

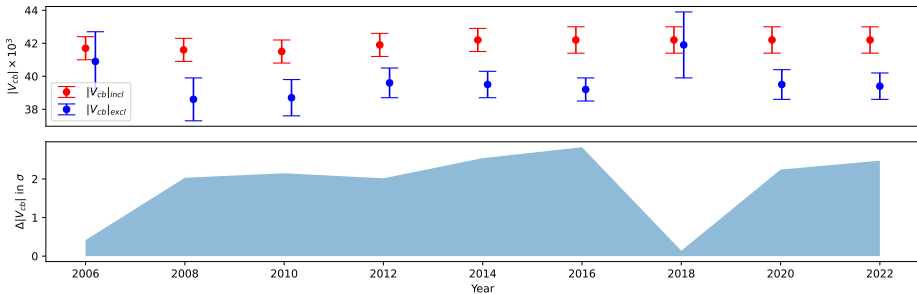
Heavy to heavy semileptonic decays in LQCD: current status

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23rd September 2024

Motivation: Tensions in $|V_{cb}|$ inclusive vs exclusive

The CKM matrix



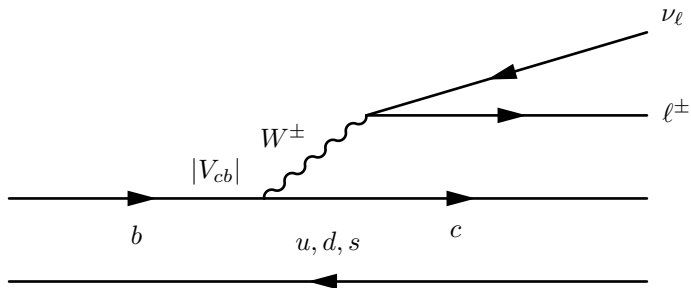
- Current values (PDG 2024):

$$|V_{cb}|_{excl} \times 10^{-3} = 39.8(6)$$

$$|V_{cb}|_{incl} \times 10^{-3} = 42.2(5)$$

- The 3σ difference between these two values shows that we have not improved much

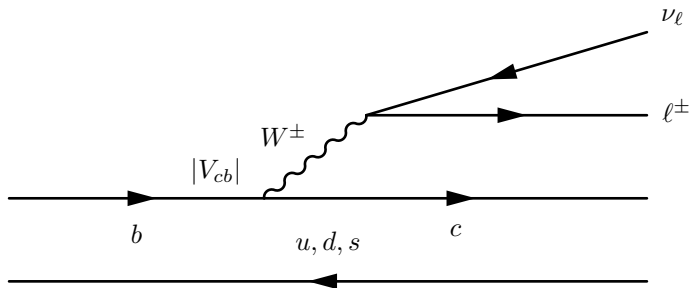
Semileptonic B decays on the lattice: Exclusive $|V_{cb}|$



$$\underbrace{\frac{d\Gamma}{dw} \left(B_{(s)} \rightarrow D_{(s)}^{(*)} \ell \bar{\nu}_\ell \right)}_{\text{Experiment}} = \underbrace{K_{B_{(s)} \rightarrow D_{(s)}^{(*)}}(w, m_\ell)}_{\text{Known factors}} \underbrace{|\mathcal{F}(w)|^2}_{\text{Theory}} \times |V_{cb}|^2, \quad w = v_{D_{(s)}^{(*)}} \cdot v_{B_{(s)}}$$

- The amplitude \mathcal{F} must be calculated in LQCD
 - Data more precise at w close to 1
- $K_{B_{(s)} \rightarrow D_{(s)}^{(*)}}(w, m_\ell) \propto (w^2 - 1)^{\frac{3}{2}, \frac{1}{2}}$ requires extrapolation of experimental data

Semileptonic B decays on the lattice: Universality ratios



$$R(D_{(s)}^{(*)}) = \frac{\int_1^{w_{\text{Max},\tau}} dw K_{B_{(s)} \rightarrow D_{(s)}^{(*)}}(w, m_\tau) |\mathcal{F}(w)|^2 \times \cancel{|V_{cb}|^2}}{\int_1^{w_{\text{Max}}} dw K_{B_{(s)} \rightarrow D_{(s)}^{(*)}}(w, 0) |\mathcal{F}(w)|^2 \times \cancel{|V_{cb}|^2}}$$

- The universality ratio depends only on the form factors
- It is possible to extract $R(D_{(s)}^{(*)})$ without experimental data!

Semileptonic B decays on the lattice: Heavy quarks

- For heavy quarks ($m_Q > \Lambda_{QCD}$), discretization errors grow as $\sim \alpha_s^k (am_Q)^n$
- Mainly two ways to address this problem
 - Effective actions (FermiLab, NRQCD...)
 - Treat the bottom as a light quark
 - Use unphysical values for m_b and extrapolate

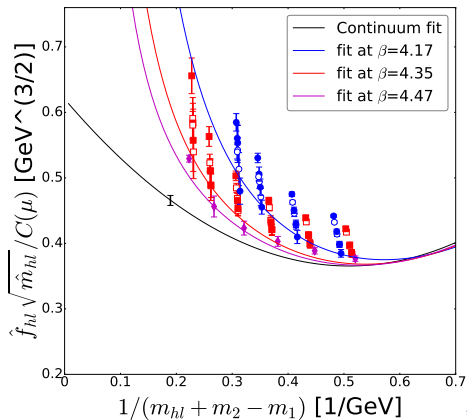
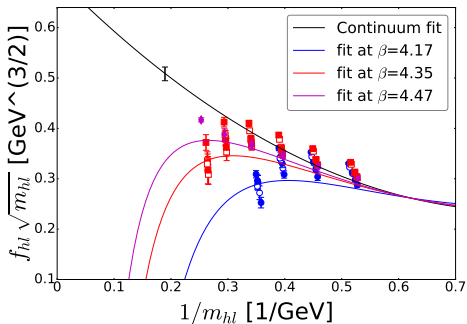
Different quark actions have different discretization errors when applied to heavy quarks

Semileptonic B decays on the lattice: Heavy quarks

- Domain-Wall fermions from JLQCD
- Errors **start** at $O(a^2 m_Q^2)$
- Data beyond $am_Q \approx 0.65$ features **large** discretization systematics
- Large correction in the extrapolation

PoS LATTICE2016 (2016) 118

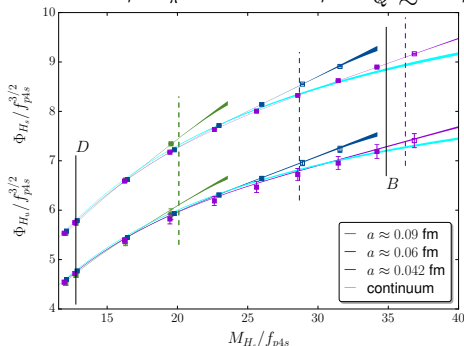
$a \approx 0.044 - 0.080$ fm, $M_\pi \approx 230 - 500$ MeV, $am_Q \lesssim 0.86$, $m_Q \lesssim 3.0$ GeV



Semileptonic B decays on the lattice: Heavy quarks

- HISQ fermions from Fermilab/MILC Phys.Rev.D 98 (2018) 7, 074512; Phys.Rev.D 107 (2023) 9, 094516
- From HPQCD Phys.Rev.D 75 (2007) 054502; Phys.Rev.D 87 (2013) 3, 034017
- Errors **start** at $O(\alpha_s v a^2 m_Q^2)$, one order of magnitude smaller than $O(a^2 m_Q^2)$
- Reasonable correction, even at large am_Q , without ap issues
- HISQ corrects at all orders, theoretical limit with fine tuning $am_Q = \pi/2$

$$a \approx 0.042 - 0.088 \text{ fm}, M_\pi \approx 135 \text{ MeV}, am_Q \lesssim 1.3, m_Q \sim m_b$$



Some statistical errors are missing

Review of lattice results

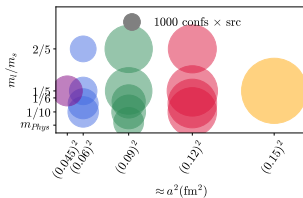
- Latest calculations and tensions (mainly $B \rightarrow D^* \ell \nu$)
- Calculations in progress
 - HPQCD $B_c \rightarrow J/\psi \ell \nu$
 - RBQCD/UKQCD $B_s \rightarrow D_s^* \ell \nu$
 - Fermilab/MILC $B_{(s)} \rightarrow D_{(s)}^* \ell \nu$
 - JLQCD $B \rightarrow D \ell \nu$
- Not covered \rightarrow Inclusive determinations from LQCD

$$B \rightarrow D^* \ell \nu$$

Review of lattice results: $B \rightarrow D^* \ell \nu$

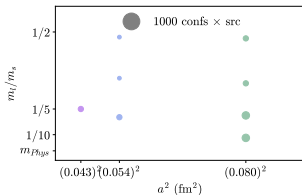
Fermilab/MILC

- ASQTAD + Fermilab



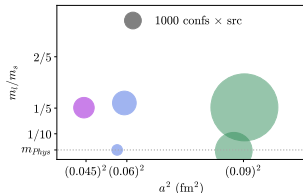
JLQCD

- DW + DW

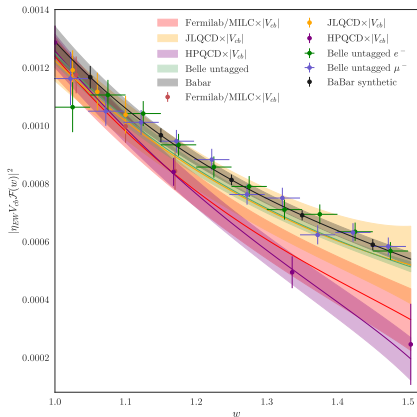


HPQCD

- HISQ + HISQ



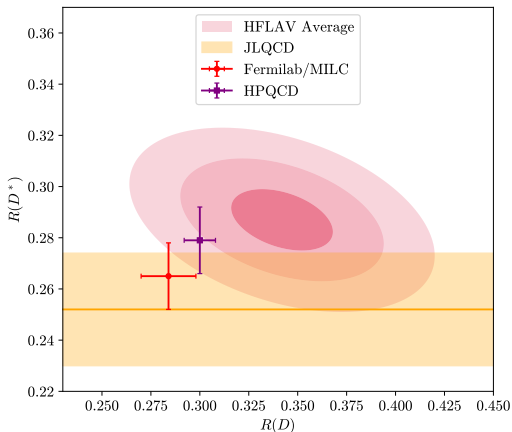
Review of lattice results: $B \rightarrow D^* \ell \nu$



$$|V_{cb}|^{\text{FM}} = 38.40(78) \times 10^{-3}$$

$$|V_{cb}|^{\text{JLQCD}} = 39.19(90) \times 10^{-3}$$

$$|V_{cb}|^{\text{HPQCD}} = 39.31(74) \times 10^{-3}$$



$$R(D^*)^{\text{FM}} = 0.265(13)$$

$$R(D^*)^{\text{JLQCD}} = 0.252(22)$$

$$R(D^*)^{\text{HPQCD}} = 0.279(13)$$

Review of lattice results: D'Agostini bias

- V_{cb} value well below the latest inclusive one (everything $\times 10^3$)

$$|V_{cb}|_{\text{excl}}^{\text{FM}} = 38.40(78) < |V_{cb}|_{\text{incl}}^{\text{BCG}} = 42.16(51)$$

Eur.Phys.J.C82 (2022), 1141; Phys.Lett.B822 (2021), 136679; JHEP10(2022)068

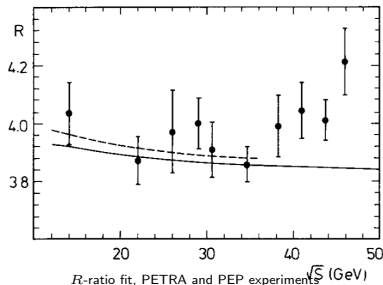
- Could this be a consequence of D'Agostini effect?

Nucl.Instrum.Meth.A346 (1994), 306

- Correlated data
- Overall normalization factor ($|V_{cb}|$)

$$\hat{V}_{cb} = \frac{\bar{V}_{cb}}{1 + \chi_{\text{Exp}}^2 \sigma_{V_{cb}}^2},$$

$$\chi^2 = \chi_{\text{LQCD}}^2 + \chi_{\text{Exp}}^2$$

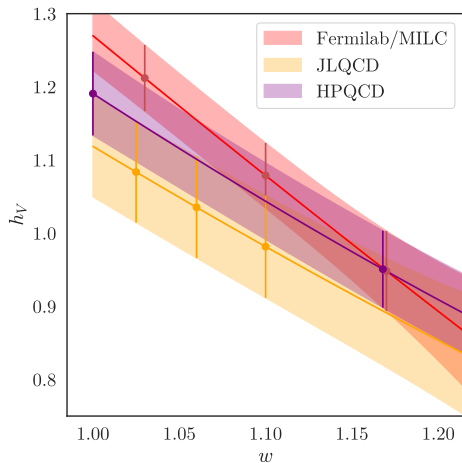
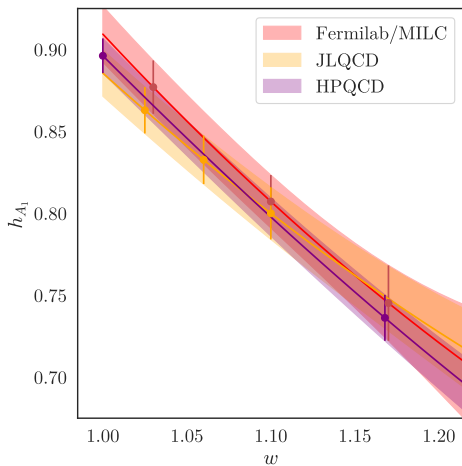


- Compare fits **Belle + all LQCD** with **Belle fits + LQCD** $\mathcal{F}(w=1)$

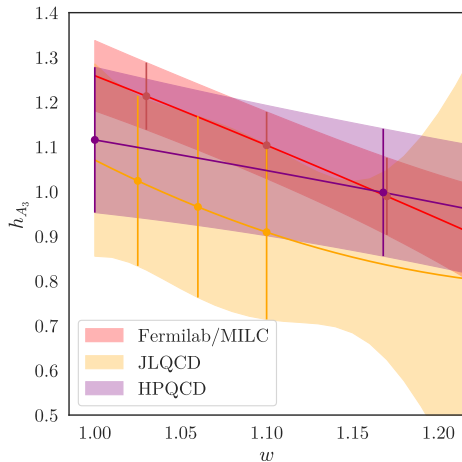
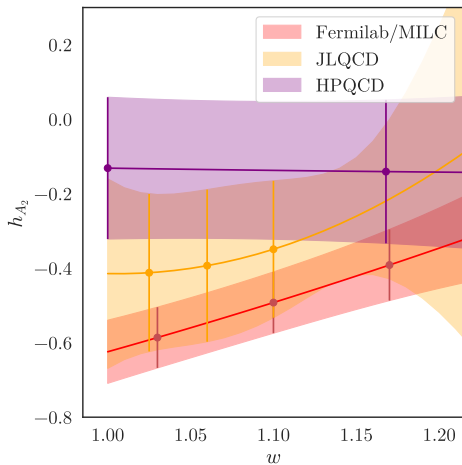
$$|V_{cb}|_{\text{All}}^{\text{FM}} = 38.60(86), \quad |V_{cb}|_{\mathcal{F}(1)}^{\text{FM}} = 38.17(78)$$

We do not see any **D'Agostini bias** in our calculation

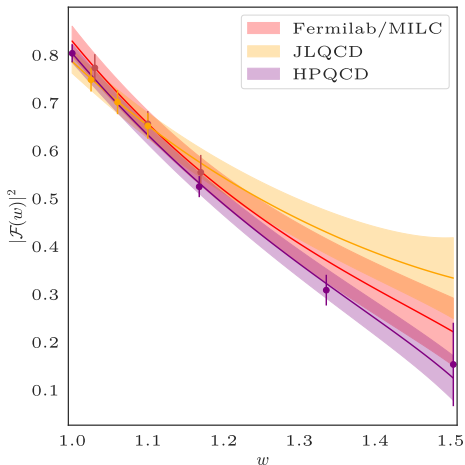
Review of lattice results: Comparison of HQET form factors



Review of lattice results: Comparison of HQET form factors



Review of lattice results: Comparison of decay amplitudes



- Maximum difference $\begin{cases} \text{Fermilab/MILC} - \text{JLQCD} & 1.02\sigma \\ \text{Fermilab/MILC} - \text{HPQCD} & 1.14\sigma \\ \text{JLQCD} - \text{HPQCD} & 2.20\sigma \end{cases}$

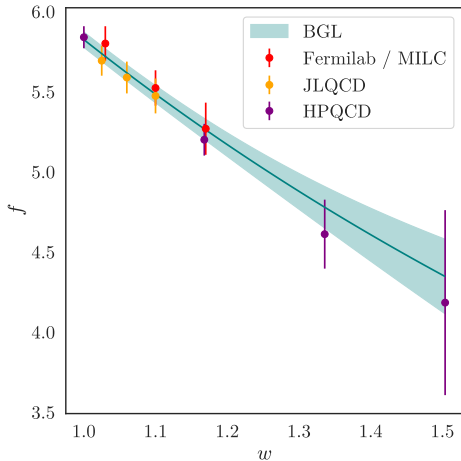
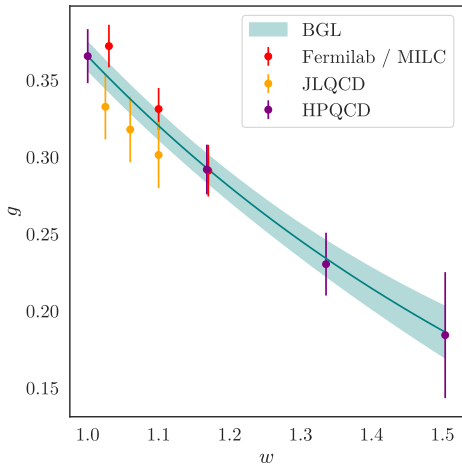
Review of lattice results: Combined fits

- Combined fits with priors 0(1)
- Kinematic constraint imposed with priors
- BGL fit 2222

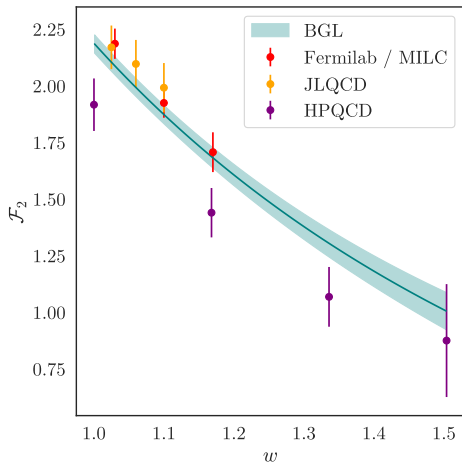
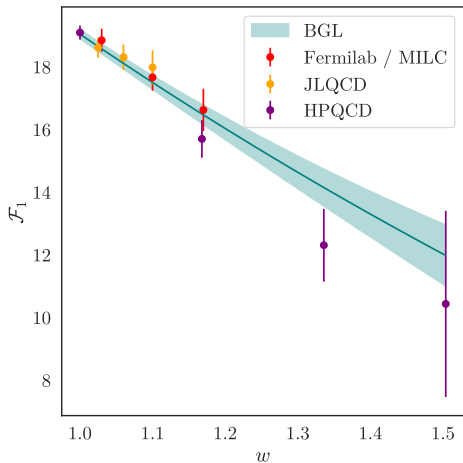
	w Constraint		w/o Constraint	
	p	$R_2(1)$	p	$R_2(1)$
MILC	0.51	1.20(12)	0.43	1.27(13)
JLQCD	0.52	0.98(19)	0.25	0.97(19)
HPQCD	0.77	1.39(16)	0.65	1.39(16)
MILC+JLQCD	0.40	1.118(97)	0.36	1.16(11)
MILC+HPQCD	0.44	1.262(93)	0.37	1.262(93)
JLQCD+HPQCD	0.73	1.18(12)	0.67	1.18(12)
All	0.56	1.193(83)	0.50	1.193(83)

- p -value of Belle untagged + BaBar BGL fit 223 is ≈ 0.04
- Combined $R(D^*) = 0.2667(57)$

Review of lattice results: Combined fits



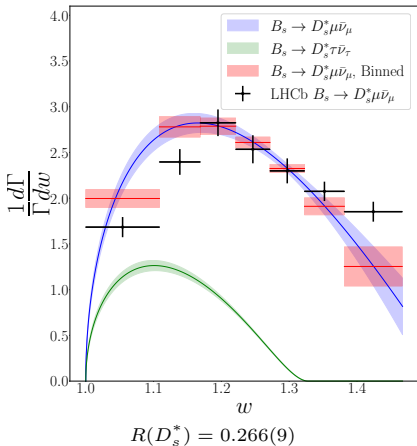
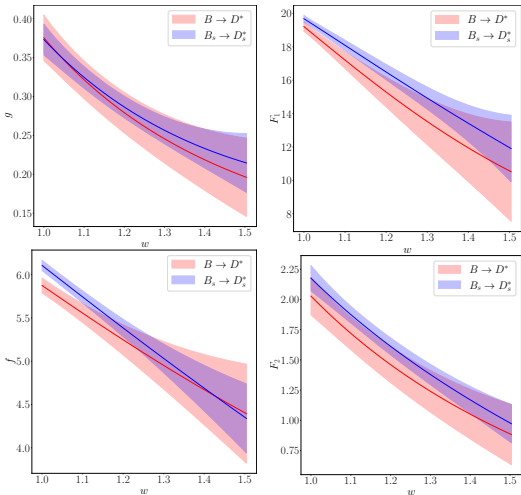
Review of lattice results: Combined fits



$$B_S \rightarrow D_S^* \ell \nu$$

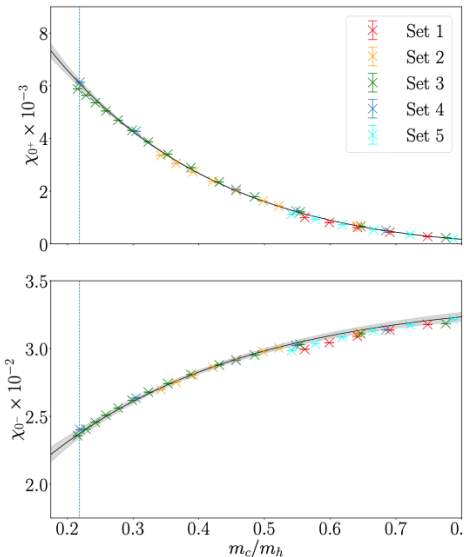
Review of lattice results: $B_s \rightarrow D_s^* \ell \nu$

• HPQCD with the same setup as $B \rightarrow D^* \ell \nu$



$$B_c \rightarrow J/\psi \ell \nu$$

Review of lattice projects: $B_c \rightarrow J\psi\ell\nu$



- HPQCD with similar setup to $B_{(s)} \rightarrow D_{(s)}^* \ell \nu$
- Update of the 2020 calculation
[Phys.Rev.D102 \(2020\), 094518](#)
- Plan to include an extra 0.03 fm ensemble
- Chiral-continuum extrapolation includes a z expansion
- Susceptibilities computed using LQCD [Phys.Rev.D104 \(2021\), 094512](#); [Phys.Rev.D110 \(2024\), 054506](#)

- Large reduction in errors
 - Large shifts wrt the previous calculation (!!)
- (Preliminary)**

$$R(J/\psi) = 0.2582(38) \rightarrow 0.2674(31)$$

$$F_L = 0.4416(92) \rightarrow 0.4510(88)$$

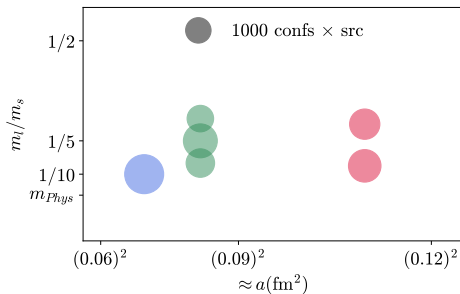
$$B_S \rightarrow D_S^* \ell \nu$$

Review of lattice projects: $B_{(s)} \rightarrow D_{(s)}^* \ell \nu$

- Different group RBC/UKQCD
- Using 6 $N_f = 2 + 1$ ensembles of sea DW quarks
- The bottom quark use an effective action
 - Good crosscheck against JLQCD
 - Potentially large systematics due to a mismatch between b and c actions

[Phys.Rev.D87 \(2018\), 054502](#)

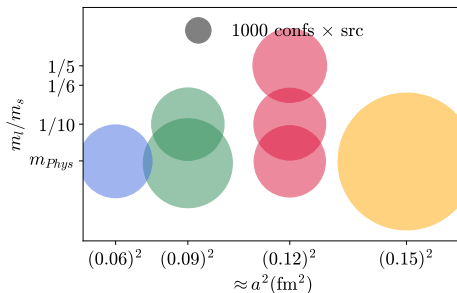
- m_π in the range 270 – 433 MeV



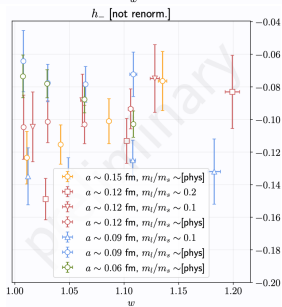
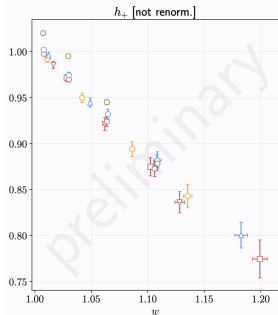
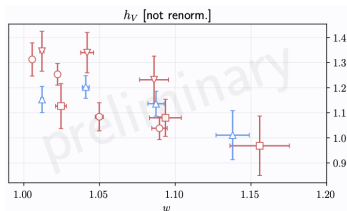
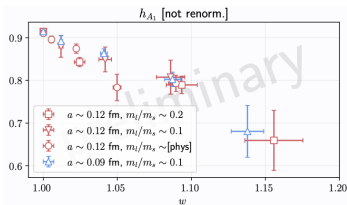
$$B_{(s)} \rightarrow D_{(s)}^{(*)} \ell\nu$$

Review of lattice projects: $B_{(s)} \rightarrow D_{(s)}^{(*)} \ell \nu$

- Fermilab/MILC calculation
- Using 7 $N_f = 2 + 1 + 1$ ensembles of sea HISQ quarks
- The heavy quarks use the Fermilab effective action
 - Correlated with a $B \rightarrow L \ell \nu$ analysis using the same data
 - Four channels in a single correlated analysis

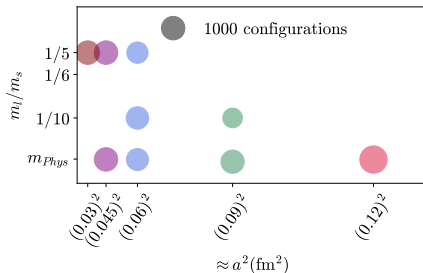


Review of lattice projects: $B_{(s)} \rightarrow D_{(s)}^{(*)} \ell \nu$

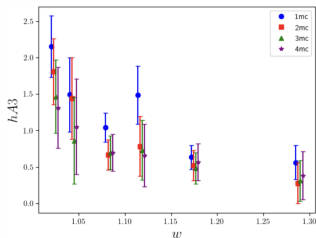
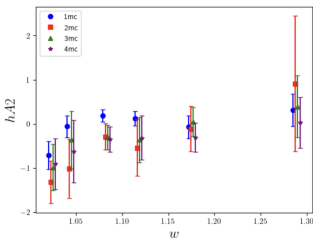
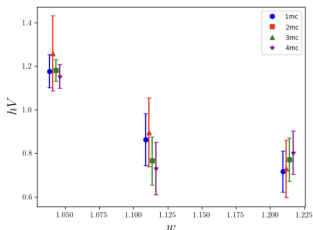
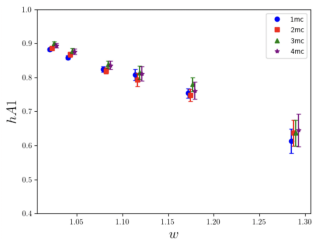


Review of lattice projects: $B_{(s)} \rightarrow D_{(s)}^{(*)} \ell \nu$

- Fermilab/MILC calculation
- Planning to use 9 $N_f = 2 + 1 + 1$ ensembles of sea HISQ quarks
- The heavy quarks use the HISQ action
 - Physical bottom mass reachable with the finest ensembles
- m_π physical in several ensembles



Review of lattice projects: $B_{(s)} \rightarrow D_{(s)}^{(*)} \ell \nu$



Preliminary results $B_s \rightarrow D_s^* \ell \nu$, statistics 24×426
 Single ensemble $a = 0.06$ fm and $m_l/m_s = \frac{1}{5}$ at different values of am_b

Thank you for your time