# Lattice-QCD Results for $|V_{ub}|$ Exclusive


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IV[th] Challenges in Semileptonic *B* Decays
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# $|V_{ub}|$ from Exclusive Semileptonic Decays

- Decays $B \to \pi l \nu$, $B_s \to K l \nu$, $B_c \to D l \nu$, $\Lambda_b \to p l \nu$, … sensitive to $|V_{ub}|$.
- Related FCNC $B \to \pi ll$, $B \to Kll$, $B_c \to D_s ll$ sensitive to BSM.
- Final-state vector meson $\rho$ ($K^*$) much more difficult: $\pi\pi$ ($K\pi$) resonance. Formalism available but no mature computations.
- Only one calculation of $\Lambda_b \to p l \nu$ [arXiv:1503.01421], discussed in first of these workshops. One calculation of $B_c \to D l \nu$ [arXiv:2108.11242].
- BaBar, Belle, Belle II for $B^{\pm}$, $B^0$ decays; LHCb for others, typically as ratio to corresponding $b \to c$ transition, e.g., $\mathrm{BR}(B_s \to K l \nu)/\mathrm{BR}(B_s \to D_s l \nu)$.
- Focus here on $B$ and $B_s$ decays.

# Form Factors

- Weak current couples to matrix element $\langle \pi | \mathscr{V}^\mu | B \rangle$ (for example).
- Decompose into sum of Lorentz-covariant forms × form factors:

$$\langle P(k)|\mathscr{V}^\mu|B(p)\rangle = f_+(q^2)\left(p^\mu + k^\mu - \frac{M_B^2 - M_P^2}{q^2} q^\mu\right) + f_0(q^2)\frac{M_B^2 - M_P^2}{q^2} q^\mu$$

$$= \sqrt{2M_B}\left[k_\perp^\mu f_\perp(E_P) + v^\mu f_\parallel(E_P)\right], \quad v = p/M_B$$

$$k_\perp = k - v\, v \cdot k$$

$$\langle P(k)|\mathscr{S}|B(p)\rangle = f_0(q^2)\frac{M_B^2 - M_P^2}{m_b - m_q}$$

$$q = p - k$$

$$\langle P(k)|\mathscr{T}^{\mu\nu}|B(p)\rangle = f_T(q^2)\frac{2}{M_B + M_P}\left(p^\mu k^\nu - p^\nu k^\mu\right)$$

BSM

with $f_\perp, f_\parallel$ being convenient in lattice QCD with $B$ in rest frame.

# Decay Rate

$$\frac{d\Gamma}{dq^2} = \frac{C_P G_F^2 |V_{ub}|^2}{3\,(2\pi)^3} \left(1 - \frac{m_\ell^2}{q^2}\right)^2 |\boldsymbol{k}| \times$$

$$m_\ell^2 \leq q^2 \leq (M_B - M_P)^2$$

$$\left[\left(1 + \frac{m_\ell^2}{2q^2}\right) |\boldsymbol{k}|^2 \left|f_+(q^2)\right|^2 + \frac{3m_\ell^2\,(M_B^2 - M_P^2)^2}{8q^2 M_B^2} \left|f_0(q^2)\right|^2\right]$$

# Form Factor Relations

$$E_P = v \cdot k$$

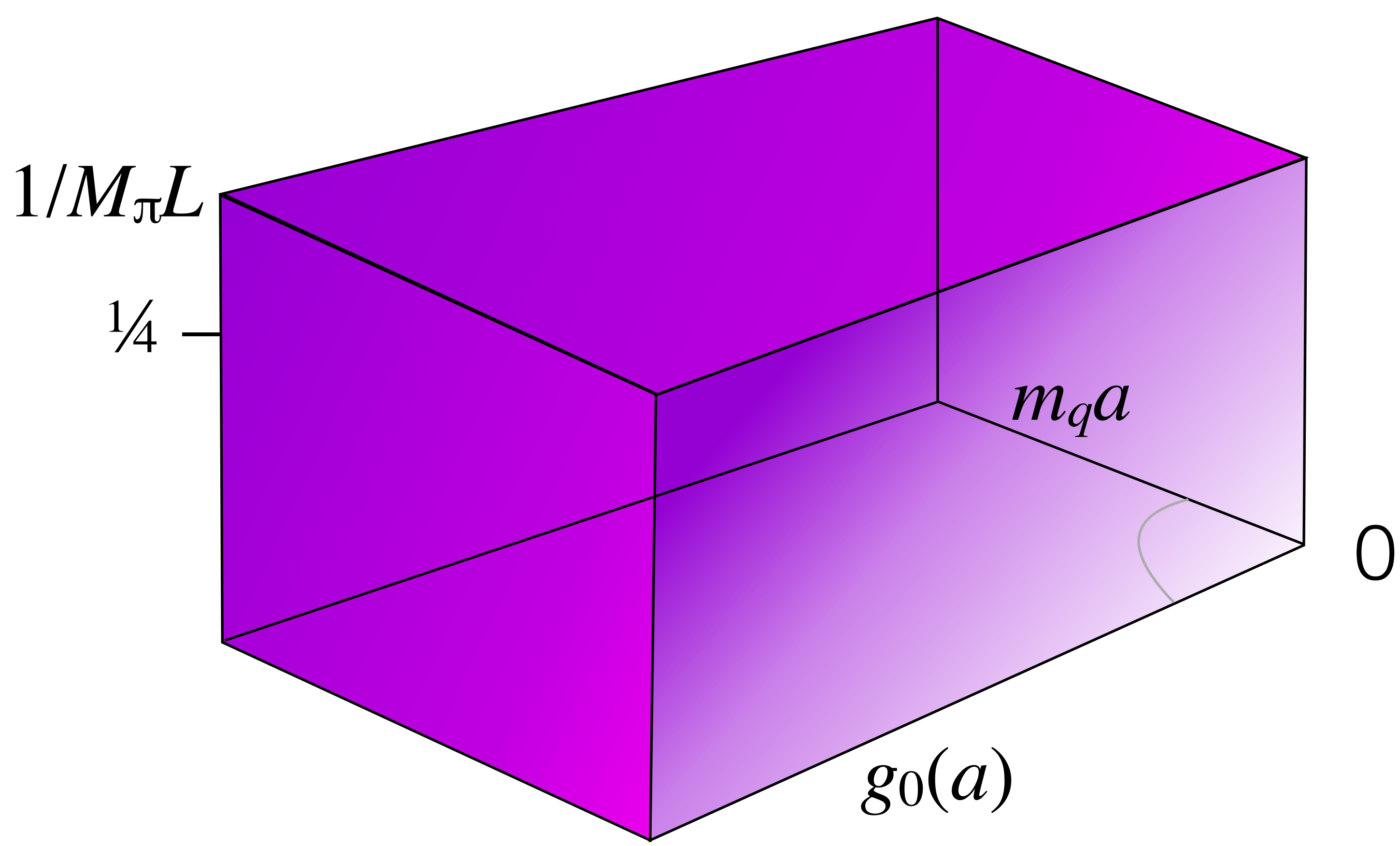
$$q^2 = M_B^2 + M_P^2 - 2M_B E_P$$

$$f_+(q^2) = \frac{1}{\sqrt{2M_B}} \left[(M_B - E_P) f_\perp(E_P) + f_\parallel(E_P)\right]$$

$$f_0(q^2) = \frac{\sqrt{2M_B}}{M_B^2 - M_P^2} \left[(M_B - E_P) f_\parallel(E_P) + (E_P^2 - M_P^2) f_\perp(E_P)\right]$$

# Lattice QCD Data

- Computer generates data in a slab of a 10-dimensional parameter space:




| Sea | Valence |
|---|---|
| | $m_u$ |
| $m'_u = m'_d$ | $m_d$ |
| $m'_s$ | $m_s$ |
| $m'_c$ | $m_c$ |
| | $m_b$ |

$m'_u \neq m'_d \Rightarrow$ 11d slab

- Combine data with effective field theories (EFT) [*e.g.*, hep-lat/0205021].

# Fits to Form Factor Data I

- Computer generates data at discrete momenta ($\Leftarrow$ finite volume).

- Chiral-continuum extrapolation $B_y \to P_{xy} l\nu$:

$$w_0^{-1/2} f_\perp(E) = \frac{c_\perp}{w_0(E + \Delta_{xy,1^-})} \left[1 + \chi\log_\perp + \text{polynomial}_\perp(E, a, m_l, m_s)\right]$$

$$w_0^{1/2} f_\parallel(E) = \frac{c_\parallel}{w_0(E + \Delta_{xy,0^+})} \left[1 + \chi\log_\parallel + \text{polynomial}_\parallel(E, a, m_l, m_s)\right]$$

- for $E_P \ll 1$ GeV, $\chi\log_\perp \neq \chi\log_\parallel$; $E_P \gtrsim 1$ GeV, $\chi\log_\perp = \chi\log_\parallel$.

- What's $\Delta_{xy,J^P}$? Pole in annihilation process $l\nu \to B_x^*(J^P) \to B_y P_{xy}$:

$$\frac{2M_{B_y}}{M^2_{B_x^*(J^P)} - q^2} \equiv \frac{1}{E_{P_{xy}} + \Delta_{xy,J^P}} \Rightarrow \Delta_{xy,J^P} = \frac{M^2_{B_x^*(J^P)} - M^2_{B_y} - M^2_{P_{xy}}}{2M_{B_y}}$$

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# Fits to Form Factor Data II

- $z$ expansion of $(f_+, f_0)$, usually with BCL [Bourrely, Caprini, Lellouch, arXiv:0807.2722]:

$$f_+(E) = \frac{1}{1 - q^2/M^2_{B_x^*(1^-)}} \sum_{k=0}^{K-1} a_k \left( z^k - (-1)^{K-k} \frac{k}{K} z^K \right)$$

$$f_0(E) = \frac{1}{1 - q^2/M^2_{B_x^*(0^+)}} \sum_{k=0}^{K-1} b_k z^k$$

$$z = \frac{\sqrt{t_{\text{cut}} - q^2} - \sqrt{t_{\text{cut}} - t_0}}{\sqrt{t_{\text{cut}} - q^2} + \sqrt{t_{\text{cut}} - t_0}}$$

  where $t_{\text{cut}} = (M_B + M_\pi)^2$ for $B \to \pi$ & $B_s \to K$ and $(M_B + M_K)^2$ for $B \to K$.

- NB: poles here are in $(f_+, f_0)$, previous slide in $(f_\perp, f_\parallel)$.

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# Outline

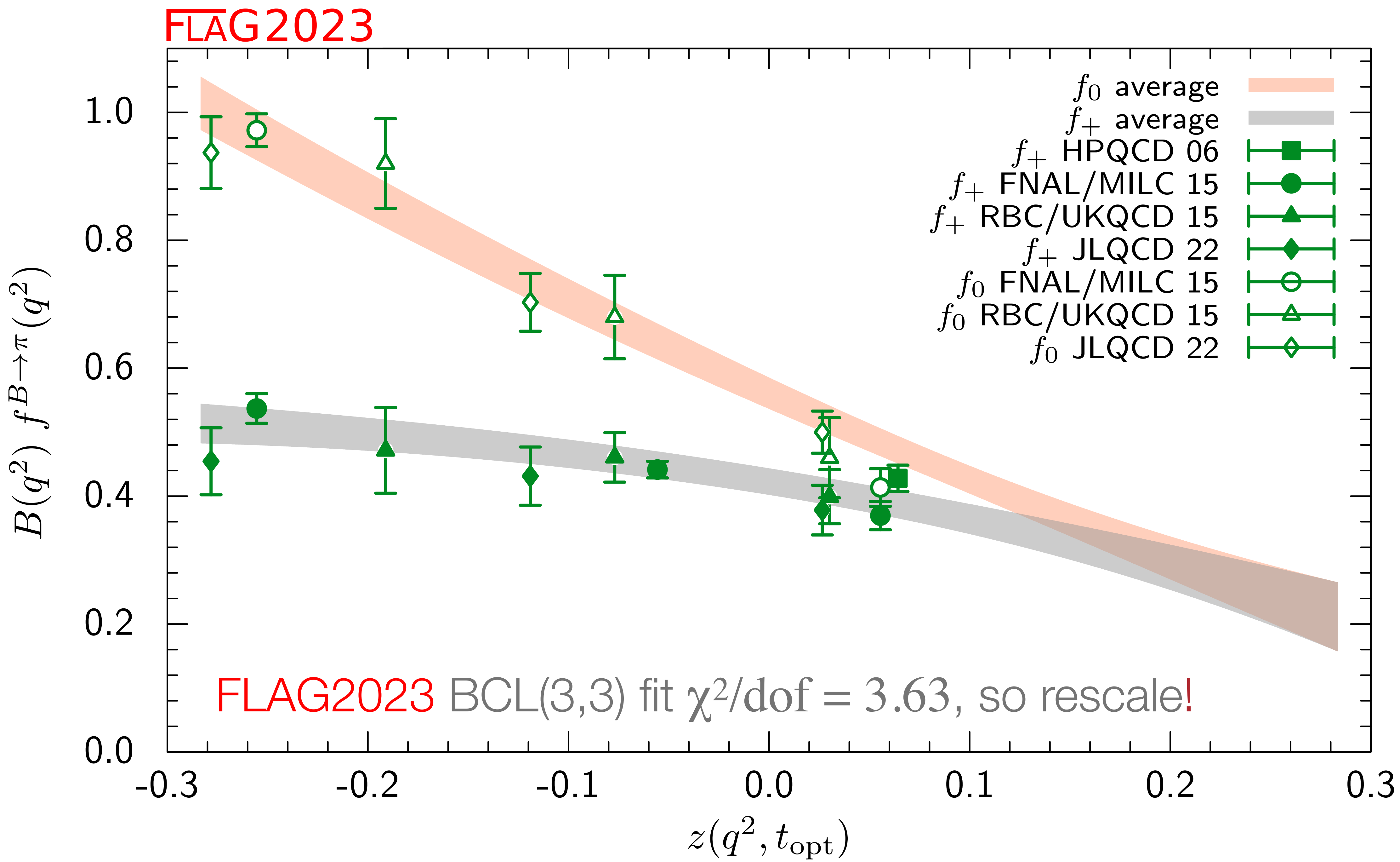
- Introduction
- Overview of published 2+1 and 2+1+1 lattice-QCD calculations
- Chiral-continuum fits: $(f_+, f_0)$ vs. $(f_\perp, f_\parallel)$
- Upcoming work (as informed by semileptonic $D$ decay)
- Final remarks

# Overview of Lattice-QCD Calculations

# $B \to \pi l \nu$ in FLAG2023

- **HPQCD 2006** [arXiv:hep-lat/0601021]: NRQCD $b$ quark on 2+1 asqtad sea (MILC); six ensembles at two lattice spacings; smallest $m_l = m_s/8$.
- **RBC/UKQCD 2015** [arXiv:1501.05373]: RHQ $b$ quark on 2+1 DWF sea; five ensembles at two lattice spacings; smallest $m_l = m_s/8$.
- **Fermilab-MILC 2015** [arXiv:1503.07839]: Fermilab $b$ quark on 2+1 asqtad sea (MILC); twelve ensembles at four lattice spacings; smallest $m_l = m_s/20$. Supersedes arXiv:0811.3640.
- **JLQCD 2022** [arXiv:2203.04938]: DWF $h$ quark, $m_h \leq 2.44 m_c$, on 2+1 DWF sea; eleven ensembles at three lattice spacings; smallest $m_l = 7m_s/80$.
- NB: Same action for Fermilab and RHQ but different trajectories to CL (unimportant).

# $B \to \pi l \nu$ in FLAG2023update



# $B_s \to K l \nu$ in FLAG2023

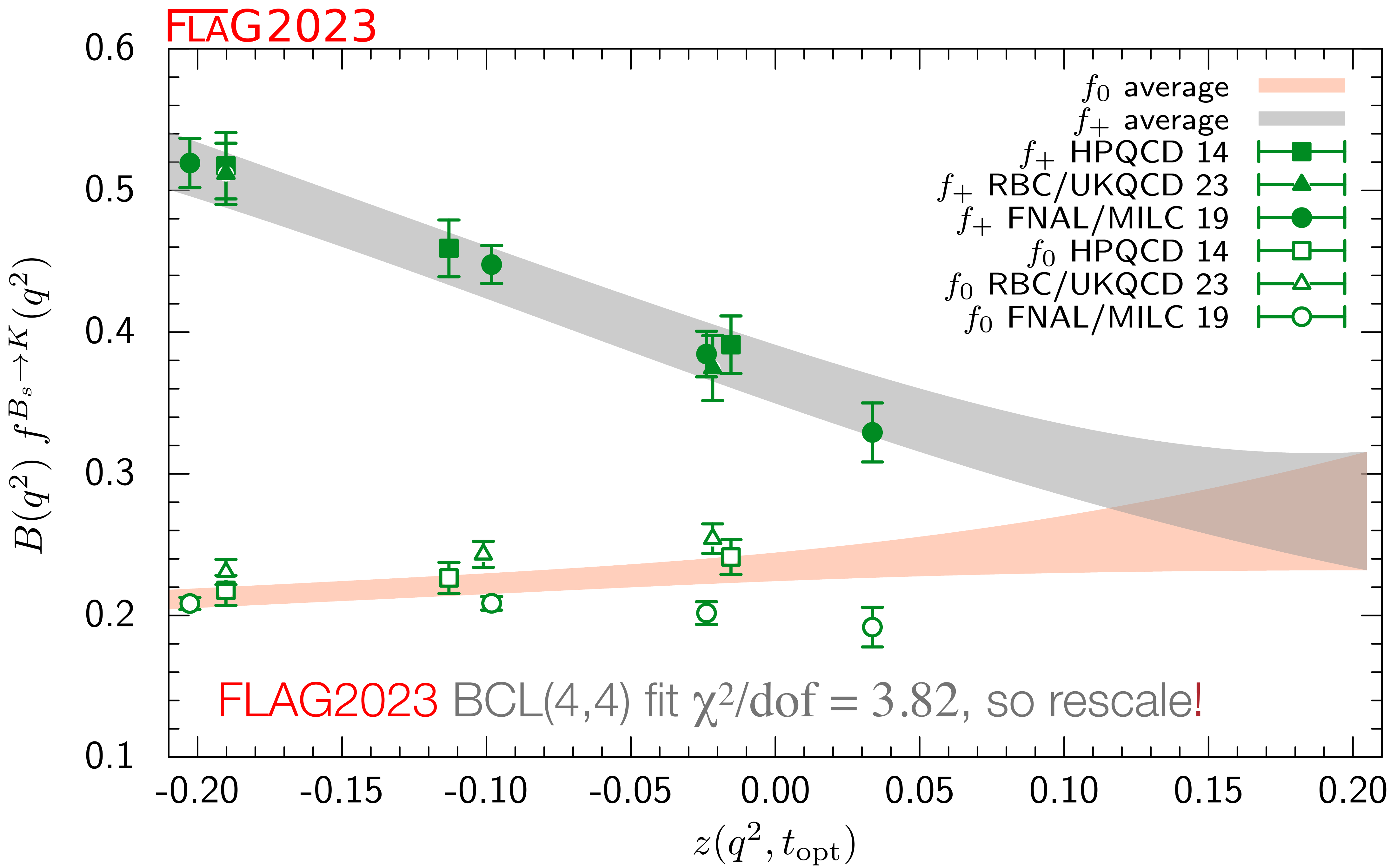
- **HPQCD 2014** [arXiv:1406.2279]: NRQCD $b$ quark on 2+1 asqtad sea (MILC); five ensembles at two lattice spacings; smallest $m_{l,\mathrm{sea}} = m_{s,\mathrm{sea}}/10$, $m_{l,\mathrm{val}} = m_{s,\mathrm{val}}/7$.

- **Fermilab-MILC 2019** [arXiv:1901.02561]: Fermilab $b$ quark on 2+1 asqtad sea (MILC); six ensembles at three lattice spacings; smallest $m_l = m_s/20$.

- **RBC/UKQCD 2023** [arXiv:2303.11280]: RHQ $b$ quark on 2+1 DWF sea; six ensembles at three lattice spacings; smallest $m_l = m_s/10$. Supersedes RBC/UKQCD 2015 [arXiv:1501.05373], says FLAG.

- NB: Same action for Fermilab and RHQ but different trajectories to CL (unimportant).

# $B_s \to K l \nu$ in FLAG2023$_{\text{update}}$



# Chiral-Continuum and $z$-Expansion Fits

# What Could Go Wrong?

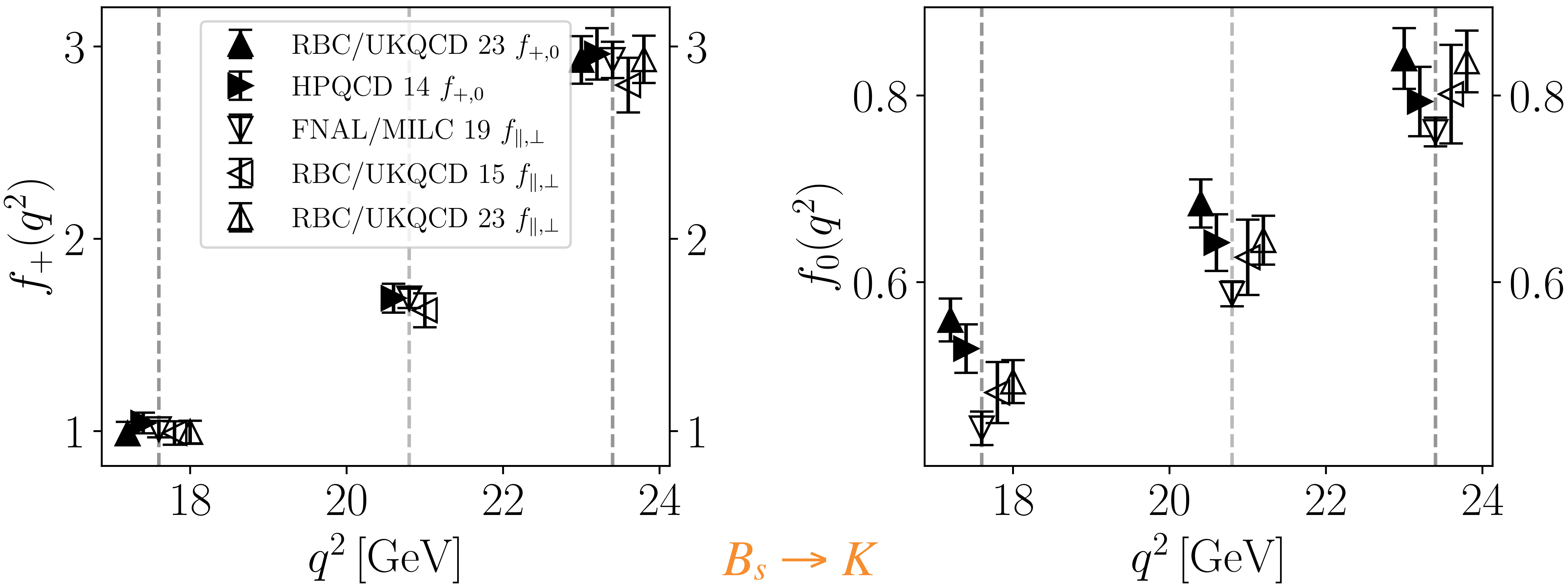
- Monte Carlo generating gauge fields? Cross-checked by some number of other quantities, so let's rule this out. {few, several, many}
- Fits to correlation functions? Cross-checked by collaborators, both by competing fits and intense scrutiny. So probably not. Alas, meaningless to compare correlator fits of Action A vs. Action B $\Leftarrow$ discretization effects.
- Insufficient data to control chiral-continuum extrapolation?
  - See number of ensembles, range of $a$, $m_l$ on previous slides. Adding a third lattice spacing has often brought significant change.
- Choices in chiral-continuum extrapolation?
  - Hard- vs. soft-pion $\chi$PT; where the poles are; $(f_+, f_0)$ vs. $(f_\perp, f_\parallel)$.

# Chiral-continuum limit with $(f_+, f_0)$ vs. $(f_\perp, f_\parallel)$

- Soft-pion formulas in heavy-meson χPT give expressions for $(f_\perp, f_\parallel)$:
  - in this limit, $f_+ \sim f_\perp$, $f_0 \sim f_\parallel$.
- Angular momentum says intermediate states in the $s$ channel are $1^-$ for $f_+$, $0^+$ for $f_0$.
- If there is a conflict, angular momentum trumps EFT:
  - higher orders in the EFT must reveal definite $J^P$ in the linear combinations of $(f_\perp, f_\parallel)$ that yield $(f_+, f_0)$.
- Also, pion/kaon is not so soft: same chiral logs in hard-pion χPT for all $f$s.

# It makes a difference

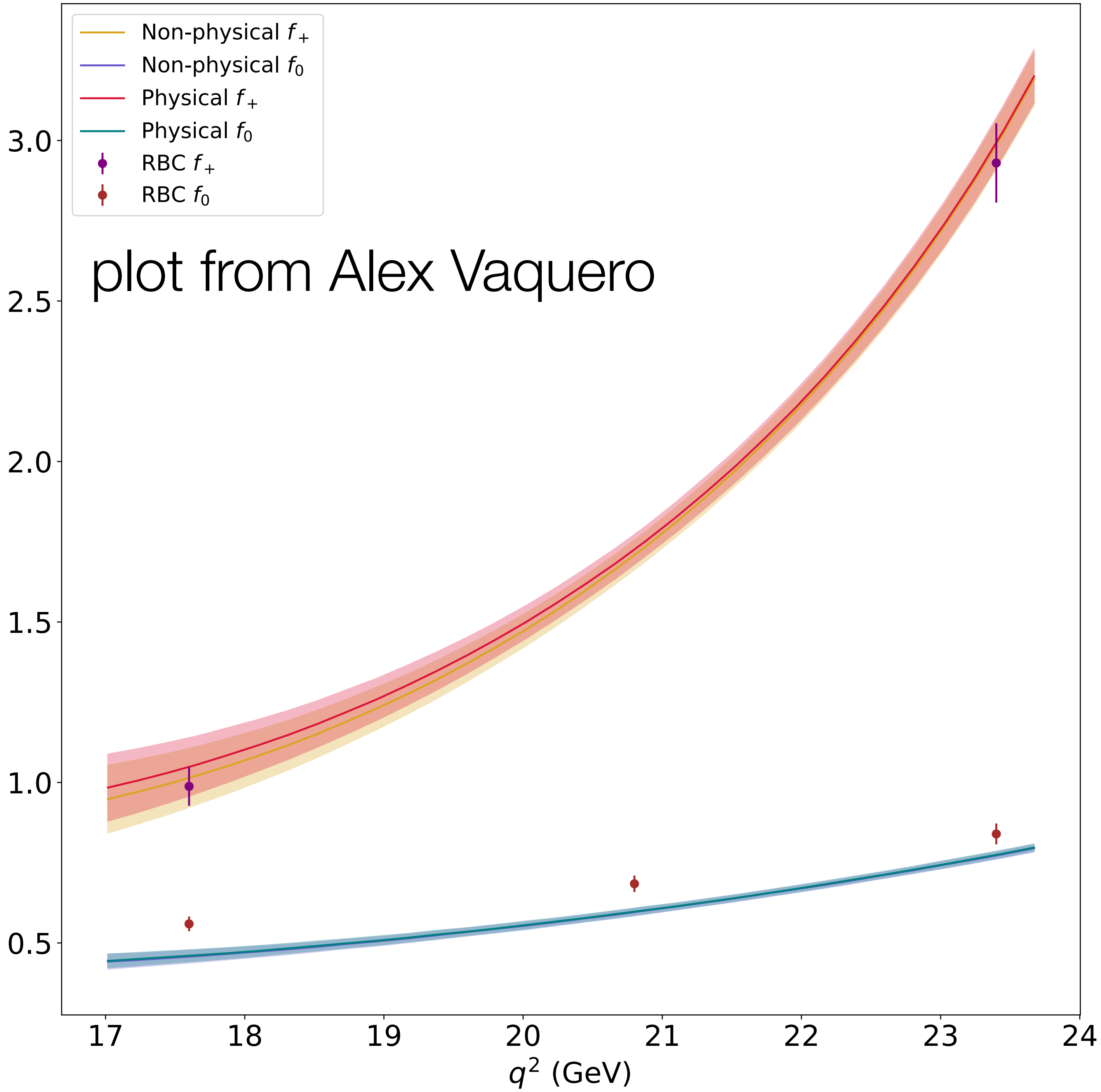
arXiv:2303.11280




- Change one thing at a time, so focus first on △ and ▲: extrapolate RBC/UKQCD 2023 with poles in ($f_\perp$, $f_\|$) or ($f_+$, $f_0$), respectively.
- Other open/solid symbols follow same convention. Note trend in $f_0$.

# Or does it?

## Vaquero archaeology of [arXiv:1901.02561](arXiv:1901.02561)

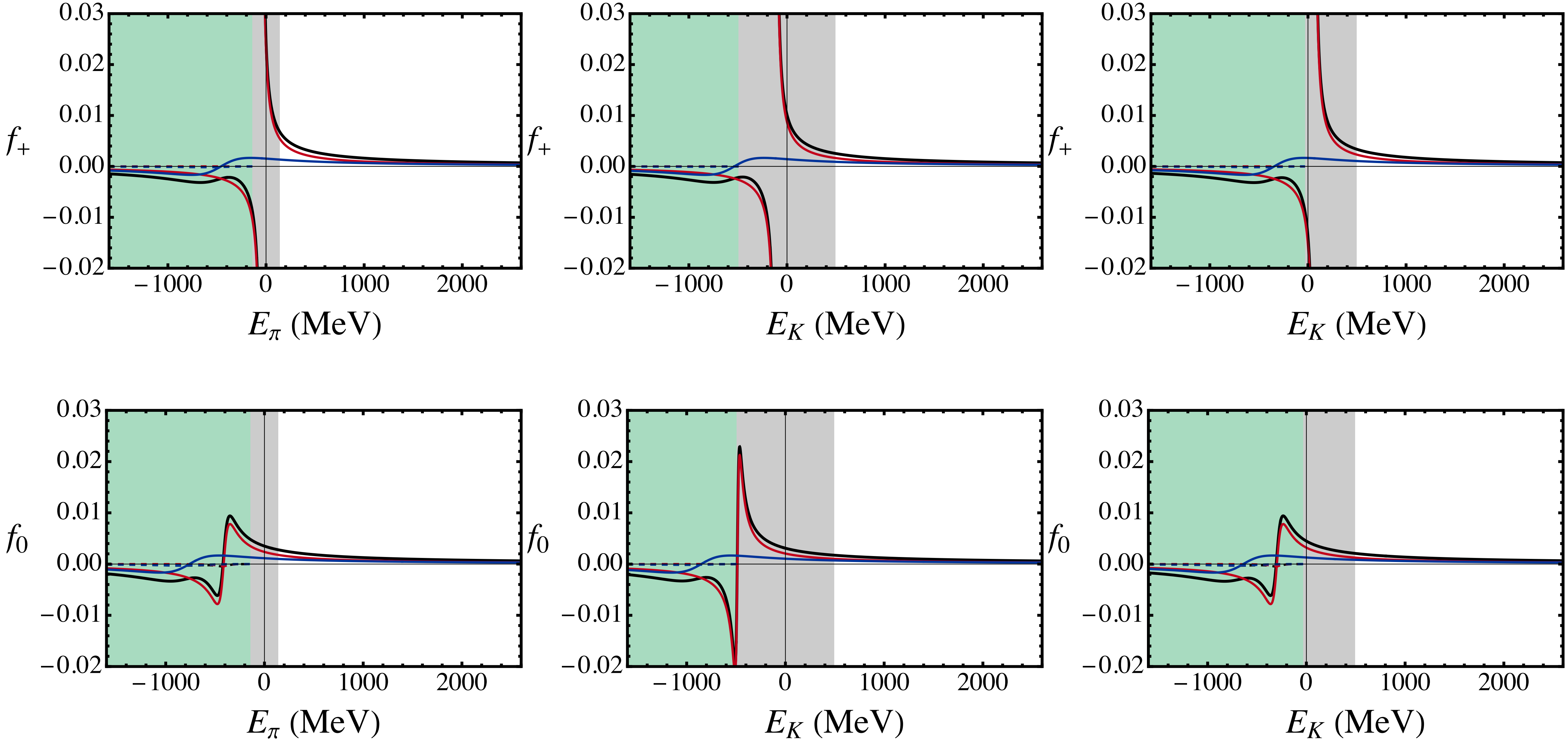



- Fermilab/MILC 2019 with poles in $(f_\perp, f_\parallel)$ or $(f_+, f_0)$ agree.
- Agreement for $f_+$ w/ RBC/UKQCD 2023.
- Clear disagreement for $f_0$ w/ RBC/UKQCD 2023.
- Pole positions differ, but tests in both papers & Alex's archaeology suggest that this difference doesn't matter.

# Pology

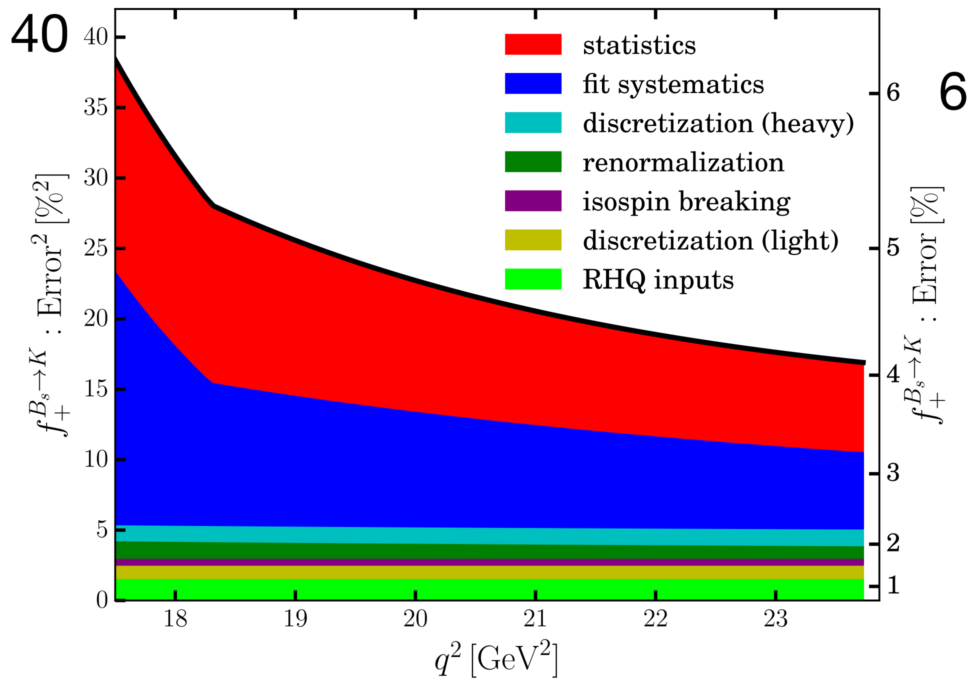
$B \to \pi l\nu$ | $B \to Kll$ | $B_s \to Kl\nu$

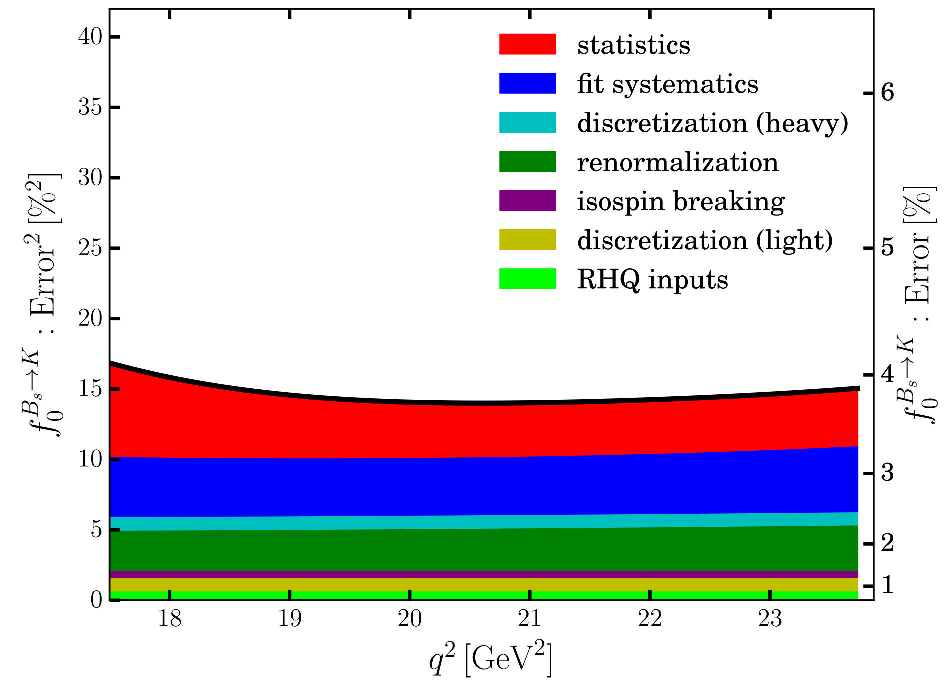
$l\nu \to B\pi$ | $ll \to BK$ | $l\nu \to B\pi$



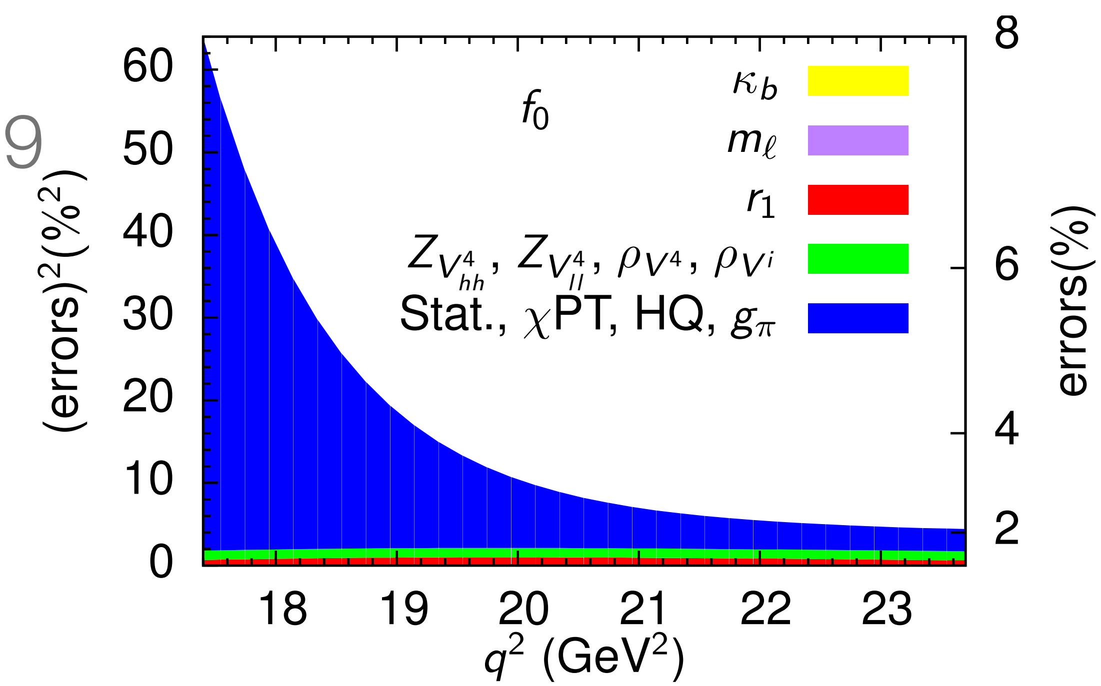
# Error Budgets

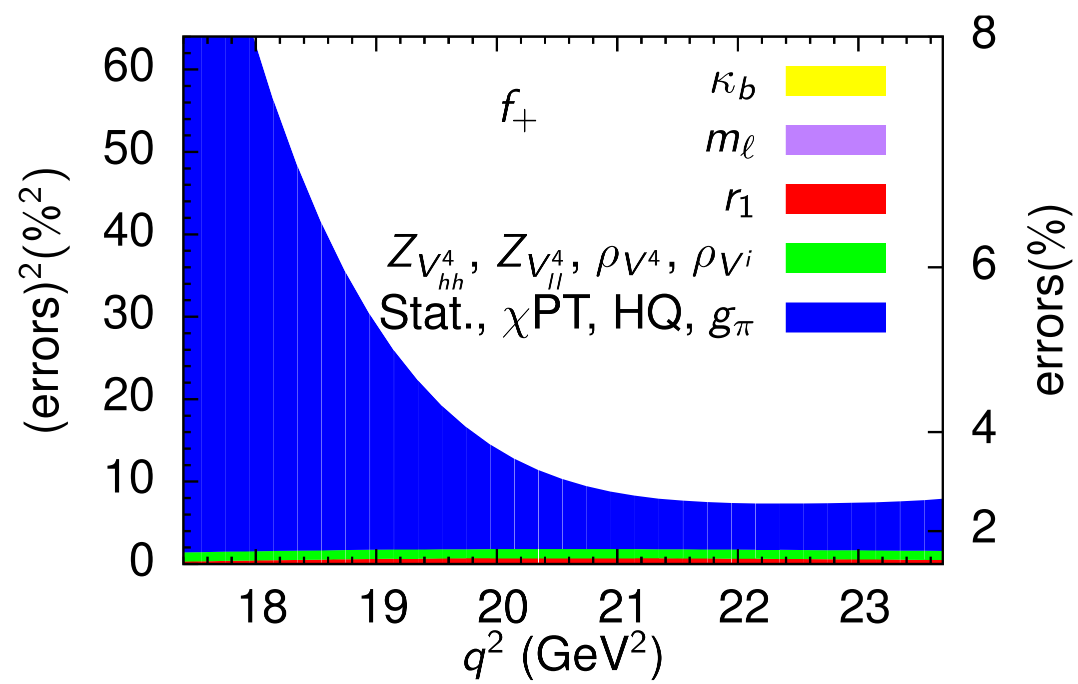
R/U 2023



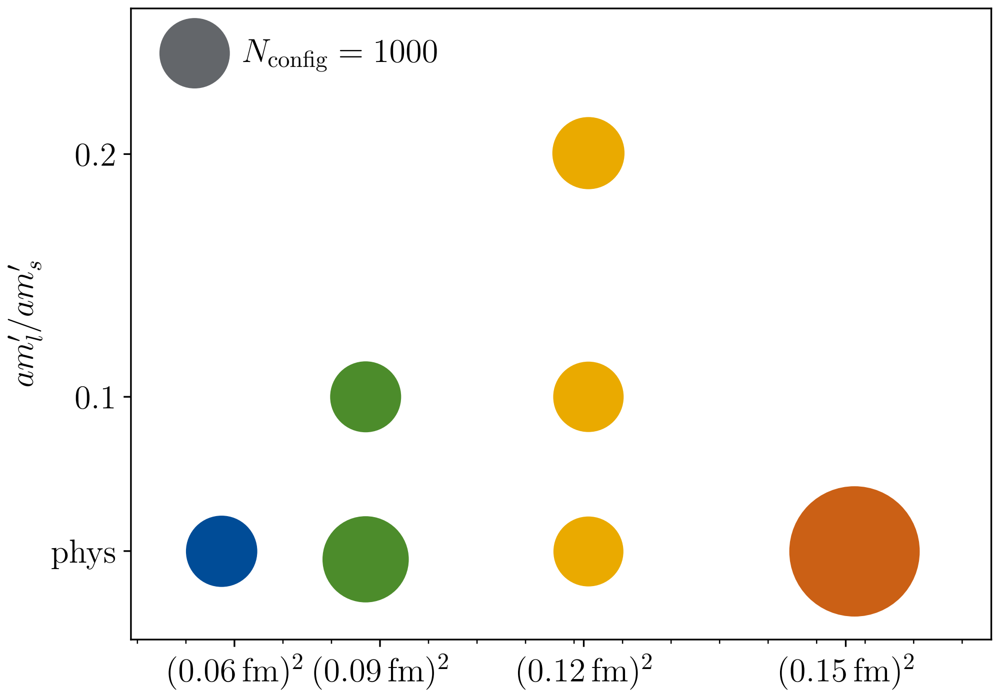

F/M 2019

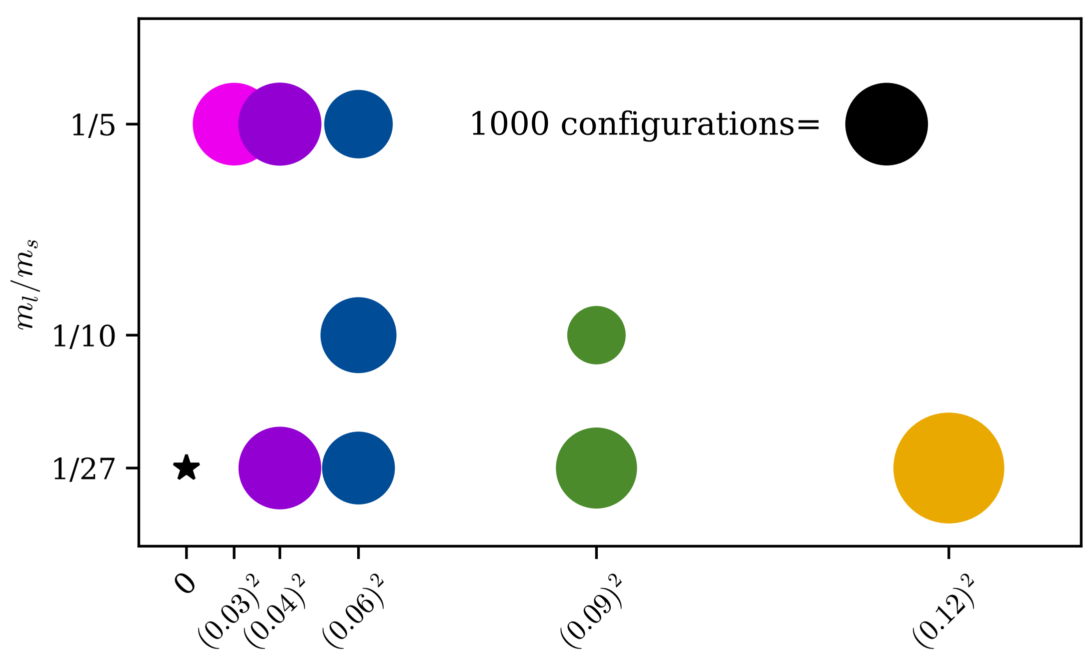


# Upcoming Lattice-QCD Calculations

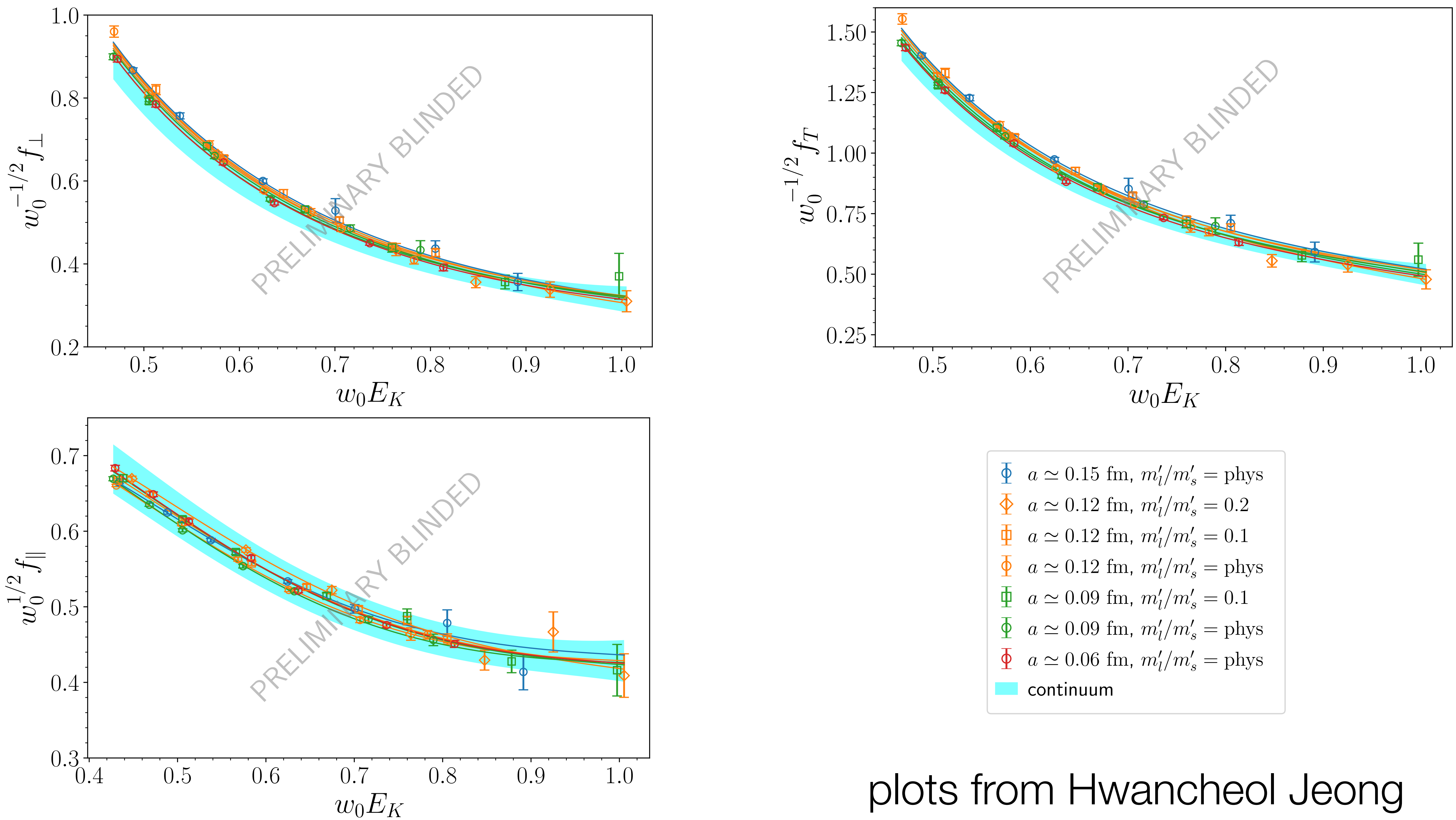
# Fermilab-MILC on 2+1+1-Flavor Ensembles



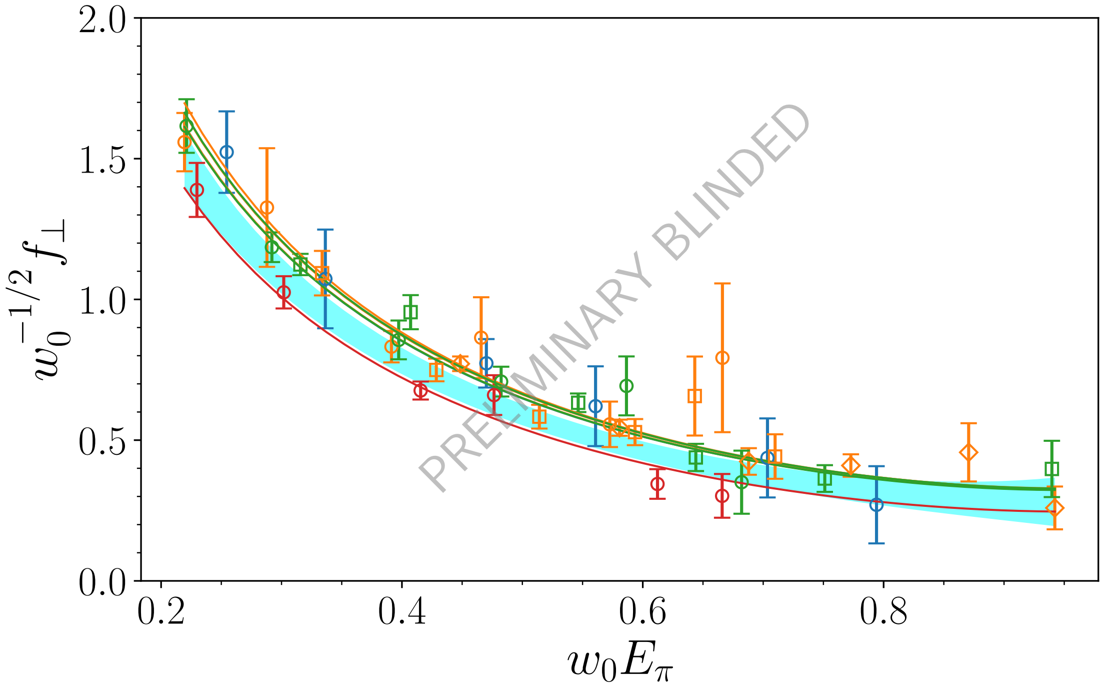

- Two projects underway, the first further along (all data generated.
- Fermilab $b$ quark on 2+1+1 HISQ sea (MILC); seven ensembles at four lattice spacings; smallest $m_l$ = physical.
- HISQ $h$ quark on 2+1+1 HISQ sea (MILC); nine ensembles at five lattice spacings; smallest $m_l$ = physical.
    - At 0.04 and 0.03 fm, $m_h = m_b$.
    - Also $f_0$ from scalar density.

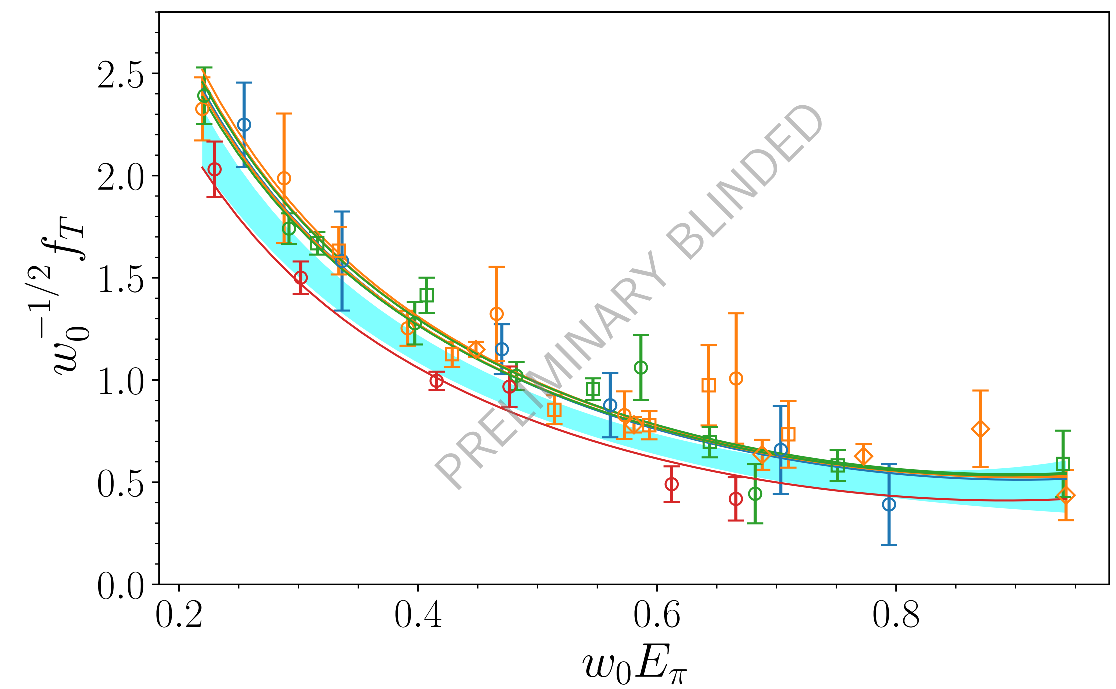
# Preview “Fermilab on HISQ” $B_s \to K l \nu$

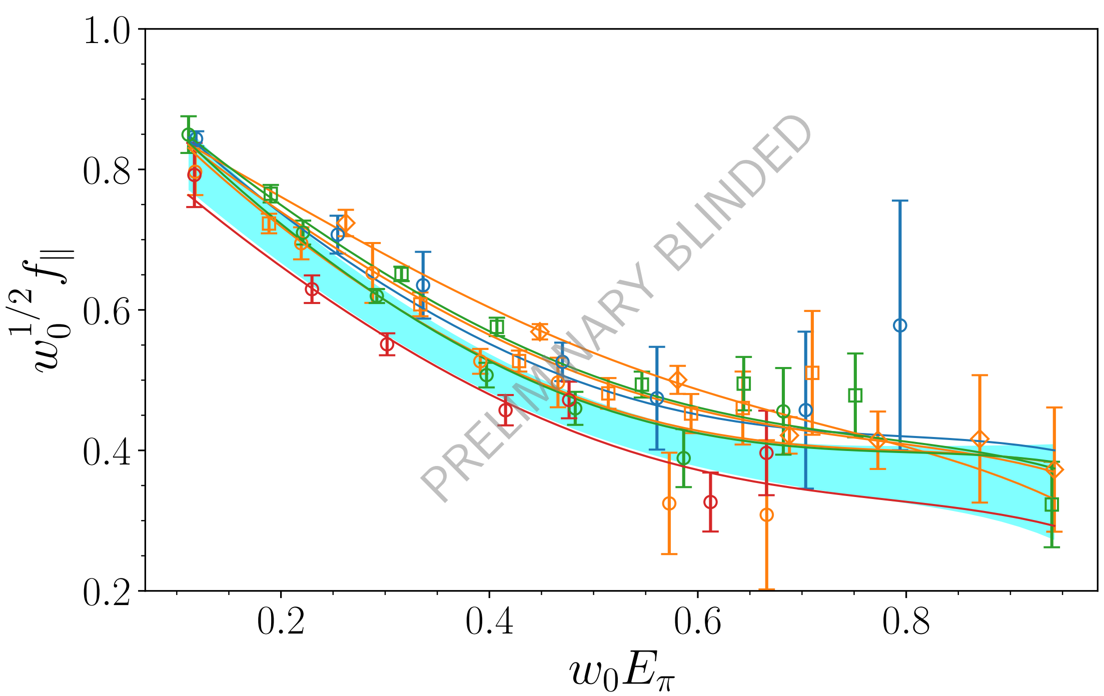


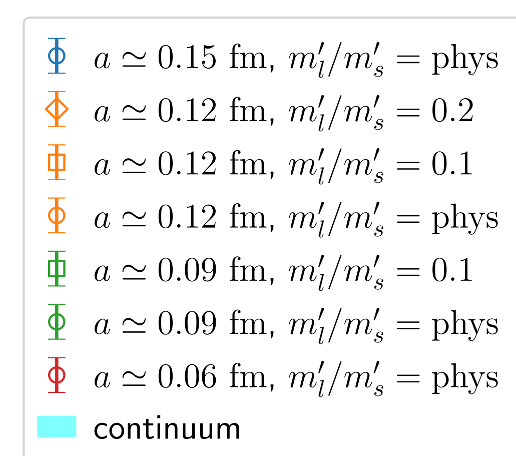

plots from Hwancheol Jeong

# Preview “Fermilab on HISQ” $B \to \pi l \nu$

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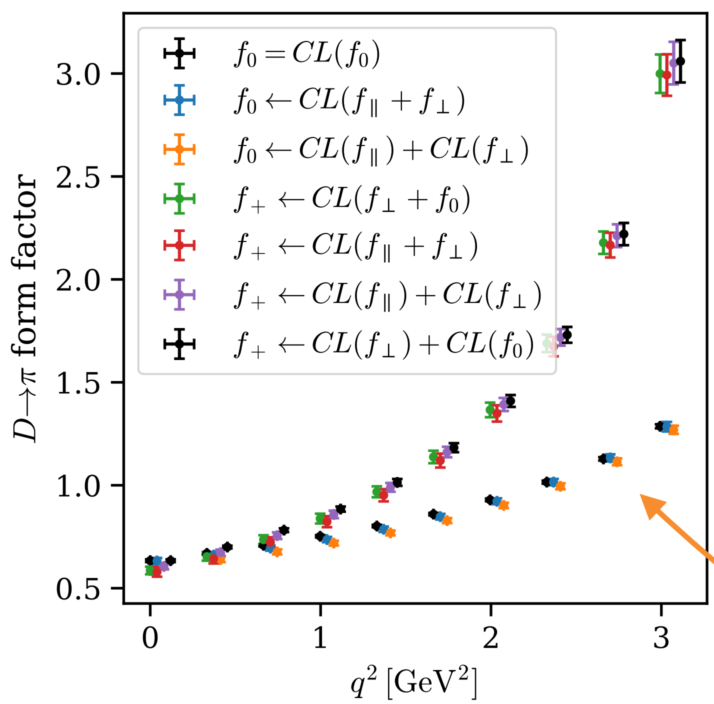
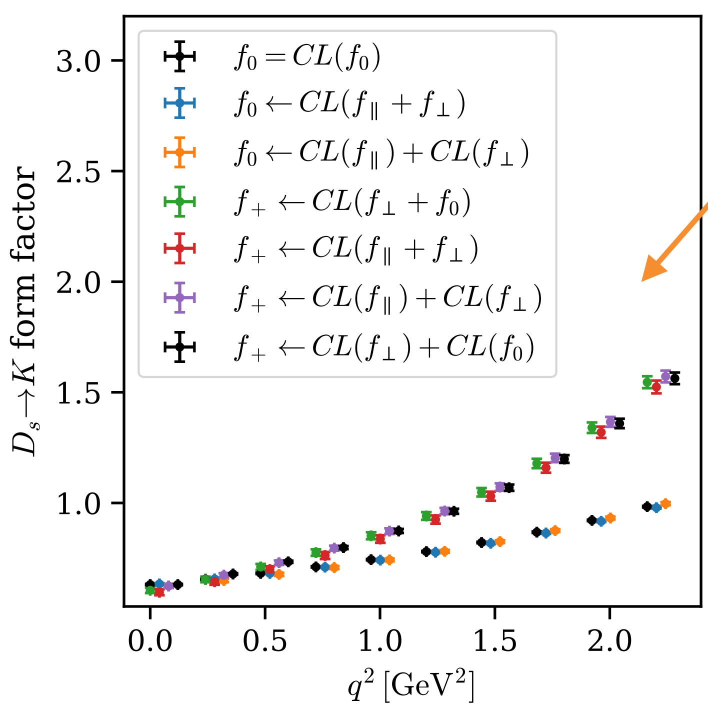
# All-HISQ $D \to \pi l\nu$, $D_s \to K l\nu$, $D \to K l\nu$




plots from
[arXiv:2212.12648](https://arxiv.org/abs/2212.12648)
seven ensembles
(three physical)

same chiral-continuum limit for various ways of doing it

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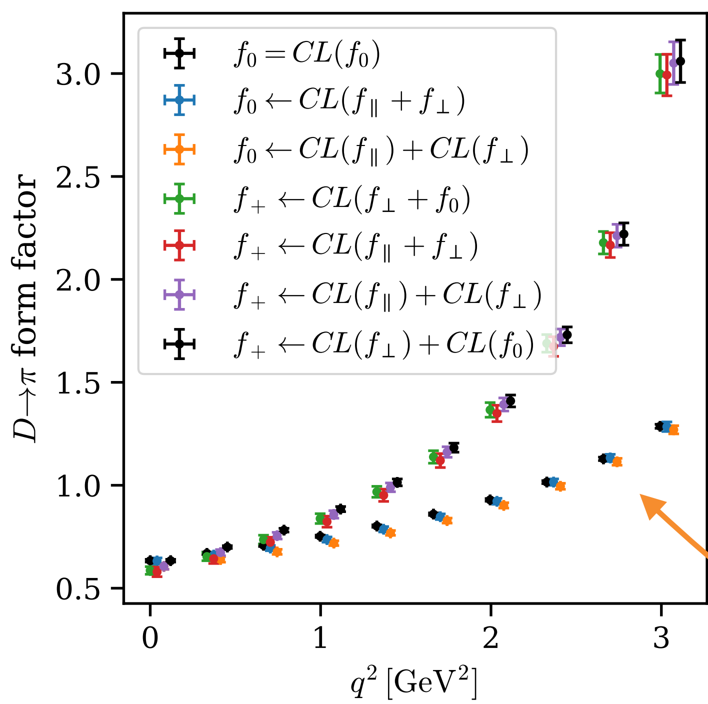


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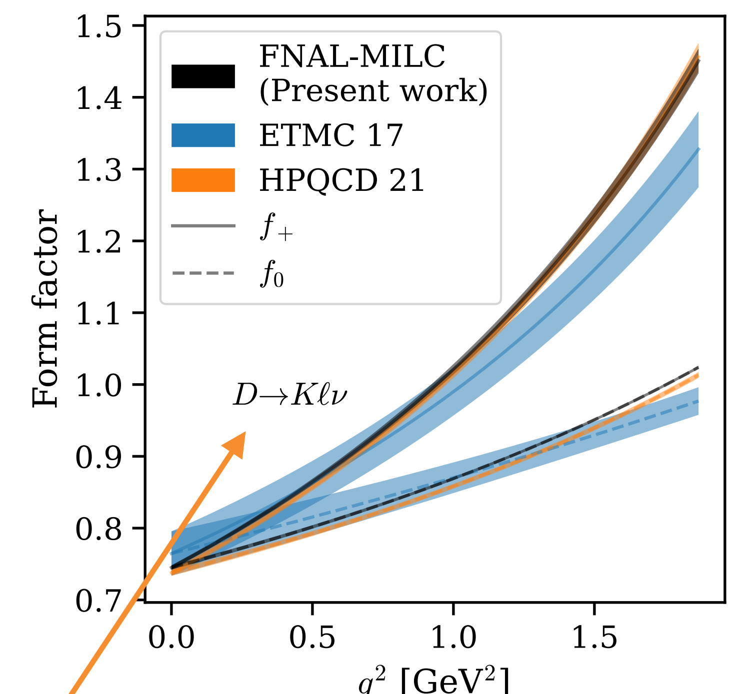


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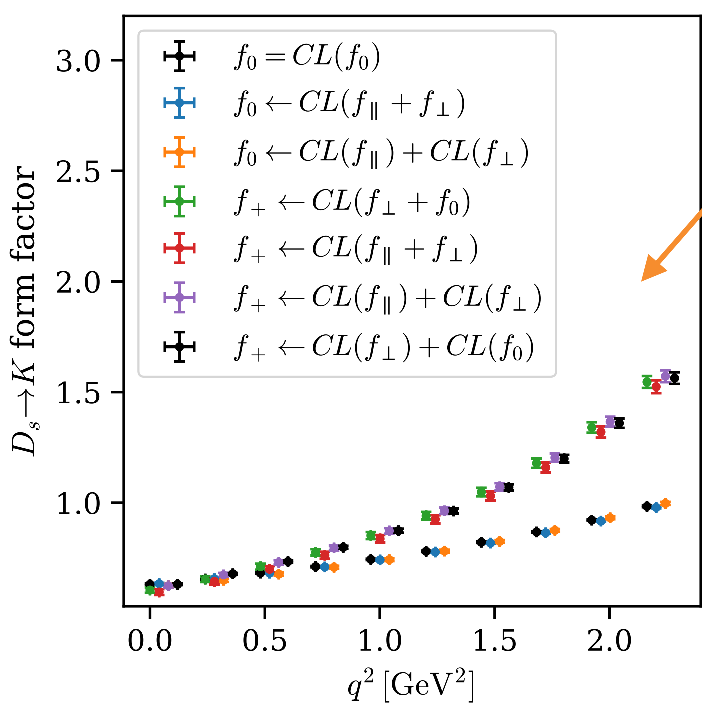


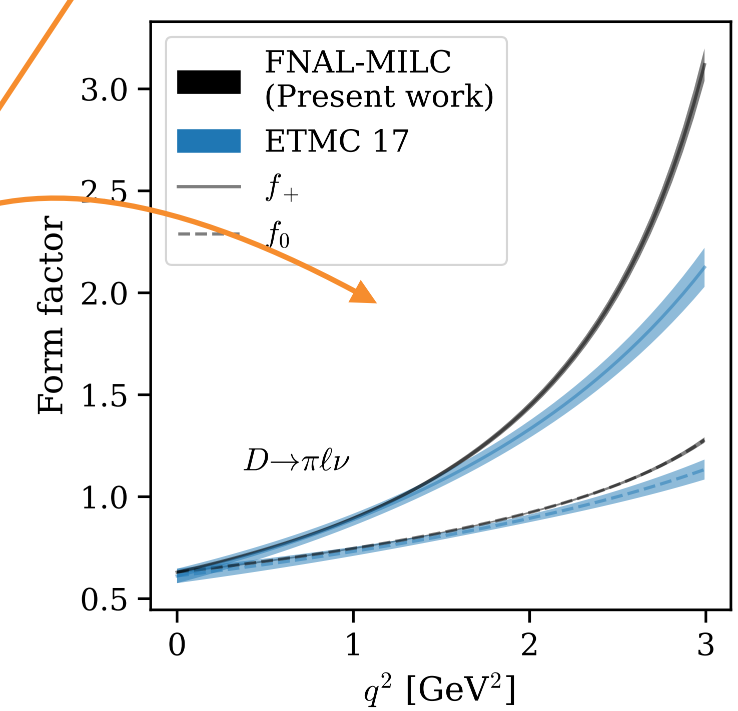

{dis}agreement with HPQCD [[arXiv:2104.09883]]

also all-HISQ

{ETM [[arXiv:1706.03017]]

15 ensembles, $m_l \geq m_s/8$,

2+1+1 tmW}



# Final Remarks

# Assessment

- Current situation unsatisfactory:
  - for $B \to \pi l \nu$, Fermilab-MILC 2015 dataset is superior to others, so (for example) being skeptical about the uncertainties of the earliest work may be warranted.
  - for $B_s \to K l \nu$, RBC/UKQCD 2023 and Fermilab-MILC 2019 datasets are just as comprehensive (though less so than Fermilab-MILC ’15 $B \to \pi$).
- I like the idea of plotting pulls of the points—maybe FLAG (actively working on next big report) can provide this.
- In 1–2 years, there will be further information; in 3–4 more precise info. Unfortunately, I don’t see $a \approx 0.03\text{–}0.04~\mathrm{fm}$ coming from anyone but MILC.