Probing strongly interacting matter with clusters

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PHQMD

The 39th edition of the Winter Workshop on Nuclear Dynamics
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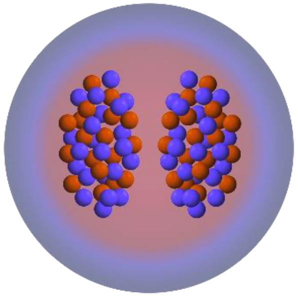
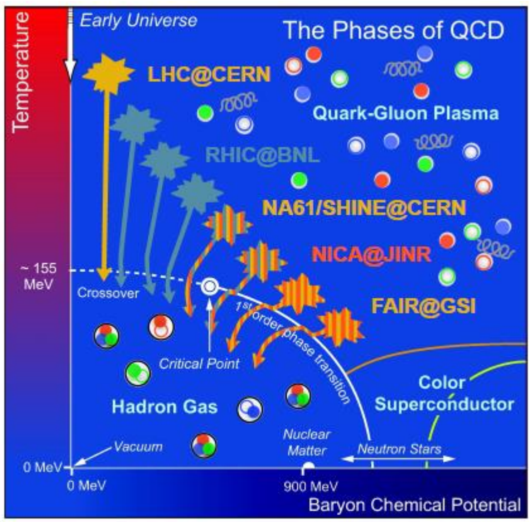


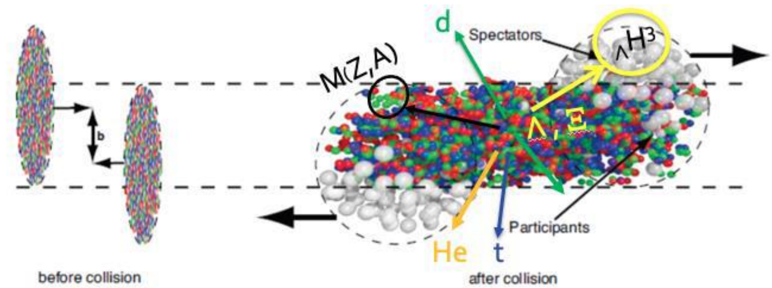
Cluster production in heavy-ion collisions

The phase diagram of QCD

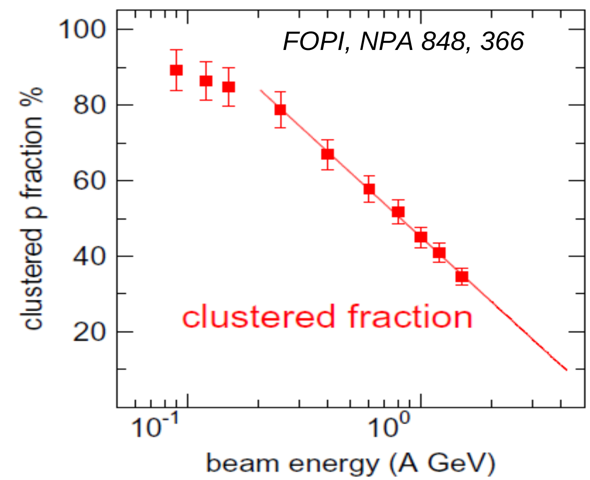


Clusters and (anti-) hypernuclei

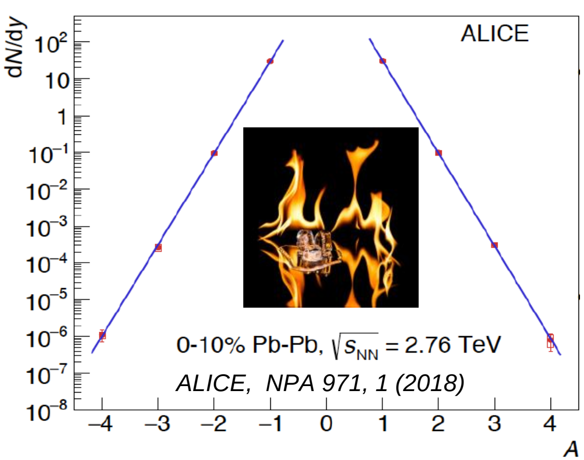
are observed experimentally at all energies



- **Clusters are very** **abundant at low energy**



Au+Au, central. midrapidity



- **High energy HIC:** **‚Ice in a fire' puzzle:** **how the weakly bound objects can be formed and survive in a hot enviroment?!**

➔ **Mechanisms of cluster formation in strongly interacting matter are not well understood**

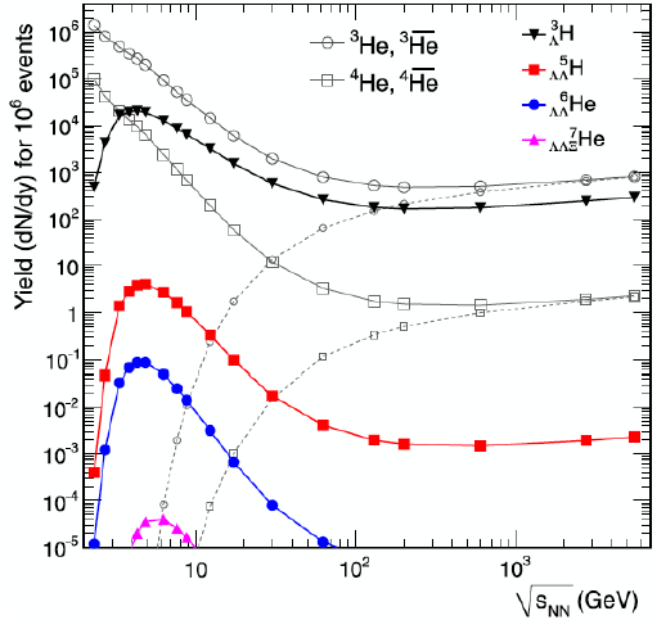
Modeling of cluster and hypernuclei formation

Existing models for cluster formation:

- **statistical model:**
  - assumption of thermal equilibrium
- **coalescence model:**
  - determination of clusters at a freeze-out time by coalescence radii in coordinate and momentum space

➔ **don't provide information on the dynamical origin of cluster formation**

A. Andronic et al., PLB 697, 203 (2011)



In order to understand the microscopic origin of cluster formation one needs a realistic model for the dynamical time evolution of the HIC

→ **transport models:**
dynamical modeling of cluster formation based on interactions:

– via potential interaction – 'potential' mechanism

-- by scattering – 'kinetic' mechanism

PHQMD

PHQMD

PHQMD: a unified n-body microscopic transport approach for the description of heavy-ion collisions and dynamical cluster formation from low to ultra-relativistic energies

Realization: combined model PHQMD = (PHSD & QMD) + (MST/SACA)

Parton-Hadron-Quantum-Molecular Dynamics

PHSD mode

Initialization → propagation of baryons:
QMD (Quantum-Molecular Dynamics)

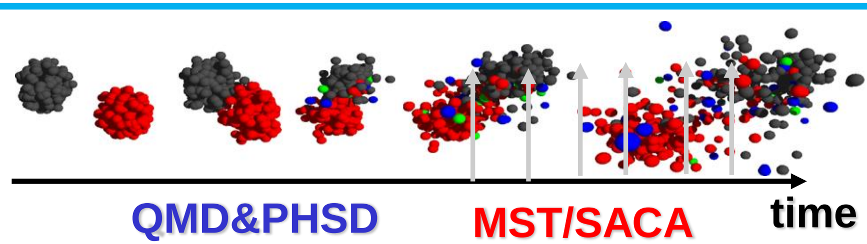
J.Aichelin, Phys. Rept. 202 (1991)

Propagation of partons (quarks, gluons) and mesons
+ collision integral = interactions of hadrons and partons (QGP)
from PHSD (Parton-Hadron-String Dynamics)

W. Cassing, E. Bratkovskaya, PRC 78 (2008) 034919; NPA831 (2009) 215

Cluster recognition:
MST (Minimum Spanning Tree)
or SACA (Simulated Annealing Clusterization Algorithm)

R. K. Puri, J. Aichelin, J.Comp. Phys. 162 (2000) 245-266



PHQMD:
J. Aichelin et al., PRC 101 (2020) 044905;
S. Gläßel et al., PRC 105 (2022) 1;
G. Coci et al., PRC 108 (2023) 1, 014902

QMD propagation (EoM)

- **Generalized Ritz variational principle:** $\delta \int_{t_1}^{t_2} dt < \psi(t)|i\frac{d}{dt} - H|\psi(t) >= 0.$

Assume that $\psi(t) = \prod_{i=1}^{N} \psi(\mathbf{r}_i, \mathbf{r}_{i0}, \mathbf{p}_{i0}, t)$ **for N particles (neglecting antisymmetrization !)**

[Aichelin, Phys. Rept. 202 (1991)]

Ansatz: trial wave function for one particle "*i*" :
Gaussian with width *L* centered at r_{i0}, p_{i0}

$$\psi(\mathbf{r}_i, \mathbf{r}_{i0}, \mathbf{p}_{i0}, t) = C \; \mathrm{e}^{-\frac{1}{4L}\left(\mathbf{r}_i - \mathbf{r}_{i0}(t) - \frac{\mathbf{p}_{i0}(t)}{m}t\right)^2} \cdot \; \mathrm{e}^{i\mathbf{p}_{i0}(t)(\mathbf{r}_i - \mathbf{r}_{i0}(t))} \cdot \; \mathrm{e}^{-i\frac{\mathbf{p}_{i0}^2(t)}{2m}t}$$

L=4.33 fm²

- **Equations-of-motion (EoM) for Gaussian centers in coordinate and momentum space:**

$$\dot{r_{i0}} = \frac{\partial\langle H\rangle}{\partial p_{i0}} \qquad \dot{p_{i0}} = -\frac{\partial\langle H\rangle}{\partial r_{i0}}$$

Hamiltonian: $H = \sum_i H_i = \sum_i (T_i + V_i) = \sum_i (T_i + \sum_{j\neq i} V_{i,j})$

2-body potential: $V_{i,j} = V(\mathbf{r_i}, \mathbf{r_j}, \mathbf{r_{i0}}, \mathbf{r_{j0}}, t)$

Local momentum dependent potential in PHQMD

PHQMD

New!

- **Nucleon-nucleon local two-body momentum dependent potential:**

$$V_{ij} = V(\mathbf{r}_i, \mathbf{r}_j, \mathbf{r}_{i0}, \mathbf{r}_{j0}, \mathbf{p}_{i0}, \mathbf{p}_{j0}, t)$$
$$= V_{\mathrm{Skyrme\ loc}} + V_{\mathrm{mom}} + V_{\mathrm{Coul}}$$
$$= \frac{1}{2}t_1\delta(\mathbf{r}_i - \mathbf{r}_j) + \frac{1}{\gamma+1}t_2\delta(\mathbf{r}_i - \mathbf{r}_j)\,\rho^{\gamma-1}(\mathbf{r}_i, \mathbf{r}_j, \mathbf{r}_{i0}, \mathbf{r}_{j0}, t)$$
$$+ V(\mathbf{r}_i, \mathbf{r}_j, \mathbf{p}_{i0}, \mathbf{p}_{j0}) + \frac{1}{2}\frac{Z_i Z_j e^2}{|\mathbf{r}_i - \mathbf{r}_j|},$$

Skyrme

momentum dependent

Coulomb

- **The single-particle potential resulting from the convolution of the distribution functions f_i and f_j with the interactions V_{Skyrme} + V_{mom} (local interactions including their momentum dependence) for symmetric nuclear matter:**

1) Skyrme potential ('static') :

$$\langle V_{Skyrme}(\mathbf{r_{i0}}, t)\rangle = \alpha\left(\frac{\rho_{int}(\mathbf{r_{i0}}, t)}{\rho_0}\right) + \beta\left(\frac{\rho_{int}(\mathbf{r_{i0}}, t)}{\rho_0}\right)^{\gamma}$$

with modified interaction density (with relativistic extension):

$$\rho_{int}(\mathbf{r_{i0}}, t) \rightarrow C\sum_j \left(\frac{4}{\pi L}\right)^{3/2} \mathrm{e}^{-\frac{4}{L}(\mathbf{r}^{\mathrm{T}}_{\mathrm{i0}}(t) - \mathbf{r}^{\mathrm{T}}_{\mathrm{j0}}(t))^2}$$
$$\times \mathrm{e}^{-\frac{4\gamma_{cm}^2}{L}(\mathbf{r}^{\mathrm{L}}_{\mathrm{i0}}(t) - \mathbf{r}^{\mathrm{L}}_{\mathrm{j0}}(t))^2},$$

Momentum dependent potential → EoS in PHQMD

New!

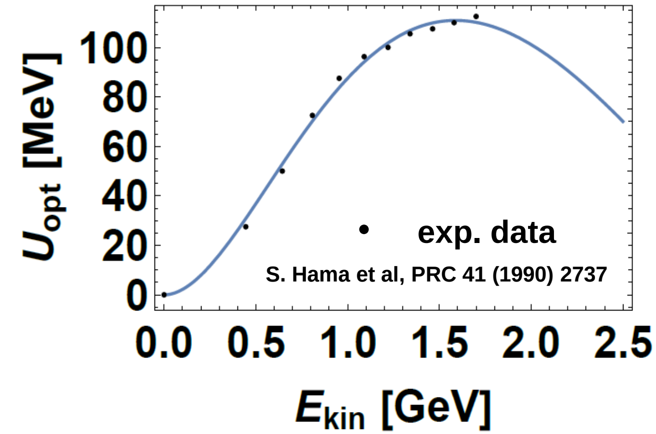
2) Momentum dependent potential :

$$V(\mathbf{r}_1, \mathbf{r}_2, \mathbf{p}_{01}, \mathbf{p}_{02}) = (a\Delta p + b\Delta p^2)\ exp[-c\sqrt{\Delta p}]\ \delta(\mathbf{r}_1 - \mathbf{r}_2)$$

$$\Delta p = \sqrt{(\mathbf{p}_{01} - \mathbf{p}_{02})^2}$$

Parameters a, b, c are fitted to the "optical" potential (Schrödinger equivalent potential U_{SEP}) extracted from elastic scattering data in pA:

$$U_{SEQ}(p) = \frac{\int^{p_F} V(\mathbf{p}-\mathbf{p}_1) dp_1^3}{\frac{4}{3}\pi p_F^3}$$



❖ **In infinite matter a potential corresponds to the EoS:**

$$E/A(\rho) = \frac{3}{5}E_F + V_{Skyrme\ stat}(\rho) + V_{mom}(\rho)$$

$$V_{mom} = (a\Delta p + b\Delta p^2)\ exp(-c\sqrt{\Delta p})\ \frac{\rho}{\rho_0}$$

$$V_{Skyrme} = \alpha\frac{\rho}{\rho_0} + \beta\frac{\rho}{\rho_0}^{\gamma}$$

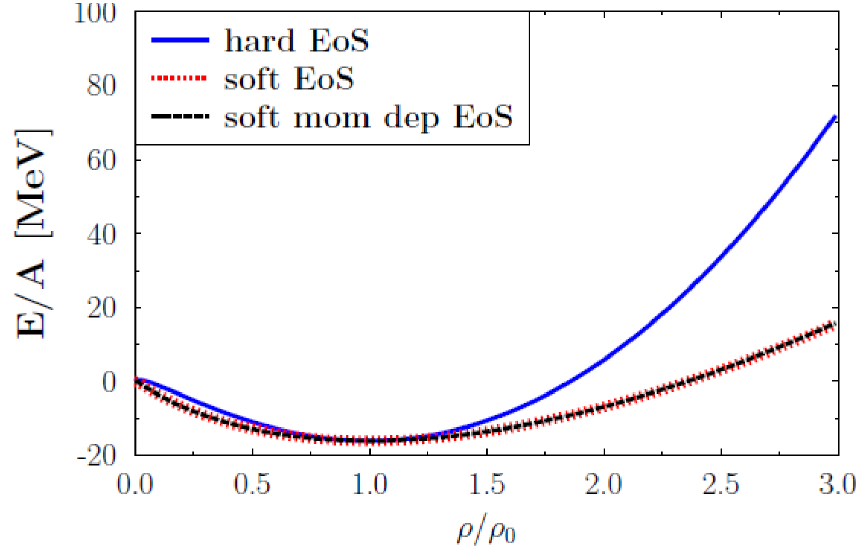
compression modulus K of nuclear matter:

$$K = -V\frac{dP}{dV} = 9\rho^2\frac{\partial^2(E/A(\rho))}{(\partial\rho)^2}|_{\rho=\rho_0}.$$

E.o.S.	$\alpha[MeV]$	$\beta[MeV]$	γ	K [MeV]
S	-383.5	329.5	1.15	200
H	-125.3	71.0	2.0	380
SM	-478.87	413.76	1.10	200

$a\ [MeV^{-1}]$	$b[MeV^{-2}]$	$c[MeV^{-1}]$
236.326	-20.73	0.901

EoS for infinite cold nuclear matter at rest



Mechanisms for cluster production in PHQMD:
I. potential interactions (MST)
&
II. kinetic reactions

PHQMD

I. Cluster recognition: Minimum Spanning Tree (MST)

R. K. Puri, J. Aichelin, J.Comp. Phys. 162 (2000) 245-266

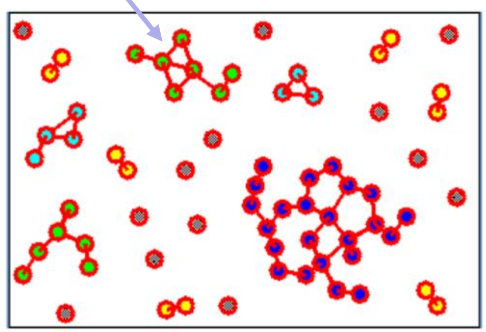
The **Minimum Spanning Tree** (MST) is a **cluster recognition** method applicable for the (asymptotic) **final states** where coordinate space correlations may only survive for bound states.

The MST algorithm searches for **accumulations of particles in coordinate space:**

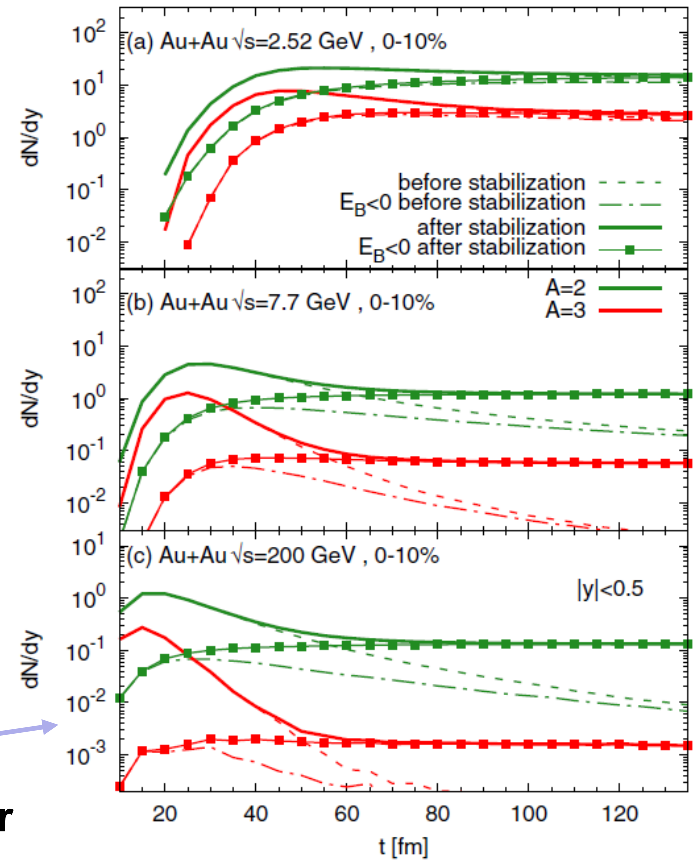
1. Two particles are **'bound'** if their **distance in the cluster rest frame** fulfills

$$|\vec{r_i} - \vec{r_j}| \leq 4 \text{ fm}$$

2. Particle is **bound to a cluster** if it **binds with at least one particle** of the cluster



* Remark: inclusion of an additional momentum cut (coalescence) leads to small changes: particles with large relative momentum are mostly not at the same position (V. Kireyeu, Phys.Rev.C 103 (2021) 5)



G. Coci et al., Phys.Rev.C 108 (2023) 014902

New: Advanced MST (aMST)

- MST **+ extra condition: $E_B<0$** **negative binding energy** for identified clusters
- **Stabilization procedure** – to correct artifacts of the semi-classical QMD: recombine the final "lost" nucleons back into cluster if they left the cluster without rescattering

II. Deuteron production by hadronic reactions

"Kinetic mechanism"
1) hadronic inelastic reactions NN ⟷ dπ , πNN ⟷ dπ , NNN ⟷ dN
2) hadronic elastic π+d, N+d reactions

SMASH: D. Oliinychenko et al., PRC 99 (2019) 044907;
J. Staudenmaier et al., PRC 104 (2021) 034908
AMPT: R.Q. Wang et al. PRC 108 (2023) 3

- **Collision rate** for hadron "*i*" is the number of reactions in the **covariant volume** $d^4x = dt*dV$
- With **test particle ansatz the transition rate for 3→2 reactions**:

W. Cassing, NPA 700 (2002) 618

$$\frac{\Delta N_{coll}[3+4+5 \to 1(d)+2]}{\Delta N_3 \Delta N_4 \Delta N_5} = P_{3,2}(\sqrt{s})$$

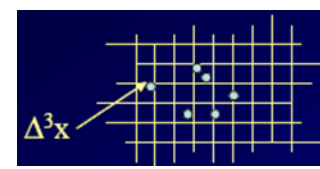
$$P_{3,2}(\sqrt{s}) = F_{spin} F_{iso} P_{2,3}(\sqrt{s}) \frac{E_1^f E_2^f}{2E_3 E_4 E_5} \frac{R_2(\sqrt{s}, m_1, m_2)}{R_3(\sqrt{s}, m_3, m_4, m_5)} \frac{1}{\Delta V_{cell}}$$

Energy and momentum of final particles

2,3-body phase space integrals [Byckling, Kajantie]

$$P_{2.3}\left(\sqrt{s}\right) = \sigma_{tot}^{2,3}(\sqrt{s}) v_{rel} \frac{\Delta t}{\Delta V_{cell}}$$

➔ solved by **stochastic method**



- Numerically tested in **"static" box:** PHQMD provides a good agreement with analytic solutions from rate equations and with SMASH for the same selection of reactions
- **New in PHQMD: π+N+N⟷ d+π** inclusion of **all possible isospin channels** allowed by total isospin T conservation ➔ enhancement of the d production

$\pi^{\pm,0} + p + n \leftrightarrow \pi^{\pm,0} + d$

$\pi^- + p + p \leftrightarrow \pi^0 + d$

$\pi^+ + n + n \leftrightarrow \pi^0 + d$

$\pi^0 + p + p \leftrightarrow \pi^+ + d$

$\pi^0 + n + n \leftrightarrow \pi^- + d$

G. Coci et al., Phys.Rev.*C* 108 (2023) 014902

Modelling finite-size effects in kinetic mechanism

G. Coci et al., PRC 108 (2023) 014902

How to account for the **quantum nature of deuteron**, i.e. for

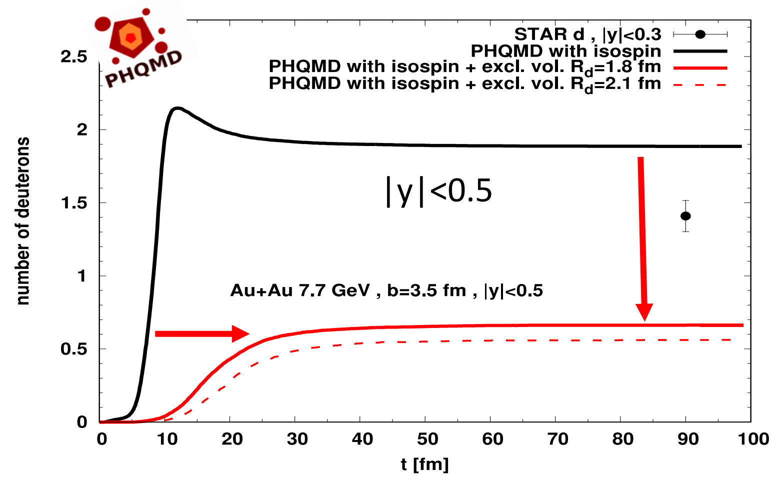
1) the **finite-size of *d* in coordinate space** (*d* is not a point-like particle) – for in-medium d production
2) the **momentum correlations of *p* and *n*** inside *d*

Realization:

1) assume that a deuteron can not be formed in a high density region, i.e. if there are other particles (hadrons or partons) inside the 'excluded volume':

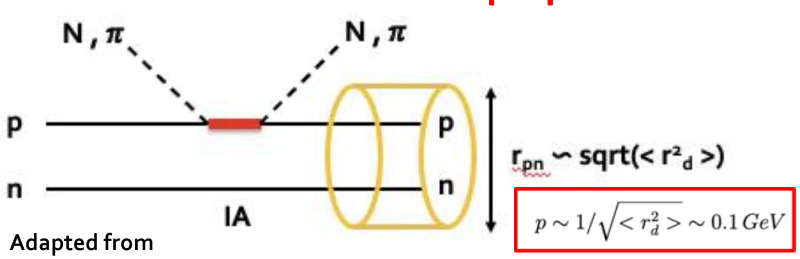
Excluded-Volume Condition: $|\vec{r}(i)^* - \vec{r}(d)^*| < R_d$

- ❑ **Strong reduction of d production**
- ❑ p_T slope is not affected by excluded volume condition

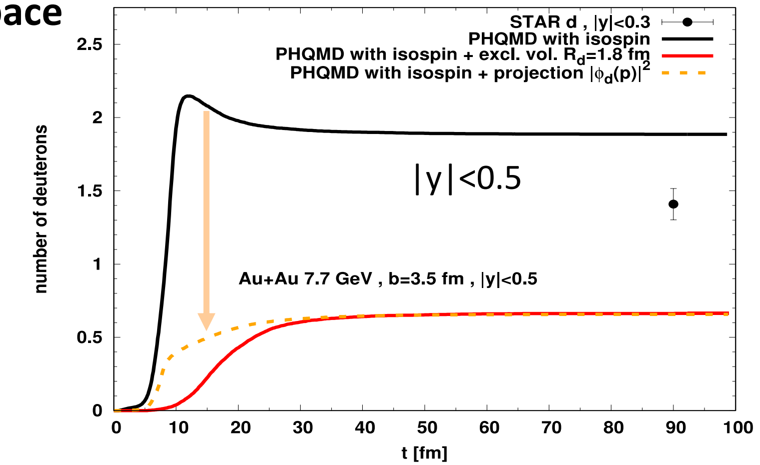


2) QM properties of deuteron must be also in momentum space

→ **momentum correlations of pn-pair**



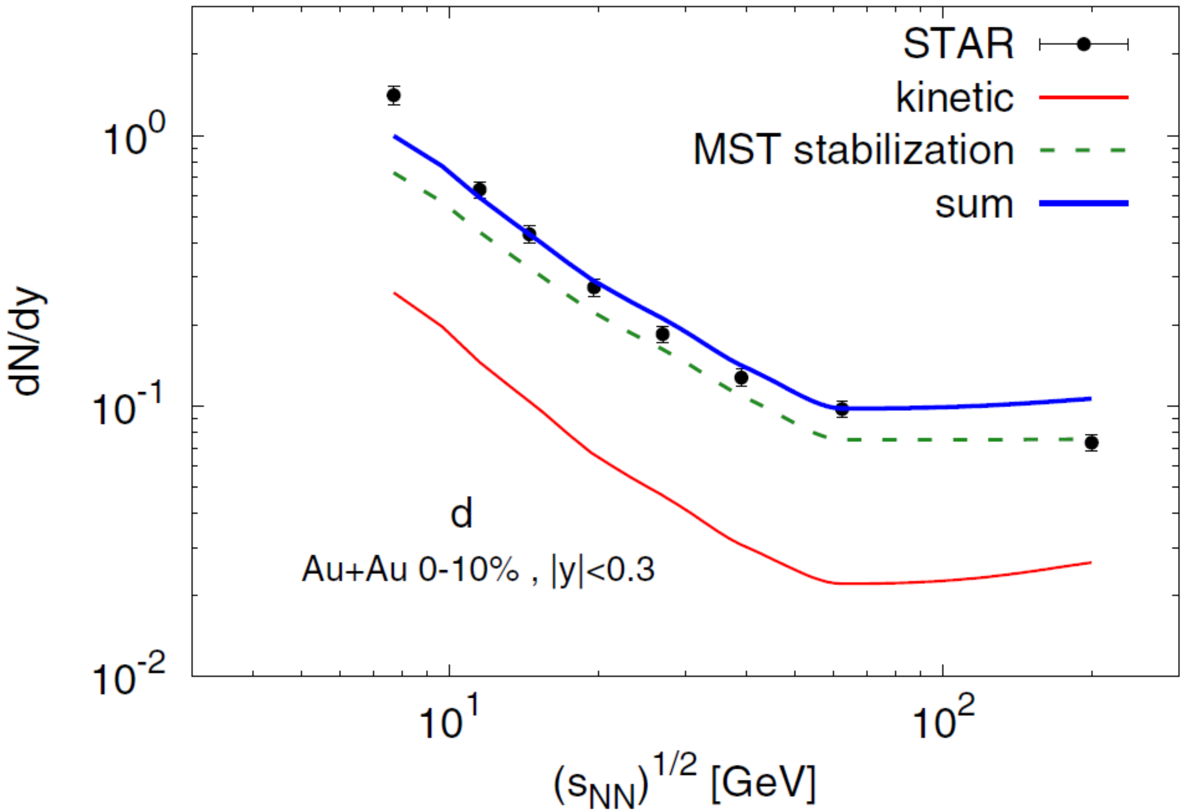
Adapted from
[Haidelbauer, Uzikov PLB 562(2003)]
[Hoftiezer et al. PRC23 (1981)]
Same spirit as AMPT [K.-J. Sun, R. Wang, C.-M. Ko et al., 2106.12742]



- ❑ **Strong reduction of d production** by projection on DWF $|\phi_d(p)|^2$

Kinetic vs. potential deuteron production

Excitation function dN/dy of deuterons at midrapidity



- **PHQMD provides a good description of STAR data**
- **The potential mechanism is dominant for d production at all energies!**

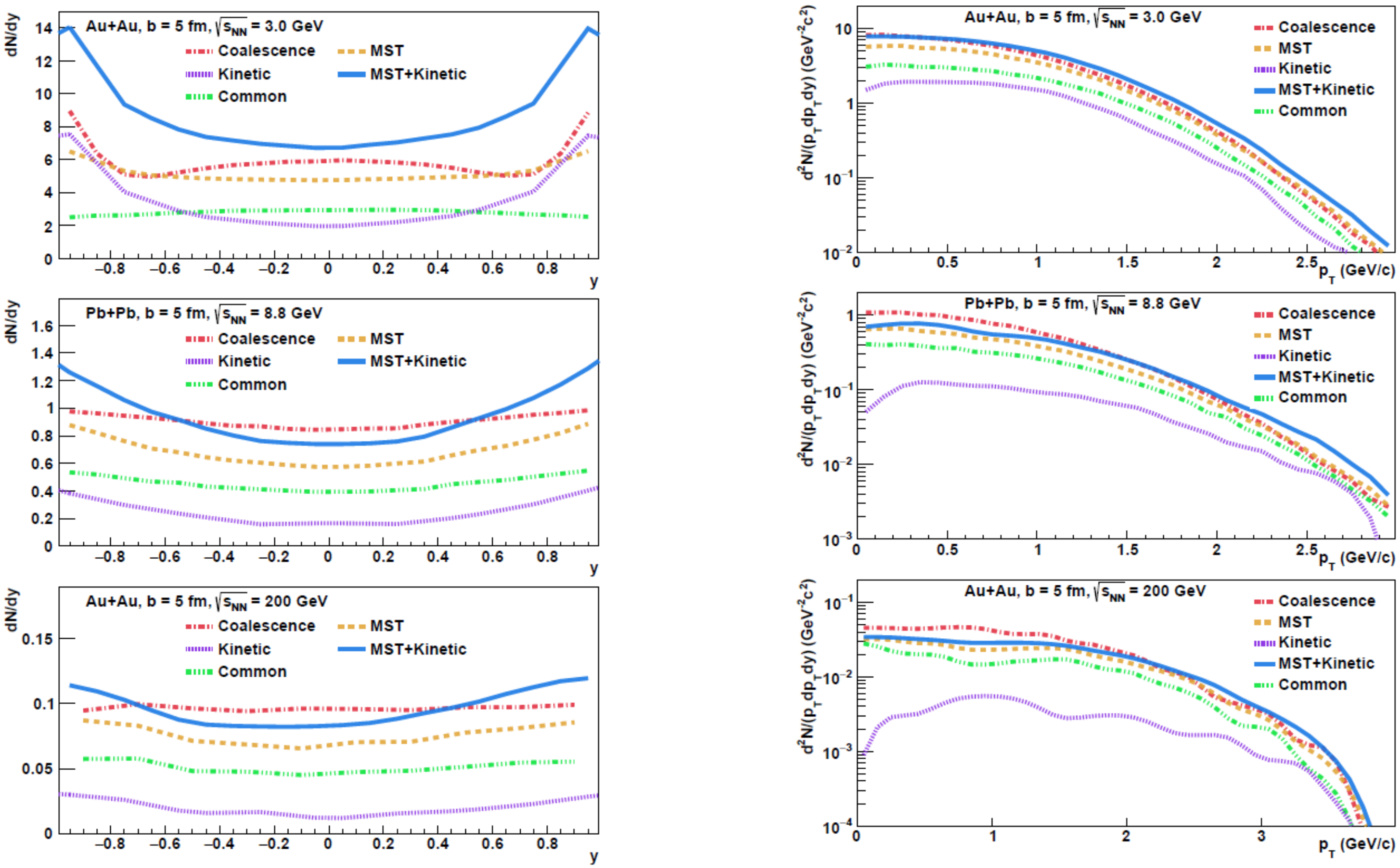
G. Coci et al., Phys.Rev.*C* 108 (2023) 014902

PHQMD

Deuterons near midrapidity - can the production mechanism be identified experimentally?

3 mechanisms: coalescence at kinetic freeze-out, kinetic and potential (MST) productions

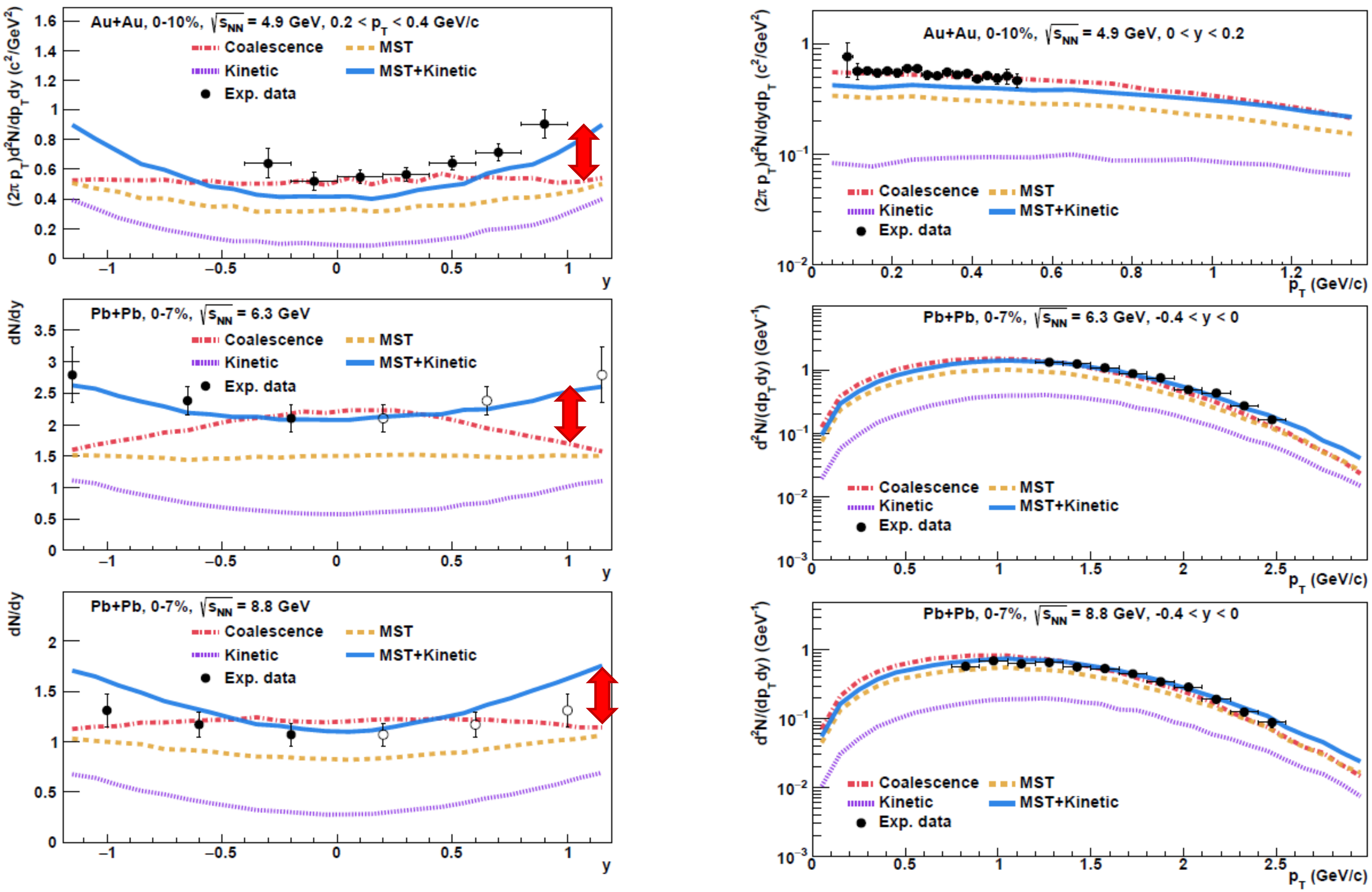
"Common": only about 20% of the MST deuterons are also identified as deuterons by coalescence



Observables, which are sensitive to the deuteron production mechanism: the rapidity distribution has a different form while the transverse momentum distribution has a different slope at low p_T

V. Kireyeu et al., 2304.12019

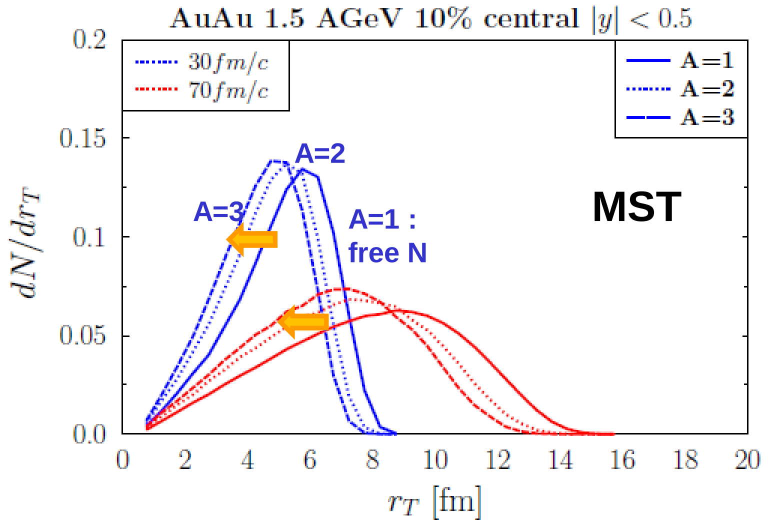
Mechanism for cluster production: theory versus experimental data

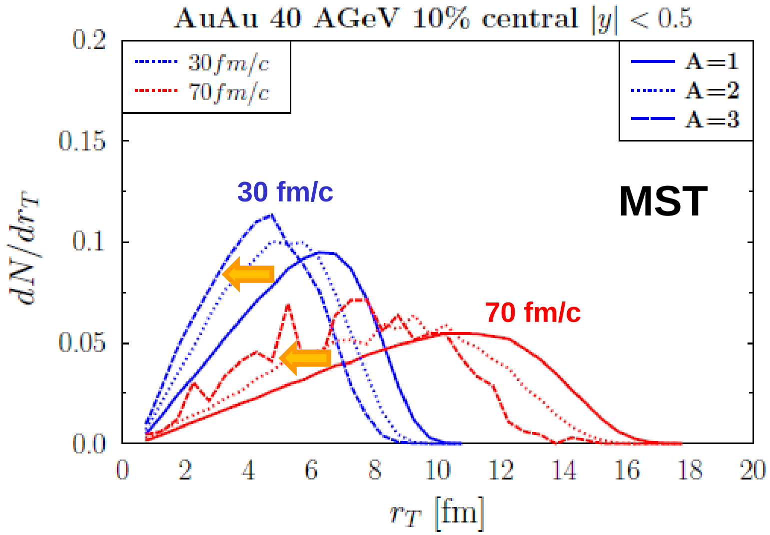


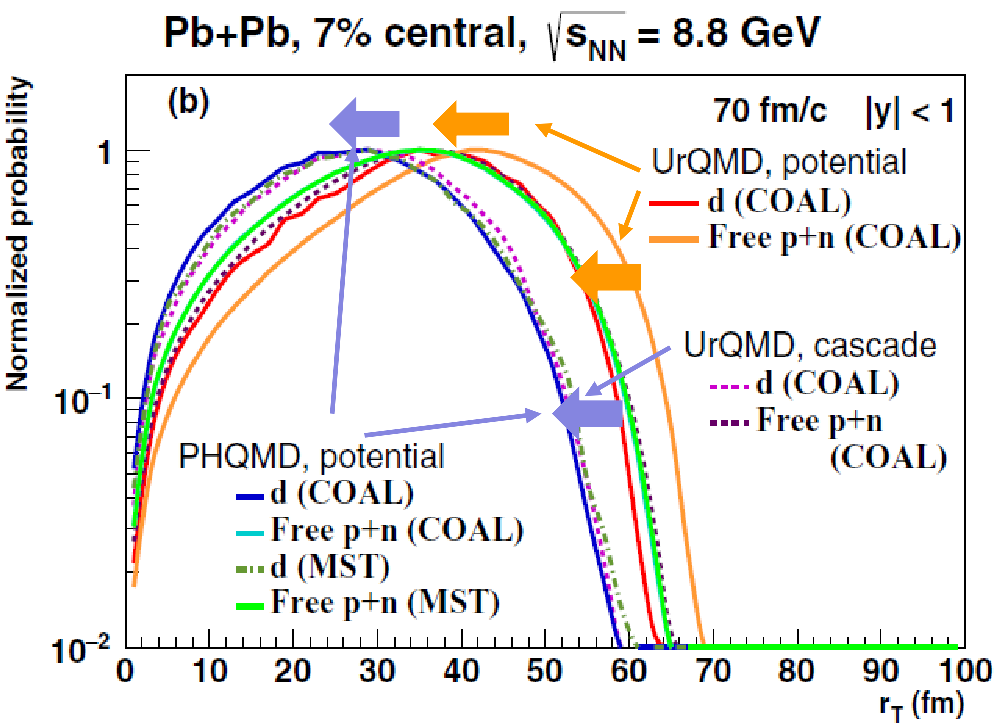
The analysis of the presently available data points tentatively to the MST + kinetic scenario but further experimental data are necessary to establish this mechanism.

V. Kireyeu et al., 2304.12019

PHQMD and UrQMD: Where clusters are formed?







➔ **Coalescence as well as the MST procedure show that the deuterons remain in transverse direction closer to the center of the heavy-ion collision than free nucleons**

➔ **deuterons are behind the fast nucleons (and pion wind)**

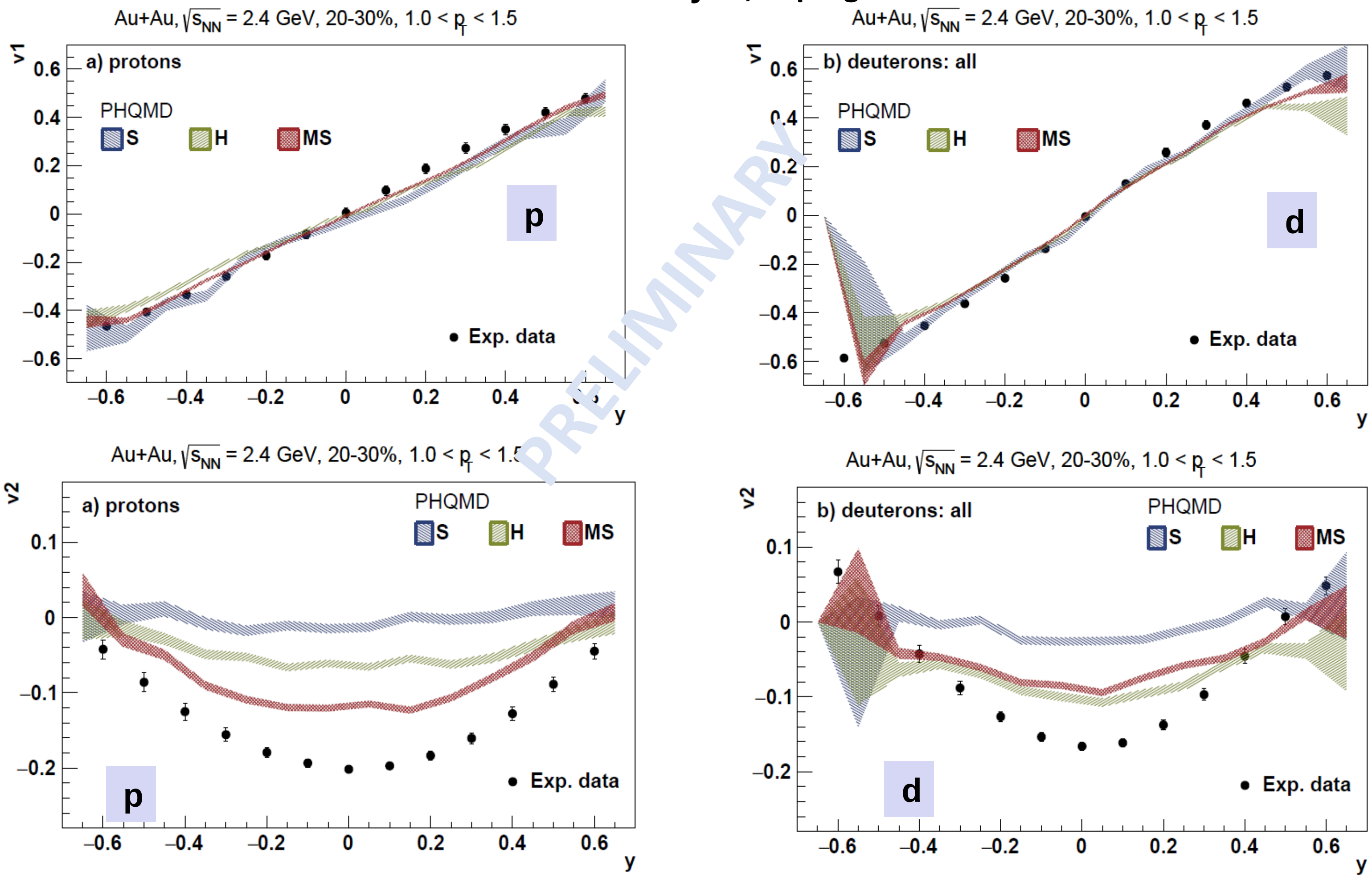
V. Kireyeu, J. Steinheimer, M. Bleicher, J. Aichelin, E.B., Phys. Rev. C 105 (2022) 044909

v_1, v_2 with different EoS
New in PHQMD: momentum dependent potential

PHQMD

EoS dependence of v_1, v_2 at SIS energies

Viktar Kireyeu, in progress



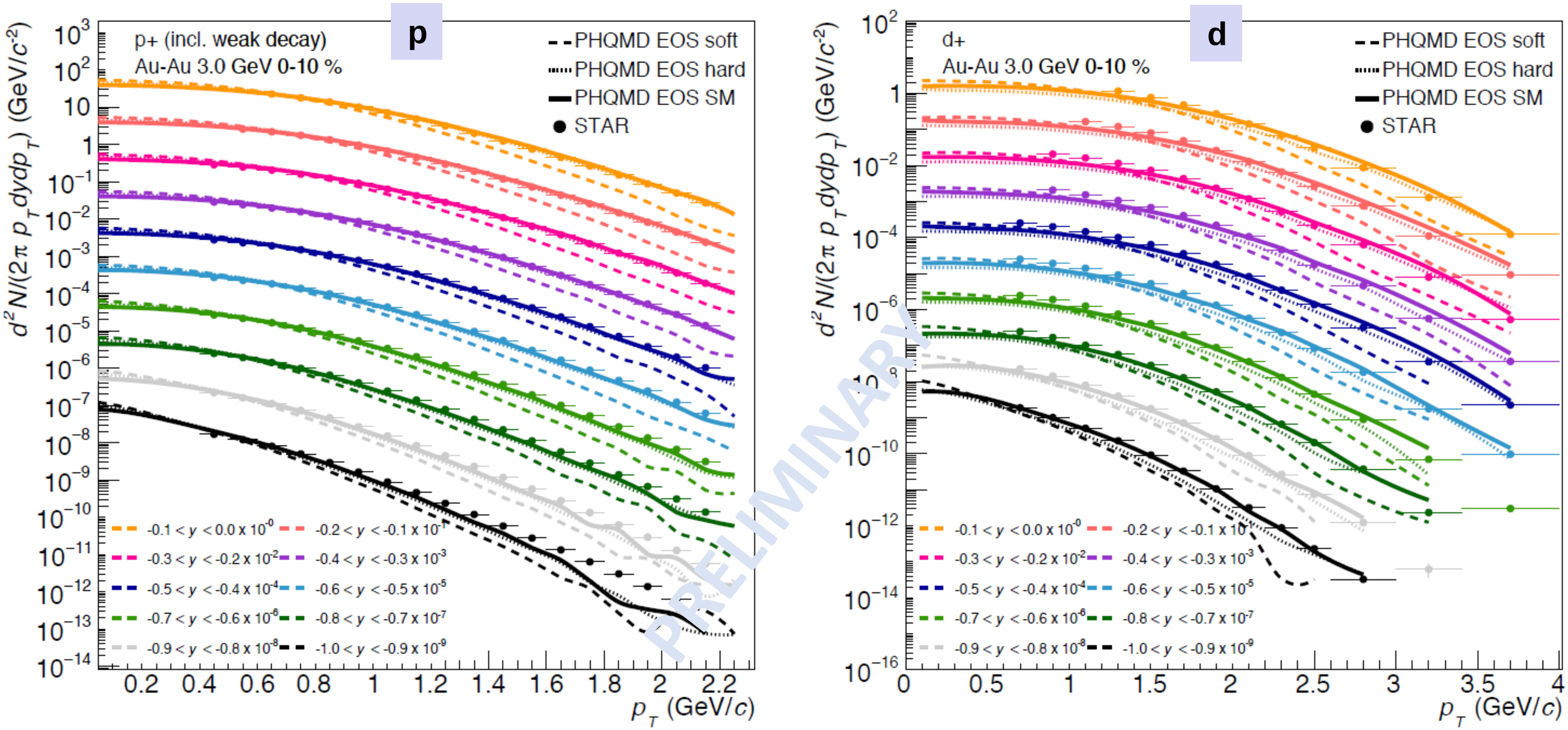
S= soft EoS, H=hard EoS, SM = soft momentum dependent EoS

HADES data: of v_1, v_2 at high p_T: 1.0 < p_T < 1.5 GeV/c

- **Strong EoS dependence of v_2**
- **HADES data favor a soft momentum dependent potential (SM)**

EoS dependence of p_T-spectra at STAR : $s^{1/2}$=3 GeV

Susanne Gläßel, in progress



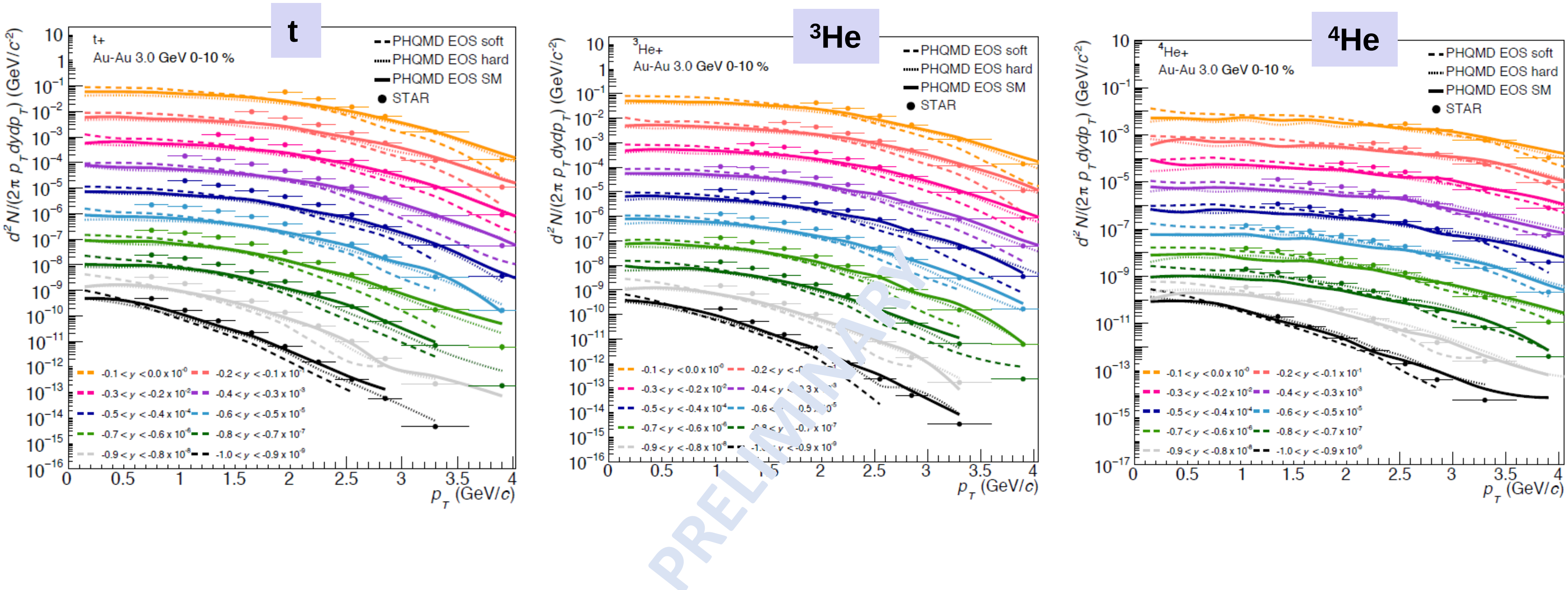
S= soft EoS, H=hard EoS, SM = soft momentum dependent EoS

- **Visible dependence of the p_T spectra of protons and deuterons EoS**
- **STAR p_T data favor a hard or soft-momentum dependent potential (H/SM)**

STAR:H. Liu et al., 2208.04650, Acta Phys.Polon.Supp. 16 (2023) 1, 148

EoS dependence of p_T-spectra at STAR : $s^{1/2}$=3 GeV

Susanne Gläßel, in progress



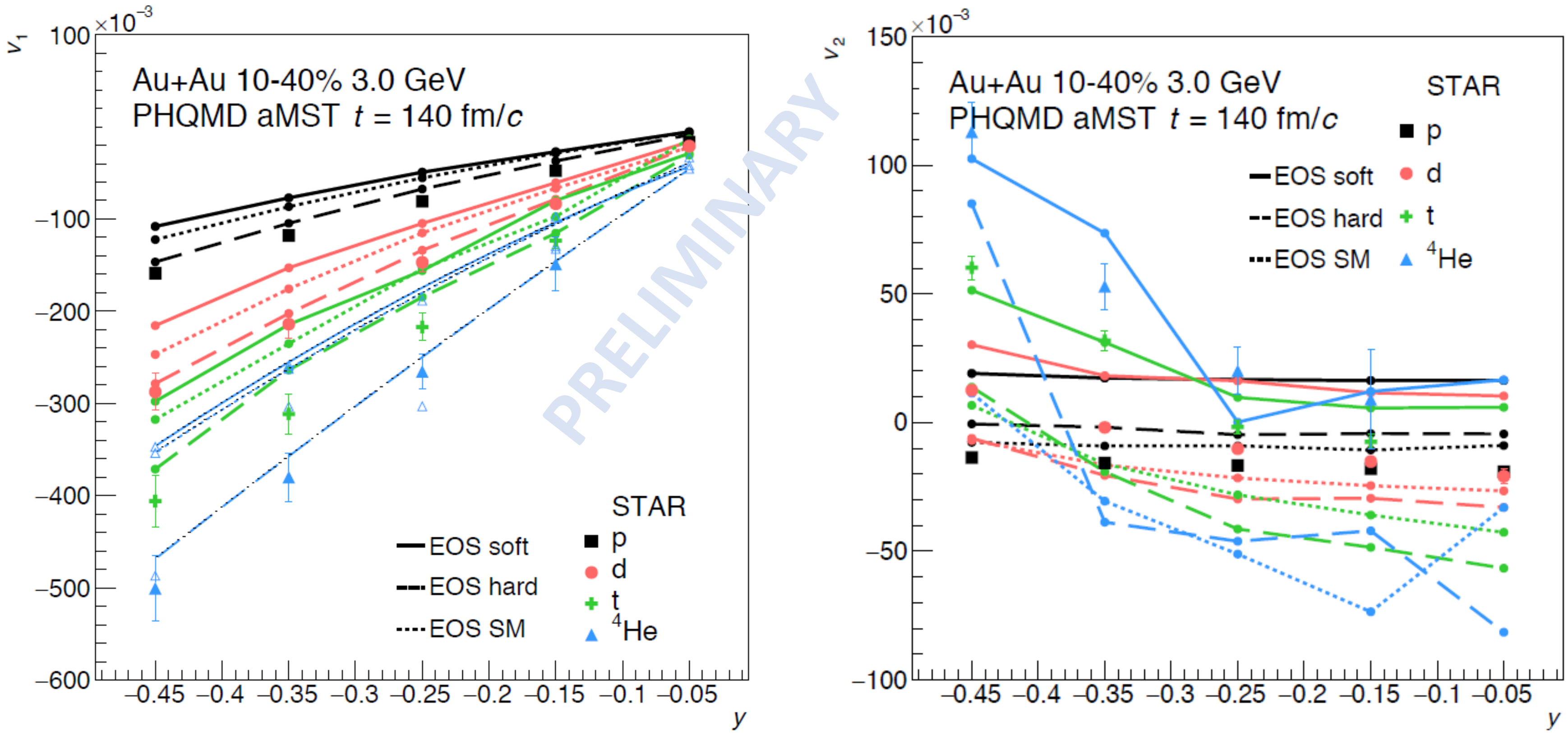
S= soft EoS, H=hard EoS, SM = soft momentum dependent EoS

- ❑ **Visible dependence of p_T spectra of t, ^{3}He, ^{4}He on EoS**
- ❑ **STAR p_T data favor a hard or soft-momentum dependent potential (H/SM)**

STAR:H. Liu et al., 2208.04650, Acta Phys.Polon.Supp. 16 (2023) 1, 148

EoS dependence of v_1, v_2 at at STAR : $s^{1/2}$=3 GeV

Susanne Gläßel, in progress



- Strong EoS dependence of v_1 , v_2
- STAR data favor a hard EoS or soft momentum dependent (H/SM)
- Influence of momentum dependent potential on flow v_n decreases with increasing energy

Summary

The PHQMD is a microscopic n-body transport approach for the description of heavy-ion dynamics and cluster and hypernuclei formation identified by Minimum Spanning Tree model

combined model PHQMD = (PHSD & QMD) & (MST | SACA)

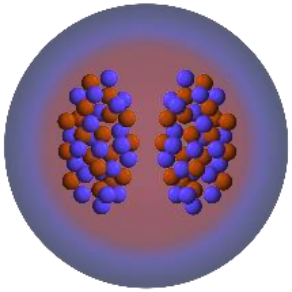
Clusters are formed dynamically

1) by potential interactions among nucleons and hyperons
   Novel development: momentum dependent potential with soft EoS
2) by kinetic mechanism for d : hadronic inelastic reactions NN $\leftrightarrow$ dπ , πNN $\leftrightarrow$ dπ , NNN $\leftrightarrow$ dN
   with inclusion of all possible isospin channels which enhance d production
   + accounting of quantum properties of d, modelled by the finite-size excluded volume effect in coordinate space and projection of relative momentum of p+n pair on d wave-function in momentum space which leads to a strong reduction of d production

- The PHQMD reproduces cluster and hypernuclei data on dN/dy and dN/dp_T as well as ratios d/p and $\bar{d}/\bar{p}$ for heavy-ion collisions from AGS to top RHIC energies.
- Measurement of dN/dy beyond mid-rapidity will allow to distinguish the mechanisms for cluster production: coalescence versus dynamical cluster production recognized by MST + kinetic mechanism for deuterons

- Strong dependencee of p_T-spectra and v_1,v_2 on EoS - soft, hard, soft-mom. dependent - at SIS energies
- The influence of U(p) decreases with increasing collision energy since the modelled U_{SEP}(p) has a maximum at energy 1.5 GeV and decreases for large p ← no exp. data for extrapolation of U_{SEP}(p) to large p!
- HADES data data on v_1,v_2 favour a soft momentum dependent potential (SM)
- STAR data at 3 GeV favour a hard EoS or SM

- Stable clusters are formed shortly after elastic and inelastic collisions have ceased and behind the front of the expanding energetic hadrons (similar results within PHQMD and UrQMD)
  → since the 'fire' is not at the same place as the 'ice', cluster can survive



Thank you for your attention !

Thanks to the Organizers !