Finite density QCD equation of state and lattice-based T′-expansion (Ising-TExS)

arXiv:2402.08636v1

Micheal KAHANGIRWE

Collaborators: Stefan A. Bass, Elena Bratkovskaya, Johannes Jahan, Pierre Moreau, Paolo Parotto, Damien Price, Claudia Ratti, Olga Soloveva and Mikhail Stephanov.

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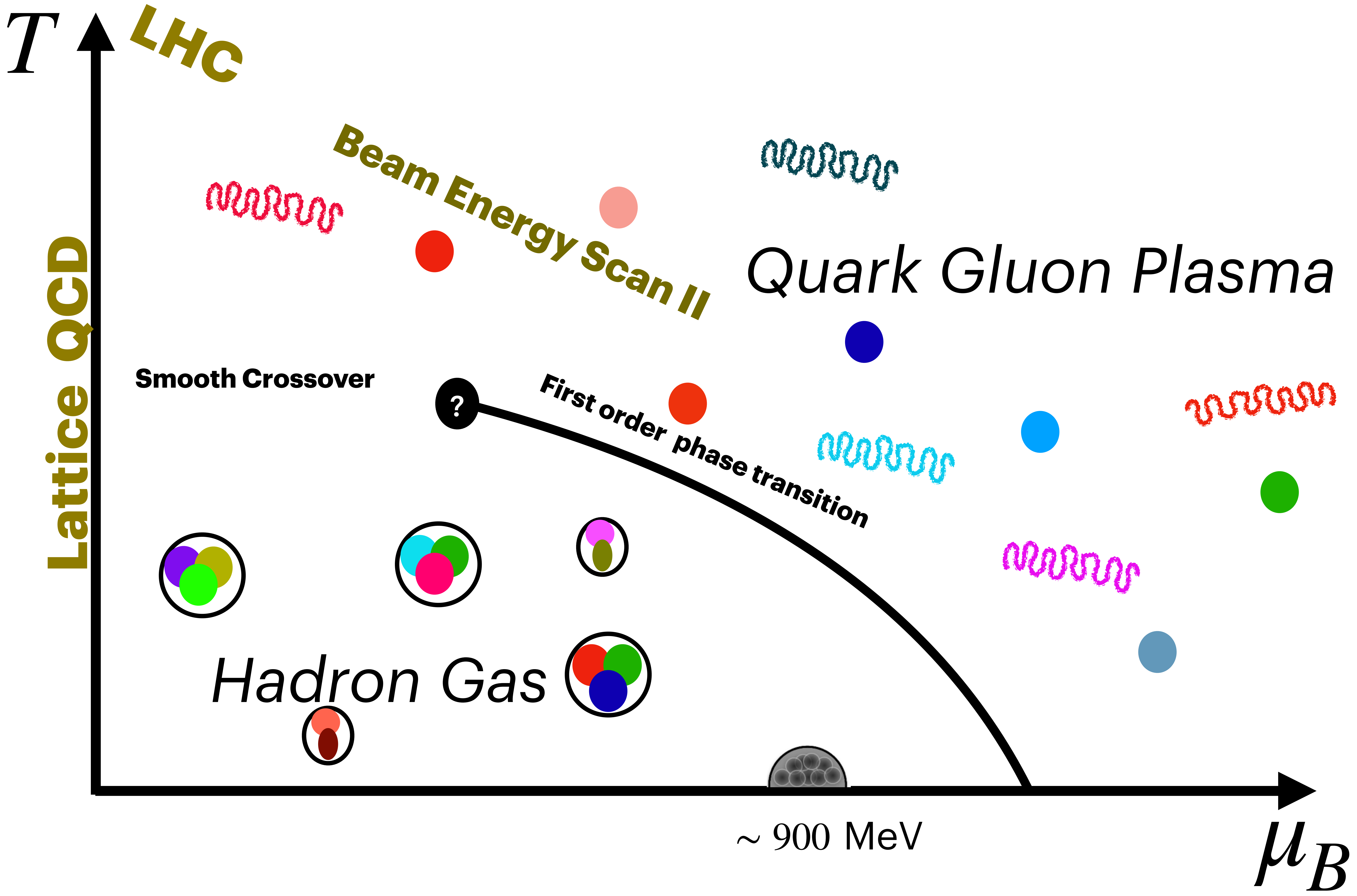
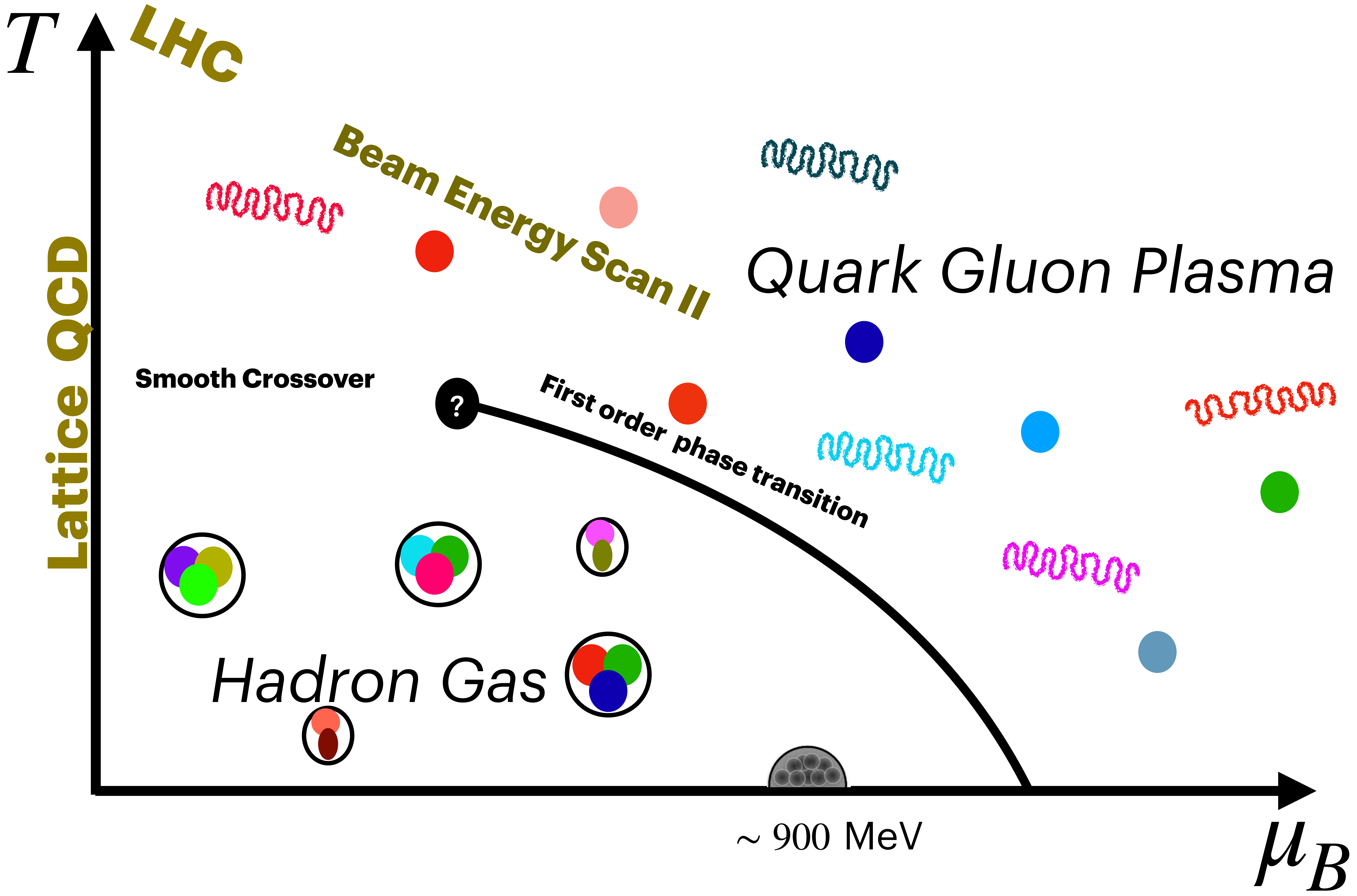
QCD Phase Diagram



Extension to 4D phase diagram

[See Johannes' Talk]

QCD Phase Diagram



Extension to 4D phase diagram

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Hydrodynamics

- Need EoS as input

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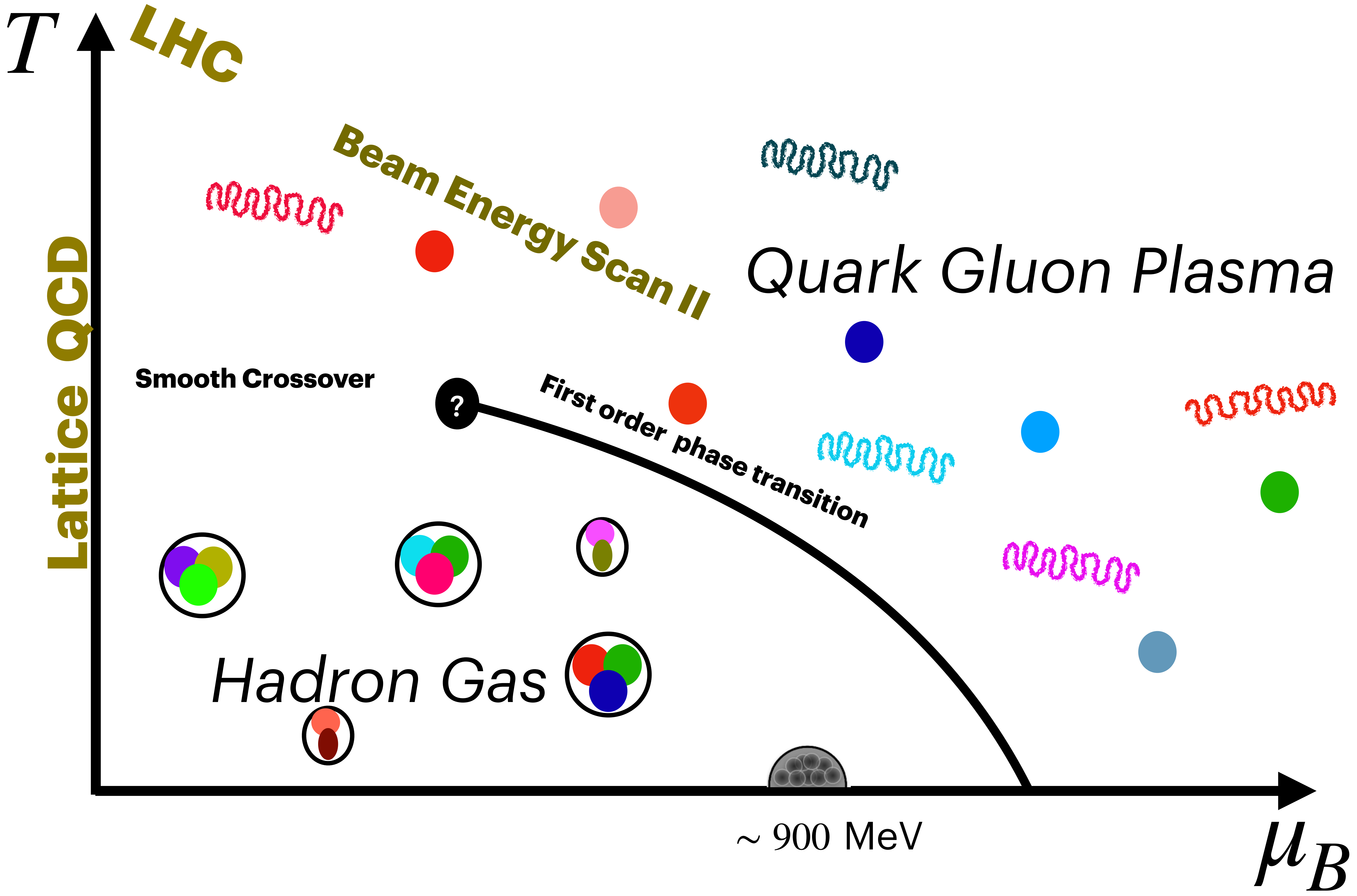

Extension to 4D phase diagram

[See Johannes' Talk]

Hydrodynamics

- Need EoS as input

Fermi sign problem

- Lattice simulations at finite μ_B are challenging

Part 1: Taylor Expansion

Taylor: Lattice QCD results

Taylor Expansion around $\mu_B = 0$

$$\frac{P(T,\mu_B)}{T^4} = \sum_{n=0}^{\infty} \frac{1}{2n!}\chi_{2n}(T,\mu_B=0)\left(\frac{\mu_B}{T}\right)^{2n}$$

[Borsanyi, S. et al High Energy Physics.9(8), 1-16.(2012)]

[Bazavov, A et al PhysRevD.95, 054504 (2017)]

$$c_n(T) = \frac{\chi_n^B(T,\mu_B=0)}{n!} = \frac{1}{n!}\left(\frac{\partial}{\partial(\mu_B/T)}\right)^n (P/T^4)\bigg|_{\mu_B=0}$$

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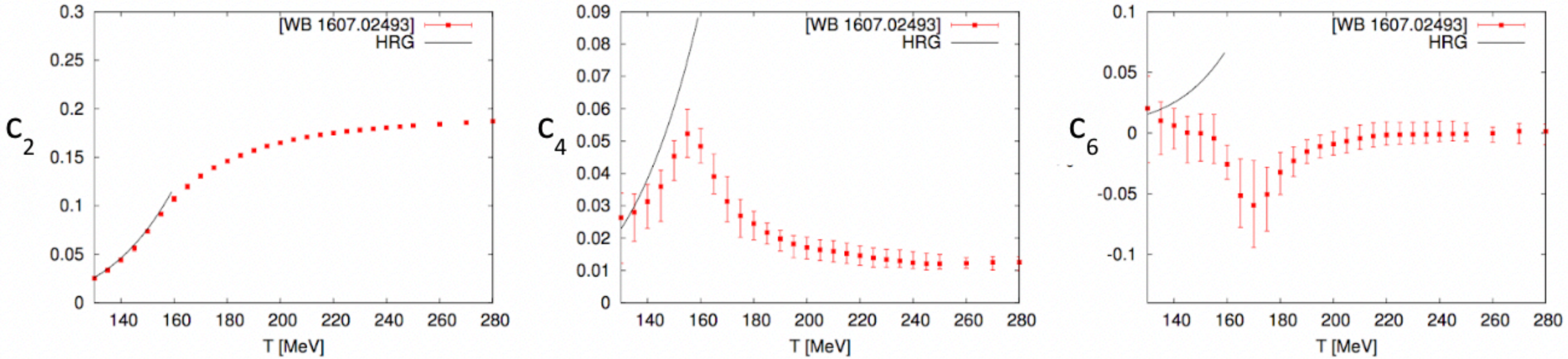
Limitations

- **Currently limited to $\frac{\mu_B}{T} \leq 3$ despite great computational power**
- **Including one more higher-order term does not remove unphysical behavior due to truncation of Taylor series**

[Bollweg, D. et al Phys.Rev.D 108 (2023) 1, 014510]

[Borsanyi , S et al arXiV:2312.07528v1. (2023)]



[WB Lattice QCD Collaboration]

Taylor: merging of lattice QCD results and critical behavior

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BEST COLLABORATION

Taylor expansion up to $\mathcal{O}((\mu_B/T)^4)$

$$\chi_n^{lat}(T) = \chi_n^{non-Ising}(T) + \frac{T_C^4}{T^4} \chi_n^{Ising}(T)$$

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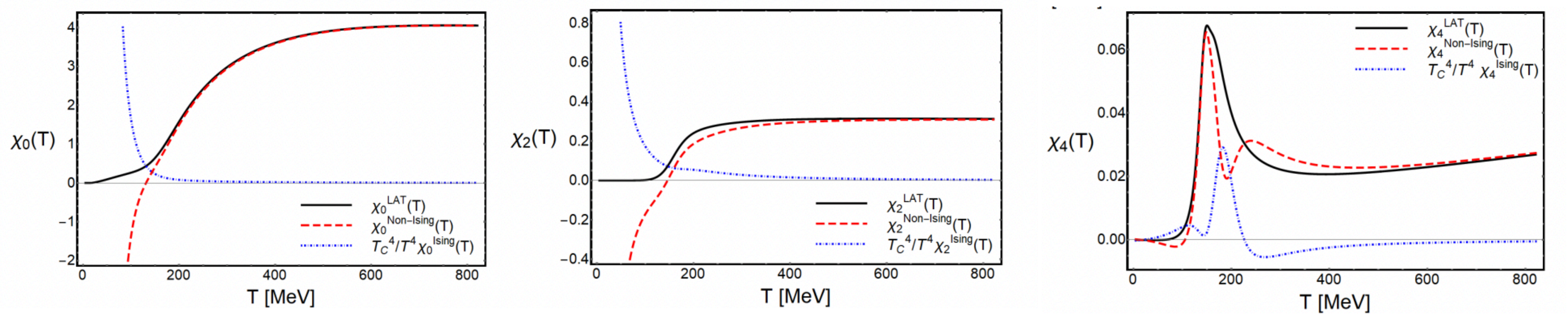


[P Parotto, et al PhysRevC. 108(1), 101.034901(2020)]

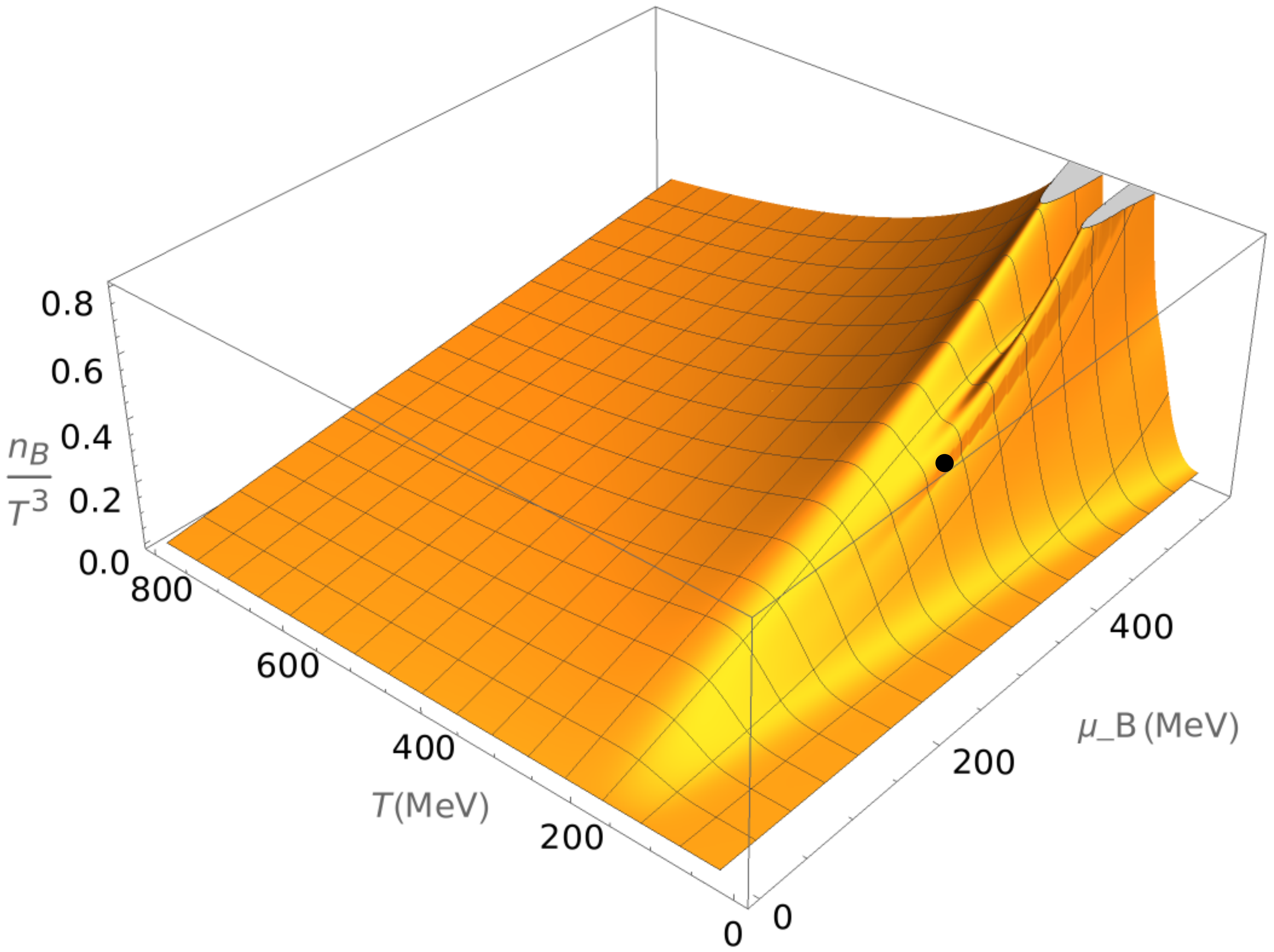
Taylor: merging of lattice QCD results and critical behavior

BEST COLLABORATION

$$\frac{n_B(T,\mu_B)}{T^3} = \frac{\partial(P/T^4)}{\partial(\mu_B/T)}$$

Critical Point at $\mu_{BC} = 350\ [MeV]$

Baryon density



- Truncated thermodynamic variables at higher $\mu_B \geq 450 MeV$ show **unphysical behavior**

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[Karthein, J, et al arXiv:2110.00622.(2021)]

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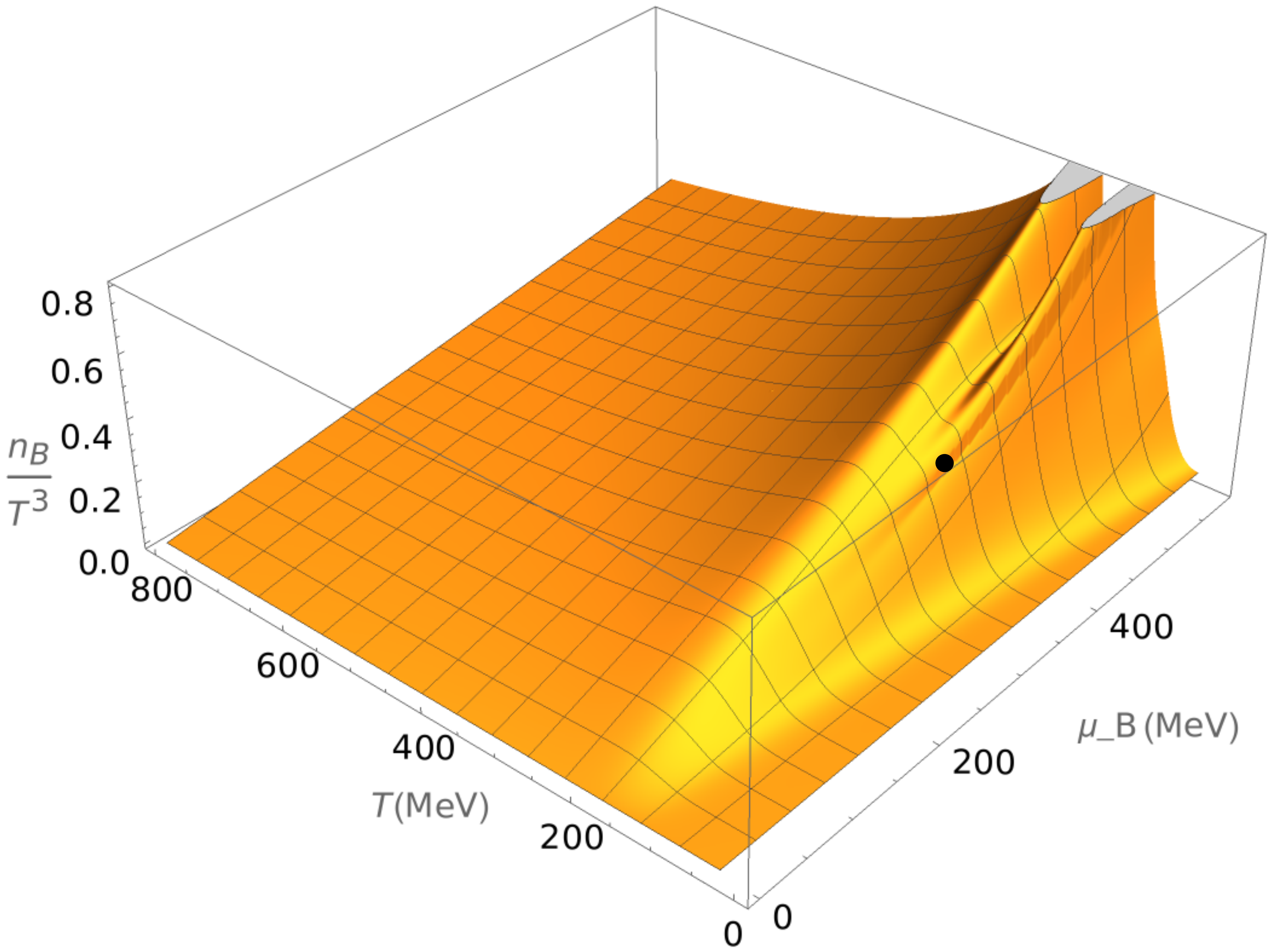

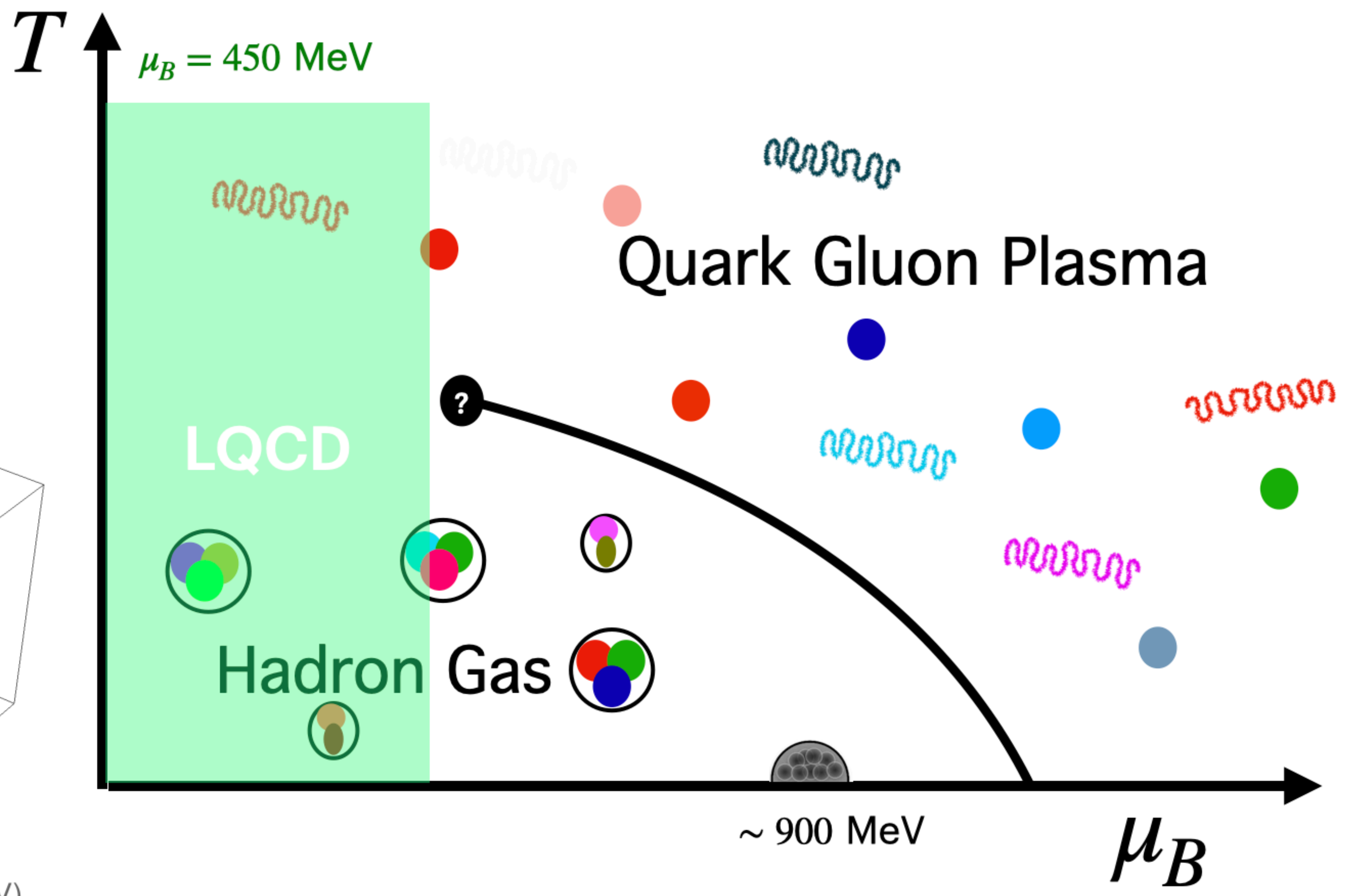

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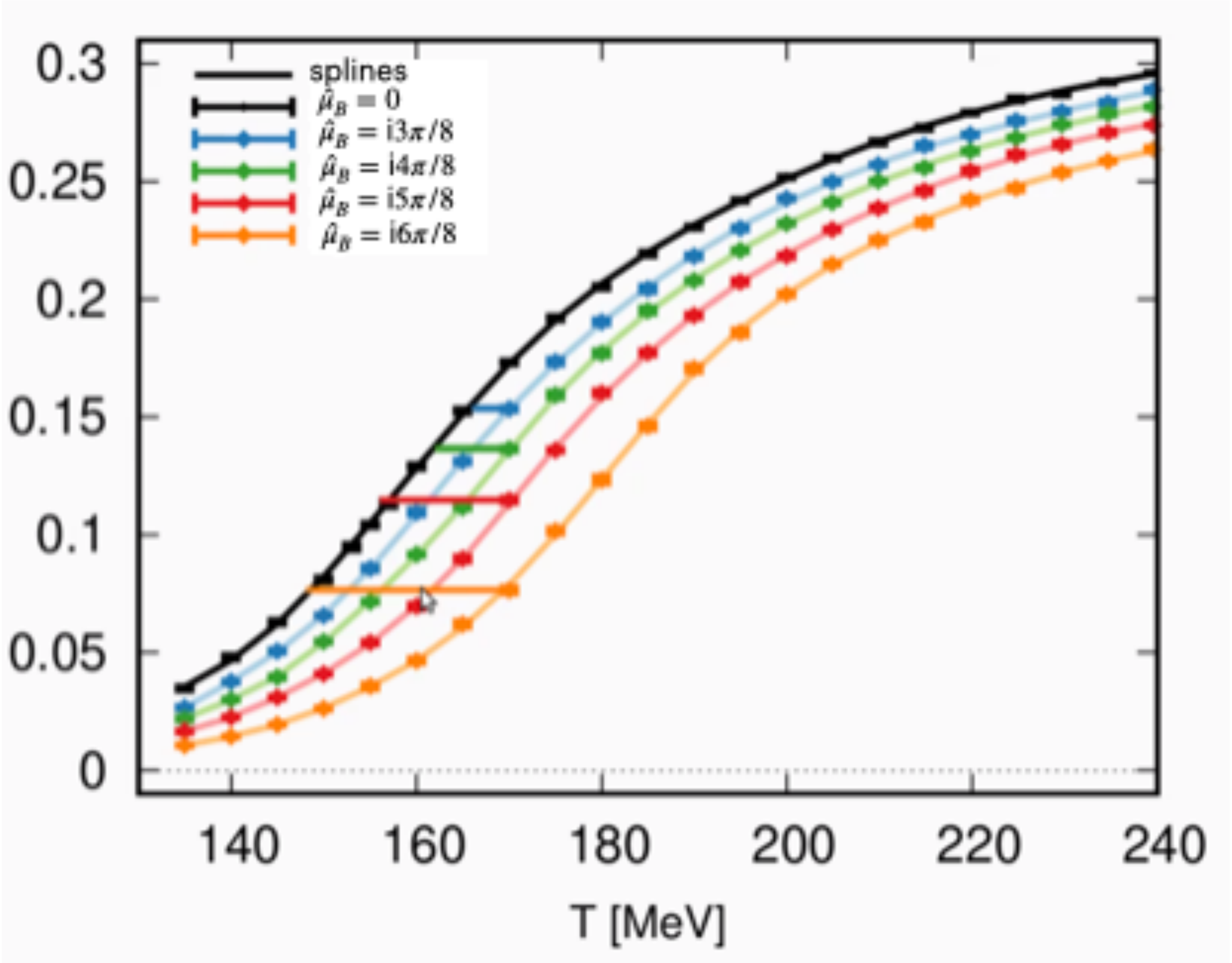
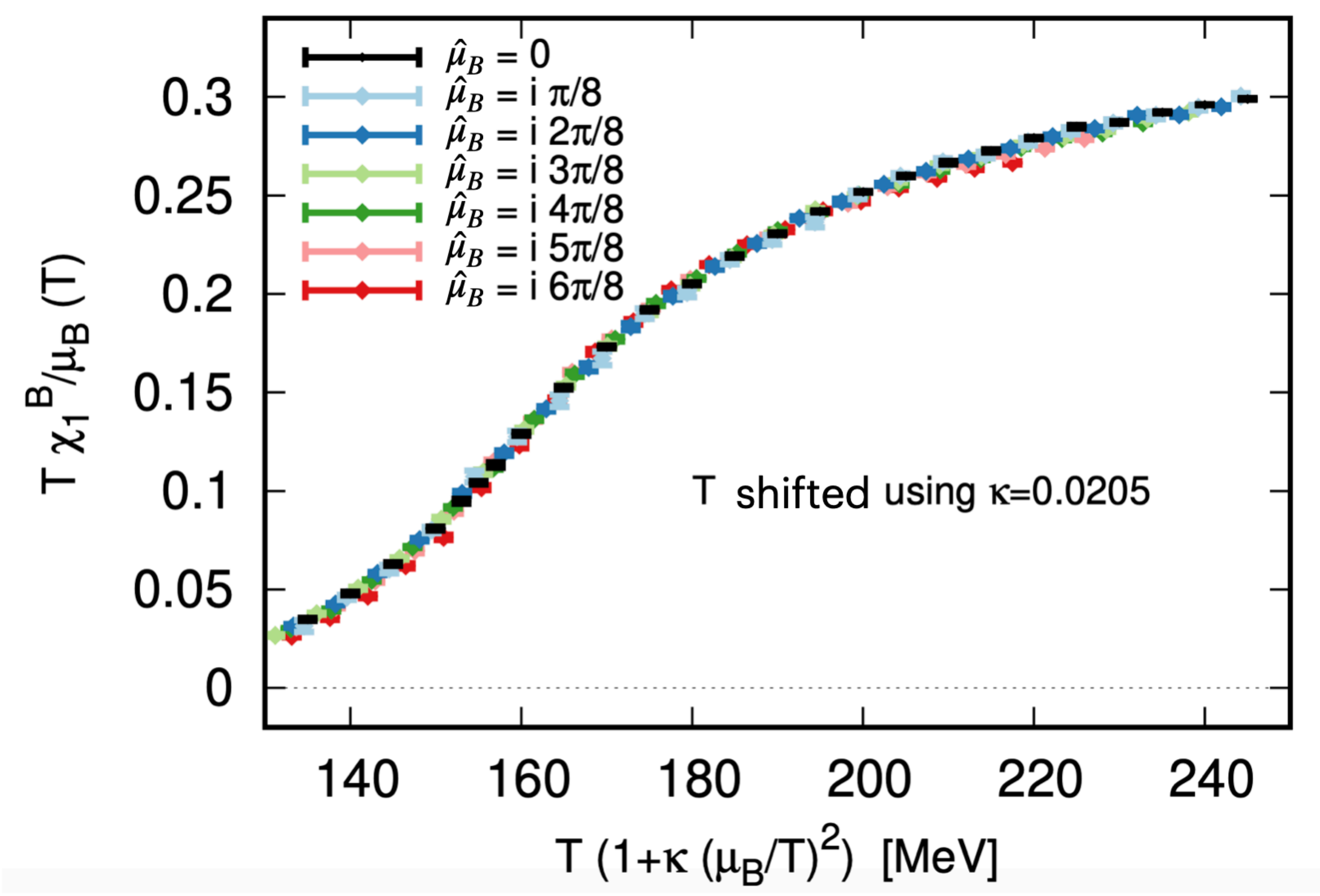
Part 2: T’ Expansion Scheme (T ExS)

T′ Expansion scheme (T ExS)

Simulating at Imaginary μ_B





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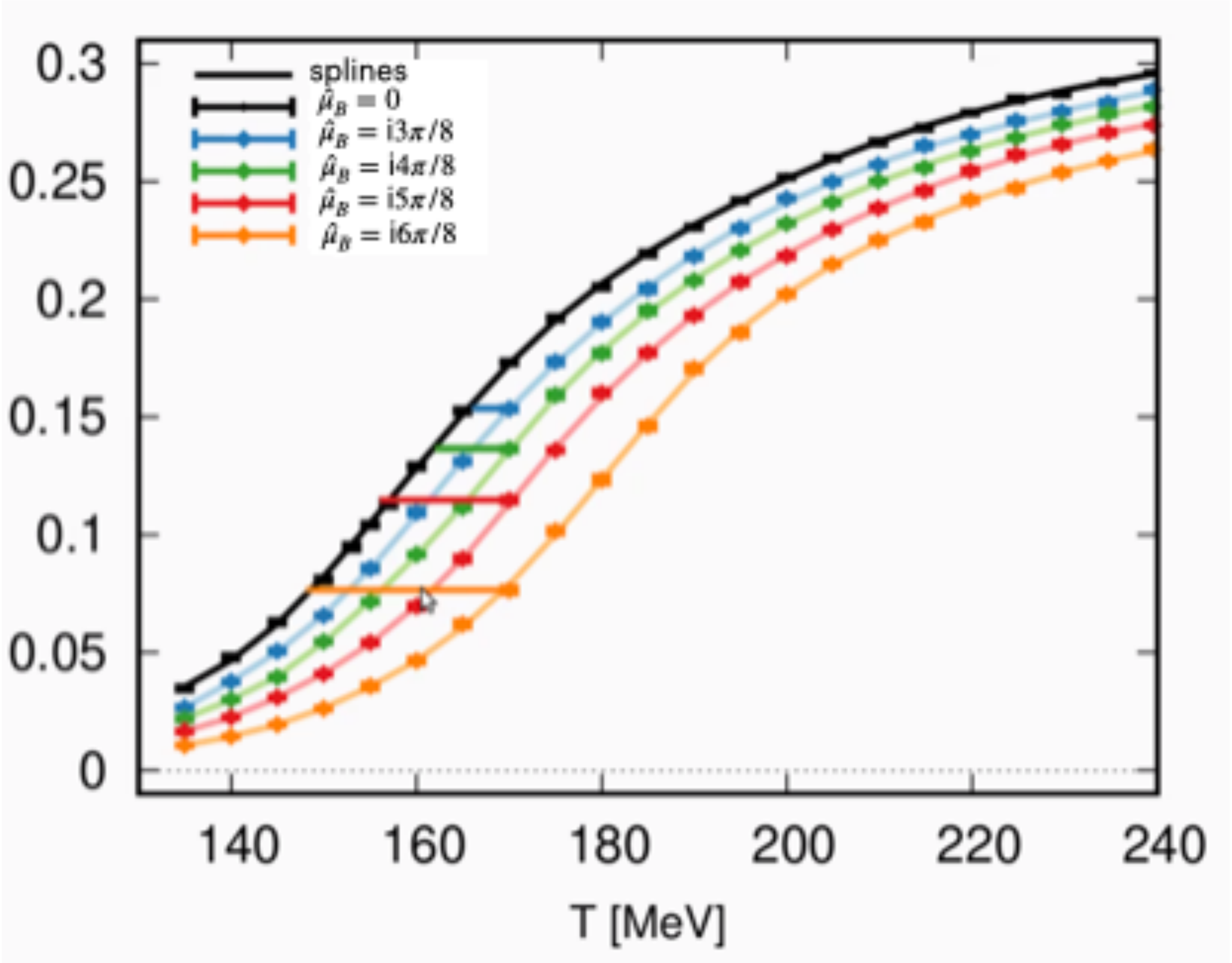

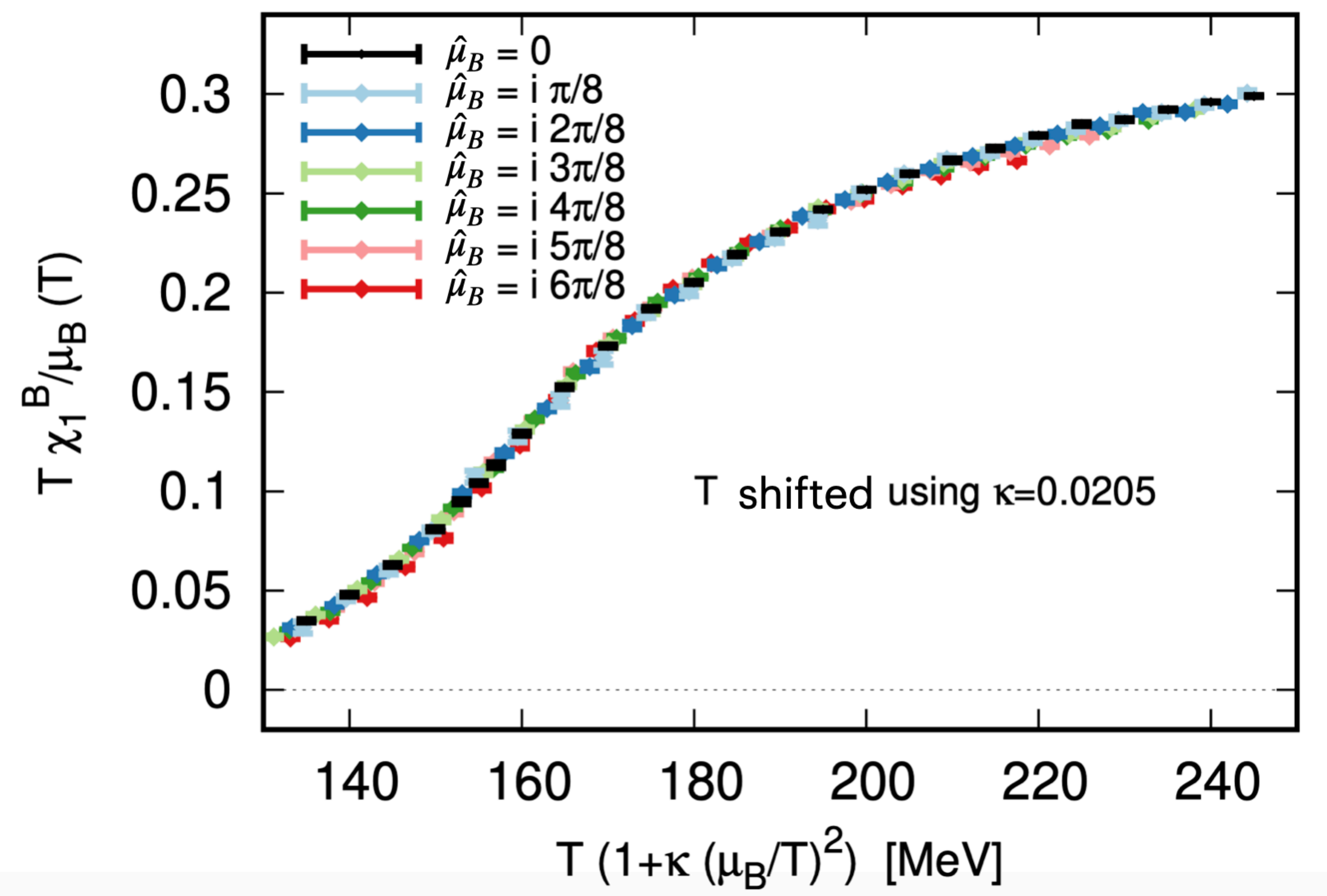

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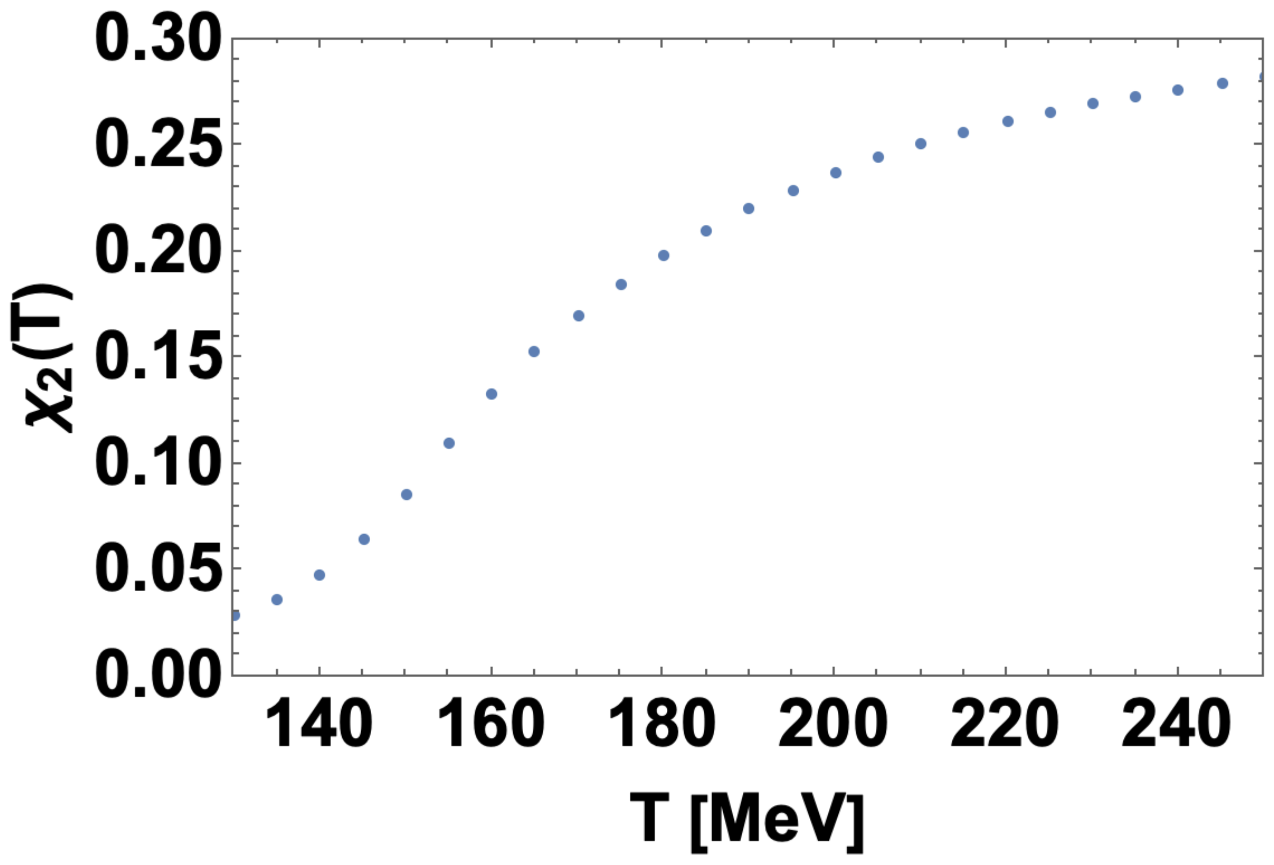

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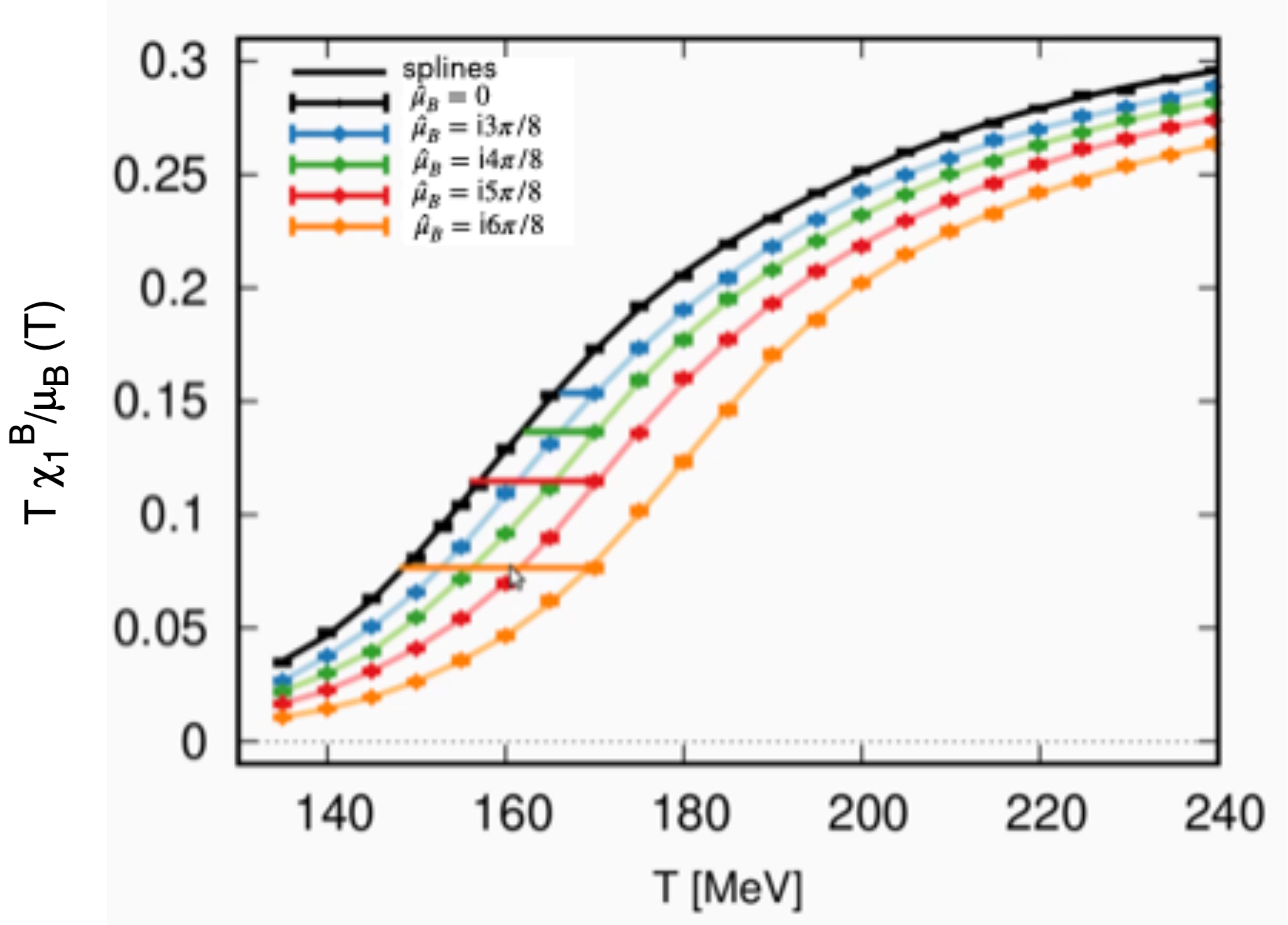

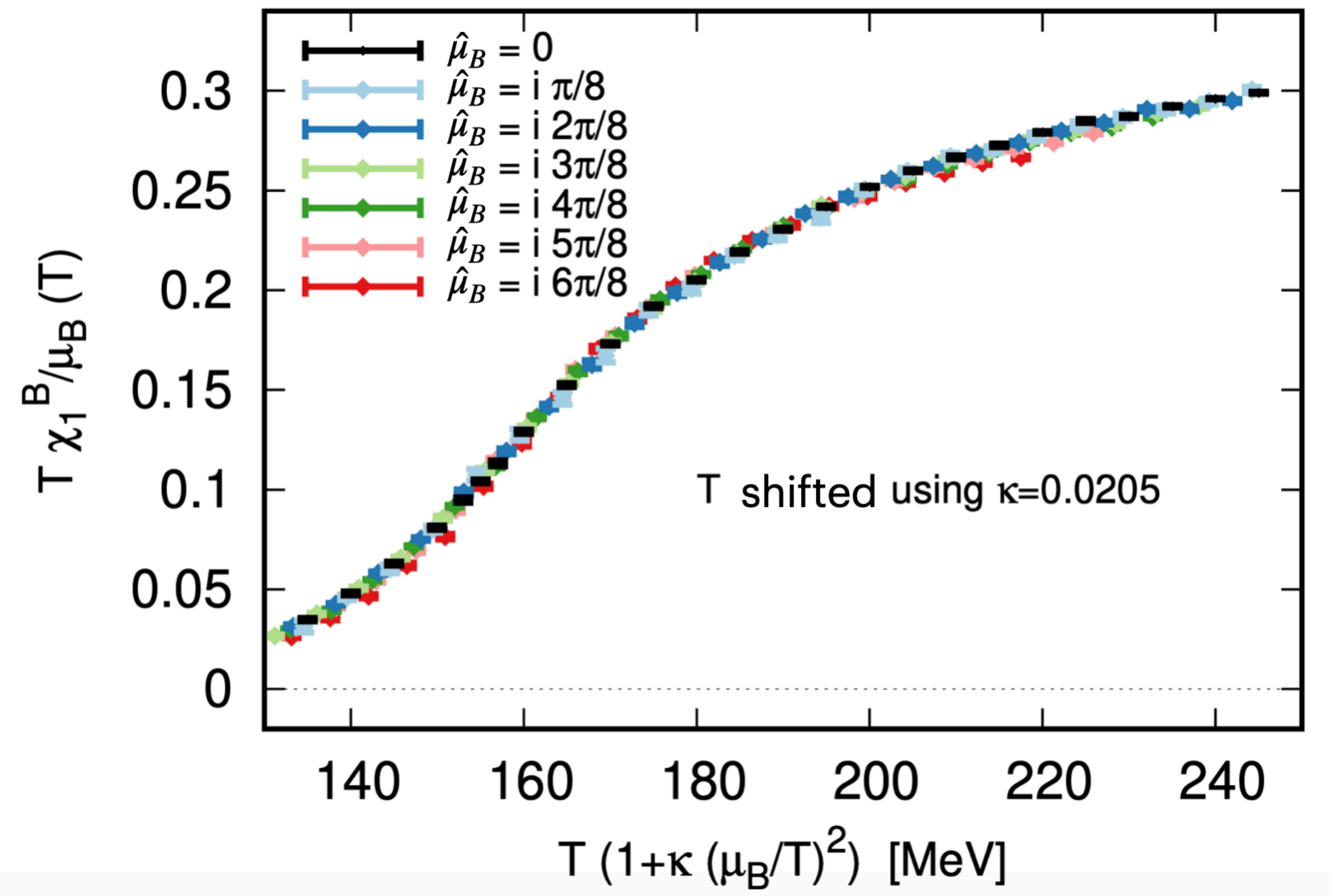

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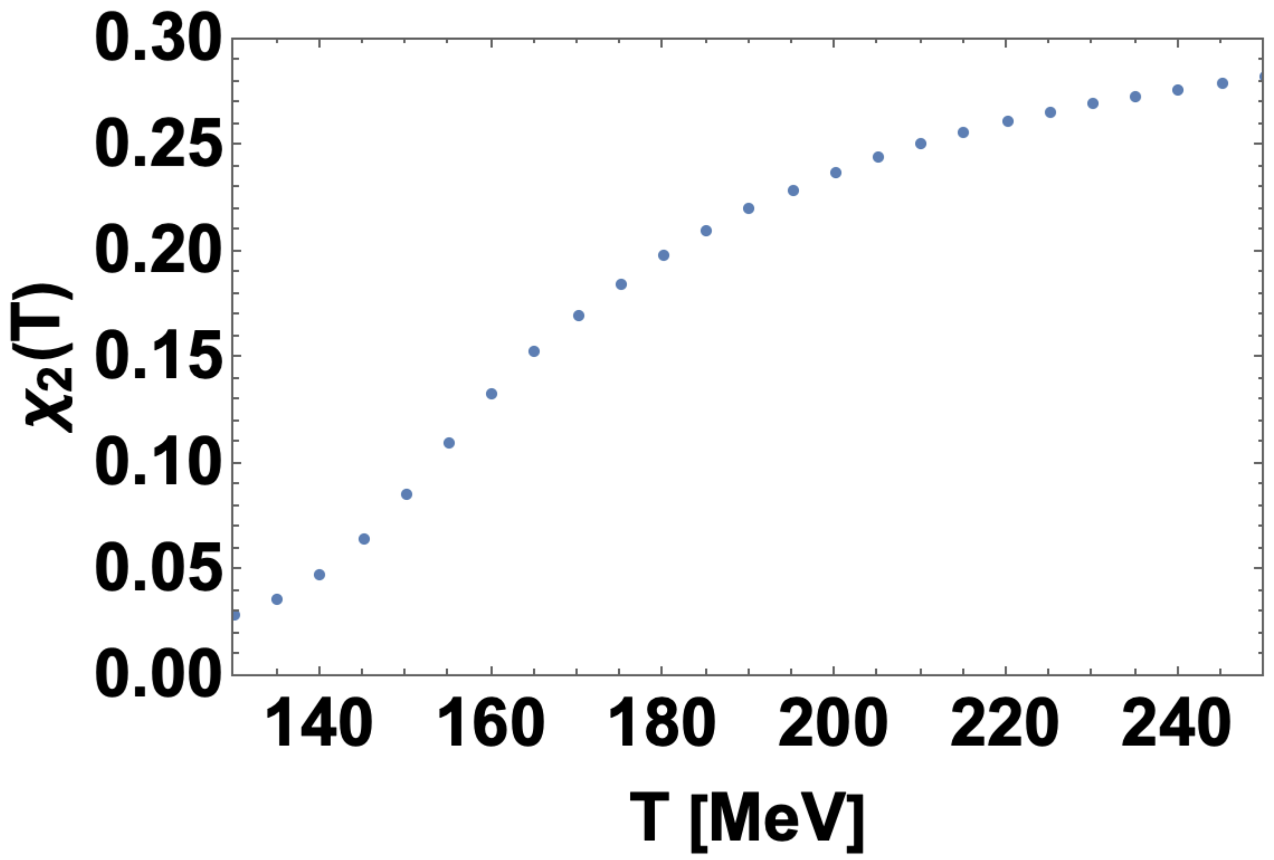

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$$T'(T,\mu_B) = T\left[1 + \kappa_2^{BB}(T)\left(\frac{\mu_B}{T}\right)^2 + \kappa_4^{BB}(T)\left(\frac{\mu_B}{T}\right)^4 + \mathcal{O}\left(\frac{\mu_B}{T}\right)^6\right]$$

T′ Expansion scheme (T ExS)

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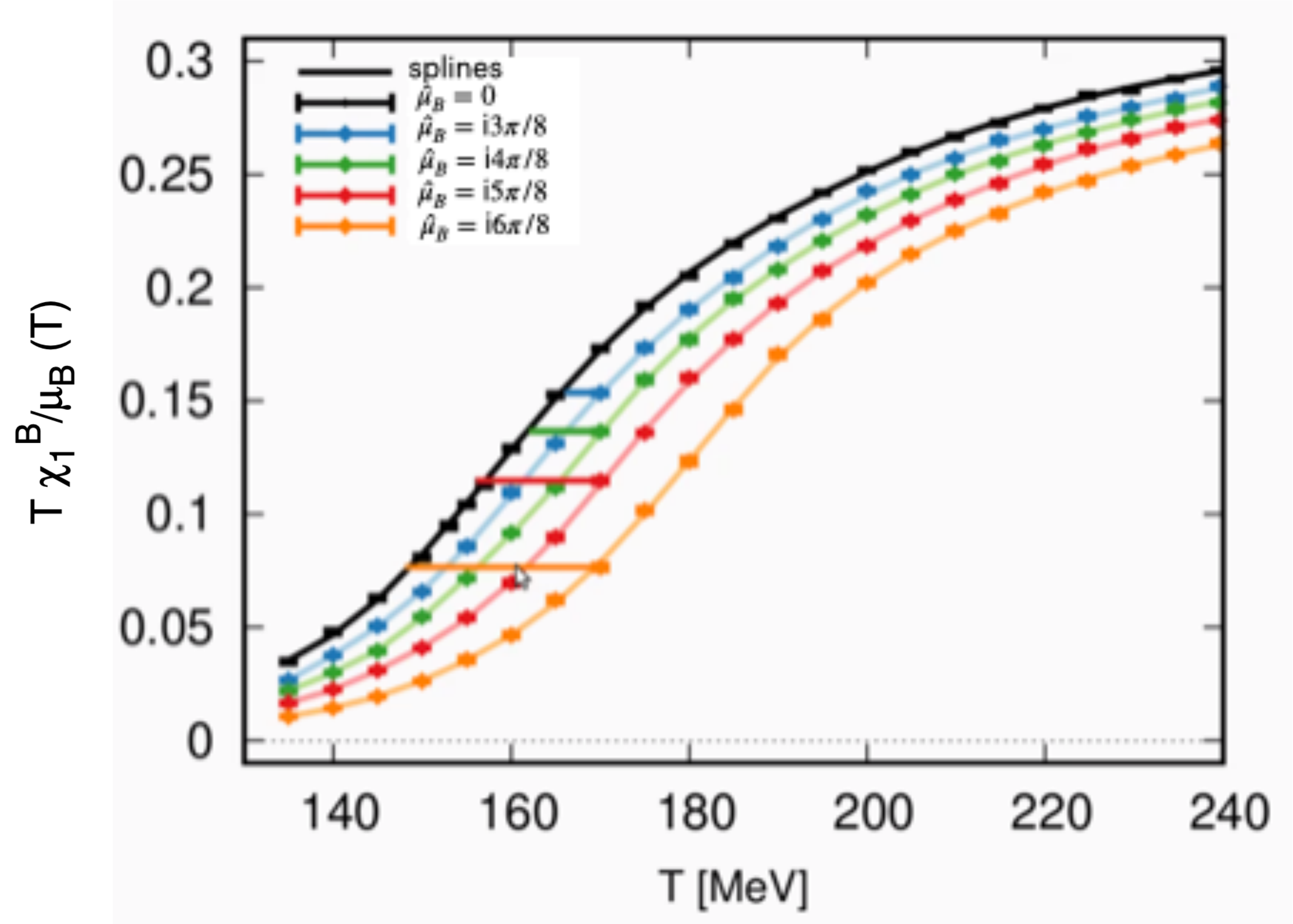

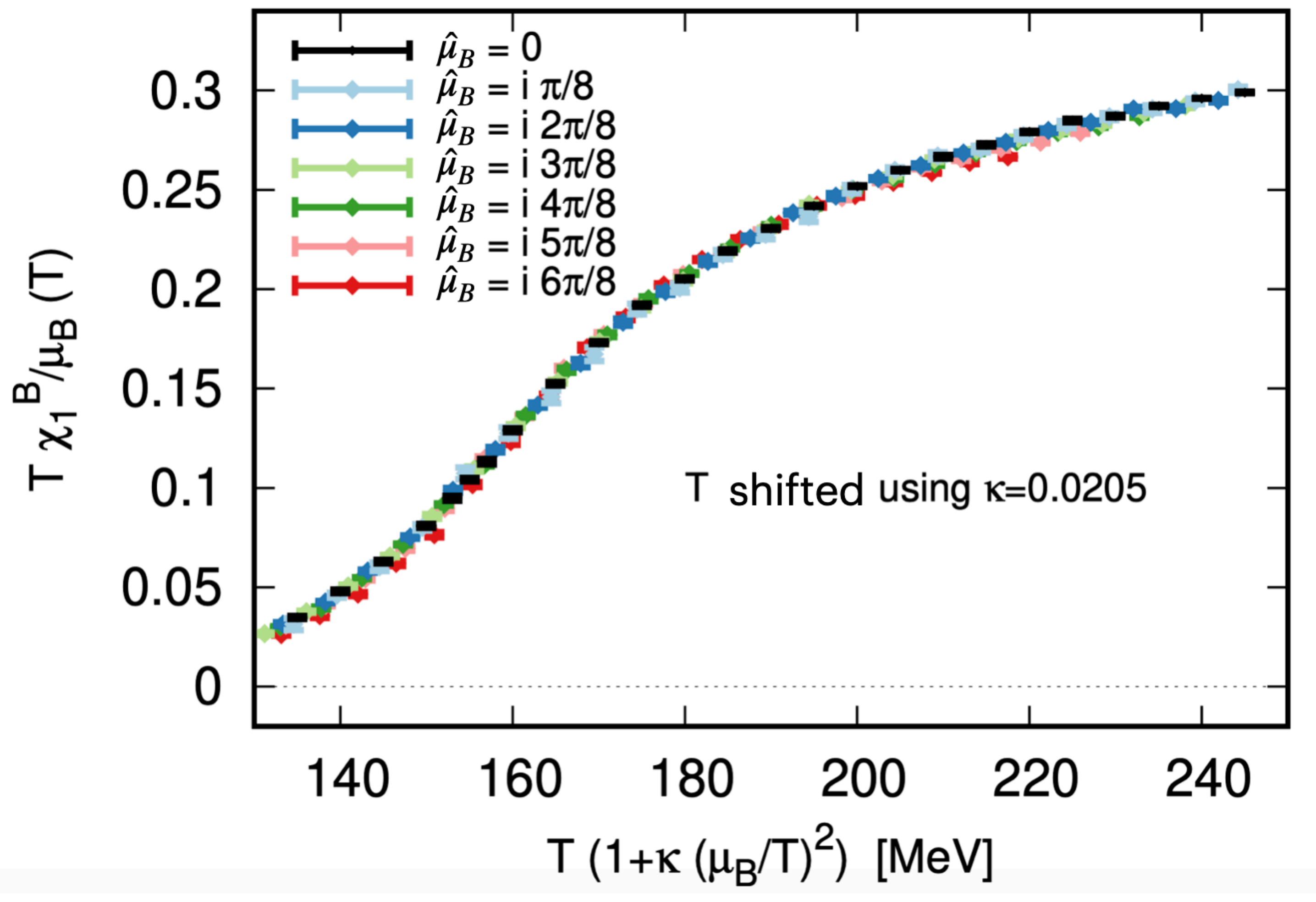

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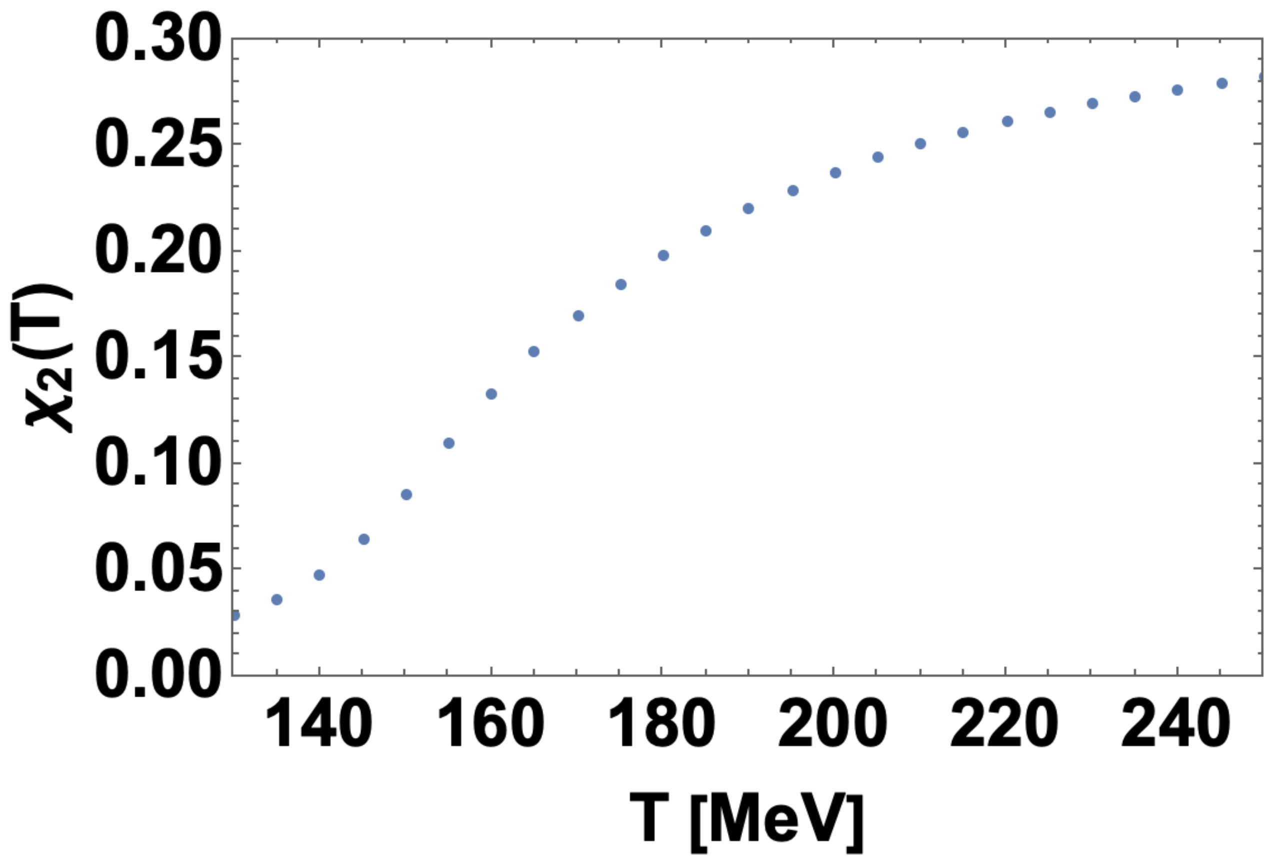

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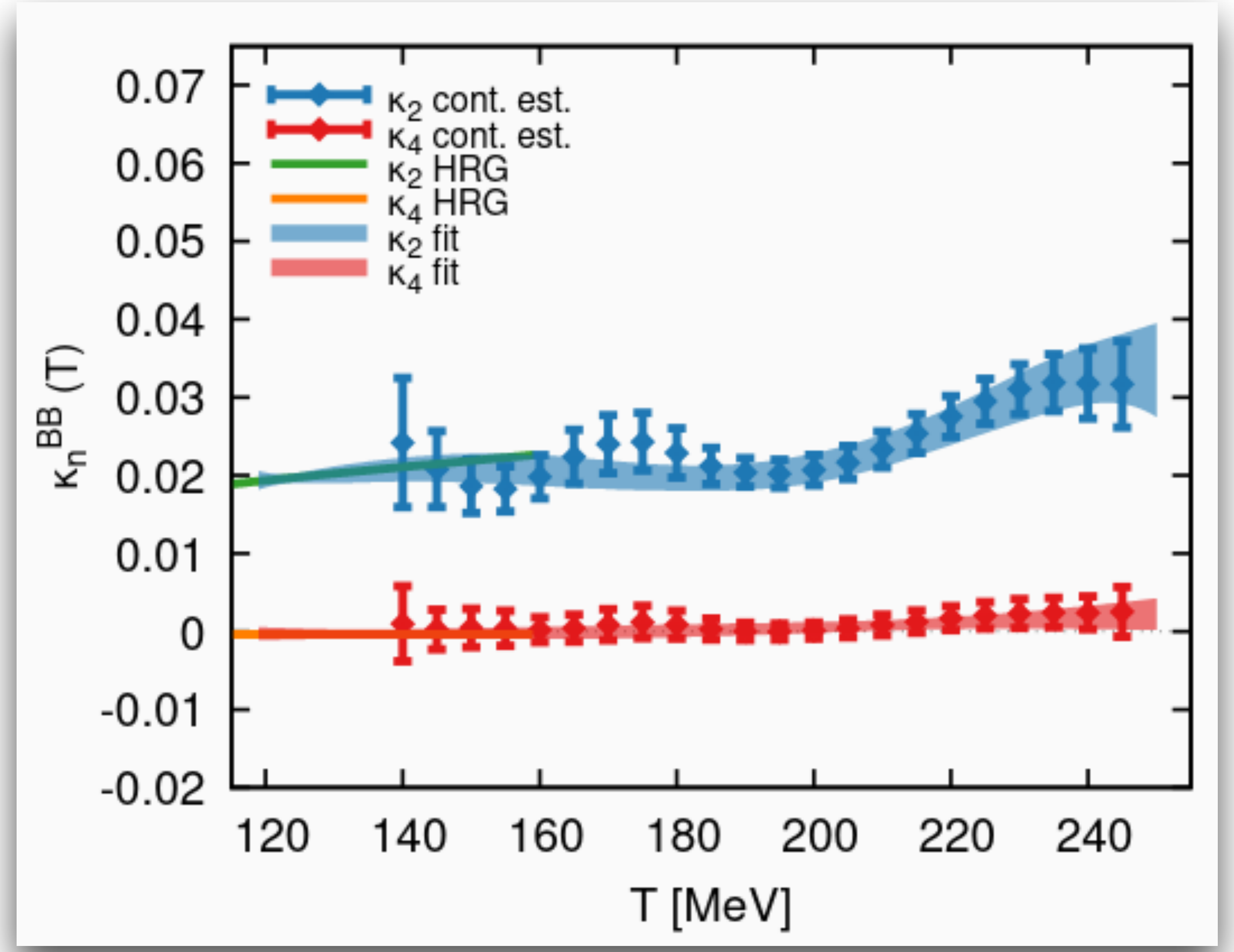
- **μ_B dependence is captured in T-rescaling.**
- **Trusted up to** $\frac{\mu_B}{T} = 3.5$

T′ Expansion scheme (T ExS)

Relationship between **Taylor expansion** and **T′ expansion**

- $$\kappa_2^{BB}(T) = \frac{1}{6T}\frac{\chi_4^B(T)}{\partial\chi_2^{\prime B}(T)}$$

- $$\kappa_4^{BB}(T) = \frac{1}{360T\chi_2^{\prime B}(T)^3}\left(3\chi_2^{\prime B2}\chi_6^B(T) - 5\chi_2^B(T)''\chi_4^B(T)^2\right)$$



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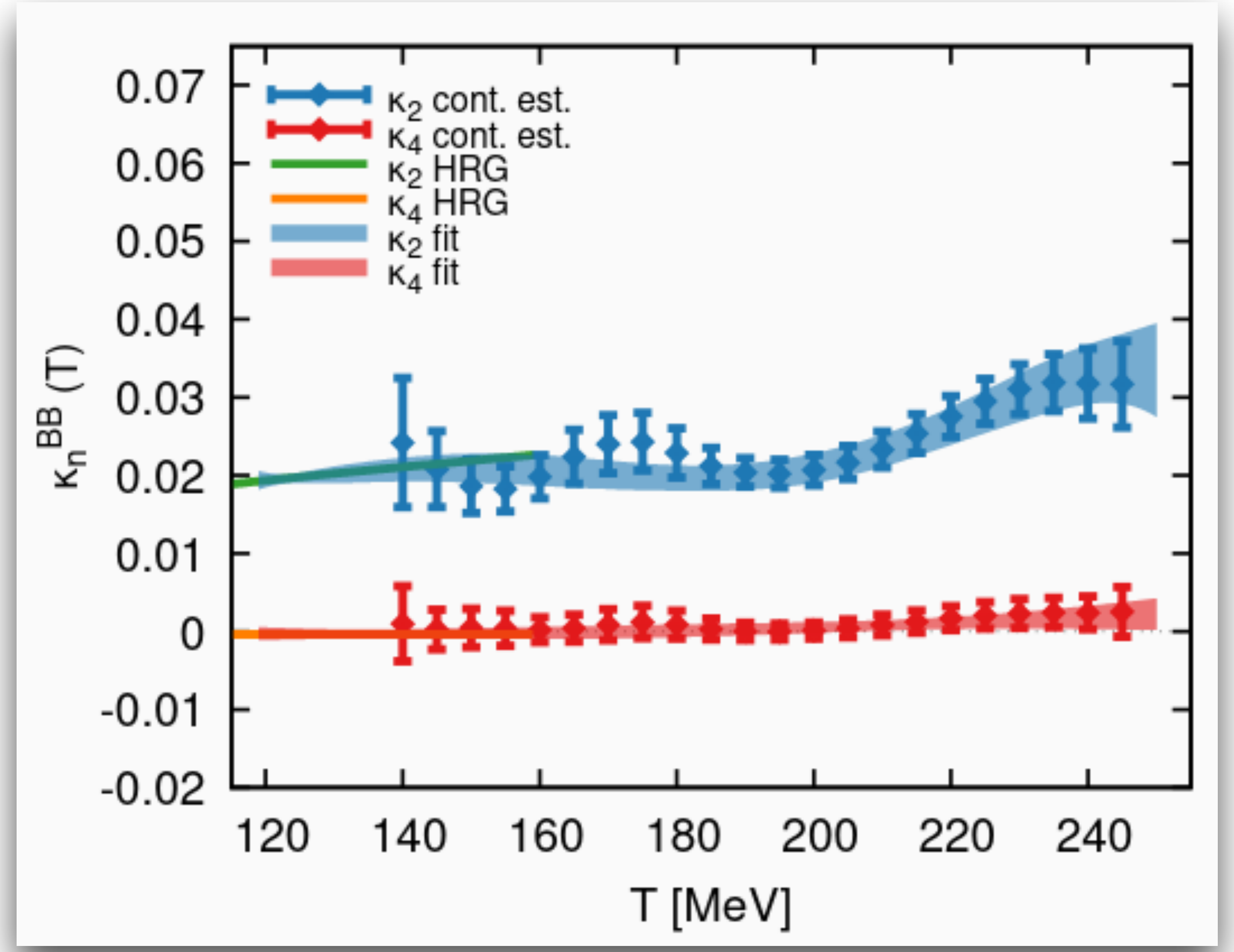

[Borsányi, S et al PRL. 108(1), 101.034901(2021)]

Pros

- $\kappa_2(T)$ **is fairly constant over a large T-Range**
- **There is a separation of scale between** $\kappa_2(T)$ **and** $\kappa_4(T)$
- $\kappa_4(T)$ **is almost zero** $\rightarrow$ **faster convergence**
- **A good agreement with HRG results at Low Temperature**

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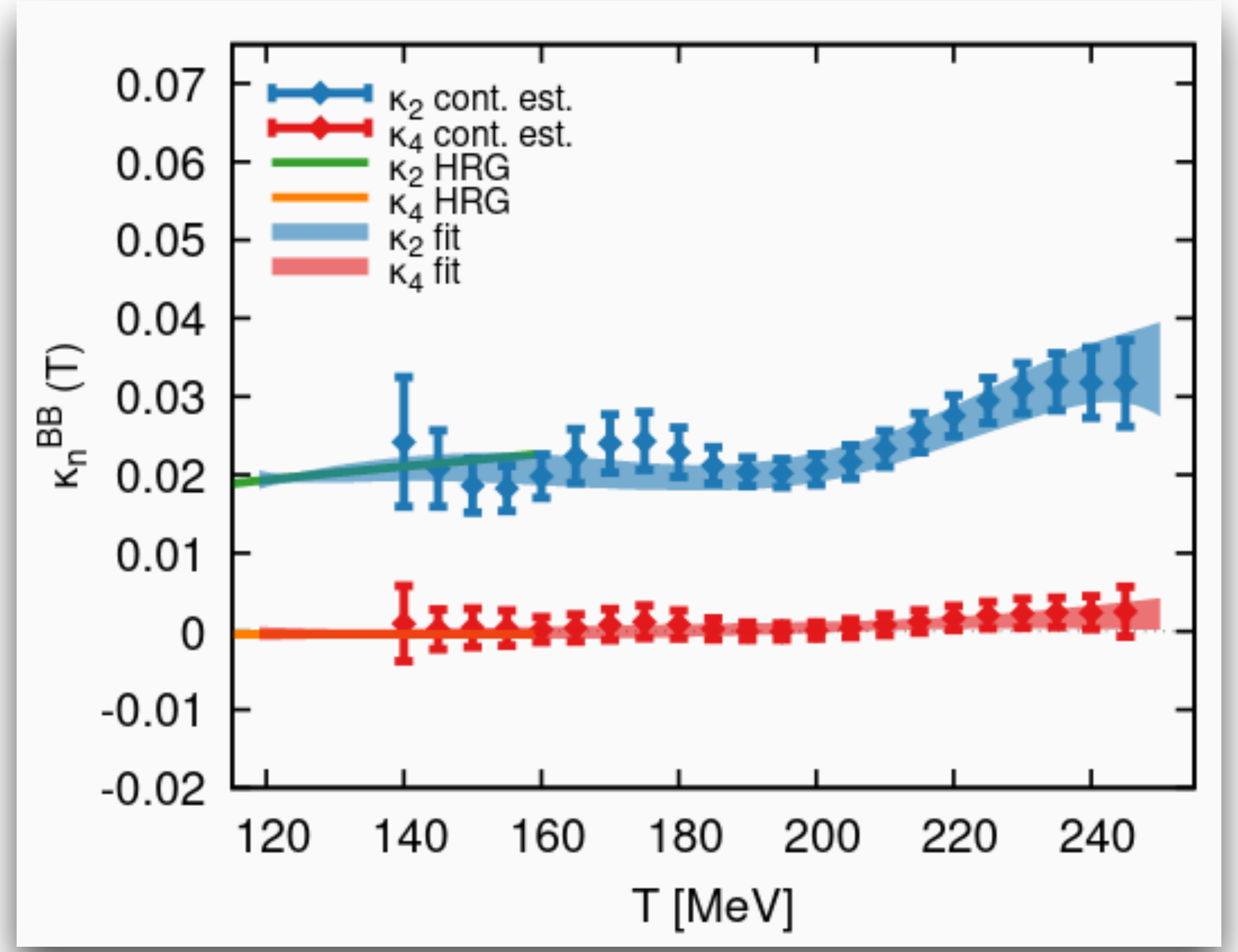

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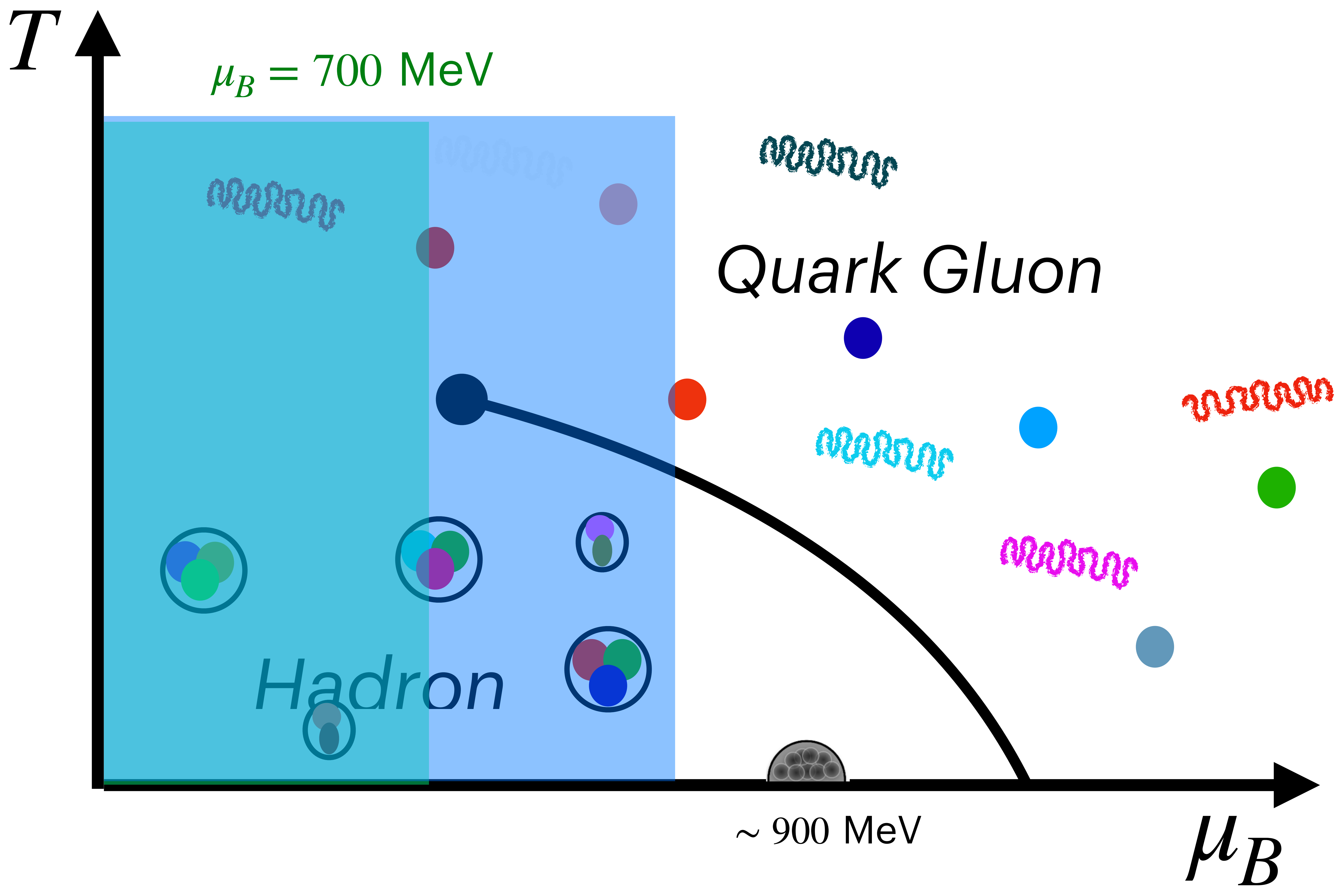


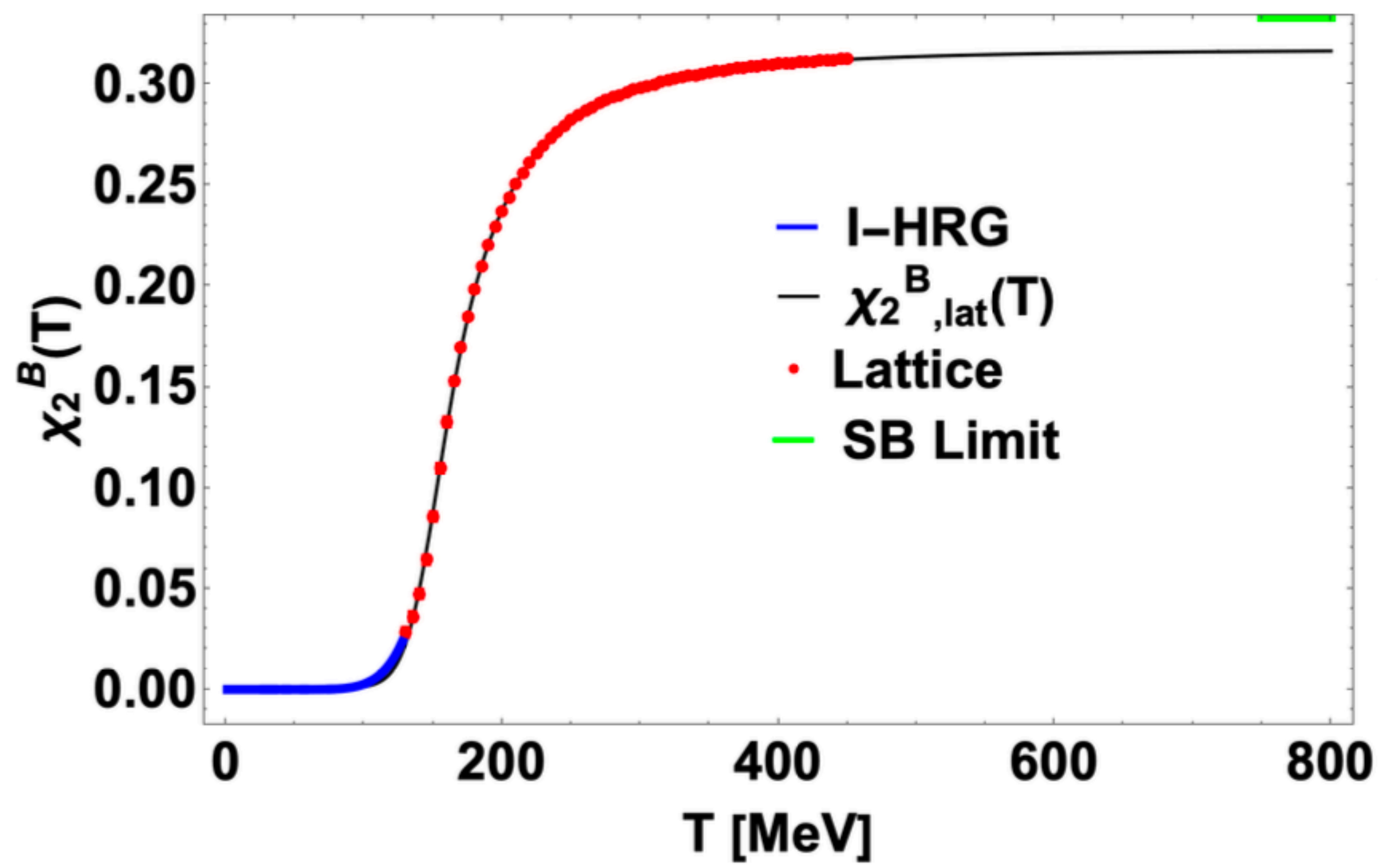
Lattice data: Parametrization

$$25 \text{ MeV} < T < 800 \text{ MeV}$$

$$\chi^B_{2,\text{lat}}(T) = \left(\frac{2m_p}{\pi x}\right)^{3/2} \frac{e^{-m_p/x}}{1+\left(\frac{x}{d_1}\right)^{d_2}} + d_3 \frac{e^{-d_4^2/x^2 - d_5^4/x^4}}{1+\left(\frac{x}{d_1}\right)^{-d_2}}$$

$x = \frac{T}{200 \text{ MeV}}$ d_i - fitting parameters

m_p - proton mass (in units of 200 MeV)



[M. K et al arXiv:2402.08636v1]

$$\kappa_2^{BB}(T) = \frac{a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + a_5 x^5}{b_0 + b_1 x + b_2 x^2 + a_5/A\, x^3}$$

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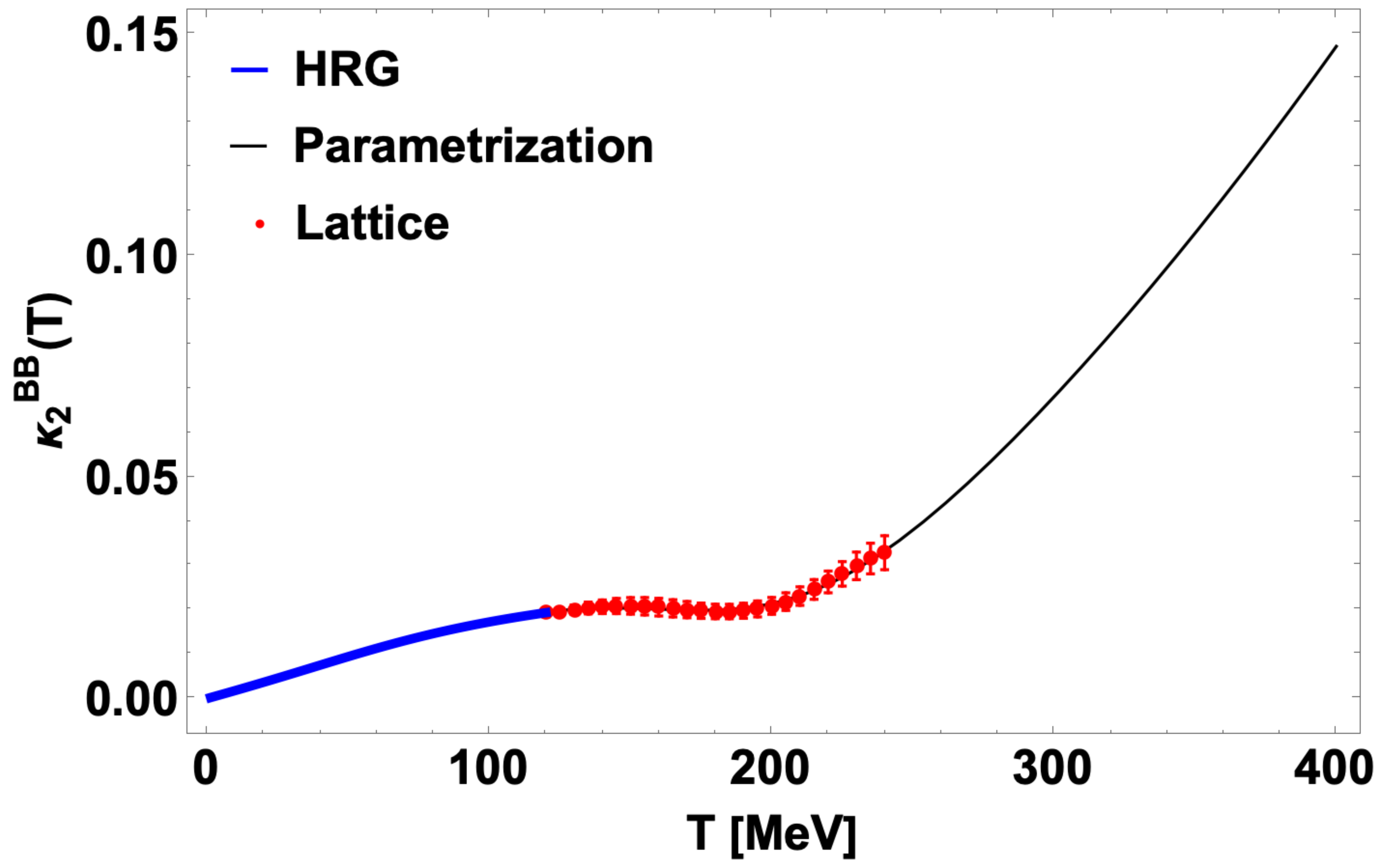


Part 3: Introducing Critical Point (3D-Ising)

EoS with a Critical point

Strategy

Scaling 3D-Ising model EoS

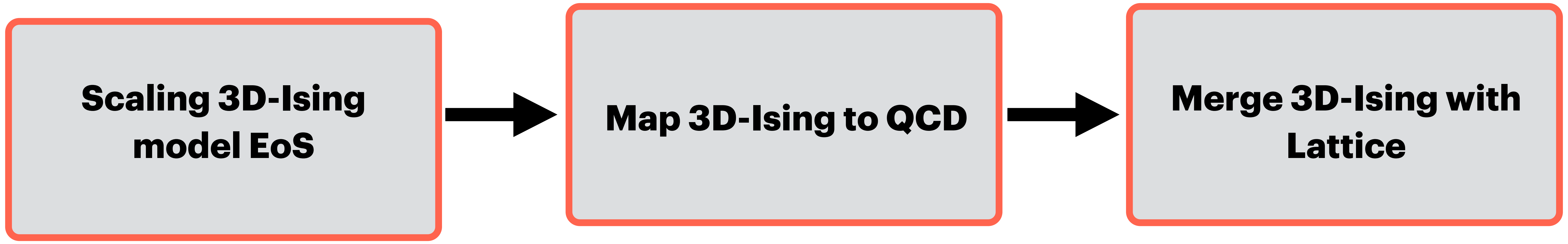
EoS with a Critical point

Strategy

Scaling 3D-Ising model EoS → Map 3D-Ising to QCD

EoS with a Critical point

Strategy



EoS with a Critical point

Strategy

Scaling 3D-Ising model EoS → Map 3D-Ising to QCD → Merge 3D-Ising with Lattice → Estimate the critical Contribution

EoS with a Critical point

Strategy

Scaling 3D-Ising model EoS → **Map 3D-Ising to QCD** → **Merge 3D-Ising with Lattice** → **Estimate the critical Contribution**

Scaling 3D-Ising Model EoS

Close to the critical point, we define a parametrization for Magnetization M, Magnetic field h, and reduced temperature

QCD Critical point is in the 3D-Ising model Universality class

$$M = M_0 R^{\beta} \theta$$

$$h = h_0 R^{\beta\delta} \tilde{h}(\theta)$$

$$r = R(1 - \theta^2)$$

$$(R \geq 0, \ |\theta| \leq \theta_0)$$

$$(R, \theta) \longmapsto (r, h)$$

$$\alpha = 0.11$$

$$\delta \sim 4.8$$

$$\beta \sim 0.326$$

$$r = \frac{T - T_C}{T_C}$$

$h \rightarrow$ **External magnetic field**

$$G(R, \theta) = h_0 M_0 R^{2-\alpha} \left[\theta \tilde{h}(\theta) - g(\theta) \right]$$

$$g(\theta) = c_0 + c_1(1 - \theta^2) + c_2(1 - \theta^2)^2 + c_3(1 - \theta^2)^3$$

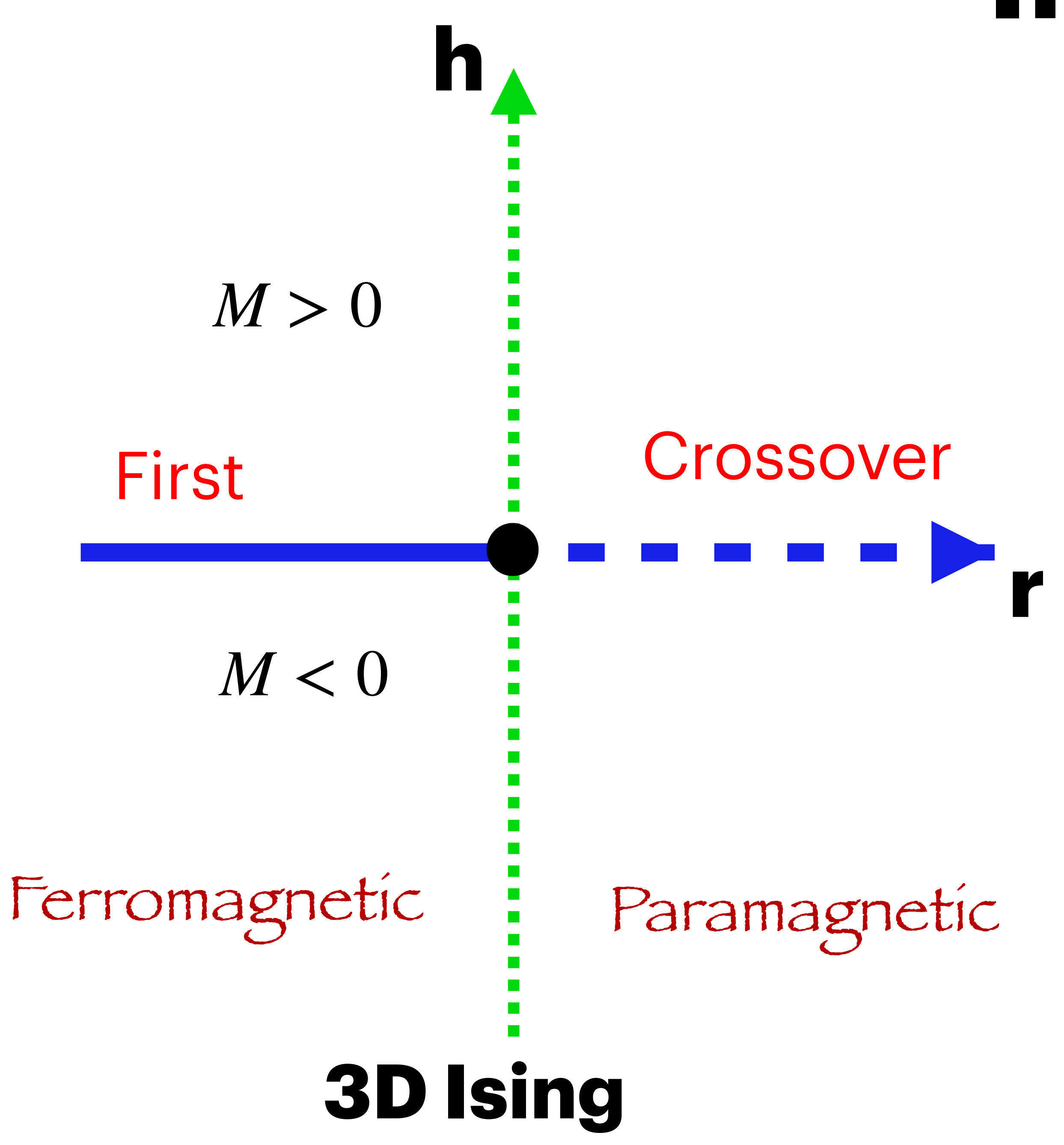
$$\tilde{h}(\theta) = (\theta + a\theta^3 + b\theta^5)$$

[Parotto et al PhysRevC.101.034901(2020)]

[Nonaka et al Physical Review C, 71(4), 044904.(2005)]

[Guida et al Nuclear Physics B, 489(3), 626-652.(1997)]

Introducing Critical Point



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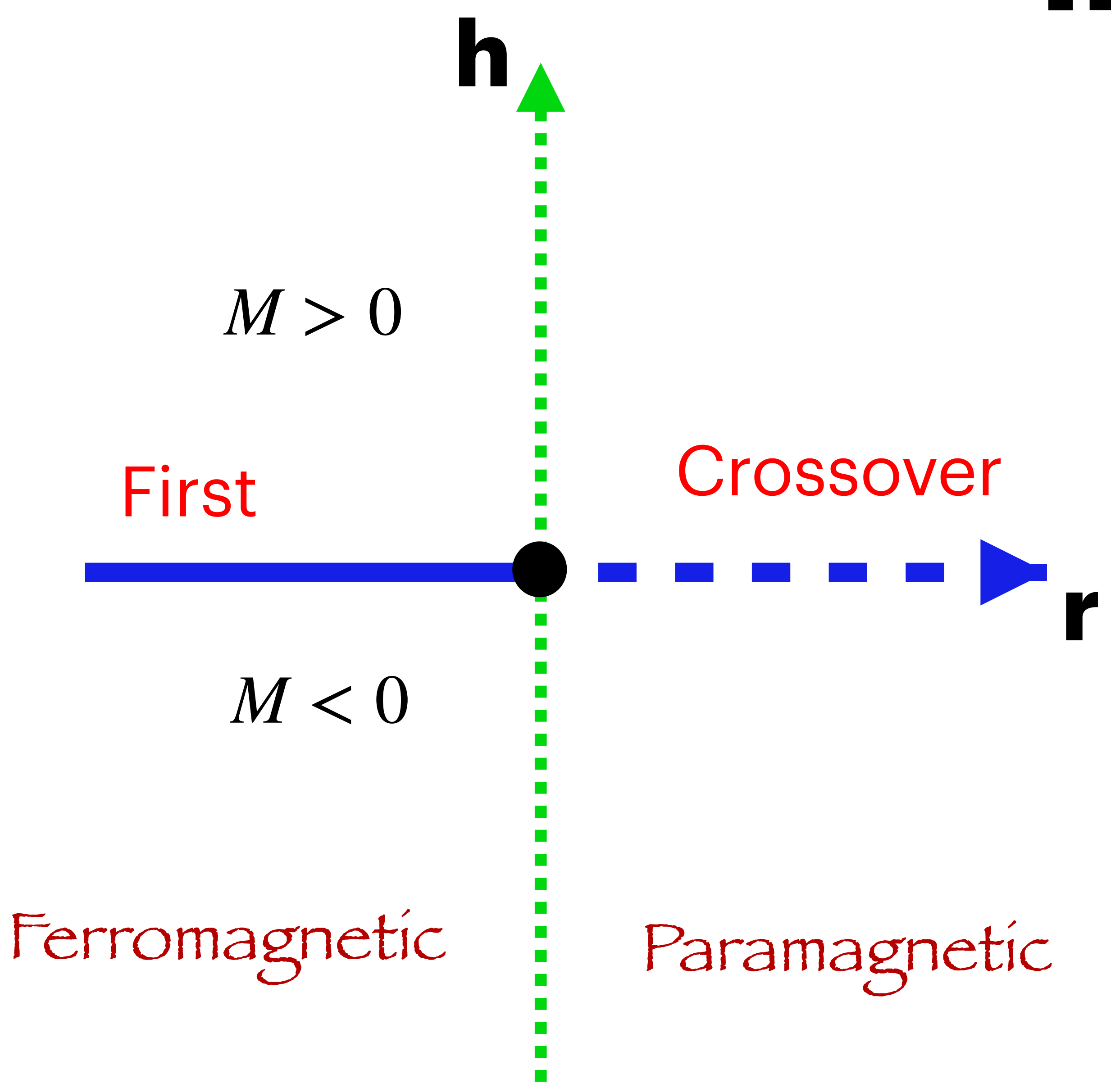
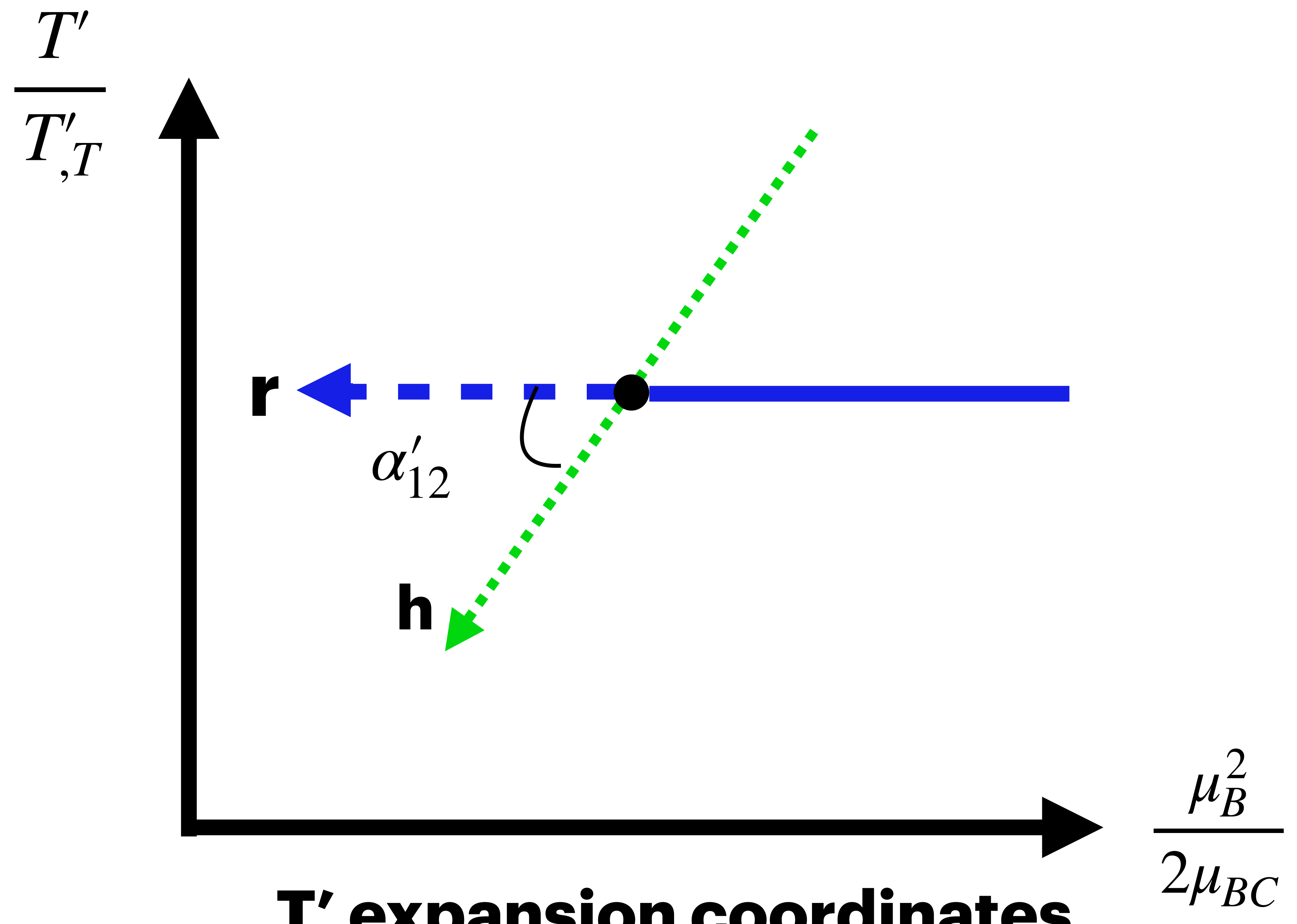


3D Ising

$$\frac{T' - T_0}{T_C T'_{,T}} = - w' \, h \sin \alpha'_{12}$$

$$\frac{\mu_B^2 - \mu_{BC}^2}{2\mu_{BC} T_C} = w' (-r \, \rho' - h \cos \alpha'_{12})$$



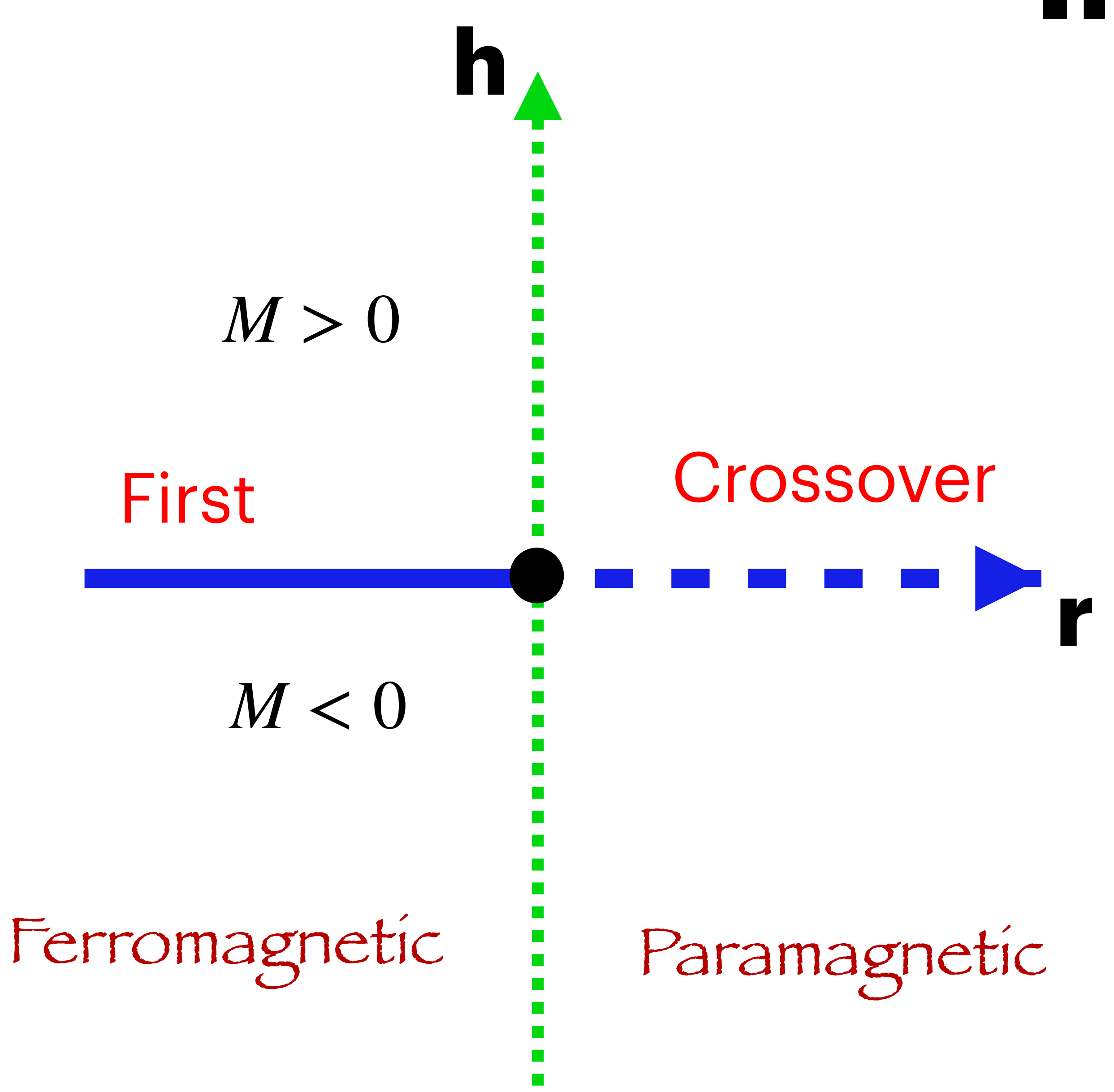
T′ expansion coordinates

$T'_{,T} = (\partial T' / \partial T)_\mu$ at the critical

T_0 Transition temperature at $\mu_B = 0$

w', ρ', α'_{12} - Free parameters

Introducing Critical Point



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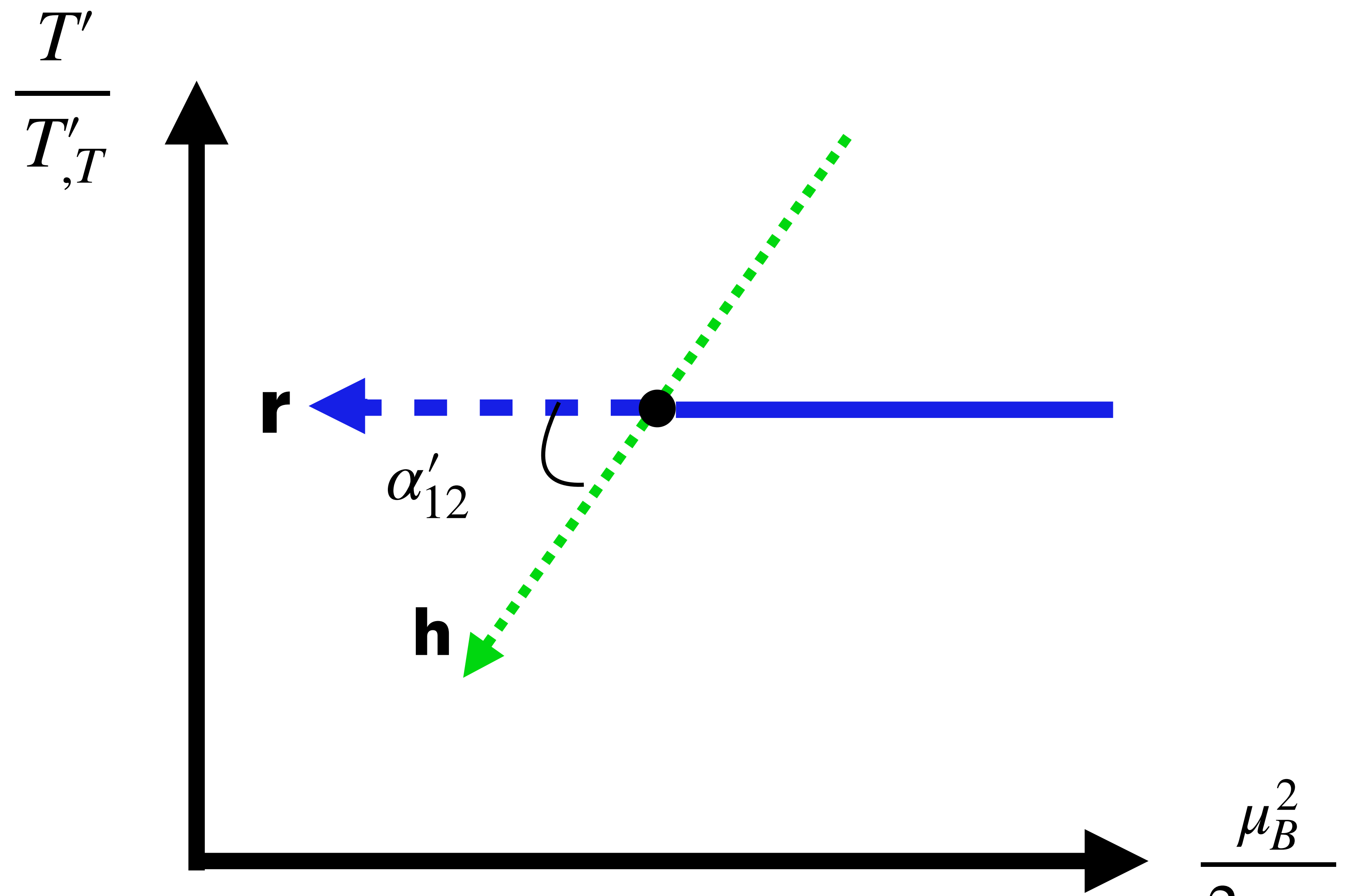

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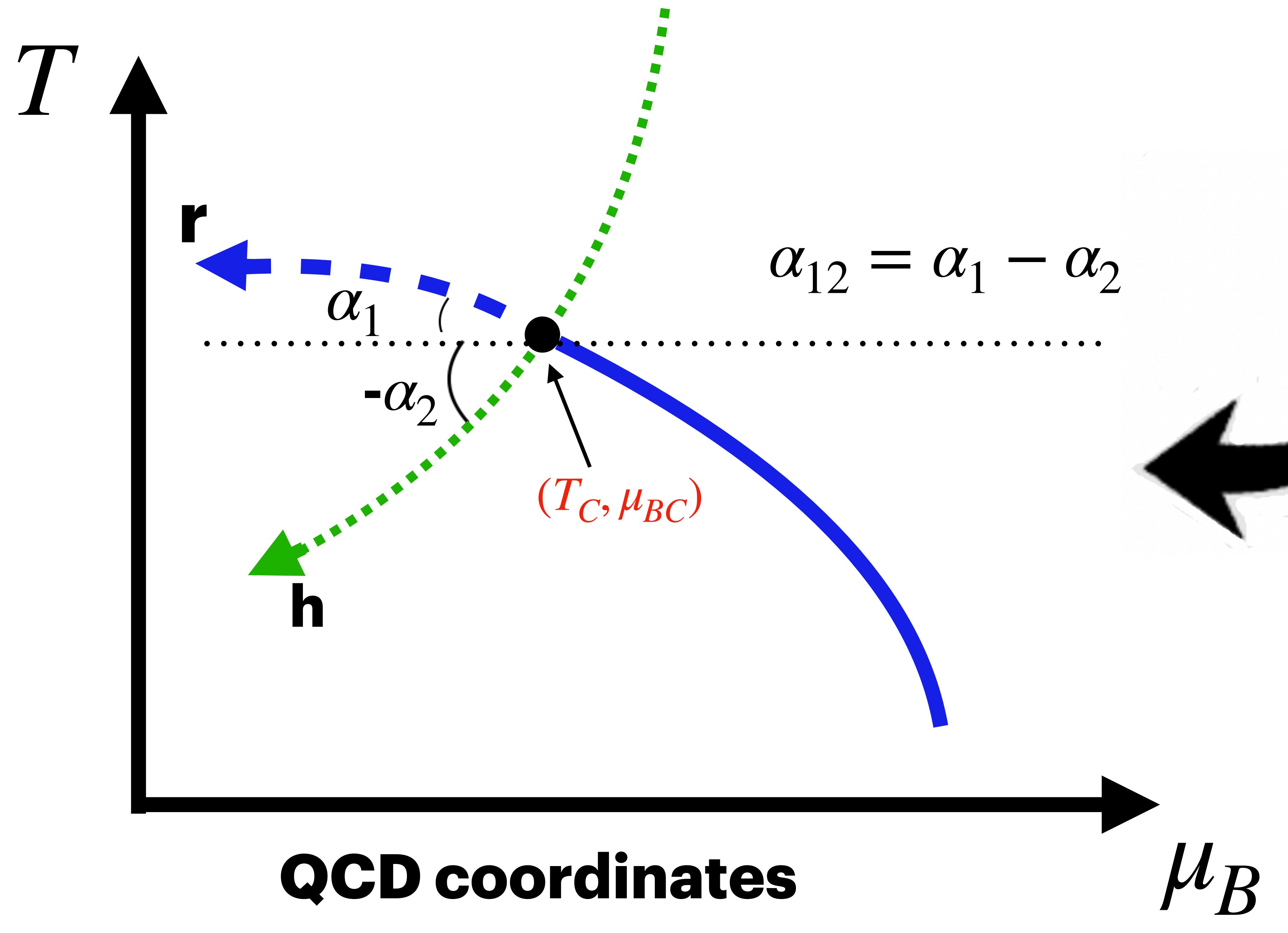

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[M. K et al arXiv:2402.08636v1]

Important relations

Relationship of TExS with BEST Mapping

$$\alpha'_{12}, w', \rho' \longrightarrow \alpha_1, \alpha_2, w, \rho$$

$$\tan \alpha'_{12} = \tan \alpha_1 - \tan \alpha_2, \qquad \rho' = \rho \frac{\cos^2 \alpha_1}{\sqrt{(\cos \alpha_1 \cos \alpha_2)^2 + (\sin \alpha_{12})^2}} \qquad w' = w \frac{1}{\cos \alpha_1} \sqrt{(\cos \alpha_1 \cos \alpha_2)^2 + (\sin \alpha_{12})^2}$$

[M. K et al arXiv:2402.08636v1]

[Parotto et al PhysRevC.101.034901(2020)]

Strength of the discontinuity

leading singular behavior of specific heat at constant pressure Cp

$$cp = T^3 \left(\frac{(s_c/n_c)\sin \alpha_1 - \cos \alpha_1}{w \sin \alpha_{12}} \right)^2 G_{hh} \left(1 + \mathcal{O}(r^{\beta\delta - 1})\right)$$

$w \sin \alpha_{12}$ -Controls the strength of the jump

G_{hh} – order parameter in Ising Model

Transition Line

$$T'[T_C, \mu_{BC}] = T_0$$

Choosing μ_{BC} fixes T_C and α_1

Slope

$$\alpha_1 = \tan^{-1}\left(\frac{2\kappa_2(T_C)\mu_{BC}}{T_C T'_{,T}}\right)$$

e.g

$$\mu_{BC} = 350 \text{ MeV}, \quad T_C = 140 \text{ MeV} \quad \text{and} \quad \alpha_1 = 6.6^0$$

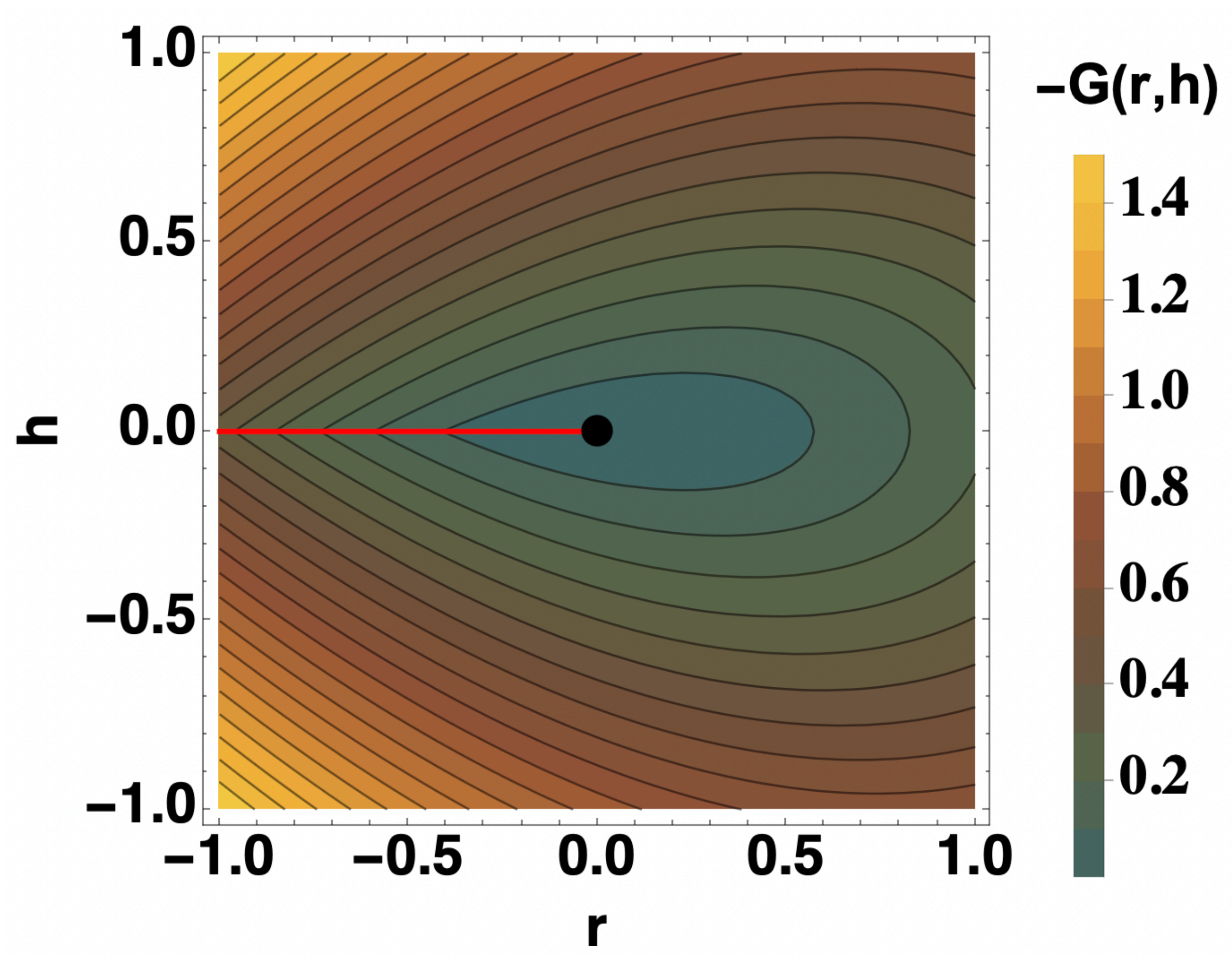
BEST COLLABORATION

$$\mu_{BC} = 500 \text{ MeV}, \quad T_C = 117 \text{ MeV} \quad \text{and} \quad \alpha_1 = 11^0$$

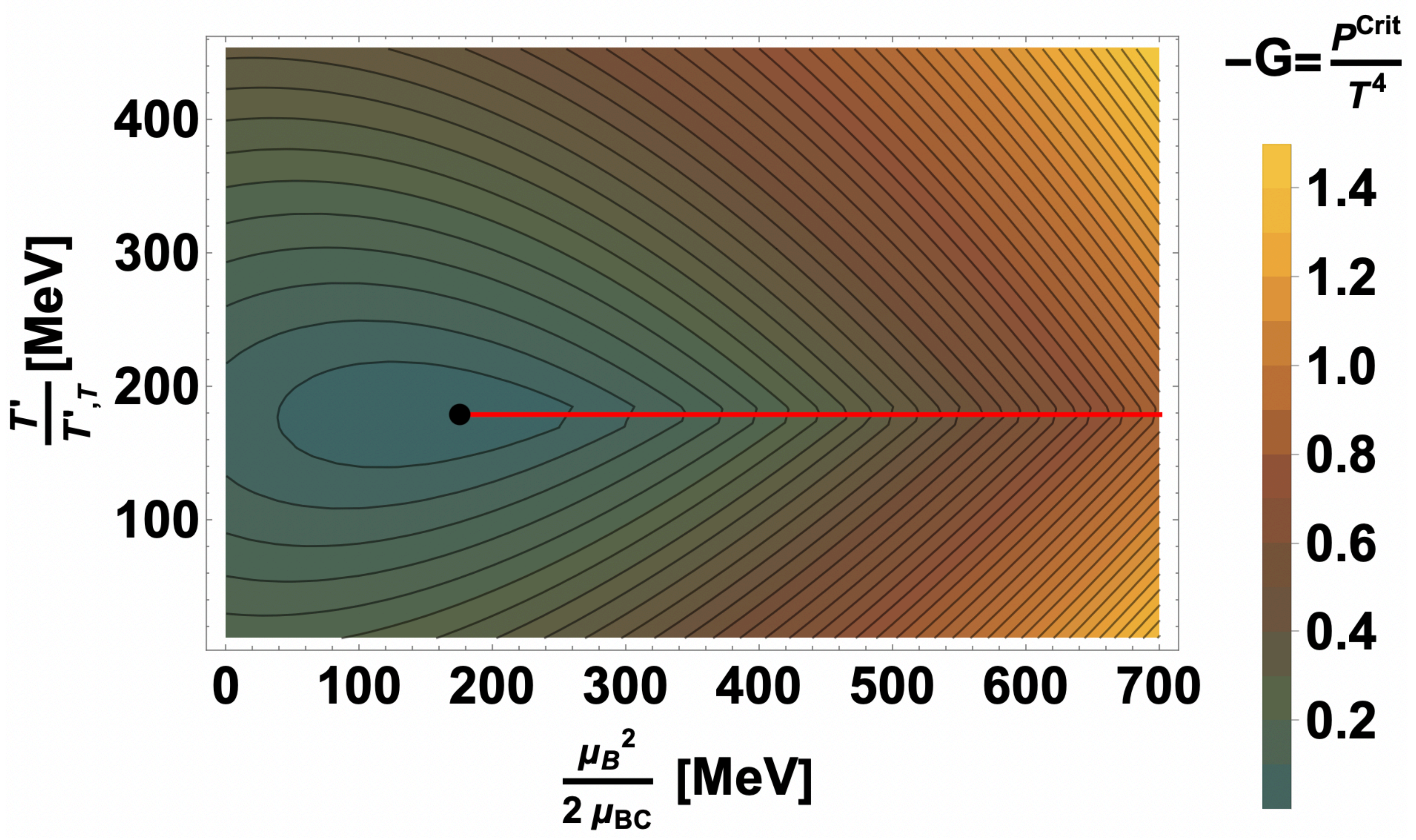
TExS

[M. K et al arXiv:2402.08636v1]

Introducing Critical Point



3D Ising



T′ Coordinates

Parameter

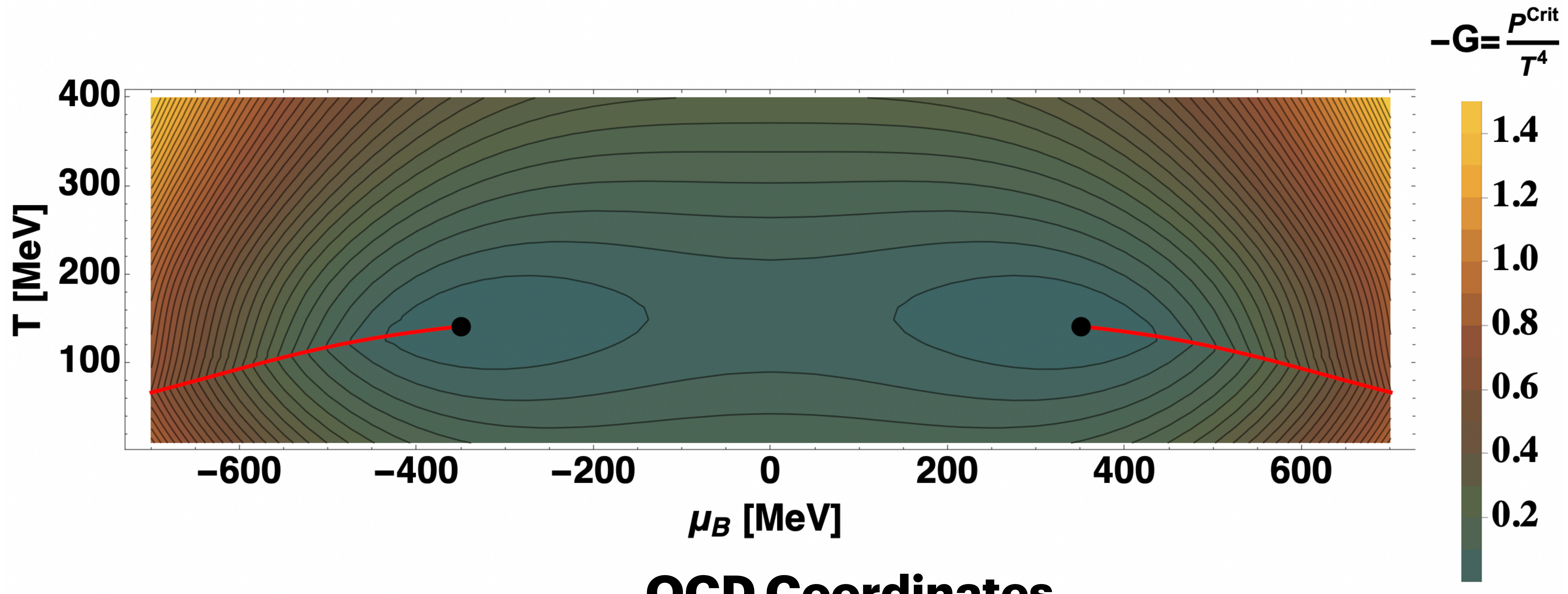
$\mu_{BC} = 350$ MeV, $T_C = 140$ MeV

$T_0 = 158$ MeV, $\alpha_1 = 6.65^0$

$\alpha_{12} = 90^0$, $\alpha_2 = \alpha_1 - \alpha_{12}$

$w = 10, \quad \rho = 0.5$

$$T_C \left[1 + \kappa(T_C) \left(\frac{\mu_{BC}}{T_C} \right)^2 \right] = T_0$$



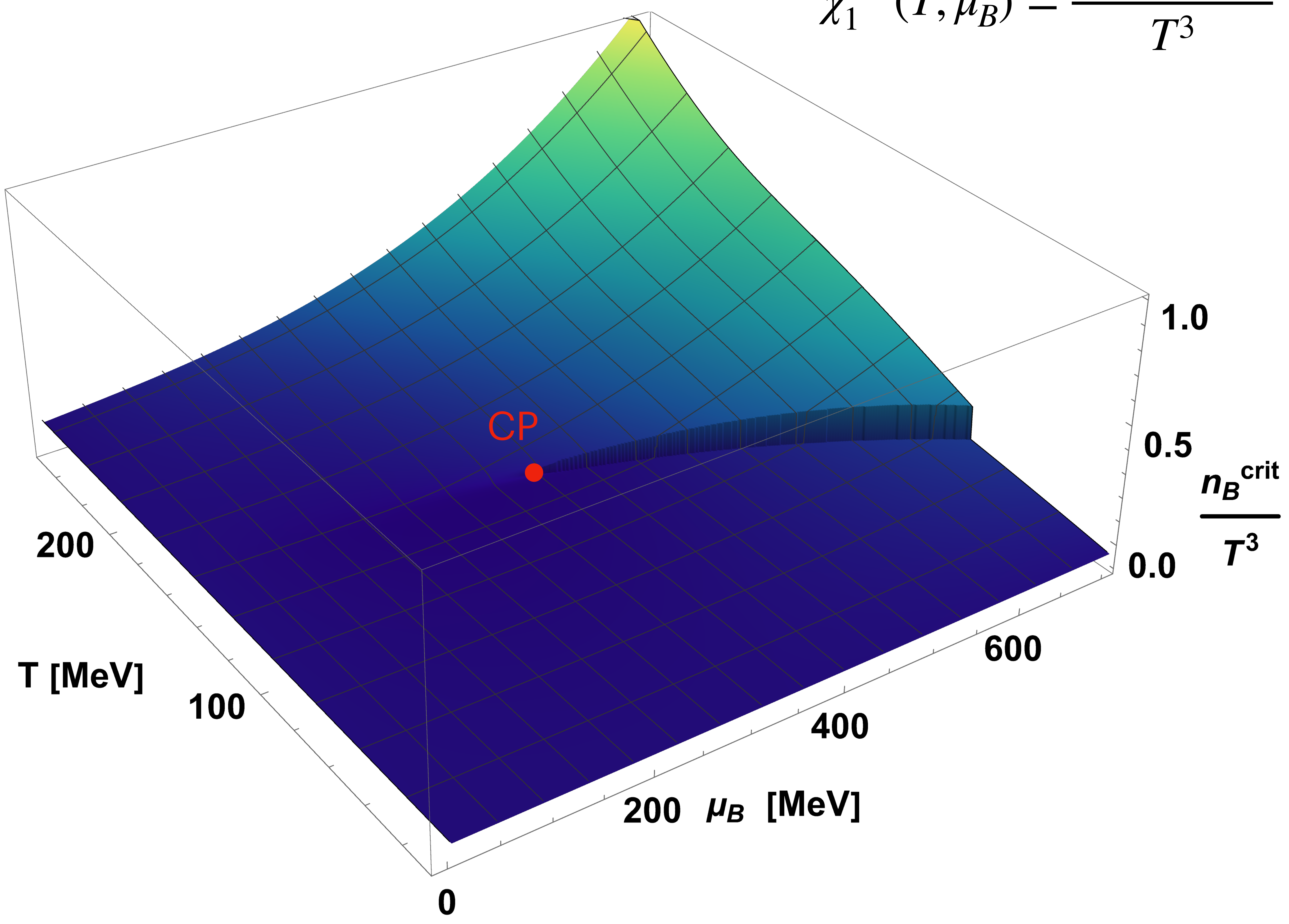
QCD Coordinates

[M. K et al arXiv:2402.08636v1]

Introducing Critical Point

Ising Baryon Density

$$\chi_1^{crit}(T,\mu_B) = \frac{n_B^{crit}(T,\mu_B)}{T^3} = \frac{\partial(P^{crit}(T,\mu_B)/T^4)}{\partial(\mu_B/T)}$$



- **Critical Point**

$w = 2$, $\rho = 2$ & $\alpha_{12} = 90$

$\mu_{BC} = 350$ MeV

$T_C = 140$ MeV

[M. K et al arXiv:2402.08636v1]

Part 4: Merging 3D Ising with T′ Expansion (Ising-TExS)

Merging Ising with Lattice (Ising-T ExS)

Full Baryon Density

$$\chi_1^B(T,\mu_B) = \frac{n_B(T,\mu_B)}{T^3} = \left(\frac{\mu_B}{T}\right)\chi_{2,lat}^B(T',0)$$

$$T': \quad T' = \underbrace{T'_{lat}(T,\mu_B)}_{\text{lower order in } (\frac{\mu_B}{T})} + \underbrace{T'_{crit}(T,\mu_B) - Taylor[T'_{crit}(T,\mu_B)]}_{\text{higher orders in } (\frac{\mu_B}{T})}$$

Lattice Term **Ising Term**

[M. K et al arXiv:2402.08636v1]

Merging Ising with Lattice (Ising-T ExS)

Full Baryon Density

$$\chi_1^B(T,\mu_B) = \frac{n_B(T,\mu_B)}{T^3} = \left(\frac{\mu_B}{T}\right)\chi^B_{2,lat}(T',0)$$

$$T': \quad T' = \underbrace{T'_{lat}(T,\mu_B)}_{\text{lower order in } (\frac{\mu_B}{T})} + \underbrace{T'_{crit}(T,\mu_B) - Taylor[T'_{crit}(T,\mu_B)]}_{\text{higher orders in } (\frac{\mu_B}{T})}$$

Lattice Term **Ising Term**

Introducing a Critical Point

$$T'_{crit}(T,\mu_B) \approx \left(\frac{\partial\chi^B_{2,lat}(T,0)}{\partial T}\bigg|_{T=T_0}\right)^{-1} \frac{n_B^{crit}(T,\mu_B)/T^3}{(\mu_B/T)} + \ldots$$

$$Taylor[T'_{crit}, n=2] \approx \left(\frac{\partial\chi^B_{2,lat}(T,0)}{\partial T}\bigg|_{T=T_0}\right)^{-1} \left[\frac{\partial(n_B^{crit}/T^3)}{\partial(\mu_B/T)}\bigg|_{\mu_B/T=0} + \frac{1}{3!}\frac{\partial^3(n_B^{crit}/T^3)}{\partial(\mu_B/T)^3}\bigg|_{\mu_B/T=0}\left(\frac{\mu_B}{T}\right)^2 + \ldots\right]$$

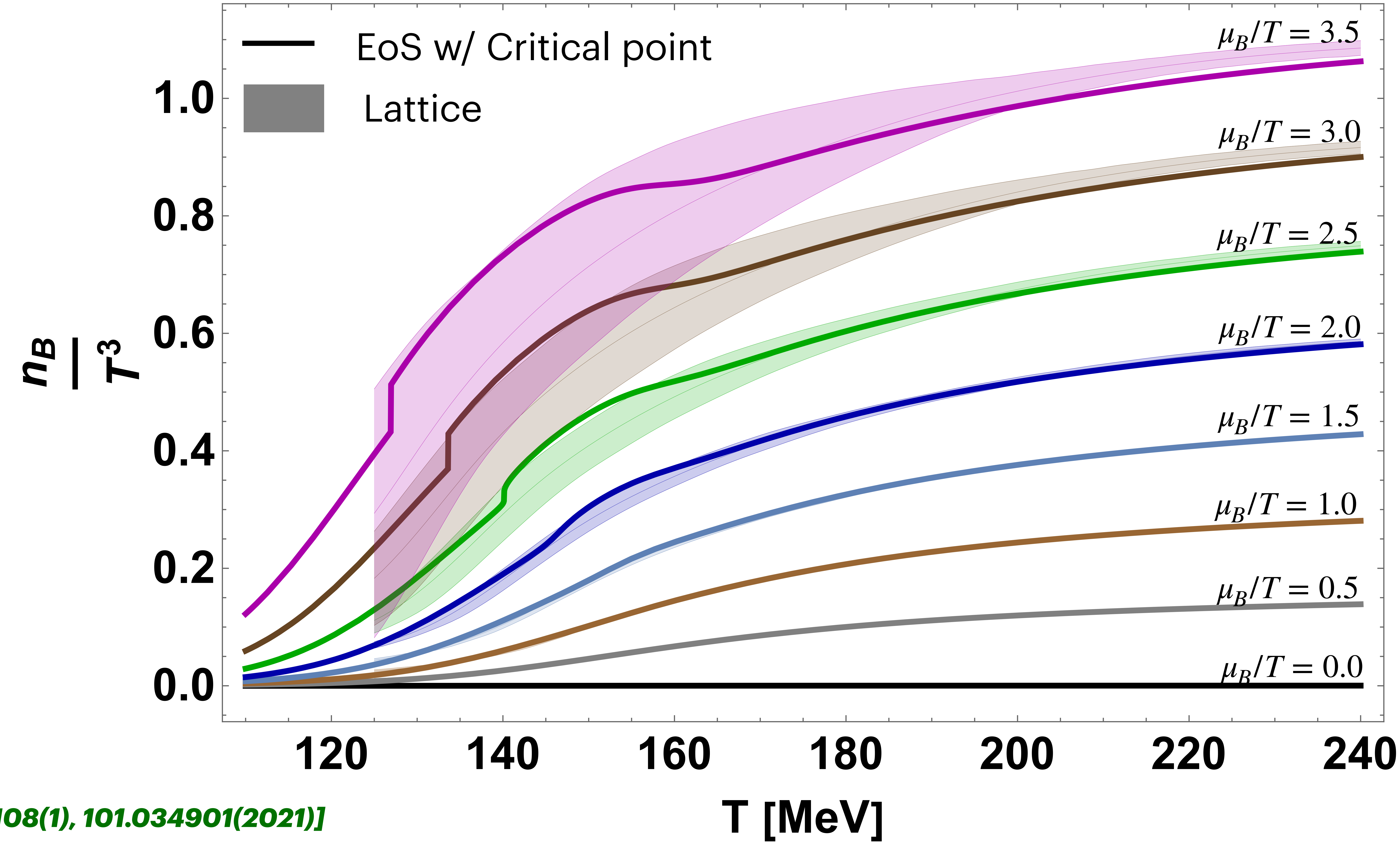
[M. K et al arXiv:2402.08636v1]

Baryon density results

Full Baryon Density at a constant $\frac{\mu_B}{T}$ compared with Lattice

Parameter choice

$\mu_B = 350$ [MeV]
$\alpha_{12} = 90^0$
$w = 2$
$\rho = 2$



[Borsányi, S et al PRL. 108(1), 101.034901(2021)]

[M. K et al arXiv:2402.08636v1]

Physical Quark masses

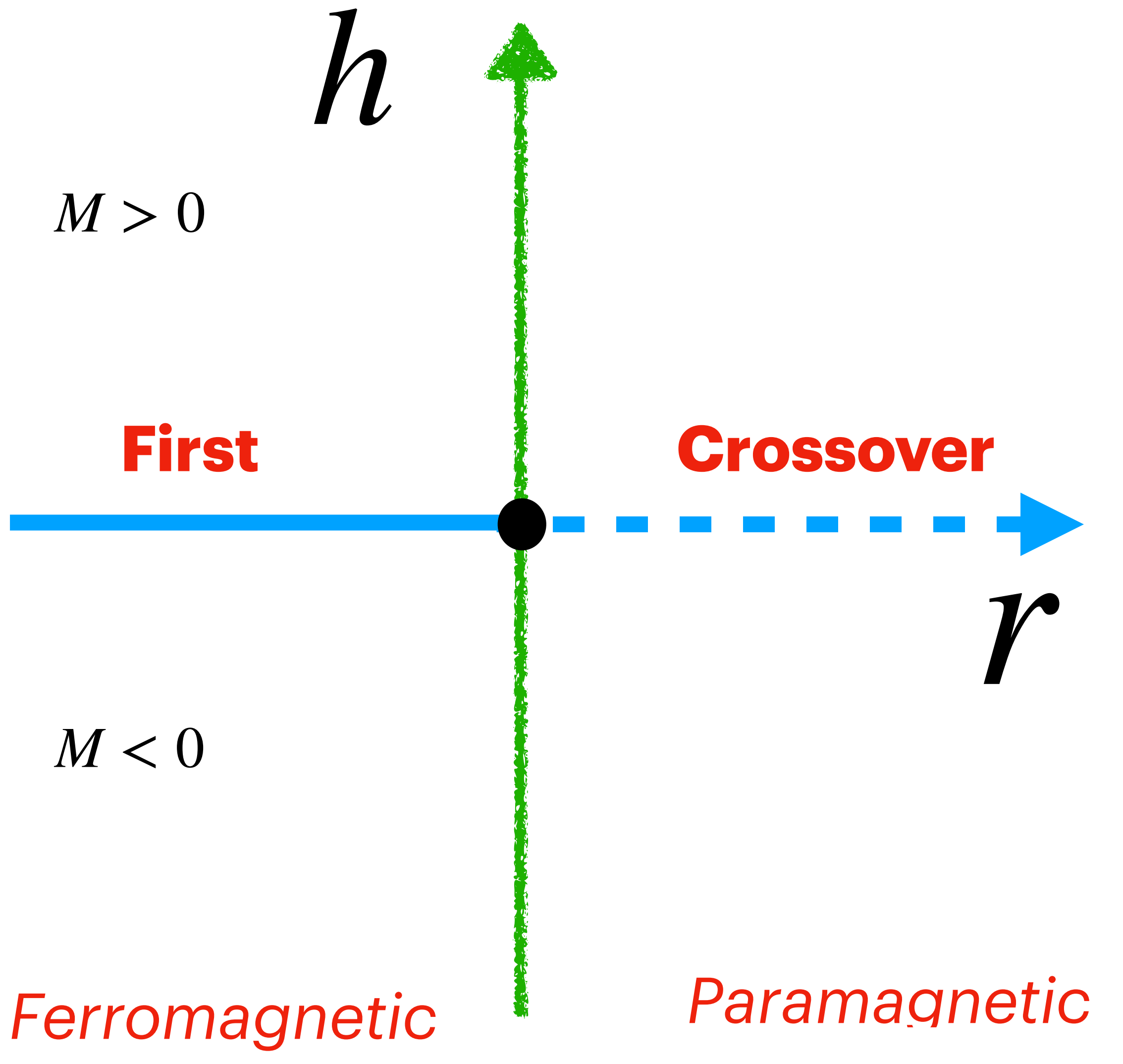
- Close to the critical point, there is a universal dependency of the mapping parameters to quark masses m_q
- Suggests the angle α_{12} between $r = 0$ and $h = 0$ lines in (T, μ_B) vanishes as $m_q^{2/5}$ $(\alpha_{12} = \alpha_1)$

Universality of the critical point mapping between Ising model and QCD at small quark mass

Maneesha Sushama Pradeep and Mikhail Stephanov
Department of Physics, University of Illinois, Chicago, IL 60607, USA
(Dated: September 21, 2019)

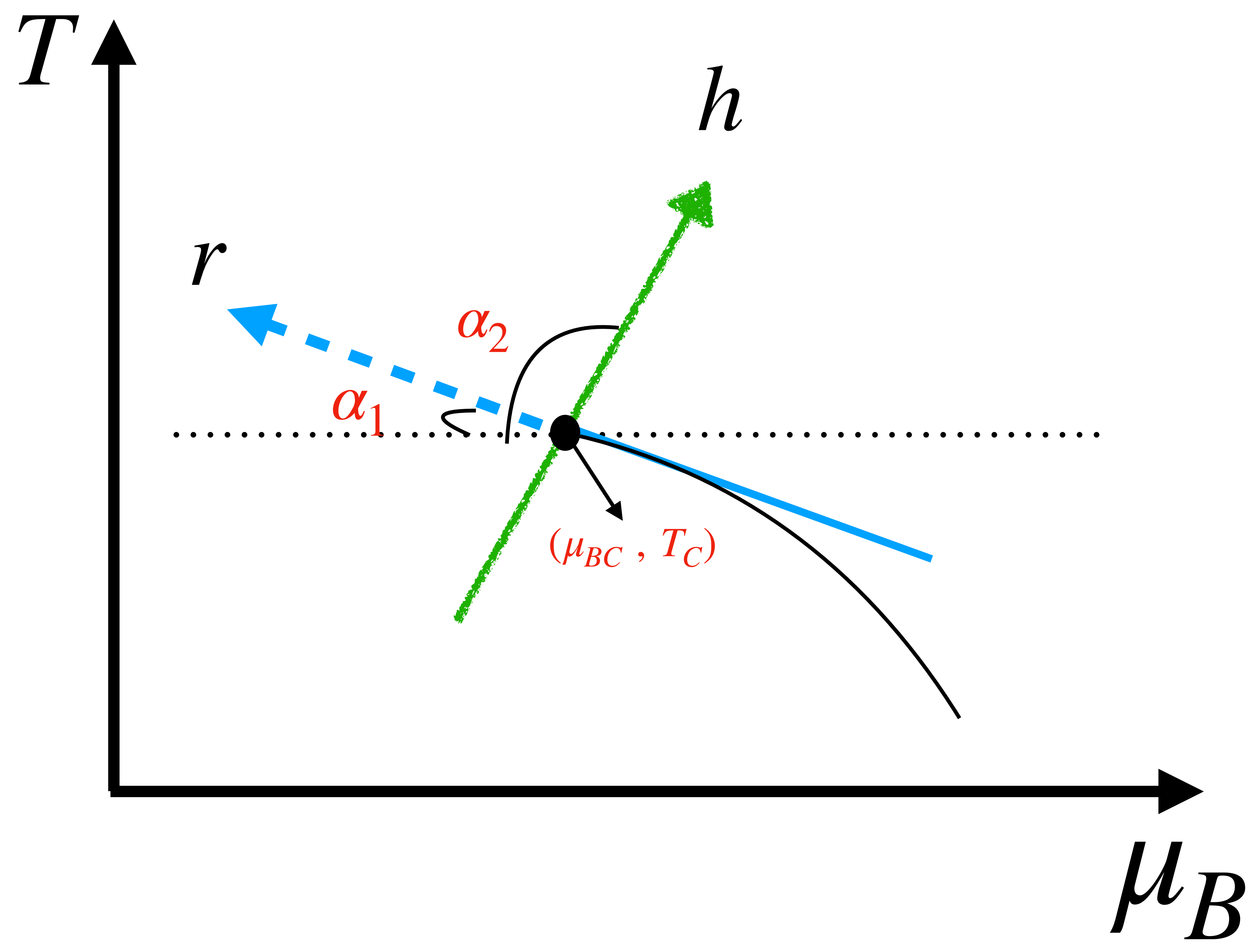
The universality of the QCD equation of state near the critical point is expressed by mapping pressure as a function of temperature T and baryon chemical potential μ in QCD to Gibbs free energy as a function of reduced temperature r and magnetic field h in the Ising model. The mapping parameters are, in general, not universal, i.e., determined by details of the microscopic dynamics, rather than by symmetries and long-distance dynamics. In this paper we point out that in the limit of small quark masses, when the critical point is close to the tricritical point, the mapping parameters show *universal* dependence on the quark mass m_q. In particular, the angle between the $r = 0$ and $h = 0$ lines in the (μ, T) plane vanishes as $m_q^{2/5}$. We discuss possible phenomenological consequences of these findings.

[Pradeep, M. S., & Stephanov, M PhysRevD . 100(5), 056003.(2019)]



$$\frac{T - T_C}{T_C} = w(r\rho \sin \alpha_1 + h \sin \alpha_2)$$

$$\frac{\mu_B - \mu_{BC}}{T_C} = w(-r\rho \cos \alpha_1 - h \cos \alpha_2)$$



BEST COLLABORATION

Thermodynamic Observables

Parameter

$\mu_{BC} = 500$ MeV

$T_C = 117$ MeV

$\alpha_{12} = \alpha_1 = 11^0$

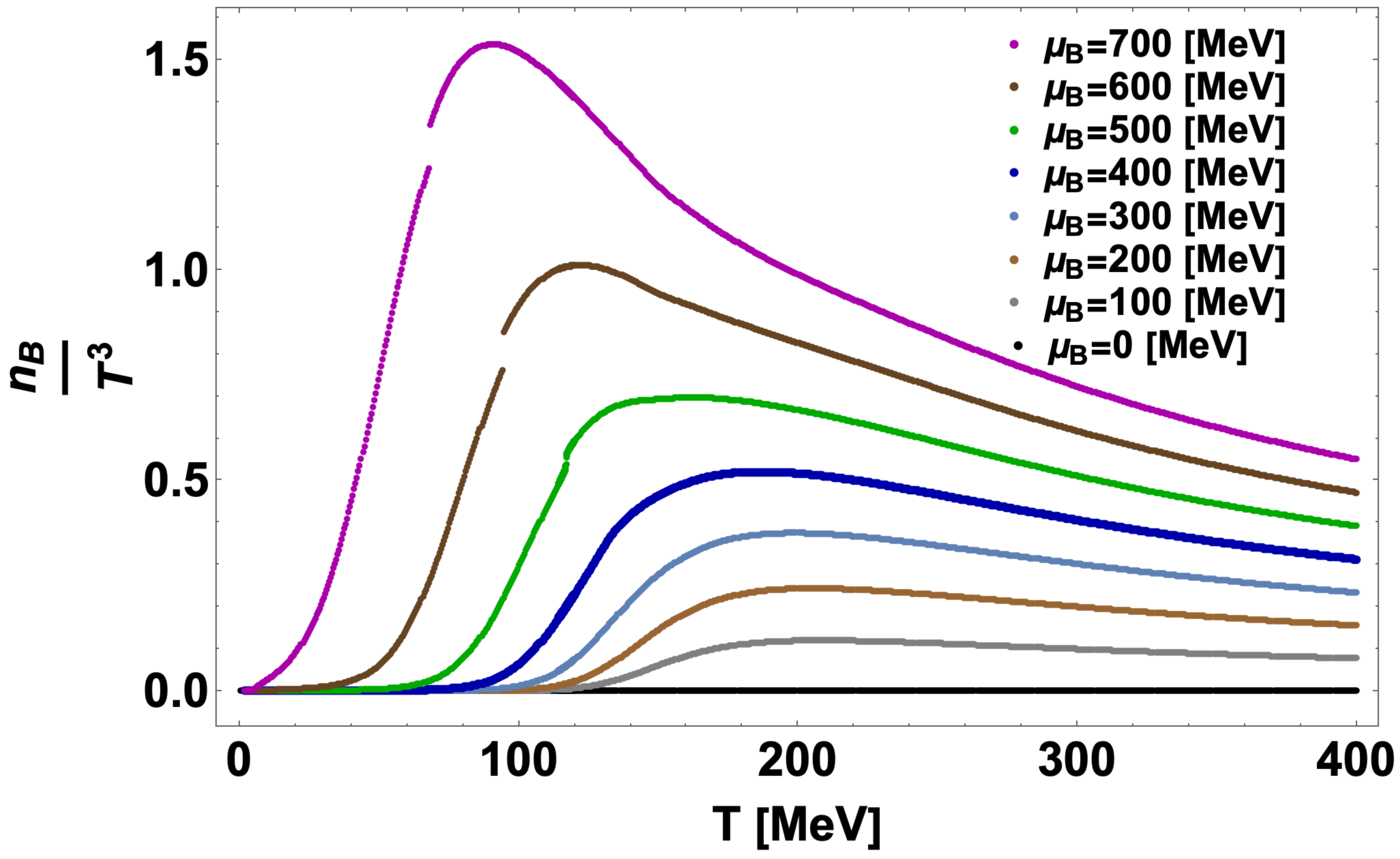
$\alpha_2 = 0^0$

$w = 15$

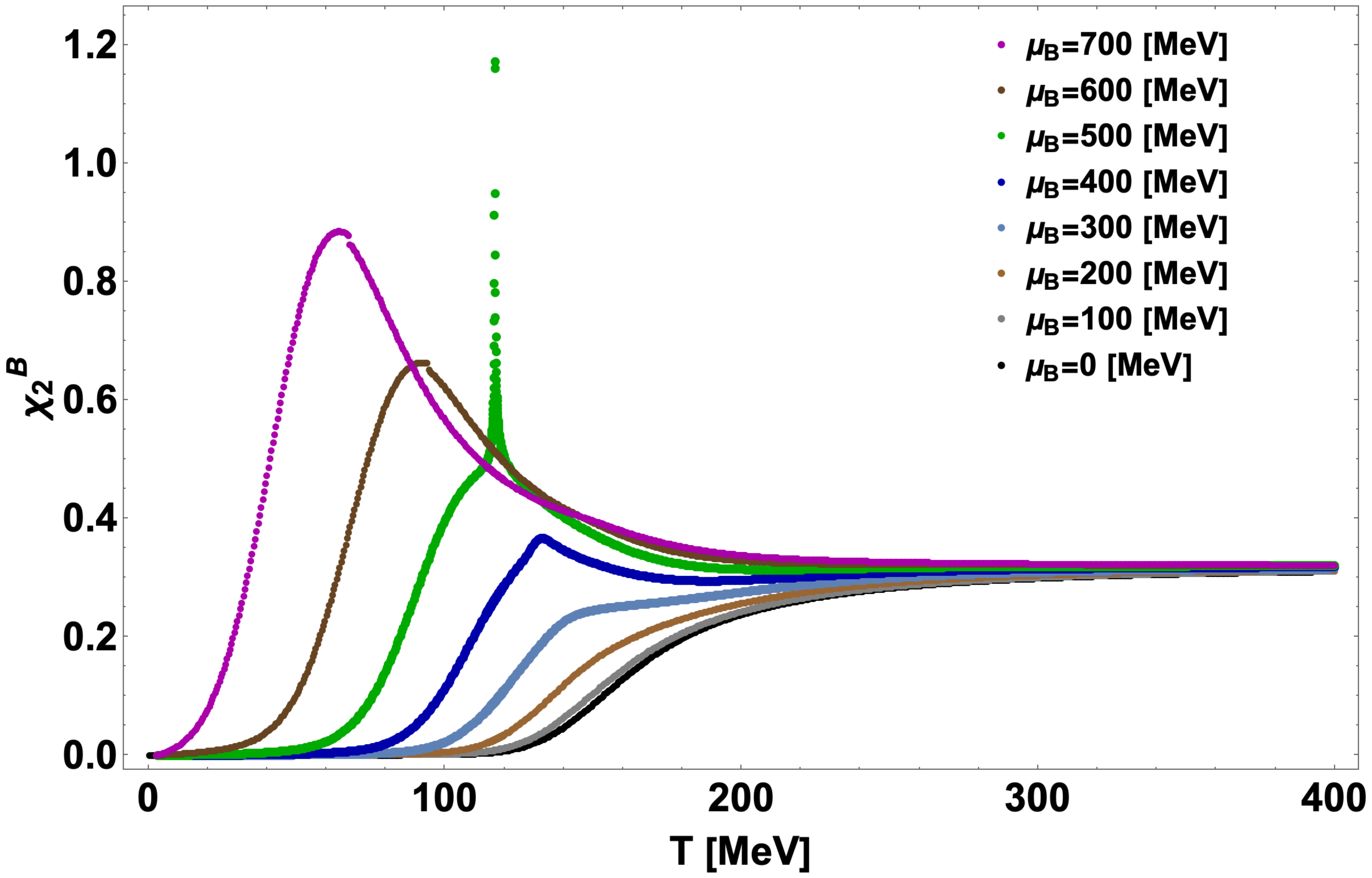
$\rho = 0.3$

$$\chi_2(T, \mu_B) = \left. \frac{\partial(n_B/T^3)}{\partial(\mu_B/T)} \right|_T$$

Baryon Density



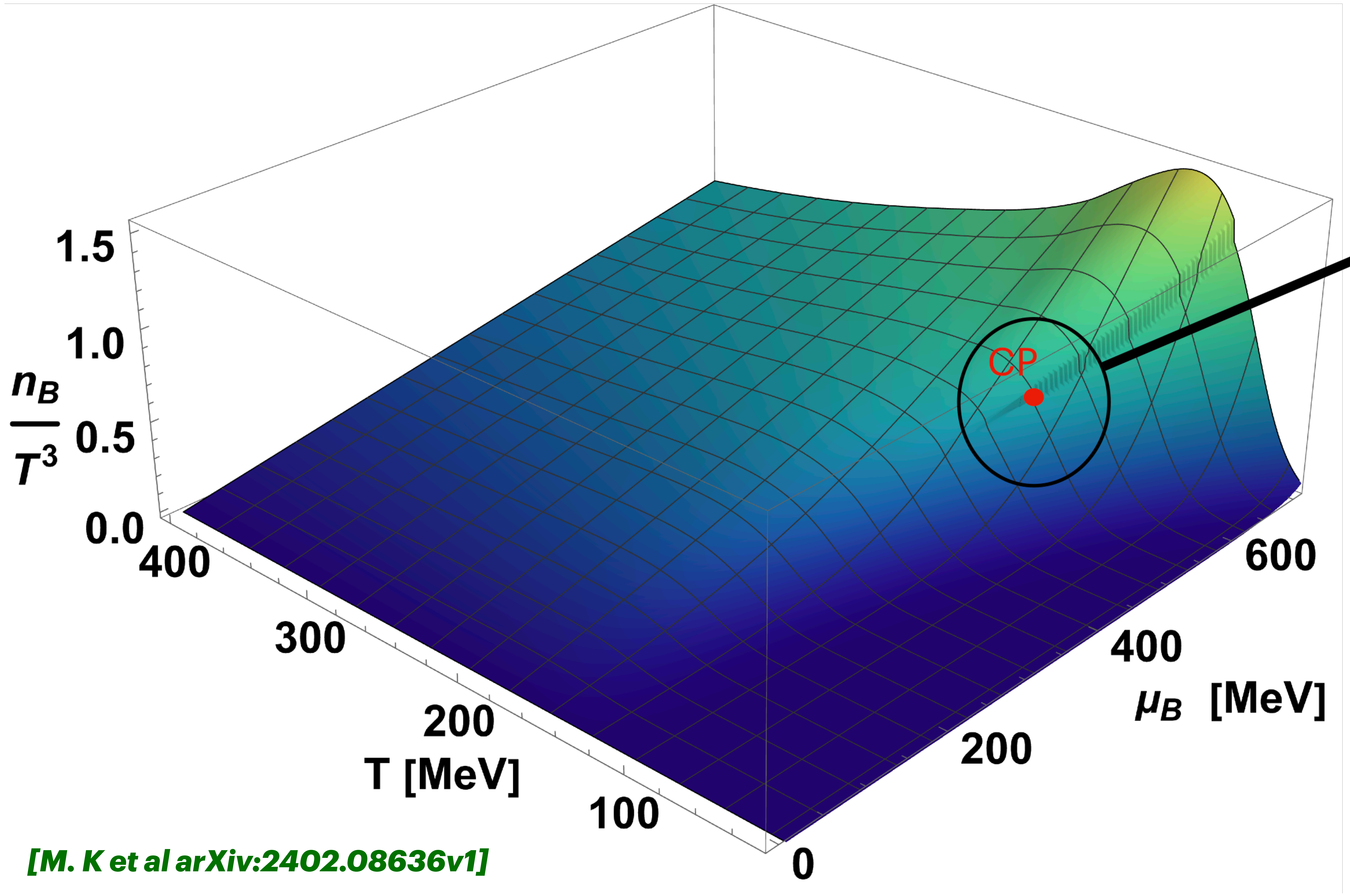
Baryon number susceptibility

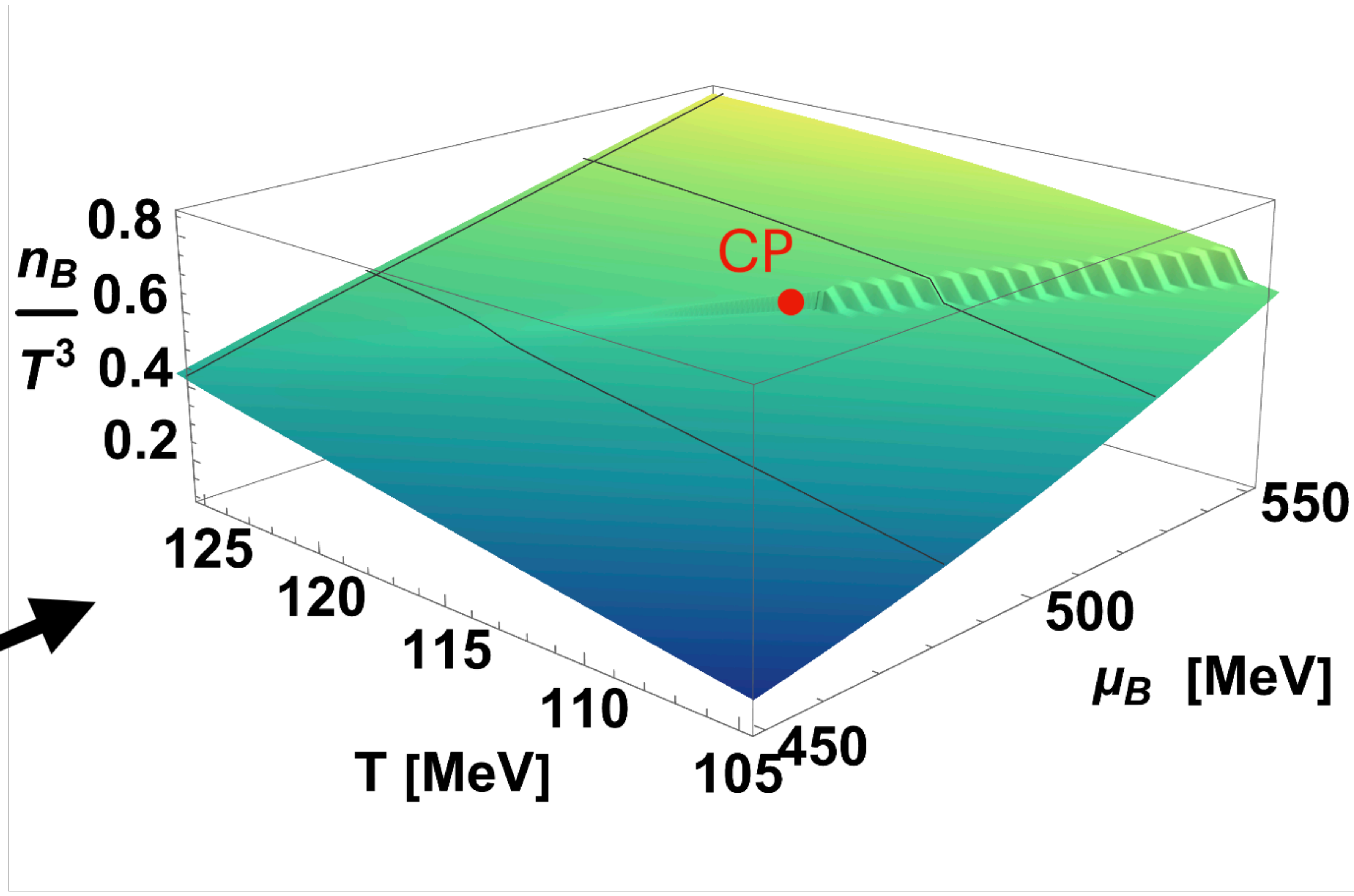


[M. K et al arXiv:2402.08636v1]

Thermodynamic Observables

Baryon Density n_B/T^3





Parameter choice

$\mu_{BC} = 500$ MeV

$T_C = 117$ MeV

$\alpha_{12} = \alpha_1 = 11^0$

$\alpha_2 = 0^0$

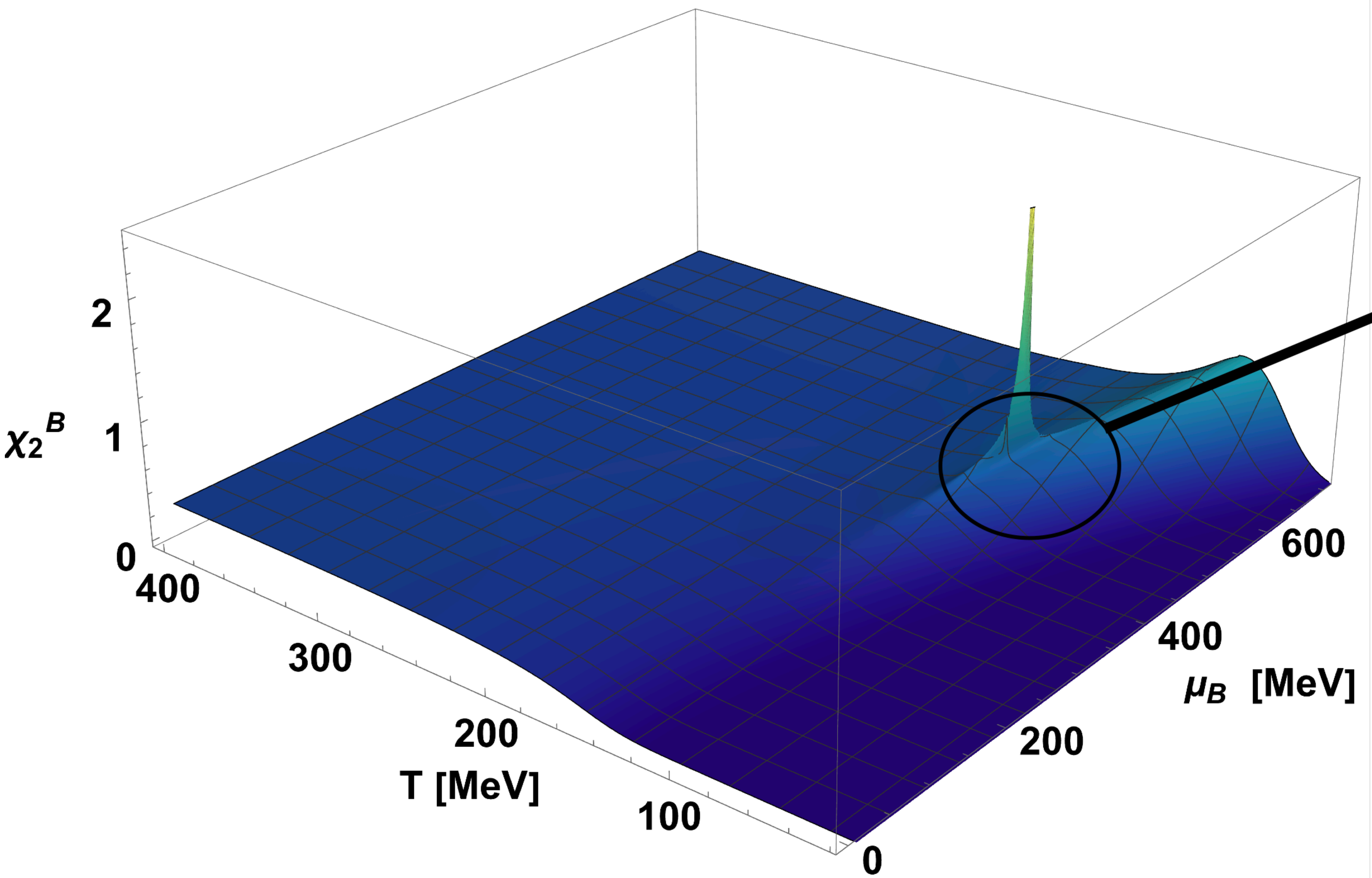
$w = 15$

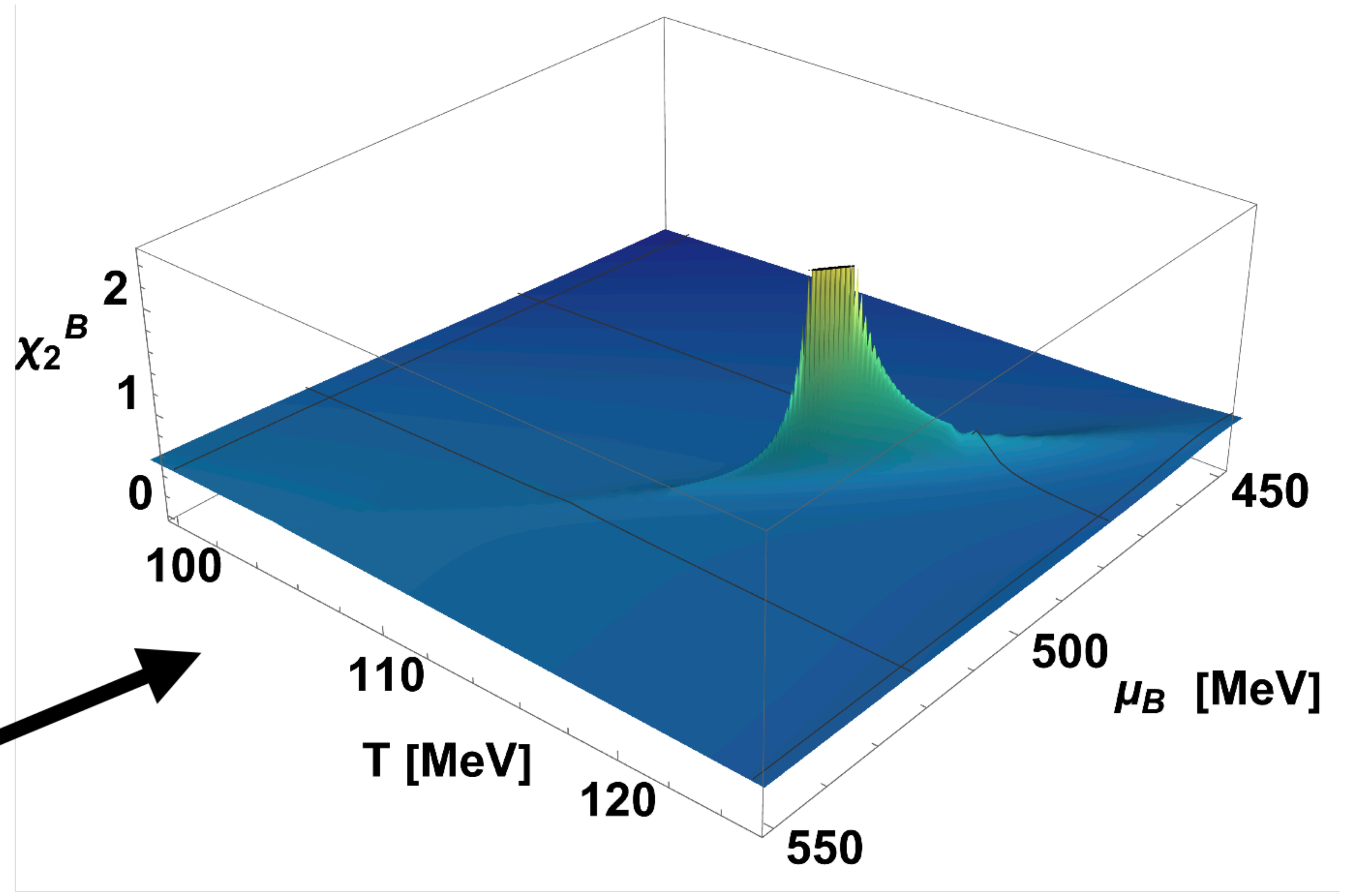
$\rho = 0.3$

[M. K et al arXiv:2402.08636v1]

Thermodynamic Observables

Baryon number susceptibility χ_2^B





Parameter choice

$\mu_{BC} = 500$ MeV

$T_C = 117$ MeV

$\alpha_{12} = \alpha_1 = 11^0$

$\alpha_2 = 0^0$

$w = 15$

$\rho = 0.3$

[M. K et al arXiv:2402.08636v1]

Thermodynamic Relations

$$\frac{P(T,\mu_B)}{T^4} = \chi^B_{0,lat}(T,0) + \frac{1}{T}\int_0^{\mu_B} d\hat{\mu}'_B \frac{n_B(T,\hat{\mu}'_B)}{T^3}$$

$$\frac{s(T,\mu_B)}{T^3} = \frac{1}{T^3}\left(\frac{\partial P}{\partial T}\right)\Bigg|_{\mu_B}$$

$$\frac{\epsilon(T,\mu_B)}{T^3} = \frac{s}{T^3} - \frac{P}{T^4} + \frac{\mu_B}{T}\frac{n_B}{T^3}$$

$$c_s^2(T,\mu_B) = \left(\frac{\partial P}{\partial \epsilon}\right)\Bigg|_{s/n}$$

Other Observables

Parameter

$\mu_{BC} = 500$ MeV

$T_C = 117$ MeV

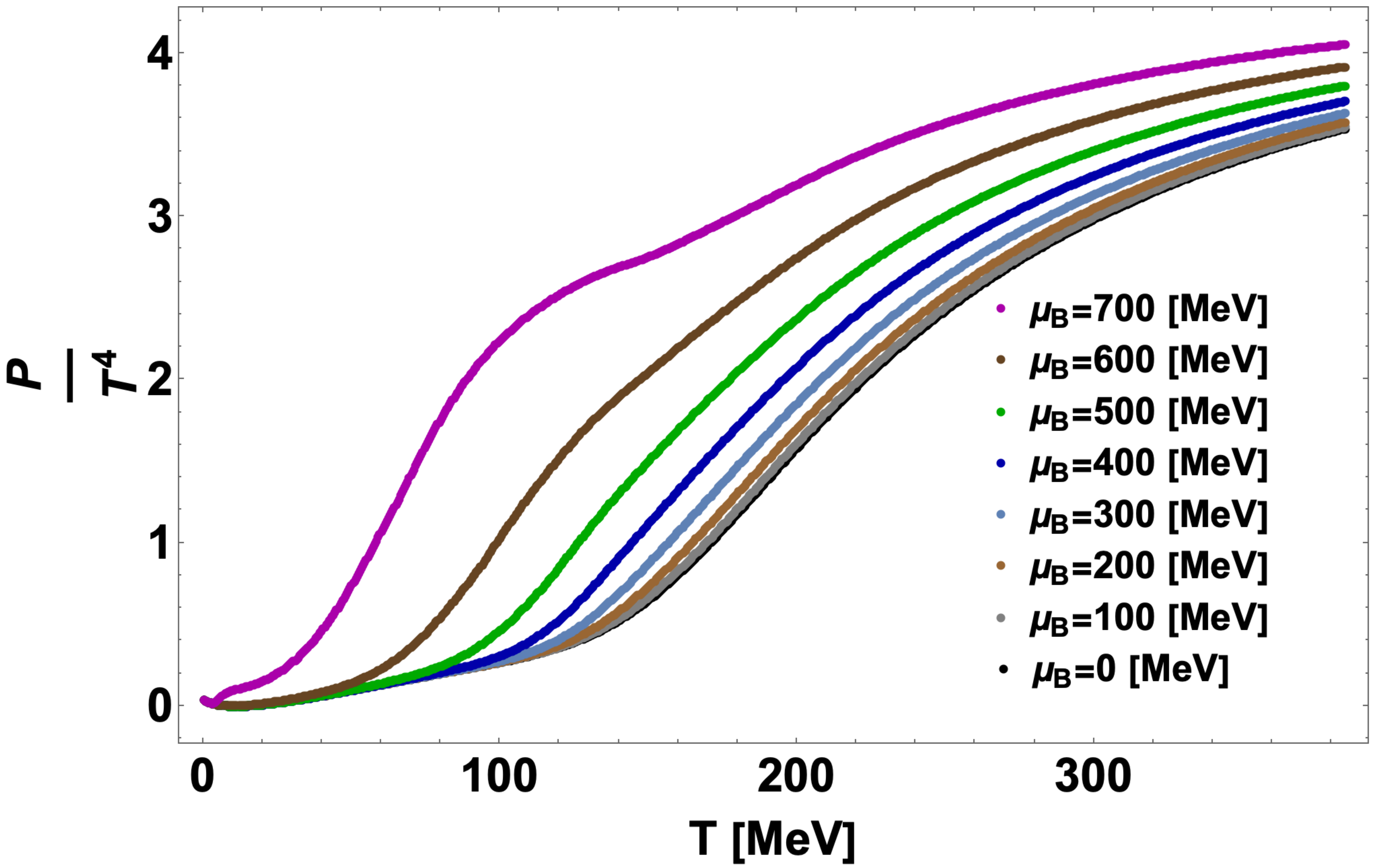
$\alpha_{12} = \alpha_1 = 11^0$

$\alpha_2 = 0^0$

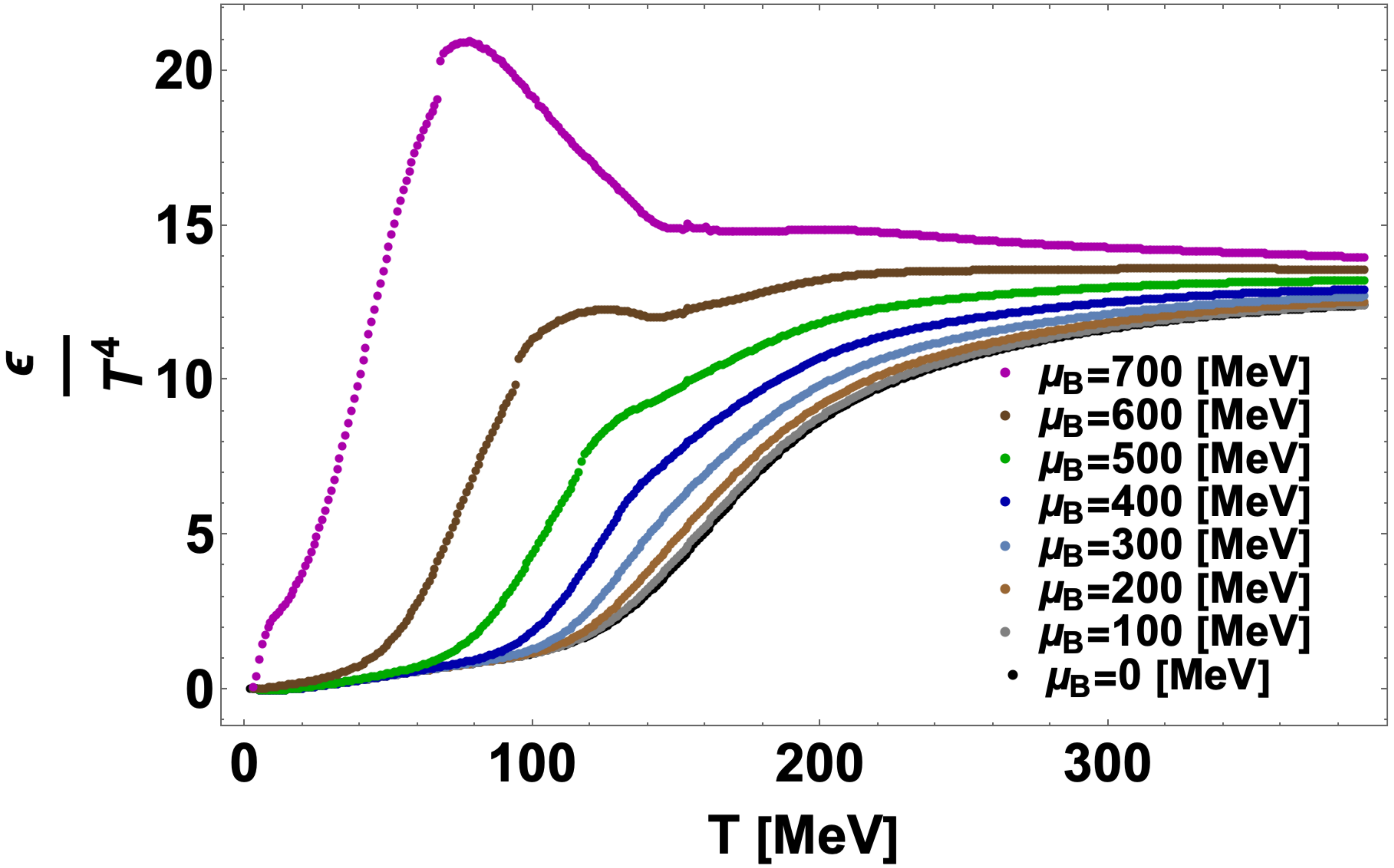
$w = 15$

$\rho = 0.3$

Pressure



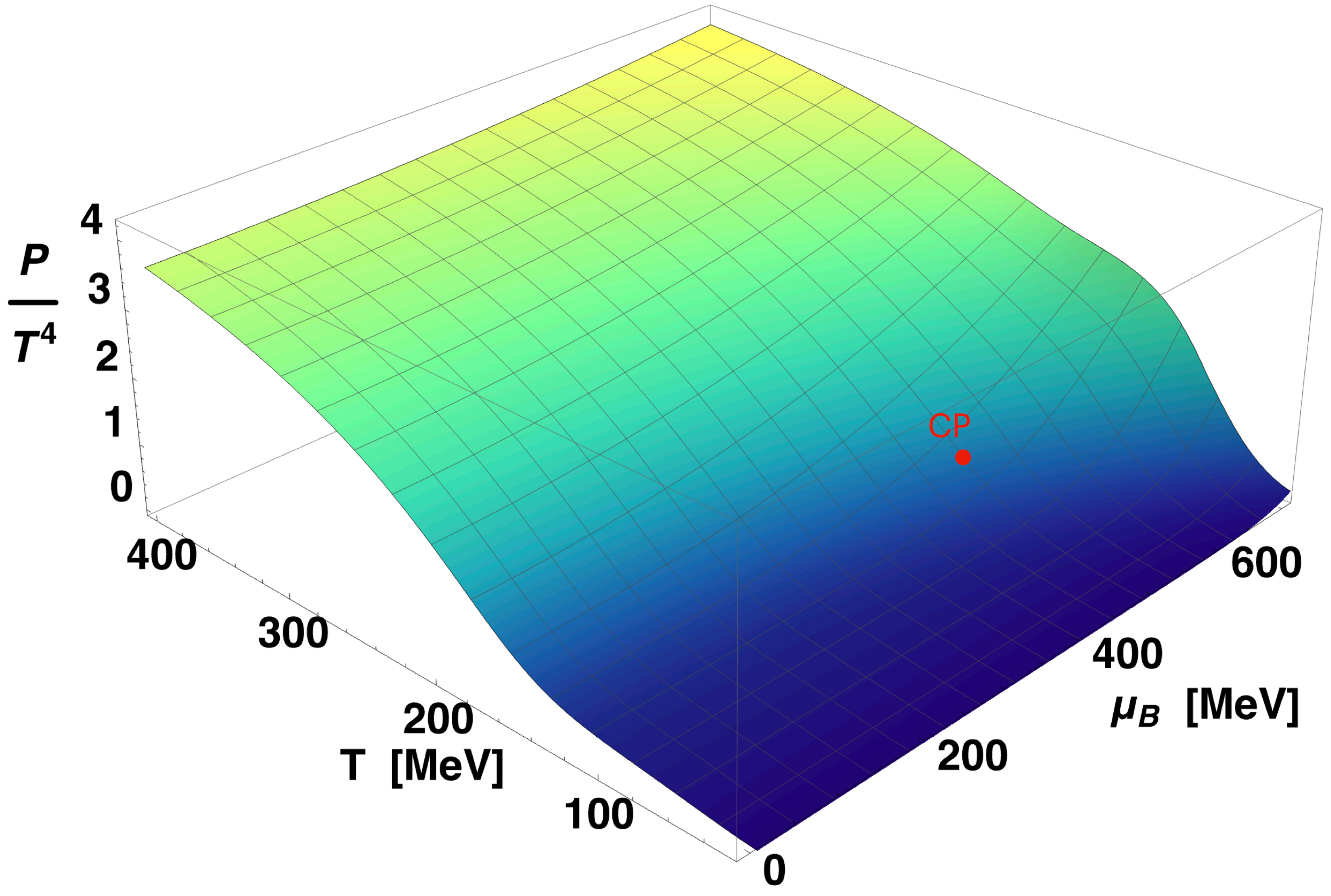
Energy density



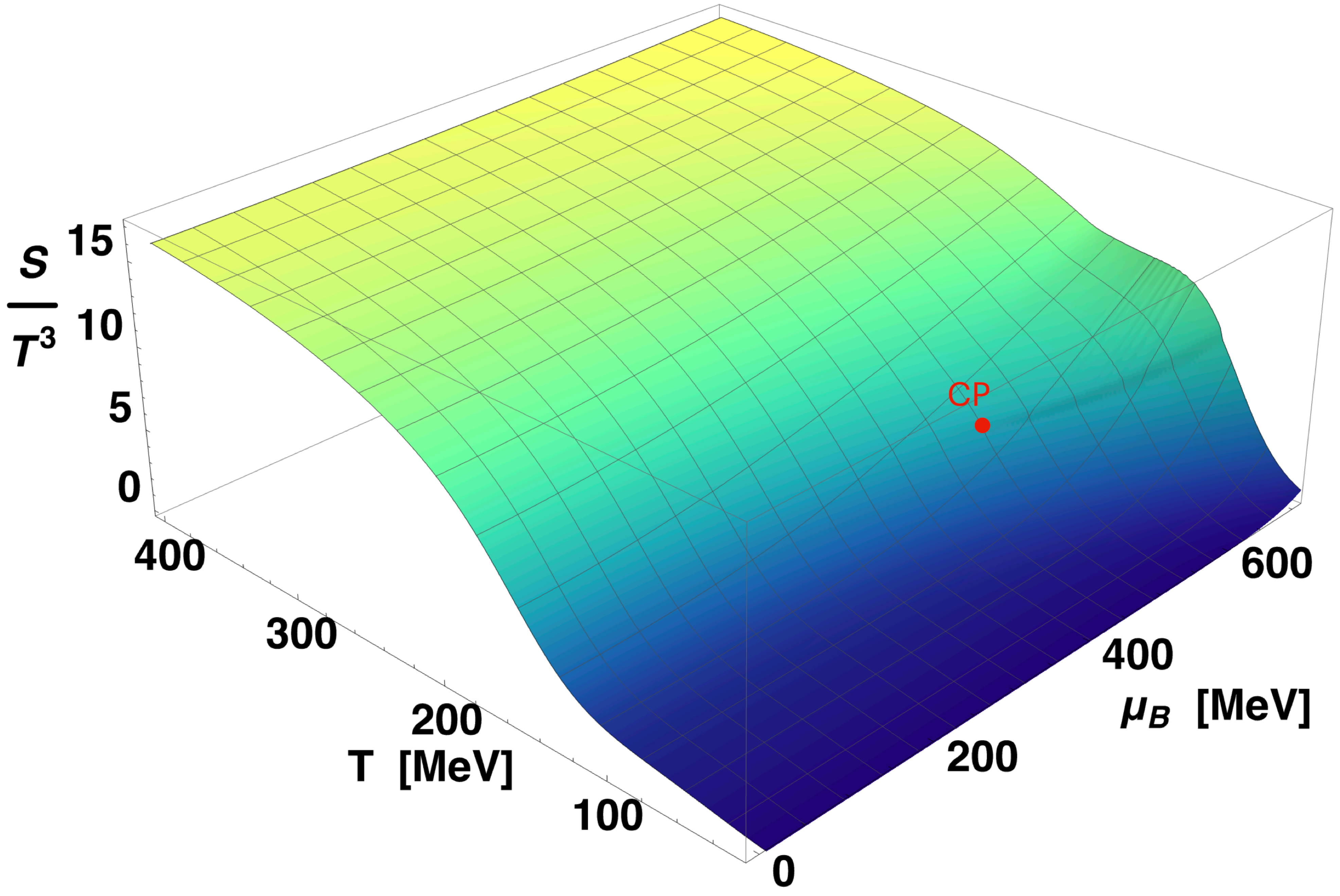
[M. K et al arXiv:2402.08636v1]

Thermodynamic Observables

Pressure



Entropy density



Parameter

$\mu_{BC} = 500$ MeV

$T_C = 117$ MeV

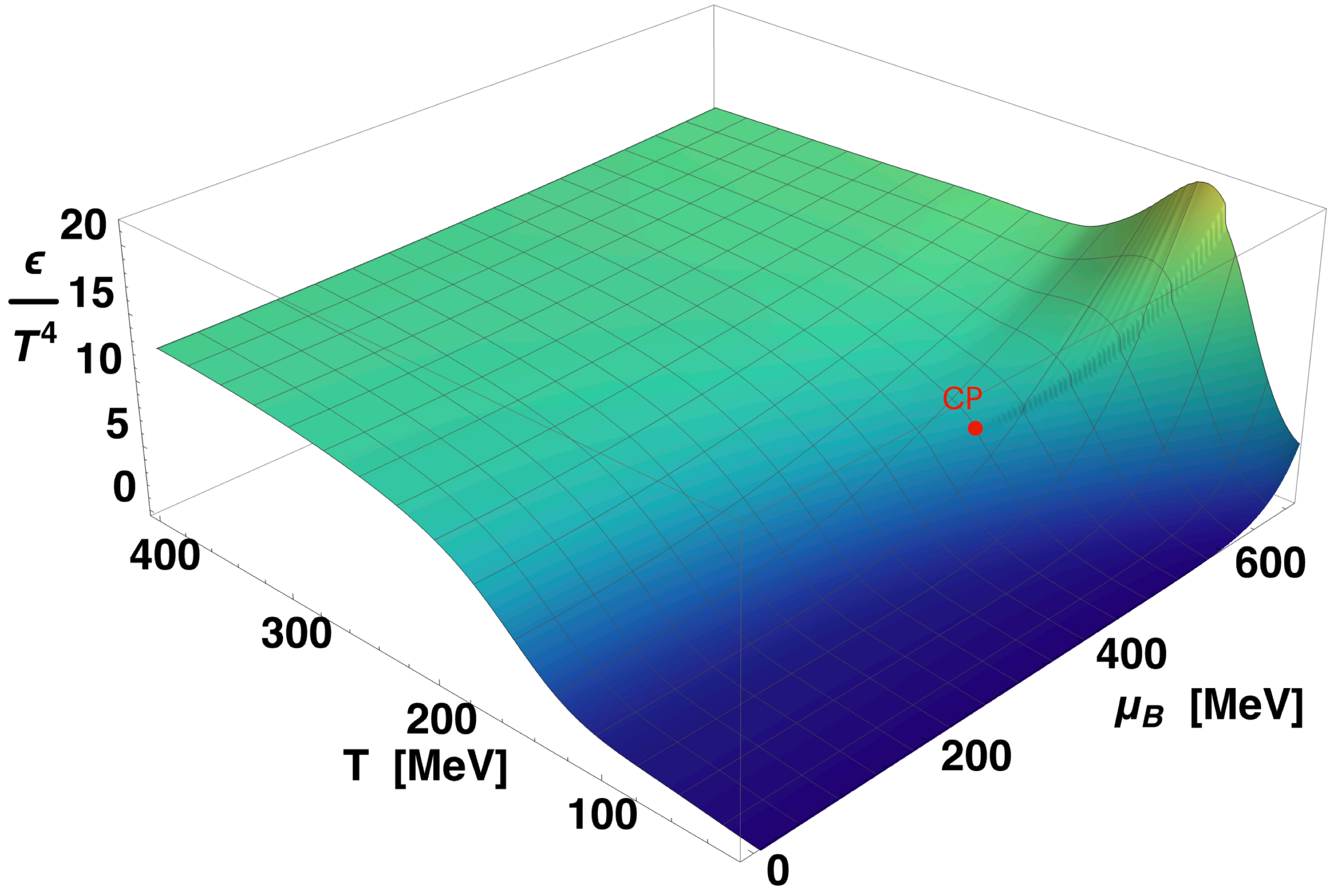
$\alpha_{12} = \alpha_1 = 11^0$

$\alpha_2 = 0^0$

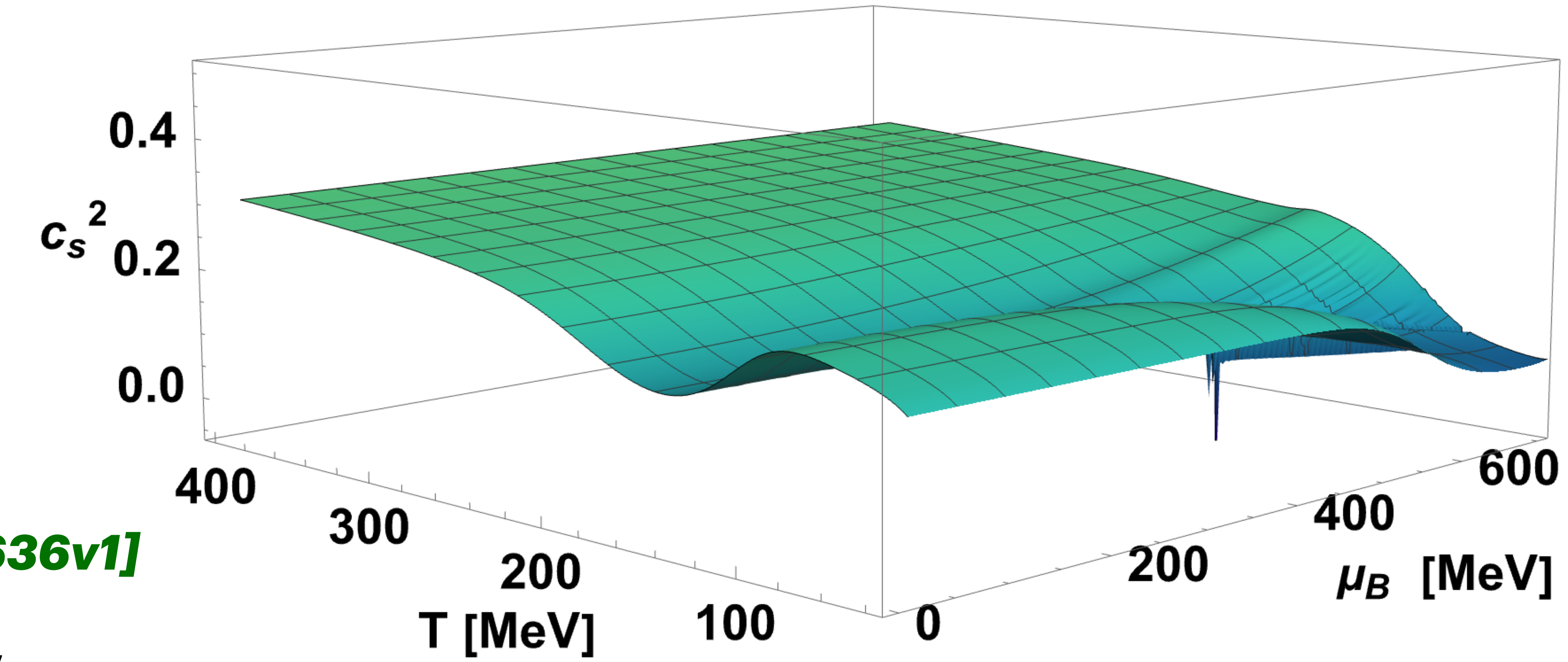
$w = 15$

$\rho = 0.3$

Entropy Density



Speed of Sound



[M. K et al arXiv:2402.08636v1]

Part 5: Constraints on the EoS

Constrains of the EoS

- Choosing μ_{BC} fixes T_C and α_1
- α_{12} is fixed physical quark masses $(\alpha_{12} = \alpha_1)$

Stability and causality

- w and ρ are fixed stability and causality

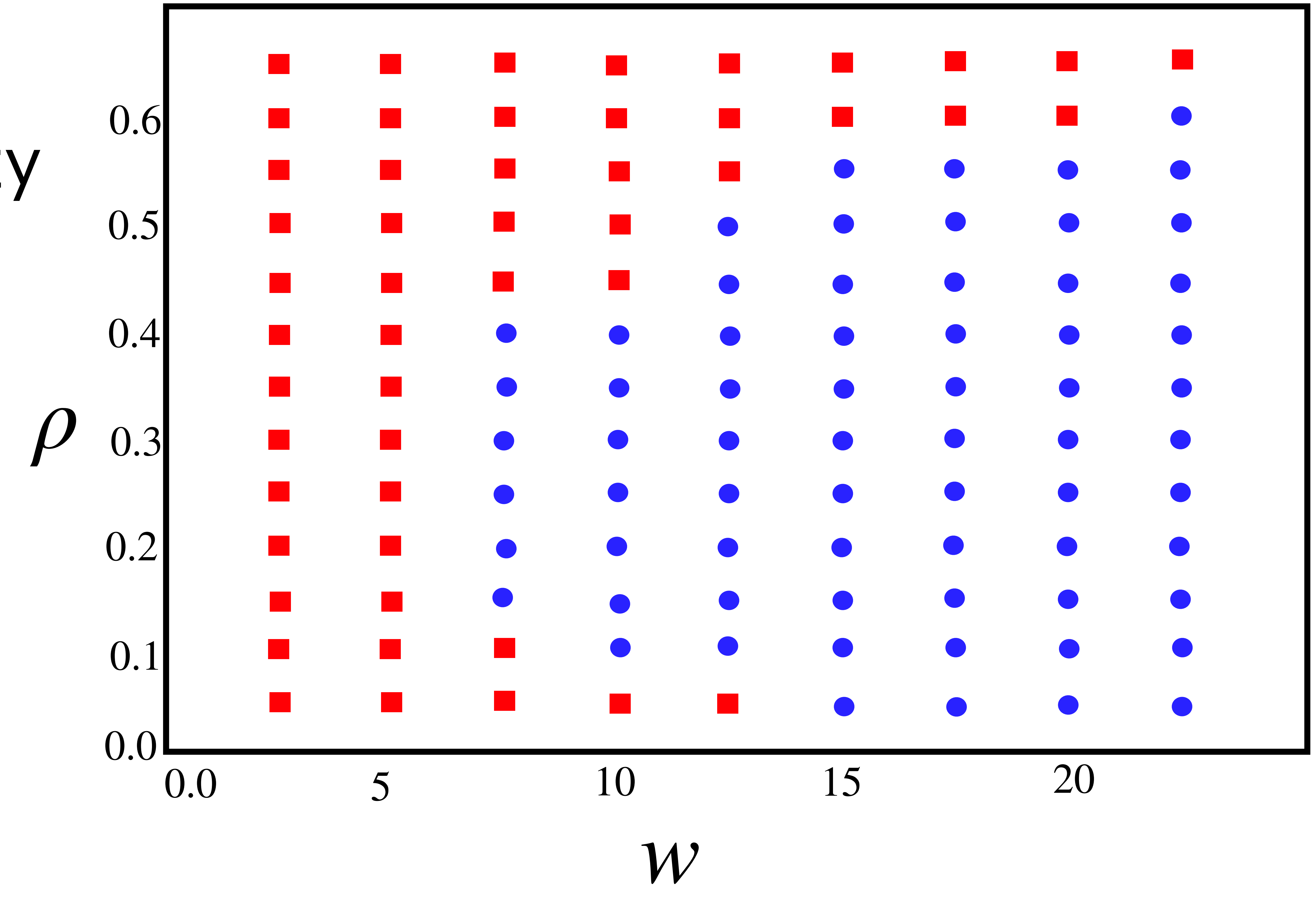
$$c_v = \left(\frac{\partial s}{\partial T}\right)\Bigg|_{n_B} > 0$$

$$\chi_2(T, \mu_B) = \left(\frac{\partial n_B}{\partial \mu_B}\right)\Bigg|_{T} > 0$$

$$0 < c_s^2(T, \mu_B) < 1$$

$\mu_{BC} = 500$ MeV

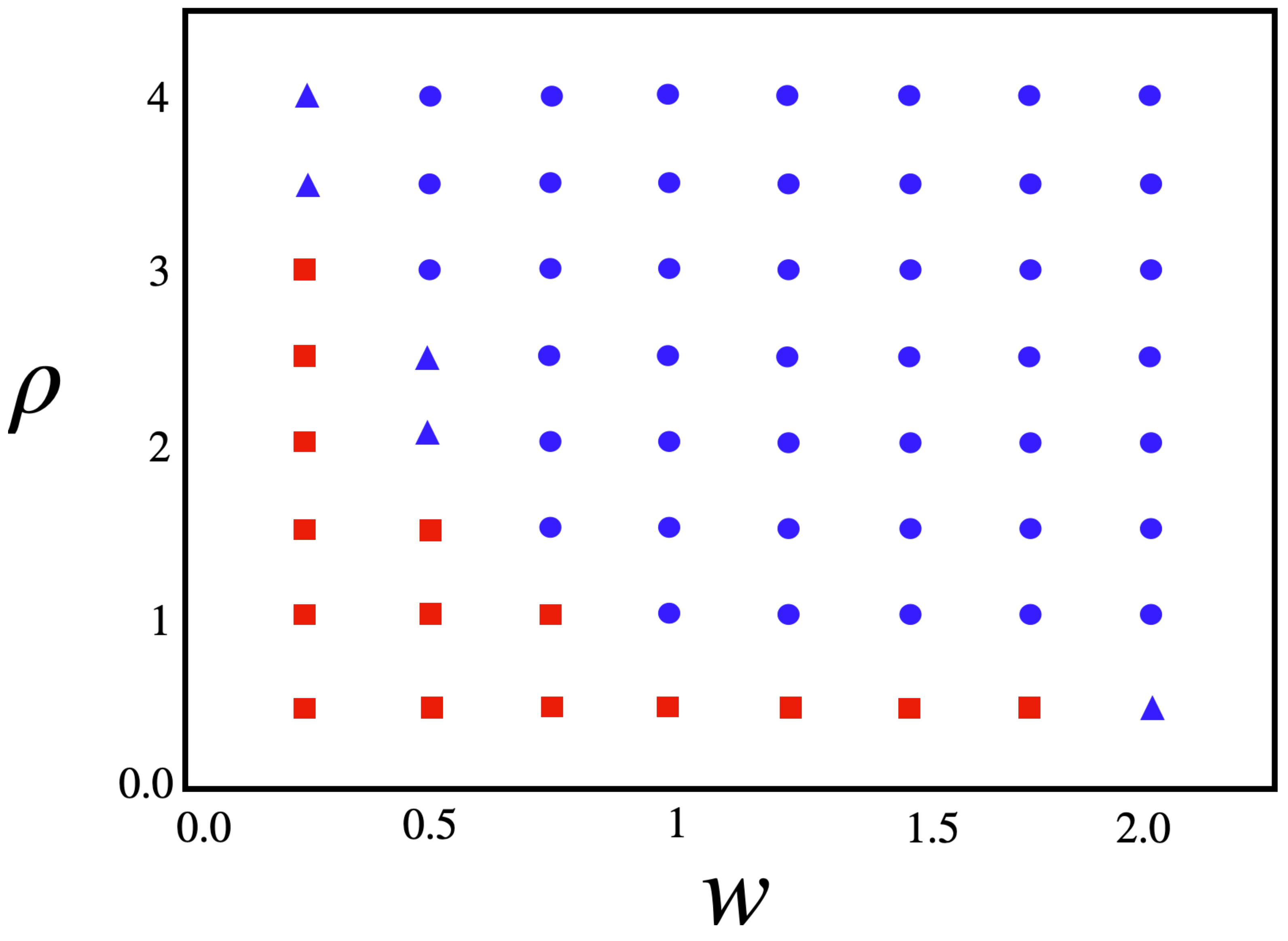
$\alpha_{12} = \alpha_1$



[M. K et al arXiv:2402.08636v1]

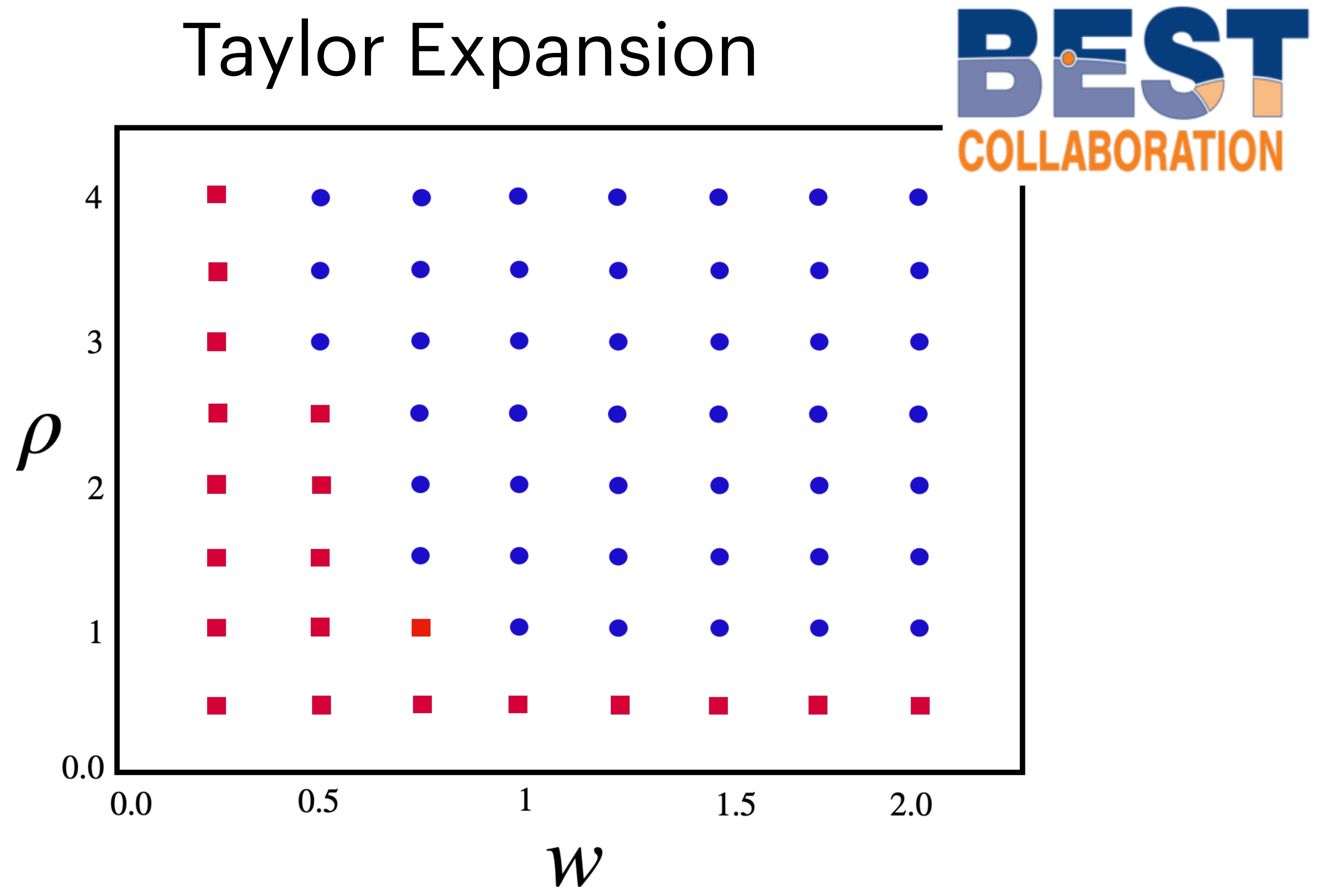
Comparison of Ising-TEXS with BEST EoS

Ising-T ExS

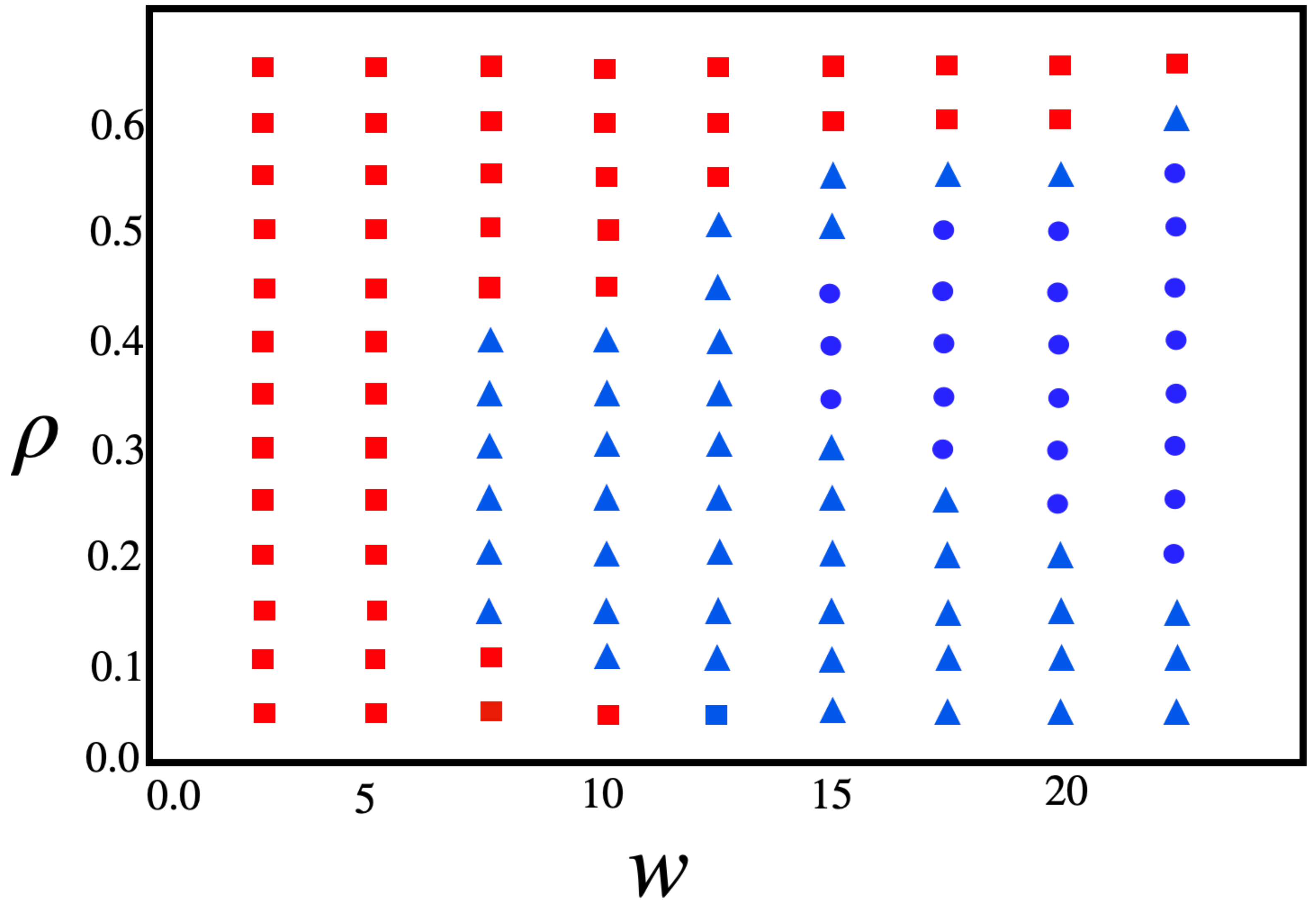


$$\mu_{BC} = 350 \text{ MeV}$$
$$\alpha_{12} = 90$$

Taylor Expansion

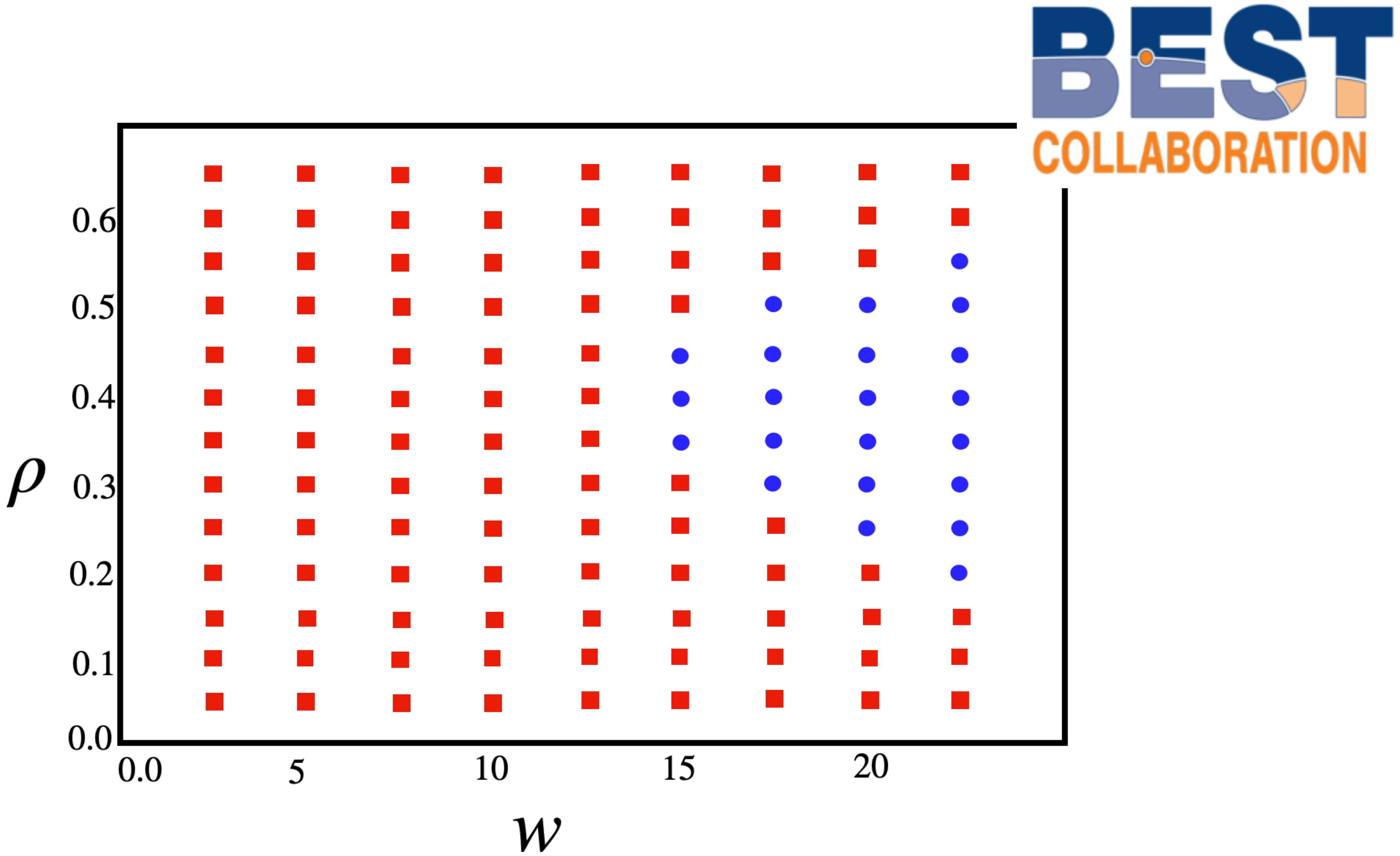


Ising-T ExS



$$\mu_{BC} = 350 \text{ MeV}$$
$$\alpha_{12} = \alpha_1$$

[M. K et al arXiv:2402.08636v1]



Summary and Conclusion

Disclaimer! : We don't predict the location of the critical point

- **We provide an enhanced coverage for family of EoS with a 3D Ising critical point from $\mu_B = [450 - 700]\ MeV$ and match lattice at low μ_B.**
- **Ising TExS EoS incorporates charge conjunction symmetry inbuilt directly from the Ising -QCD mapping**
- **Ising TExS has ability to be constrained to reproduce physical quark masses**
- **Ising TExS has adjustable parameters and can used it as input in hydrodynamical simulations to compare with the data from the Experiment (Beam Energy Scan II)**

M. K, Steffen A. Bass, Elena Bratkovskaya, Johannes Jahan, Pierre Moreau, Paolo Parotto, Damien Price, Claudia Ratti, Olga Soloveva and Mikhail Stephanov, arXiv:2402.08636v1

Code

Thank you for listening !

Back up !

Relationship of TExS with BEST Mapping

Formula's

$$T - T_C = T_C w(r\rho \sin\alpha_1 + h \sin\alpha_2)$$

$$\mu_B - \mu_{BC} = T_C w(-r\rho \cos\alpha_1 + h \cos\alpha_2)$$

$$\frac{1}{T'_{,T}} \frac{\Delta T'}{\Delta \mu_B} = \frac{\Delta T}{\Delta \mu_B} + \frac{2\kappa_2(T)\mu_B}{T'_{,T} T}$$

At $h = 0$

$$\tan\alpha_1 = \frac{2\kappa_2(T_C)\mu_{BC}}{T'_{,T} T_C}$$

At $r = 0$

$$\tan\alpha'_{12} = \tan\alpha_1 - \tan\alpha_2$$

Other relations can be found from

Simple trigonometric relation