 V_{\text{CKM}} &= (S_L^u)^\dagger S_L^d.
 \end{aligned}$$

Resulting relations among HEFT/LEFT $LLLL$ Wilson Coefficients

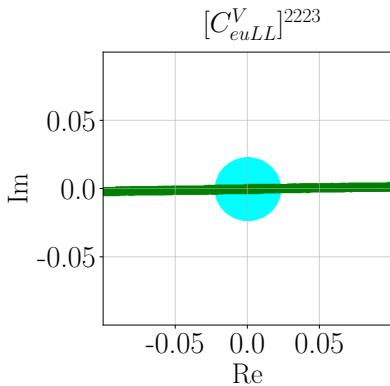
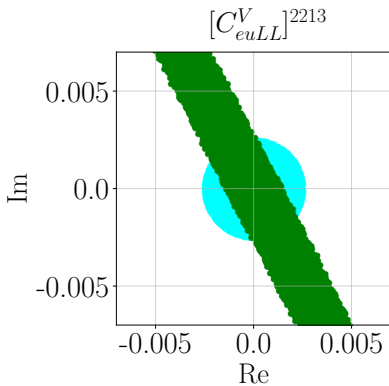
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	$V_{ik}^\dagger [\hat{\mathbf{c}}_{LL}^V]^{\alpha\beta kj} = [\hat{\mathbf{c}}_{edLL}^V]^{\alpha\rho ij} U_{\rho\beta}^\dagger - U_{\alpha\sigma}^\dagger [\mathbf{c}_{\nu dLL}^V]^{\sigma\beta ij}$	162 (81)

[S. Karmakar, A. Dighe, R. S. Gupta, arXiv:2404.10061]

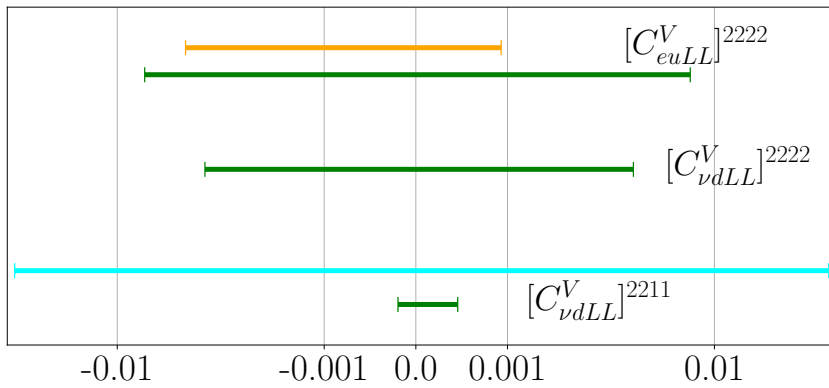
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SMEFT predictions: Indirect bounds on $(\bar{\mu}\gamma^\sigma\mu)(\bar{u}\gamma_\sigma u)$, $(\bar{\nu}\gamma^\sigma\nu)(\bar{d}\gamma_\sigma d)$

Indirect bounds on C_{euLL}^V and C_{vdLL}^V



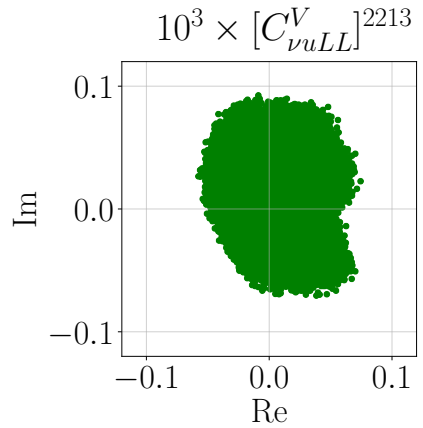
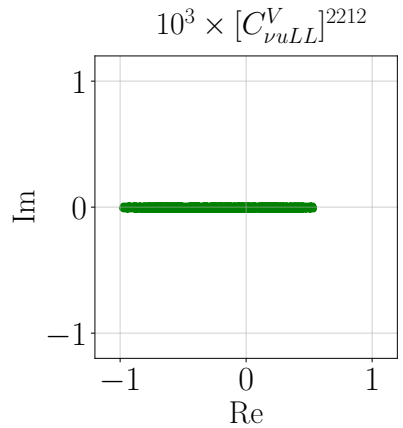
$t \rightarrow u\mu\mu$
 $t \rightarrow c\mu\mu$



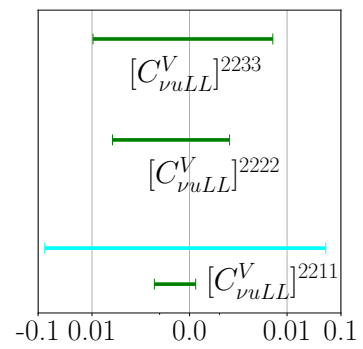
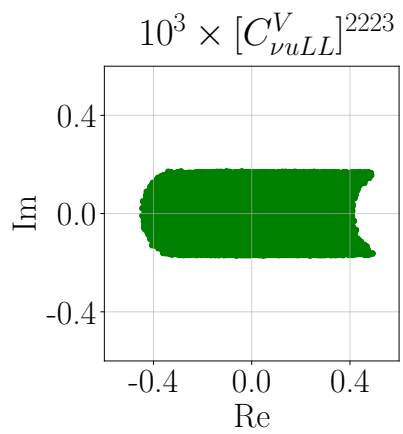
$cc \rightarrow \mu\mu$
 $ss \rightarrow \nu\nu$
 $dd \rightarrow \nu\nu$

SMEFT predictions: Indirect bounds on $(\bar{\nu}\gamma^\sigma\nu)(\bar{u}\gamma_\sigma u)$

Indirect bounds on $C_{\nu u LL}^V$



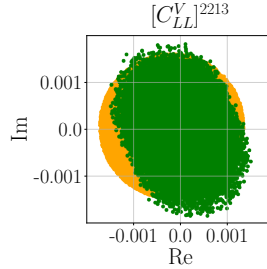
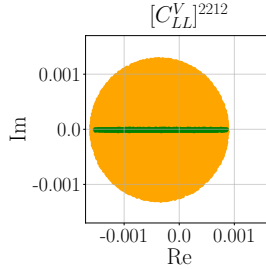
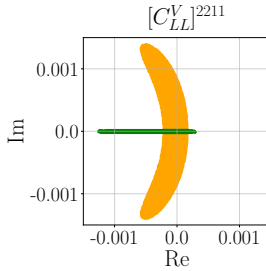
$D \rightarrow \pi\nu\nu$
 $t \rightarrow u\nu\nu$



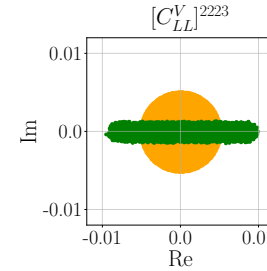
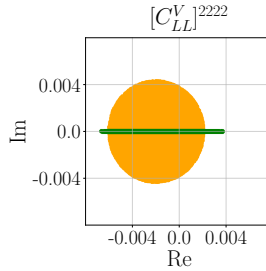
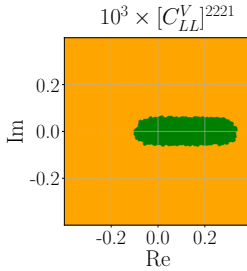
$t \rightarrow c\nu\nu$
 $tt \rightarrow \nu\nu$
 $cc \rightarrow \nu\nu$
 $uu \rightarrow \nu\nu$

SMEFT predictions: Indirect bounds on $(\bar{\mu}\gamma^\sigma\nu)(\bar{u}\gamma_\sigma d)$

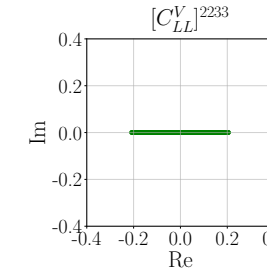
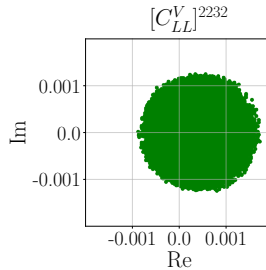
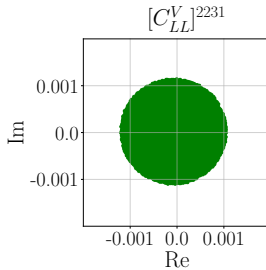
Indirect bounds on C_{LL}^V



$\pi \rightarrow \mu\nu$
 $K \rightarrow \pi\mu\nu$
 $B \rightarrow \pi\mu\nu$



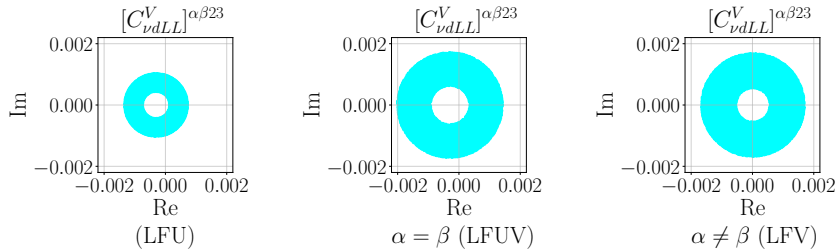
$D \rightarrow \pi\mu\nu$
 $D \rightarrow K\mu\nu$
 $B \rightarrow D\mu\nu$



$t \rightarrow d\mu\nu$
 $t \rightarrow s\mu\nu$
 $t \rightarrow b\mu\nu$

[S. Karmakar, A. Dighe, R. S. Gupta, arXiv:2404.10061]

Observed excess in $B \rightarrow K\nu\nu$:



$$[C_{euLL}^V]^{\alpha\beta ij} = V_{i2}[C_{\nu dLL}^V]^{\alpha\beta 23}V_{3j}^\dagger + \dots$$

$$\begin{aligned} \text{For } i = 2, j = 3, \quad & [C_{euLL}^V]^{\alpha\beta 23} \sim 0.97[C_{\nu dLL}^V]^{\alpha\beta 23}. \\ \text{For } i = 1, j = 3, \quad & [C_{euLL}^V]^{\alpha\beta 13} \sim 0.22[C_{\nu dLL}^V]^{\alpha\beta 23} \end{aligned}$$

\Rightarrow Possible excess in $t \rightarrow ce^\alpha e^\beta$, $t \rightarrow ue^\alpha e^\beta$

$$[C_{LL}^V]^{\alpha\beta i3} = V_{i2}([C_{edLL}^V]^{\alpha\beta 23} - [C_{\nu dLL}^V]^{\alpha\beta 23})V_{3j}^\dagger.$$

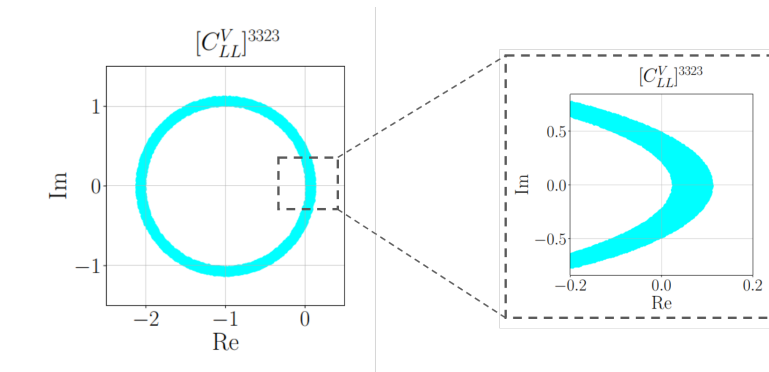
\Rightarrow Possible excess in $b \rightarrow cl\nu$, $b \rightarrow ul\nu$

[R. Bause, H. Gisbert, and G. Hiller, *PhysRevD.109.015006*]

[S. Bhattacharya, S. Jahedi, S. Nandi and A. Sarkar, *arXiv:2312.14872*]

[S. Karmakar, A. Dighe, R. S. Gupta, *arXiv:2404.10061*]

$R(D^{(*)})$ anomalies:



$$[C_{LL}^V]^{3323} = V_{cd} \left[[C_{edLL}^V]^{3313} - [C_{\nu dLL}^V]^{3313} \right] + V_{cs} \left[[C_{edLL}^V]^{3323} - [C_{\nu dLL}^V]^{3323} \right] + V_{cs} \left[[C_{edLL}^V]^{3333} - [C_{\nu dLL}^V]^{3333} \right]$$

- Possible NP in $b \rightarrow d\tau\tau$, $b \rightarrow s\tau\tau$, $b \rightarrow d\nu\nu$ and $b \rightarrow s\nu\nu$
- These possible NP effects can manifest in $B \rightarrow \tau\tau$, $B_s \rightarrow \tau\tau$, $B \rightarrow K^{(*)}\tau\tau$, $B \rightarrow K^{(*)}\nu\nu$ etc.

[R. Alonso, B. Grinstein and J. Martin Camalich, *JHEP10(2015)184*]

[A. Crivellin, D. Müller and T. Ota, *JHEP09(2017)040*]

[S. Karmakar, A. Dighe, R. S. Gupta, *arXiv:2404.10061*]

- Systematic exploration of SMEFT predictions for all semileptonic operators taking the full expansion of the CKM matrix.
- These prediction are independent of any assumptions about the alignment of the mass and flavor bases for the quarks.
- Implications of the violation of SMEFT predictions:
 - Physics beyond UV4f
 - Large contribution from dimension-8 SMEFT operators
 - Physics beyond SMEFT
- Next: Exploring possible physics beyond SMEFT with two examples:
 - 1 $b \rightarrow s\tau\tau$
 - 2 $b \rightarrow c\tau\nu$

Identifying effects beyond SMEFT in $b \rightarrow s\tau\tau$

EFT for processes involving $b \rightarrow s\tau\tau$ channel

$$\mathcal{H}^{\text{eff}} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \frac{\alpha_e}{4\pi} \left(\sum_i C_i O_i + \sum_j C'_j O'_j \right),$$

where the scalar and pseudoscalar operators are

$$O_S^{(\prime)} = [\bar{s} P_R(L) b] [\ell\ell], \quad O_P^{(\prime)} = [\bar{s} P_R(L) b] [\ell\gamma_5\ell].$$

SMEFT predictions : $C_S = -C_P$, and $C'_S = C'_P$.

[O. Catà and M. Jung, *PhysRevD.92.055018*]

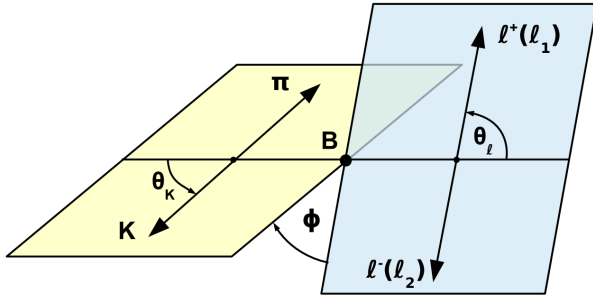
Non-SMEFT effect can be parameterized as

$$C_S + C_P \equiv \Delta C, \quad C'_S - C'_P \equiv \Delta C'.$$

We consider the following scenarios

- ① SM,
- ② VA: where NP is present only in vector operators,
- ③ SP: where NP is present only in scalar operators with, $\Delta C^{(\prime)} = 0$
- ④ $\widetilde{\text{SP}}$: where NP is present only in scalar operators with $\Delta C^{(\prime)} \neq 0$.

$$\frac{d^4\Gamma}{dq^2 d\cos\theta_l d\cos\theta_V d\phi} = \frac{9}{32\pi} I(q^2, \theta_l, \theta_V, \phi),$$



[J. Gratrex, M. Hopfer and R. Zwicky, Phys.Rev.D 93(2016)054008]

$$\begin{aligned} I(q^2, \theta_l, \theta_V, \phi) &= I_1^s \sin^2 \theta_V + I_1^c \cos^2 \theta_V \\ &+ (I_2^s \sin^2 \theta_V + I_2^c \cos^2 \theta_V) \cos 2\theta_l \\ &+ I_3 \sin^2 \theta_V \sin^2 \theta_l \cos 2\phi \\ &+ I_4 \sin 2\theta_V \sin 2\theta_l \cos \phi \\ &+ I_5 \sin 2\theta_V \sin \theta_l \cos \phi \\ &+ (I_6^s \sin^2 \theta_V + I_6^c \cos^2 \theta_V) \cos \theta_l \\ &+ I_7 \sin 2\theta_V \sin \theta_l \sin \phi + I_8 \sin 2\theta_V \sin 2\theta_l \sin \phi \\ &+ I_9 \sin^2 \theta_V \sin^2 \theta_l \sin 2\phi, \end{aligned}$$

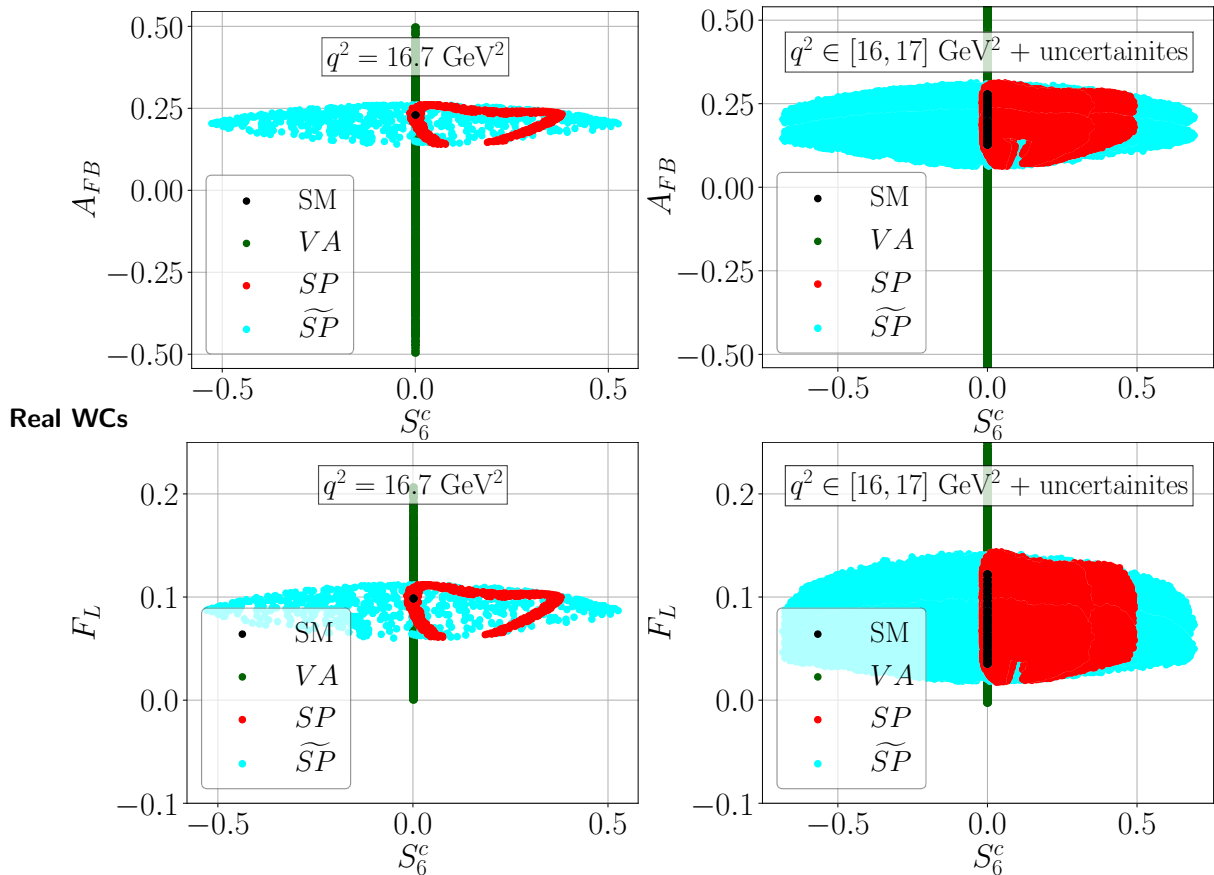
$$C_S + C_P \equiv \Delta C, \quad C'_S - C'_P \equiv \Delta C'$$

$$S_i^{(a)} = \frac{(I_i^{(a)} + \bar{I}_i^{(a)})}{d(\Gamma + \bar{\Gamma})/dq^2},$$

$$A_{FB} = \frac{3}{8}(2S_6^s + S_6^c), \quad F_L = S_1^c.$$

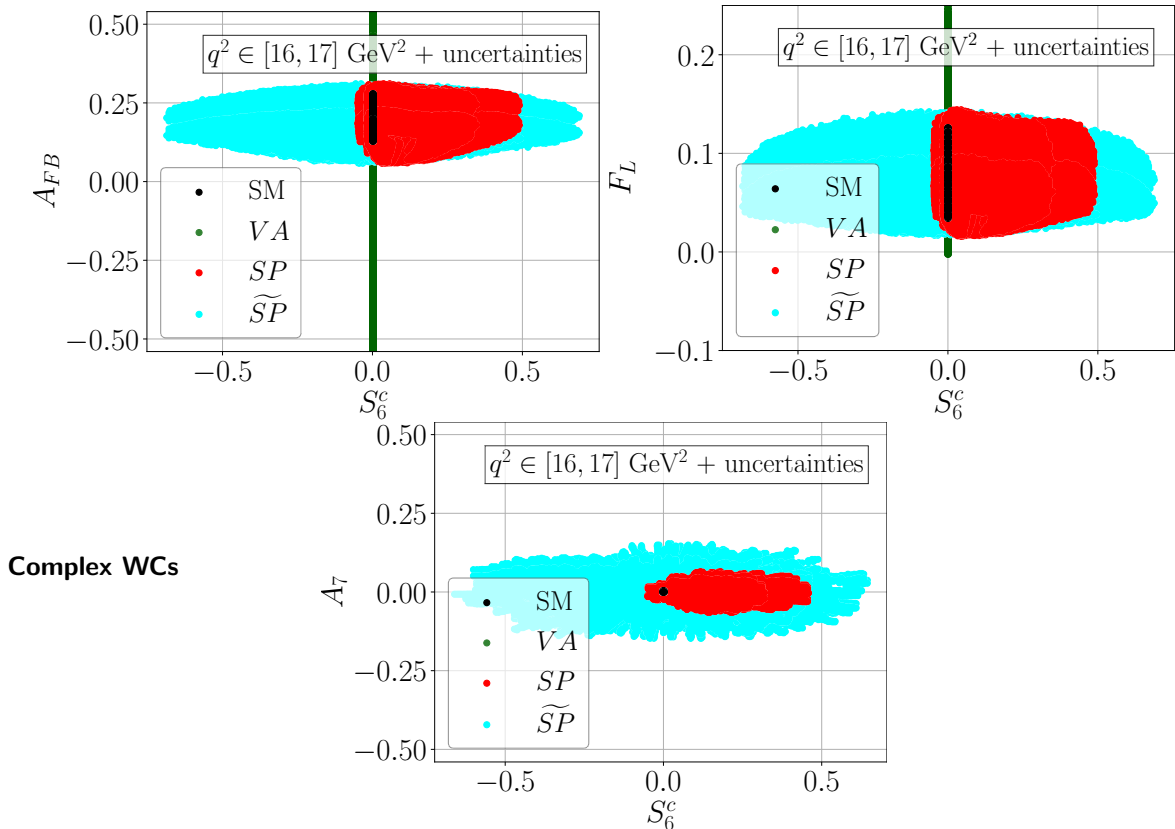
NP WCs	Sensitive observables
$C_9^{(l)}, C_{10}^{(l)}$	$S_1^{s,c}, S_2^{s,c}, S_3, S_4, S_5, S_6^s, A_7$ $A_{FB}, \mathcal{B}(B \rightarrow K^* \tau^+ \tau^-)$
C_{S-}	$S_1^c + S_2^c, S_6^c, A_{FB}$ $\mathcal{B}(B_s \rightarrow \tau^+ \tau^-)$
C_{P-}	F_L $\mathcal{B}(B_s \rightarrow \tau^+ \tau^-)$
C_{S+}, C_{P+}	$\mathcal{B}(B \rightarrow K \tau^+ \tau^-)$

Beyond-SMEFT effects in $B \rightarrow K^{*0} \tau^+ \tau^-$ angular observables



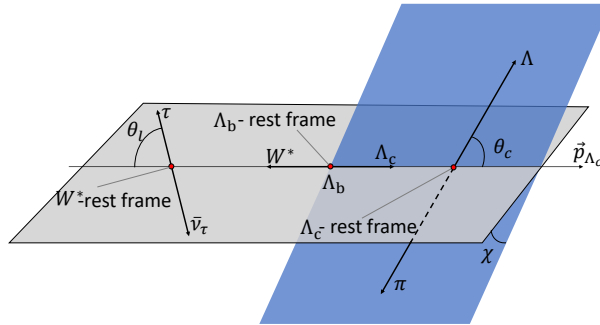
[S. Karmakar, A. Dighe, arXiv:2408.13069]

Beyond-SMEFT effects in $B \rightarrow K^{*0} \tau^+ \tau^-$ angular observables



[S. Karmakar, A. Dighe, arXiv:2408.13069]

Identifying effects beyond SMEFT in $b \rightarrow c\tau\nu_\tau$ channel



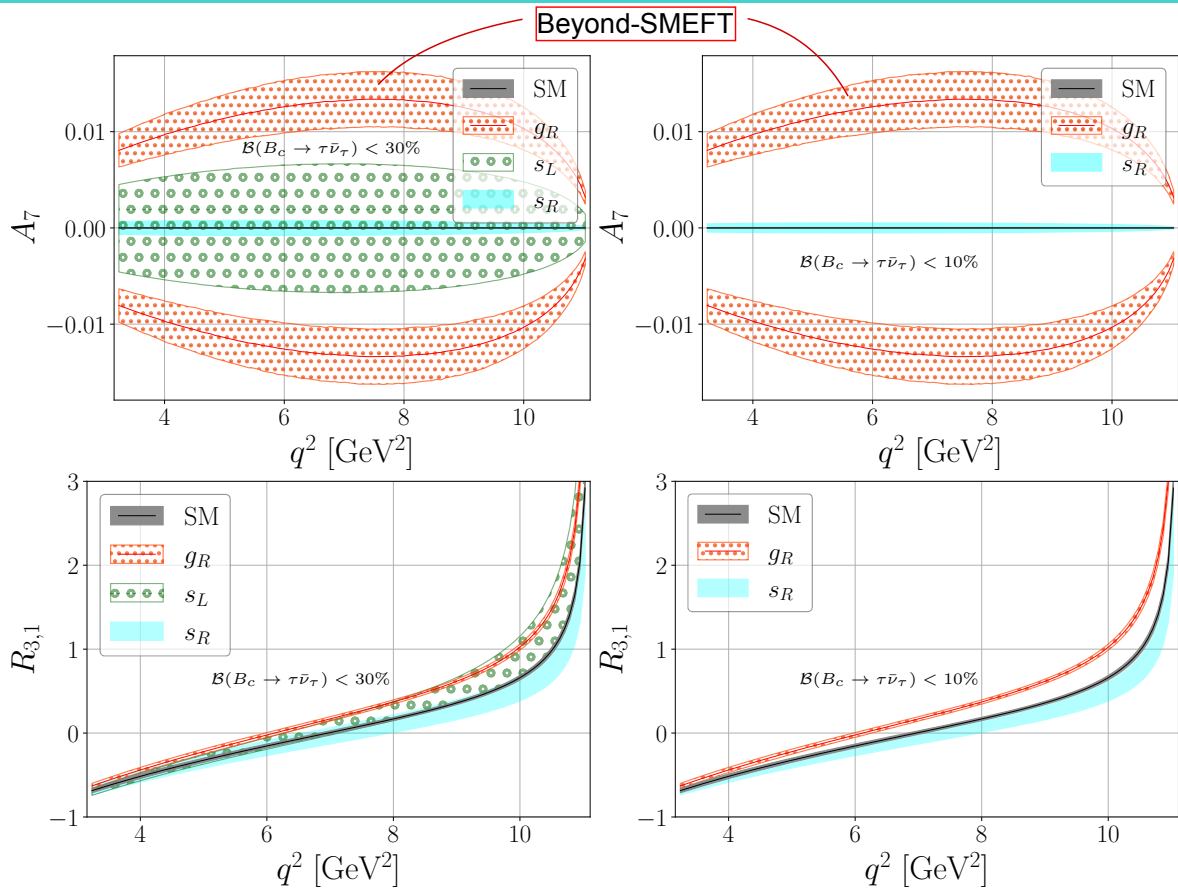
$$\begin{aligned}
 & \frac{1}{(d\Gamma/dq^2)} \frac{d\Gamma}{dq^2 d\cos\theta_c d\cos\theta_l d\chi} \\
 & = A_0 + A_1 \cos\theta_c + A_2 \cos\theta_l \\
 & \quad + A_3 \cos\theta_c \cos\theta_l + A_4 \cos^2\theta_l \\
 & \quad + A_5 \cos\theta_c \cos^2\theta_l \\
 & \quad + A_6 \sin\theta_c \sin\theta_l \cos\chi \\
 & \quad + A_7 \sin\theta_c \sin\theta_l \sin\chi \\
 & \quad + A_8 \sin\theta_c \sin\theta_l \cos\theta_l \cos\chi \\
 & \quad + A_9 \sin\theta_c \sin\theta_l \cos\theta_l \sin\chi .
 \end{aligned}$$

$$O_V^{LR} \equiv (\bar{\tau}\gamma^\mu P_L \nu_\tau)(\bar{c}\gamma_\mu P_R b)$$

- Large contribution coming from O_V^{LR} would imply effects beyond SMEFT.
- Our goal is to find angular observables in $\Lambda_b \rightarrow \Lambda_c(\rightarrow \Lambda\pi)\tau\nu_\tau$ that can distinguish effects of large O_V^{LR} .

[C.P. Burgess, S. Hamoudou, J. Kumar and D. London, PhysRevD.105.073008]

Beyond-SMEFT effects in angular observables in $\Lambda_b \rightarrow \Lambda_c(\rightarrow \Lambda\pi)\tau\bar{\nu}_\tau$



[S. Karmakar, S. Chattopadhyay, A. Dighe, *PhysRevD.110.015010*]

- We find 17 classes (2223 with generation indices) of relations among LEFT WCs based on the $SU(2)_L \times U(1)_Y$ invariance of SMEFT.
- Based on these relations, we find indirect bounds on WCs which are in some cases weakly constrained in direct experiments.
- The relations and the indirect bounds do not depend on any NP flavour assumption.

- Violation of these relations implies existence of physics beyond SMEFT.
- Effects beyond SMEFT can be probed indirectly in low energy flavour physics observables.
- We find the effectiveness of different angular observables in $B \rightarrow K^* \tau^+ \tau^-$ and $\Lambda_b \rightarrow \Lambda_c (\rightarrow \Lambda \pi) \tau \nu_\tau$ decay, which can distinguish non-SMEFT effects from other NP scenarios present within SMEFT.

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Thank you for your attention!

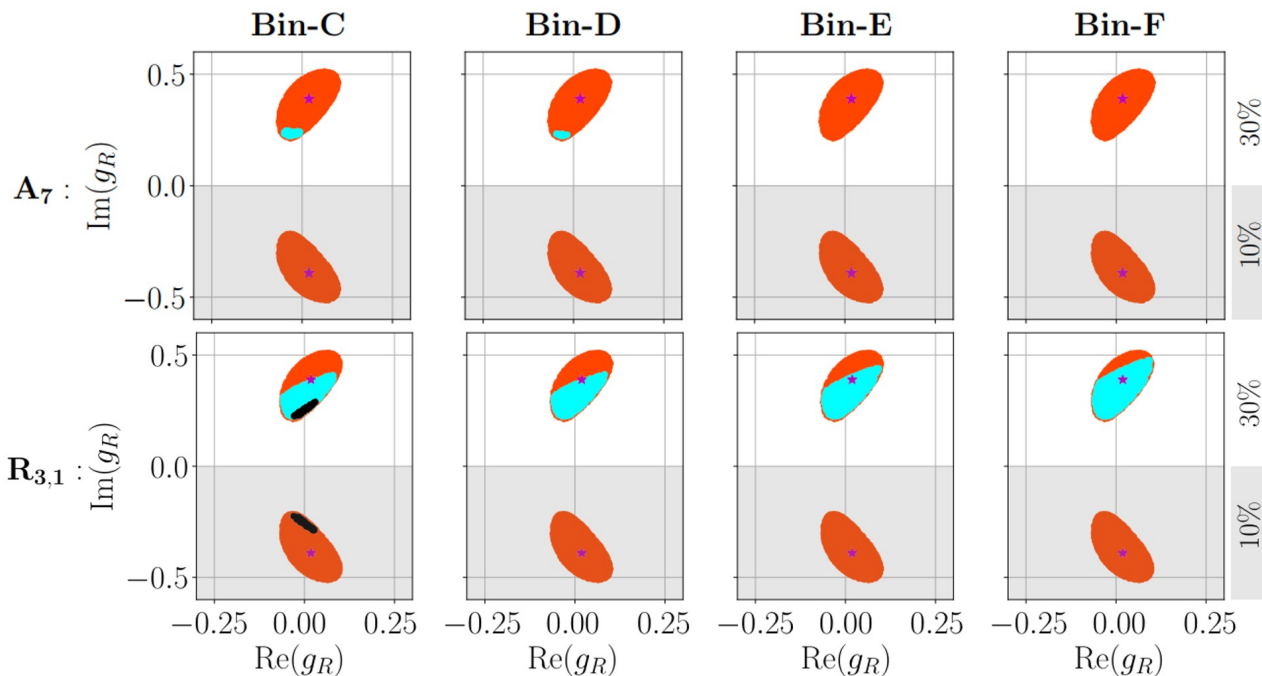
Backup: Relations among the HEFT/LEFT WCs in UV4f scenario

Category	Analytic relations	Count
<i>LLLL</i>	$V_{ik}^\dagger [\hat{\mathbf{c}}_{euLL}^V]^{\alpha\beta kl} V_{lj} = U_{\alpha\rho}^\dagger [\hat{\mathbf{c}}_{\nu dLL}^V]^{\rho\sigma ij} U_{\sigma\beta}$	81 (45)
	$V_{ik} [\hat{\mathbf{c}}_{edLL}^V]^{\alpha\beta kl} V_{lj}^\dagger = U_{\alpha\rho}^\dagger [\hat{\mathbf{c}}_{\nu uLL}^V]^{\rho\sigma ij} U_{\sigma\beta}$	81 (45)
	$V_{ik}^\dagger [\hat{\mathbf{c}}_{LL}^V]^{\alpha\beta kj} = [\hat{\mathbf{c}}_{edLL}^V]^{\alpha\rho ij} U_{\rho\beta}^\dagger - U_{\alpha\sigma}^\dagger [\hat{\mathbf{c}}_{\nu dLL}^V]^{\sigma\beta ij}$	162 (81)
<i>RRRR</i>	No relations	
<i>LLRR</i>	$[\hat{\mathbf{c}}_{edLR}^V]^{\alpha\beta ij} = U_{\alpha\rho}^\dagger [\hat{\mathbf{c}}_{\nu dLR}^V]^{\rho\sigma ij} U_{\rho\beta}$	81 (45)
	$[\hat{\mathbf{c}}_{euLR}^V]^{\alpha\beta ij} = U_{\alpha\rho}^\dagger [\hat{\mathbf{c}}_{\nu uLR}^V]^{\rho\sigma ij} U_{\rho\beta}$	81 (45)
	$[\hat{\mathbf{c}}_{LR}^V]^{\alpha\beta ij} = 0$	162 (81)
<i>RRLL</i>	$[\hat{\mathbf{c}}_{edRL}^V]^{\alpha\beta ij} = V_{ik}^\dagger [\hat{\mathbf{c}}_{euRL}^V]^{\rho\sigma kl} V_{lj}$	81 (45)

Backup: Relations among the HEFT/LEFT WCs in UV4f scenario

Category	Analytic relations	Count
Scalar (d_R)	$V_{ik} [\hat{\mathbf{c}}_{ed,RLLR}^S]^{\alpha\beta kj} = [\hat{\mathbf{c}}_{RLLR}^S]^{\alpha\rho ij} U_{\rho\beta}$	162 (81)
	$[\hat{\mathbf{c}}_{ed,RLLR}^S]^{\alpha\beta ij} = 0$	162 (81)
Scalar (u_R)	$[\hat{\mathbf{c}}_{eu,RLRL}^S]^{\alpha\beta ik} V_{kj} = -[\hat{\mathbf{c}}_{RLRL}^S]^{\alpha\rho ij} U_{\rho\beta}$	162 (81)
	$[\hat{\mathbf{c}}_{eu,RLLR}^S]^{\alpha\beta ij} = 0$	162 (81)
Tensor (d_R)	$[\hat{\mathbf{c}}_{ed,all}^T]^{\alpha\beta ij} = 0$	324 (162)
	$[\hat{\mathbf{c}}_{RLLR}^T]^{\alpha\beta ij} = 0$	162 (81)
Tensor (u_R)	$[\hat{\mathbf{c}}_{eu,RLRL}^T]^{\alpha\beta ik} V_{kj} = -[\hat{\mathbf{c}}_{RLRL}^T]^{\alpha\rho ij} U_{\rho\beta}$	162 (81)
	$[\hat{\mathbf{c}}_{eu,RLLR}^T]^{\alpha\beta ij} = 0$	162 (81)
Z and W^\pm	$[\hat{\mathbf{c}}_{ud_L W}]^{ij} = \frac{1}{\sqrt{2}} \cos \theta_w ([\hat{\mathbf{c}}_{u_L Z}]^{ik} V_{kj} - V_{ik} [\hat{\mathbf{c}}_{d_L Z}]^{kj})$	18 (9)
	$[\hat{\mathbf{c}}_{e\nu_L W}]^{\alpha\rho} U_{\rho\beta} = \frac{1}{\sqrt{2}} \cos \theta_w ([\hat{\mathbf{c}}_{e_L Z}]^{\alpha\beta} - U_{\alpha\rho}^\dagger [\hat{\mathbf{c}}_{\nu_L Z}]^{\rho\sigma} U_{\sigma\beta})$	18 (9)

Backup: Beyond-SMEFT effects in angular observables in $\Lambda_b \rightarrow \Lambda_c(\rightarrow \Lambda\pi)\tau\bar{\nu}_\tau$



$B \rightarrow K^*$ form factors from [A. Bharucha, D.M. Straub and R. Zwicky - 2015]

$\Lambda_b \rightarrow \Lambda_c$ form factors from [W. Detmold, C. Lehner and S. Meiri - 2015]