# DISCUSSION ON INCLUSIVE $b \to c$ DECAYS

**Paolo Gambino & Christoph Schwanda**

Challenges in semileptonic B decays, Vienna, 26.09.2024

# $D_{(s)}$ INCLUSIVE SEMILEPTONIC DECAYS


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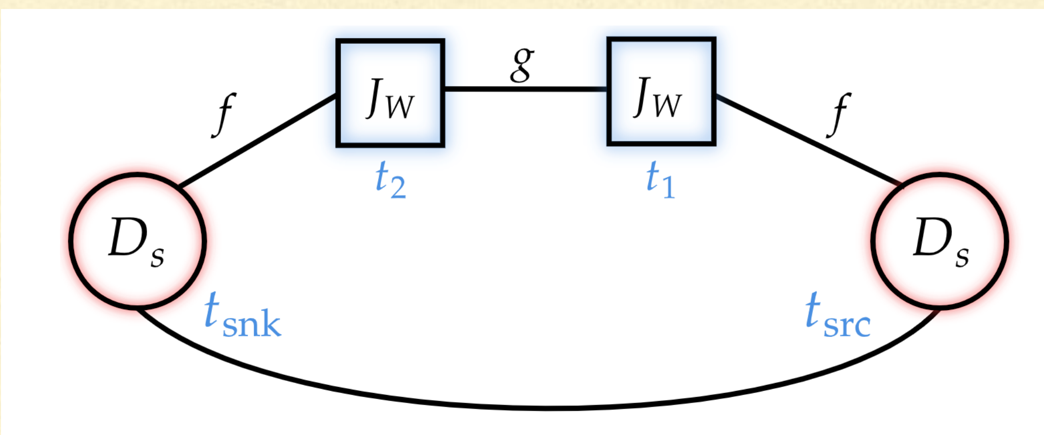
Validation of the GH method on experimental data

First full-fledged study of systematic uncertainties

Multiple lattice spacings and volumes, high statistics

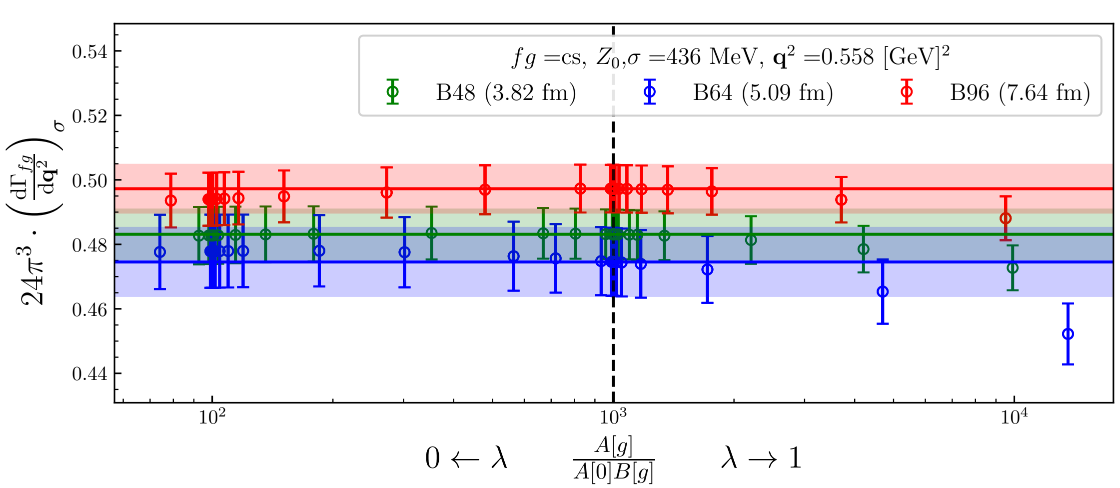
Hansen-Lupo-Tantalo to extract spectral function

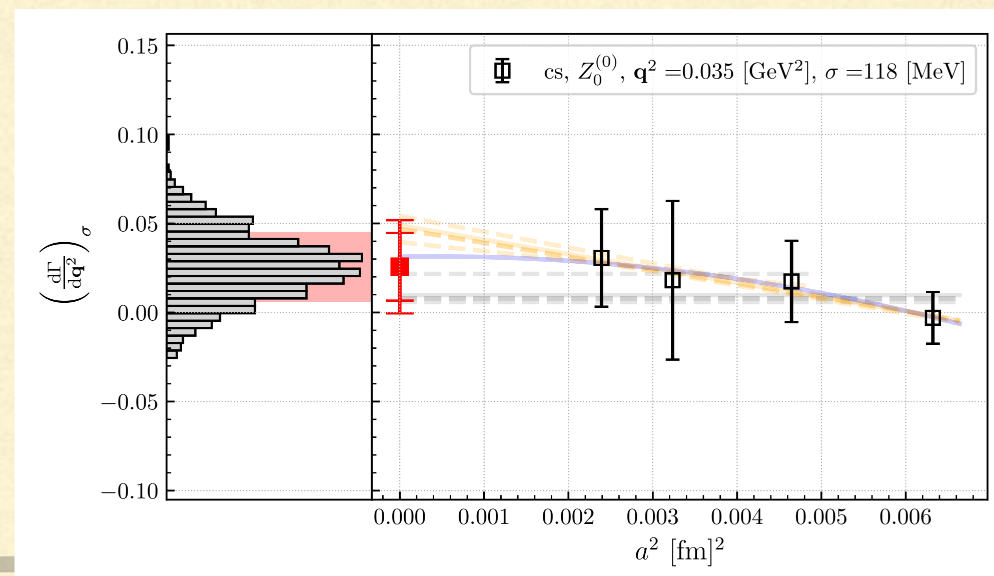



| name | $L$ [fm] | $a$ [fm] | $M_\pi$ [MeV] |
|---|---|---|---|
| B48 | 3.82 | 0.080 | $\approx 135$ |
| B64 | 5.10 | 0.080 | $\approx 135$ |
| B96 | 7.64 | 0.080 | $\approx 135$ |
| C80 | 5.46 | 0.068 | $\approx 135$ |
| D96 | 5.46 | 0.057 | $\approx 135$ |
| E112 | 5.48 | 0.049 | $\approx 135$ |

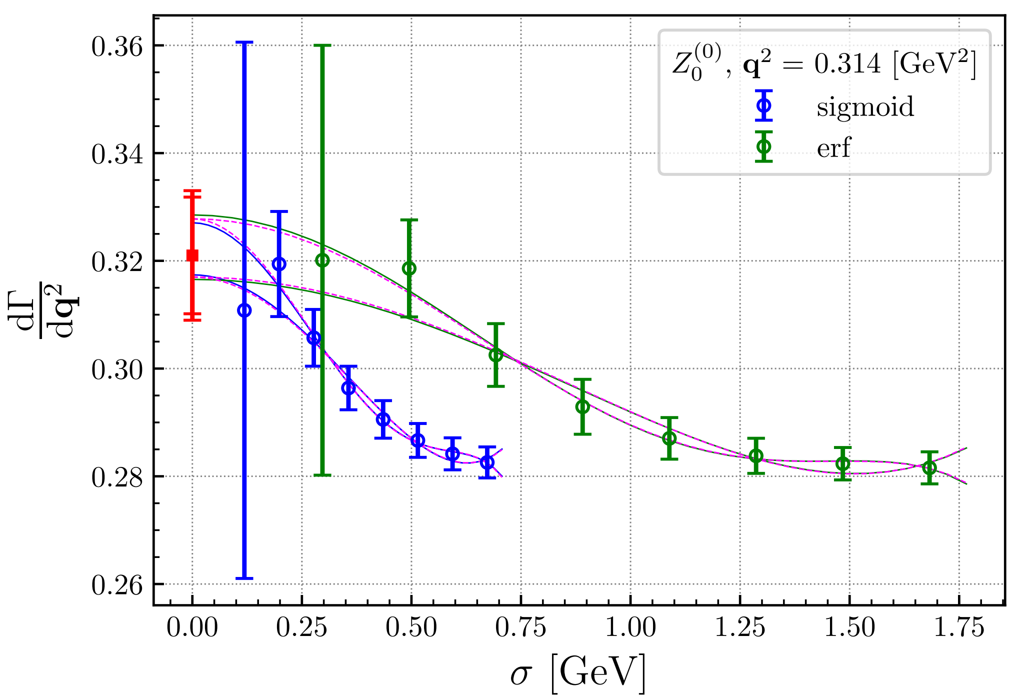
- ETMC-configurations
- $\mathcal{O}(a)$ and clover improved
- $N_f = 2+1+1$
- ten momenta per ensemble
- three decay channels
- two smearing kernels
- $\mathcal{O}(10)$ values of $\sigma$

Flat volume dependence, HLT result stable



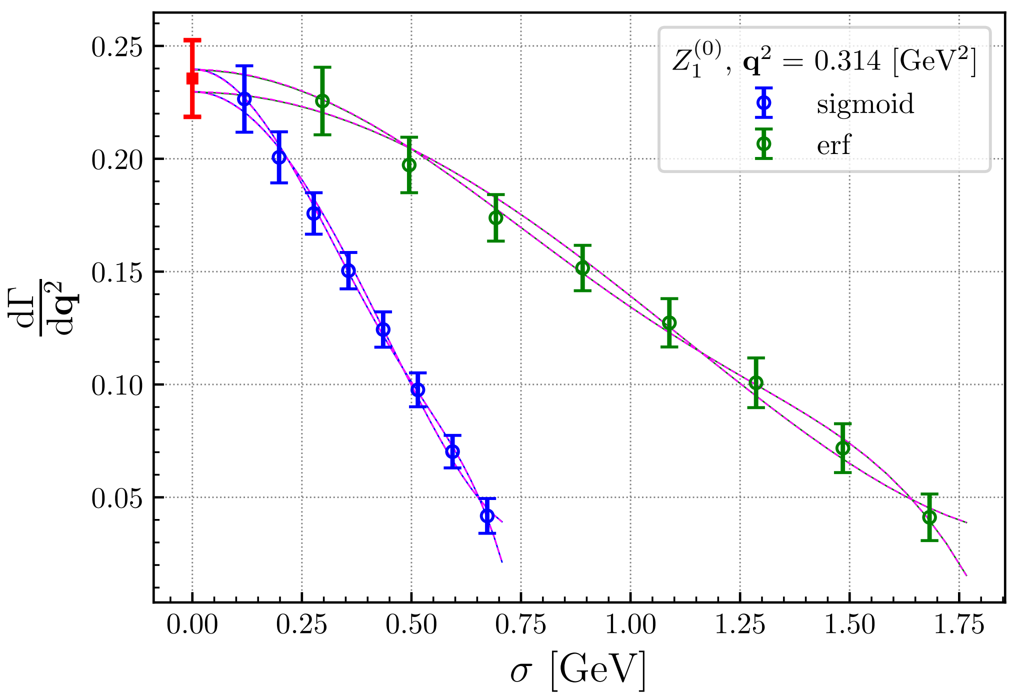
# $D_{(s)}$ INCLUSIVE SEMILEPTONIC DECAYS




- combination of two kernels
- good agreement between kernels
- smooth extrapolations for all contributions
- even powers of $\sigma$

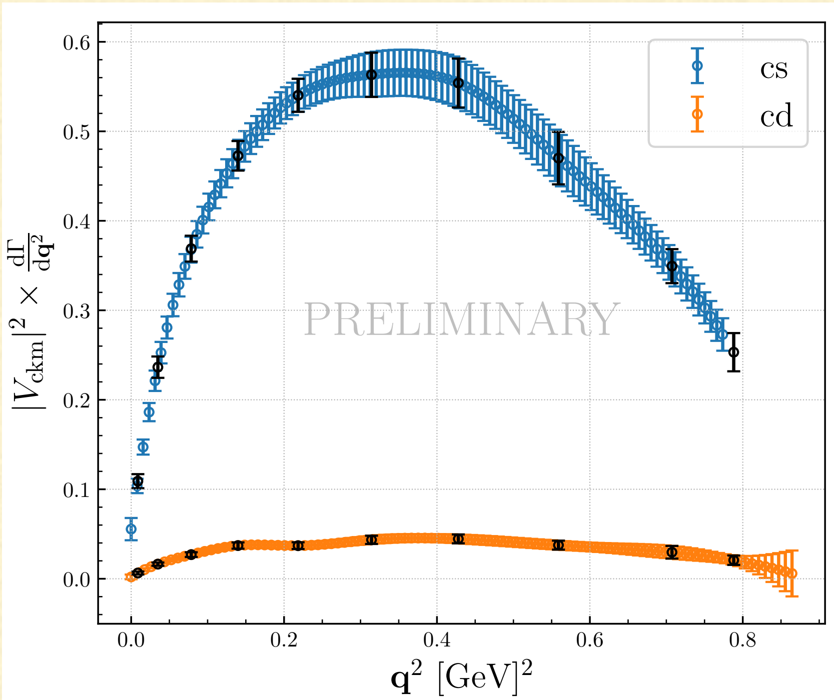
smearing the kinematic theta function

NB we expect a quadratic dependence on $\boldsymbol{\sigma}$



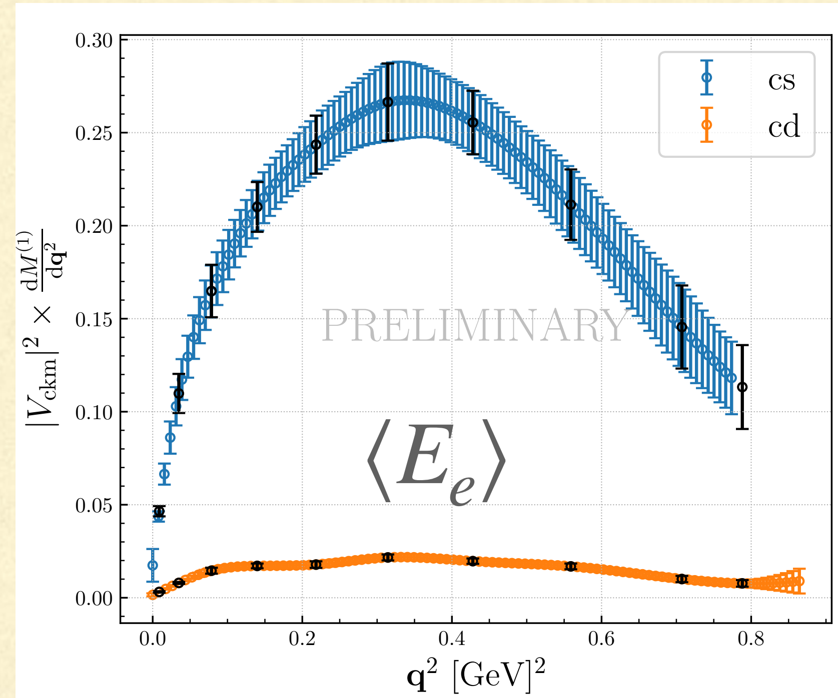

This is after continuum limit but inverting the order leads to compatible results

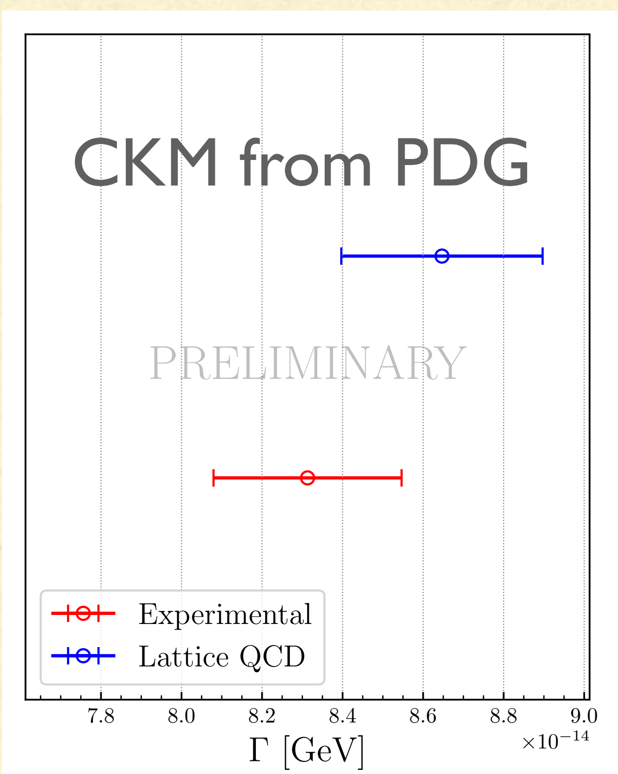
# $D_{(s)}$ INCLUSIVE SEMILEPTONIC DECAYS




Connected diagrams only

Weak Annihilation (disconnected diagrams) in progress

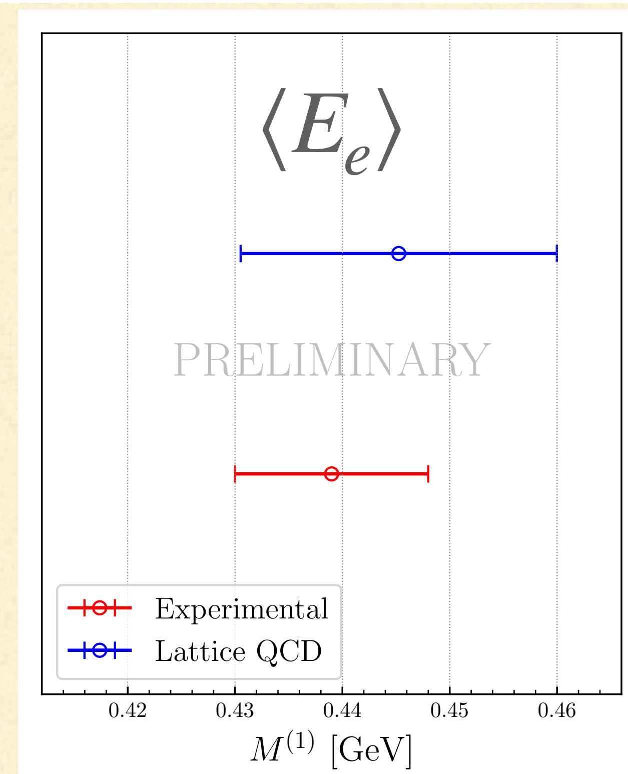



2.5% error on $\Gamma$

3.3% error on $\langle E_e \rangle$

Exp from BES-III



# $|V_{cb}|$ inclusive — experimental points

- In general, there is good consistency between different $E_\ell$, $M_X^2$ and $q^2$ moment measurements
  - Except for the normalization — see next slide
- Correlations
  - Currently experimental correlations between $E_l$, $M_X^2$ und $q^2$ moments at the same experiment are unknown, any impact?
  - It would be desirable to determine $E_l$, $M_X^2$ und $q^2$ moments simultaneously in the same analysis and determine experimental correlations precisely

# Semileptonic branching fraction

BaBar [Phys. Rev. D 95, 072001 (2017)]

*BABAR*-PUB-16/006
SLAC-PUB-16855

**Measurement of the inclusive electron spectrum from $B$ meson decays and determination of $|V_{ub}|$**

Based on the full *BABAR* data sample of 466.5 million $B\overline{B}$ pairs, we present measurements of the electron spectrum from semileptonic $B$ meson decays. We fit the inclusive electron spectrum to distinguish Cabibbo-Kobayashi-Maskawa (CKM) suppressed $B \to X_u e\nu$ decays from the CKM-favored $B \to X_c e\nu$ decays, and from various other backgrounds, and determine the total semileptonic branching fraction $\mathcal{B}(B \to X e\nu) = (10.34 \pm 0.04_{\rm stat} \pm 0.26_{\rm syst})\%$, averaged over $B^{\pm}$ and $B^0$ mesons. We determine the spectrum and branching fraction for charmless $B \to X_u e\nu$ decays and extract the CKM element $|V_{ub}|$, by relying on four different QCD calculations based on the

Gael yesterday

| $m_b^{\rm kin}$ | $\overline{m}_c(2\,\text{GeV})$ | $\mu_\pi^2$ | $\mu_G^2$ | $\rho_D^3$ | $\rho_{LS}^3$ | BR$_{c\ell\nu}$ | $10^3\lvert V_{cb}\rvert$ |
|---|---|---|---|---|---|---|---|
| 4.572 | 1.090 | 0.430 | 0.282 | 0.161 | −0.091 | 10.61 | 41.83 |
| 0.012 | 0.010 | 0.040 | 0.048 | 0.018 | 0.089 | 0.15 | 0.47 |
| 1 | 0.389 | −0.229 | 0.561 | −0.025 | −0.181 | −0.062 | −0.422 |
| | 1 | 0.019 | −0.238 | −0.030 | 0.083 | 0.033 | 0.076 |
| | | 1 | −0.097 | 0.536 | 0.262 | 0.142 | 0.334 |
| | | | 1 | −0.261 | 0.006 | 0.006 | −0.260 |
| | | | | 1 | −0.019 | 0.022 | 0.139 |
| | | | | | 1 | −0.011 | 0.067 |
| | | | | | | 1 | 0.697 |
| | | | | | | | 1 |

- ~1$\sigma$ shift of central value…

# A FEW ADDITIONAL POINTS

- ***Experiment***: more precise measurement of moments and BR at Belle II with correlations between different kinds of moments, clarify Belle II vs Belle discrepancy in $q^2$ moments, new observables ($A_{FB}$, quantities computable on the lattice with optimal uncertainty), improved QED treatment (at least with/without PHOTOS?)

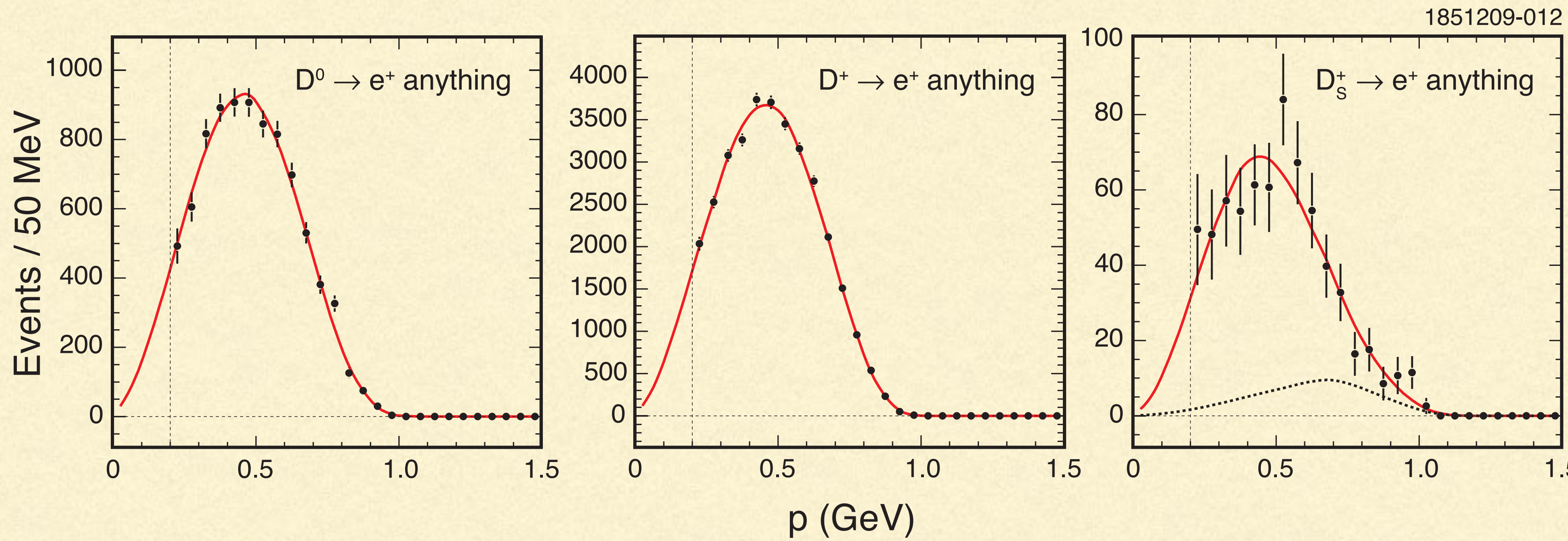
- ***Theory***: analytic (or numerically more accurate) calculation of $O(\alpha_s^2)$ corrections to lept and hadr moments, $O(\alpha_s \rho_D^3/m_b^3)$ to lept and hadr moments, QED effects in $q^2$ and hadronic moments, reasonable uncertainties and their correlations…

- ***Interplay with lattice calculations***: in the mid term look for complementarity with exp data, and new directions in parameters space (lattice as virtual lab: new observables, V, A, S, P currents,…)

BACK-UP

# $D_{(s)}$ INCLUSIVE SEMILEPTONIC DECAYS

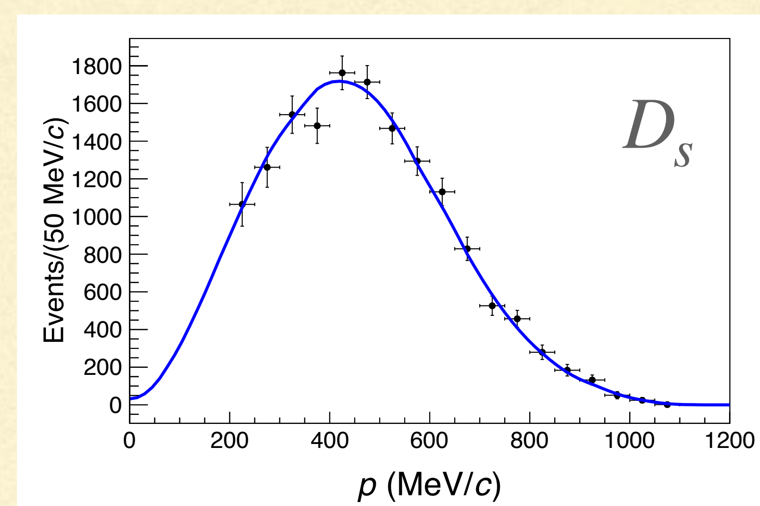
*Cleo-c*
*0912.4232*




*$D_{(s)}$ decay offer the opportunity to* ***validate the method on experimental data.***

*Easiest to extrapolate the exp spectrum*
*then compute total width (0.5% uncertainty)*
*mean $\langle E_e \rangle$ (0.5%) and variance (1%) for $D^+$,*
*with $D_s$ a bit less precise*

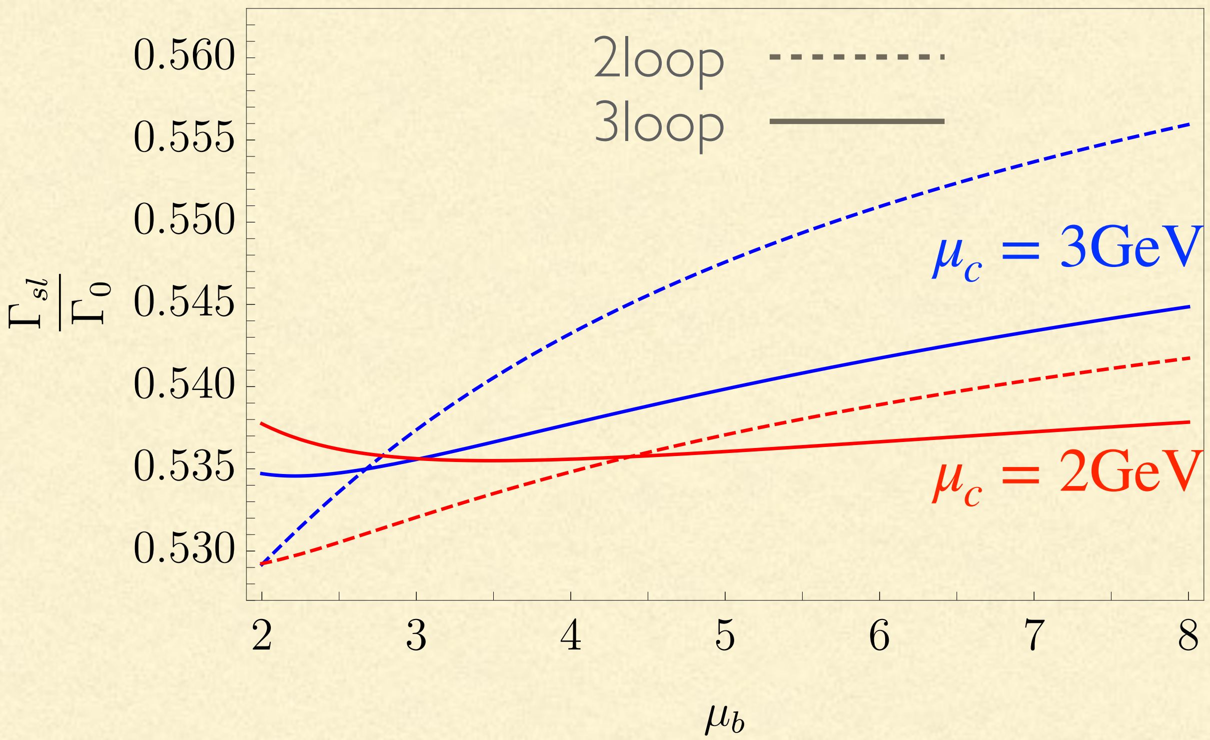
*slow convergence (?) of HQE in charm decays*

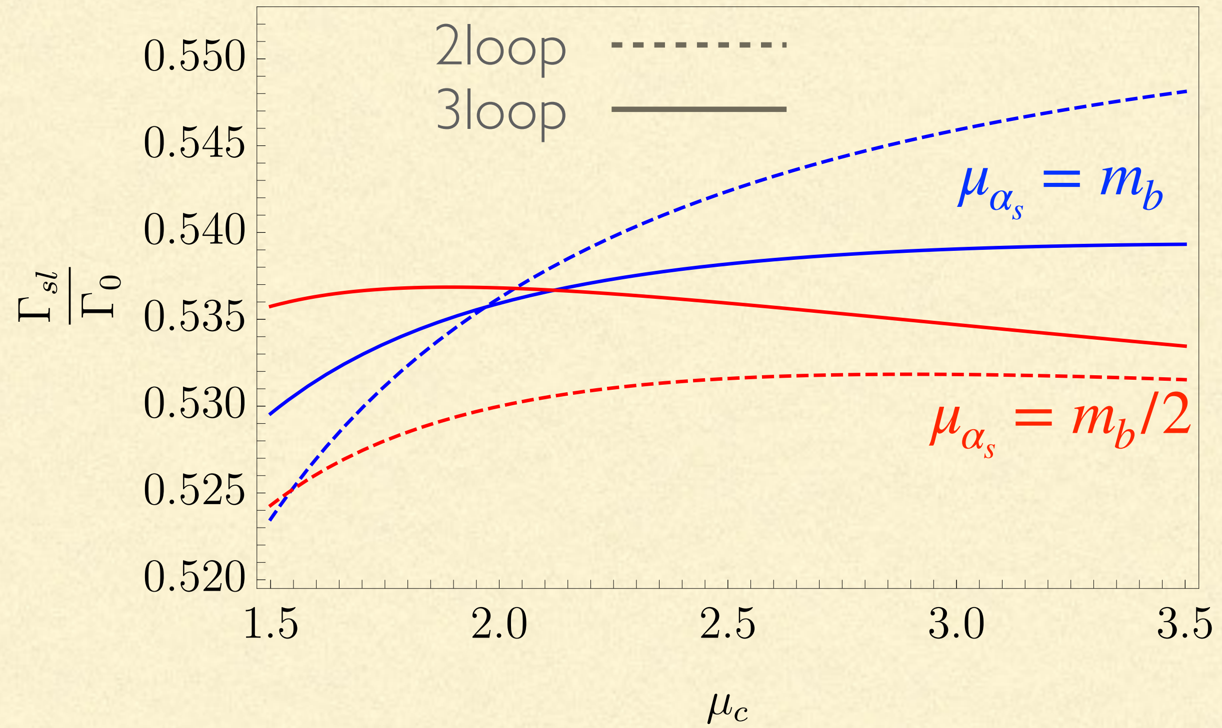



*BES III*
*2104.07311*

# RESIDUAL UNCERTAINTY on $\Gamma_{sl}$

Bordone, Capdevila, PG, 2107.00604




Similar reduction in $\mu_{kin}$ dependence. Purely perturbative uncertainty $\pm 0.7\,\%$ (max spread), central values at $\mu_c = 2\mathrm{GeV}, \mu_{\alpha_s} = m_b/2$.

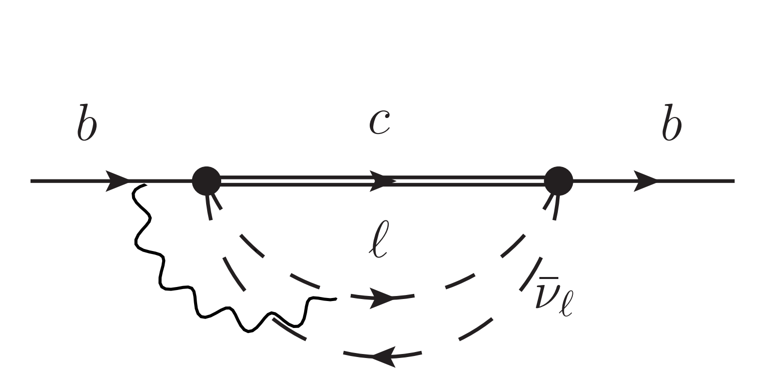
$O(\alpha_s/m_b^2, \alpha_s/m_b^3)$ effects in the width are known. Additional uncertainty from higher power corrections, soft charm effects of $O(\alpha_s/m_b^3 m_c)$, duality violation.

***Conservatively: 1.2% overall theory uncertainty in $\Gamma_{sl}$ (a ~50% reduction)***

Interplay with fit to semileptonic moments, known only to $O(\alpha_s^2, \alpha_s\Lambda^2/m_b^2)$

# QED CORRECTIONS

Bigi, Bordone, Haisch, Piccione PG
2309.02849

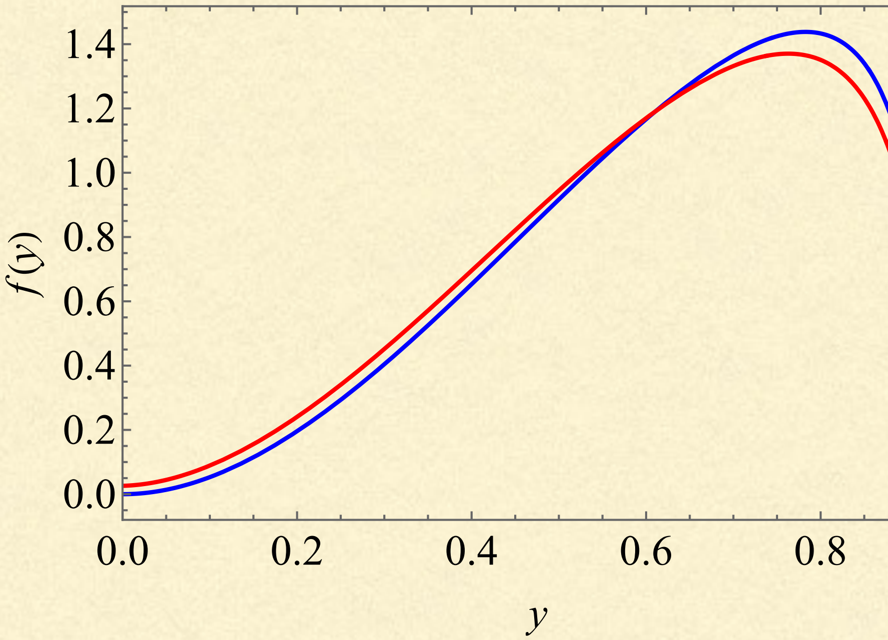



In the presence of photons, ***OPE valid only for total width*** and moments that do not resolve charged lepton or hadron properties ($E_\ell, q^2, E_X \dots$). Expect mass singularities and $O(\alpha\Lambda/m_b)$ corrections.

***Leading logs*** $\alpha \ln m_e/m_b$ can be easily computed for simple observables using structure function approach, for ex the lepton energy spectrum

$$\left(\frac{d\Gamma}{dy}\right)^{(1)} = \frac{\alpha}{2\pi} \ln \frac{m_b^2}{m_\ell^2} \int_y^1 \frac{dx}{x} P_{\ell\ell}^{(0)}\left(\frac{y}{x}\right) \left(\frac{d\Gamma}{dx}\right)^{(0)}$$

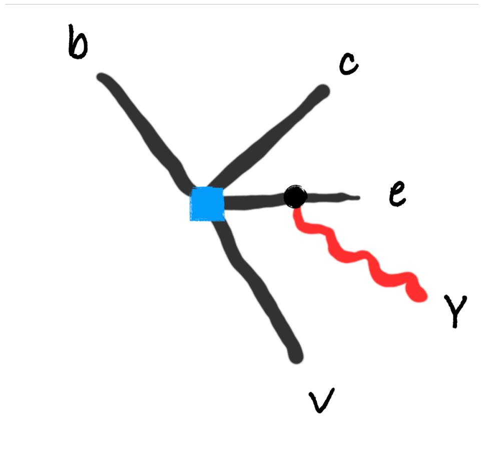
$$P_{\ell\ell}^{(0)}(z) = \left[\frac{1+z^2}{1-z}\right]_+$$




Electron energy spectrum

# QED Leading contributions

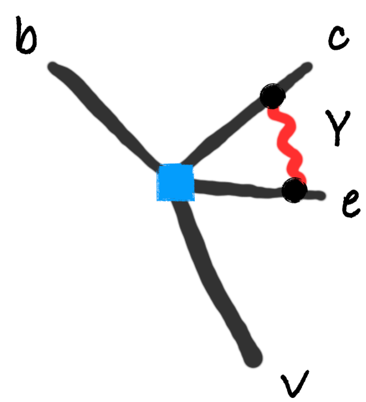
1. **Collinear logs**: captured by splitting functions




also at subleading power!

$$\sim \frac{\alpha_e}{\pi} \log \frac{m_b^2}{m_e^2}$$

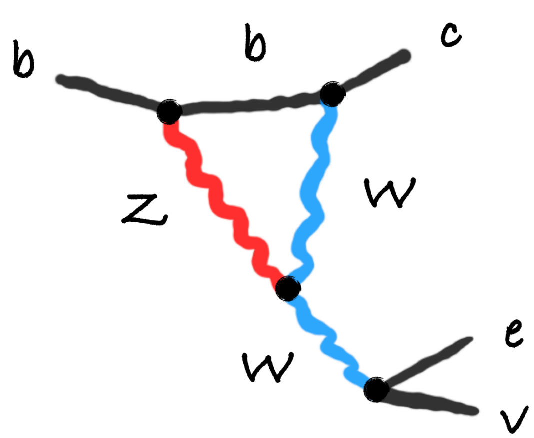
2. **Threshold effects** or Coulomb terms




$$\sim \frac{4\pi\alpha_e}{9}$$

discontinuity at threshold

3. **Wilson Coefficient**



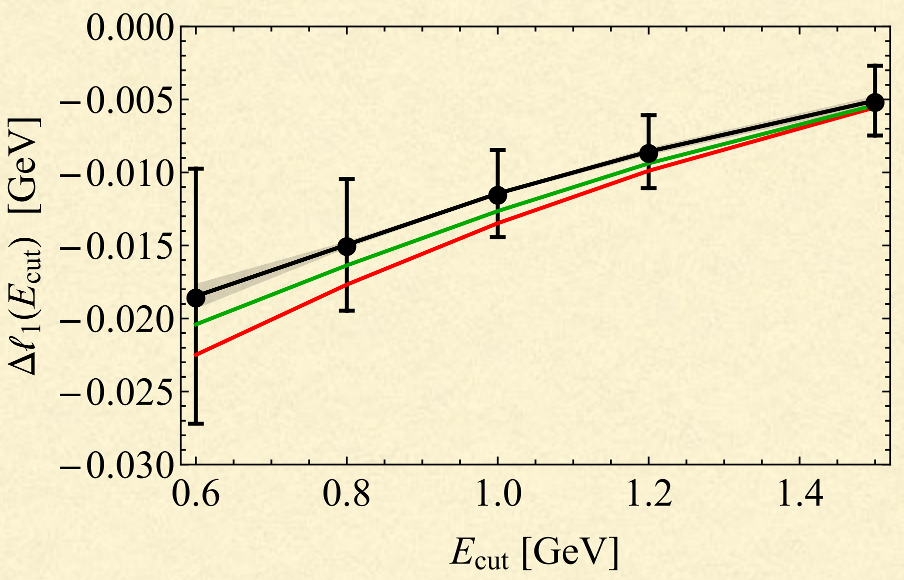

$$\sim \frac{\alpha_e}{\pi}\left[\ln \frac{M_Z^2}{\mu^2} - \frac{11}{6}\right]$$

# COMPLETE $O(\alpha)$ EFFECTS IN LEPTONIC SPECTRUM

Typical measurements are completely inclusive, $B \to X_c \ell \nu(\gamma)$, but QED radiation is ***subtracted*** by experiments using ***PHOTOS*** (soft-collinear photon radiation to MC final states).

Small but non-negligible differences with *PHOTOS* in BaBar leptonic moments hep-ex/0403030

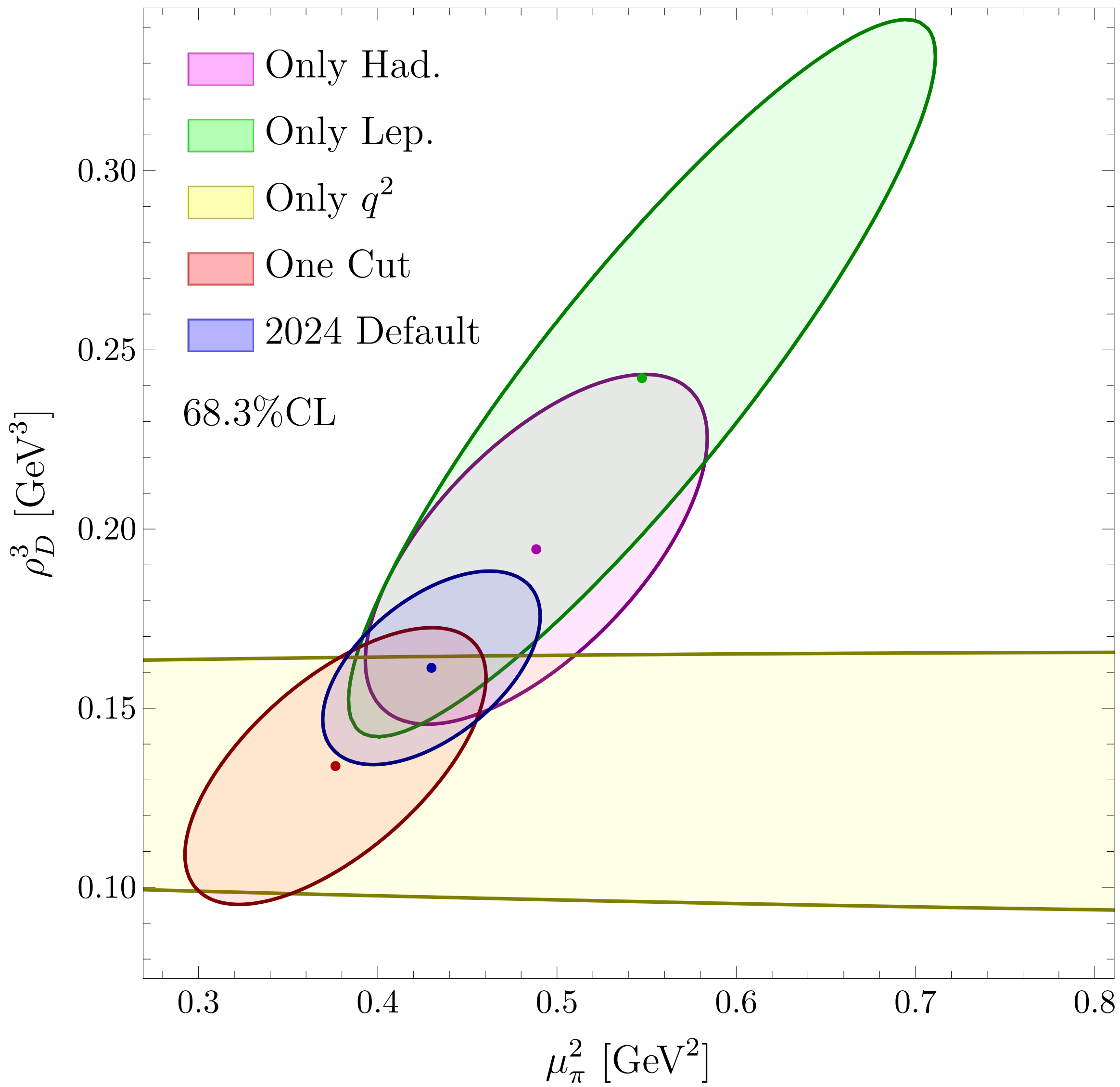
| $E_{\rm cut}$ | $\delta{\rm BR}_{\rm incl}^{\rm BaBar}$ | $\delta{\rm BR}_{\rm incl}^{\rm LL}$ | $\delta{\rm BR}_{\rm incl}^{\rm NLL}$ | $\delta{\rm BR}_{\rm incl}^{\alpha}$ | $\delta{\rm BR}_{\rm incl}^{1/m_b^2}$ | $\delta{\rm BR}_{\rm incl}$ | $\sigma$ |
|---|---|---|---|---|---|---|---|
| 0.6 | −1.26% | −1.92% | −1.95% | −0.54% | −0.50% | −0.45% | +0.34 |
| 0.8 | −1.87% | −2.88% | −2.91% | −1.36% | −1.29% | −1.22% | +0.30 |
| 1.0 | −2.66% | −4.03% | −4.04% | −2.38% | −2.26% | −2.15% | +0.25 |
| 1.2 | −3.56% | −5.43% | −5.41% | −3.65% | −3.43% | −3.27% | +0.14 |
| 1.5 | −5.22% | −8.41% | −8.26% | −6.37% | −5.73% | −5.39% | −0.09 |

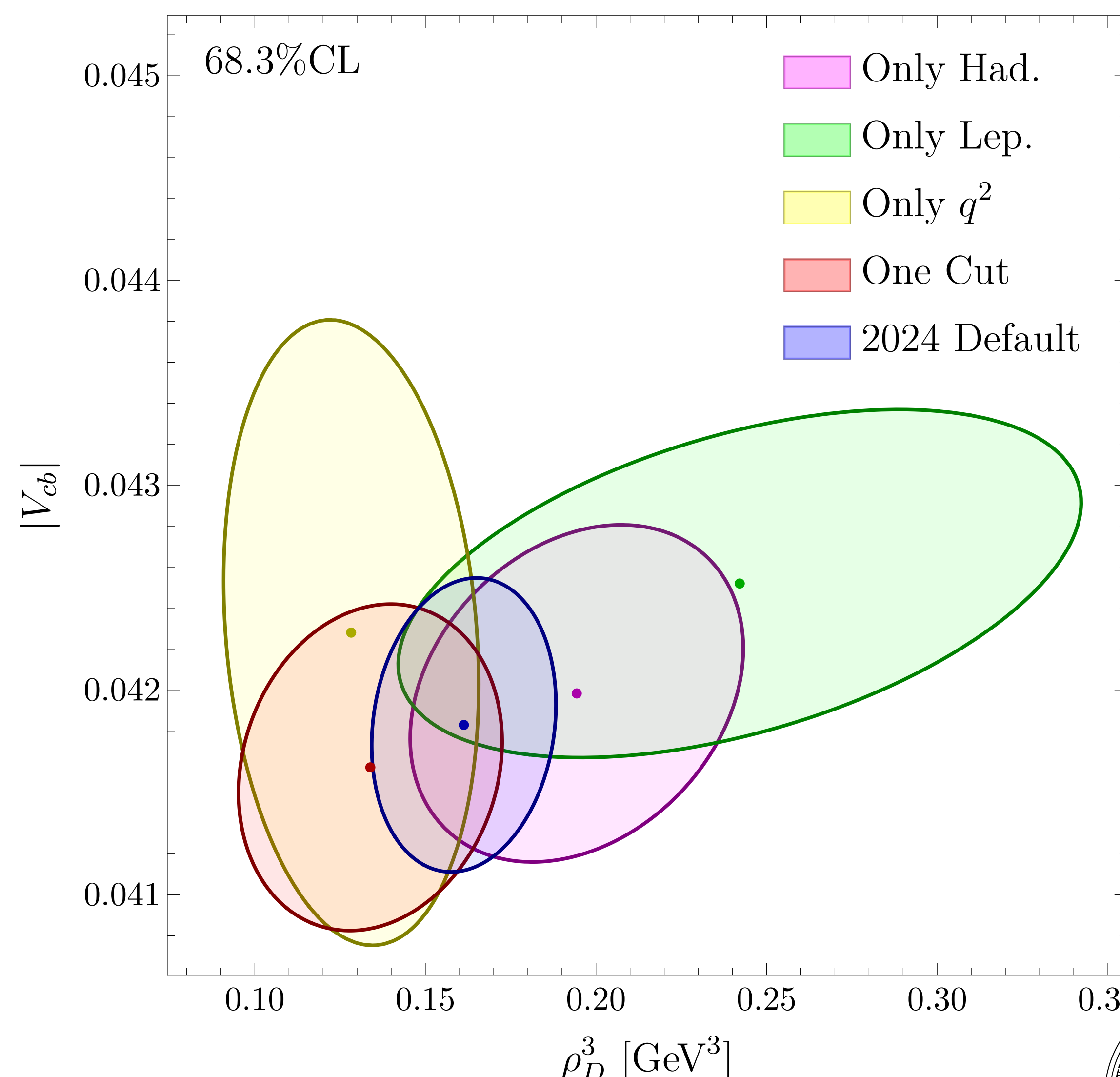



The black curve corresponds to the correction obtained by BaBar using PHOTOS, while the red (green) curve corresponds to our QED prediction including the LL terms (all QED corrections). The grey band represents the systematic uncertainty on the PHOTOS bremsstrahlungs corrections that BaBar quotes, while the black error bars correspond to the total uncertainties of the QED corrected BaBar results.

## ~0.2% reduction in $V_{cb}$

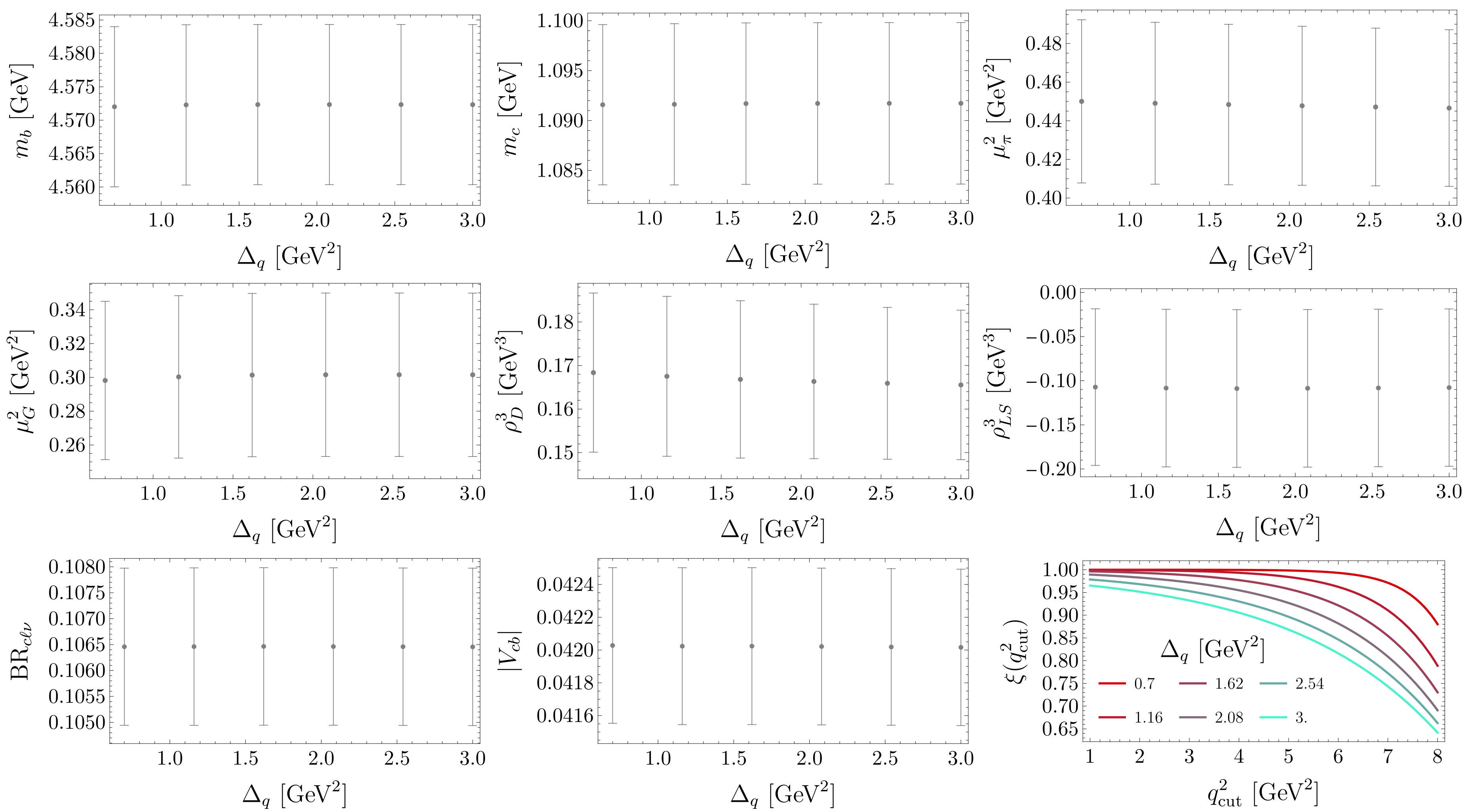
# Fit Results (PRELIMINARY)




Theory correlations are no longer an issue (IMHO)

# Theoretical Correlations




Correlations between different central moments set to 0

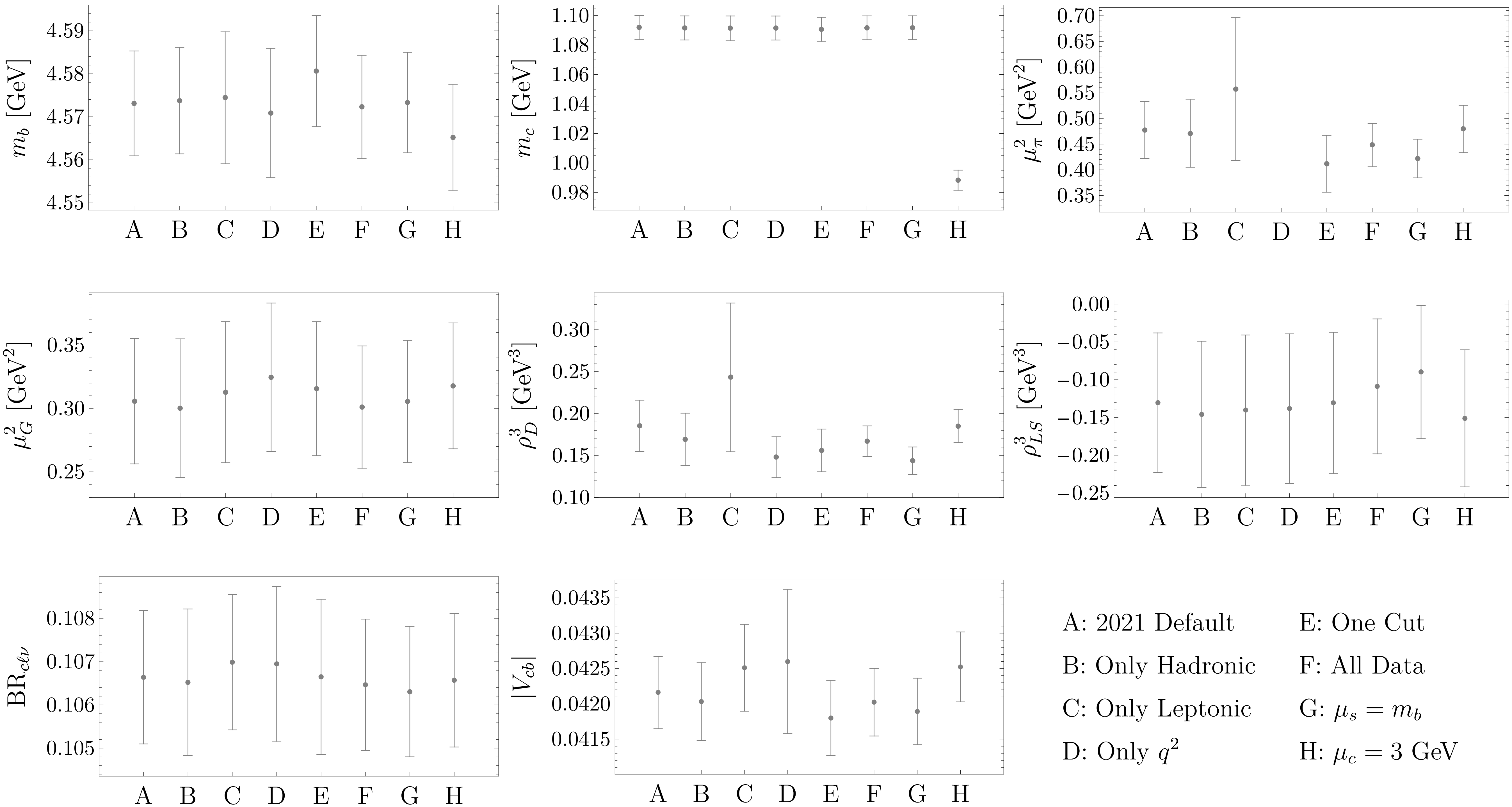
Correlations between same moments at 0.5 GeV$^2$ distance in $q^2_{\rm cut}$:

$$\xi(q^2_{\rm cut}) = 1 - \frac{1}{2} e^{-\frac{9\text{GeV}^2 - q^2_{\rm cut}}{\Delta_q}}$$

$q^2_{\rm cut}$ dependent to take into account spectrum endpoint

this concerns $q^2$ moments only…

Finauri, PG 2310.20324



# HIGHER ORDER CORRECTIONS TO MOMENTS

- complete $O(\alpha_s)$ corrections to triple differential
  Aquila, Ridolfi, PG, Trott, Czarnecki, Jezabek, Kuhn, …

- complete $O(\alpha_s^2)$ corrections to leptonic, hadronic (*partly numerical*), $q^2$ moments at arbitrary cuts
  Biswas, Melnikov, Czarnecki, Pak, PG, Fael, Herren

- $O(\alpha_s^3)$ corrections to leptonic, hadronic, $q^2$ moments *without cuts*
  Fael, Schoenwald, Steinhauser

- complete $O(\alpha_s \Lambda^2/m_b^2)$ corrections to triple differential, $O(\alpha_s \Lambda^3/m_b^3)$ to width and $q^2$ moments
  Alberti, Healey, Nandi, PG, Becher, Lunghi, Mannel, Moreno, Pivovarov

- power corrections of $O(\Lambda^2/m_b^2)$ and $O(\Lambda^3/m_b^3)$ to triple differential, $O(\Lambda^4/m_b^4)$ and $O(\Lambda^5/m^5)$ for moments
  Manohar, Wise, Blok, Koyrakh, Shifman, Vainshtein, Grimm, Kapustin, Mannel, Turzcyk, Uraltsev, Milutin, Vos

# HIGHER POWER CORRECTIONS

Proliferation of non-pert parameters starting 1/m$^4$: 9 at dim 7, 18 at dim 8

In principle relevant: HQE contains $O(1/m_b^n 1/m_c^k)$

Mannel,Turczyk,Uraltsev 1009.4622

**Lowest Lying State Saturation Approx (LLSA)** truncating

$$\langle B|O_1 O_2|B\rangle = \sum_n \langle B|O_1|n\rangle\langle n|O_2|B\rangle$$

see also Heinonen,Mannel 1407.4384

and relating higher dimensional to lower dimensional matrix elements, e.g.

$$\rho_D^3 = \epsilon\,\mu_\pi^2 \qquad \rho_{LS}^3 = -\epsilon\,\mu_G^2 \qquad \epsilon \sim 0.4\text{GeV}$$

$\epsilon$ excitation energy to P-wave states. LLSA might set the scale of effect, but large corrections to LLSA have been found in some cases 1206.2296

We use LLSA as loose constraint or priors (60% gaussian uncertainty, dimensional estimate for vanishing matrix elements) in a fit including higher powers.

still without $q^2$ moments!

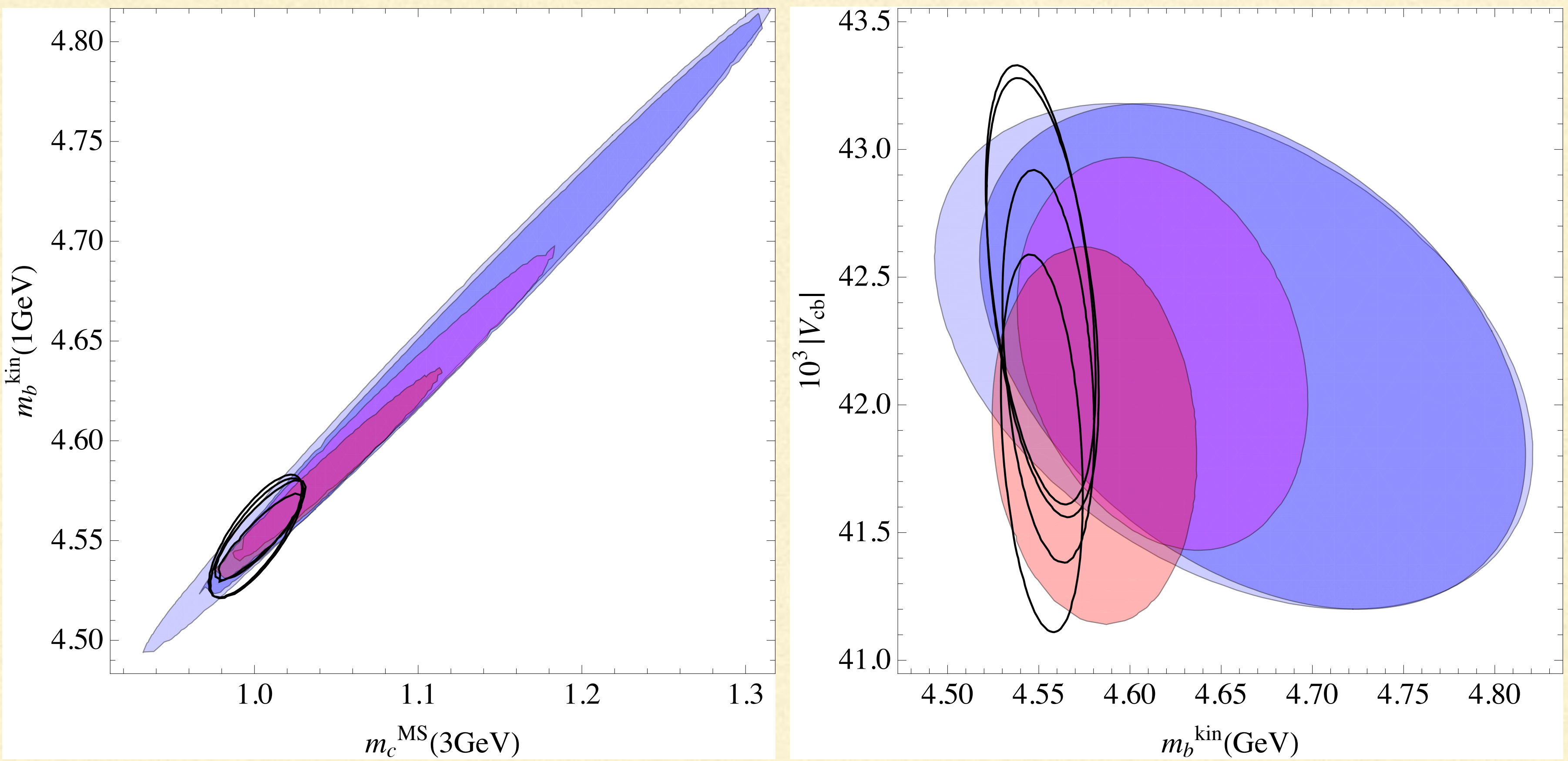
$$|V_{cb}| = 42.00(53) \times 10^{-3}$$

Bordone, Capdevila, PG, 2107.00604

***Update of 1606.06174***

# HEAVY QUARK MASSES AND THEORETICAL CORRELATIONS

C. Schwanda, PG 2013



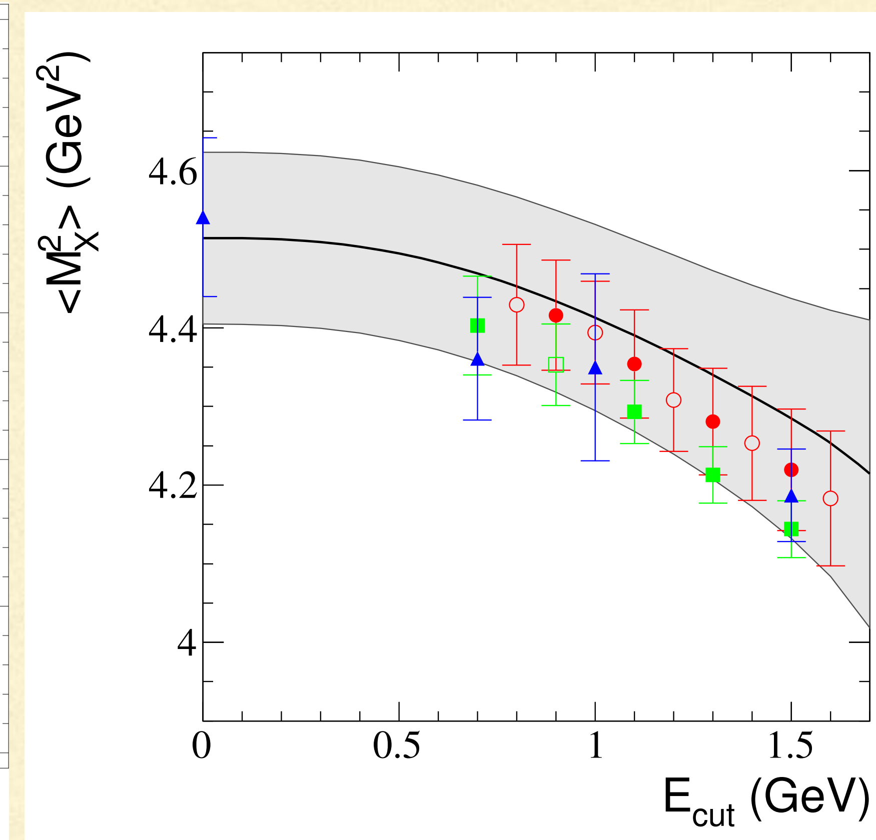




Fig. 3. Two-dimensional projections of the fits performed with different assumptions for the theoretical correlations. The orange, magenta, blue, light blue 1-sigma regions correspond to the four scenarios considered in [58]. The black contours show the same regions when the $m_c$ constraint of Ref. [59] is employed.