Modern nuclear and astrophysical constraints of dense matter in a renormalized chiral approach

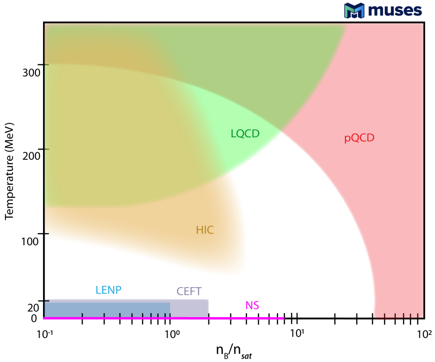
Rajesh Kumar[1],
Yuhan Wang[1], Nikolas Cruz Camacho, Arvind Kumar,
Jacquelyn Noronha-Hostler, Veronica Dexheimer[1]

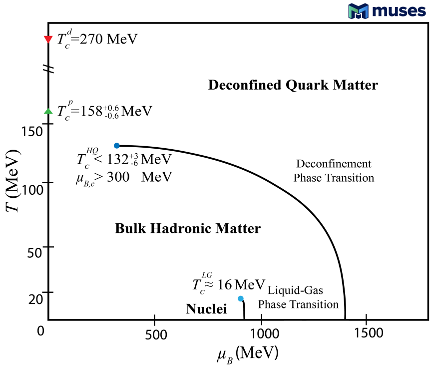
[1]Center for Nuclear Research, Department of Physics, Kent State University, OH

Winter Workshop on Nuclear Dynamics 2024, Jackson WY
February 15, 2024

Plan of presentation

QCD Phase Diagram as per Modern Theories and Experiments



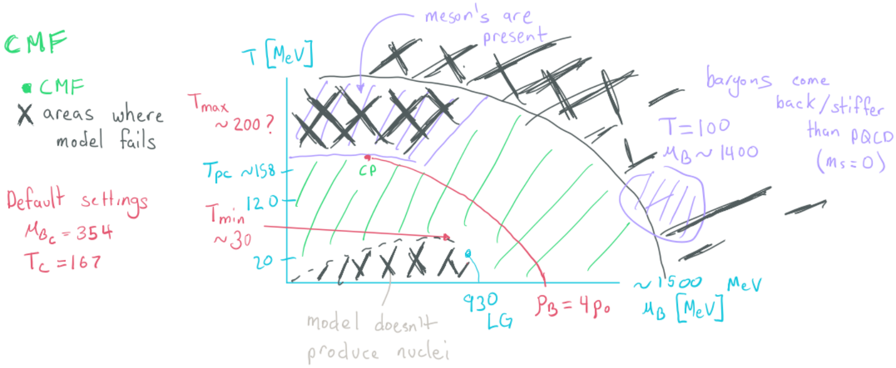


RK, VD et al., Theoretical and Experimental Constraints for the Equation of State of Dense and Hot Matter (MUSES Collaboration), arXiv:2303.17021.

RK, VD et al., Modern nuclear and astrophysical constraints of dense matter in a renormalized chiral approach , arXiv:2401.12944.

To include thermal mesons

QCD Phase Diagram as per CMF Model (credits: Angel Nava, University of Houston@MUSES Collaboration Meeting 2022)



Chiral Mean-Field (CMF) Model

- Quarks and hadrons interactions are mediated via the exchange of scalar (σ, ζ and δ) and vector (ω, ϕ and ρ) mesons.
- A comprehensive equation of state encompassing the baryon octet, decuplet, and quarks.
- CMF is a non-linear extension of the sigma model and is fitted to agree with low- and high-energy physics data.
- Uses Mean-Field Approximation (MFA),i.e., $\langle\sigma\rangle=\sigma^0$, $\langle\omega_\mu\rangle=\omega^0$ and $\langle\pi_i\rangle=0$.
- CMF uses a Polyakov loop-inspired deconfinement potential to describe the deconfinement phase transition.

  V. A. Dexheimer and S. Schramm, Novel approach to modeling hybrid stars, Phys. Rev. C 81, 045201 (2010).

Chiral Mean-Field Model

The chiral mean-field Lagrangian is written as

$$\mathcal{L}_{\rm CMF} = \mathcal{L}_{\rm kin} + \mathcal{L}_{\rm int} + \mathcal{L}_{\rm scal} + \mathcal{L}_{\rm vec} + \mathcal{L}_{\rm SB} - U_{\Phi}.$$

where $\mathcal{L}_{\rm kin}$ stands for kinetic, $\mathcal{L}_{\rm int}$ for meson-baryon interactions, $\mathcal{L}_{\rm scal}$ for scalar self interactions, $\mathcal{L}_{\rm vec}$ for vector self interactions, $\mathcal{L}_{\rm SB}$ for explicit symmetry breaking, and U_{Φ} is a Polyakov loop inspired potential may be written as

$$U_{\Phi} = \left(a_0 T^4 + a_1 \mu_B^4 + a_2 T^2 \mu_B^2\right) \Phi^2 + a_3 T_0^4 \ln\left(1 - 6\Phi^2 + 8\Phi^3 - 3\Phi^4\right).$$

RK, VD et al., Modern nuclear and astrophysical constraints of dense matter in a renormalized chiral approach , arXiv:2401.12944.

Chiral Mean-Field Model

The chiral mean-field Lagrangian is written as

$$\mathcal{L}_{\mathrm{CMF}} = \mathcal{L}_{\mathrm{kin}} + \mathcal{L}_{\mathrm{int}} + \mathcal{L}_{\mathrm{scal}} + \mathcal{L}_{\mathrm{vec}} + \mathcal{L}_{\mathrm{SB}} - U_{\Phi}.$$

where $\mathcal{L}_{\mathrm{kin}}$ stands for kinetic, $\mathcal{L}_{\mathrm{int}}$ for meson-baryon interactions, $\mathcal{L}_{\mathrm{scal}}$ for scalar self interactions, $\mathcal{L}_{\mathrm{vec}}$ for vector self interactions, $\mathcal{L}_{\mathrm{SB}}$ for explicit symmetry breaking, and U_{Φ} is a Polyakov loop inspired potential may be written as

$$U_{\Phi} = \left(a_0 T^4 + a_1 \mu_B^4 + a_2 T^2 \mu_B^2\right) \Phi^2 + a_3 T_0^4 \ln\left(1 - 6\Phi^2 + 8\Phi^3 - 3\Phi^4\right).$$

RK, VD et al., Modern nuclear and astrophysical constraints of dense matter in a renormalized chiral approach , arXiv:2401.12944.

Before adding thermal mesons need to fix the vector meson mass degeneracy

The renormalized ($\sim$) vector meson Lagrangian is

$$\tilde{\mathcal{L}}_{\text{vec}} = \tilde{\mathcal{L}}^{\text{kin}}_{\text{vec}} + \tilde{\mathcal{L}}^{\text{m}}_{\text{vec}},$$

where the kinetic term is

$$\tilde{\mathcal{L}}^{\text{kin}}_{\text{vec}} = -\frac{1}{4}\left(\left(\tilde{V}^{\mu\nu}_{\rho}\right)^2 + \left(\tilde{V}^{\mu\nu}_{K^*}\right)^2 + \left(\tilde{V}^{\mu\nu}_{\omega}\right)^2 + \left(\tilde{V}^{\mu\nu}_{\phi}\right)^2\right).$$

The simplest mass term with degenerate mass

$$\tilde{\mathcal{L}}^{m}_{\text{vec}} = \frac{1}{2} m_V^2 \operatorname{Tr} \tilde{V}_{\mu}\tilde{V}^{\mu} = \frac{1}{2} m_V^2 \left(\tilde{\omega}^2 + \tilde{\phi}^2 + \tilde{\rho}^2 + \tilde{K}^{*2}\right).$$

The chiral invariant term is added to break the vector nonet mass degeneracy

$$\tilde{\mathcal{L}}^{\text{CI}}_{\text{vec}} = \frac{1}{4}\mu \operatorname{Tr}\left[\tilde{V}_{\mu\nu}\tilde{V}^{\mu\nu}\langle X\rangle^2\right].$$

Mass degeneracy of vector meson nonet

Comparing the modified renormalized Lagrangian $\tilde{\mathcal{L}}_{\rm vec} = \tilde{\mathcal{L}}^{\rm kin}_{\rm vec} + \tilde{\mathcal{L}}^{\rm m}_{\rm vec} + \tilde{\mathcal{L}}^{\rm CI}_{\rm vec}$ with the unrenormalized one $\mathcal{L}_{\rm vec} = \mathcal{L}^{\rm kin}_{\rm vec} + \mathcal{L}^{\rm m}_{\rm vec}$, gives

$$m^2_{K^*} = Z_{K^*} m^2_V\,, \quad m^2_{\omega/\rho} = Z_{\omega/\rho} m^2_V\,, \quad m^2_\phi = Z_\phi m^2_V\,,$$
$$\tilde{\xi} = Z^{1/2}_\xi \xi\,, \quad \xi = \rho, \omega, K^*, \phi.$$

Meson	ω	ρ	K^*	ϕ
Old Mass (MeV)	687.33	687.33	687.33	687.33
New Mass (MeV)	770.87	770.87	865.89	1007.76

where

$$Z^{-1}_\rho = Z^{-1}_\omega = \left(1 - \mu\frac{\sigma_0^2}{2}\right), \quad Z^{-1}_\phi = \left(1 - \mu\zeta_0^2\right), \quad Z^{-1}_{K^*} = \left(1 - \frac{1}{2}\mu\left(\frac{\sigma_0^2}{2} + \zeta_0^2\right)\right).$$

Self-interaction term for vector mesons

The net vector Lagrangian with the self-interactive term is now

$$\mathcal{L}_{\text{vec}} = \mathcal{L}_{\text{vec}}^{\text{kin}} + \mathcal{L}_{\text{vec}}^{\text{m}} + \mathcal{L}_{\text{vec}}^{\text{SI}} .$$

The different possible chiral invariant renormalized self-interaction (SI) terms of the vector mesons considered are

$$^{\text{R}}\text{C1:}\ \tilde{\mathcal{L}}_{\text{vec}}^{\text{SI}} = 2\tilde{g}_4 \,\text{Tr}\left(\tilde{V}^4\right),$$

$$^{\text{R}}\text{C2:}\ \tilde{\mathcal{L}}_{\text{vec}}^{\text{SI}} = \tilde{g}_4 \left[\frac{3}{2}\left[\text{Tr}\left(\tilde{V}^2\right)\right]^2 - \text{Tr}\left(\tilde{V}^4\right)\right],$$

$$^{\text{R}}\text{C3:}\ \tilde{\mathcal{L}}_{\text{vec}}^{\text{SI}} = \tilde{g}_4 \left[\text{Tr}\left(\tilde{V}^2\right)\right]^2,$$

$$^{\text{R}}\text{C4:}\ \tilde{\mathcal{L}}_{\text{vec}}^{\text{SI}} = \tilde{g}_4 \frac{[\text{Tr}(\tilde{V})]^4}{4} .$$

Self-interaction term for vector mesons

The renormalized self-interaction term in a simplified version ($^{\mathrm{R}}$C1-$^{\mathrm{R}}$C4) reads

$$^{\mathrm{R}}\mathrm{C1}: \mathcal{L}_{\mathrm{vec}}^{\mathrm{SI}} = g_4\left(\omega^4 + 6\frac{Z_\rho}{Z_\omega}\omega^2\rho^2 + \left(\frac{Z_\rho}{Z_\omega}\right)^2\rho^4 + 2\left(\frac{Z_\phi}{Z_\omega}\right)^2\phi^4\right),$$

$$^{\mathrm{R}}\mathrm{C2}: \mathcal{L}_{\mathrm{vec}}^{\mathrm{SI}} = g_4\left(\omega^4 + \left(\frac{Z_\rho}{Z_\omega}\right)^2\rho^4 + \left(\frac{Z_\phi}{Z_\omega}\right)^2\frac{\phi^4}{2} + 3\left(\frac{Z_\rho}{Z_\omega}\frac{Z_\phi}{Z_\omega}\right)\rho^2\phi^2 + 3\left(\frac{Z_\phi}{Z_\omega}\right)\omega^2\phi^2\right),$$

$$^{\mathrm{R}}\mathrm{C3}: \mathcal{L}_{\mathrm{vec}}^{\mathrm{SI}} = g_4\left(\omega^4 + 2\frac{Z_\rho}{Z_\omega}\omega^2\rho^2 + \left(\frac{Z_\rho}{Z_\omega}\right)^2\rho^4 + 2\frac{Z_\phi}{Z_\omega}\omega^2\phi^2 + \left(\frac{Z_\phi}{Z_\omega}\right)^2\phi^4 + 2\left(\frac{Z_\rho}{Z_\omega}\frac{Z_\phi}{Z_\omega}\right)\rho^2\phi^2\right),$$

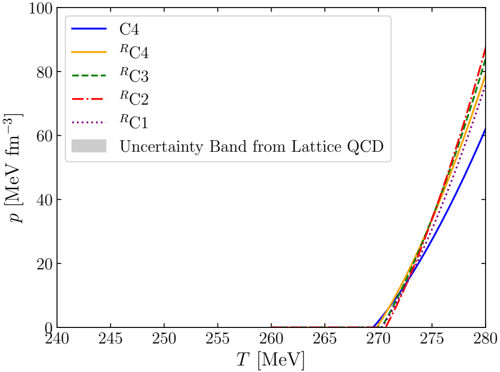
$$^{\mathrm{R}}\mathrm{C4}: \mathcal{L}_{\mathrm{vec}}^{\mathrm{SI}} = g_4\left(\omega^4 + 2\sqrt{2}\left(\frac{Z_\phi}{Z_\omega}\right)^{1/2}\omega^3\phi + 3\left(\frac{Z_\phi}{Z_\omega}\right)\omega^2\phi^2 + \sqrt{2}\left(\frac{Z_\phi}{Z_\omega}\right)^{3/2}\omega\phi^3 + \frac{1}{4}\left(\frac{Z_\phi}{Z_\omega}\right)^2\phi^4\right).$$

The renormalization process required us to refit our model to the nuclear saturation properties, the latest first principle theories, and observational constraints.

Parameters used to fit the constraints

Parameter	Term	Used to constrain
g_1^V, g_8^V, α_V, g_4	$\mathcal{L}_{\text{int}}+\mathcal{L}_{\text{vec}}^{\text{SI}}$	$g_{N\phi}=0$, $g_1^V=\sqrt{6}g_8^V$, $n_{\text{sat}}\approx 0.15$ fm $^{-3}$, $B^{\text{sat}}/A\approx -15.70$ MeV, $E_{\text{sym}}^{\text{sat}}\approx 28.9$ MeV, $66\leq L^{\text{sat}}(\text{MeV})\leq 87$, $275\leq K(\text{MeV})\leq 305$
m_V, μ	$\mathcal{L}_{\text{vec}}^{\text{m}}+\mathcal{L}_{\text{vec}}^{\text{CI}}$	$m_\omega=770.87$ MeV, $m_\rho=770.87$ MeV, $m_\phi=1007.76$ MeV
m_3	$\mathcal{L}_{\text{SB}}$	$U_\Lambda\approx -28$ MeV
a_0		$T_c^d\approx 270$ MeV
a_1		$n_{B,c}^d\approx 3.5\ n_{\text{sat}}$
a_2		$T_c^{\text{HQ}}>135$ MeV, $\mu_{B,c}>400$ MeV
a_3	U_Φ	$\Phi\in 0,1$
T_0(gauge)		T_c^d, $\Phi\in 0,1$
T_0(quarks)		$T_c^p\approx 159$ MeV, $\Phi\in 0,1$
$g_{q\Phi}$, $g_{B\Phi}$		T_c^p

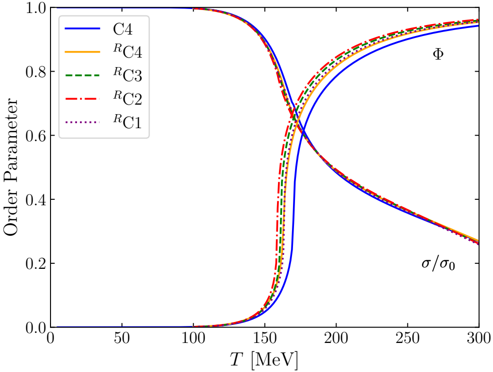
Deconfinement phase transition for the pure gauge case for different renormalized couplings (RCs) at $\mu_B = 0$



- First order phase transition indicating the deconfined gluons at T ∼ 270 MeV.
  G. Boyd et al., Thermodynamics of SU(3) lattice gauge theory, Nucl. Phys. B 469, 419 (1996)
- In CMF model, mesons exchange and Φ fields mimics the deconfined gluons.

RK, VD et al., Modern nuclear and astrophysical constraints of dense matter in a renormalized chiral approach , arXiv:2401.12944.

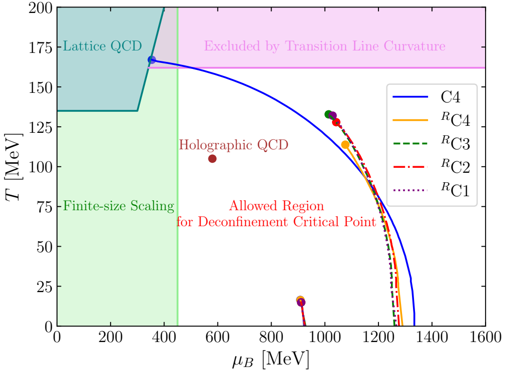
Crossover transition temperature represented by the change in σ and Φ for different RCs at $\mu_B = \mu_S = \mu_Q = 0$



- From lattice QCD, the pseudo critical transition temperature T_c^p=158 ± 0.6 MeV.
  S. Borsanyi et al., QCD Crossover at Finite Chemical Potential from Lattice Simulations, Phys. Rev. Lett. 125, 052001 (2020)
- In the CMF model, the maximum change in the order parameter ($\frac{d\sigma}{dT} \sim \frac{d\Phi}{dT}$) occurs around T_c^p =161 MeV for all RCs.
- In the previous CMF fit, T_c^p =171 MeV for C4.

RK, VD et al., Modern nuclear and astrophysical constraints of dense matter in a renormalized chiral approach , arXiv:2401.12944.

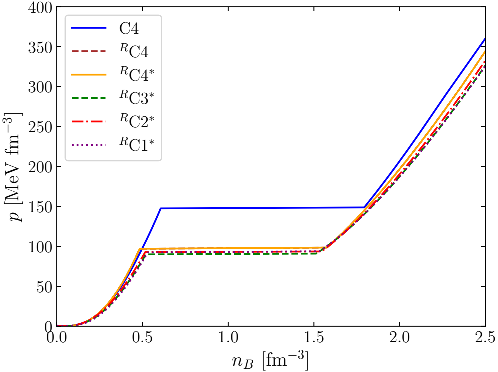
Hadron-quark as well as liquid-gas coexistence lines and respective critical points for $\mu_Q = 0$ and zero net strangeness



- As per lattice QCD results, $\mu_{B_c} > 300$ MeV, with $T_c^{HQ} < 132^{+3}_{-6}$ MeV.
  S. Borsanyi et al., QCD Crossover at Finite Chemical Potential from Lattice Simulations, Phys. Rev. Lett. 125, 052001 (2020)
  H. T. Ding et al. (HotQCD), Chiral Phase Transition Temperature in (2+1)-Flavor QCD, Phys. Rev. Lett. 123, 062002 (2019)
- From old CMF fit, $\mu_{B_c} = 354$ MeV and $T_c^{HQ} = 167$ MeV.
- We get critical point of LG phase transition $T_c^{\text{LG}} \approx 16$ MeV.

RK, VD et al., Modern nuclear and astrophysical constraints of dense matter in a renormalized chiral approach , arXiv:2401.12944.

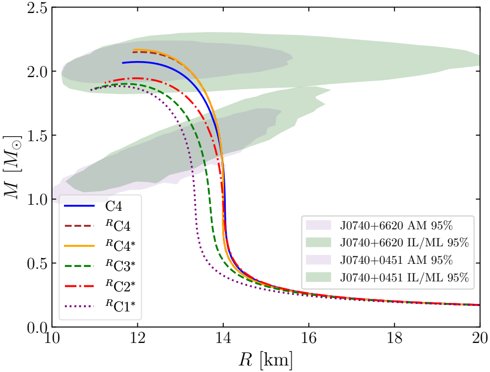
Equation of state for neutron-star matter at $T = 0$ for different RCs.



- EoS is beta equilibrated and charge neutral.
- "*' means no hyperons.
- A smaller number density jump during the phase transition compared to the old C4 scheme is obtained.
- A lower value of $n^{d}_{B,c} \approx 3.4\, n_{\rm sat}$ is obtained.
- the $^{\rm R}$C4 coupling schemes exhibit stiffer pressures compared to the old C4 scheme.
- In the $^{\rm R}$C4 coupling scheme the stiffness is almost the same, independently of the presence of hyperons.

RK, VD et al., Modern nuclear and astrophysical constraints of dense matter in a renormalized chiral approach , arXiv:2401.12944.

Mass-radius curve for neutron star matter with different renormalized vector couplings



- Including the EoS with a large jump would show a kink (instability) in the MR diagram therefore it is shown for only hadronic matter.
- This problem can be solved by the different choice of deconfinement potential.
- Without hyperons, we observe that $^{\mathrm{R}}$C4 yields the highest M_{max}..
- Inclusion of hyperons results in a slight reduction in M_{max} for $^{\mathrm{R}}$C4, but it still remains higher than the other coupling schemes.

RK, VD et al., Modern nuclear and astrophysical constraints of dense matter in a renormalized chiral approach , arXiv:2401.12944.

Conclusions

- Fixed the vector mesons mass degeneracy through renormalization of vector meson fields.
- This paves the way for the addition of interacting mesons within the CMF model.
- Refitted the model using constraints from lattice QCD, stellar observations, and nuclear physics.
- Redefinition of vector fields plays a significant role in reproducing neutron stars with higher masses compared to the previous fit.

Estimation of error in the model

- Adding interacting thermal mesons' contribution will improve the constraints reproduced within the model, especially at finite T and μ_B.
- Low-energy nuclear physics observations are not well-constrained, therefore more wider range of constraints should be studied.
  RK, VD et al., Effects of hyperon potentials and symmetry energy in quark deconfinement , Physics Letters B 849, 138475 (2024).
- Optimizing CMF code by transforming it into C++ from Fortran within MUSES (with Nikolas Cruz from UIUC).
- Use statistical methods, such as Bayesian analysis, to constrain model parameters within the MUSES collaboration (See Claudia's talk for more about MUSES).

Outlook

- Add fluctuations in the CMF model by going beyond mean-field approximation within the NP3M collaboration (with Joaquin Grefa, KSU).
- Add in-medium meson masses and thermal meson condensation at low temperatures.
- Comparison of partial pressure of mesons from lattice QCD.

Regulating the meson contribution in the quark phase

$$M_B^* = g_{B\sigma}\sigma + g_{B\delta}\tau_3\delta + g_{B\zeta}\zeta + M_{0_B} + g_{B\Phi}\Phi^2.$$

$$M_q^* = g_{q\sigma}\sigma + g_{q\delta}\tau_3\delta + g_{q\zeta}\zeta + M_{0_q} + g_{q\Phi}(1-\Phi).$$

$$M_M^*(\Phi) = ?$$

Acknowledgements

muses

NSF

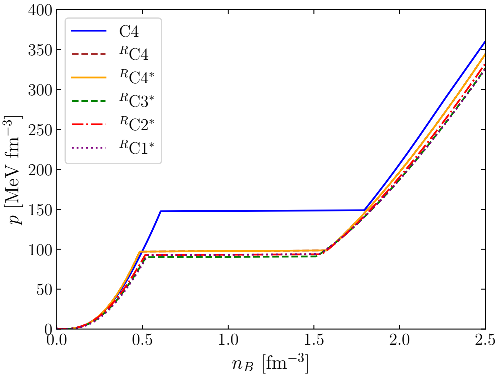
NP3M

Center for Nuclear Research

Kent State University · Ohio · 1910

University of Illinois Urbana-Champaign

Backup: Accessing the stability of neutron star



- The speed of hadron↔quark conversion, which could in turn make hybrid stars unstable.
- If the surface tension of quark matter is below a certain threshold, a mixture of phases appears, which enhances stellar stability.
- Alternatively, by altering the deconfined potential (for example, from $a_1\mu_B^4$ to $a_1'\mu_B^2$) to make it less responsive to the μ_B.
- This facilitates producing stable hybrid stars without mixed phases with small jump.

  V. Dexheimer et al., GW190814 as a massive rapidly rotating neutron star with exotic degrees of freedom, Phys. Rev. C 103, 025808 (2021)