Winter Workshop Nuclear Dynamics — 2024

Dynamical fluctuations in Heavy Ion Collisions

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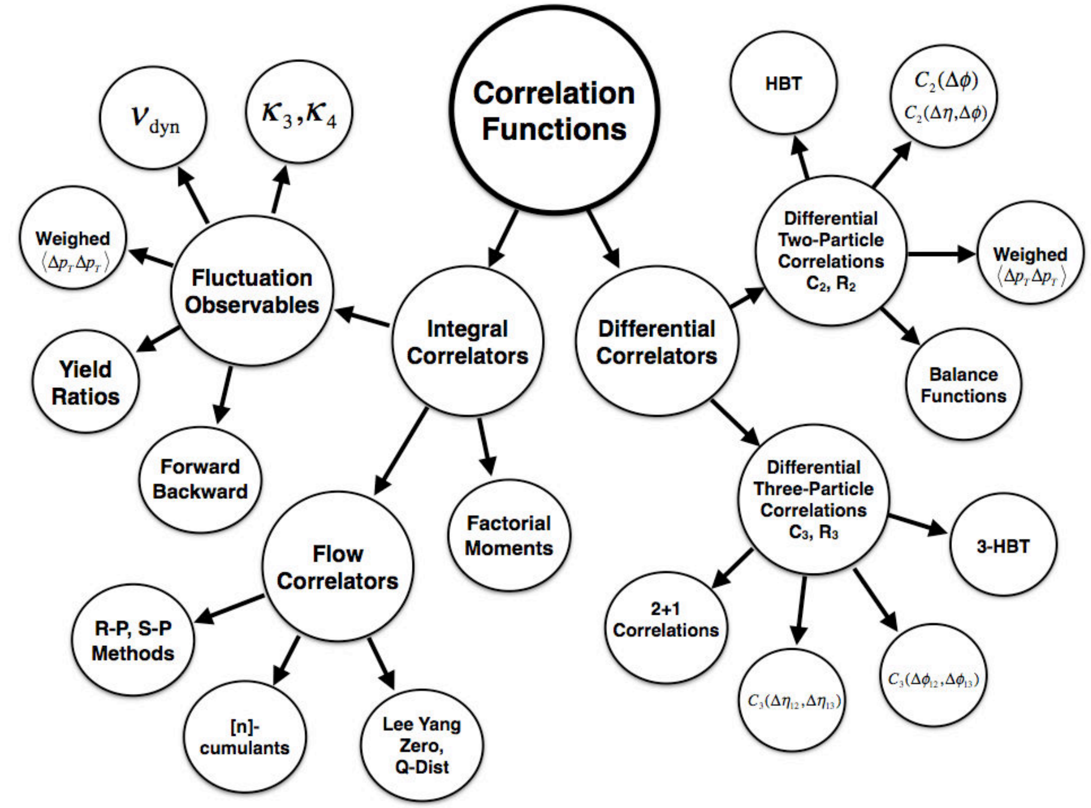
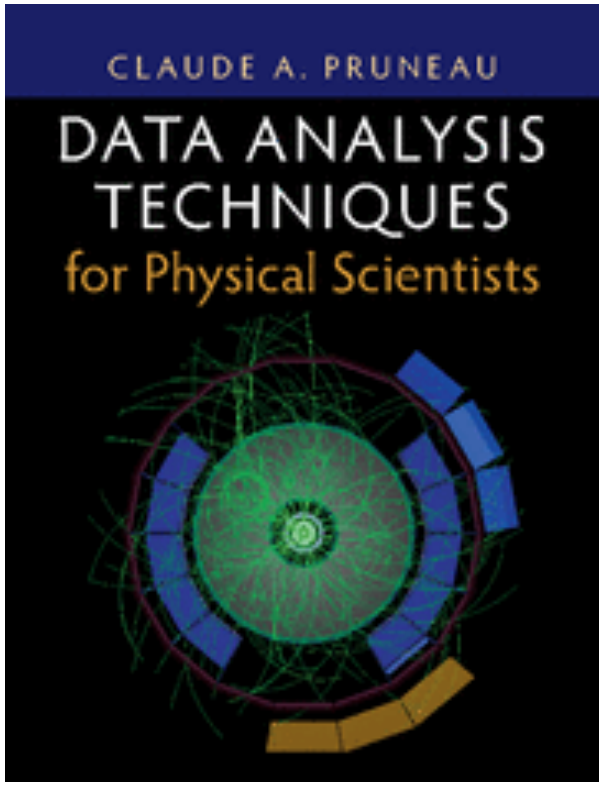
Outline

- Why measure fluctuations?
- Selected Results
  - Net Charge,
  - Net Baryon
  - Cross Species Relative Yield Fluctuations
  - Kaon Isospin fluctuations (Search for Strange DCCs)
- Summary

Fluctuations vs. Correlations

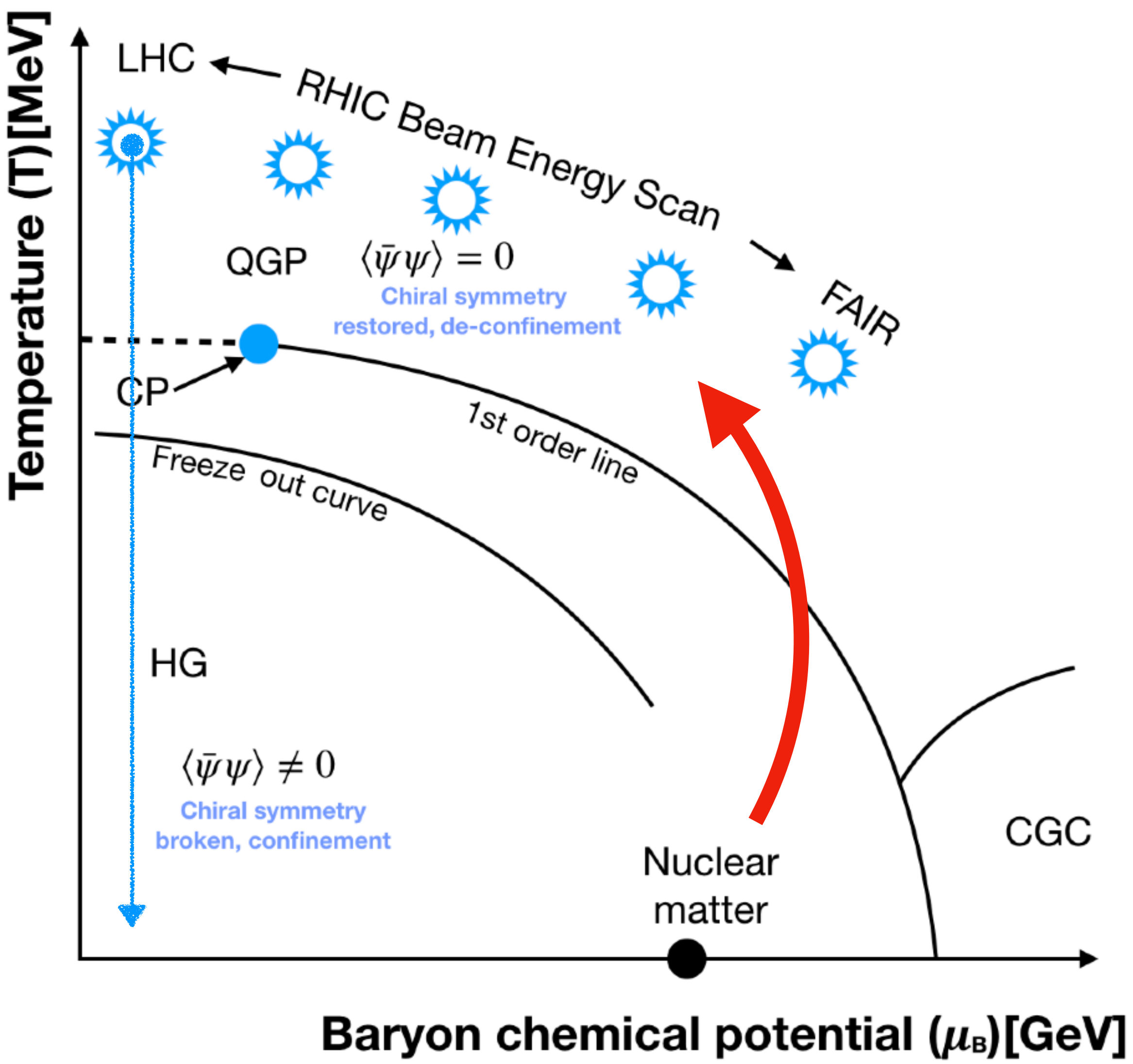
- **Fluctuations ↔ Correlations**
  - Two facets of the same thing.
- **Tools**
  - **Integral Correlations** (Fluctuations)
    - **Moments, Factorial Moments** of discrete & continuous observables.
    - **Cumulants, Factorial Cumulants** of discrete & continuous observables.
  - **Differential correlation functions**.
    - Number correlations $R_2^{CI/CD}$
    - Pt Correlations $P_2^{CI/CD}$, $G_2^{CI/CD}$
    - Balance Functions B_2





Cambridge Press, 2017

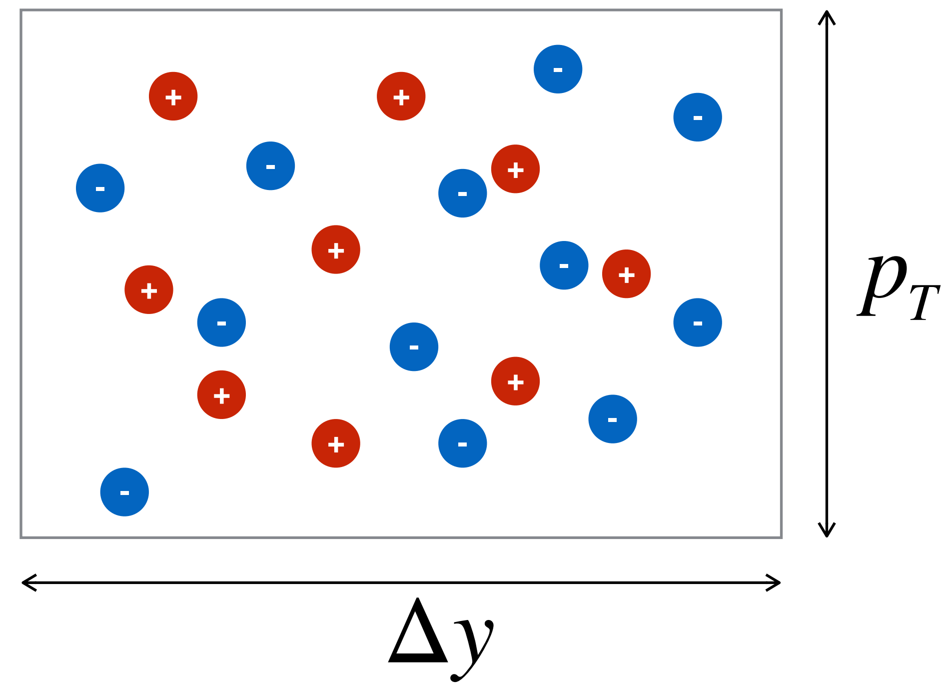
Dynamical fluctuations in Heavy Ion Collisions



- **Two QCD Transitions**:
  - **Confinement/Deconfinement**
    - **Change in Charge Fluctuations**
    - **Change in susceptibilities → net charge, strange, baryon fluctuations**
    - **Temperature fluctuations**
  - ***Chiral Symmetry***
    - Broken in hadron phase/partially restored in QGP state.
    - Consequence: **Disoriented Chiral Condensates (DCC)**
  - Search for DCCs.
- Collisions Dynamics
  - Initial state fluctuations
    - pT fluctuations
    - pT vs v_n correlations
    - And more …

Theoretical Predictions

- Expect number of charged particles (N_+ vs. N_-) to fluctuate **event-by-event**
  - **Fluctuations of $Q = N_+ - N_-$ in A-A collisions (in volume/acceptance)**.
  - **Hadrons: ± 1 Quarks: $\pm 1/3, \pm 2/3$.**
  - At equal number of particles, a **quark phase** should feature **smaller net charge fluctuations** than a **hadron phase**.



QGP Signature: suppression of net charge fluctuations (variance).

$$\omega_Q \equiv \frac{\langle \Delta Q^2 \rangle}{\langle N_{CH} \rangle}$$

$$Q = N_+ - N_-$$

$$N_{CH} = N_+ + N_-$$

- Koch et al. [1,3,4]:

$$R = \frac{N_+}{N_-}$$

$$D \equiv \langle N_{CH} \rangle \langle \Delta R^2 \rangle$$

$$D \equiv 4 \frac{\langle \Delta Q^2 \rangle}{\langle N_{CH} \rangle} = 4\omega_Q$$

[1] S. Jeon, et al, Phys. Rev. Lett. 85, 2076 (2000).
[2] M. Asakawa, et al., Phys. Rev. Lett. 85, 2072 (2000).
[3] M. Bleicher, et al, Phys. Rev. C 62, 061902 (2002).
[4] S. Jeon, et al., Phys. Rev. Lett. 83, 5435 (1999).
[5] E. V. Shuryak, M. A. Stephanov, Phys. Rev. C 63, 064903 (2001).
[6] M. A. Aziz, S. Gavin, Phys. Rev. C 70, 034905 (2004).

Suppression of fluctuations

$$Q = N_+ - N_- \qquad N_{CH} = N_+ + N_- \qquad R = \frac{N_+}{N_-}$$

$$D \equiv \langle N_{CH} \rangle \langle \Delta R^2 \rangle$$

Process	$\omega_Q \equiv \frac{\langle \Delta Q^2 \rangle}{\langle N_{CH} \rangle}$	$D \equiv \langle N_{CH} \rangle \langle \Delta R^2 \rangle$
Thermalized QGP w/ Fast Hadronization	<0.25	~1
Hadron Gas (Resonances)	~0.7	~2.8
Poisson Emission i.e. no correlations	1	4

Nominally provides a clean cut distinction between QGP and HG!!!

[1] S. Jeon, et al, Phys. Rev. Lett. 85, 2076 (2000).
[2] M. Bleicher, et al, Phys. Rev. C 62, 061902 (2002).
[3] S. Jeon, et al., Phys. Rev. Lett. 83, 5435 (1999).

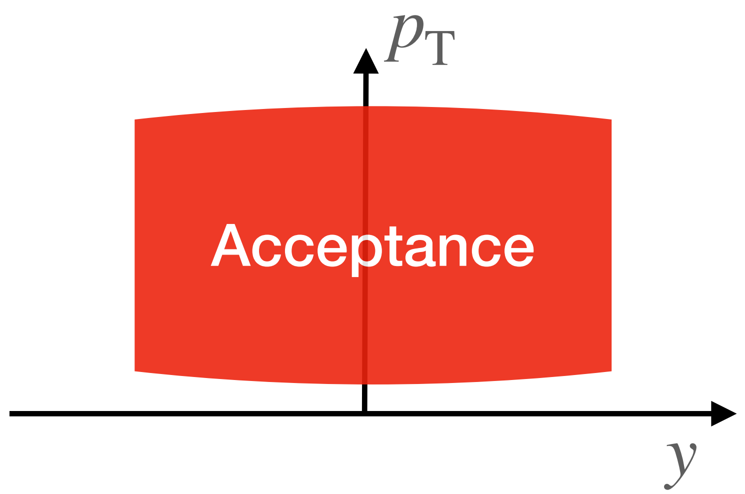
C.P., S.G., S.V. - PRC 66, 44904 (2002)

Nu-Dyn Observable

Definition (as fluctuations)

$$\nu_{\rm dyn} = \frac{\langle N_+(N_+ - 1)\rangle}{\langle N_+\rangle^2} + \frac{\langle N_-(N_- - 1)\rangle}{\langle N_-\rangle^2} - 2\frac{\langle N_+ N_-\rangle}{\langle N_+\rangle\langle N_-\rangle}$$

$\nu_{\rm dyn} = 0$ in the absence of correlations



Definition as Correlator (Normalized Cumulants)

$$\langle N_{\rm ch}\rangle \nu_{\rm dyn} = D - 4$$

$$\nu_{\rm dyn} = R_2^{++} + R_2^{--} - 2R_2^{+-}$$

$$R_2^{\alpha\beta} = \frac{\langle N_\alpha(N_\beta - \delta_{\alpha\beta})\rangle}{\langle N_\alpha\rangle\langle N_\beta\rangle} - 1$$

$$\langle N_\alpha\rangle = \int_\Omega \rho_1^\alpha(\vec{p})d\vec{p}$$

$$\langle N_\alpha(N_\beta - \delta_{\alpha\beta})\rangle = \int_\Omega \rho_2^{\alpha\beta}(\vec{p}_1,\vec{p}_2)d\vec{p}_1 d\vec{p}_2$$

Process	$\omega_Q \equiv \frac{\langle \Delta Q^2\rangle}{\langle N_{CH}\rangle}$	$D \equiv \langle N_{CH}\rangle\langle \Delta R^2\rangle$	$\langle N_{\rm ch}\rangle \nu_{\rm dyn}$
QGP w/ Fast Hadronization	<0.25	~1	-3
Hadron Gas (Resonances)	~0.7	~2.8	-1.2
Poisson Emission i.e. no correlations	1	4	0

Scaling vs. Multiplicity

$$\nu_{\rm dyn}^{AA} = \frac{1}{N}\nu_{\rm dyn}^{pp}$$

ALICE, Phys. Rev. Lett. 110 (2013) 152301

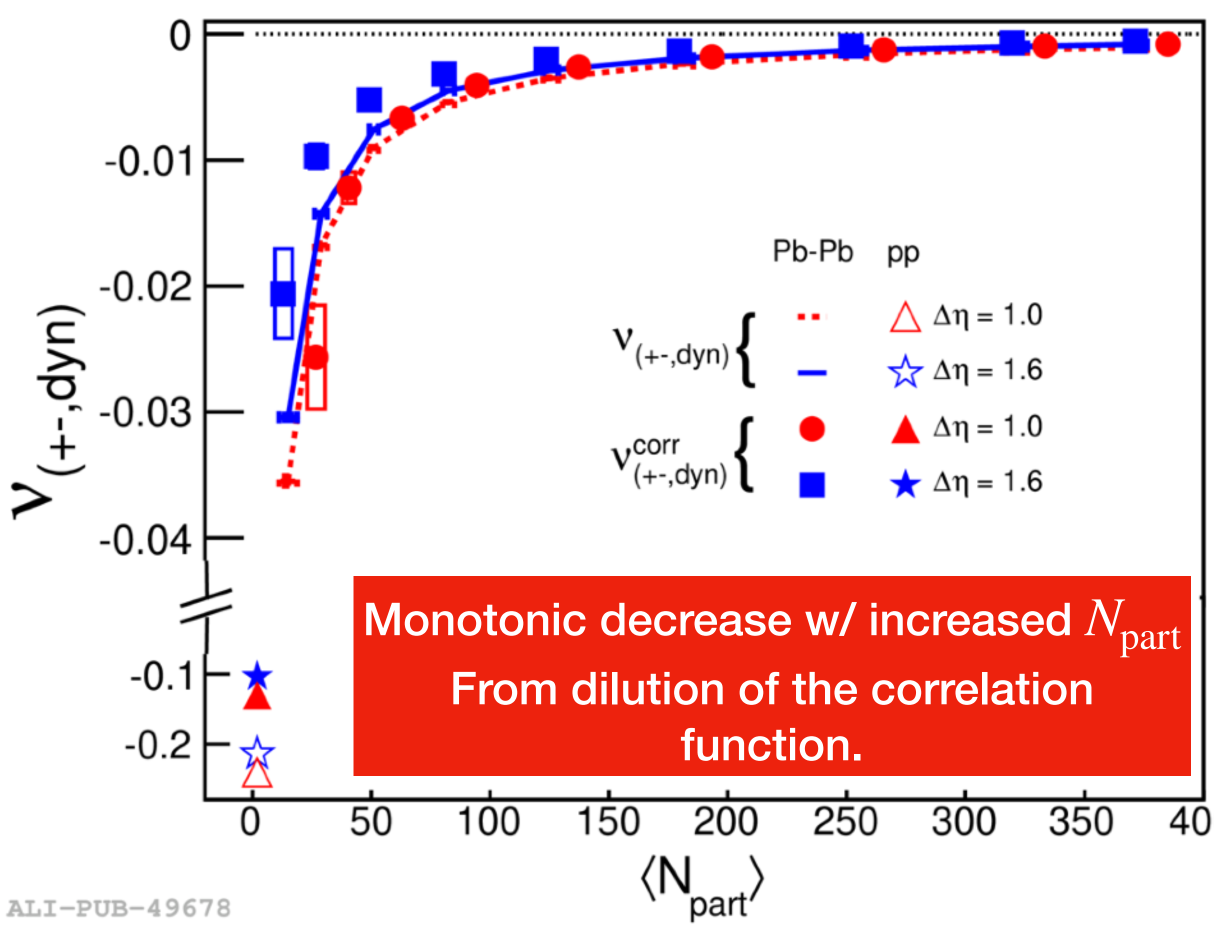
ALICE Results: Pb—Pb @ $\sqrt{s_{NN}} = 2.76$ TeV

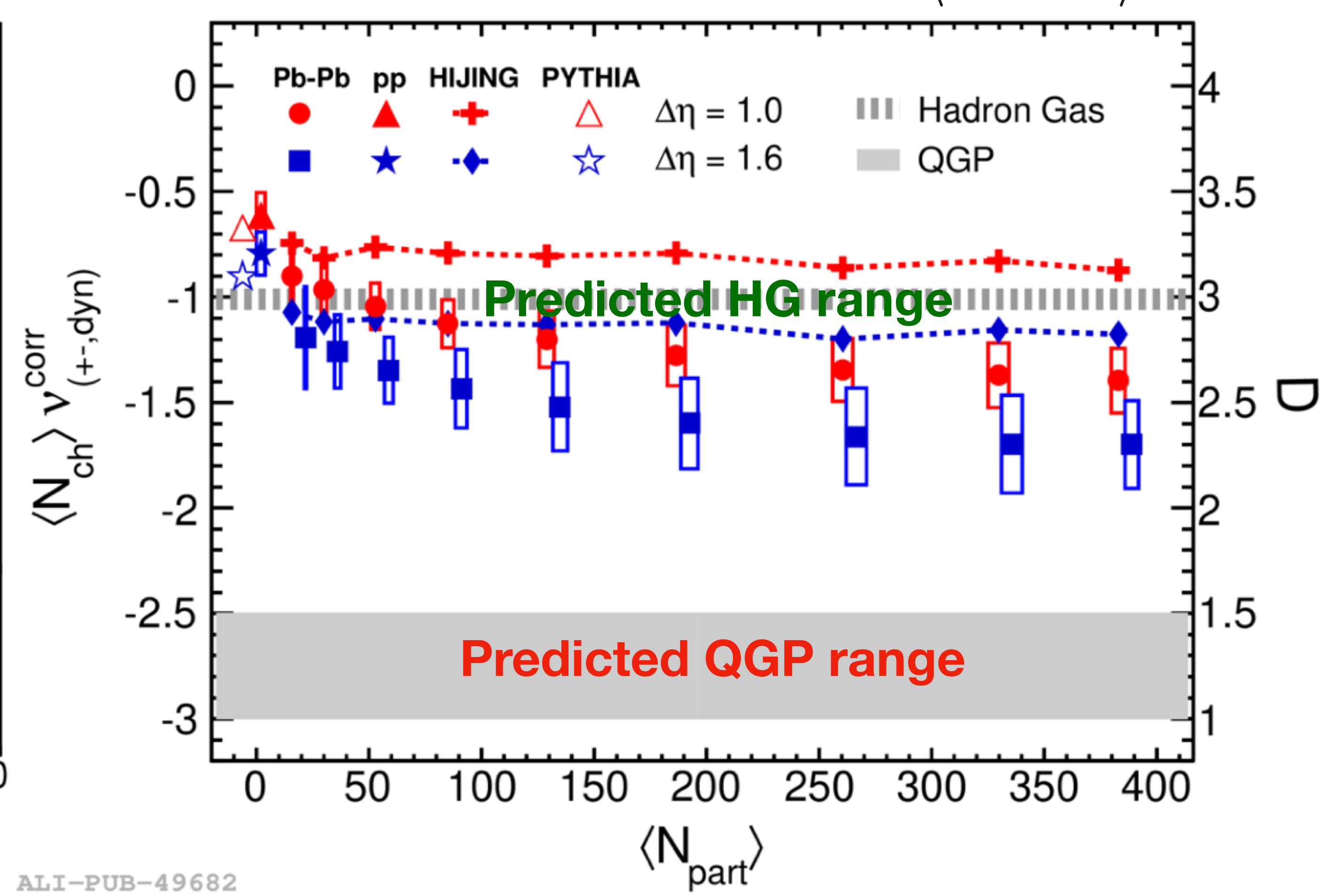
13×10^{6} minimum-bias collisions

$|\eta| < 0.8;\ 0.2\ \mathrm{GeV}/c < p_T < 5.0\ \mathrm{GeV}/c$

$$\langle N_{CH} \rangle \nu_{\rm dyn} = D - 4$$

$$\nu^{\rm corr}_{(+-,{\rm dyn})} = \nu_{(+-,{\rm dyn})} + \frac{4}{\langle N_{\rm total} \rangle}.$$





Trivial Scaling Dilution

$$\nu^{(m)}_{\rm dyn} = \frac{\nu_{\rm dyn}}{\langle N_{\rm ch} \rangle}$$

Decrease of D vs. $N_{\rm part}$ possibly due to radial flow vs. $N_{\rm part}$
Alternatively: diffusion of charged hadrons in rapidity.

No smoking gun for deconfinement!

ALICE, Eur.Phys.J.C 79 (2019) 3

Motivations/Method

ALICE

- QCD: At **sufficiently high-energy density** nuclear matter transforms into a deconfined state — Quark-Gluon Plasma (QGP) [1, 2].
- **Signatures:**
  - **Enhancement of fluctuations of the number of produced particles in the hadronic final state of relativistic heavy-ion collisions [3–5].**
  - Event-by-event **fluctuations and correlations may show critical behavior** near the *phase boundary*, including the crossover region where there is no thermal singularity
- A **correlation analysis of event-by-event abundances of pions, kaons and protons** produced in Pb–Pb collisions at LHC energies ***may provide a connection to fluctuations of globally conserved quantities such as electric charge, strangeness and baryon number, and therefore shed light on the phase structure of strongly interacting matter*** [6].
- **Method:**
  - **Measure** $\nu_{\text{dyn}} = R_2^{\alpha\alpha} + R_2^{\beta\beta} - 2R_2^{\alpha\beta}$
    - α, β: Particle species of interest: e.g., $\pi^{\pm}$, $K^{\pm}$, $p\bar{p}$
    - Vs. collision centrality.

$$R_2^{\alpha\beta} = \frac{\langle N_\alpha (N_\beta - \delta_{\alpha\beta})\rangle}{\langle N_\alpha\rangle\langle N_\beta\rangle} - 1$$

N_α, N_β: multiplicities of species α and β in a specific measurement acceptance.

[1] J. C. Collins et al., Phys. Rev. Lett. 34 (1975) 1353.
[2] E. V. Shuryak, Phys. Rept. 61 (1980) 71–158.
[3] M. A. Stephanov, et al., Phys. Rev. Lett. 81 (1998) 4816–4819.
[4] E. V. Shuryak and M. A. Stephanov, Phys. Rev. C63 (2001) 064903.
[5] A. Bazavov et al., Phys. Rev. D86 (2012) 034509.
[6] V. Koch, Relativistic Heavy Ion Physics, R. Stock, ed., pp. 626 2010.

ALICE, Eur.Phys.J.C 79 (2019) 3

Results: Centrality Dependence

ALICE

Pb—Pb collision at $\sqrt{s_{NN}} = 2.76$ TeV

$$\nu_{dyn}^{\alpha\beta} = R_2^{\alpha\alpha} + R_2^{\beta\beta} - 2R_2^{\alpha\beta}$$

Based on N_{ch} in V0 detectors	Based on N_{ch} in TPC	π, K	π, p	p, K
Centrality (%)	$\langle dN_{ch}/d\eta \rangle$	$\nu_{dyn}[\pi, K]$ (10^{-3})	$\nu_{dyn}[\pi, p]$ (10^{-3})	$\nu_{dyn}[p, K]$ (10^{-3})
0–5	1601±60	1.35 ±0.08 ±0.25	0.59 ±0.08 ±0.13	0.59 ±0.08 ±0.13
5–10	1294±49	1.22 ±0.08 ±0.22	0.19 ±0.08 ±0.06	0.46 ±0.10 ±0.11
10–20	966±37	1.35 ±0.08 ±0.21	0.38 ±0.08 ±0.12	0.98 ±0.10 ±0.17
20–30	649±23	1.69 ±0.09 ±0.21	0.29 ±0.09 ±0.15	1.76 ±0.13 ±0.34
30–40	426±15	2.27 ±0.11 ±0.25	0.01 ±0.18 ±0.18	2.39 ±0.24 ±0.40
40–50	261±9	3.52 ±0.16 ±0.37	-0.49 ±0.18 ±0.22	3.64 ±0.32 ±0.57
50–60	149±6	6.43 ±0.26 ±0.96	-1.38 ±0.24 ±0.29	6.54 ±0.47 ±0.92
60–70	76±4	11.91 ±0.53 ±2.1	-4.90 ±0.58 ±0.56	10.34 ±1.0 ±1.8
70–80	35±2	29.99 ±1.2 ±4.0	-16.02 ±1.5 ±1.1	17.93 ±2.0 ±3.3
		Always > 0	Changes sign!	Always > 0

Always positive means: $R_2^{\alpha\alpha} + R_2^{\beta\beta} > 2R_2^{\alpha\beta}$

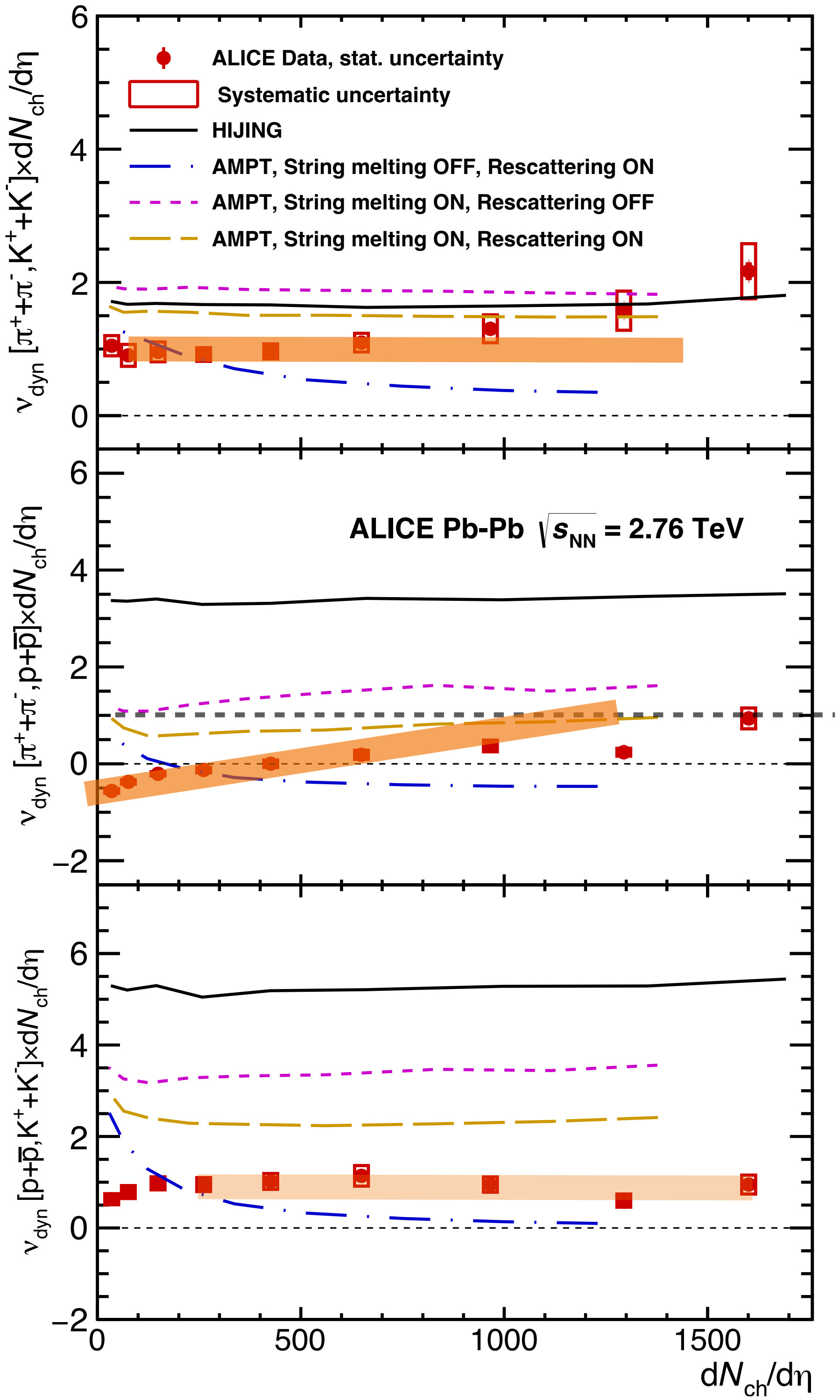
Changes sign: $R_2^{\alpha\alpha} + R_2^{\beta\beta} > 2R_2^{\alpha\beta}$ in mid to most central collisions

$R_2^{\alpha\alpha} + R_2^{\beta\beta} < 2R_2^{\alpha\beta}$ in mid to peripheral collisions

"Anomalous behavior" in π, p: Strongly violates 1/N scaling.

Results: Scaled Centrality Dependence

Scaled values: $\nu_{\rm dyn}^{\alpha\beta} \times \frac{{\rm d}N_{\rm ch}}{{\rm d}\eta}$



$\pi^\pm$ vs. $K^\pm$

Always positive: $R_2^{AA} + R_2^{BB} > 2R_2^{AB}$

Nearly constant from peripheral to mid-central, rises in central collisions

$\pi^\pm$ vs. $p\bar{p}$

$R_2^{AA} + R_2^{BB} > 2R_2^{AB}$ Changes sign: $R_2^{AA} + R_2^{BB} < 2R_2^{AB}$

in peripheral to mid central collisions

Significant Anomaly! Is the measurement correct?

$p\bar{p}$ vs. $K^\pm$

Always positive: $R_2^{AA} + R_2^{BB} > 2R_2^{AB}$

Nearly constant from peripheral to central collisions

HIJING and AMPT do NOT match the observed values.

More comprehensive models required - to be studied!

QCD at High Temperature

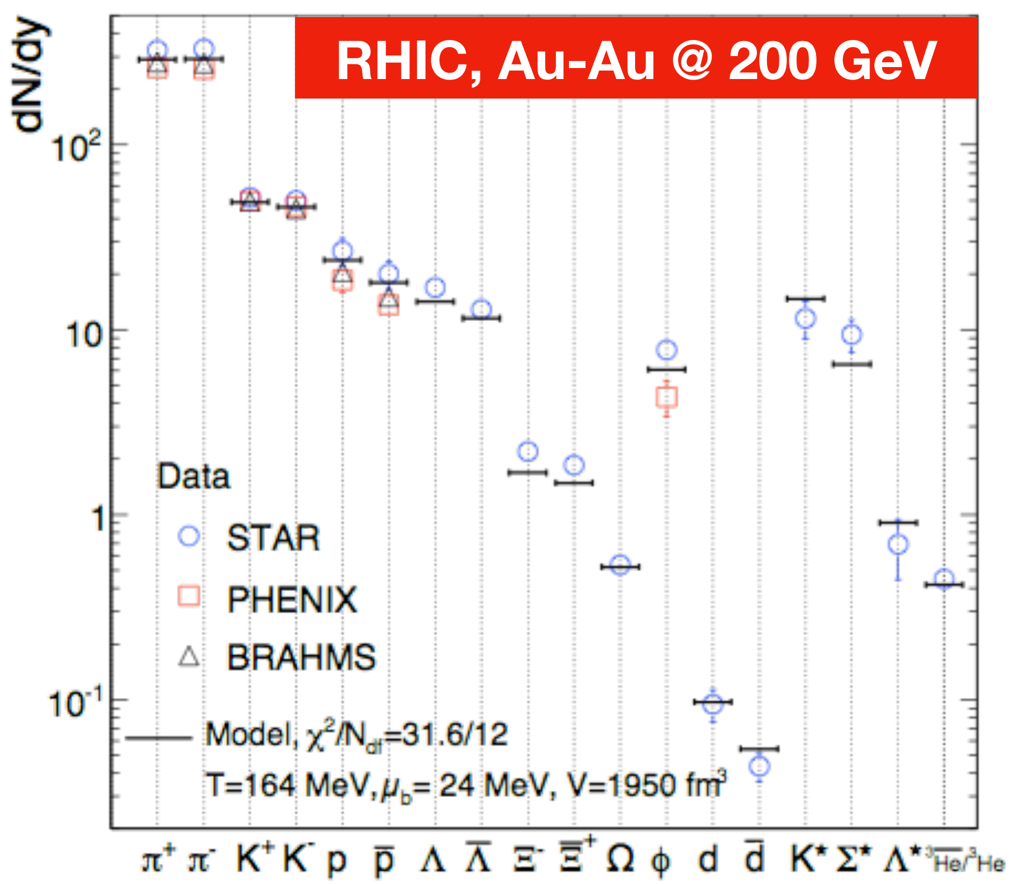
GCE Partition Function:

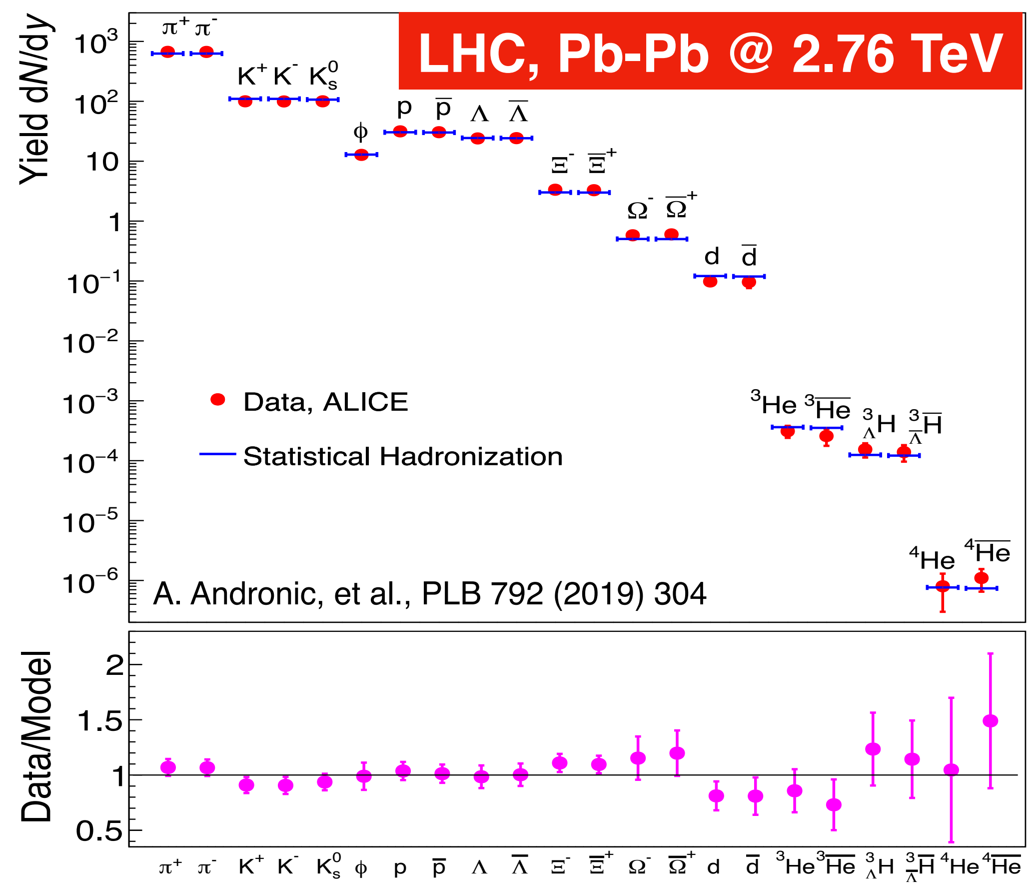
$$Z(V, T, \mu_B, \mu_Q, \mu_S) = \mathrm{Tr}\left[e^{-\beta\left(H - \sum_i \mu_i N_i\right)} \right]$$

$\beta = 1/T$, w/ T : System Temperature
H : Hamiltonian
μ_i: Chemical potentials
N_i: Conserved number operators

Particle density

$$N_i = -T\frac{\partial \ln Z_i}{\partial \mu} = \frac{g_i V}{2\pi^2}\int_0^\infty \frac{p^2 \mathrm{d}p}{\exp[(E_i - \mu_i)/T] \pm 1}$$





HRG Model w/ parameters T, μ_B, V
W/ "feed-downs": E&M, Strong Decays: e.g., $\Delta \rightarrow p(n) + \pi$, $\rho \rightarrow \pi + \pi$,../
Fit to ratios: Volume V cancels out

Thermal HG models predict observed abundances with spectacular precision

QCD at Finite Temperature

Susceptibilities: $\chi_{ijk}^{BQS} = \frac{\partial^{i+j+k}[P/T^4]}{\partial\hat{\mu}_B^i \partial\hat{\mu}_Q^j \partial\hat{\mu}_S^k}$ w/ $\hat{\mu}_q \equiv \mu_q/T,\ q = B, Q, S$

Diagonal/Non-diagonal Cumulants: $C_{ijk}^{BQS} = \frac{\partial^{i+j+k}}{\partial\hat{\mu}_B^i \partial\hat{\mu}_Q^j \partial\hat{\mu}_S^k} \ln\left[Z(V, T, \mu_B, \mu_Q, \mu_S)\right] = VT^3\chi_{ijk}^{BQS}(T, \mu_B, \mu_Q, \mu_S)$

Diagonal Cumulants:

$$M_q = \langle N_q \rangle = VT^3\chi_1^q$$

$$\sigma_2^q = C_2^q = \langle (\Delta N_q)^2 \rangle = VT^3\chi_2^q$$

$$C_3^q = \langle (\Delta N_q)^3 \rangle = VT^3\chi_3^q$$

$$C_4^q = \langle (\Delta N_q)^4 \rangle - 3\langle (\Delta N_q)^2 \rangle^2 = VT^3\chi_4^q$$

Skewness:

$$S_q = \frac{\langle (\Delta N_q)^3 \rangle}{\langle (\Delta N_q)^2 \rangle^{3/2}} = \frac{C_3^q}{(\sigma_2^q)^{3/2}}$$

Kurtosis:

$$\kappa_q = \frac{\langle (\Delta N_q)^4 \rangle}{\langle (\Delta N_q)^2 \rangle^2} - 3 = \frac{C_4^q}{(\sigma_2^q)^2}$$

To avoid ambiguities associated with the unknown volume V, consider ratios of cumulants:

$$\frac{\sigma_2^q}{M_q} = \frac{C_2^q}{M_q} = \frac{\chi_2^q}{\chi_1^q}$$

$$S_q\sigma_2^q = \frac{C_3^q}{C_2^q} = \frac{\chi_3^q}{\chi_2^q}$$

$$\kappa_q\sigma_2^q = \frac{C_4^q}{C_2^q} = \frac{\chi_4^q}{\chi_2^q}$$

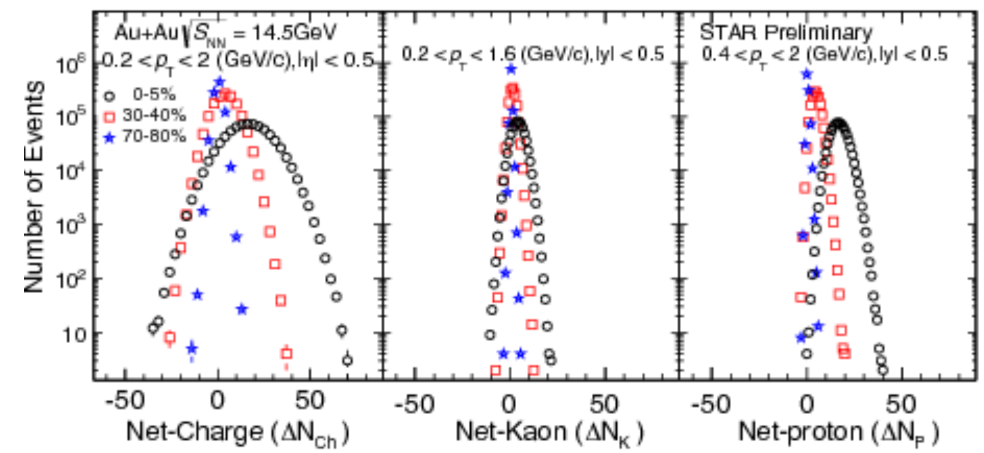
$$\frac{\kappa_q\sigma_2^q}{S_q} = \frac{C_4^q}{C_3^q} = \frac{\chi_4^q}{\chi_3^q}$$

Measurements of C_n^q nominally yield ratios of susceptibilities.

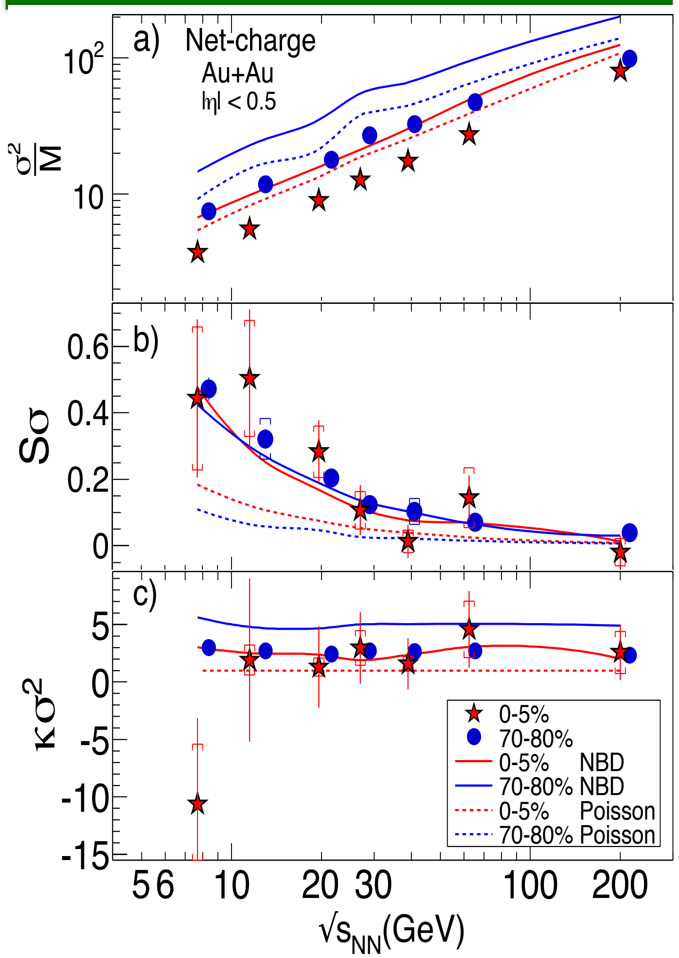
Should provide ability to prove properties of QCD matter near QGP-HG phase transition

Goals and RHIC Results

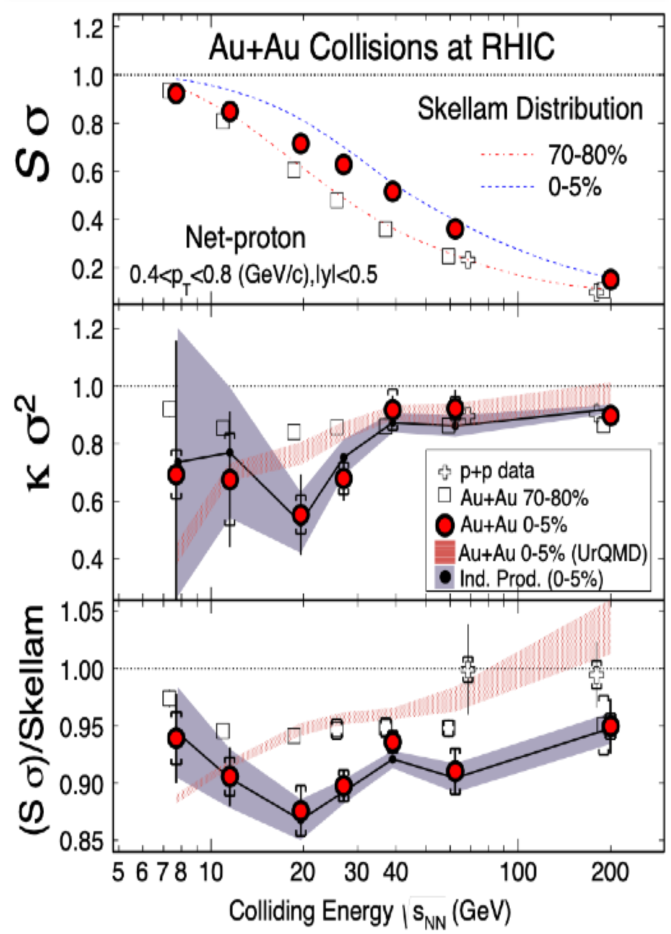
- (Ultimate) Goal:
  - **Experimental test** of Lattice QCD (LQCD) predictions on second and higher order cumulants of net-charge, net-strangeness, net-baryon distributions to search for critical behavior near the QCD phase boundary.
- At RHIC, **search for critical point — end point of 1st order transitions**



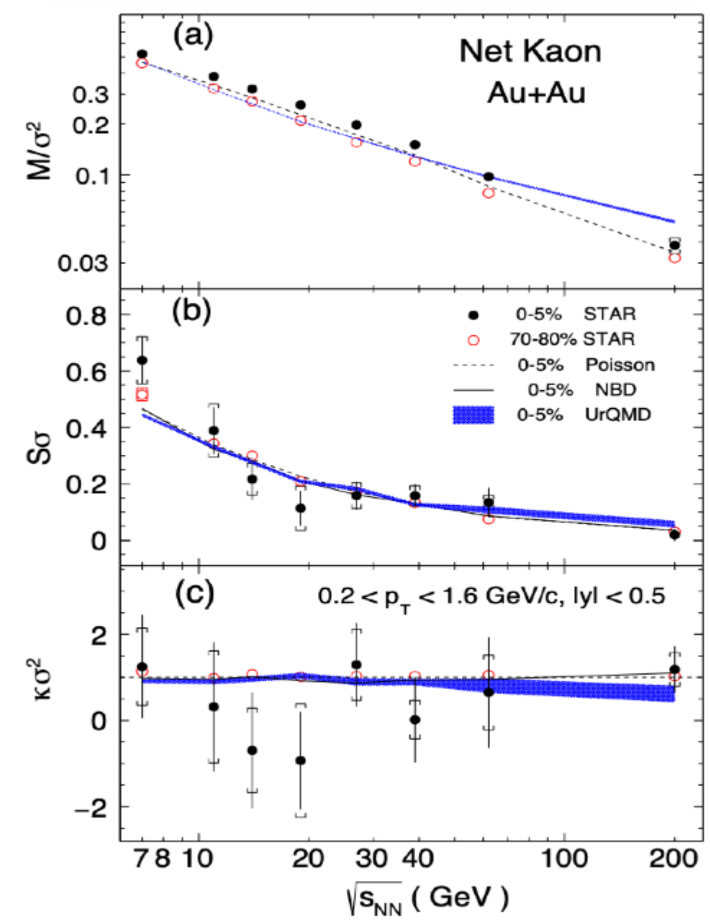
STAR: arXiv:1402.1558



STAR: arXiv:1309.5681



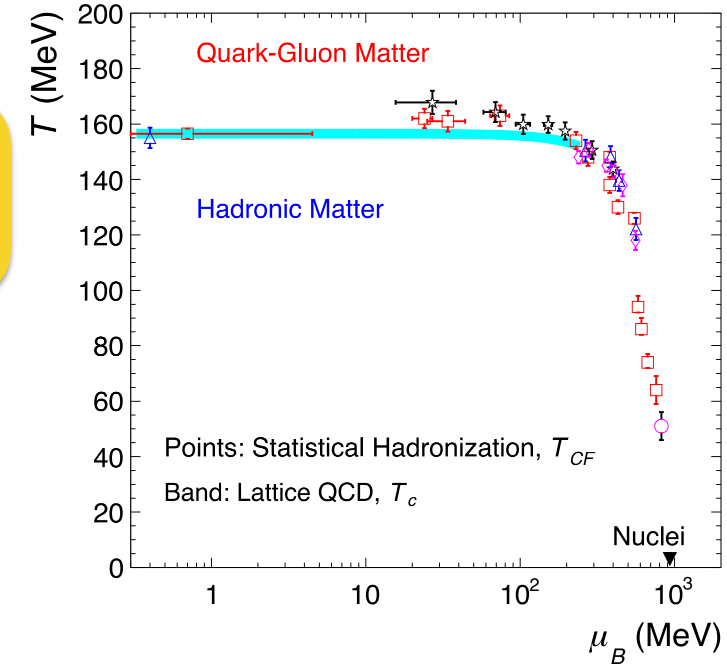
STAR: arXiv:1709.00773



C.P., Phys.Rev.C 100 (2019) 3, 034905
CP, VG, SB AM, e-Print: 2310.07618

Motivations

- Vanishing light quark masses (LHC): **2nd order phase transition**: HG ↔ QGP [3].
- Realistic quark masses: **smooth cross over** [4, 5].
- Can still probe critical phenomena at LHC (vanishing baryon chemical potential) [6].
- LQCD calculations [4, 5] exhibit strong signal for **pseudo-critical chiral temperature T ~156 MeV**
  - w/ T in agreement with the chemical freeze-out temperature extracted from analysis of hadron multiplicities [7, 8] measured by ALICE. Strongly interacting matter created in central collisions of Pb nuclei at LHC energies freezes out very near the chiral phase transition line!?!
- Hence, the singularities arising from the second order phase transition could be captured by measuring fluctuations of conserved charges such as the net-baryon number.
  - Evaluated within Hadron Resonance Gas (HRG), net-baryon distributions coincide with the Skellam distribution [9, 10].
  - LQCD also predicts a Skellam distribution at T~156 MeV for the second cumulants of net-baryons.
  - Fourth cumulants of net-baryons from LQCD are significantly below the corresponding Skellam baseline [11, 12]. Investigate QCD phase transitions w/ measurements of **fluctuations of conserved charges** s(e.g., electric charge, baryon number, etc) [1, 2].



[1] V. Koch, Relativistic Heavy Ion Physics, R. Stock, ed. 2010.
[2] STAR, Phys. Rev. Lett. 112 (2014) 032302.
[3] M. A. Stephanov, Prog. Theor. Phys. Suppl. 153 (2004) 139–156.
[4] A. Bazavov et al., Phys. Rev. D85 (2012) 054503.
[5] S. Borsanyi, et al., JHEP 10 (2018) 205.
[6] B. Friman, et al., Eur. Phys. J. C71 (2011) 1694.
[7] A. Andronic,et al., Nature 561 no. 7723, (2018) 321–330.
[8] A. Andronic, et al., Phys. Lett. B792 (2019) 304–309.
[9] K. Redlich,Central Eur. J. Phys. 10 (2012) 1254–1257.
[10] J. G. Skellam, J. of the Royal Statistical Society A109(3) (1946) 296.
[11] A. Bazavov et al., Phys. Lett. B795 (2019) 15–21.
[12] A. Bazavov et al., Phys. Rev. D95 no. 5, (2017) 054504.

ALICE, PLB 807 (2020) 135564

Method/Definitions

Number of baryons & anti-baryons in a given event: $n_{\mathrm{B}}, n_{\bar{\mathrm{B}}}$

Net baryon number: $\Delta n_{\mathrm{B}} = n_{\mathrm{B}} - n_{\bar{\mathrm{B}}}$

Probability of $n_{\mathrm{B}}, n_{\bar{\mathrm{B}}}$: $P(n_{\mathrm{B}}, n_{\bar{\mathrm{B}}})$

Net baryon cumulants:

$$\kappa_1(\Delta n_{\mathrm{B}}) = \sum_{\Delta n_{\mathrm{B}}=-\infty}^{\infty} \Delta n_{\mathrm{B}} P(\Delta n_{\mathrm{B}}) = \langle \Delta n_{\mathrm{B}} \rangle.$$

$$\kappa_2(\Delta n_{\mathrm{B}}) = \sum_{\Delta n_{\mathrm{B}}=-\infty}^{\infty} \left(\Delta n_{\mathrm{B}} - \langle \Delta n_{\mathrm{B}} \rangle\right)^2 P(\Delta n_{\mathrm{B}}) = \langle \left(\Delta n_{\mathrm{B}} - \langle \Delta n_{\mathrm{B}} \rangle\right)^2 \rangle$$

$$\kappa_2(\Delta n_{\mathrm{B}}) = \kappa_2(n_{\mathrm{B}}) + \kappa_2(n_{\bar{\mathrm{B}}}) - 2\left(\langle n_{\mathrm{B}} n_{\bar{\mathrm{B}}} \rangle - \langle n_{\mathrm{B}} \rangle \langle n_{\bar{\mathrm{B}}} \rangle\right)$$

$Cov[n_{\mathrm{B}}, n_{\bar{\mathrm{B}}}] = \langle n_{\mathrm{B}} n_{\bar{\mathrm{B}}} \rangle - \langle n_{\mathrm{B}} \rangle \langle n_{\bar{\mathrm{B}}} \rangle = 0$ in absence of correlations.

For **vanishing correlations**:

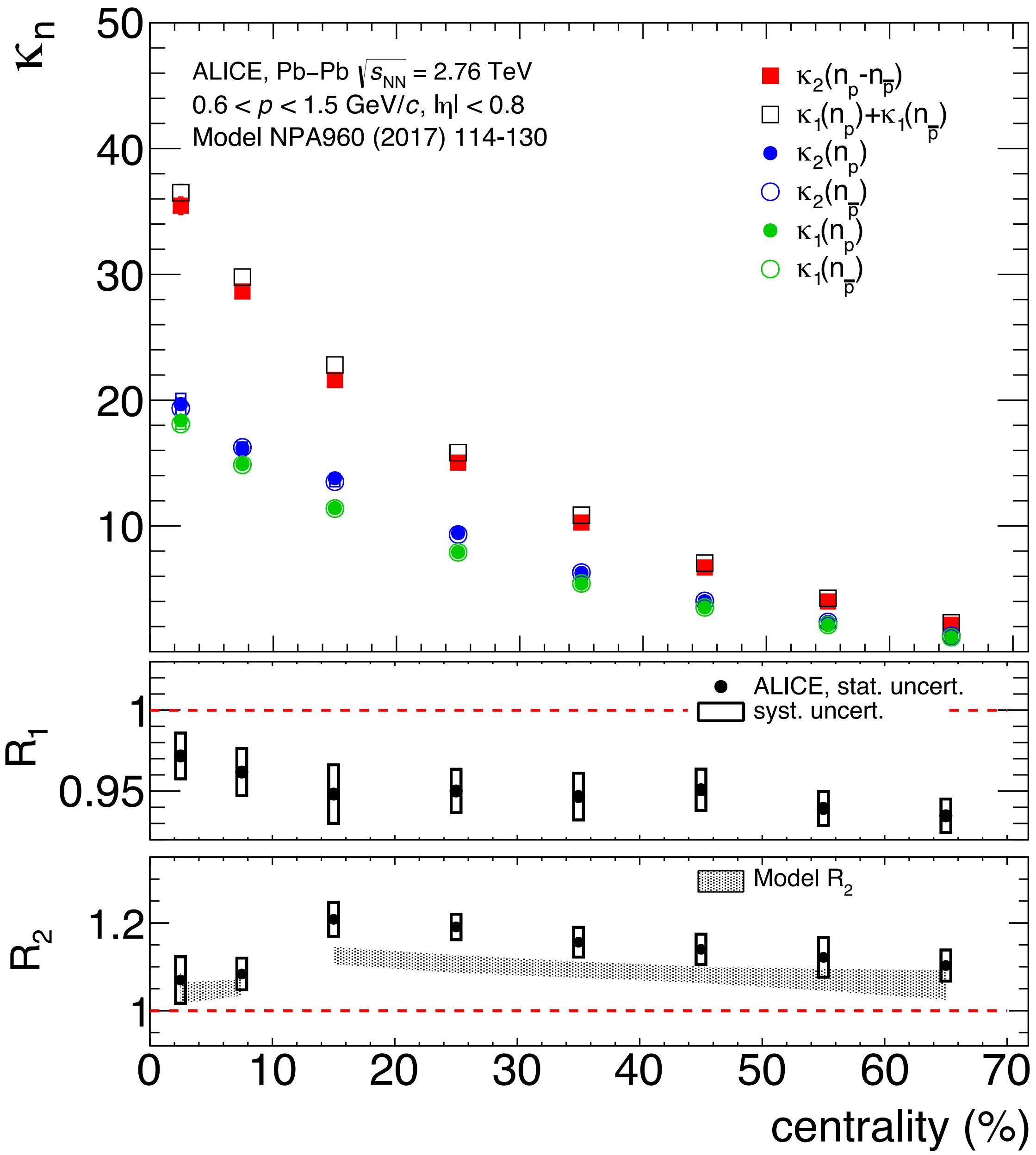
$$\kappa_2(\Delta n_{\mathrm{B}}) = \kappa_1(n_{\mathrm{B}}) + \kappa_1(n_{\bar{\mathrm{B}}})$$

Using **net-proton fluctuations as a proxy to net baryons**.

Protons identified based on dE/dx and TOF w/ **Identity Method** [1,2]

[1] Gorenstein et al. *Phys. Rev.* C83 (2011) 054907
[2] Pruneau, Phys.Rev. C 96 (2017) 5, 054902

Results



- Cumulants are **extensive quantities**,
  - i.e., proportional to the system volume.
- Explains the centrality dependence of all cumulants,
- Consider normalized cumulants to suppress system size dependence.

$$R_1 = \kappa_2(n_p - n_{\bar{p}})/\langle n_p + n_{\bar{p}}\rangle,$$
$$R_2 = \kappa_2(n_p)/\langle n_p\rangle.$$

Deviations from unity

Deviations from unity twice as large as for R_1

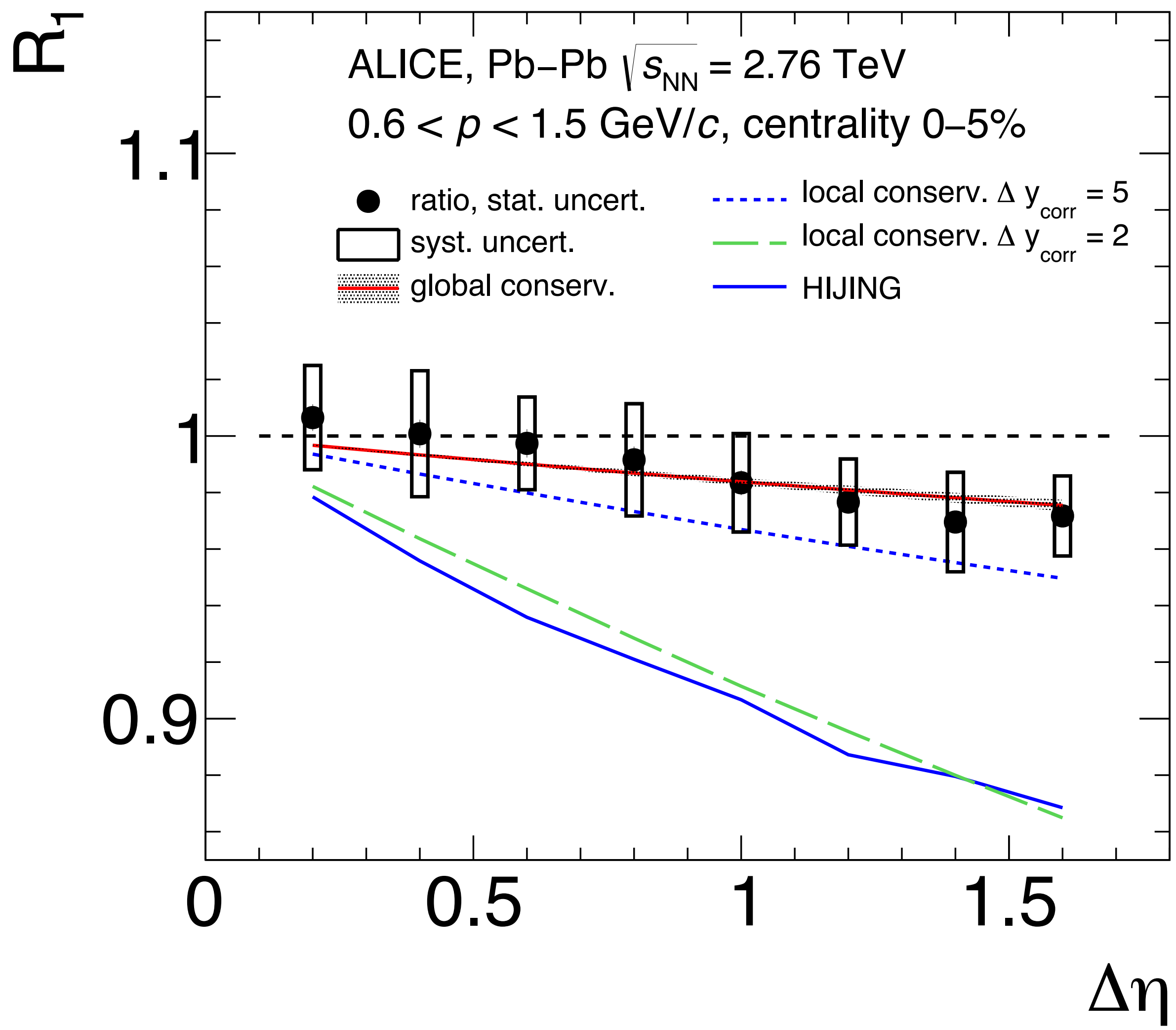
Results are compared with predictions from a model constructed recently [1], in which participant fluctuations are included following the analysis of the ALICE centrality selection. Within uncertainties, the model predictions are consistent with the measured R_2 values, lending support to the interpretation that volume fluctuations are largely at the origin of the observed deviation.

[1] PBM et al, Nucl. Phys. A960 (2017) 114–130

ALICE, PLB 807 (2020) 135564

Results vs. Acceptance

$$R_1 = \kappa_2(n_{\rm p} - n_{\bar{\rm p}})/\langle n_{\rm p} + n_{\bar{\rm p}}\rangle$$



Authors:
Linear behavior predicted based on the assumption of **global baryon number conservation (?!?)** [1, 2, 3], which induces correlations between protons and antiprotons leading to the following dependence on the acceptance factor α.

$$R_1 = 1 - \alpha,$$

where $\alpha = \langle n_p\rangle/\langle N_B^{4\pi}\rangle,$

$\langle n_p\rangle$ measured
$\langle N_B^{4\pi}\rangle$ estimated from HIJING, AMPT simulations

But one can also show that baryon conservation dominates the strength of the cumulant [4].

[1] PBM et al, Nucl. Phys. A960 (2017) 114–130,
[2] S. Mrowczynski, Phys. Rev. C66 (2002) 024904
[3] A. Bzdak,et al., Phys. Rev. C87 (2013) 014901
[4] Pruneau, Phys.Rev.C 100 (2019) 3, 034905

C.P., Phys.Rev.C 100 (2019) 3, 034905

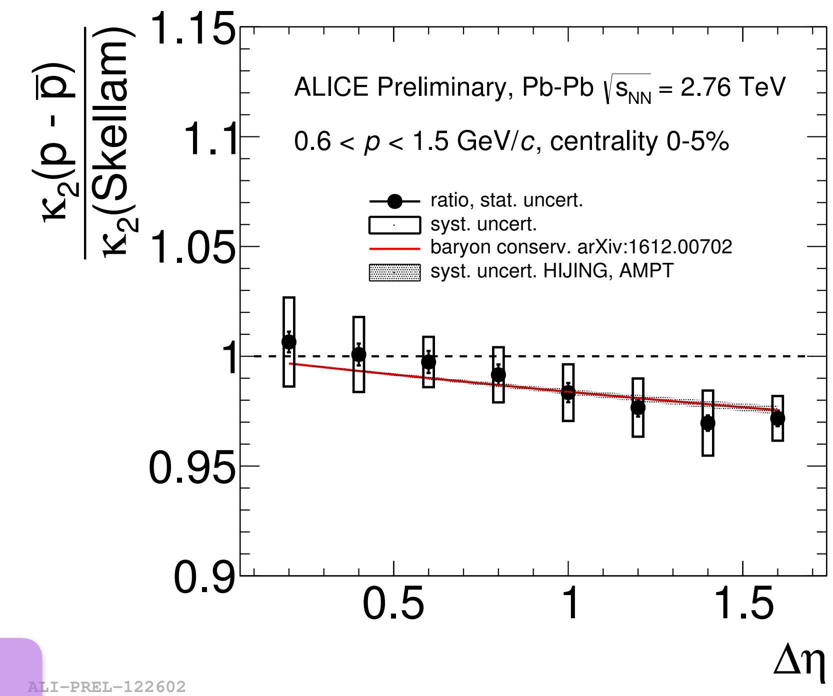
Baryon Number Conservation

Second order cumulant : $\kappa_2(\Delta N_p) = F_1^p + F_1^{\bar{p}} + F_2^{p,p} + F_2^{\bar{p},\bar{p}} - 2F_2^{p,\bar{p}}$

Poisson limit (Skellam) : $\kappa_2^{Skellam}(\Delta N_p) = F_1^p + F_1^{\bar{p}} = \langle N_p \rangle + \langle N_{\bar{p}} \rangle$

Ratio of $\kappa_2(\Delta N_p)$ to Skellam :

$$r_{\Delta N_p} \equiv \frac{\kappa_2(\Delta N_p)}{\kappa_2^{\text{Skellam}}(\Delta N_p)} = 1 + \frac{F_2^{p,p} + F_2^{\bar{p},\bar{p}} - 2F_2^{p,\bar{p}}}{F_1^p + F_1^{\bar{p}}}.$$



LHC: $\langle N_p \rangle \approx \langle N_{\bar{p}} \rangle$:

$$r_{\Delta N_p} = 1 + \frac{F_1^p}{2}\left[R_2^{p,p} + R_2^{\bar{p},\bar{p}} - 2R_2^{p,\bar{p}}\right],$$

Consequently :

$$r_{\Delta N_p} = 1 + \frac{1}{4}\frac{dN_T}{d\eta}\Delta\eta\nu_{\text{dyn}}^{p,\bar{p}},$$

$$r_{\Delta N_p} \approx 1 - a\Delta\eta$$

Trivial result:

Correlations exist: non Poisson behavior obtained from ν_{dyn} vs. $\Delta\eta$...

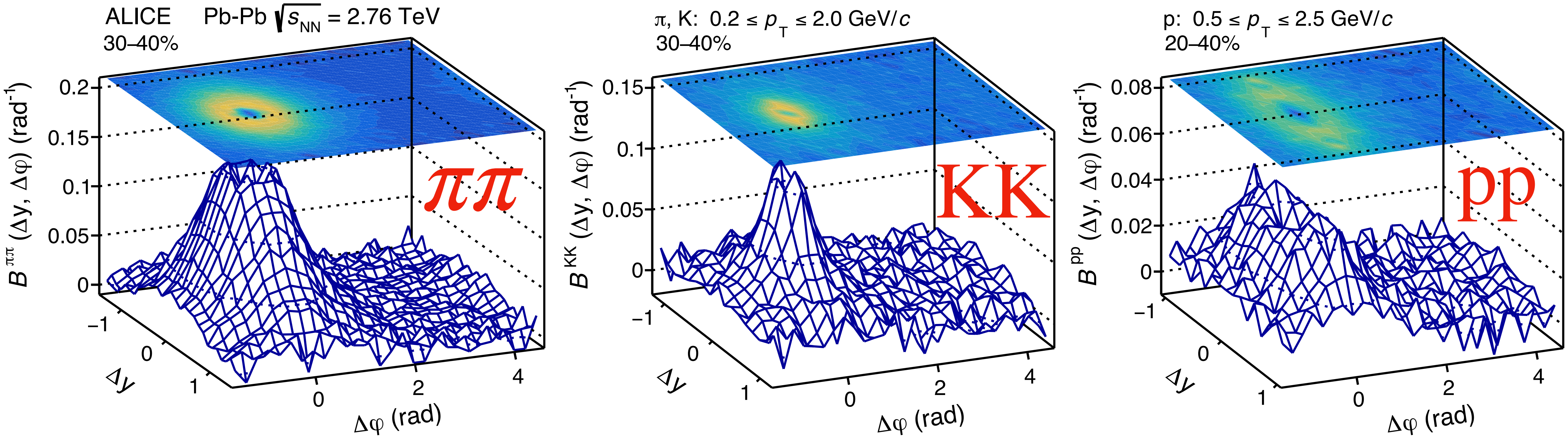
ALICE, Phys. Lett. B 833 (2022) 137338

Vs. Balance Functions (BF)

$$B^{p,p}(\Delta y) = -\frac{\Delta\eta}{4}\frac{dN_T}{d\eta}\left\{R_2^{p,p}(\Delta y) + R_2^{\bar{p},\bar{p}}(\Delta y) - 2R_2^{p,\bar{p}}(\Delta y)\right\}$$

Integral of the Balance Function

$$I_{p,\bar{p}}(\Omega) = -\frac{1}{4}\frac{dN_T}{d\eta}\Delta\eta \times \nu_{\rm dyn}^{p,\bar{p}}(\Omega)$$



- BF: Near-side peaked -> Not enough time to diffuse and "randomize"
- $\nu_{\rm dyn}^{p,\bar{p}}$ is related to the integral of Balance Function [1], determined by…
  - **width of the acceptance**
  - **collision dynamics**
- **Exercise caution in the interpretation of cumulants and their ratios…**

[1] C.P., Phys.Rev.C 100 (2019) 3, 034905

Sigma Model - Pion & Kaon Sectors

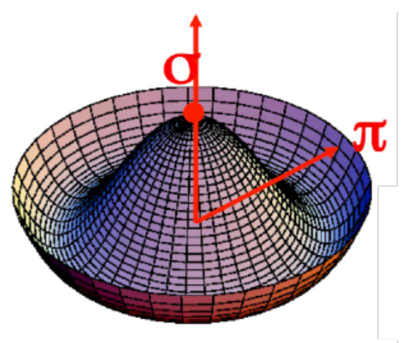
For 2nd order phase transition in QCD:
Landau-Ginzburg free energy w/ 2 massless quarks

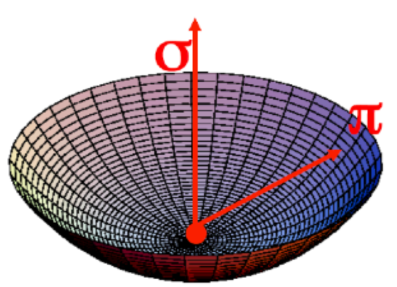
K.L. Kowalski, C.C. Taylor, hep-ph/9211282
K. Rajagopal, F. Wilczek, Nucl.Phys. B399 (1995) 395

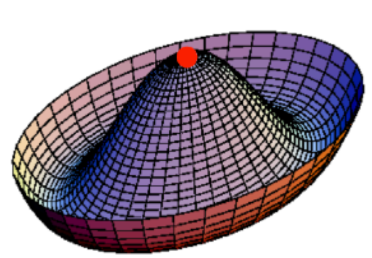
$$F = \int d^3x \left[\frac{1}{2} \partial^i \phi^\alpha \partial_i \phi^\alpha \frac{\mu^2}{2} \phi^\alpha \phi_\alpha \frac{\lambda}{4} \left(\phi^\alpha \phi_\alpha\right)^2 \right]$$

μ : renormalized mass (T)
λ: strength of coupling

Rewritten in terms of σ and $\vec{\pi}$ fields ("Mexican-hat" potential).







- **Condensates**:
  - 2 flavors: $\sigma \propto \langle \bar{u}u + \bar{d}d \rangle$
  - 3 flavors: $\sigma \propto \cos\theta \langle \bar{u}u + \bar{d}d \rangle + \sin\theta \langle \bar{s}s \rangle$
- **"Normal Vacuum"**:
  - π^+, π^-, π^0 equally probable.
  - $K^+, K^-, K^0, \bar{K}^0$ equally probable
- **Chiral symmetry restored at high-*T***
- **Quenching to low-*T***:
  - **New field "orientation"**
  - **Disoriented Chiral Condensate (DCC)**

Kaon Isospin Fluctuations

- Condensate w/ 3 light flavors (u,d) [1,2]:

$$\sigma \propto \cos\theta\langle \bar{u}u + \bar{d}d\rangle + \sin\theta\langle \bar{s}s\rangle$$

- Neutral Kaon Fraction:

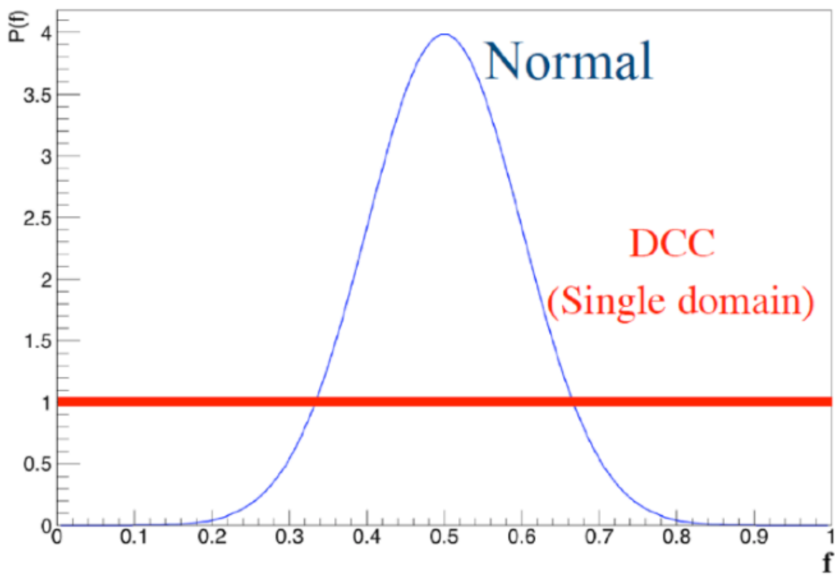
$$f = \frac{N_{K^0} + N_{\bar{K}^0}}{N_{K^0} + N_{\bar{K}^0} + N_{K^-} + N_{K^+}}$$

- **Model Expectations**
  - Normal Vacuum: **Binomial distribution**

$$P(f) = B(1/2; N) \text{ — “Normal”}$$

  - Strange DCC: **Uniform distribution**

$$P(f) = 1$$



[1] Randrup et al, PRC 59 (1999) 3329
[2] J. Kapusta, S.M.H. Wong PRL 86 (2001) 4251

ALICE, Phys. Lett. B 832 (2022) 137242

Kaon Isospin Fluctuations

ALICE

Integral correlators: $R_{\alpha\beta} = \frac{\langle N_\alpha (N_\beta - \delta_{\alpha\beta})\rangle}{\langle N_\alpha\rangle\langle N_\beta\rangle}$

N_α , N_β : Number of particles of types α, β within acceptance

Nu-dyn: $\nu_{dyn}^{\alpha\beta} = R_{\alpha\alpha} + R_{\beta\beta} - 2R_{\alpha\beta}$

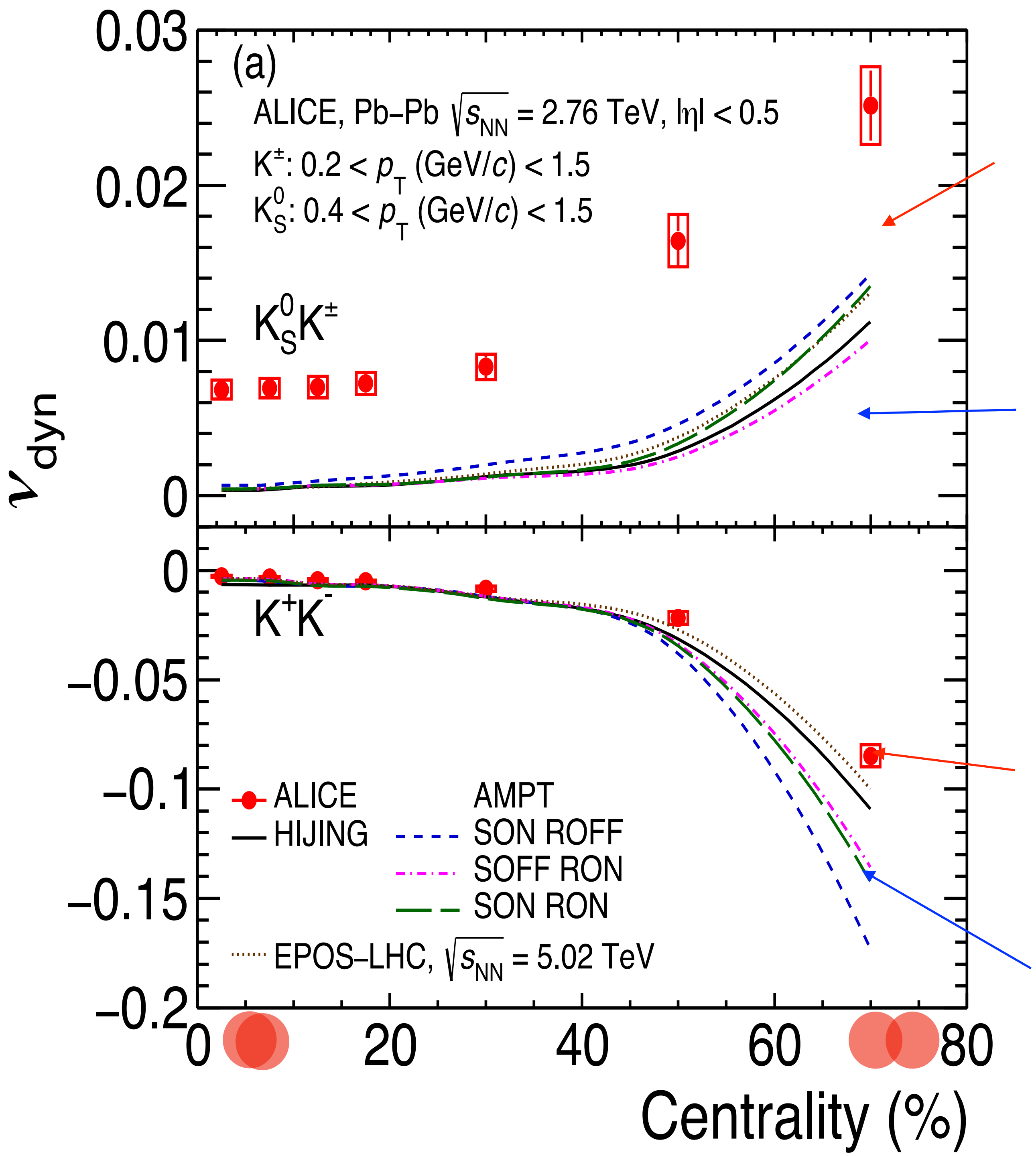
Charged Kaons: $\nu_{dyn}^{K^+K^-} = R_{K^+K^+} + R_{K^-K^-} - 2R_{K^+K^-}$

Neutral vs. Charged Kaons: $\nu_{dyn}^{K_s^0 K^\pm} = R_{K^\pm K^\pm} + R_{K_s^0 K_s^0} - 2R_{K^\pm K_s^0} = R_{cc} + R_{00} - 2R_{c0}$

- $R_{\alpha\beta}$ and $\nu_{\text{dyn}}^{\alpha\beta}$ are robust observables (approx. independent of efficiencies)
- Measure relative strength of **charged-charged** (cc), **neutral-neutral** (00), and **charged-neutral** (c0) kaon correlations.
- **Independent source scaling (n sources):** $R_{\alpha\beta}^{(n)} = \frac{1}{n} R_{\alpha\beta}^{(1)}$ $\quad \nu_{dyn}^{(n)} = \frac{1}{n}\nu_{dyn}^{(1)}$
- Proposed as indicator of anomalous production of kaon isospin fluctuations - a signal of DCCs [3]
- Shown to be sensitive to small or multiple DCCs [4]

[1] C.P., S. Gavin, S. Voloshin, **PRC 66 2002) 044904**
[2] C.P., S. Gavin, S. Voloshin, **Nucl.Phys.A 715 (2003) 661**.
[3] Gavin, Kapusta, PRC 65 (2002) 054910
[4] R. Nayak, S. Dash, C.P., PRC 004900 (2020).

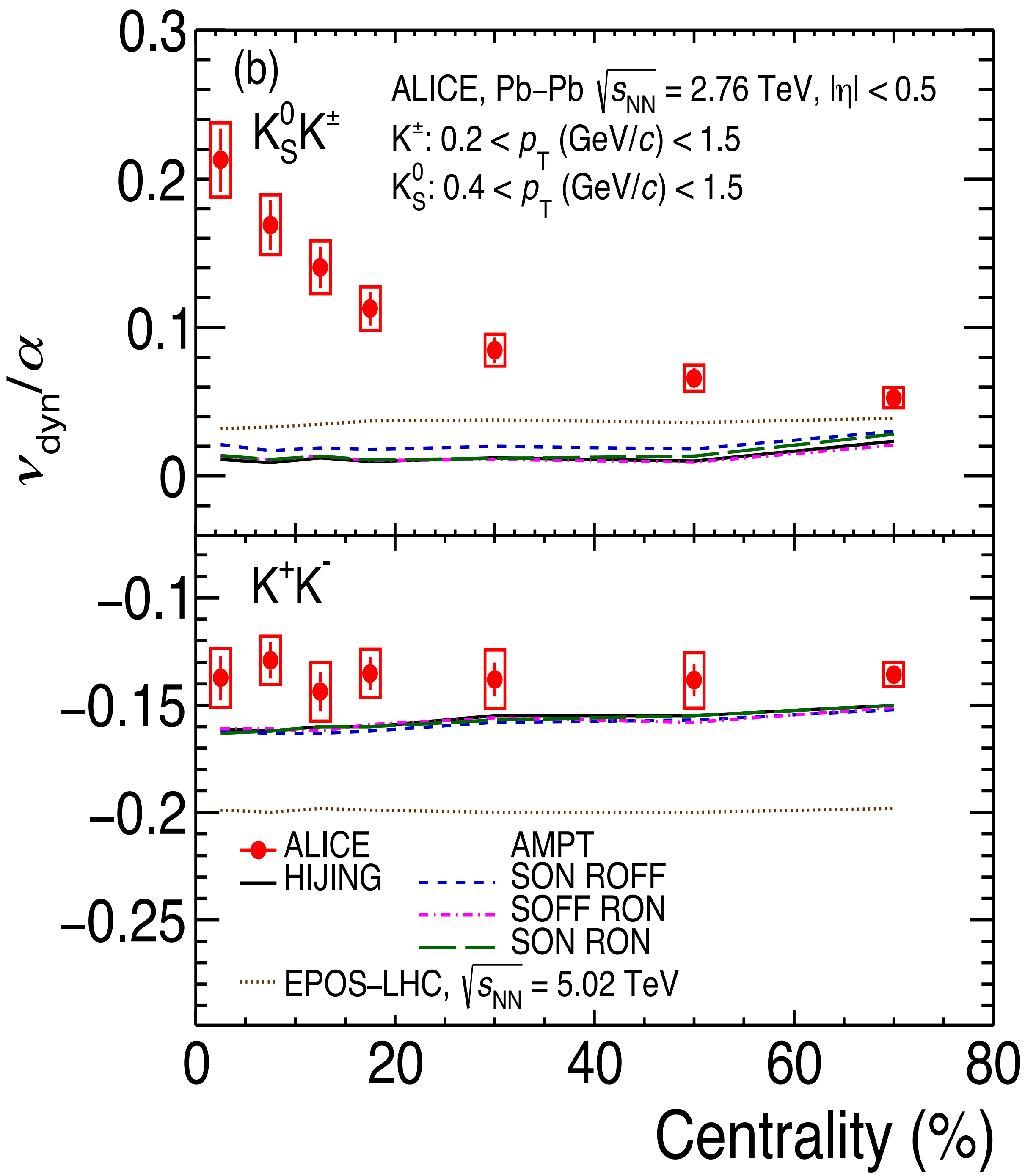
Kaon Isospin Fluctuations



- $\nu_{dyn}[K^0_s K^{\pm}]$
  - Data:
    - $R_{cc} + R_{00} > 2R_{c0}$.
    - "Flattening behavior" in central collisions
  - HIJING/AMPT/EPOS:
- $\nu_{dyn}[K^+_s K^-]$
  - Data:
    - $R_{++} + R_{--} < 2R_{+-}$
    - Pair creation dominance
  - HIJING/AMPT/EPOS:
    - "Similar" centrality dependence
    - HIJING (1/n) Close to data

Kaon Isospin Fluctuations

$\alpha = \langle N_{K_s^0} \rangle^{-1} + \langle N_{K^\pm} \rangle^{-1}$



- $\nu_{dyn}[K_s^0 K^\pm]/\alpha$
  - Data:
    - Strong correlation (excess) in central collisions
    - Strong 1/N scaling violation
    - “Anomalous” fluctuations
  - HIJING/AMPT/EPOS:
    - (Nearly) Flat as expected from 1/N scaling.
- $\nu_{dyn}[K_s^+ K^-]/\alpha$
  - Data:
    - Very small dependence on centrality
    - Approximate 1/N scaling
  - HIJING/AMPT/EPOS:
    - Very small dependence on centrality
    - (Expected) approximate 1/N scaling — no rescattering, no significant radial flow.
    - Magnitude slightly off relative to data — “decent” agreement.

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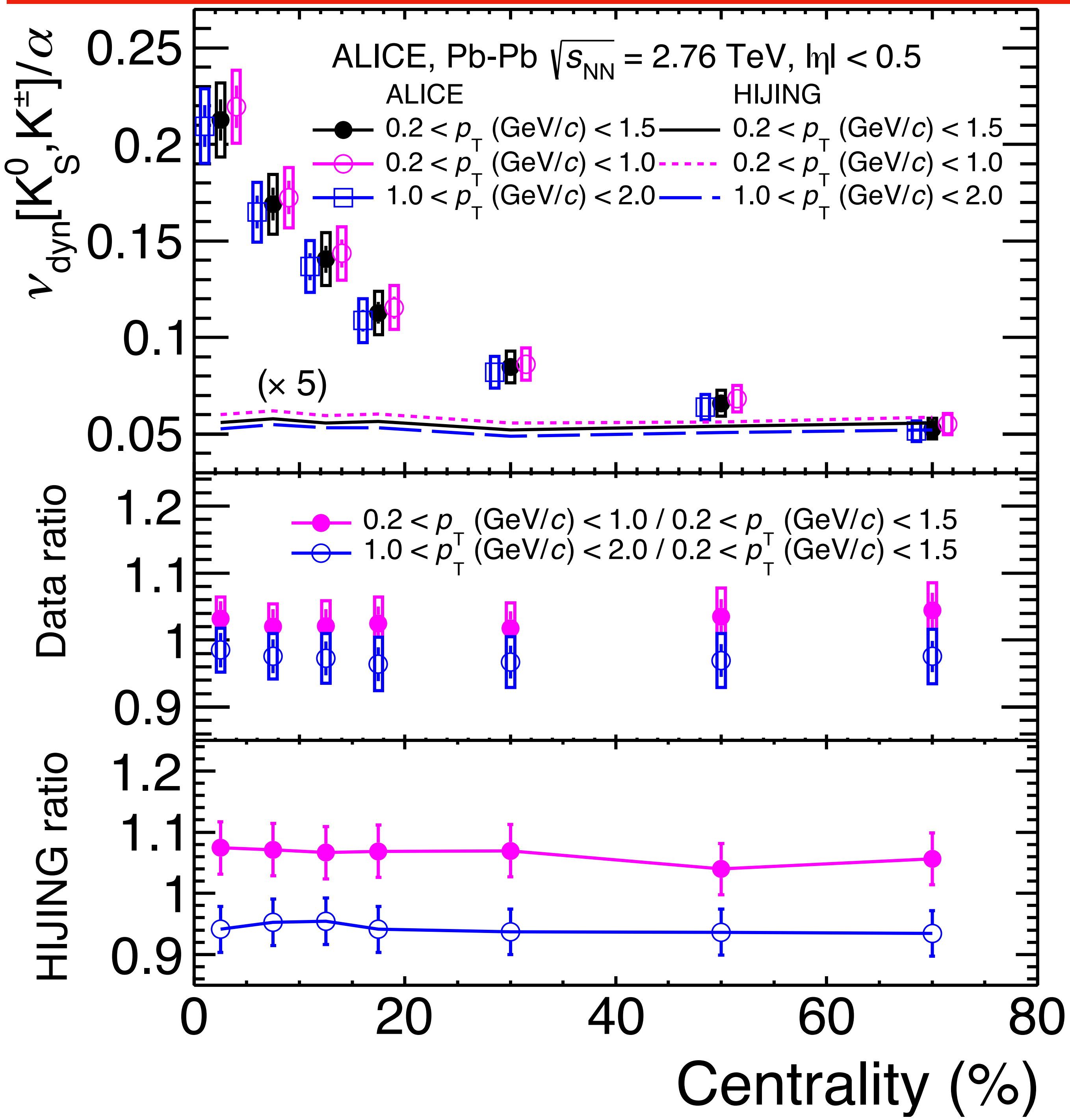
p_T Range Dependence ??

DCC expected to be more prominent at lower p_T

Study p_T dependence of $\nu_{dyn}[K_s^0 K^{\pm}]$



- $\nu_{dyn}[K_s^0 K^{\pm}]$
- Data:
  - Scaling violation in both p_T ranges
  - Marginally weaker at "higher" p_T
  - p_T dependence within systematic errors.
- HIJING:
  - Amplitude exhibits small (finite) dependence on p_T range
- **No evidence for a DCC "surge" at low p_T.**

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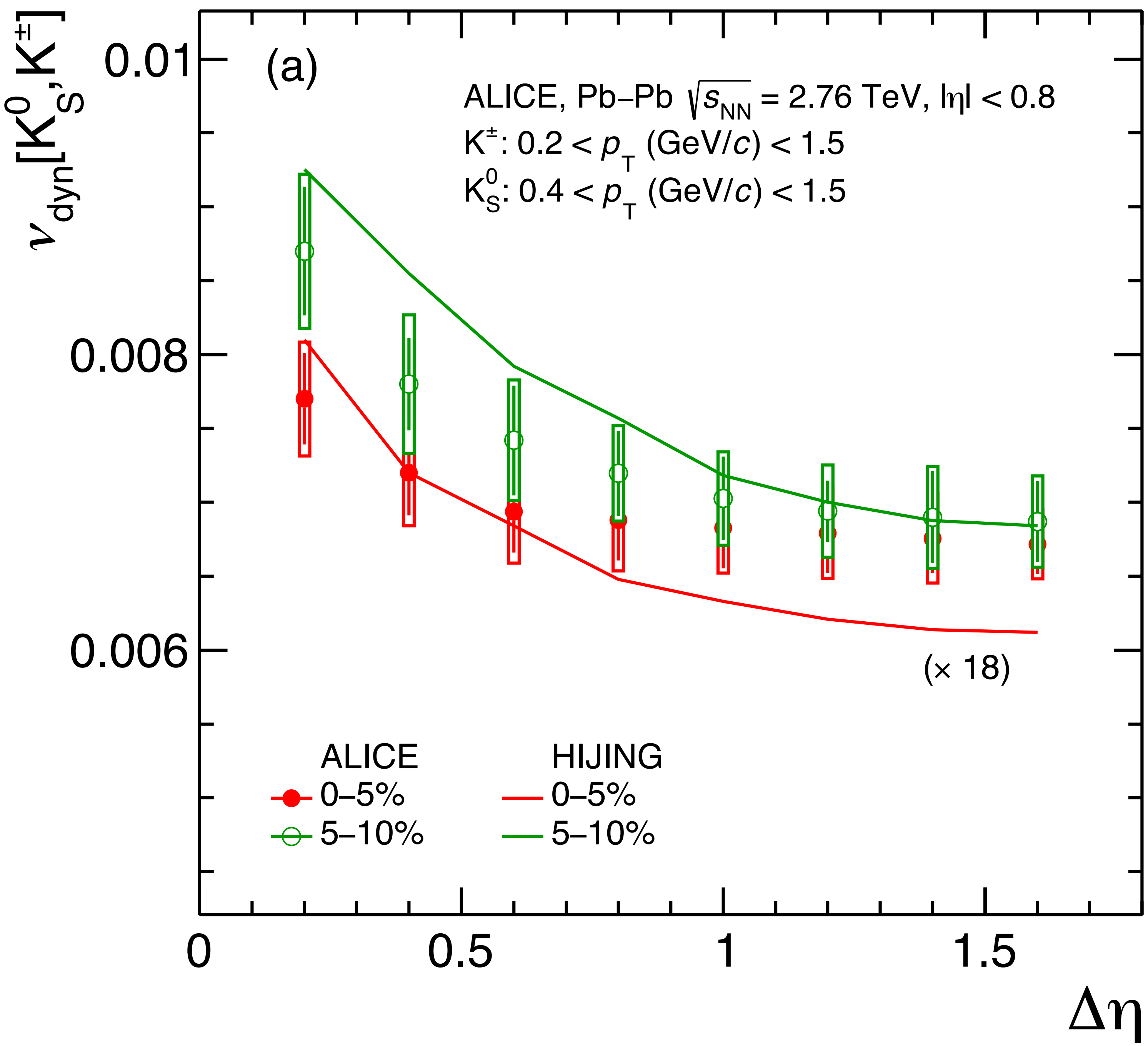
Rapidity Range Dependence



Δη correlation features "narrow" peak atop broad distribution.
Not readily compatible with DCC production.

DCC & DIC Theoretical Models

J. Kapusta, S. Pratt, M., Singh, Phys.Rev.C 107 (2023) 014913.

- DCC: Disoriented Chiral Condensate Model
- Examined several scenarios of kaon production, e.g., charge conservation effects, Bose symmetrization, resonance decays, degenerate kaons from condensates.
- Concluded condensates provide the only way to explain ALICE results.

J. Kapusta, S. Pratt, M., Singh, 2306.13280 [hep-ph]

- DIC: Disoriented Isospin Condensate Model

- ***"If the scalar condensate, which is typically associated with chiral symmetry, is accompanied by an isospin=1 field, then the combination can produce large fluctuations where $\langle \bar{u}u \rangle \neq \langle \bar{d}d \rangle$.***
- **Hadronizing strange and anti-strange quarks might then strongly fluctuate between charged ($u\bar{s}$, $s\bar{u}$) and neutral ($d\bar{s}$ or $s\bar{d}$) kaons"**

Summary

- Presented studies of integral correlations — particularly fluctuations of conserved quantities.
- Models typically fair poorly relative to the measured data —need for increased focus and ideas from the theory community.
- Many possible extensions to these studies
    - Based on more recent data, new collision systems and beam energies
    - Higher moments,
    - Identified particles, including weakly decaying particles
    - Also consider more differential measurements — correlation functions and balance functions.

Thank you for your attention

Summary

- Presented several studies of integral correlations — including fluctuations of conserved quantities.
- Models typically fair poorly relative to the measured data
  - Need for increased focus and ideas from the theory community.
  - Need to check w/ more modern models (e.g., EPOS4)
- Many possible extensions to these studies
  - Lots of new data, new collision systems and beam energies
  - Higher moments,
  - Identified particles, including weakly decaying particles
  - Also consider more differential measurements — correlation functions and balance functions.

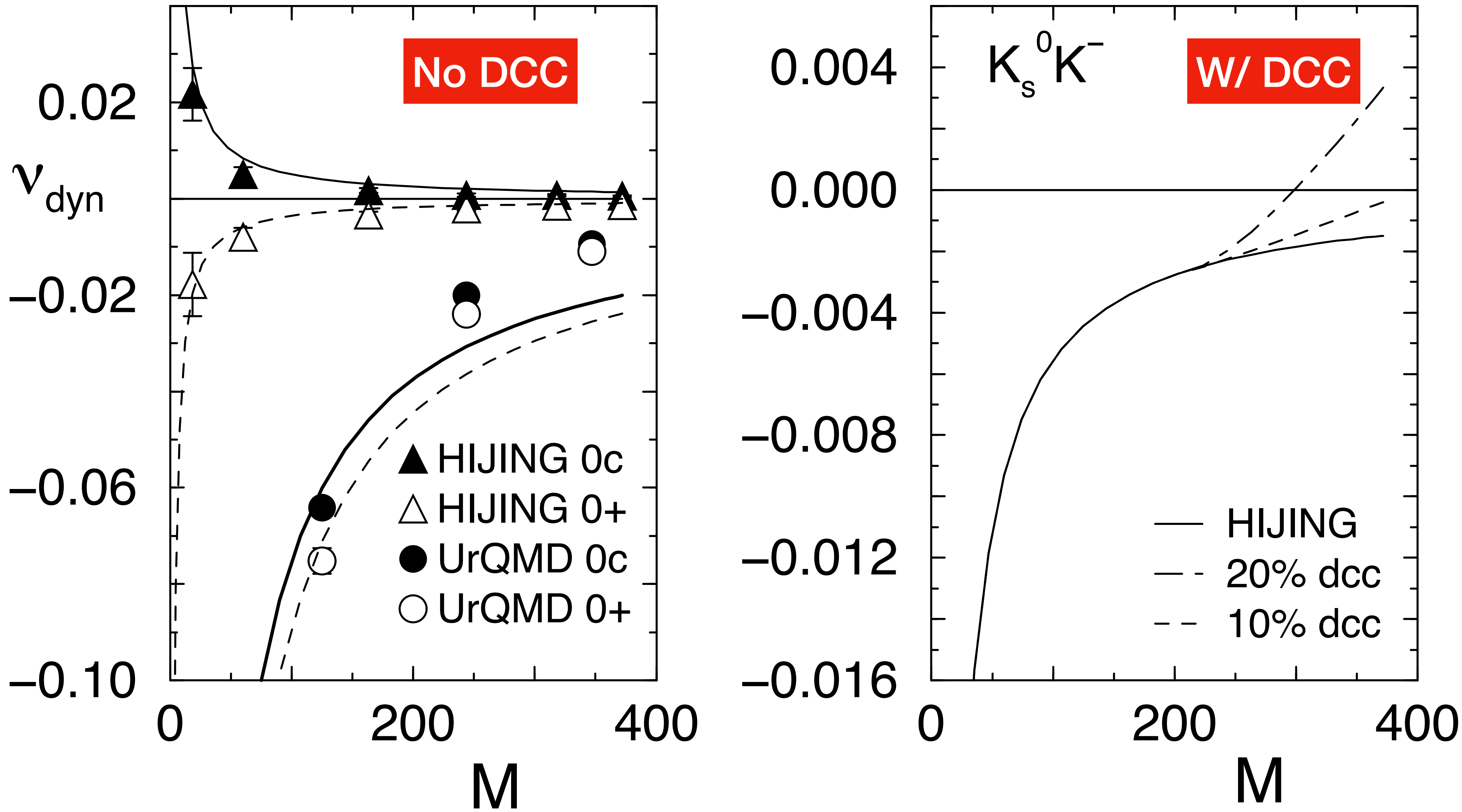
Thank you for your attention

Additional Material

DCC in Kaon Sector Detectable w/ $\nu_{\rm dyn}[K_s^0, K^{\pm}]$

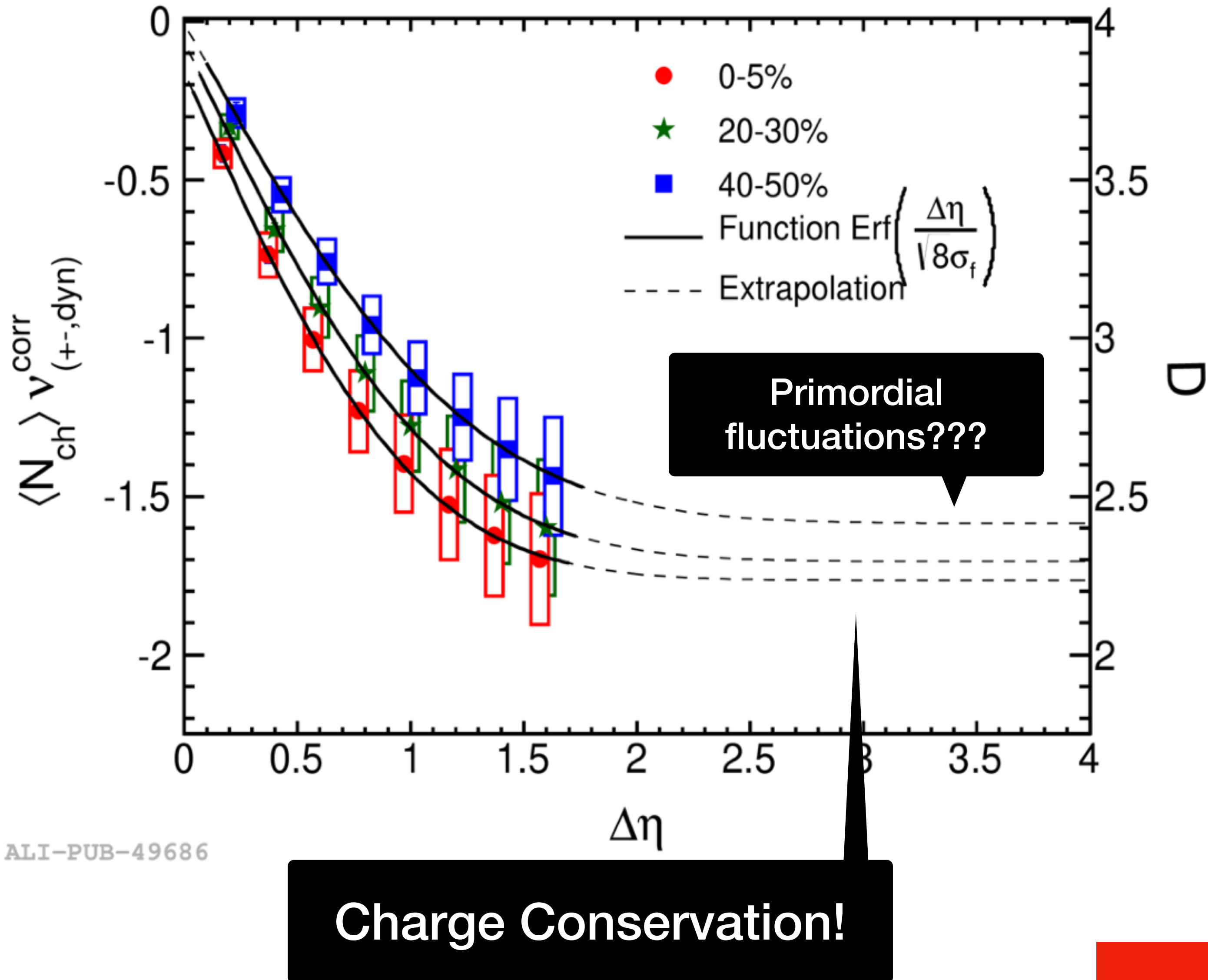
S. Gavin, J. Kapusta PRC 65 (2002) 054910
S. Gavin, et al., Nucl.Phys.A 715 (2003) 657, J.Phys.G 30 (2004) S271

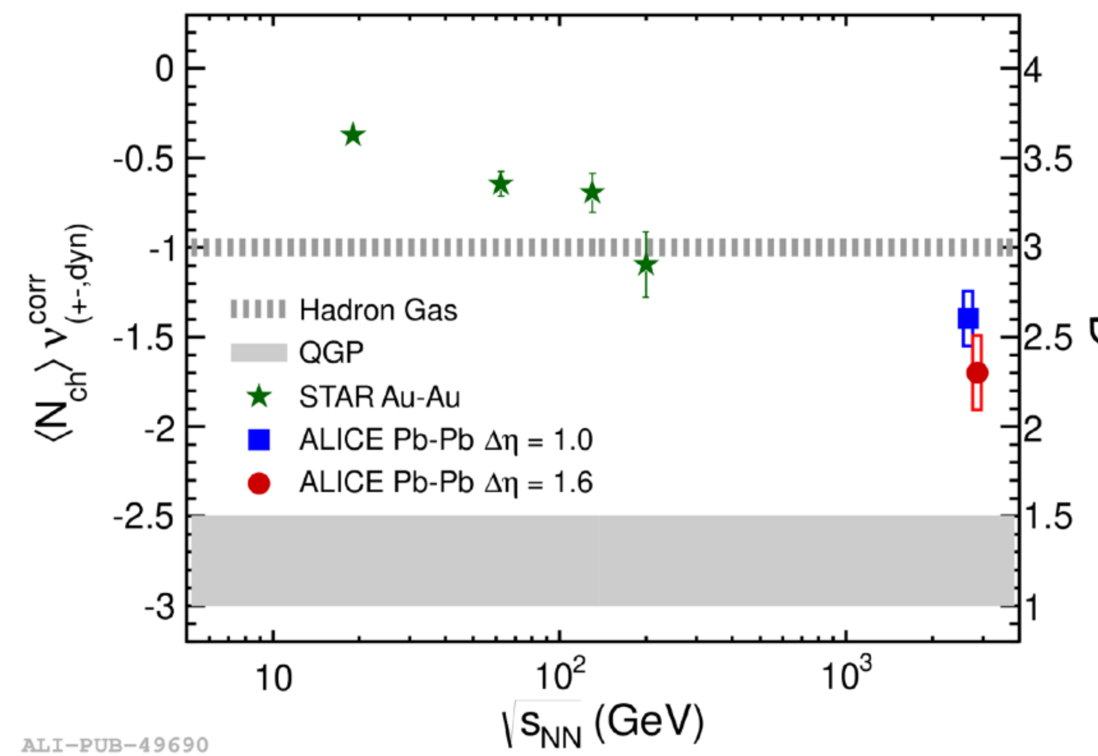
Kaon isospin fluctuations measurable with $\nu_{\rm dyn}$ observable.



ALICE, Phys. Rev. Lett. 110 (2013) 152301

ALICE Results: Pb—Pb @ $\sqrt{s_{NN}} = 2.76$ TeV



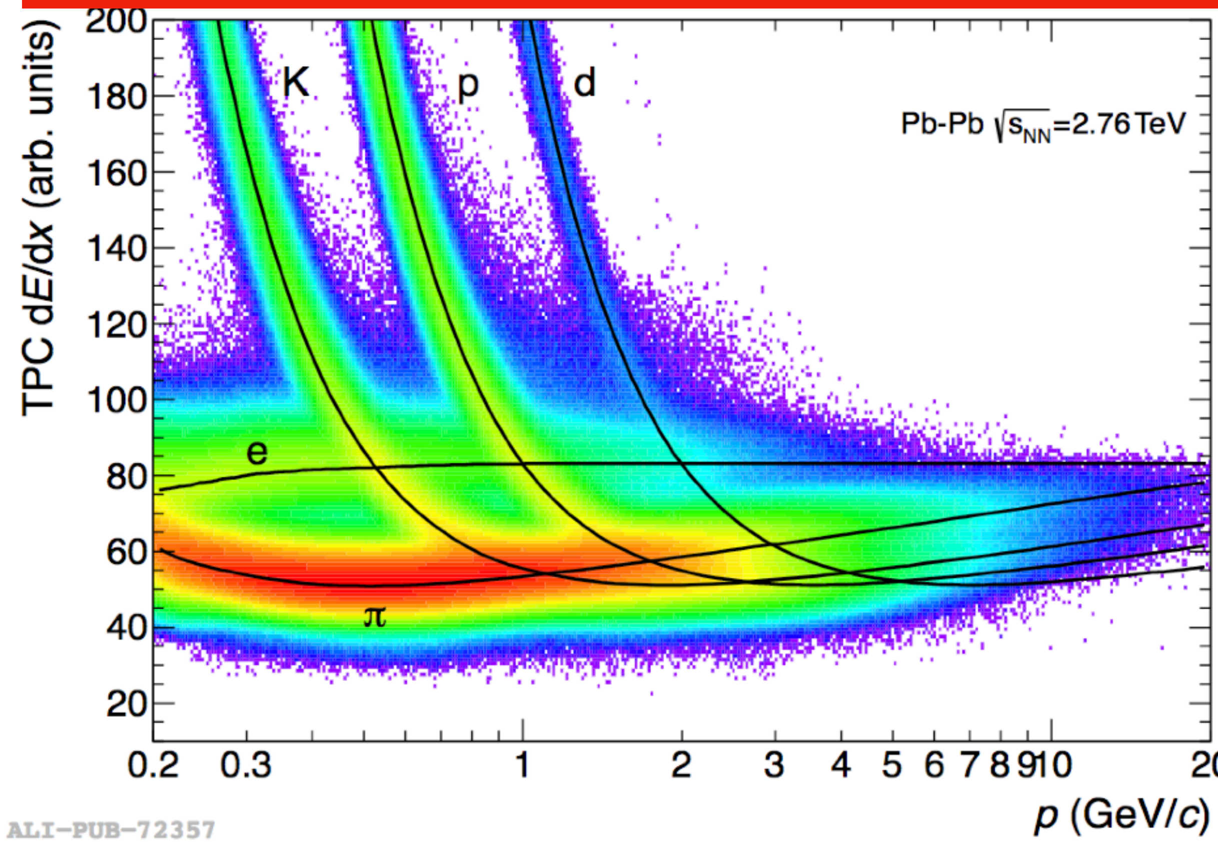


Scaled nu-dyn vs. beam energy trends towards $D \sim 1$ with increasing $\sqrt{s}$??
QGP or some other (trivial) effect ??

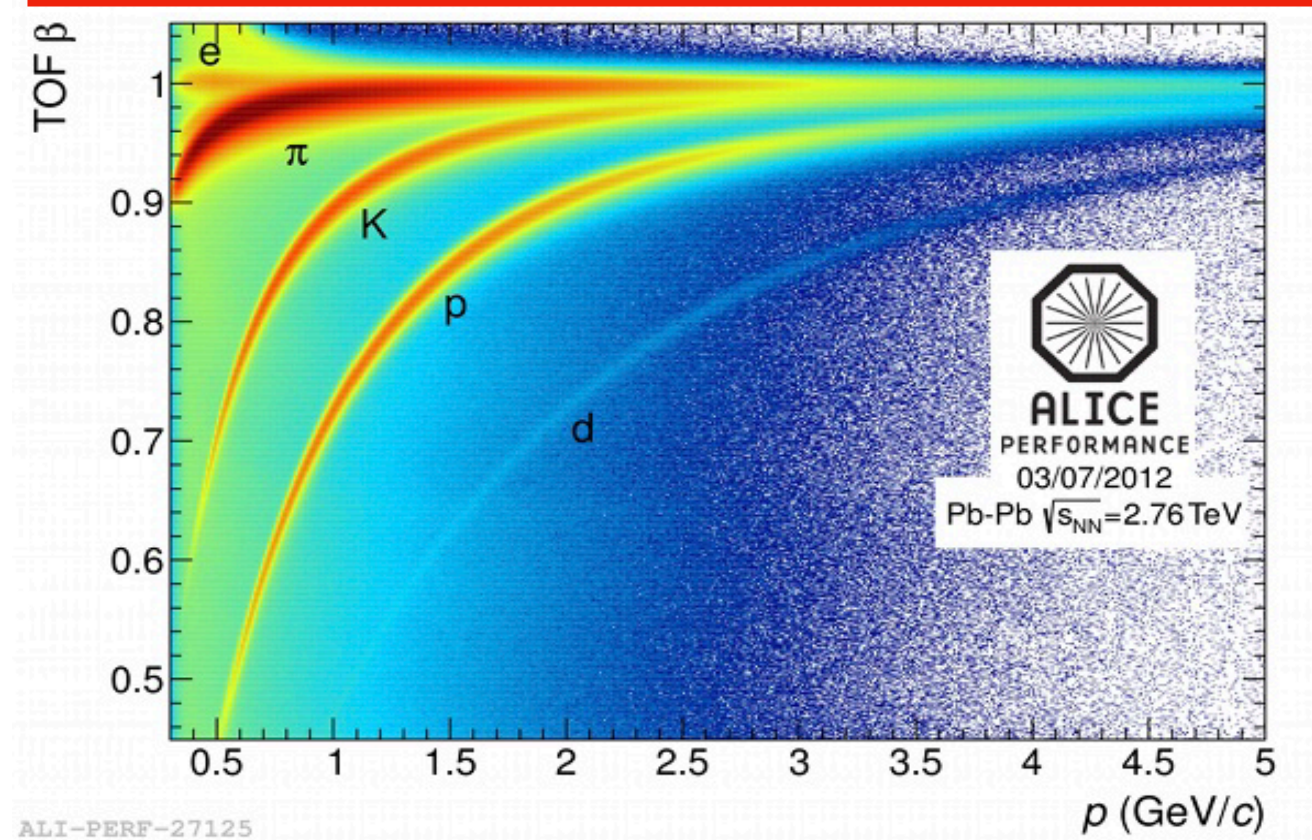
- **Opportunities for new measurements?**
  - **PbPb at 5.02 TeV, XeXe at 5.4 TeV, etc**
  - My take: **Consider differential measurements instead —> balance functions**.

$\pi^{\pm}$, K$^{\pm}$, p, $\bar{\mathrm{p}}$ identification/classification

TPC: $\mathrm{d}E/\mathrm{d}x$ vs p



TOF velocity vs p



$n\sigma$ method:

- Event-by-event Counting
- Candidates: $n\sigma$ method
  - $$n\sigma = \frac{1}{\sigma\left(\frac{\mathrm{d}E}{\mathrm{d}x}\right)}\left[\left|\frac{\mathrm{d}E}{\mathrm{d}x}\right|_{\text{measured}} - \left|\frac{\mathrm{d}E}{\mathrm{d}x}\right|_{\text{particle}}\right]$$
  - Similarly w/ TOF signal.
  - **Contamination 1-3 %**

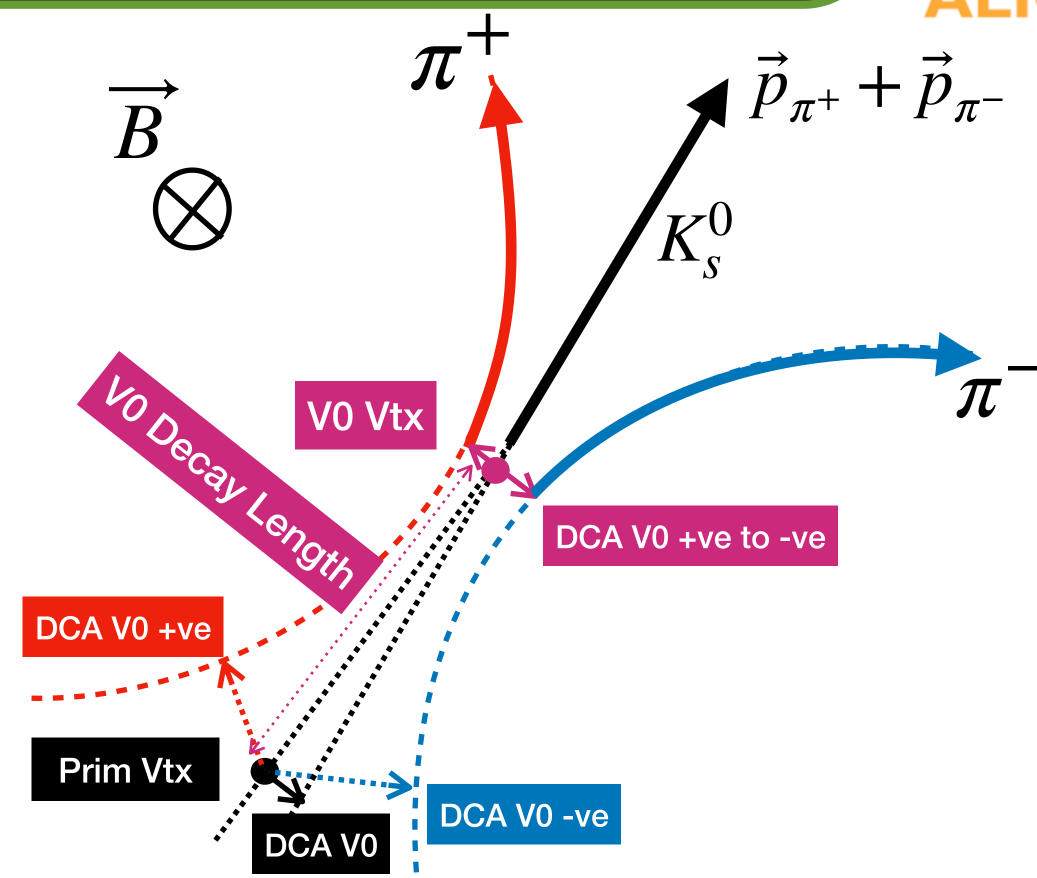
Identity method:

[1] M. Gazdzicki et al., Phys. Rev.C 83 (2011) 054907
[2] M.I. Gorenstein, Phys.Rev.C 84 (2011) 024902
[3] A. Rustamov, Phys.Rev.C 86 (2012) 044906
[4] C. Pruneau, Phys.Rev.C 96 (2017) 5, 054902
[5] C. Pruneau, Alice Ohlson, Phys.Rev.C 98 (2018) 1, 014905

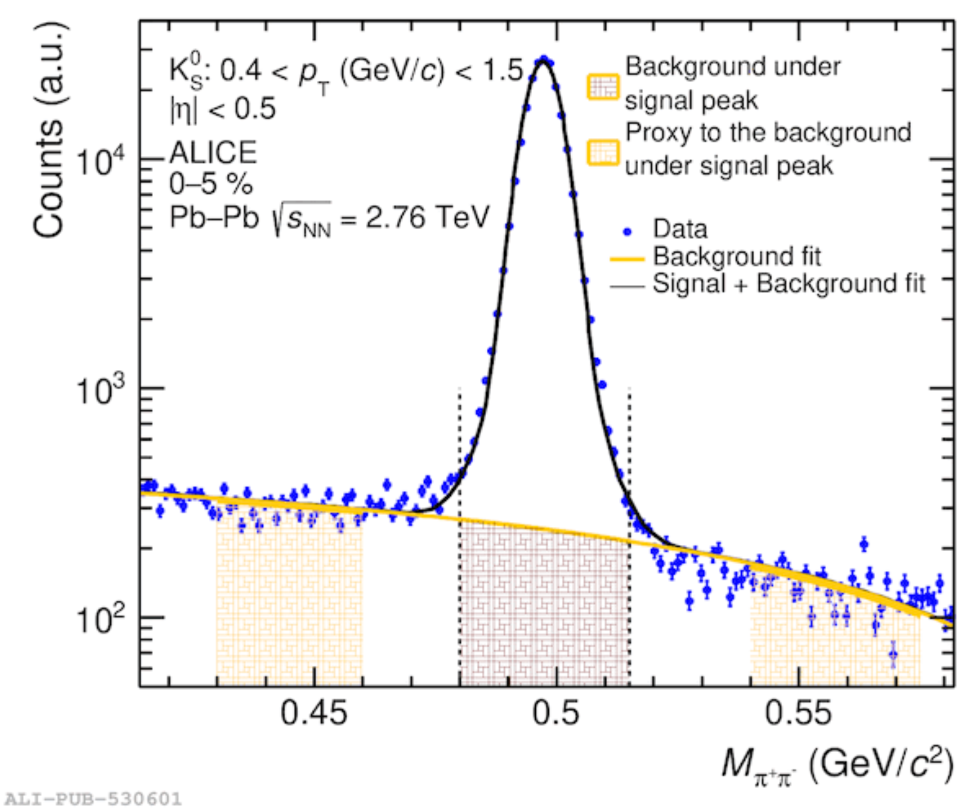
K_s^0 identification & selection

ALICE, Phys. Lett. B 832 (2022) 137242

- Standard ALICE topological (V0) selection criteria,
  - See backup for details.
- Invariant mass selection,
- Kinematic selection:
  - $|y| < 0.5$,
  - $0.4 < p_T < 1.5$ GeV/c.



- Event-by-event Counting:
- Candidates:
  - $0.48 < M_{inv}(\pi^+\pi^-) < 0.515$ GeV/c^2
  - **Contamination 1-4 %**
- Background (fluctuations) estimate:
  - From side bands



ALICE, Phys. Lett. B 832 (2022) 137242

Combinatorial background and correction

Single yields:

N_c : Number of $K^{\pm}$

N_s : Number of signal K^0_s

N_b : Number of background pairs

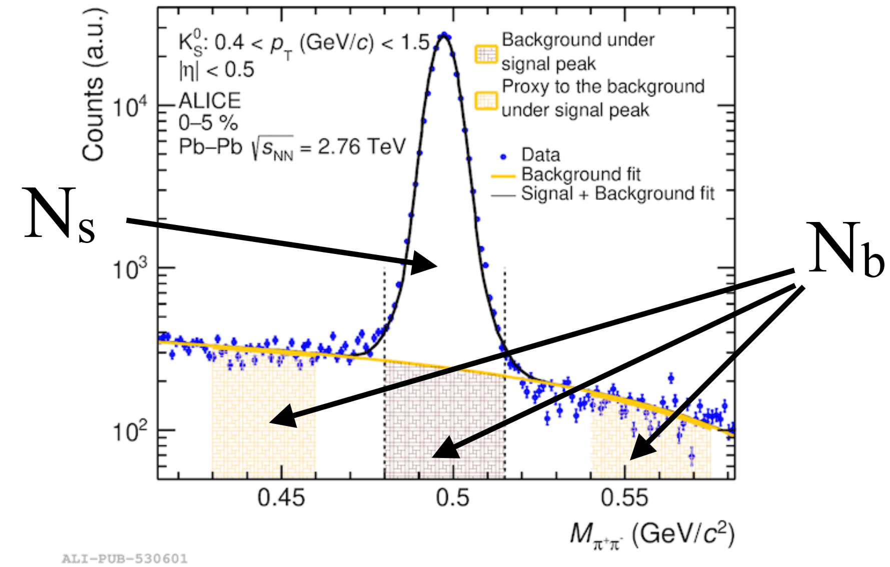
$N_0 = N_s + N_b; \quad f_b = N_b / N_0$

Pair yields

$N_{00} = N_{ss} + N_{bb} + 2N_{sb}$

$N_{ss} = N_{00} - N_{bb} - 2N_{sb}$

$N_{sc} = N_{0c} - N_{bc}$



Use “side windows” to estimate yield of background in the signal region. Example:

$$\frac{< N_s(N_s-1) >}{< N_s >^2} = \frac{< N_0(N_0-1) >}{< N_0 >^2} - \frac{2f}{(1-f)^2}\frac{< N_0 N_b >}{< N_0 >< N_b >} + \frac{f^2}{(1-f)^2}\frac{< N_b N_b >}{< N_b(N_b-1) >< N_b >}$$

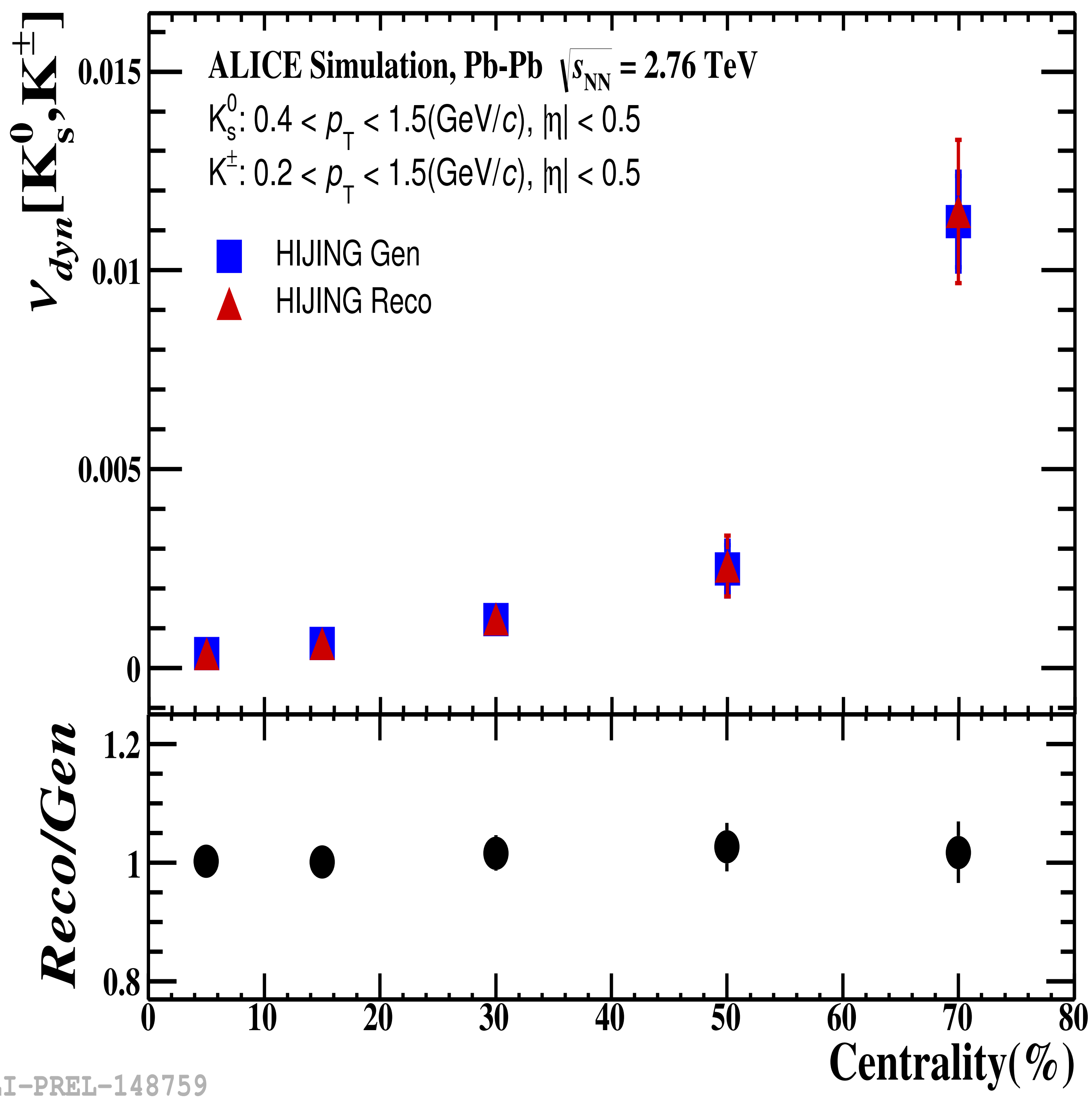
Corrected ν_{dyn} based on side mass windows

$$\nu_{dyn}^{corrected} = \frac{< N_c(N_c-1) >}{< N_c >^2} + \frac{< N_s(N_s-1) >}{< N_s >^2} - 2\frac{< N_c N_s >}{< N_c >< N_s >}$$

ALICE, Phys. Lett. B 832 (2022) 137242

Closure test

- Test performed with HIJING + ALICE/GEANT
- Analysis done at
  - Generator level (Gen)
  - GEANT processed + full reconstruction (Reco)



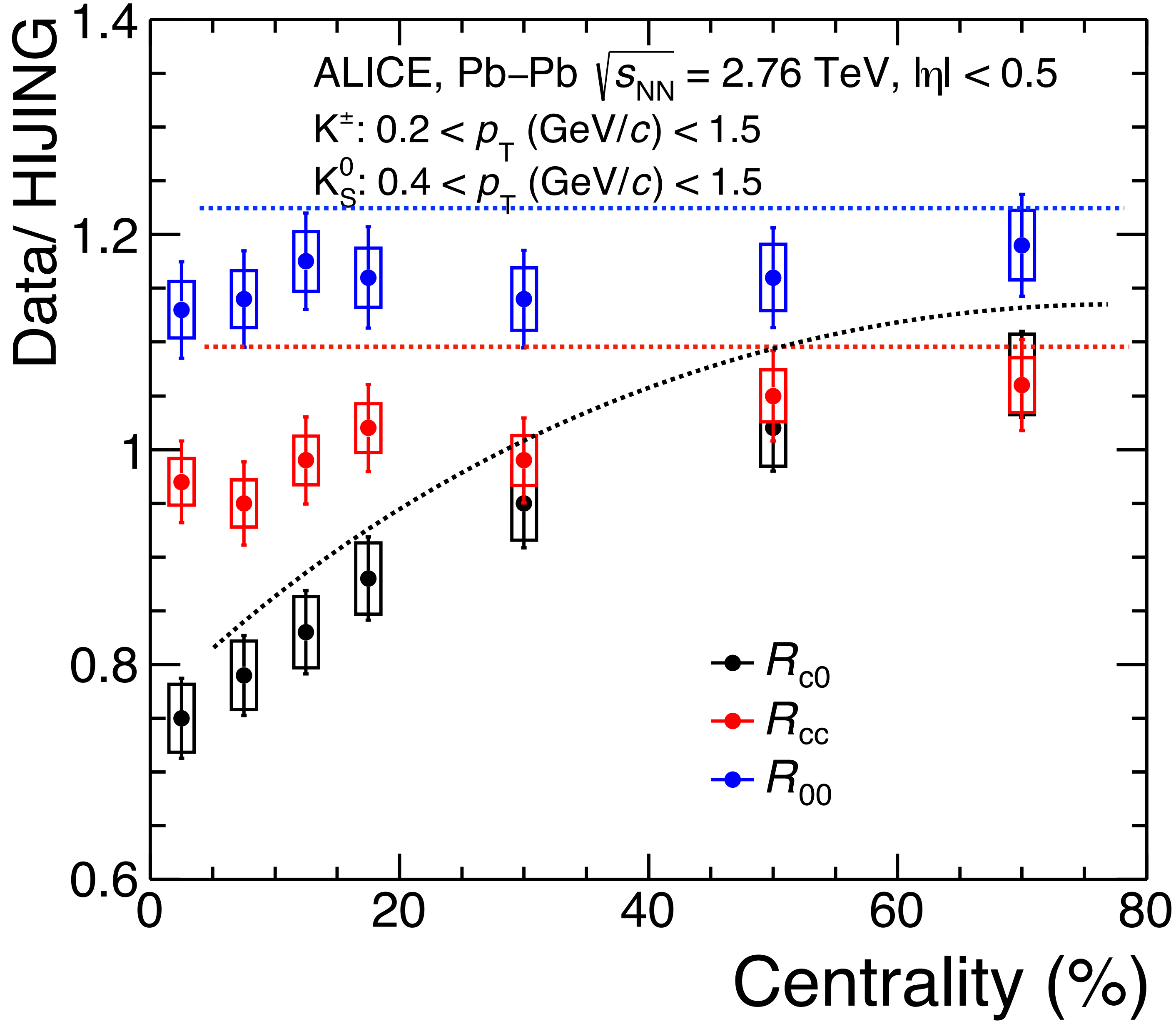
ALICE, Phys. Lett. B 832 (2022) 137242

Where is the excess (scaling violation) from?

Exploit HIJING approximate 1/N scaling

Study ratios of data to HIJING for three terms of ν_{dyn}

$$\nu_{dyn}^{K^+K^-} = R_{00} + R_{cc} - 2R_{c0}$$



- Variance terms have little to no centrality dependence.
  - Approx. 1/N scaling.
- Covariance term varies by more than 20% with centrality.
- Excess of ν_{dyn} in central collisions from the covariance term.
  - **Expected from fluctuations caused by DCC fluctuations.**

Measurements & Observables

- Φ_q: S. Mrowczynski, Phys.Rev.C 66 (2002) 024904.
- $\nu_{\rm dyn}$ methods Paper - C.P., S.G., S.V. - PRC 66, 44904 (2002).
- Some key Measurements…
  - **PHENIX** (130 GeV) - K. Adcox, et al., Phys.Rev.Lett. 89, 082301 (2002). → $\omega_Q \equiv \frac{\langle \Delta Q^2 \rangle}{\langle N_{CH} \rangle}$
  - **STAR**
    - AuAu @ 130 GeV, PRC68, 044905 (2003).
    - AuAu @ 20, 130, 200 GeV, Phys.Rev.C 79 (2009) 024906.
    - Higher-moments: *Nucl.Phys.A* 830 (2009) 555C-558C.

    → ν_{dyn}
  - **NA49** (20, 30, 40, 80, 158 A GeV), *Phys.Rev.C* 70 (2004) 064903. → Φ_q
  - **CERES** J.Phys.G 30 (2004) S1371-S1376.
  - **ALICE**, Phys. Rev. Lett. 110 (2013) 152301. → ν_{dyn}

Mathematical Foundation

- Consider ``identical" systems characterized by a **discrete or continuous observable X.**
  - Assume X fluctuates event-by-event according to a **probability distribution function (discrete) or a probability density function:** $p(x)$.
- **Moments** of X of order $n = 1,2, \ldots$ defined as **expectation value**

$$m_1' \equiv \mathrm{E}[x] \equiv \langle x \rangle = \int_{-\infty}^{\infty} x\, p(x)\mathrm{d}x$$

mean of X.

$$m_2' \equiv \mathrm{E}[x^2] \equiv \langle x^2 \rangle = \int_{-\infty}^{\infty} x^2\, p(x)\mathrm{d}x$$

$$m_3' \equiv \mathrm{E}[x^3] \equiv \langle x^3 \rangle = \int_{-\infty}^{\infty} x^3\, p(x)\mathrm{d}x$$

etc

- **Centered moments**, for $n \geq 2$:

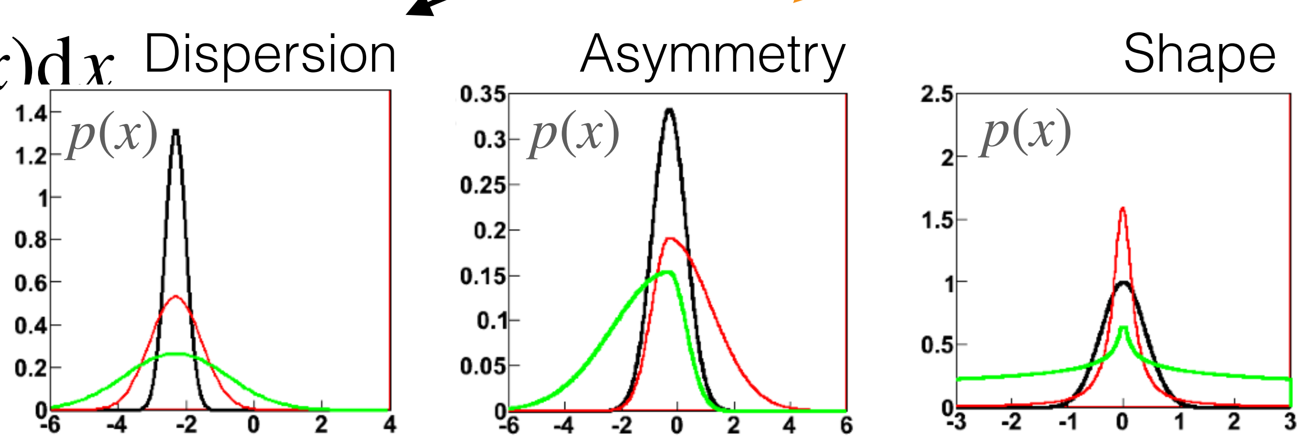
$$m_2 \equiv \langle \Delta x^2 \rangle = \int_{-\infty}^{\infty} \left(x - \langle x \rangle\right)^2\, p(x)\mathrm{d}x$$

variance of X: Var[X] or σ^2.

$$m_3 \equiv \langle \Delta x^3 \rangle = \int_{-\infty}^{\infty} \left(x - \langle x \rangle\right)^3\, p(x)\mathrm{d}x$$

$$m_4 \equiv \langle \Delta x^4 \rangle = \int_{-\infty}^{\infty} \left(x - \langle x \rangle\right)^4\, p(x)\mathrm{d}x$$

m_3, m_4 related to **skewness and kurtosis**…



Net-Charge Fluctuations

- **Poisson Distribution** (Fluctuations w/ no correlations)

$$P(n|\mu) = \frac{\mu^n e^{-\mu}}{n!} \quad \Rightarrow \quad \langle n \rangle = \mu, \quad \mathrm{Var}[n] = \langle \Delta n^2 \rangle = \mu$$

- **Poisson Limit** for the variance and covariance of N_α, N_β:

$$Var[N_\alpha] \equiv \langle \Delta N_\alpha^{\,2} \rangle = \langle N_\alpha \rangle$$

$$Cov[N_\alpha, N_\beta] \equiv \langle \Delta N_\alpha \Delta N_\beta \rangle = 0$$

Finite but "trivial" because it is devoid of correlations and **information**

- Poisson Limit of "nu"

$$\frac{\langle \Delta R^2 \rangle}{\langle R \rangle^2} = \frac{\langle N_\alpha \rangle}{\langle N_\alpha \rangle^2} + \frac{\langle N_\beta \rangle}{\langle N_\beta \rangle^2} - 2 \times 0 \quad \Rightarrow \quad \nu_{\text{Poisson}} = \frac{1}{\langle N_\alpha \rangle} + \frac{1}{\langle N_\beta \rangle}$$

Generating Functions

- Method to systematically compute moments in terms of a moment generating function $M(t)$:

$$M(t) \equiv \mathrm{E}[e^{tx}] = \int_{-\infty}^{\infty} e^{tx} p(x) \mathrm{d}x$$

- Expressing e^{tx} as Taylor series, one verifies:

$$m'_n = \left. \frac{\partial^n M(t)}{\partial t^n} \right|_{t=0}$$

Derivatives are first calculated at order n and then evaluated at $t = 0$.

- Also convenient to introduce a cumulant generating function according to :

$$G(t) \equiv \log\left(\mathrm{E}[e^{tx}]\right) = \log M(t)$$

- Cumulants, often denoted C_n, are then computing according to

$$C_n = \left. \frac{\partial^n G(t)}{\partial t^n} \right|_{t=0}$$

- One similarly defines joint cumulants of multiple variables and joint generating functions.

See my textbook for details.

Cumulants

- Given $G(t) = \log[M(t)]$, it is straightforward to express cumulants in terms of moments (and conversely) :

$$C_1 = m'_1$$
$$C_2 = m'_2 - {m_1}'^2$$
$$C_3 = m'_3 - 3m'_2 m'_1 + 2{m_1}'^3$$
$$C_4 = m'_4 - 4m'_3 m'_1 - 3{m_2}'^2 + 12 m'_2 {m_1}'^2 - 6{m_1}'^4$$

… and so on …

- Measurements of **fluctuations of net quantum numbers**
  - **Cumulants C_n of Net** Charge, Strangeness, Baryon number.
    - **Related to QGP susceptibilities χ_q^n.**
    - **Search for critical point of nuclear matter.**
- Measurements of particle **multiplicity fluctuations**
  - **Cumulants C_2 of $N_{\rm ch}$.**
    - **Nominally related to the compressibility of nuclear matter.**

Generating Functions

- **Densities**

$$\rho_1^{\alpha}(\vec{p}) = \frac{\mathrm{d}N}{\mathrm{d}\vec{p}} \qquad \rho_2^{\alpha\beta}(\vec{p}_1,\vec{p}_2) = \frac{\mathrm{d}^2N}{\mathrm{d}\vec{p}_1\mathrm{d}\vec{p}_2}, \ldots$$

- **Factorial Moments are denoted f_n,**
  - Integrals of n-particle densities $\rho_n(\vec{p}_1, \ldots, \vec{p}_n)$

$$f_1^{\alpha} \equiv \int_{\Omega} \rho_1^{\alpha}(\vec{p})\mathrm{d}\vec{p} = \langle N^{\alpha} \rangle$$

→ **Average number of particles**

$$f_2^{\alpha\beta} \equiv \int_{\Omega} \rho_2^{\alpha\beta}(\vec{p}_1,\vec{p}_2)\mathrm{d}\vec{p}_1\mathrm{d}\vec{p}_2 = \langle N^{\alpha}(N^{\beta} - \delta_{\alpha\beta}) \rangle$$

→ **Average number of pairs**

$$f_3^{\alpha\beta\gamma} \equiv \int_{\Omega} \rho_3^{\alpha\beta\gamma}(\vec{p}_1,\vec{p}_2,\vec{p}_3)\mathrm{d}\vec{p}_1\mathrm{d}\vec{p}_2\mathrm{d}\vec{p}_3 = \langle N^{\alpha}(N^{\beta} - \delta_{\alpha\beta})(N^{\gamma} - \delta_{\alpha\gamma} - \delta_{\beta\gamma}) \rangle$$

→ **Average number of triplets**

And so on…

- Factorial moments generating functions:

$$G_{\mathrm{f}}(\vec{s}) \equiv \left\langle \prod_{\alpha=1}^{K} s_{\alpha}^{N_{\alpha}} \right\rangle = \sum_{\vec{N}} \left(\prod_{\alpha=1}^{K} (s_{\alpha})^{N_{\alpha}} \right) P(\vec{N})$$

  - s_{α} represent K distinct auxiliary variables associated with the K stochastic yields N_{α}, $\alpha = 1,\ldots,K$

- Introducing $\partial_{s_{\alpha}} = \partial/\partial s_{\alpha}$, one verifies

$$f_{\nu_1\nu_2\ldots\nu_K} = \partial_{s_K}^{\nu_K}\cdots\partial_{s_2}^{\nu_1}\partial_{s_1}^{\nu_1} G_{\mathrm{f}} \Big|_{\vec{s}=1}$$

corresponding to mixed factorial moments of order $\nu_1, \nu_2, \ldots, \nu_K$ for species 1 to K.

Factorial Cumulant and Generating Functions

- Provide **another tool to characterize particle production**
  - **Directly related to differential correlation functions**
- Definition of the **factorial cumulant generating function**:

$$G_{\rm F}(\vec{s}) \equiv \ln G_{\rm f}(\vec{s}) = \ln \sum_{\vec{N}} \left(\prod_{\alpha=1}^{K} (s_\alpha)^{N_\alpha} \right) P(\vec{N}),$$

- Factorial cumulants are defined and computed according to

$$F_{\nu_1\nu_2\ldots\nu_K} \equiv \partial_{s_K}^{\nu_K}\cdots\partial_{s_2}^{\nu_2}\partial_{s_1}^{\nu_1} G_{\rm F}\Big|_{\vec{s}=1}$$

Singles:

$$F_1^\alpha = \partial_s G_{\rm F}\Big|_{\vec{s}=1} = \langle N^\alpha \rangle$$

Pairs:

$$F_2^{\alpha\beta} = \partial_{s_\alpha}\partial_{s_\beta} G_{\rm F}\Big|_{\vec{s}=1}$$
$$= f_2^{\alpha\beta} - f_1^\alpha f_1^\beta = \langle N^\alpha (N^\beta - \delta_{\alpha\beta})\rangle - \langle N^\alpha \rangle \langle N^\beta \rangle$$

Triplets:

$$\begin{aligned} F_3^{\alpha\beta\gamma} = & \langle N^\alpha (N^\beta - \delta_{\alpha\beta})(N^\gamma - \delta_{\alpha\gamma} - \delta_{\beta\gamma})\rangle \\ & - \langle N^\alpha (N^\beta - \delta_{\alpha\beta})\rangle \langle N^\gamma \rangle - \langle N^\alpha (N^\gamma - \delta_{\alpha\gamma})\rangle \langle N^\beta \rangle \\ & - \langle N^\beta (N^\gamma - \delta_{\beta\gamma})\rangle \langle N^\alpha \rangle + 2\langle N^\alpha \rangle \langle N^\beta \rangle \langle N^\gamma \rangle \end{aligned}$$

And so on…

- **Factorial Cumulants are of interest because they identically vanish in the absence of correlations.**
- **Simple cumulants do not have that property.**

Normalized Factorial Cumulants

It is also useful to define **normalized factorial cumulants**

$$R_2^{\alpha\beta} = \frac{F_2^{\alpha\beta}}{F_1^{\alpha} F_1^{\beta}} = \frac{\langle N^\alpha \left(N^\beta - 1\right)\rangle - \langle N^\alpha\rangle\langle N^\beta\rangle}{\langle N^\alpha\rangle\langle N^\beta\rangle}$$

$$R_2^{\alpha\beta} = \frac{\langle N^\alpha \left(N^\beta - 1\right)\rangle}{\langle N^\alpha\rangle\langle N^\beta\rangle} - 1$$

And similarly for higher orders: $R_3^{\alpha\beta\gamma} = \frac{F_3^{\alpha\beta}}{F_1^{\alpha} F_1^{\beta} F_1^{\gamma}}$

…

Normalized Factorial Cumulants also identically vanish in the absence of correlation (Poisson statistics)

- Measurements of **fluctuations of relative species abundances**.
  - e.g., $K^\pm$ vs. $\pi^\pm$.
  - **With ν_{dyn}:**
  - $\nu_{\text{dyn}}^{\pi\text{K}} = R_2^{\pi\pi} + R_2^{\text{KK}} - 2R_2^{\pi\text{K}}$
  - **Related to the proximity of phase boundary and critical point.**
- Measurements of $\pi^\pm$ vs. π^0 and $K^\pm$ vs. K^0 **fluctuations**.
  - **With ν_{dyn}:**
    $\nu_{\text{dyn}}^{\text{K}^\pm\text{K}_s^0} = R_2^{\text{K}^\pm\text{K}^\pm} + R_2^{\text{K}_s^0\text{K}_s^0} - 2R_2^{\text{K}^\pm\text{K}_s^0}$
  - **Possible signatures of the production of disoriented chiral condensates**.
- Measurements of average (event-wise) pT fluctuations.
  - **Related to temperature fluctuations and the specific heat of nuclear matter.**

1/N Scaliing

By construction, the **factorial cumulants F_n scale in proportion to the number m of (non interacting) sources**.

With m sources of particle and non interacting particle from these sources, one has

$$F_n^{(m)} = mF_n^{(1)}$$

$F_n^{(1)}$: factorial cumulants for a single source
$F_n^{(m)}$: factorial cumulants for a superposition of m independent sources.

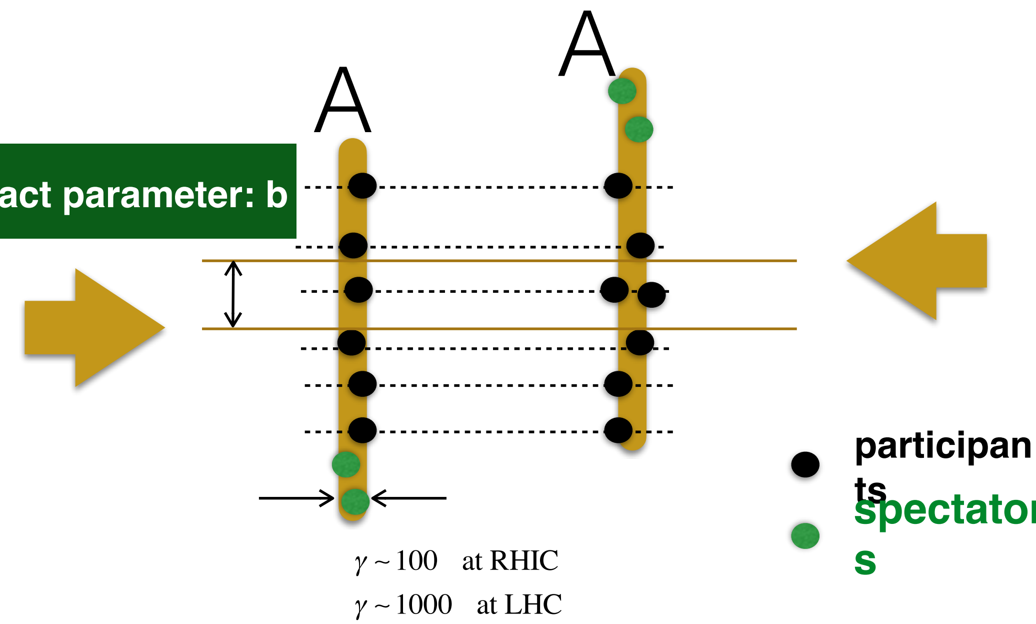
By construction of R_2, one gets:

$$R_2^{(m)} = \frac{F_2^{(m)}}{F_1^{(m)}F_1^{(m)}} = \frac{1}{m}R_2^{(1)}$$

1/m referred to as dilution of the correlation strength in the presence of a superposition of m independent and identical sources (on average).

By construction, $\nu_{\rm dyn}$ obeys the same scaling:

$$\nu_{\rm dyn}^{(m)} = \frac{1}{m}\nu_{\rm dyn}^{(1)}$$



$$R_2^{(m)} = \frac{R_2^{(1)}}{\langle N_{\rm ch}\rangle}$$

$$\nu_{\rm dyn}^{(m)} = \frac{\nu_{\rm dyn}}{\langle N_{\rm ch}\rangle}$$

DCC or DIC?

J. Kapusta, S. Pratt, M., Singh, 2306.13280 [hep-ph]

- $I = 0$ iso-singlet: $(\langle \bar{u}u \rangle + \langle \bar{d}d \rangle)/\sqrt{2}$
  - Lowest excitation: $f_0(500)$ or σ meson.
- $I = 1$ iso-triplet: $\langle \bar{d}u \rangle, (\langle \bar{u}u \rangle - \langle \bar{d}d \rangle)/\sqrt{2}, \langle \bar{u}d \rangle$
  - Lowest excitations: a_0^+, a_0^0, a_0^-, i.e, $a_0(980)$

- If only $I = 0$ field were present,
  - It should **couple equally to charged and neutral kaons**.
- If both $I = 1, I_3 = 0$ and $I = 0, I_3 = 0$ contribute in similar amounts,
  - They could combine to form nearly all $\langle \bar{u}u \rangle$ or all $\langle \bar{d}d \rangle$ condensates.
  - Provides seed for the formation of charged and neutral kaons, respectively… leading to isospin kaon fluctuations.
  - Authors given concrete estimates of number of DIC needed to explain measured values given the observed number of kaons.
- ***"Although the DIC mechanism investigated here is speculative, it seems to be the least questionable explanation for the ALICE measurement of $\nu_{\rm dyn}$ thus far".***

Past theoretical and experimental studies

- **Found 138 publications on DCCs in inspirehep.net**
- Mostly theoretical works on DCCs in the **pion sector**
  - *Evidences for New Type of Cosmic Ray Nuclear Interactions Named CENTAURO*, M. Tamada, Nuovo Cim B41 (1977) 245.
  - *Explosive Quark Matter and the CENTAURO Event*, J.D. Bjorken and L. McLerran, PRD 20 (1979) 2353.
  - ***Baked Alaska,*** J.D. Bjorken et al. SLAC-PUB-6109.
- **Few theoretical works on strange DCCs**
  - *Is anomalous production of Omega and anti-Omega evidence for disoriented chiral condensates?*, J. Kapusta, *et al.*, PRL 86 (2001) 4251.
  - ***Kaon and pion fluctuations from small disoriented chiral condensates***, S.Gavin, J. Kapusta, PRC 65 (2002) 054910.
  - Strange disoriented chiral condensate, S. Gavin, 18th WWND (2002).
- Very few experimental searches — **All with negative or non-conclusive results**:
  - Minimax @ Tevatron: J.D. Bjorken *et al.*, PRD 55 (1997) 5667; T.C. Brooks et al., PRD 61 (2000) 032003.
  - WA98 @ SPS: T. K. Nayak, Nucl.Phys. A 638 (1998) 249c; M.M. Aggarwal, PLB 701 (2001) 300.
  - STAR @ RHIC: S.M. Dogra *et al.*, J. Phys.G 35 (2008) 104094.
  - E864 @ AGS: P. Fachini (Thesis, Wayne State), *et al.,* APS Meeting, (1999).