Updating the Factorization approach to partial decay rates in inclusive $\bar{B} \to X_u \ell \bar{\nu}$

Björn O. Lange (Siegen University)

in collaboratin with
Thomas Mannel, Kevin Olschewski, K. Keri Vos

"BLNP" based on
"Factorization and Shape-Function Effects in Inclusive B-Meson Decays"
S.W.Bosch, B.O.L., M.Neubert, G.Paz, 2004

"Theory of Charmless Inclusive B Decays and the Extraction of V_{ub}"
B.O.L., M.Neubert, G.Paz, 2005

UNIVERSITÄT SIEGEN

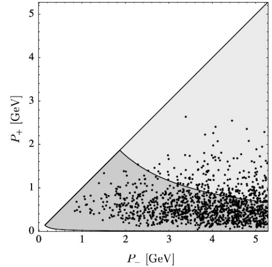
Wien, 26. September 2024

Triple differential decay rate

$$P_+ = E_X - |\vec{P}_X|$$

$$P_- = E_X + |\vec{P}_X|$$

$$P_\ell = M_B - 2E_\ell$$



$$\frac{d^3\Gamma}{dP_+ dP_- dP_\ell} = \frac{G_F^2 |V_{ub}^2|}{16\pi^3} \sum_{i=1}^{3} \varphi_i(P_+, P_-, P_\ell) \mathcal{F}_i(P_+, P_-) \,,$$

where all capitalised variables are hadronic, and

$$\varphi_1 = (M_B - P_+)(P_- - P_\ell)(M_B - P_- + P_\ell - P_+) \,, \quad \text{for example}$$

phase-space neglecting pion mass

$$0 \leq P_+ \leq P_\ell \leq P_- \leq M_B$$

What is BLNP?

- predicts **partial rates** for cuts, not differential spectra everywhere.
- optimizes the calculation for the peak region,
  where $P_- \sim m_b$ and $P_+ \sim \Lambda_{\mathrm{QCD}}$.
- organizes the calculation in this power counting, in the HQ Limit

$$\mathcal{F}_i = \mathcal{F}_i^{(0)} + \frac{1}{(M_B - P_+)} \mathcal{F}_i^{(1)} + \frac{1}{(M_B - P_+)^2} \mathcal{F}_i^{(2)} + \dots$$

  [convenient because of φ_i, but both could be re-expanded using
  $M_B - P_+ = m_b - p_+$. This would then introduce partonic variables.]

- factorizes $\mathcal{F}_i^{(j)}$ via multi-step matching
  QCD (weak eff. Ham.) $\to$ SCET$_I$ $\to$ HQET, e.g.

$$\mathcal{F}_i^{(0)} = H_i(\mu_F) J_i(\mu_F) \otimes \hat{S}(\mu_F)$$

- evolves the ingredient functions to their "natural scales" μ_h, μ_i, μ_0.
  Resums "large logs" of their ratios.
  [maybe not necessary, but certainly not wrong.]

What is BLNP?

After specifying a cut and phase-space integration,
partial rate is a weighted integral over shape functions, e.g.

$$\Gamma_{\mathrm{cut}}^{(0)} = \frac{G_F^2 |V_{ub}|^2}{16\pi^3} \int_0^{\Delta_{\mathrm{cut}}} d\hat{\omega}\ \hat{S}(\hat{\omega})\, T_{\mathrm{cut}}(\hat{\omega}, \Delta_{\mathrm{cut}})\ ,$$

and requires the non-perturbative shape functions as an input.

We'll get back to that...

What is BLNP?

The old work had the following level of accuracy:

[Bosch, BOL, Neubert, Paz, 2004, 2005]

- count $\alpha_s/\pi \sim \Lambda_{\mathrm{QCD}}/m_b$
- Leading Power at NLO in RG-improved perturbation theory:
  - 1-loop matching hard- and jet-function
  - 2-loop anomalous dimension
  - 3-loop cusp anomalous dimension
- Subleading Power: LO
  - tree-level: 4 subleading shape functions
  - plus certain 4-quark operator contributions
  - leading 1-loop RG-running factor
  - unfactorized 1-loop "kinematic corrections"
- Subsubleading Power: "residual hadronic corrections"
  - only norms of subsubleading shape functions considered

What is BLNP?

Moments of the shape functions with a large cutoff Λ_{UV} have been expanded in a local OPE up to dimension-5 operators.

Input HQ parameters

$$\bar{\Lambda} \quad , \quad \mu_\pi^2 \quad , \quad \mu_G^2 \quad ,$$

but partial rates in the shape-function region are also quite sensitive to higher moments. Thus the need for models.

Updating efforts

There is a Smörgasbord of progress, this is only a partial list.

Perturbative side

- 2-loop ingredients at Leading-Power
  - hard functions [Bonciani, Ferroglia, 2008; Asatrian, Greub, Pecjak, 2008; Beneke, Huber, Li, 2009; Bell, 2009]
  - jet functions [T.Becher, M.Neubert, PLB637, 2006]
  - partonic shape function [T.Becher, M.Neubert, PLB633, 2006]
- 3-loop anomalous dimensions [C.Greub, M.Neubert, B.D.Pecjak, EPJC65, 2010, and references therein]
- 4-loop cusp anomalous dimension [J.M.Henn, G.P.Korchemsky, B.Mistlberger, JHEP04, 2020]
- Even the 3-loop hard function has recently been calculated. [M.Fael, T.Huber, F.Lange, J.Müeller, K.Schönwald, M.Steinhauser, PRD110, 2024]
- 1-loop at subleading power [T.Ewerth, P.Gambino, S.Nandi, NPB830, 2010]
  Beyond tree-level there are many more independent subleading shape functions. [R.J.Hill, T.Becher, S.J.Lee, M.Neubert, JHEP 0407, 2004]

Updating efforts

Heavy-Quark Expansion

- [T.Mannel, S.Turczyk, N.Uraltsev, JHEP11, 2010]
- [P.Gambino, K.J.Healy, S.Turczyk, PLB763, 2016]
- [A.Gunawardana, G.Paz, 1702.08904]
- [T.Mannel, K.K.Vos, JHEP06, 2018]

and many more, including present company not listed (apologies!)

A road block, however, is the hadron mass formula

$$M_H = m_Q + \bar{\Lambda} + \frac{\mu_\pi^2 - \frac{d_H}{3}\mu_G^2}{2m_Q} + \dots \quad ?$$

Updating plan

We decided on the following framework:

- use the kinetic scheme: high accuracy
  [M.Fael, K. Schönwald, M.Steinhauser, 2020/21]
- count $\alpha_s^2 \sim \Lambda_{\mathrm{QCD}}/m_b$.
- implement NNLO in RG-improved perturbation theory at leading power.
- improve the leading-power shape function by including ρ_D^3, ρ_{LS}^3 in the modelling.
- update the power corrections.

Expectations

The perturbative error is reduced. [C.Greub, M.Neubert, B.D.Pecjak, EPJC65, 2010]
Quote from their publication on the **leading-power** predictions of partial rates:

- $P_+ < 0.66$ GeV:

	$\Gamma_u^{(0)}$	μ_h	μ_i
NLO	60.37	${}^{+3.52}_{-3.37}$	${}^{+3.81}_{-6.67}$
NNLO	52.92	${}^{+1.46}_{-1.72}$	${}^{+0.09}_{-2.79}$

- $M_X < 1.7$ GeV:

	$\Gamma_u^{(0)}$	μ_h	μ_i
NLO	61.86	${}^{+3.23}_{-3.21}$	${}^{+1.89}_{-5.50}$
NNLO	52.26	${}^{+1.05}_{-1.40}$	${}^{+0.65}_{-3.90}$

- $E_l > 2.0$ GeV:

	$\Gamma_u^{(0)}$	μ_h	μ_i
NLO	25.98	${}^{+1.63}_{-1.61}$	${}^{+1.69}_{-2.81}$
NNLO	22.61	${}^{+0.63}_{-0.78}$	${}^{+0.01}_{-0.94}$

- $P_+ < 0.66$ GeV:

Fixed-Order	$\Gamma_u^{(0)}$	μ
NLO	49.11	${}^{+5.43}_{-9.41}$
NNLO	49.53	${}^{+0.13}_{-4.01}$

- $M_X < 1.7$ GeV:

Fixed-Order	$\Gamma_u^{(0)}$	μ
NLO	51.81	${}^{+3.69}_{-8.62}$
NNLO	50.47	${}^{+0.01}_{-2.62}$

- $E_l > 2.0$ GeV:

Fixed-Order	$\Gamma_u^{(0)}$	μ
NLO	21.01	${}^{+2.04}_{-3.54}$
NNLO	20.99	${}^{+0.04}_{-1.43}$

"[Resummation] is not strictly necessary"

A few details on modelling
the leading shape function

Modelling the leading shape function

What we know about the shape function

- The anomalous dimension of the light-cone operator is determined by the renormalization of the "partonic shape function", $S_{\rm part}(\omega, \mu_0)$, which is distribution-valued.
- Cutoff-moments of the shape function can be expanded in a local OPE using the partonic shape function.

$$\langle \hat{S} \rangle_n^{(\Lambda_{UV})} = \int_0^{\Lambda_{UV}} \hat{\omega}^n \, \hat{S}(\hat{\omega}, \mu_0) \, d\hat{\omega} = \text{OPE in } 1/\Lambda_{UV} \, .$$

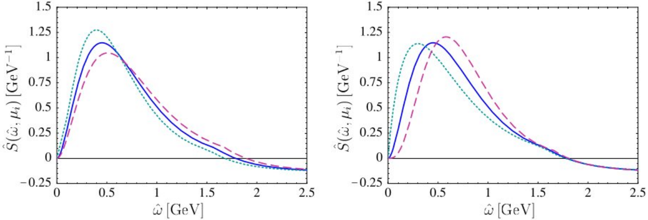
- It follows that the "tail" is given by

$$\hat{S}(\hat{\omega} \gg \Lambda_{\rm QCD}, \mu_0) = \frac{d}{d\Lambda_{UV}} \langle \hat{S} \rangle_0^{(\Lambda_{uv})} \Big|_{\Lambda_{uv} \to \hat{\omega}}$$

- The bulk, $\hat{\omega} \sim \Lambda_{\rm QCD}$, is non-perturbative.
- All HQE parameters in the pole scheme shall be eliminated in favour of those in a renormalon-free scheme.

Ansätze for modelling

BLNP Ansatz: use the shape-function scheme and glue on the tail where it goes through zero (if needed at all).



Small cusp where tail starts.
Negligible, exponentially suppressed contribution in tail region.

Modelling the leading shape function

SIMBA Ansatz: [Z.Ligeti, I.W.Stewart, F.J.Tackmann, PRD78, 2008]

$$\hat{S}(\hat{\omega}, \mu_0) = \int_0^{\hat{\omega}} S_{\rm part}(\hat{\omega} - \hat{k}, \mu_0)\, \hat{F}_{\rm pol}(\hat{k})\, d\hat{k}$$

- correct μ_0 dependence automatic.
- correct moments and tail automatic.
- moments of $\hat{F}_{\rm pol}$ exist without cutoff. Thus exponential tail?

Cutoff-dependent moments of this convolution Ansatz:

$$\langle S_{\rm part} \otimes \hat{F}_{\rm pol} \rangle_n^{(\Lambda_{UV})} = \Lambda_{UV}^n \sum_{i=0}^{\infty} K_i^{(n)} \left(\ln \frac{\Lambda_{UV}}{\mu_0} \right) \frac{\langle \hat{F}_{\rm pol} \rangle_i^{(\Lambda_{UV})}}{\Lambda_{UV}^i}$$

matched onto local operators:

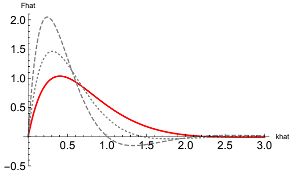
- for $n = 0, 1, 2$ it suffices to use 2-point functions. The sum of all diagrams vanish for the local operators. It follows that $\langle \hat{F}_{\rm pol} \rangle_n^{(\Lambda_{UV})}$ reduce to the tree-level expressions $1,\ \bar{\Lambda},\ \bar{\Lambda}^2 + \mu_\pi^2/3$.
- for $n = 3$ the tree-level value is $\bar{\Lambda}^3 + \bar{\Lambda}\mu_\pi^2 + \rho_D^3/3$, but radiative corrections require also matching with 3- and 4-point functions. This has not been done.

Modelling the shape function: the "renormalon battle"

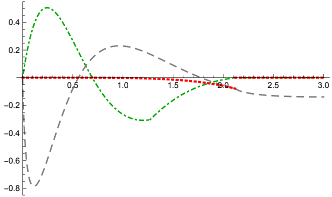
SIMBA Ansatz is attractive, but there is trouble:

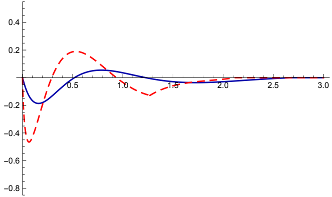
1. This is **not** a factorization of long- and short-distance physics!
2. $S_{\rm part} \otimes \hat{F}_{\rm pol}$ produces $\alpha_s(\mu_0)\ln(\hat{\omega}/\mu_0)$ logs, which become large as $\hat{\omega} \to 0$. [if you see an α_s in the bulk, **you** put it there.]
3. Need for scheme change inside the bulk.

$$\hat{F}_{\rm pol} = \hat{F}_{\rm kin} + \sum_j c_j(\alpha_s)\hat{g}_j = \hat{F}_{\rm kin} + \hat{G}(\alpha_s)$$



4. they need to battle it out. 1-loop "radiative corrections":





Our modelling Ansatz

We propose the following ansatz, which combines both BLNP and SIMBA:

$$\hat{S}(\hat{\omega}, \mu_0) = [S_{\rm part}(\mu_0) \otimes \hat{F}_{\rm pol}](\hat{\omega}) + \hat{S}_{\rm Null}(\hat{\omega}, \mu_0)$$

1. The modelling functions $\hat{F}_{\rm pol}$ and $\hat{F}_{\rm kin}$ are **compact**.
2. There is an additional function $\hat{S}_{\rm Null}$ that has zero first few moments and lifts the α_s contributions out of the bulk region.

- $\hat{F}_{\rm kin}(\hat{k})$ has support only on $[0, \hat{k}_c]$, with $\hat{k}_c \sim$ few times $\Lambda_{\rm QCD}$. This implies
  - $\langle \hat{F}_{\rm kin} \rangle_n^{(\Lambda_{UV})} = \langle \hat{F}_{\rm kin} \rangle_n^{(\hat{k}_c)}$ for all $\Lambda_{UV} \geq \hat{k}_c$.
  - Those moments do **not** grow factorially with n

    (as an exponential tail would: $\int x^n e^{-x} = n!$)
- $\hat{S}_{\rm Null}$ allows us to **not** model the renormalon battle.

Construction by adding and subtracting

1. The renormalon fight happens in the privacy of its own curly brackets. We do not model that part explicitely.

$$\begin{aligned} S_{\rm part} \otimes \hat{F}_{\rm pol} &= S_{\rm part} \otimes (\hat{F}_{\rm kin} + \hat{G}) \\ &= \hat{F}_{\rm kin} + \Big\{ (S_{\rm part} - \delta) \otimes F_{\rm kin} + S_{\rm part} \otimes G \Big\} \end{aligned}$$

2. Next, we model the low $\hat{\omega}$ region of $\hat{S}_{\rm tail}$ to our liking (mostly zero). This makes the contribution inside square brackets compact.

$$\begin{aligned} S_{\rm part} \otimes \hat{F}_{\rm pol} &= \hat{F}_{\rm kin} + \Big\{ S_{\rm part} \otimes \hat{F}_{\rm pol} - \hat{F}_{\rm kin} \Big\} \\ &= \hat{F}_{\rm kin} + \hat{S}_{\rm tail}(\mu_t) + \Big[S_{\rm part} \otimes \hat{F}_{\rm pol} - \hat{F}_{\rm kin} - \hat{S}_{\rm tail}(\mu_t) \Big] \end{aligned}$$

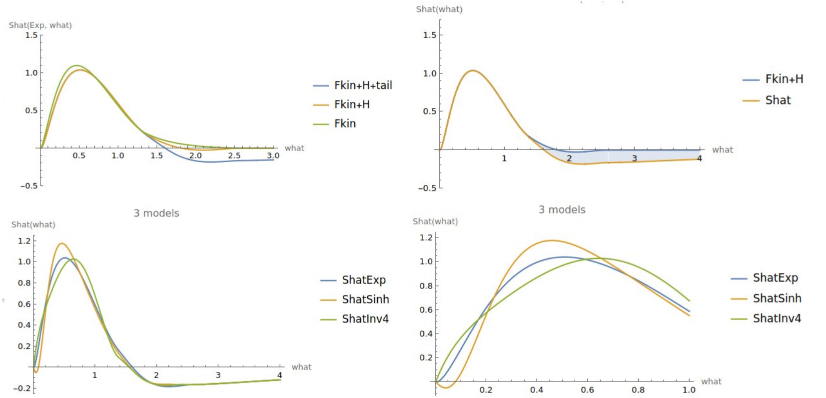
3. We know the moments of the square brackets to 2-loop order. Now we add and subtract a nonperturbative function $\hat{H}$ with exactly those moments. This lifts α_s from the low $\hat{\omega}$ region.

$$S_{\rm part} \otimes \hat{F}_{\rm pol} = \hat{F}_{\rm kin} + \hat{H} + \hat{S}_{\rm tail}(\mu_t) + \underbrace{\Big[S_{\rm part} \otimes \hat{F}_{\rm pol} - \hat{F}_{\rm kin} - \hat{S}_{\rm tail}(\mu_t) \Big] - \hat{H}}_{-\hat{S}_{\rm Null}}$$

Examples

Smoother version of “glue on the tail”.

$$\hat{S} = S_{\rm part} \otimes \hat{F}_{\rm pol} + \hat{S}_{\rm Null} = \hat{F}_{\rm kin} + \hat{H} + \hat{S}_{\rm tail}$$



Shape function uncertainty

Combinatorics

- $\mathcal{O}(10)$ different functional forms for $\hat{F}_{\rm kin}$
- $\mathcal{O}(10)$ different functional forms for $\hat{H}$
- So far only 1 transition model for $\hat{S}_{\rm tail}(\hat{\omega} < \mu_t)$, but μ_t can be chosen.

Hundreds of models

for example: $\hat{H}(\hat{k}) = \frac{1}{k_0} w(\frac{\hat{k}}{k_0}) p(\frac{\hat{k}}{k_0})$ with $w(x) = x^\alpha \sinh(x_c - x)^\beta$.

Coefficients of polynomial p adjusted to moment constraints.

SF uncertainty estimates

- envelope of scan?
- variance of scan?

You can always add a (reasonable) multiple of some $\hat{S}_{\rm Null}$ and still fulfill all constraints.

What does “reasonable” mean?
Check the next higher unknown moment?

What's my bias?

Challenges in semileptonic B decays

- **Ad hoc assumptions:** we find that **compactness** for some part of $\hat{S}$ is better motivated than **positive definiteness**.
- (orthogonal) polynomials have zeros and lead to peaks and valleys. But inclusive means we don't resolve resonances.
- my bias is: $\hat{S}$ **should be boring** in the bulk. How do I quantify boringness? What's my metric?

$$\int_0^{\Lambda_{UV}} d\hat{\omega}\,\sqrt{1+\hat{S}'(\hat{\omega})^2} \qquad \text{minimal?}$$

Last comment: RG evolution acts as a smoothing filter, makes the shape more boring (good!). It also mixes the moments. Either you live with a little α_s in the bulk, or you construct $\hat{S}_{Null}$ anew.

Summary

BLNP mandatory maintenance

- reduced perturbative uncertainty at NNLO.
- reduced SF uncertainty by including ρ_D^3
- updating power corrections