New Insights into the Properties of Matter at High Baryon Density

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Work in collaboration with
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See also work with:
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Mass and radii of observed neutron stars and data from neutron star collisions give
an excellent determination of the equation of state of strongly interacting matter

Such equations of state must be hard
The sound velocity squared is greater than or of the order of 1/3 at only a few times nuclear matter density
This is NOT what one expects from a 1st or 2nd order phase transition
Relativistic degrees of freedom appear to be important

After a short review, will discuss a field theoretical method to include both quark and nucleon degrees of freedom in a consistent field theoretical formalism

Neutron Star Matter and Some Conjectures on Scale Invariance

From observations of neutron stars masses and radii, one gets very good information about the zero temperature equation of state of nuclear matter

One equates the outward force of matter arising from pressure inward force of gravity. This gives a general relativistic equation of hydrostatic equilibrium.

For a specific equation of state, one obtains a relationship between radii and neutron star masses

Equations of state may be characterized by two dimensionless numbers

Sound velocity:

$$v_s^2 = \frac{dP}{de}$$

and the trace of the stress energy tensor scaled by the energy density

$$\Delta = \frac{1}{3} - \frac{P}{e}$$

$$P = -\frac{dE}{dV}$$

In a scale invariant theory at zero temperature:

$$E \sim (N/V)^{4/3} V \sim N^{4/3} V^{-1/3}$$

$$P = \frac{1}{3}\frac{E}{V} = \frac{1}{3}e$$

$$v_s^2 = \frac{dP}{de} = \frac{1}{3}$$

$$\Delta = \frac{1}{3} - \frac{p}{e} = 0$$

The trace of the stress energy tensor is taken to be a measure of scale invariance. It is anomalous in QCD.

$$T^\mu_\mu = -\frac{\beta(g)}{g}(E^2 - B^2) + m_q(1+\gamma_q)\overline{\psi}\psi$$

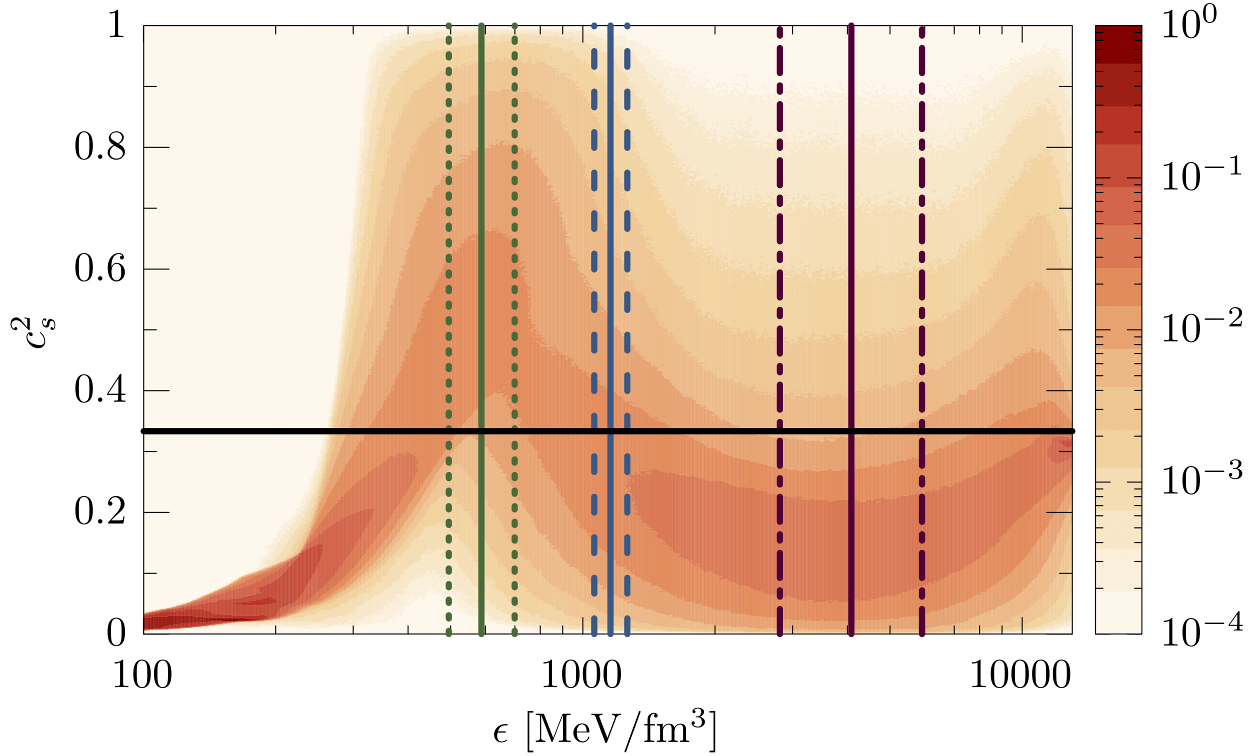
In this equation, the beta function of QCD is negative, and the fermion term is from quarks. The fermion term vanishes in the chiral limit.

If we take matrix elements of single particle states

$$< p \mid T^\mu_\mu \mid p > \sim p^2 = m^2 \geq = 0$$

In the chiral limit, this implies E > B, as we expect for massive quarks, except for the pion, which is very tightly. bound

For dilute systems, the trace anomaly is positive, as it is at high density for a quark gas. In general, we expect it to be positive, except possibly for small effects due to pion condensate, if they exist



L. McLerran, M. Marczenko, K. Redlich and C. Sasaki

Y. Fujimoto, K. Fukushima,
K. Murase

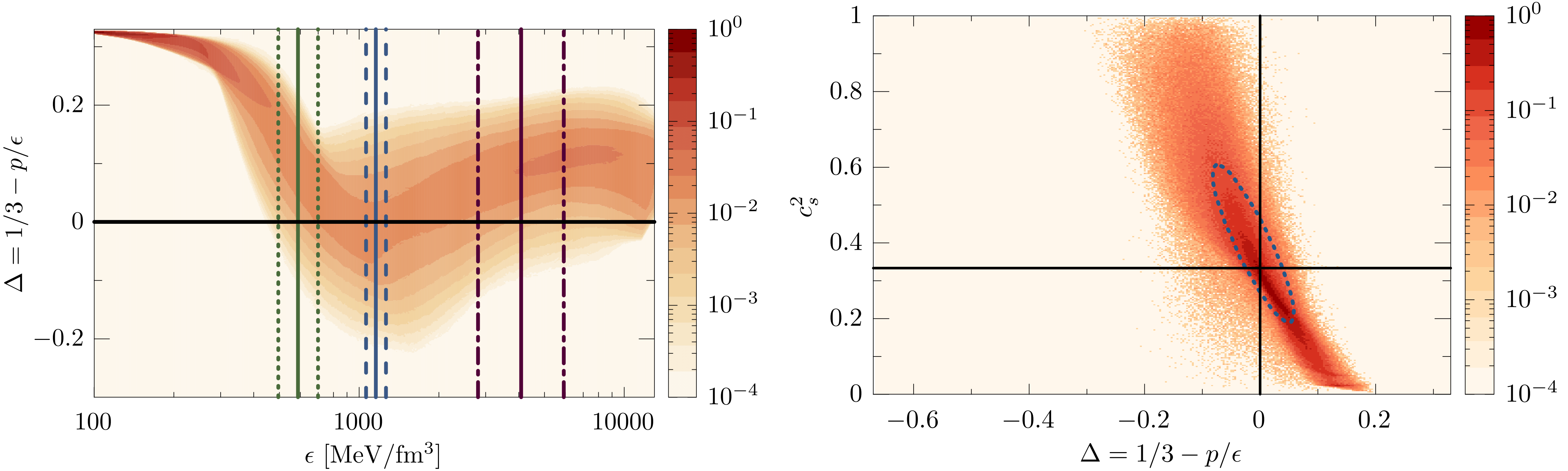
Tews, Carlson, Gandolfi and Reddy; Kojo; Annala, Gorda, Kurkela and Vuorinen

As a result of LIGO experiments, and more precise measurement of neutron star masses and radii, the equation of state of nuclear matter at a few times nuclear matter density is tightly constrained

Sound velocity approaches and perhaps exceeds

$$v_s^2 = 1/3$$

at a few times nuclear matter density

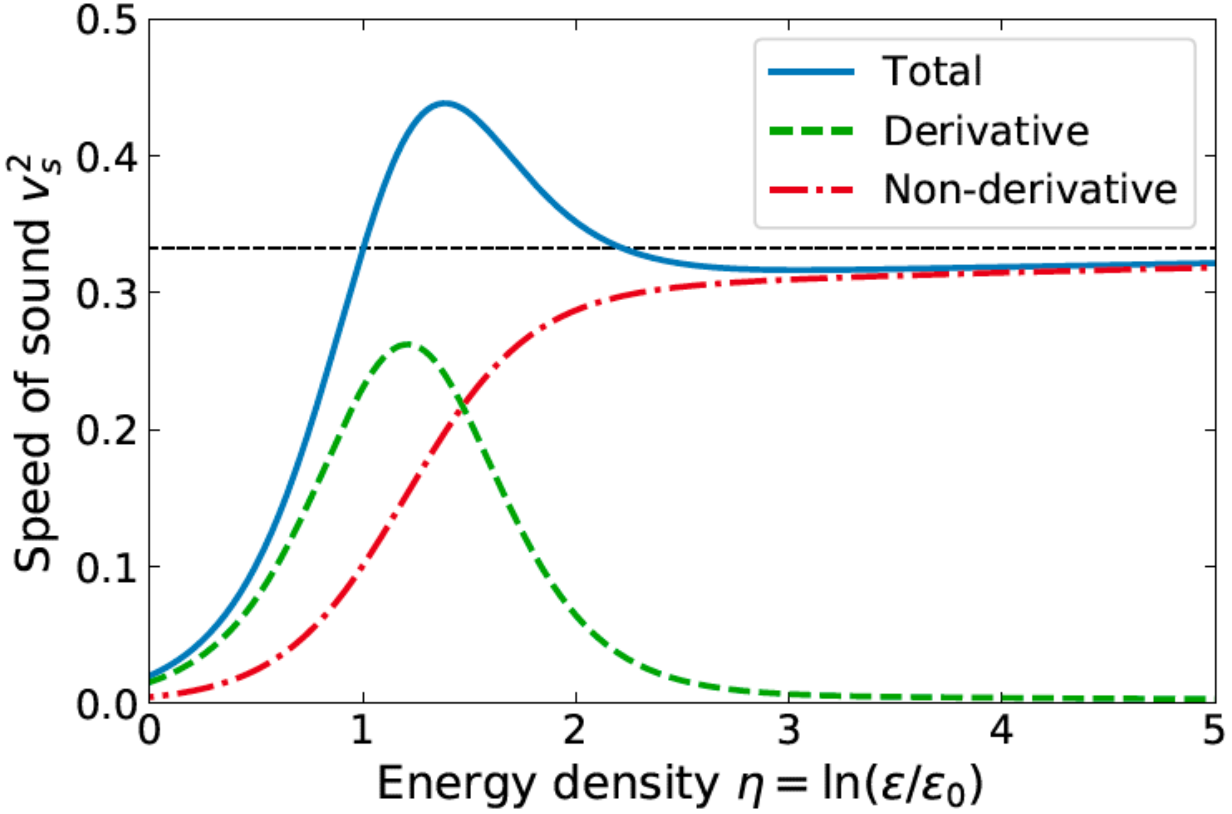


Also, the trace anomaly is approaching zero at highest densities in neutrons stars,
where also the sound velocity squared approaches 1/3

Matter is strongly interacting and conformal:
Probably some form of quark matter

$$v_s^2 = v_{s,\,(\mathrm{deriv})}^2 + v_{s,\,(\mathrm{non\text{-}deriv})}^2 \equiv \varepsilon \frac{d}{d\varepsilon}\left(\frac{P}{\varepsilon}\right) + \frac{P}{\varepsilon}$$

Work with Y. Fujimoto, K. Fukushima and M. Praszalowicz



Peak in sound velocity is because system rapidly approaches scale invariant limit

The rapid rise of the energy density signals the transition between nucleon and quark degrees of freedom

Quarkyonic matter has the properties needed to describe this behavior

Sound velocity of order one has important consequences

For zero temperature Fermi gas:

$$\frac{n_B}{\mu_B dn_B/d\mu_B} = v_s^2$$

where the baryon chemical potential includes the effects of nucleon mass

$$\frac{\delta\mu_B}{\mu_B} \sim v_s^2 \; \frac{\delta n_B}{n_B}$$

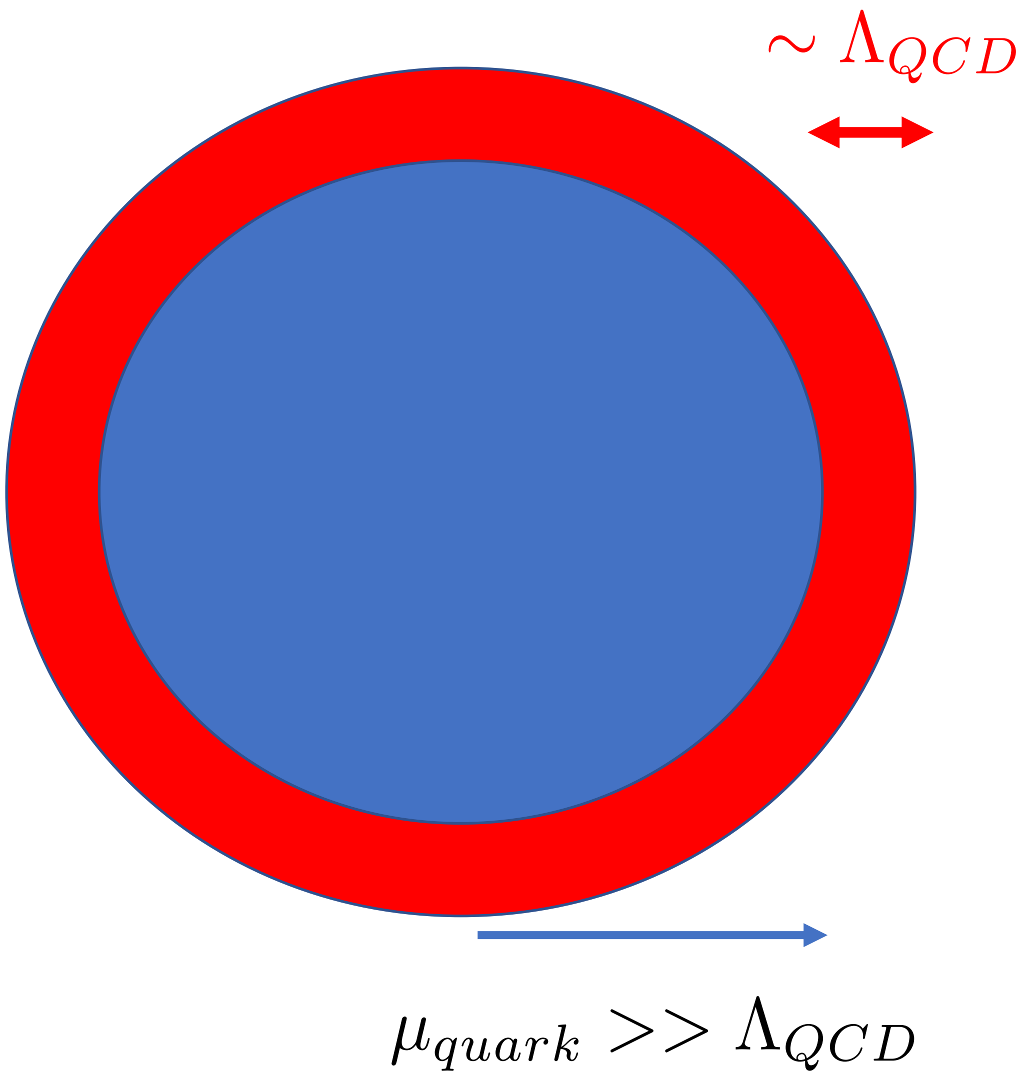
So if the sound velocity is of order one, an order one change in the baryon density generates a change in the baryon number chemical potential of order the nucleon mass

For nuclear matter densities

$$\mu_B - M \sim \frac{\Lambda_{QCD}^2}{2M} \sim 100 \;\; MeV$$

Large sound velocities will require very large intrinsic energy scales, and a partial occupation of available nucleon phase space because density is not changing much while Fermi energy changes a lot

Fermi Surface is Non-perturbative



Fermi Surface
Interactions sensitive to infrared
Degrees of freedom: baryons, mesons and glueballs

Fermi Sea: Dominated by exchange interactions which are less sensitive to IR.
Degrees of freedom are quarks

Relation between quark and nucleon Fermi momenta

$$m_q = m_N / N_c$$

$$\mu_q = \mu_N / N_c$$

$$k_q^2 = \mu_q^2 - m_q^2 = \mu_n^2/N_c^2 - M_N^2/N_c^2 = k_N^2/N_c^2$$

For 2 flavors of nucleons

$$n_B^N = 4 \int^{k_N} \frac{d^3k}{(2\pi)^3} = \frac{2}{3\pi^2} k_N^3$$

For two flavors of quarks

$$n_q^N = \frac{1}{N_c} 4N_c \int^{k_q} \frac{d^3k}{(2\pi)^3} = \frac{2}{3\pi^2} k_q^3$$

To get any baryons from the quarks at the bottom of the shell of nucleons, need the quark fermi momentum to be of the order of the QCD scale, so that the nucleons in the shell are relativistic. Naturally driven to the conformal behavior. The chemical potential of the quarks must jump up from a small value to a typical QCD scale

If there is a continuous transition then the baryon density will have to remain fixed, so the chemical potential will change by of order N_c. The sound velocity is changing for a very non-relativistic system to a very relativistic one.

$$\epsilon_B = M_N \Lambda^3_{QCD} \sim N_c \Lambda^4_{QCD}$$

$$\epsilon_Q \sim \Lambda_{QCD} n_q \sim N_c \Lambda^4_{QCD}$$

$$P \sim k_F^5 / M_N$$

The pressure on the other hand must jump by order N_c squared

$$P \sim \epsilon_q$$

Energy density and density fixed, but pressure and chemical potential jump.

A first order phase transition has pressure and chemical potential fixed, but energy density and density jump

An Explicit Quantum Mechanical Theory of Quarkyonic Matter

T. Kojo; Y. Fujimoto., LM and T. Kojo

Let occupation number density for nucleons and quarks be

$$f_q, f_N \qquad 0 \leq f_q, f_N \leq 1$$

A duality relation (nucleons are composed of quarks)

$$f_q(k) = \int [dp]\ K(k - p/N_c) f_N(p)$$

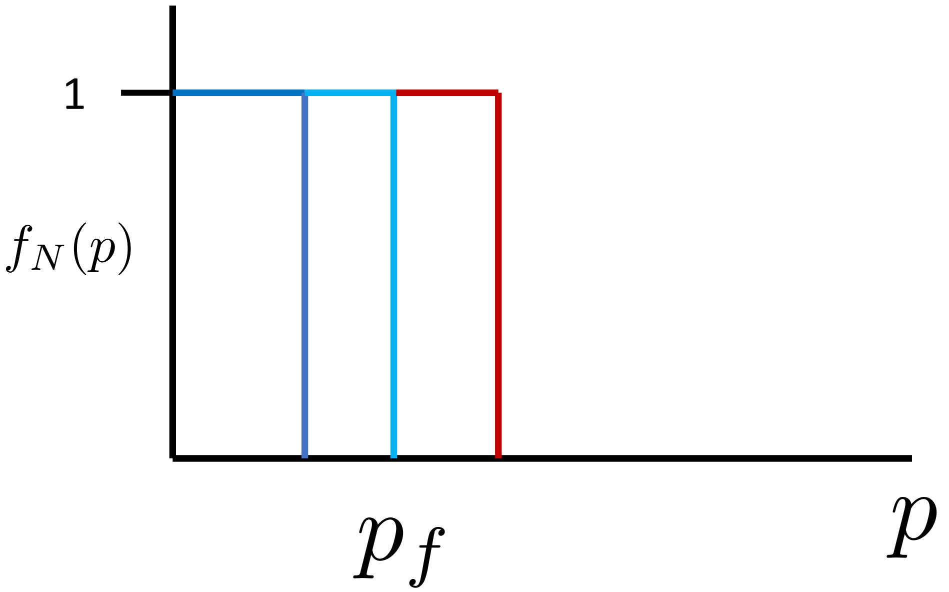
$$\int [dk]\ K(k) = 1$$

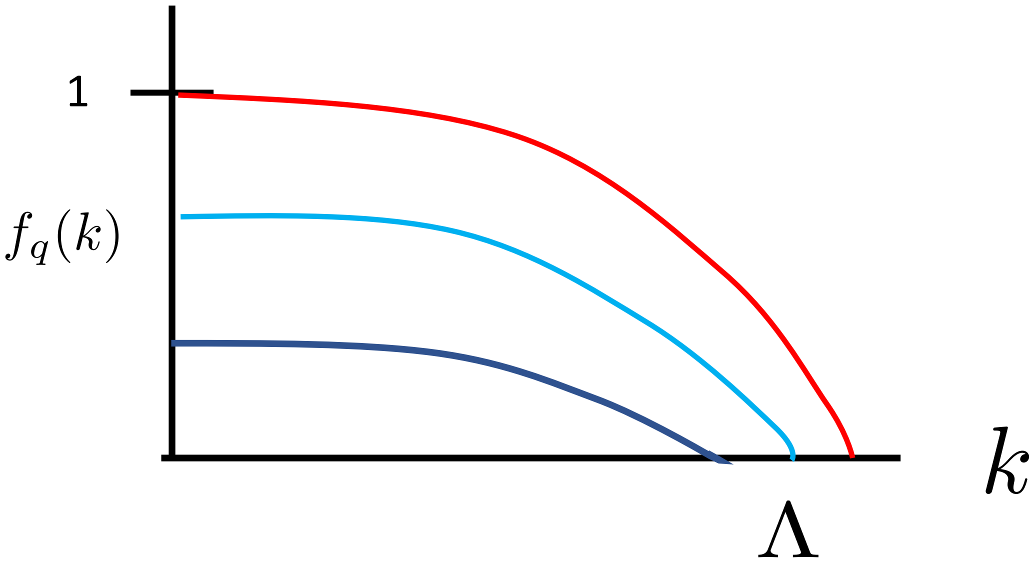
First: Free theory of nucleon and quarks (except for duality relation)

This is a solvable theory with non-trivial solution with two phases:
A nucleonic phase and a quarkyonic phase

Solution looks like:

Low density:





$$1 = \int [dp]\ K(k - p/N_c) f_N(p)$$

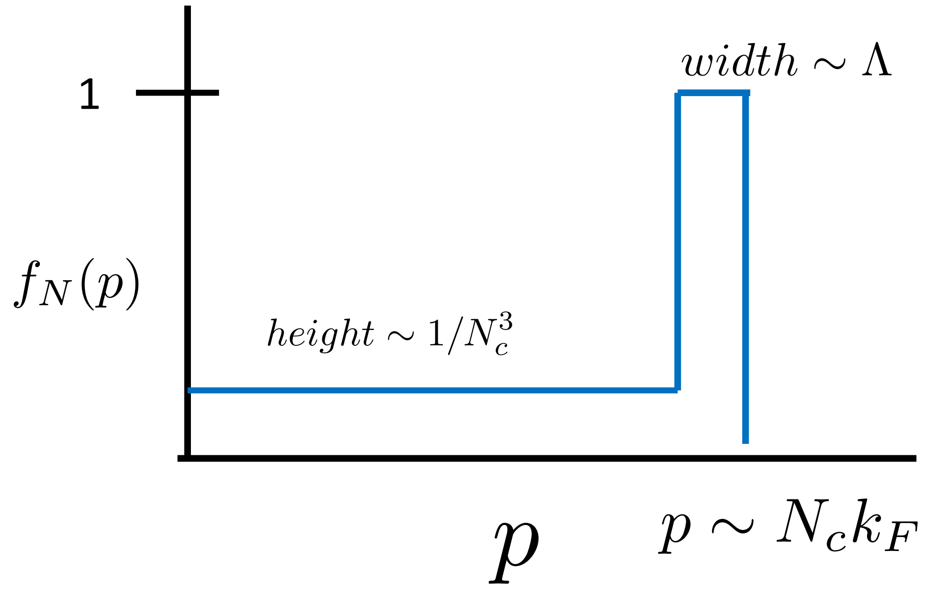
$$\sim K(0) \int [dp]\ f_N(p) = \kappa \frac{n_n}{\Lambda^3}$$

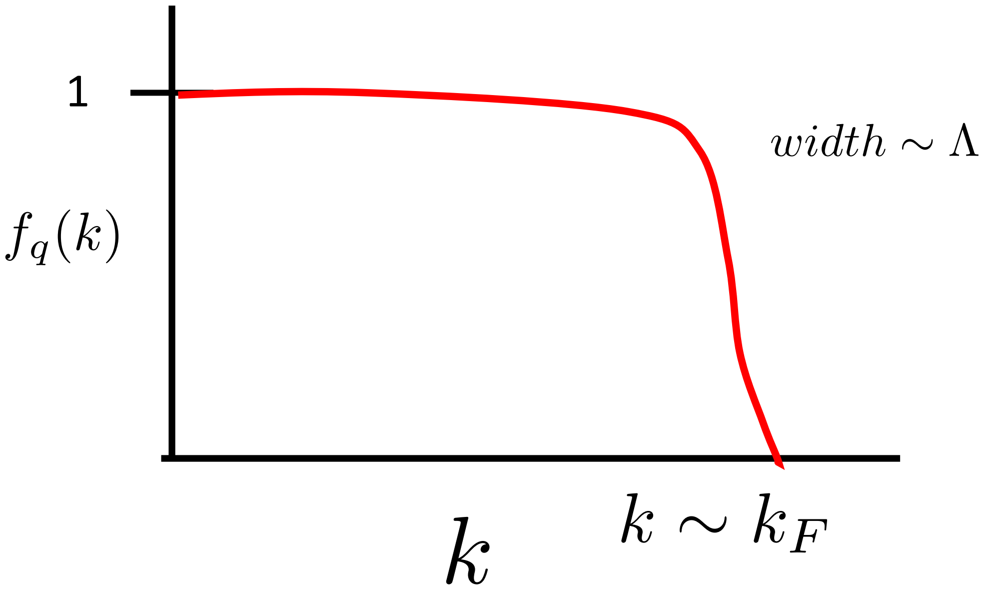
Width of quark distribution determined by intrinsic confinement scale of quarks inside of nucleons

$$\Lambda$$

$$n_N^{crit} \sim \Lambda^3$$

High Density:





At high densities, a thin shell of saturated baryon matter forms surrounded by an underoccupied distribution of nucleons

The quarks make a filled sea of nucleons with an underoccupied tail where the shell of nucleons are not Pauli blocked

How do we understand the correspondence of the underoccupied nucleon sea and the filled quark Fermi sea?

With a filled quark Fermis sea, one can add no more nucleon than can be composed by the quark Fermis sea. The density of baryon number for quarks of momentum k is

$$n_B = \frac{1}{N_c} n_q \sim k_q^3$$

The baryon number of nucleons composed of such quarks has momentum

$$k_N = N_c k_q$$

So that a filled Fermi sea of nucleons would have

$$n_N = N_c^3 k_q^3 = N_c^3 n_q$$

Is much more than the possible number of states which can be made from quarks. Therefore the composite nucleon sea is underoccupied.

Before the transition:

$$v_s^2 \sim \Lambda^2/M_N^2 \sim 1/N_c^2$$

Equation of state is soft

After the transition:

$$v_s^2 \sim \Lambda^2/M_Q^2 \sim 1$$

Equation of state is hard

There may be is a sharp the sound velocity at the transition density. For free quarks and nucleons, the peak might be too sharp and violates causality. In teractions that smooth the discontinuous changes will cure this.

A simple choice of kernel and explicit duality

$$f_q(k) = \int [dp]\; K(k - p/N_c) f_N(p)$$

$$K(k) = \frac{1}{4\pi\Lambda^2} \frac{e^{\frac{-|k|}{\Lambda}}}{|\, k \,|}$$

$$\left\{ -\nabla_K^2 + \frac{1}{\Lambda^2} \right\} K(k) = \frac{1}{\Lambda^2} \delta(k)$$

$$(-\nabla_k^2 + \frac{1}{\Lambda^2}) f_q(k) = \frac{N_c^3}{\Lambda^2} f_N(N_c k)$$

$$\epsilon = \int [dp] \sqrt{p^2 + M_n^2} f_N(p) = N_c \int [dk]\; E_q(k) f_q(k)$$

$$E_q(k) = \sqrt{k^2 + M_q^2} - \frac{M_q^2 \Lambda^2}{(k^2 + M_q^2)^{3/2}}$$

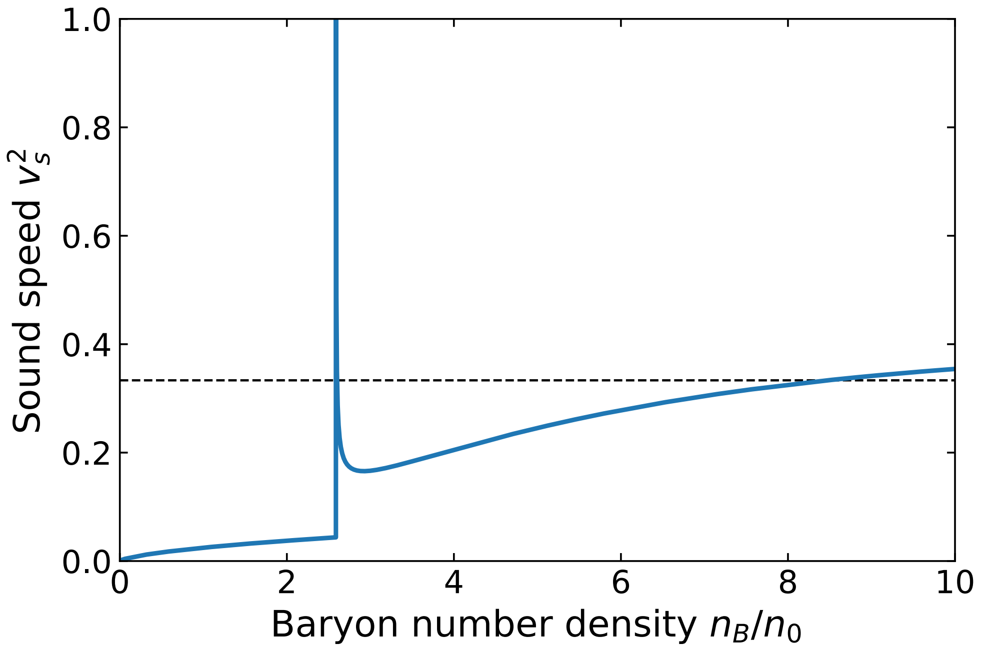
Free theory of quarks with modified kinetic energy term

Low density phase nuclear matter phase:

$$n_B \leq \frac{\Lambda^3}{N_c^{3/2}}$$

(N_c factor comes from IR singularity of kernel)

High density quarkyonic phase which can be weakened by very strong interactions. Quarkyonic phase turns off at some high density



To do list:

Hyperons and Isospin Dependence
Finite temperature
Relation between saturation density of quarkyonic matter and saturation density of nuclear matter
How to do transport theory
Regularizing surfaces
Lorentz covariant wavefunctions