Triple-Differential Yields from Heavy-Ion Collisions Through Reaction-Plane Deblurring

Pawel Danielewicz

Facility for Rare Isotope Beams, Michigan State University, USA

39th Winter Workshop on Nuclear Dynamics

Jackson, Wyoming, Feb 11-17, 2024

Paradigm: Triple-Differential Yields from Data

Distributions for *Fixed Direction of Reaction Plane* from Theory and Experiment

no control over plane

What is it?!

FRIB

Paradigm: Triple-Differential Yields from Data

Distributions for *Fixed Direction of Reaction Plane* from Theory and Experiment

no control over plane

some control, v_n

Still not clear what the system is. . .

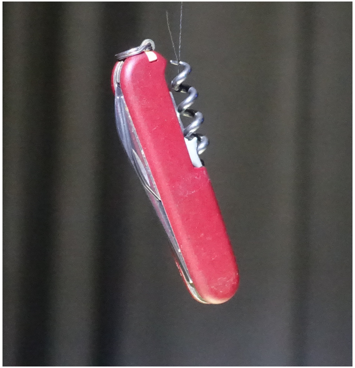
FRIB

Paradigm: Triple-Differential Yields from Data

Distributions for *Fixed Direction of Reaction Plane* from Theory and Experiment

no control over plane

some control, v_n



full control, $\frac{d^3N}{dp^3}$

FRIB

Claim: You can go from center to right panel through deblurring

Deblurring by Example

Budd, *Crime Fighting Math*, plus.maths.org magazine

Blurred Photo of Moving Car

Deblurred

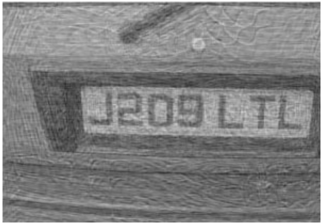
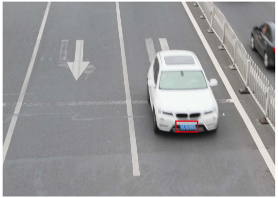


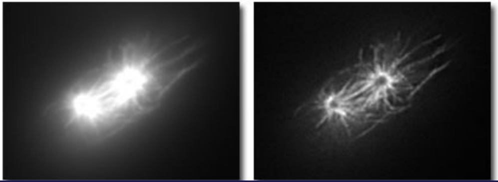
Photo of Parked Car

Fast Moving



Deblurring in Optical Microscopy

Before and After Nearest Neighbor Deconvolution Analysis



Correcting f/Distortions Due to Apparatus or Method

Detector efficiency ϵ, n measured ptcle number, N actual number

$$N \simeq \frac{1}{\epsilon}\, n$$

Typical energy loss in thick target $\overline{\Delta E}$ for detected particle

$$E_{\rm prod} \simeq E_{\rm det} + \overline{\Delta E}$$

General problem stated probabilistically, with $P(\zeta|\xi)$ - probability to measure ptcle characteristic to be ζ when it is actually ξ

$$n(\zeta) = \int {\rm d}\xi \, P(\zeta|\xi)\, N(\xi)$$

For small distortions, P finite only when ζ little different from ξ. Optical terminology: P - blurring or transfer function.

Bayesian Deblurring

Distorted $n(\zeta)$ measured, while pristine $N(\xi)$ sought:

$$n(\zeta) = \int \mathrm{d}\xi \, P(\zeta|\xi) \, N(\xi)$$

$P(\zeta|\xi)$ - probability that ptcle with ζ detected while it really has characteristic ξ, understood given the method/apparatus, can be simulated (Geant4) & can depend on N

$Q(\xi|\zeta)$ - unknown complementary probability that ptcle has characteristic ξ while measured at ζ

Bayesian relation: number of times ptcle has characteristic in $\mathrm{d}\xi$ while measured in $\mathrm{d}\zeta$ is

$$P(\zeta|\xi) \, N(\xi) \, \mathrm{d}\xi \, \mathrm{d}\zeta = Q(\xi|\zeta) \, n(\zeta) \, \mathrm{d}\xi \, \mathrm{d}\zeta$$

Hence $$N(\xi) = \frac{\int \mathrm{d}\zeta \, Q(\xi|\zeta) \, n(\zeta)}{\int \mathrm{d}\zeta' \, P(\zeta'|\xi)} \, , \quad Q(\xi|\zeta) = \frac{P(\zeta|\xi) \, N(\xi)}{\int \mathrm{d}\xi' \, P(\zeta|\xi') \, N(\xi')}$$

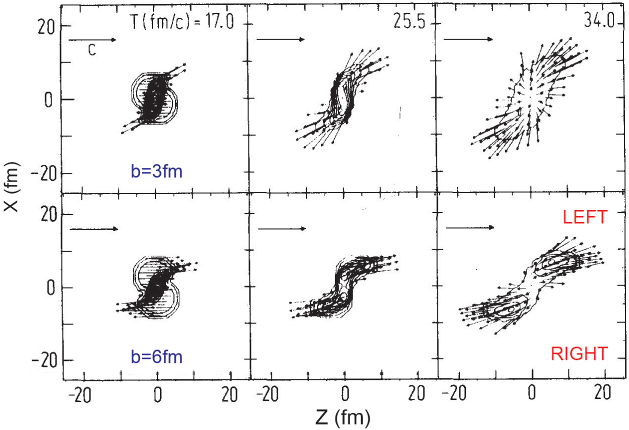
Richardson-Lucy method solves eqs iteratively till stabilization

FRIB

Side Focus in Hydrodynamic Calculations

Matter dispersed in the final stage, but most likely direction of motion away from the beam, e.g., in the calculations by Buchwald for Nb + Nb at 400 MeV/nucl

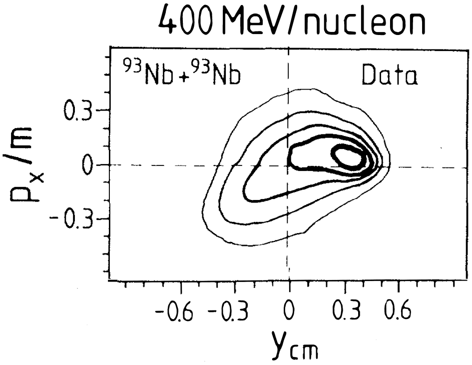
Stöcker&Greiner Phys.Rep. 137(86)277



Can this be seen experimentally??

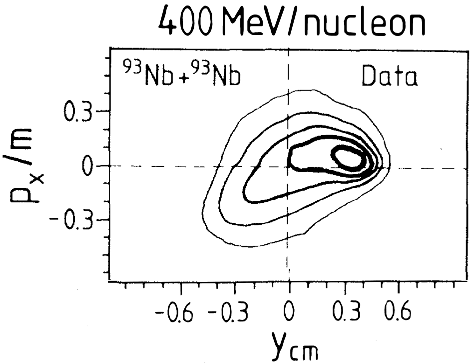
1984 Claim

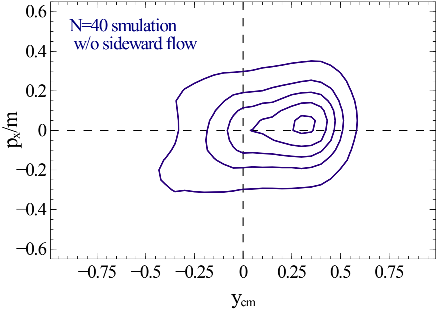
Gustafsson *et al.* PRL 18(84)1590 Plastic Ball Group claims to see preferential emission away from the beam axis, in $d^3N_{ch}/dy\,d^2p^{\perp}$ for 400 MeV/nucl Nb + Nb collisions, when determining reaction plane from flow tensor, $\mathbf{S}^{\perp z} = \sum_{\nu} \mathbf{p}_{\nu}^{\perp}\, p^z/2m_{\nu}$



1984 Claim

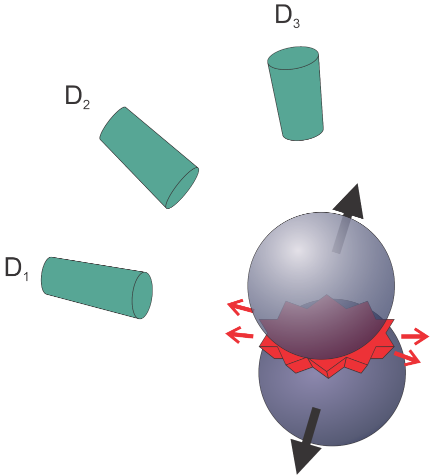
Gustafsson *et al.* PRL 18(84)1590 Plastic Ball Group claims to see preferential emission away from the beam axis, in $d^3N_{ch}/dy\, d^2p^{\perp}$ for 400 MeV/nucl Nb + Nb collisions, when determining reaction plane from flow tensor, $\mathbf{S}^{\perp z} = \sum_\nu \mathbf{p}_\nu^{\perp}\, p^z/2m_\nu$





The observation can be explained by particle self-correlation, w/o invoking transverse collective movement

Estimating Reaction-Plane Direction w/o Self-Correlation



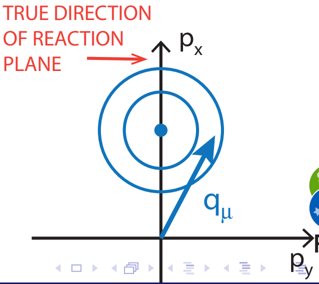
Plane direction f/particle μ estimated with

$$\mathbf{q}_\mu = \frac{1}{N}\sum_{\nu\neq\mu}\omega_\nu\,\mathbf{p}_\nu^\perp \qquad \omega_\nu = \begin{cases} +1, & \text{if } p_\nu^z > 0 \\ -1, & \text{if } p_\nu^z < 0 \end{cases}$$

N - measured particle multiplicity; other ptcles in the event used as reference for μ

PD&Odyniec PLB157(85)146

Problem: Reference vector $\mathbf{q}_\mu$ Gaussian fluctuates around true plane direction, blurring features



Current Solution: Angular Moments of Distributions

Solution: average angular moments (azimuthal Fourier coefficients)

$$v_n = \langle \cos n\phi \rangle$$

ϕ - angle relative to true reaction plane

Voloshin&Zhang ZfPhC70(1996)665

v_n derived from average scalar products/contractions, e.g.,

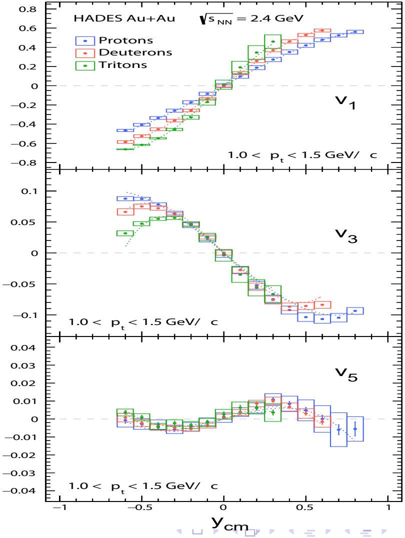
$$\langle \mathbf{p}_\mu^\perp \cdot \mathbf{q}_\mu \rangle \simeq p^\perp \langle q^x \rangle \langle \cos\phi \rangle$$

for different $p^\perp$, y and ptcle ID

Problem: unclear physics in v_n especially for higher n

1.23 GeV/nucl Au + Au $b \simeq 6$ fm

HADES PRL125(2020)262301

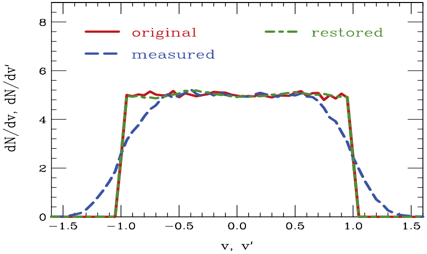


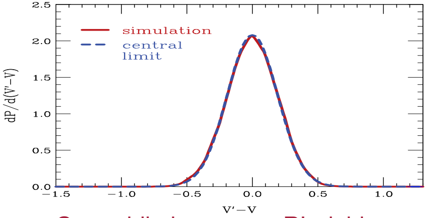
Schematic 1D Model

Proposition: Carry out as good determination of 3D info as you can & refine with deblurring. ~~v_n~~?

First 1D deblurring test. Projectile at unknown velocity V deexcites emitting $N = 10$ ptcles distributed with box-like dN/dv in projectile cm. Task: Measuring ptcles in lab, determine dN/dv. Cm velocity V' estimated from remaining ptcles, so V' & dN/dv' smeared:

$$\frac{dN}{dv'} = \int dV' \, \frac{dP}{dV'} \, \frac{dN}{dv}$$



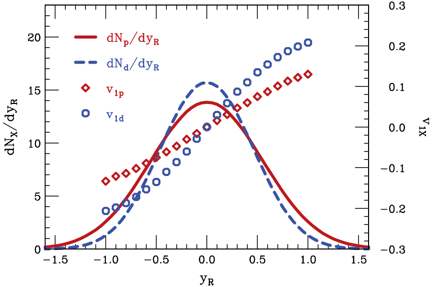


→Central-limit smear + RL deblur

PD&Kurata-Nishimura PRC105(2022)034608

3D Model for Collisions

Customary thermal model with flow, N, d, t, ^{3}He, ^{4}He. $\langle Z_{Tot} \rangle = 50$
Rapidity dstr, temperature & flow typical for semicentral collisions at 300 MeV/nucl

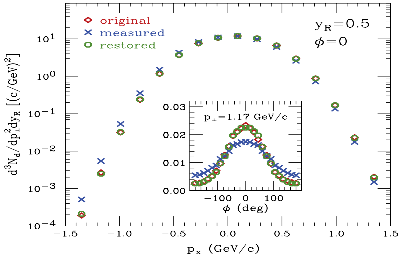


$$\frac{dN}{d\phi'} = \int d\Phi' \, \frac{dP}{d\Phi'} \, \frac{dN}{d\phi}$$

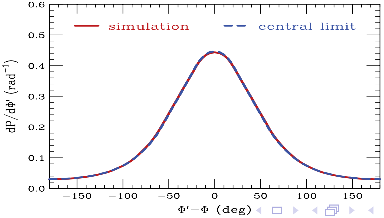
$$\phi' + \Phi' = \phi + \Phi$$

RL deblur + central-limit
Strong anisotropies restored!

Triple differential spectrum in reaction plane:



Uncertainty in reaction plane:



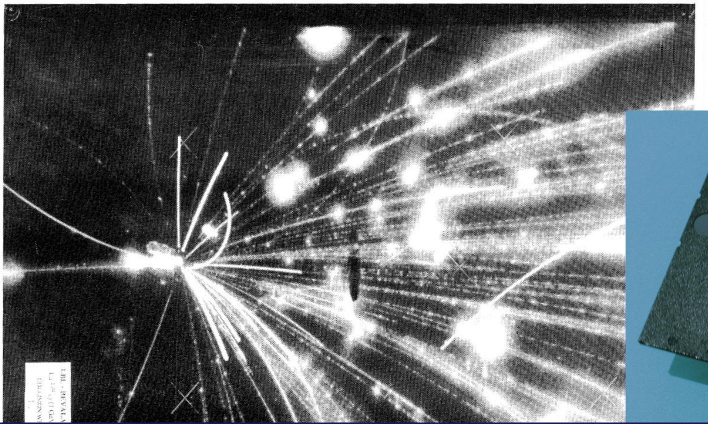
FRIB

Ar + KCl @ 1.8 GeV/nucl

Ströbele PRC 27(83)1349

495 events from Streamer Chamber, $b \lesssim 2.4\,\mathrm{fm}$

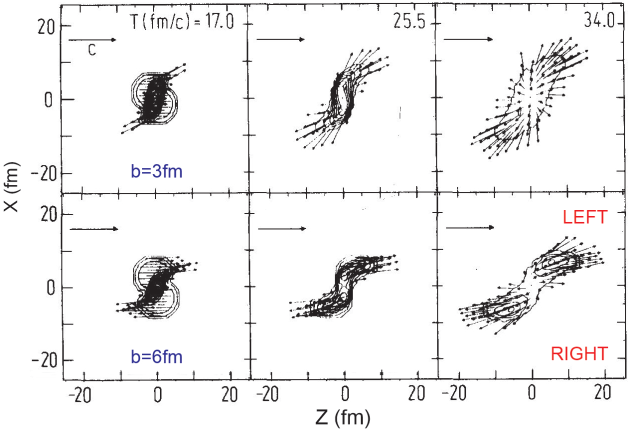
PD&Odyniec PLB 157(85)146



Reminder: Hydrodynamic Calculations

Matter dispersed in the final stage, but most likely direction of motion away from the beam, e.g., in the calculations by Buchwald for Nb + Nb at 400 MeV/nucl

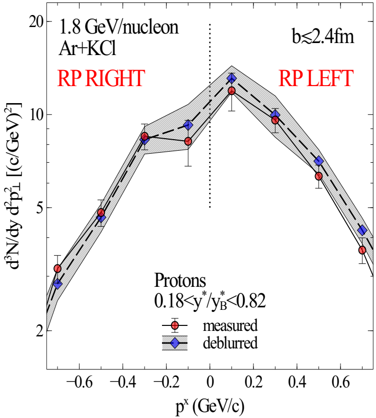
Stöcker&Greiner Phys.Rep. 137(86)277



Can this be seen experimentally??

Side-Focus in Ar + KCl 1.8 GeV/nucl?

protons

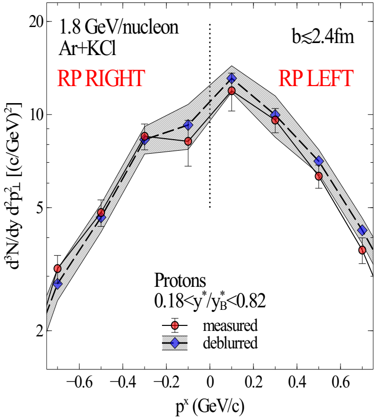


Particles in the forward hemisphere, $y^* \sim 0.5 y_B^*$

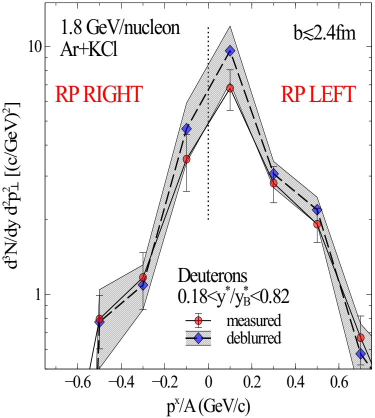
PD, Ströbele, Nzabahimana PRC108(23)L051603

Side-Focus in Ar + KCl 1.8 GeV/nucl?

protons



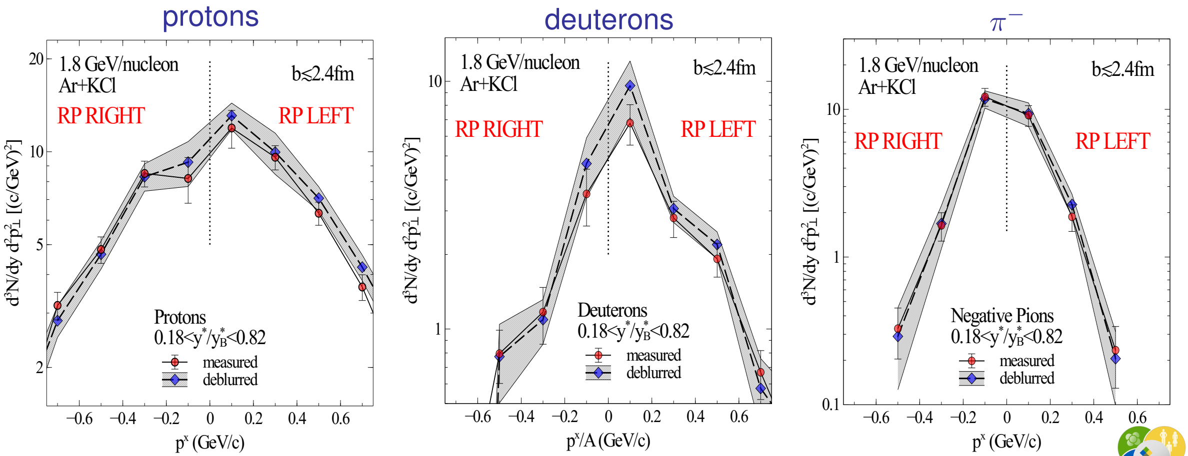
deuterons



Particles in the forward hemisphere, $y^* \sim 0.5 y_B^*$

PD, Ströbele, Nzabahimana PRC108(23)L051603

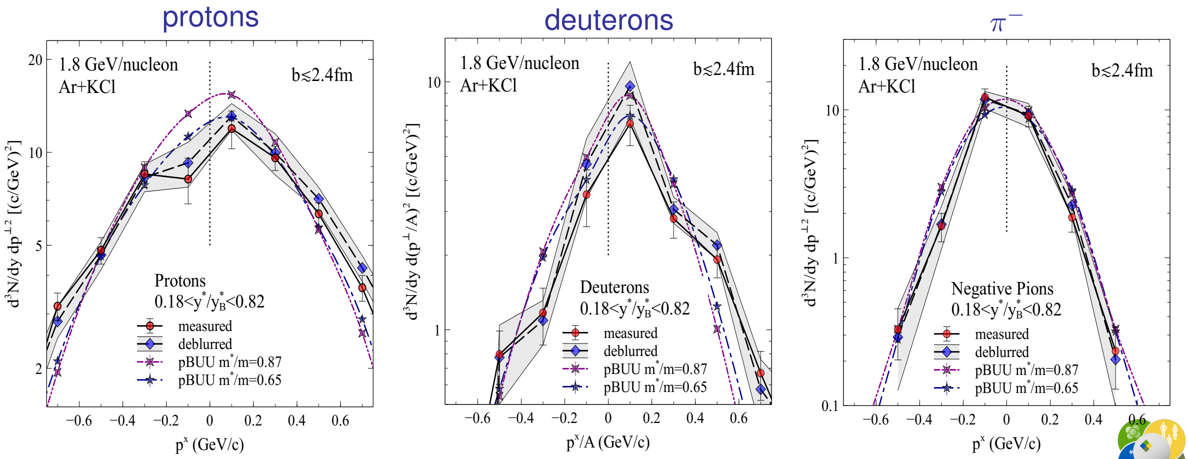
Side-Focus in Ar + KCl 1.8 GeV/nucl



Particles in the forward hemisphere, $y^* \sim 0.5 y_B^*$

PD, Ströbele, Nzabahimana PRC108(23)L051603

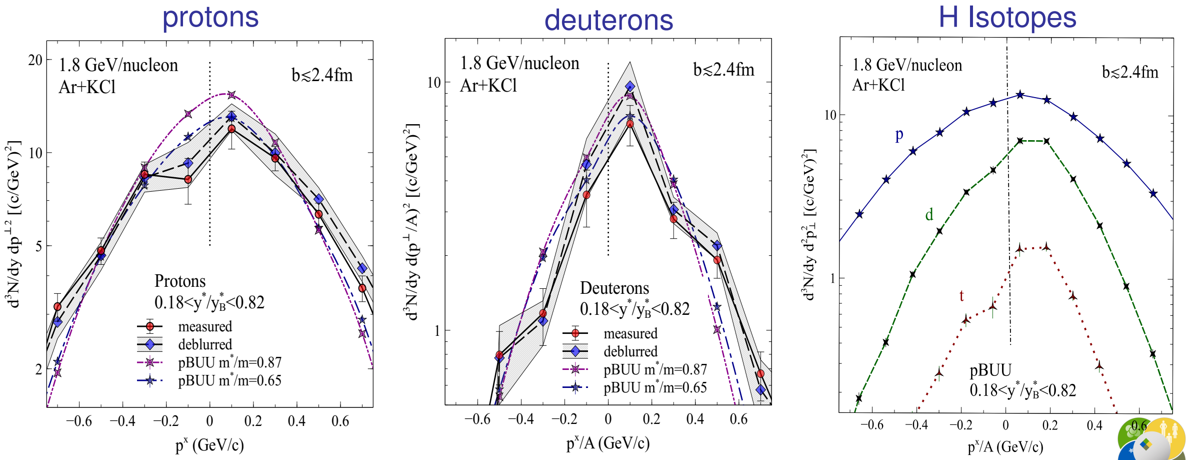
Side-Focus: Experiment vs Theory



Transport Calculations: sensitivity to mean-field *p*-dependence in Ar+KCl

FRIB

Side-Focus: Experiment & Theory



Transverse boost $v^x \sim 0.1\,c$

What's Behind Deblurring's Success?

Singular value decomposition f/forward conditional probability:

$$P_{ij} = \sum_{n} \sigma_n \, U_{ni} \, V_{nj} \qquad \Rightarrow \qquad Q_{ji} \stackrel{?}{=} \sum_{n} \sigma_n^{-1} \, V_{nj} \, U_{ni}$$

i - measurement, j - reality, Q - backward conditional probability. Instability??

Plain Reaction-Plane Deblurring:

$$U_n(\varphi) = V_n(\varphi) = \begin{cases} \frac{1}{\sqrt{2\pi}}, & n = 0 \\ \frac{\cos(n\varphi)}{\sqrt{\pi}}, & n > 0 \end{cases}$$

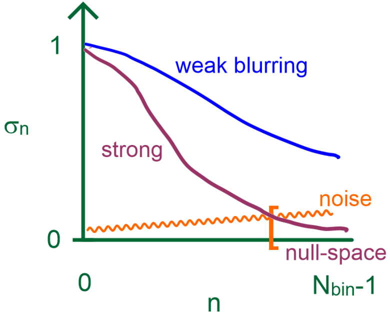
$$\sigma_n = \langle \cos(n\Delta\Phi) \rangle$$

with $\Delta\Phi$ estimated-true reaction plane deviation

Detector effects yield more complicated vectors

Positivity + regularization stabilize restoration!

Hansen *et al. Deblurring Images* 2006; Sinethemba Mamba, PD *to be submitted*



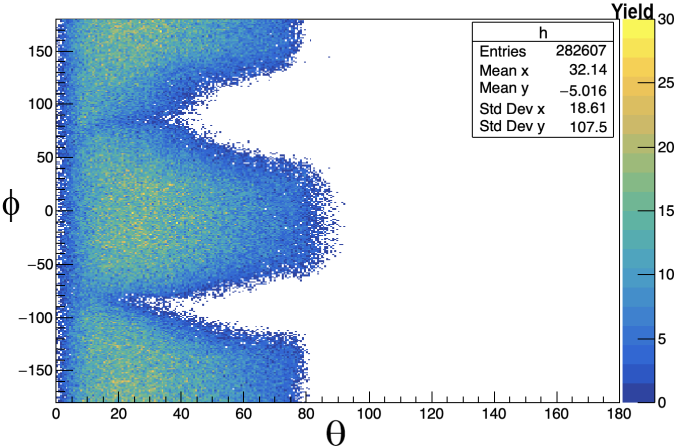
FRIB

Restoration with Inefficiencies

SΠRIT@RIKEN Time-Projection Chamber

Sn + Sn @ 270 MeV/nucl

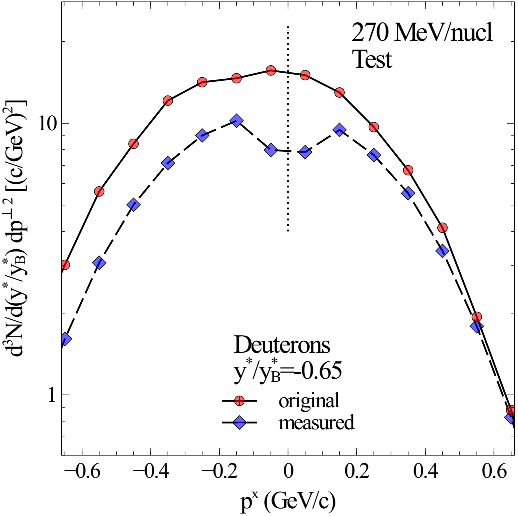
Proton distribution in lab angles



Strong azimuthally-asymmetric inefficiencies for slow particles and at small polar angles

FRIB

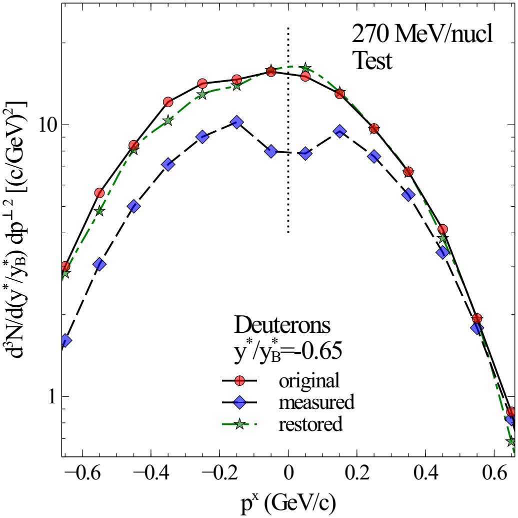
Simulated Restoration f/SΠRIT TPC in Backward CM Hemisphere



Preliminary (minimal statistics)

Flow model ran forward through efficiency simulator for the SΠRIT TPC: not only particles lost but also reaction-plane effects

Simulated Restoration f/SΠRIT TPC in Backward CM Hemisphere



Preliminary (minimal statistics)

Flow model ran forward through efficiency simulator for the SΠRIT TPC: not only particles lost but also reaction-plane effects - restored through deblurring

Conclusions

- Reaction-plane fluctuations made us concentrate at intermediate energies on azimuthal moments w/unclear physics content for higher orders
- Deblurring, common in optics, enables accessing 3D distributions associated w/true reaction plane, when reaction-plane effects are strong enough
- Side focus in Ar + KCl collisions at 1.8 GeV/nucl with $v^x \sim 0.1\,c$, visible with just $\sim$ 500 collision events, is just an example of what may be achieved!
- Other nuclear problems where deblurring started producing results: $^{26}\text{O} \rightarrow ^{24}\text{O} + n + n$ decay, source-imaging from 2-particle correlations in HIC

PD&Kurata-Nishimura PRC105(22)034608; Nzabahimana *et al.* PRC107(23)064315;

PD *et al.* PRC108(23)L051603; Adamczewski-Musch *et al.* PRL125(20)262301 - v_n reconstruction

Berkowitz Physics 15(22)s26

https://www.energy.gov/science/np/articles/deblurring-can-reveal-3d-features-heavy-ion-collisions

Supported by US Department of Energy under Grant US DE-SC0019209

FRIB