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Recent progress in quarkonia dynamics with open quantum systems

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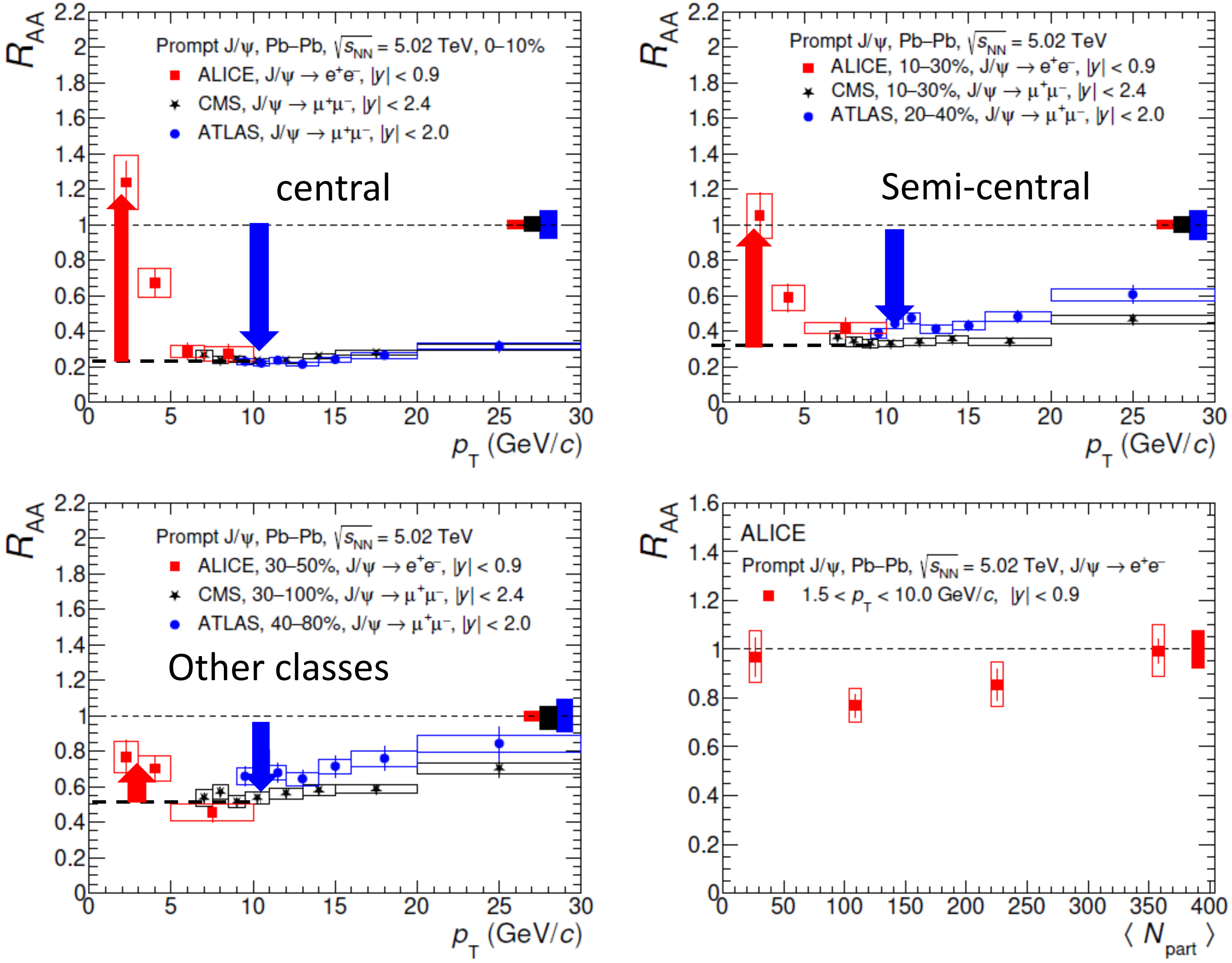
Mainly based on 2402.04488 (and references therein)

and Pays de la Loire

Quarkonia as a probe of the QGP created in ultrarelativistic systems



ALICE Collab. JHEP02 (2024) 066

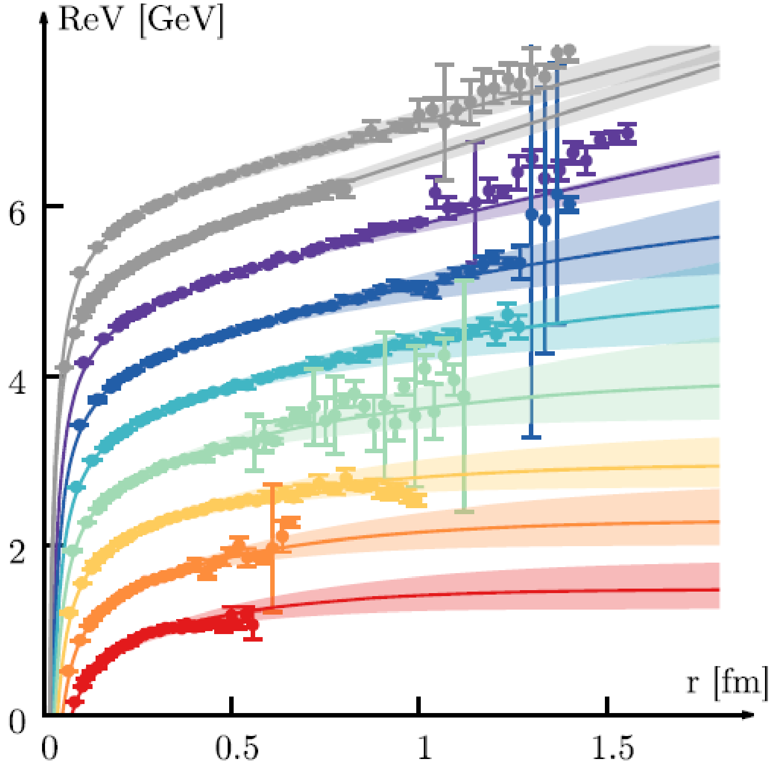
- Quarkonia production in AA strongly affected by the presence of the QGP => good probe of the QGP properties on small scales ($1/M_Q$)
- Increasing suppression with centrality at intermediate and high p_T
- Increasing yield with centrality at low p_T
- Increasing experimental precision => need for the models to gain in accuracy

The 3 pillars of quarkonia production in AA

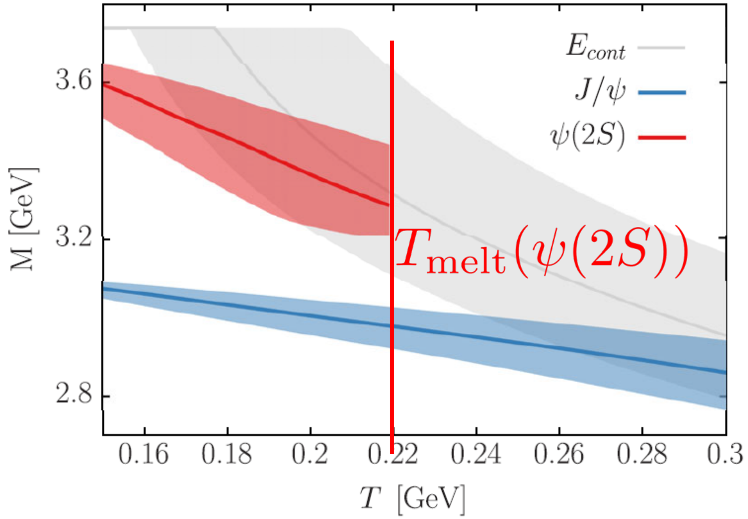
Implicitly in the pNRQD EFT.

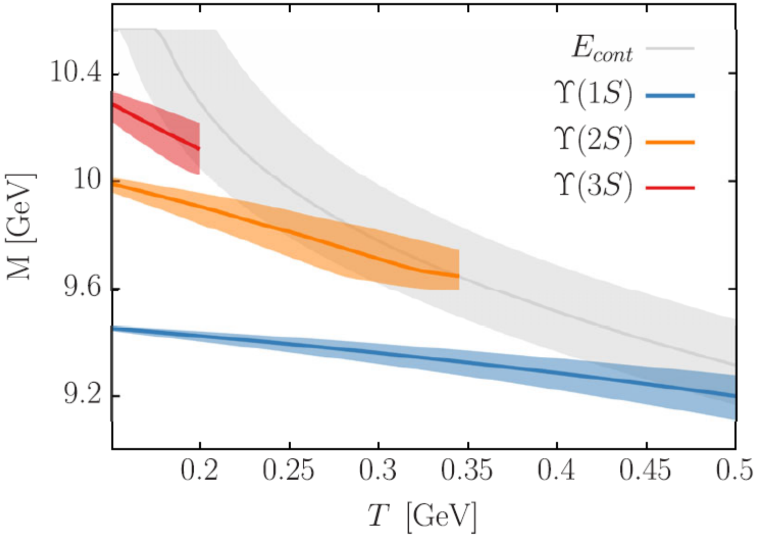
Screening of the real potential

At T=0, well described by the Cornell potential: $V(r) = -\frac{\alpha}{r} + Kr$



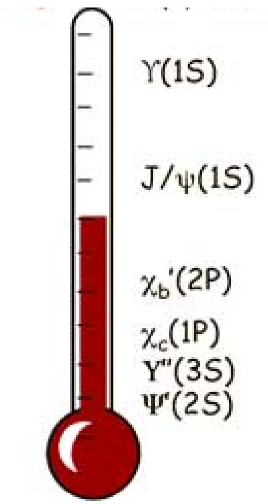
D. Lafferty, A. Rothkopf , Phys. Rev. D 101, 056010 (2020)





Probing new state of matter in AA collision; Original idea by Matsui and Satz (86) :

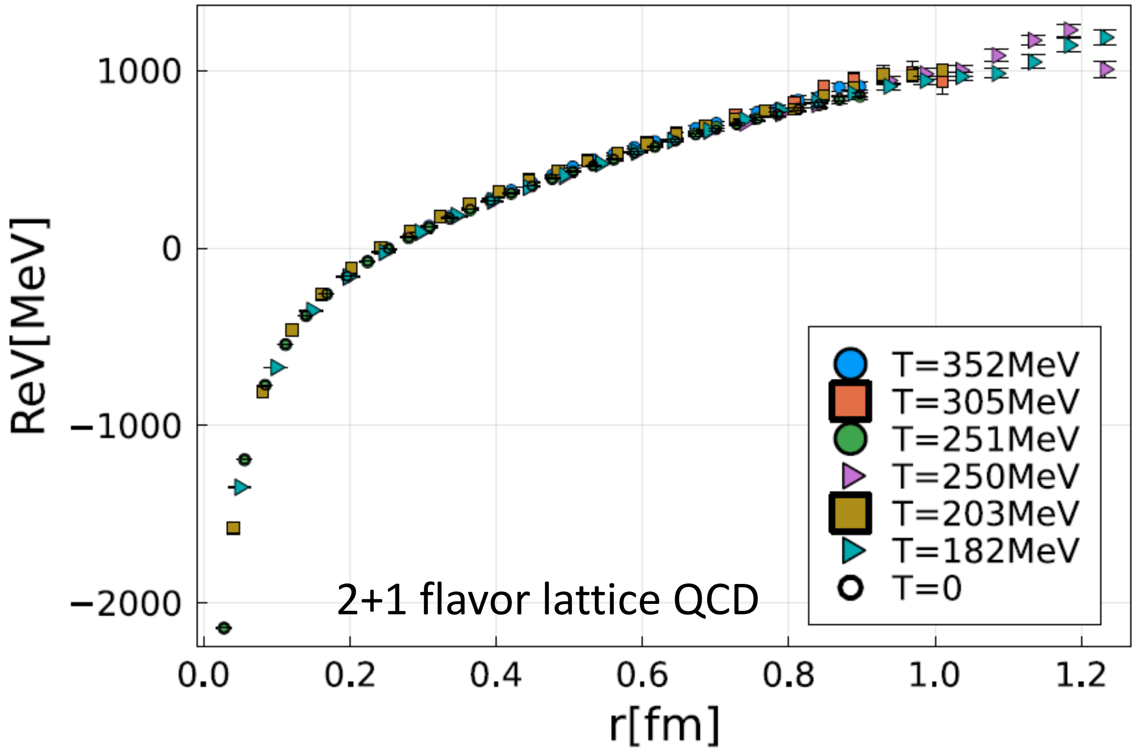
SEQUENTIAL SUPPRESSION; Quarkonia as early QGP thermometer



Advertized as a motivation in hundreds of talks (and papers) since then

Screening of the real potential

Recent news : the real potential is not screened at temperatures reached in AA collisions !!!



Bazazov et al 2023 (Hot QCD collaboration)

How to define properly a “potential” on the lattice ?

Historically : thermodynamical potential like the free energy (in presence of a static dipole) or the total internal energy.

Modern approach : evaluate the Wilson loop and connect it to the r-dependent spectral density

$$W(\tau, r, T) = \int_{-\infty}^{+\infty} d\omega e^{-\omega\tau} \rho_r(\omega, T)$$

A “peak” contribution in the spectral density modelled as

$$\rho_r^{\text{peak}}(\omega, T) = \frac{1}{\pi} \text{Im} \frac{A_r(T)}{\omega - \text{Re}V(r,T) - i\Gamma(\omega, r, T)}$$

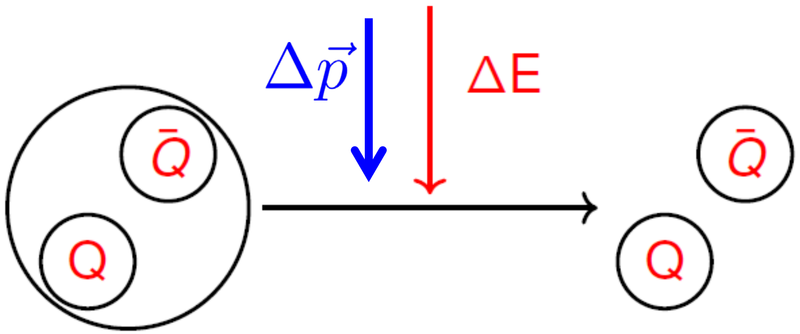
=> Lattice data then unfolded with this Ansatz.

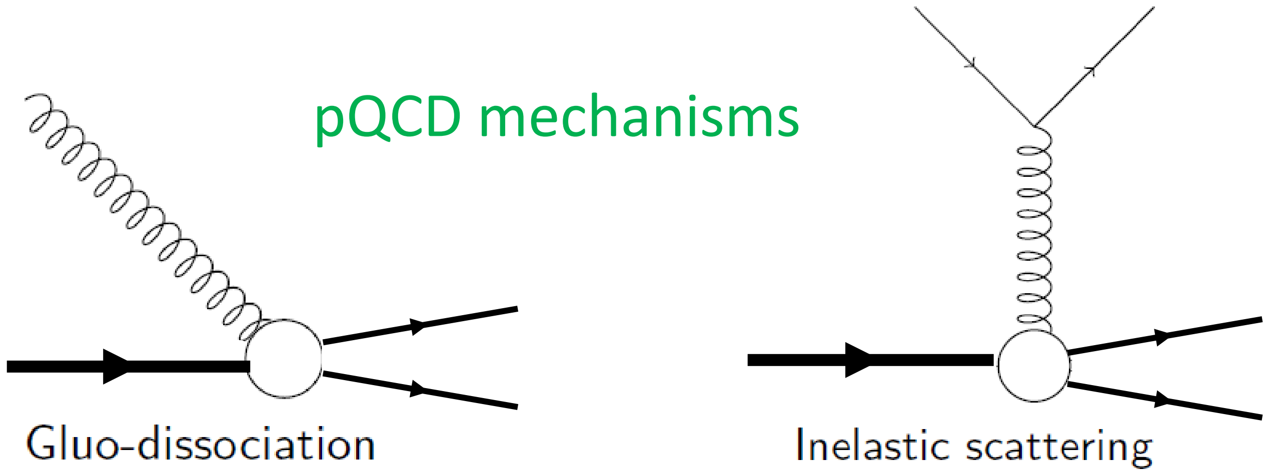
May not be the end of the story … makes quarkonia suppression in AA even more interesting !

Collisions with the QGP

- Besides arguments based on the Debye mass / screening, it was pointed out already in the 90's that interactions with partons in the QGP could lead to dissociation of bound states (whose spectral function thus acquire some width Γ corresponding to the dissociation rate)

- Energy-momentum exchange with the QGP (gluo-dissociation, q – quarkonia quasi elastic scattering)





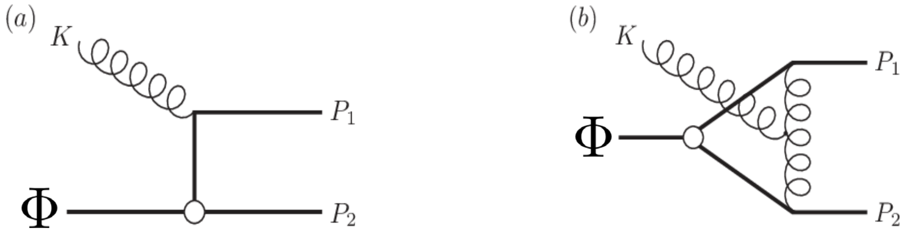
- => pair dissociation => **Suppression**

- ⬄ loss of probability of the quarkonia … Often described by some imaginary potential W in modern approaches

A central quantity: the dissociation rate Γ

Many approaches

pQCD view (Bhanot & Peskin), later on consolidated by NRQCD (Brambilla & Vairo)



$$\Phi + g \to Q + \bar{Q}$$

Dissociation cross section σ

$$\Gamma_\Phi(T) = \langle \sigma n_g \rangle_T$$

Other mechanisms : $x + \Phi \to x + Q + \bar{Q}$

QFT/Lattice QCD

Time correlator

$$\mathcal{C}_>(t,\vec{r}) \approx \langle \psi(t,\tfrac{\vec{r}}{2})\bar{\psi}(t,-\tfrac{\vec{r}}{2})\psi(0,0)\bar{\psi}(0,0)\rangle$$

Satisfies Schroedinger equation with complex potential V+iW . Breakthrough by Laine et al. (2006)

$$\Gamma_\Phi(T) = -2\langle \Phi | W | \Phi \rangle$$

Concept better suited at it genuinely encodes the “in medium” propagation

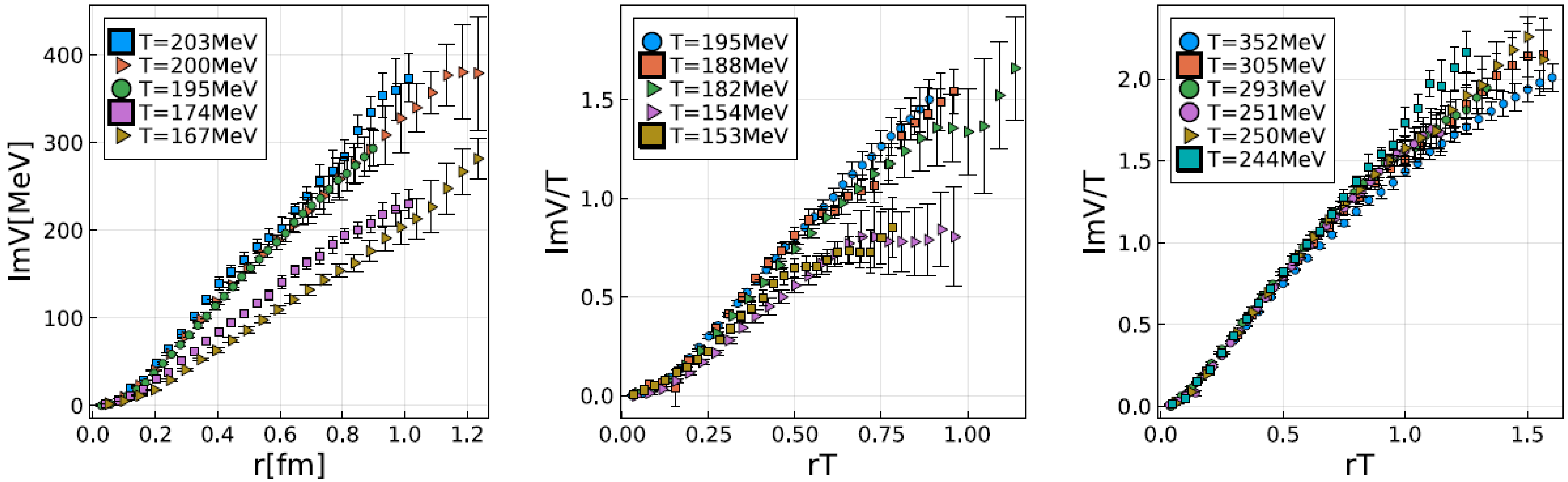
=> Simple decay law : $\text{Probability survival} = \exp\left(-\int_{t_0}^{t_{\rm fin}} \Gamma(T(t))dt\right)$

A central quantity: the dissociation rate Γ

Recent lQCD calculations of W(r) = Im(V(r)) (at ω=0)

$$\rho_r^{\text{peak}}(\omega,T) = \frac{1}{\pi}\text{Im}\frac{A_r(T)}{\omega - \text{Re}V(r,T) - i\Gamma(\omega,r,T)}$$

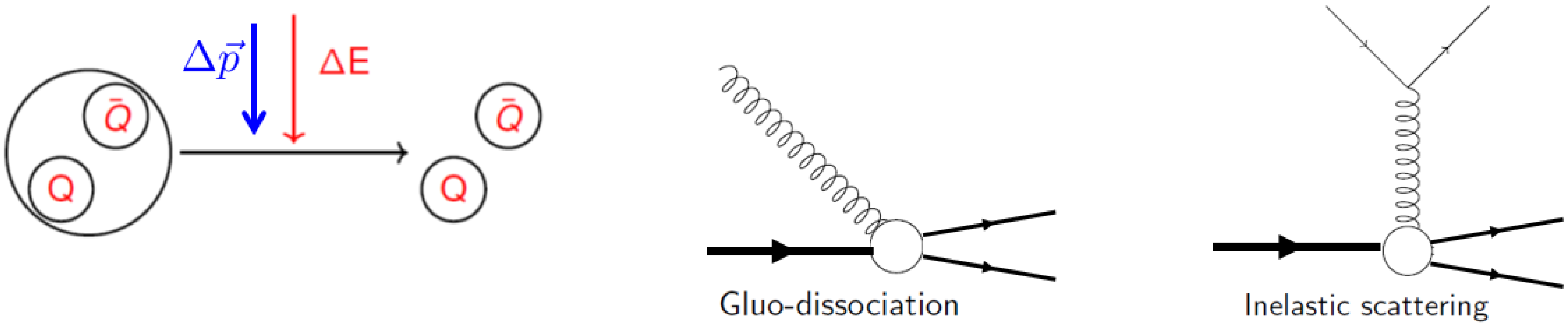
Bazazov et al 2023 (Hot QCD collaboration)

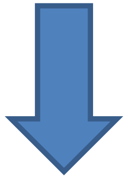


- Nice r T scaling
- Dipole structure at small r, no saturation seen at “large” r

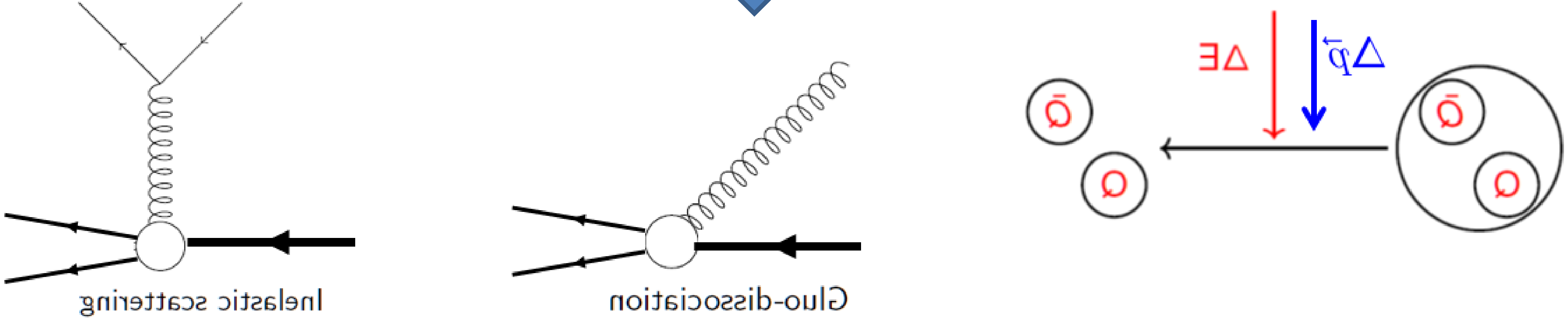
(Re)generation

Detailed balance :





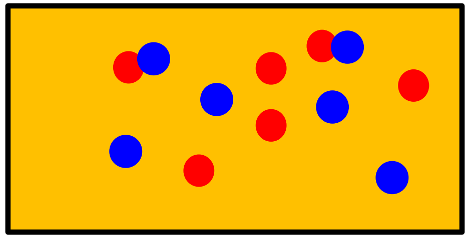
Reverse mechanisms :



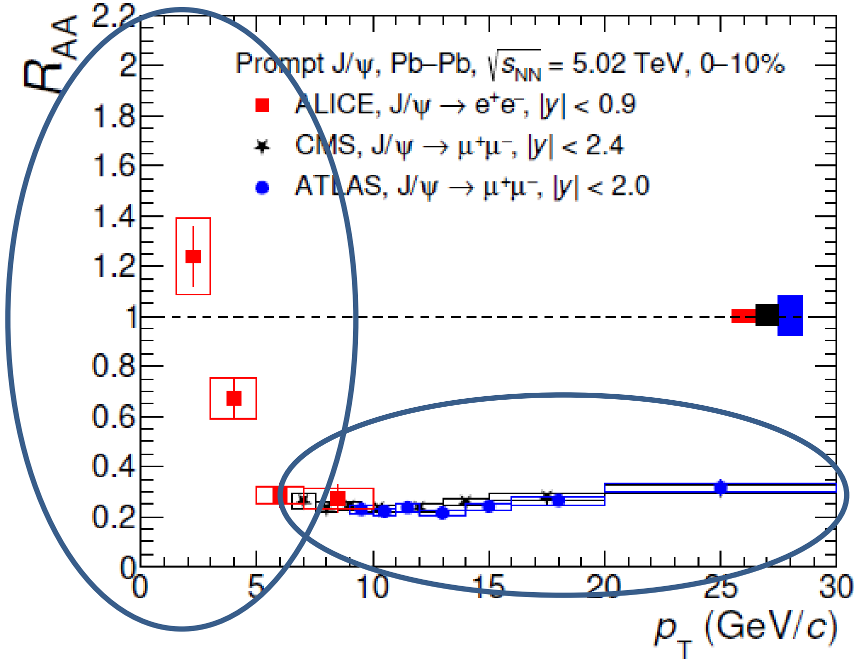
… become quite effective when heavy quarks are dense in the (x,p) phase space

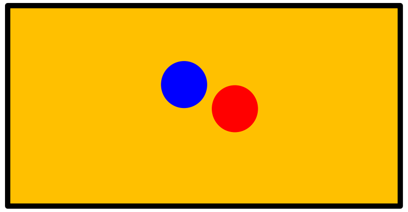
(Re)generation

ALICE Collab. JHEP02 (2024) 066



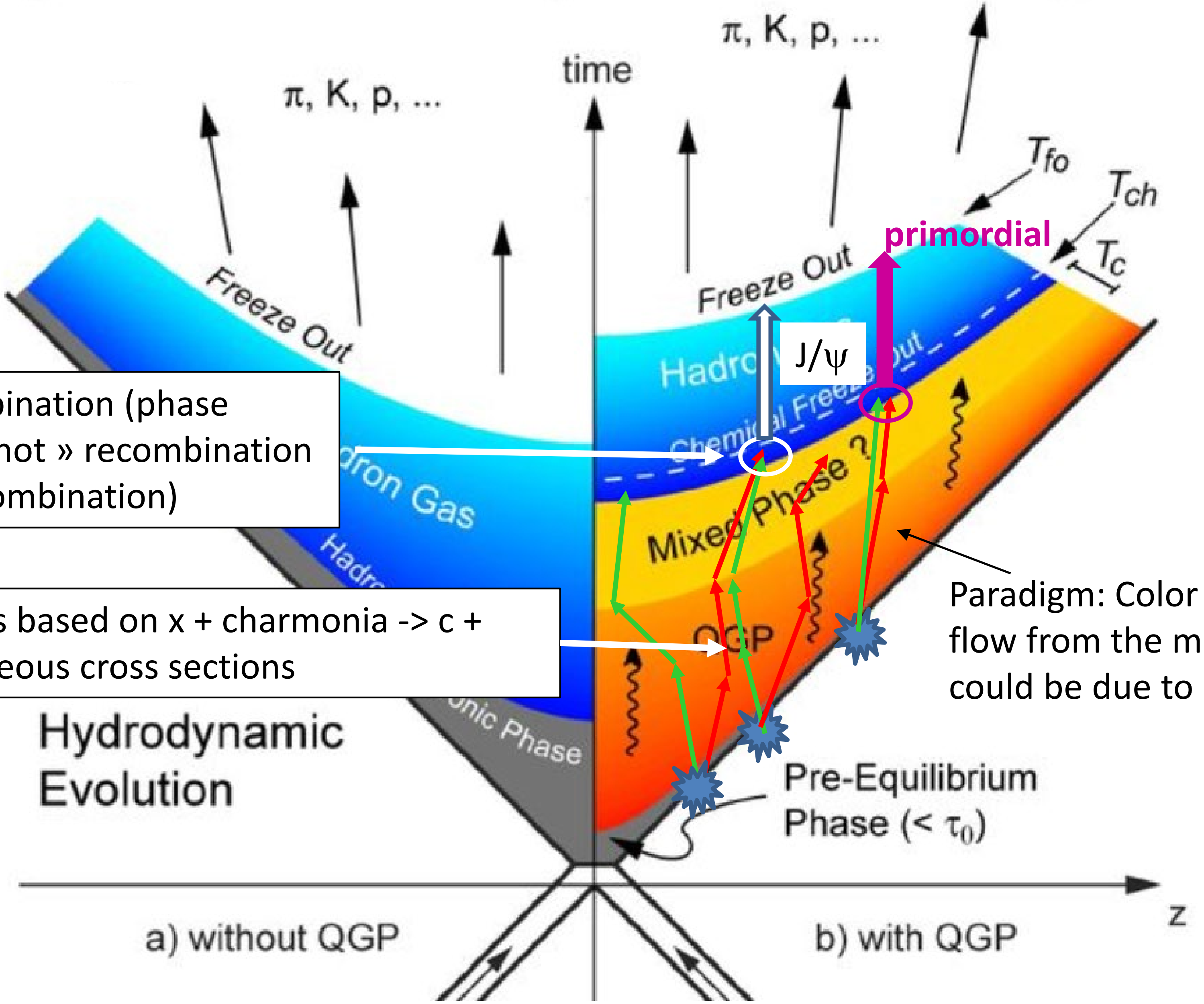
Dense phase





Dilute phase

Quarkonia as a probe of the QGP created in ultrarelativistic systems

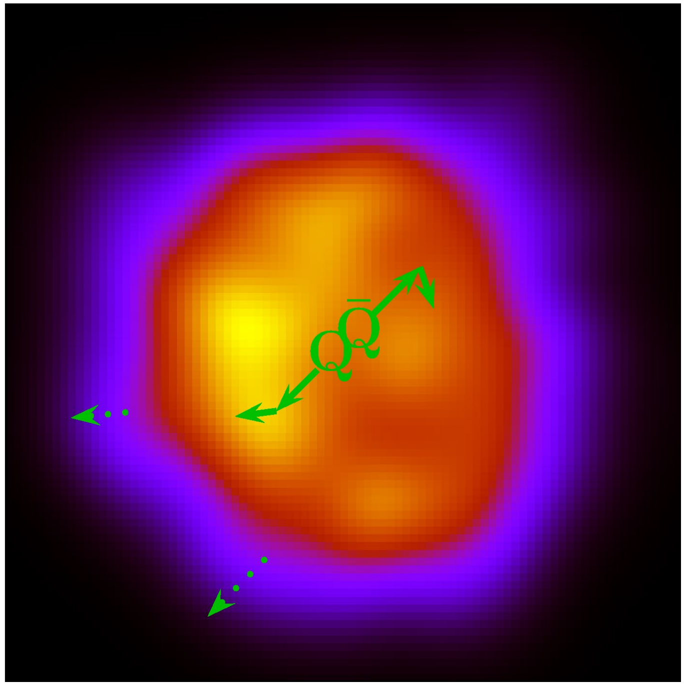


Statistical recombination (phase space) or « one shot » recombination (coalescence recombination)

Rate equations based on x + charmonia -> c + cbar instantaneous cross sections

Paradigm: Color singlet => no flow from the medium… but could be due to CO* -> CS

What is a quarkonia… in a hot QGP medium ?

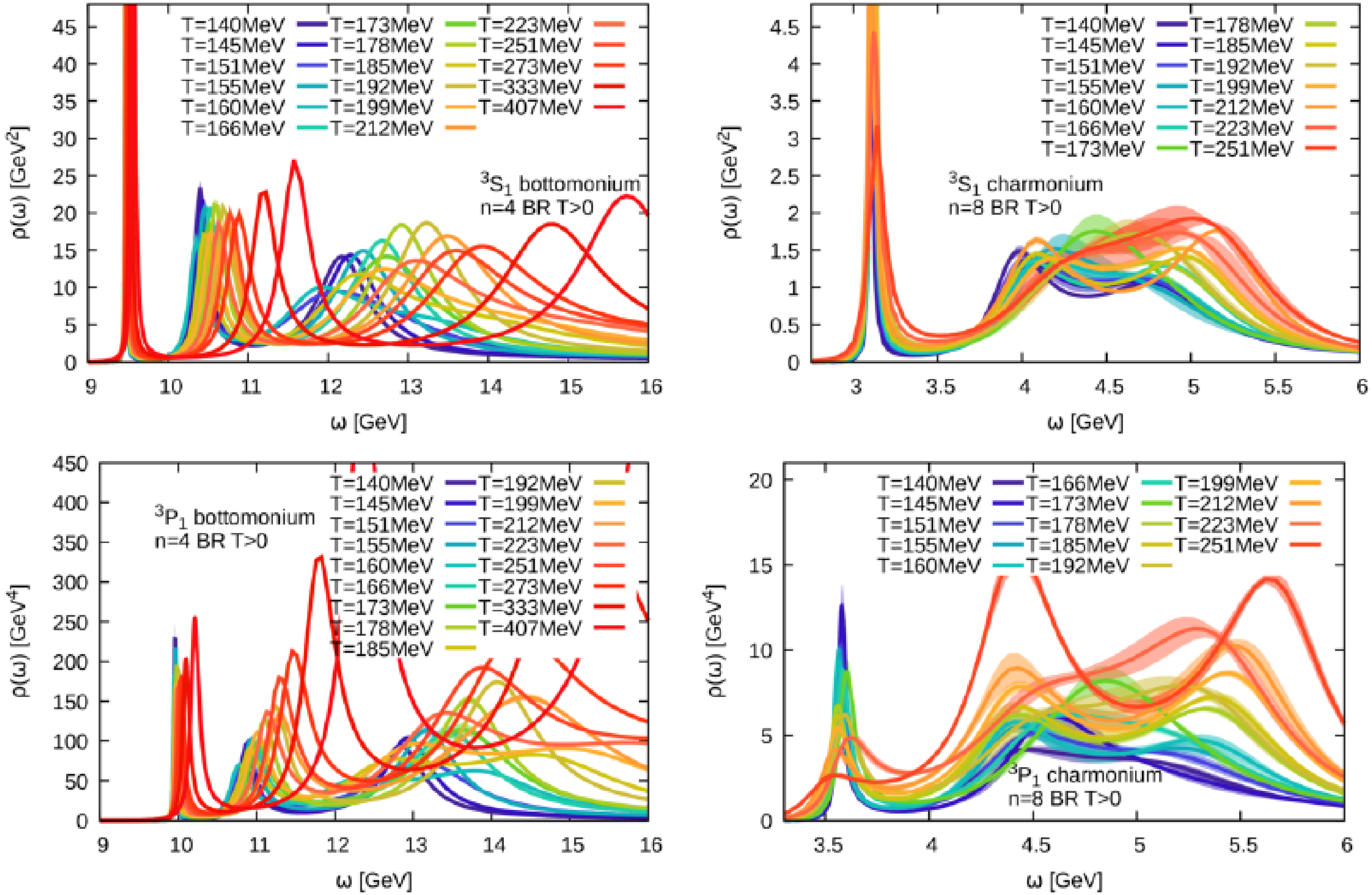


Answer may vary depending on how hot is the QGP, and how long you observe

- Not to high T, not too long : Same as in vacuum (see Maxim's talk) + some external perturbation
- If not : probably better to speak a $Q\bar{Q}$ pair
- When is it legitimate to speak of a bound state ?... And deal with it as such in the transport theory. Answer may vary depending on the fundamental ingredients

lQCD perspective : spectral function

Kim et al, JHEP11(2018)088



Many such kind of results in the literature

Rich structure : broadening and mass shift. What are the underlying “ingredients’” ?

The present challenges for Quarkonium modelling in URHIC

Meet the higher and higher precision of experimental data (already beyond the present model uncertainties)

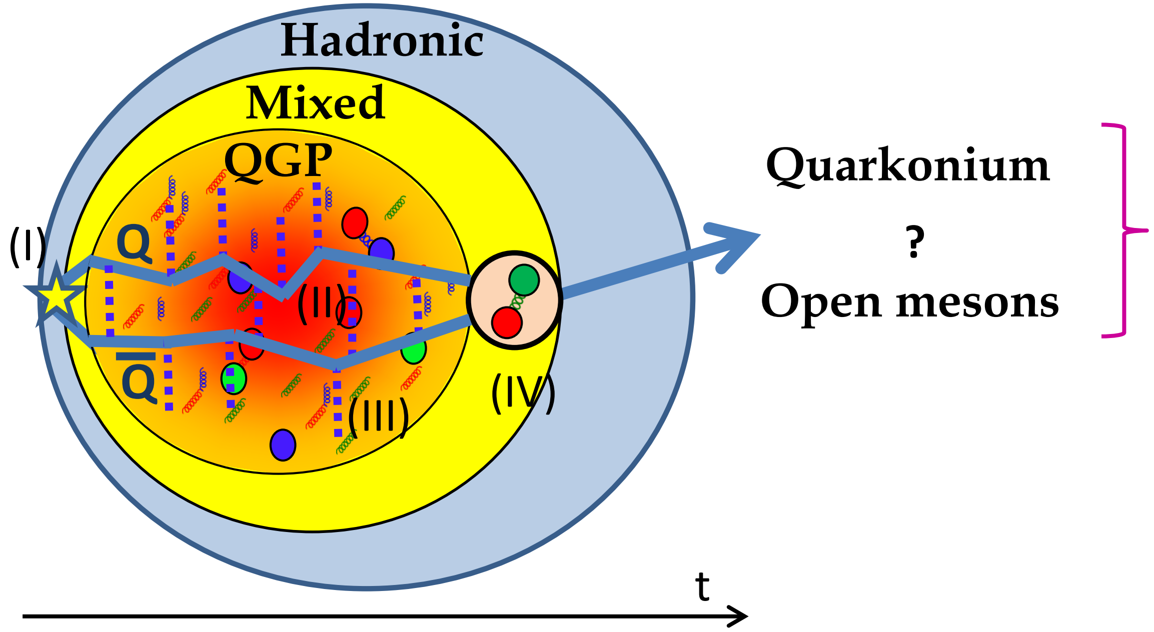
Unravel the Q-Qbar interactions under the influence of the surrounding QGP and with the QGP

Develop a scheme able to deal with the evolution of one (or many) $Q\bar{Q}$ pair(s) in a QGP, fulfilling all fundamental principles (quantum features, gauge invariance, equilibration,…)

Need for lQCD constraints / inputs

Ultimately, go beyond the "one team – one model" paradigm

The desired full-quantum scheme



Strictly speaking, only resolved at the end of the evolution

Beware of quantum coherences during the whole evolution !

Especially at early time…

In practice, what counts is the so-called decoherence time, not the “Heisenberg time”

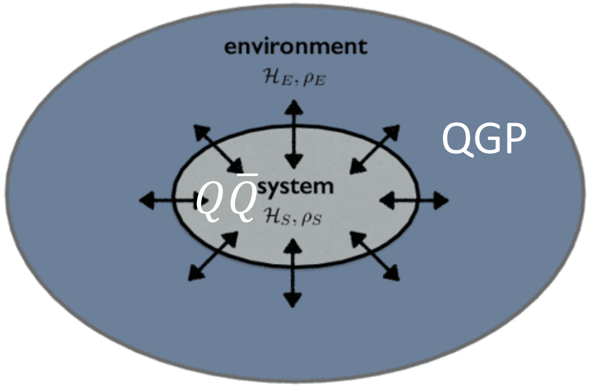
Complicated QFT problem (also due to the evolving nature of the QGP that mixes several scales)… only started to be addressed at face value recently

How to proceed ?

1) Initial state
2) (Screened) interaction between both HQ
3) Interactions with surrounding QGP partons
4) Projection on the final quarkonia

First incomplete QM treatments dating back to Blaizot & Ollitrault, Thews, Cugnon and Gossiaux; early 90’s

Open Quantum Systems & Quantum Master Equations



Quite generally, the system ($Q\bar{Q}$) builds correlation with the environment thanks to the Hamiltonian

$$\hat{H} = \hat{H}_S^{(0)} + \hat{H}_E + \hat{H}_{\rm int}$$

Von Neumann equation for the total density operator $\hat{\rho}$

System + environment

$$\hat{\rho}(t=0) = \hat{\rho}_S(t=0) \otimes \hat{\rho}_E$$

$$\frac{d\hat{\rho}}{dt} = -i[\hat{H}, \hat{\rho}] \longrightarrow$$

Evolution of the total system

$$\hat{\rho}(t) = \hat{U}(t,0)\,[\hat{\rho}_S(t=0) \otimes \hat{\rho}_E]\,\hat{U}^\dagger(t,0)$$

The environment acts as an "observer" of the system ⇔ $H_{\rm int}$ builds entanglement (see talk from Ian Low):

$$\frac{1}{\sqrt{2}}(|\psi_1\rangle + |\psi_2\rangle) \otimes |E_0\rangle \to \frac{1}{\sqrt{2}}(|\psi_1\rangle \otimes |E_1\rangle + |\psi_2\rangle \otimes |E_2\rangle)$$

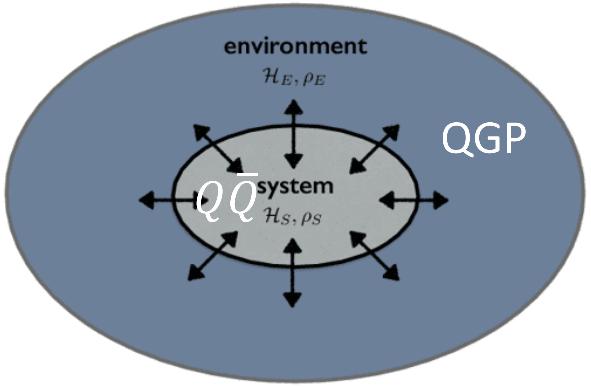
But we do not observe / consider the environment => trace the density matrix on the env. dof. :

<u>t=0</u> : $\hat{\rho}_S = \frac{\langle E_0|E_0\rangle}{2}(|\psi_1\rangle\langle\psi_1| + |\psi_2\rangle\langle\psi_2| + |\psi_1\rangle\langle\psi_2| + |\psi_2\rangle\langle\psi_1|)$ $\quad \hat{\rho}_S = \frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$ pure state

<u>Finite t</u> : $\hat{\rho}_S^{\rm red} = \frac{1}{2}(|\psi_1\rangle\langle\psi_1| + |\psi_2\rangle\langle\psi_2| + \langle E_2|E_1\rangle|\psi_1\rangle\langle\psi_2| + \langle E_1|E_2\rangle|\psi_2\rangle\langle\psi_1|)$

In the so-called "**preferred basis**" : $\langle E_1|E_2\rangle \propto e^{-\frac{t}{t_d}}$ (env. acquires increasing information about the system and realizes it corresponds to 2 different states)

Open Quantum Systems & Quantum Master Equations



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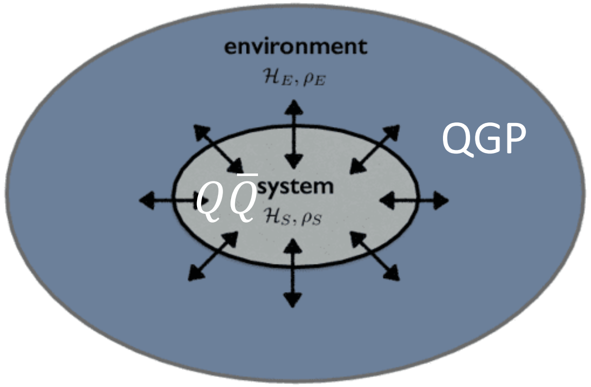
⇓

$\underline{t \gg t_d}$: $\hat{\rho}_S^{\rm red} \approx \frac{1}{2}(|\psi_1\rangle\langle\psi_1| + |\psi_2\rangle\langle\psi_2|)$ $\quad \hat{\rho}_S^{\rm red} \approx \frac{1}{2}\begin{pmatrix} 1 & \approx 0 \\ \approx 0 & 1 \end{pmatrix}$ Appears as quasi classical mixed state

Important to consider quantum coherence in the early time of the evolution and also to work in the preferred basis!

Open Quantum Systems & Quantum Master Equations



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Evolution of the total system

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Trace out environment degrees of freedom =>
Reduced density operator $\hat{\rho}_S$

System

$$\hat{\rho}_S^{\text{red}}(t=0) = \hat{\rho}_S(t=0)$$

Evolution of the system

$$\hat{\rho}_S^{\text{red}}(t) = \text{tr}_E\left[\hat{U}(t,0)\,[\hat{\rho}_S(t=0) \otimes \hat{\rho}_E]\,\hat{U}^\dagger(t,0)\right]$$

Evolution of the reduced density operator: $\frac{\mathrm{d}\hat{\rho}_S^{\text{red}}}{\mathrm{d}t} = \mathcal{L}[\hat{\rho}_S^{\text{red}}]$ (linear mapping)

However, $\mathcal{L}[\cdot]$ is generically a non local superoperator in time

Good, formally, but very involved to solve

Inspired from Yao Int. J. of Mod. Phys. A, Vol. 36, No. 20, 2130010 (2021)

Pictorial summary

τ_E: environment autocorrelation time

$$\tau_E \approx \frac{1}{m_D} = \frac{1}{CT}$$

τ_S: subsystem ($Q\bar{Q}$) intrinsic time scale

τ_R: subsystem ($Q\bar{Q}$) relaxation time

Subsystem + environment: von Neumann equation

↓ Trace out environment

Subsystem: non-unitary, time-irreversible evolution

↓ Weak syst-environment coupling + Markovian limit $\tau_E \ll \tau_R$

Redfield equation

Smallest time scales wins it all !

$\tau_S \ll \tau_R$ — Quantum Optical Regime → **Lindblad equation**

$\tau_E \ll \tau_S$ — Quantum Brownian Motion → **Lindblad equation** (conserving, hermitic, positive-defined)

Not the same pref. basis !

Pref. Basis: Eigenstates of the HQ Hamiltonian

Pref. Basis: Phase space densities

↓ Wigner transform + gradient expansion

Boltzmann equation — Rate equations: ⇔ transport models

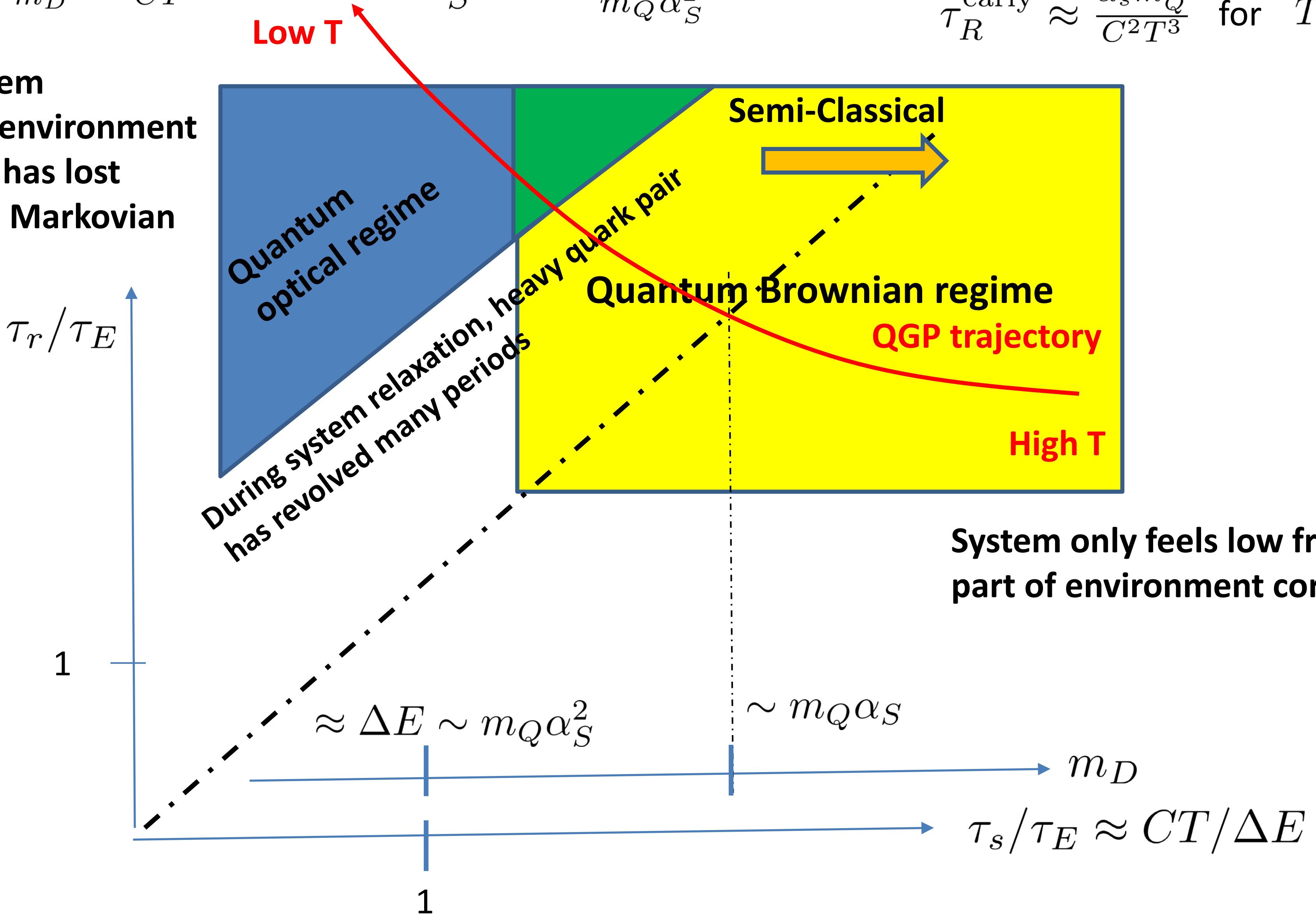
Fokker-Planck equation — Good method for many $c\bar{c}$ pairs

Semi-classical approx : density matrix ≈ diagonal

QCD time scales

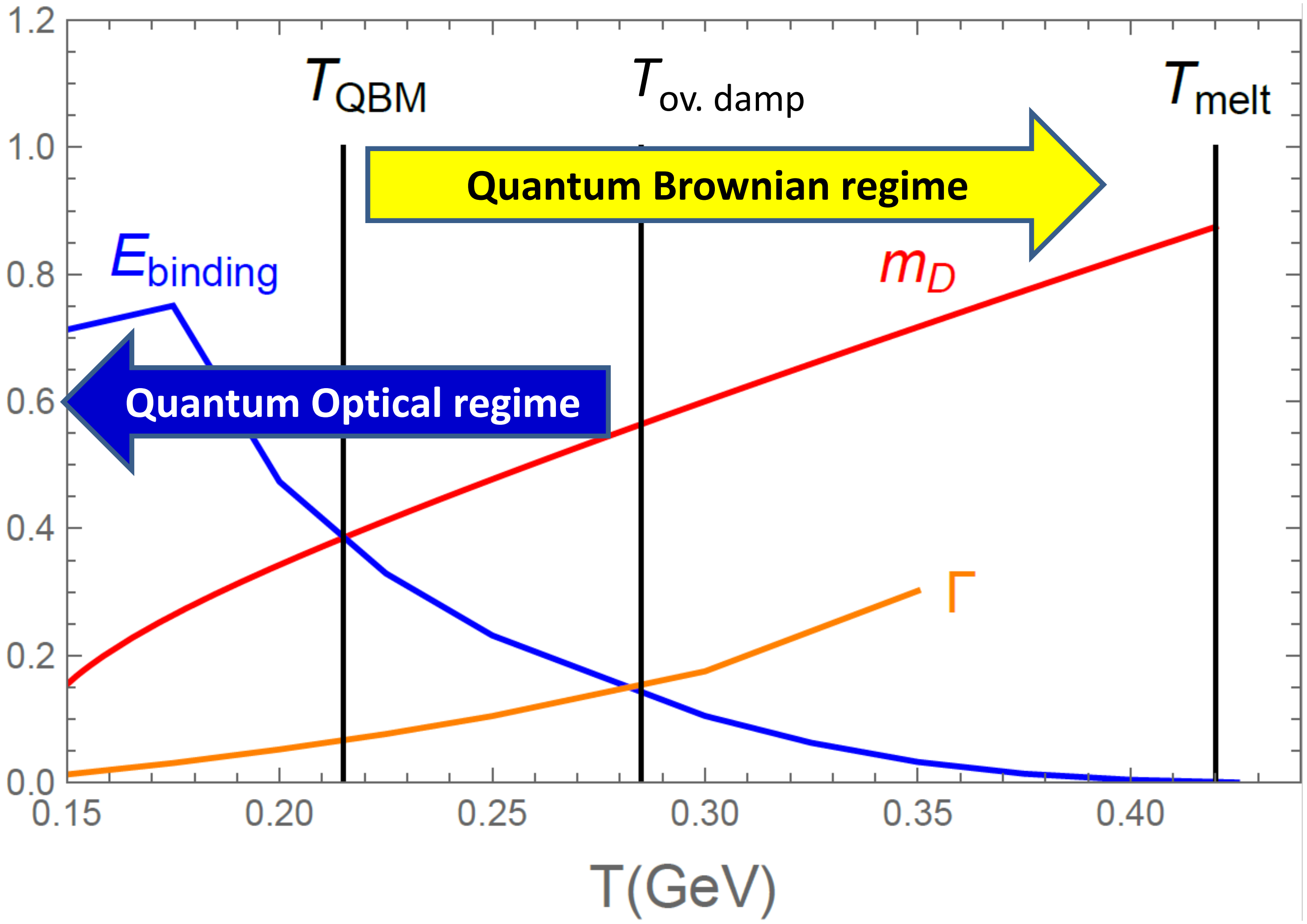
$\tau_E \approx \frac{1}{m_D} = \frac{1}{CT}$ $\tau_S^{\text{early}} \approx \frac{1}{m_Q \alpha_S^2}$ $\tau_R^{\text{early}} \approx \frac{\alpha_s m_Q^2}{C^2 T^3}$ for $T \lesssim \frac{m_Q \alpha_S}{C}$

During system relaxation, environment correlation has lost memory => Markovian process



System only feels low frequency part of environment correlation

Two types of dynamical modelling



$c\bar{c}$ pair

Numbers extracted from a specific potential model : Katz et al, Phys. Rev. D 101, 056010 (2020)

Quantum Brownian Motion : The Blaizot-Escobedo QME

JP Blaizot & MA Escobedo, JHEP06(2018)034

Coulomb gauge, $H_{\rm int} = -g\int_r A_0^a(\mathbf{r})n^a(\mathbf{r})$ (NR, weak coupling)

Compact form: $\frac{\mathrm{d}\mathcal{D}_Q}{\mathrm{d}t} = \mathcal{L}\mathcal{D}_Q$ with $\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_1 + \mathcal{L}_2 + \mathcal{L}_3 + \cdots$ Series expansion in τ_E/τ_S

$$\mathcal{L}_0\,\mathcal{D}_Q \equiv -i[H_Q, \mathcal{D}_Q],$$

$$\mathcal{L}_1\,\mathcal{D}_Q \equiv -\frac{i}{2}\int_{\boldsymbol{x}\boldsymbol{x}'} V(\boldsymbol{x}-\boldsymbol{x}')[n^a_{\boldsymbol{x}}n^a_{\boldsymbol{x}'}, \mathcal{D}_Q],$$

Mean field hamiltonian (unitary evolution)

$$\mathcal{L}_2\,\mathcal{D}_Q \equiv \frac{1}{2}\int_{\boldsymbol{x}\boldsymbol{x}'} W(\boldsymbol{x}-\boldsymbol{x}')\left(\{n^a_{\boldsymbol{x}}n^a_{\boldsymbol{x}'}, \mathcal{D}_Q\} - 2n^a_{\boldsymbol{x}}\mathcal{D}_Q n^a_{\boldsymbol{x}'}\right),$$

Fluctuations, Linblad form

$$\mathcal{L}_3\,\mathcal{D}_Q \equiv \frac{i}{4T}\int_{\boldsymbol{x}\boldsymbol{x}'} W(\boldsymbol{x}-\boldsymbol{x}')\left([n^a_{\boldsymbol{x}}, \dot{n}^a_{\boldsymbol{x}'}\mathcal{D}_Q] + [n^a_{\boldsymbol{x}}, \mathcal{D}_Q\dot{n}^a_{\boldsymbol{x}'}]\right)$$

Dissipation

External "ingredients" : complex potential V + i W

N.B. : Friction is NOT of the Linbladian form => the evolution breaks positivity.

Positivity and Linblad form can be restored at the price **of extra subleading terms** :

$$\left\{\left(n^a_{\mathbf{x}} - \frac{i}{4T}\dot{n}^a_{\mathbf{x}}\right)\left(n^a_{\mathbf{x}'} + \frac{i}{4T}\dot{n}^a_{\mathbf{x}'}\right), \mathcal{D}_{Q\bar{Q}}\right\} - 2\left(n^a_{\mathbf{x}} + \frac{i}{4T}\dot{n}^a_{\mathbf{x}}\right)\mathcal{D}_{Q\bar{Q}}\left(n^a_{\mathbf{x}'} - \frac{i}{4T}\dot{n}^a_{\mathbf{x}'}\right)$$

$\mathcal{L}_4$

Application to QED-like and QCD for both cases of 1 body and 2 body densities…, **SC approximation**

The Blaizot-Escobedo QME for a single $Q\bar{Q}$ pair

2 coupled color representations (singlet & octet)

Unitary $\mathcal{L}_0 + \mathcal{L}_1$ and "loss terms" ($\mathcal{L}_2 + \mathcal{L}_3 + \mathcal{L}_4$) for the singlet

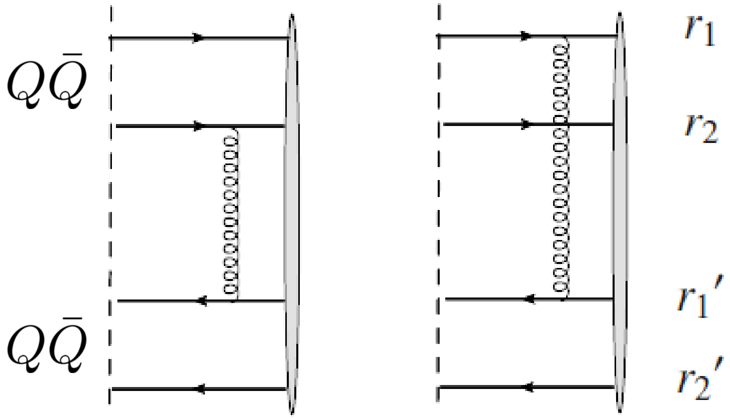
octet -> singlet transition from non unitary $\mathcal{L}_2 + \mathcal{L}_3 + \mathcal{L}_4$

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix} \mathcal{D}_\mathrm{s} \\ \mathcal{D}_\mathrm{o} \end{pmatrix} = \underbrace{\begin{pmatrix} \mathcal{L}^\mathrm{ss} & \mathcal{L}^\mathrm{so} \\ \mathcal{L}^\mathrm{os} & \mathcal{L}^\mathrm{oo} \end{pmatrix}}_{\mathcal{L}} \begin{pmatrix} \mathcal{D}_\mathrm{s} \\ \mathcal{D}_\mathrm{o} \end{pmatrix}$$

singlet -> octet transition from non unitary op.

Unitary $\mathcal{L}_0 + \mathcal{L}_1$ and "loss terms" ($\mathcal{L}_2 + \mathcal{L}_3 + \mathcal{L}_4$) for the singlet

Contributions to the Schwinger-Keldysh contour



- Scattering from gluons change the color representation : o <-> s

$$\mathcal{D}_{Q\bar{Q}} = \begin{pmatrix} \mathcal{D}_\mathrm{s} \\ \mathcal{D}_\mathrm{o} \end{pmatrix}$$

- No binding potential in the octet chanel => « large » energy gap

Our ongoing projects

Our Goals:

- Gain insight on the quarkonium dynamics inside the QGP by **solving exactly the B-E equations** for a single $Q\bar{Q}$ pair (without performing the semi-slassical approximation):
  - Evolution of the density matrix
  - Evolution of states probabilities over time
  - Singlet-octet transitions
  - Color relaxation time
  - …
- Understand the asymptotic limit of the QME
- Comparison with the semi-classical approach for a various range of QGP temperatures (should be fine at large temperature… but down to ?)
- Possibly design improved algorithm for intermediate temperatures

$\Rightarrow$ **Restrict to 1D for computer memory issues**

$\Rightarrow$ **Restrict to relative coordinates s (cm integrated out)**

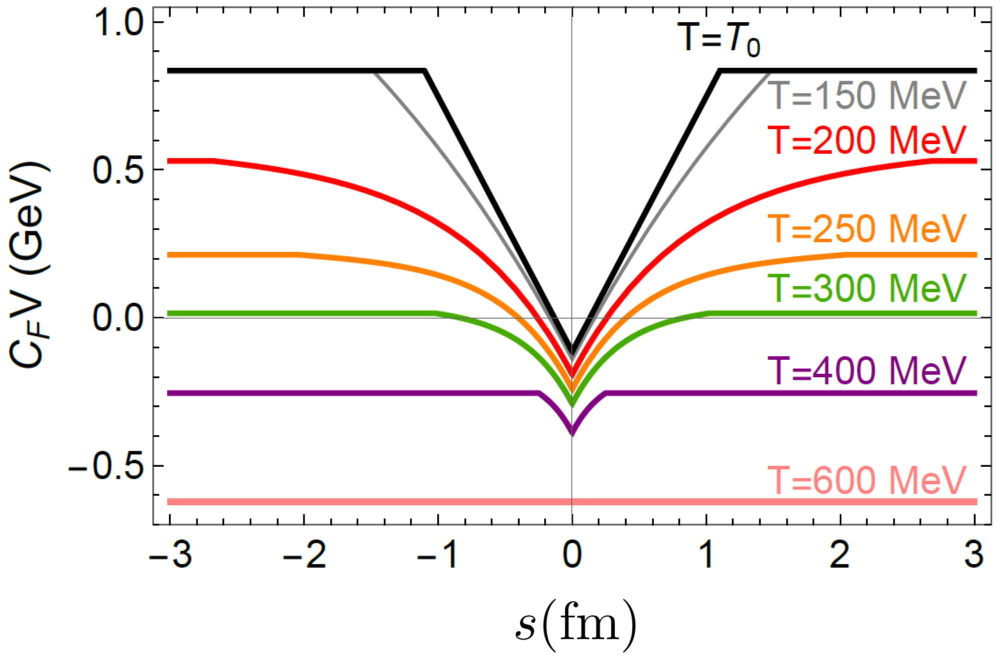
Further implementation features

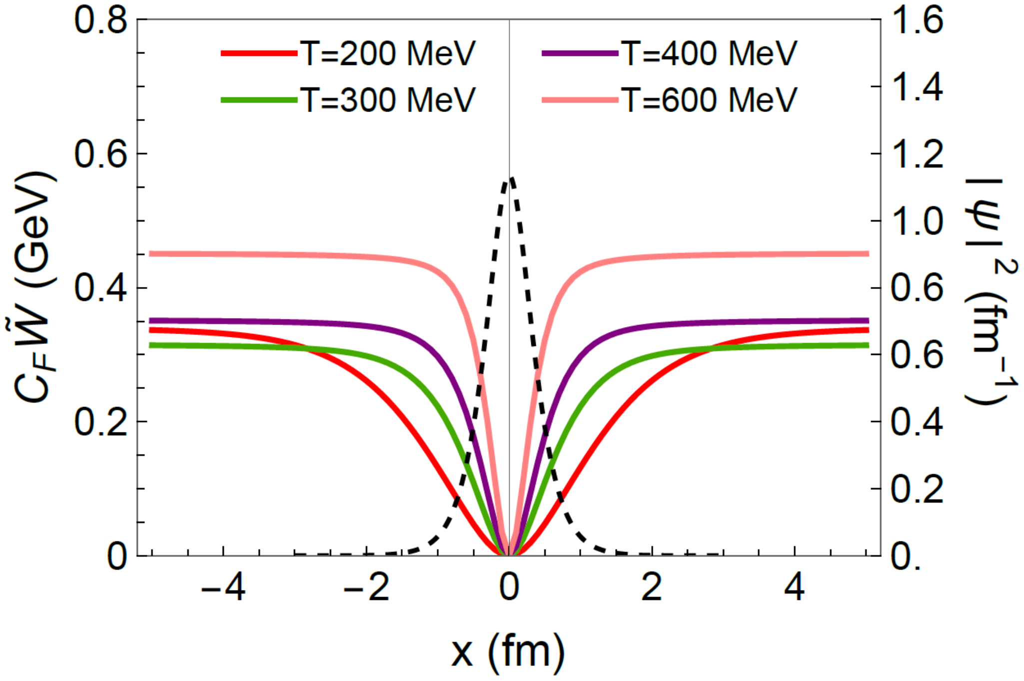
- 1D grid for both $s \in [-s_{\max}, +s_{\max}]$ and $s' \in [-s_{\max}, +s_{\max}]$

!!! Not the radial decomposition of $\mathcal{D}_{c\bar{c}}(\vec{s}, \vec{s}\,')$ which is more cumbersome

Even states will be considered as « S like » while odd states will be considered as « P like » states

Need to design a realistic 1D bona fide potential V + i W (based on 3D lQCD results, tuned to reproduce 3D mass spectra and decay widths) + proper regularization of W preserving the Linbladian structure



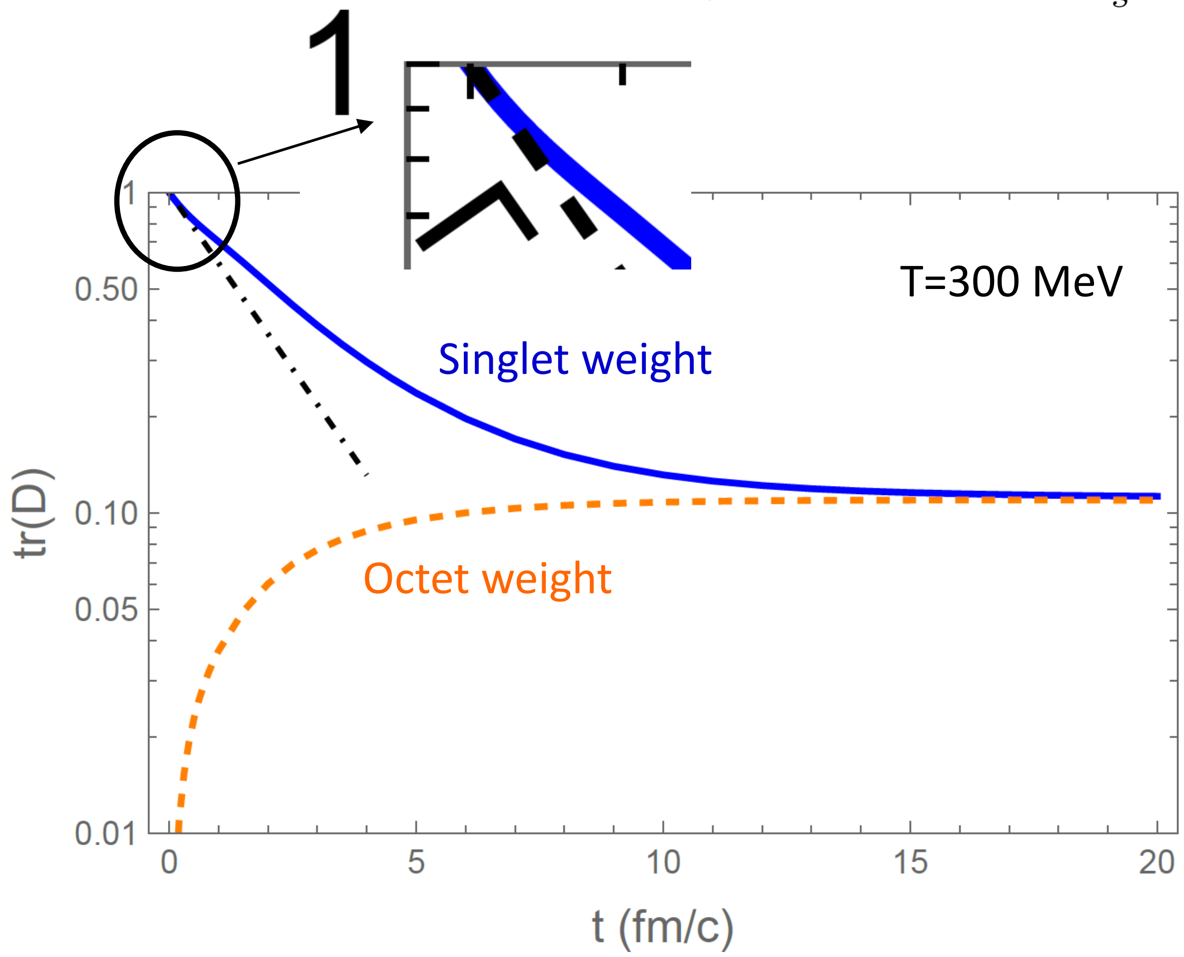


1D potential from R. Katz, S. Delorme & PBG, Eur. Phys. J. A (2022) 58:198

Some selected results for 1 c-cbar system

Color Dynamics : Singlet – octet weights:

- Starting from a singlet 1S-like, one expects some equilibration / thermalisation -> asymptotic values : $D_s^{\rm eq} = D_o^{\rm eq} = \frac{1}{9}$ $\quad 1 = (1+8) \times \frac{1}{9}$

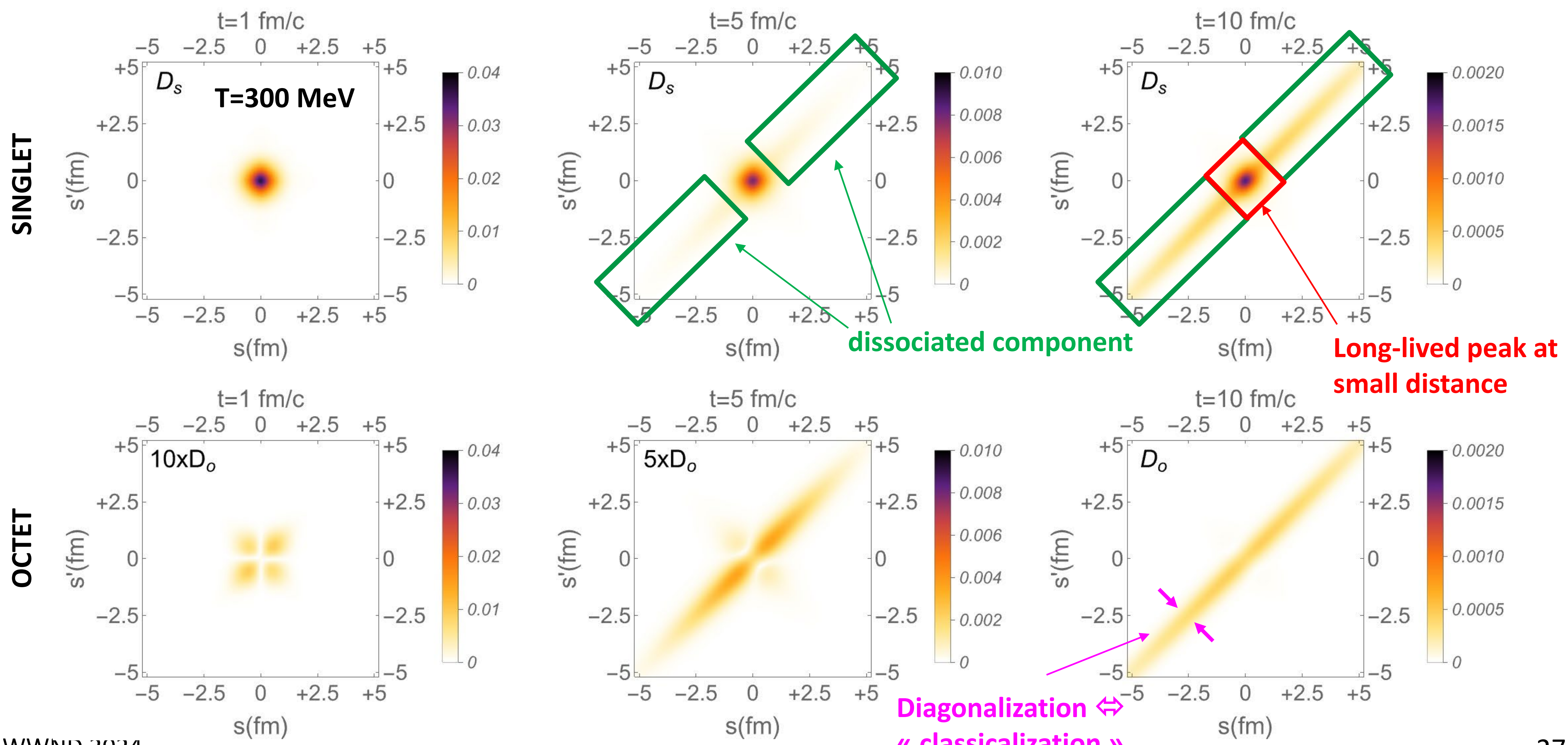


- At early times : linear decay α (1-Γ_{1S}t), as octet population corresponds to the 1S dissociation…
- But rather soon (less then 1fm/c), deviation from the exponential law. Uncommon feature of master equations, understood from the non commutativity

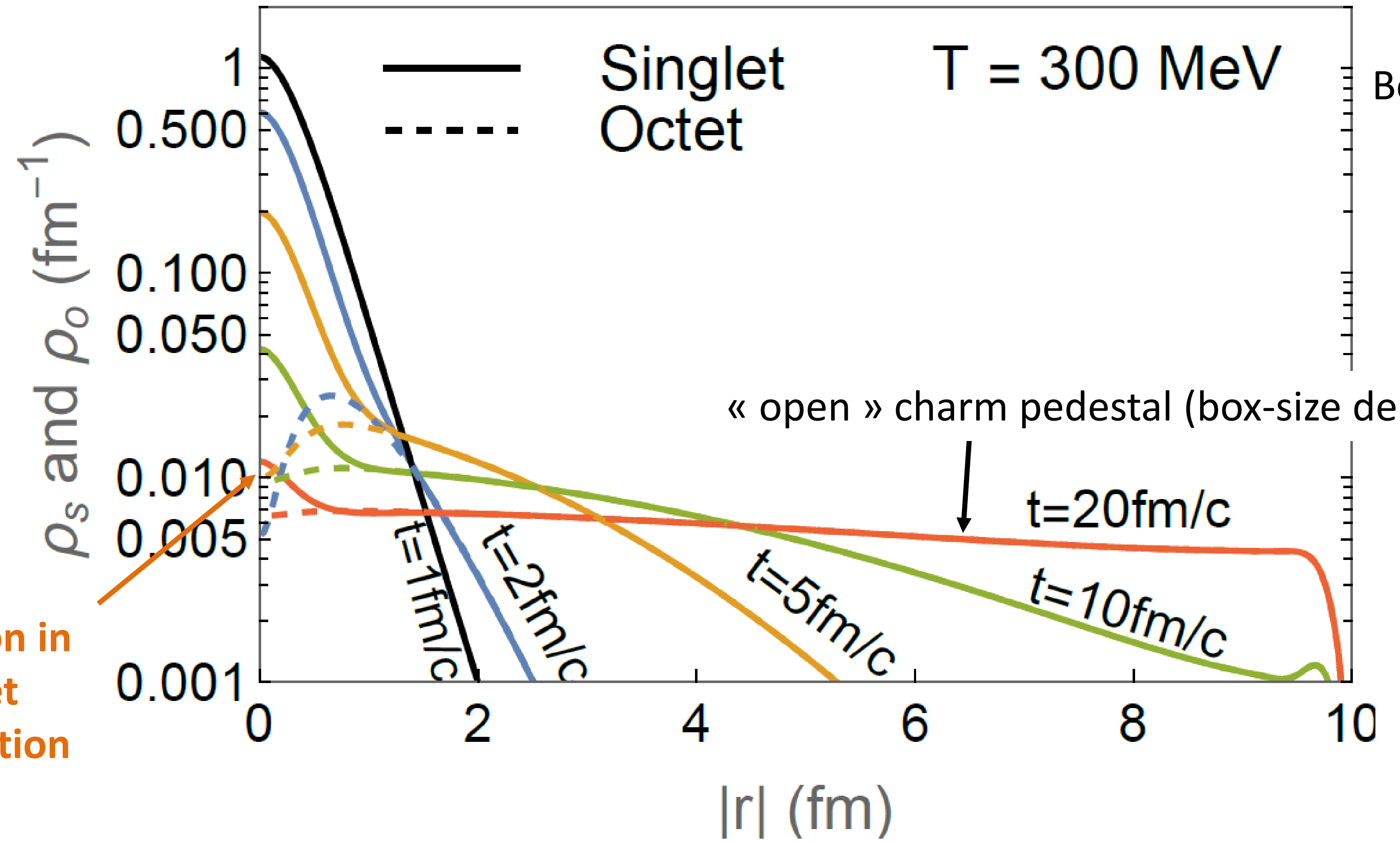
$$\int_s \exp(-\mathcal{L}^{\rm ss} t) \neq \exp(-\int_s \mathcal{L}^{\rm ss} t)$$

- Color appears to thermalize on time scales ≈ QGP life time, not instantaneoulsy !
- Phenomenological consequence: $c\bar{c}$ pairs can interact with the surrounding QGP as an octet => energy loss of the cm.

Time-evolution of the density matrix



Time-evolution of the spatial density

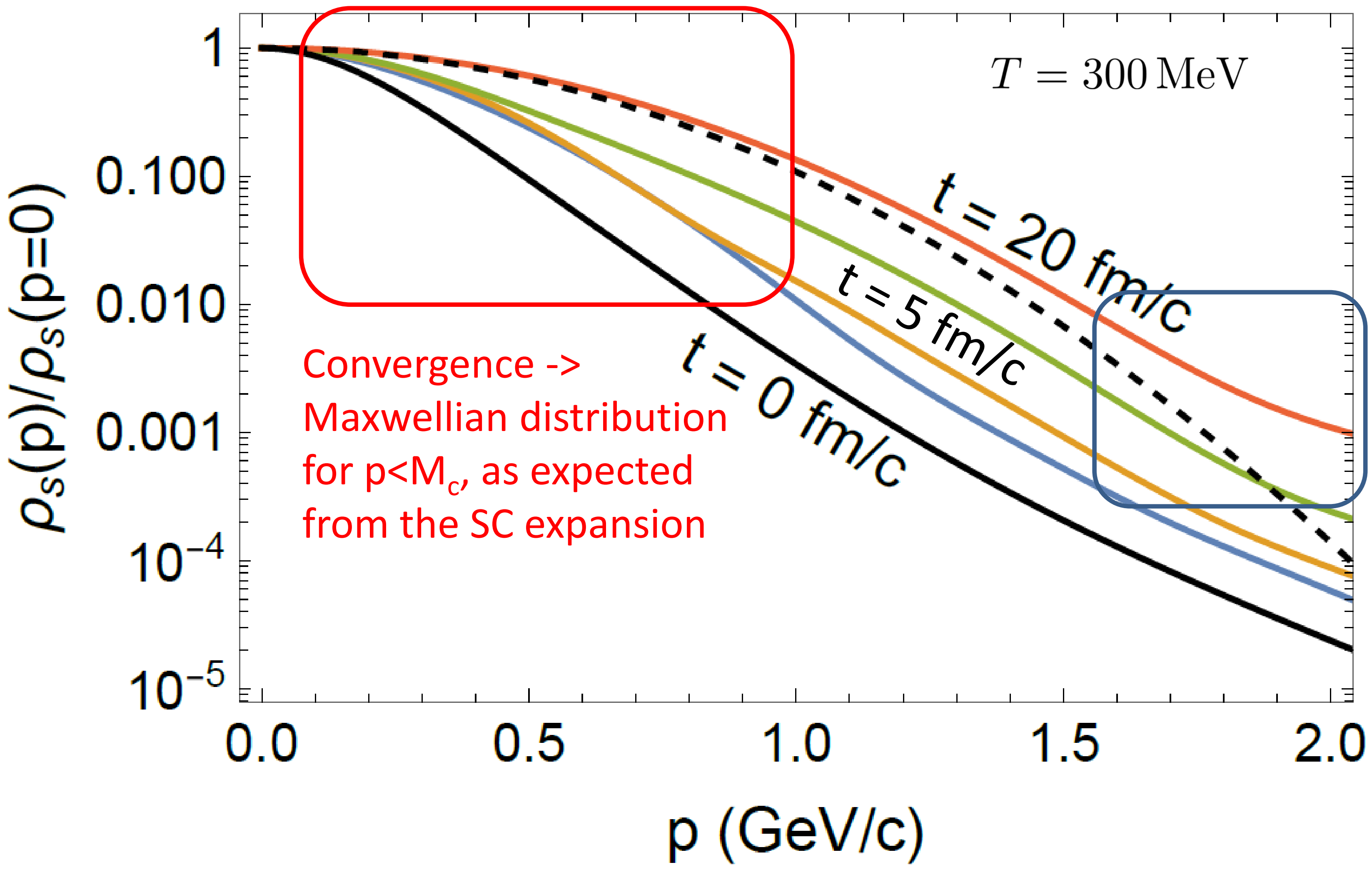


Initial state: 1S singlet

Box : $(s, s') \in [-10\text{fm}, 10\text{fm}]^2$

- Both “bound” and “dissociated” components are consistently described at the quantum level.
- Some $c\bar{c}$ stay close at intermediate distance and evolve -> attractive region (“dilute recombination”)

Time-evolution of the momentum density

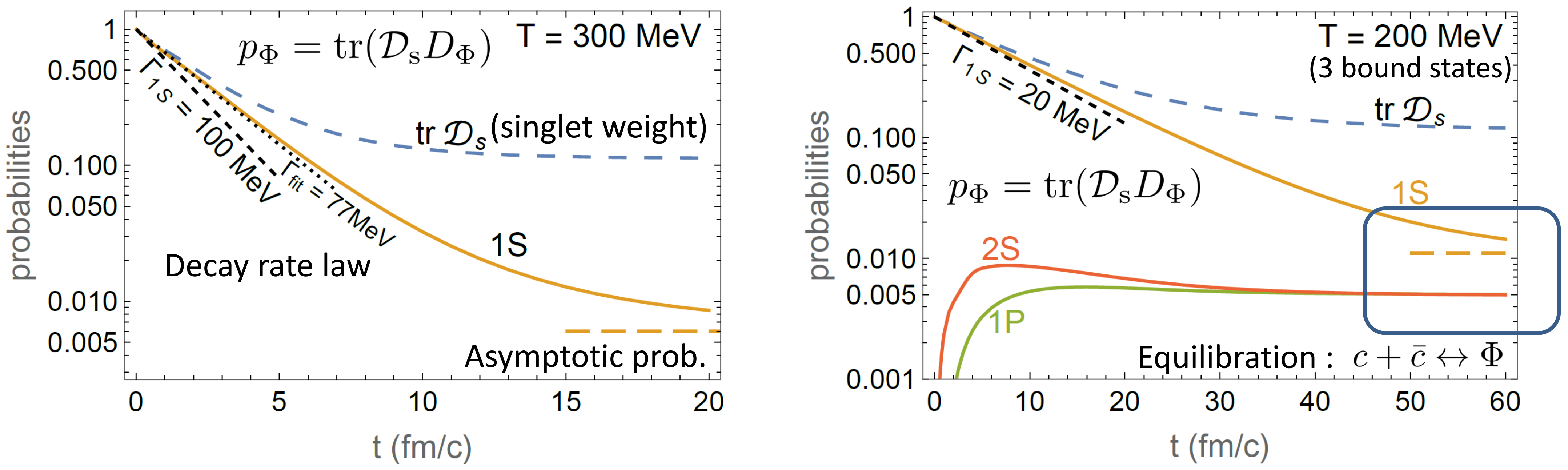


Spurious for $p>M_c$ (coming from the mandatory regularization of the imaginary potential…room for improvement) … which is not the main focus of the (NR) model

Mostly sensitive to the distribution at large relative distance (integrated on r)

Results for projection on *in-medium states*

<u>« in-medium states »</u> = eigenstates of the screened potential Hamiltonian at a given T (<> vacuum states)

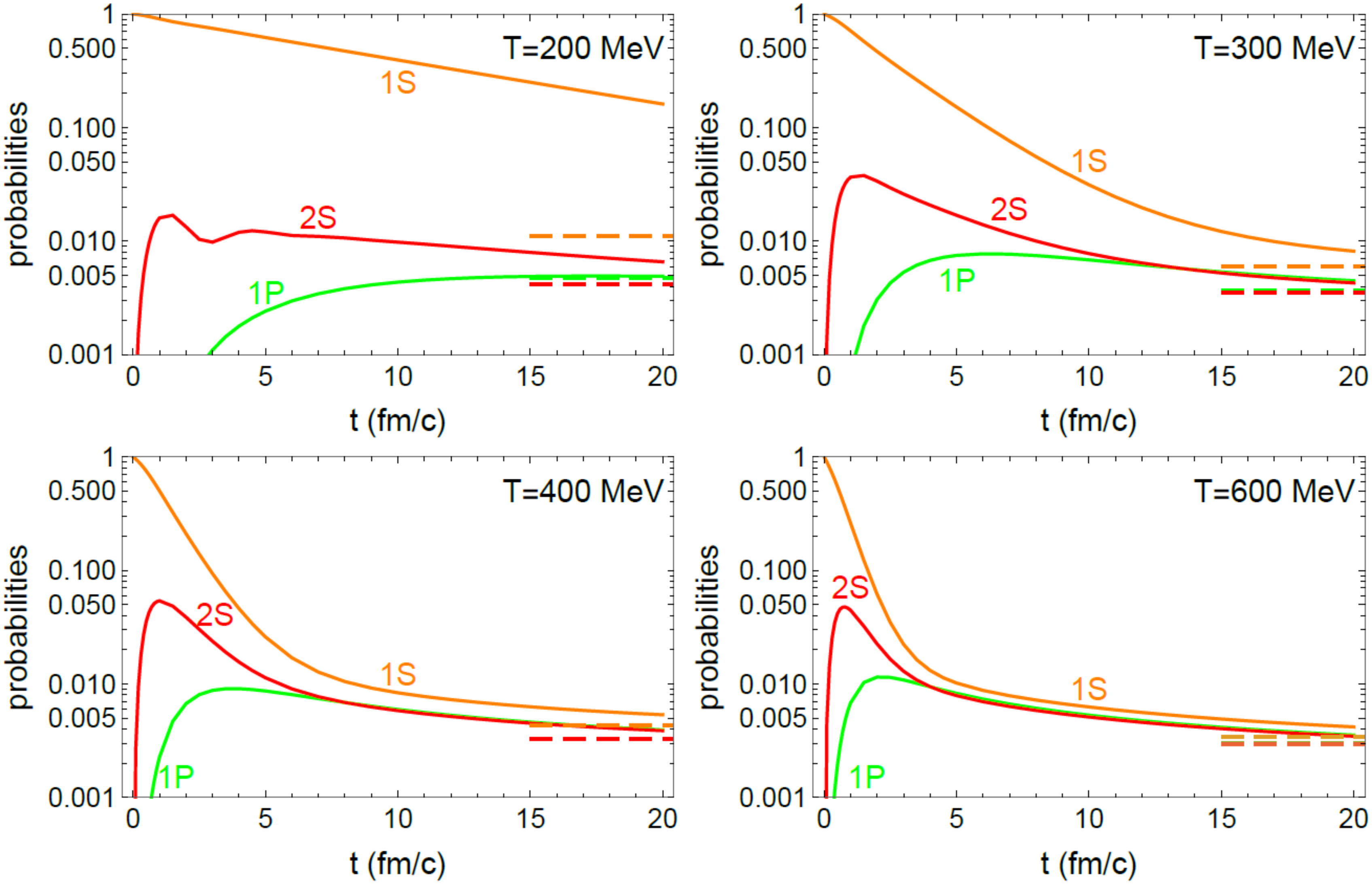


- At early times, $\mathcal{L}_3 \ll \mathcal{L}_2$: fluctuations dominate… higher states (and continuum) population
- At late times, $\mathcal{L}_3 \sim \mathcal{L}_2$ leading to asymptotic distribution of states.
- 1S evolution at intermediate time well described by exp. decay rate law, but with Γ_{fit} 20% smaller than the exact decay rate Γ_{1S} calculated from the QME (from the first derivative of p_Φ)
- 1P and 2S generated from 1S show a more complex behavior, not governed by their own decay rate !!! They quickly reach their (common) asymptotic value.

Results for projection on vacuum states

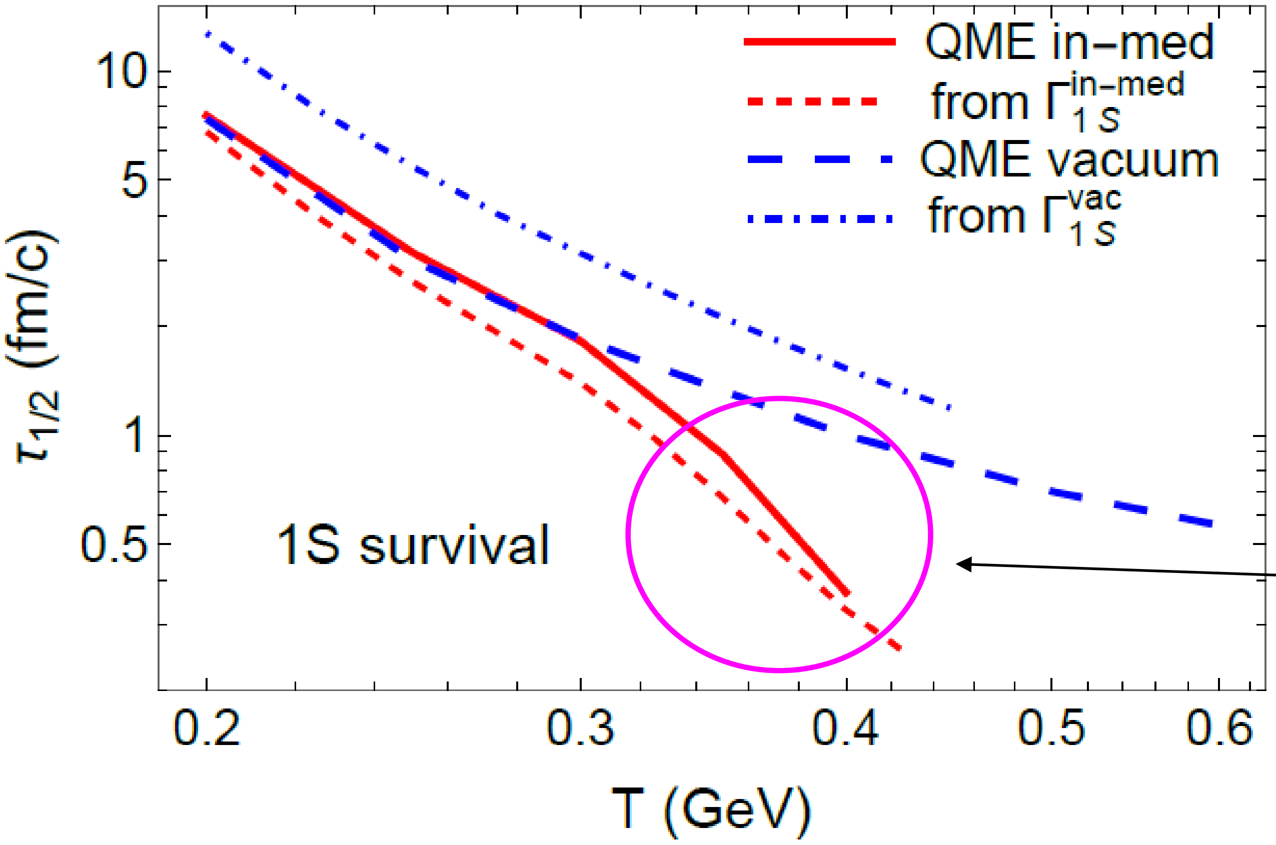
Initial State : 1S-vacuum

$$p_\Phi = \mathrm{tr}(\mathcal{D}_s D_\Phi)$$



- Natural evolution for 1S-like suppression, from low to high QGP temperature
- Excited states partly driven by the ground state at intermediate time, after some transient stage… Rise and fall of the 2S
- ⇔ **Common trend** driven by the evolution of the density matrix in coordinate space.

Half-lifes extracted from QME evolution



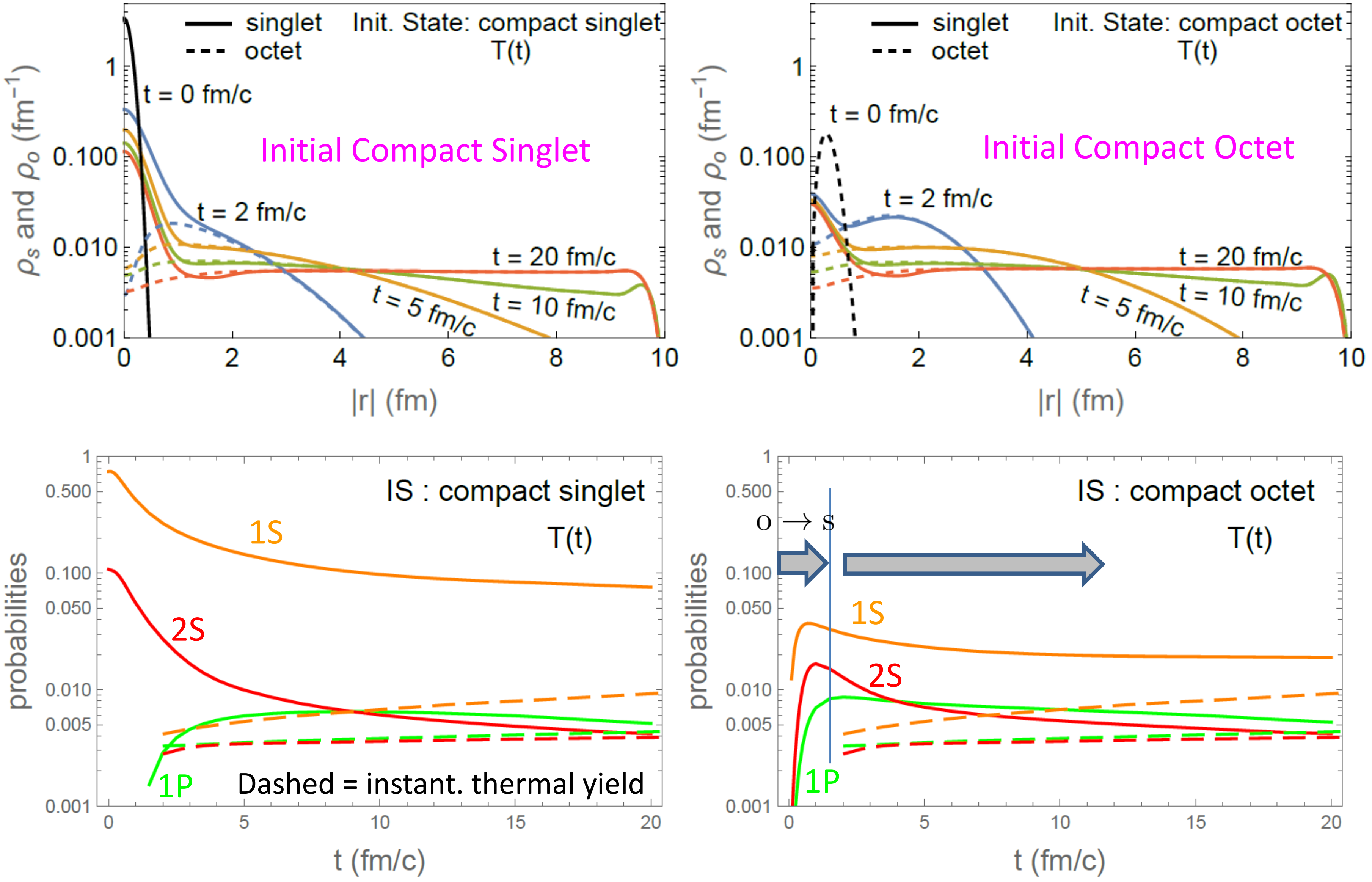
“run (escape) for your life !” zone

Interesting information for phenomenology but does not substitute to the genuine calculation

Results in a Bjorken-like T(t) starting from compact states

$$T(t) = T_0 \times \left(\frac{\tau_0}{t+\tau_0}\right)^{\frac{1}{3}}$$

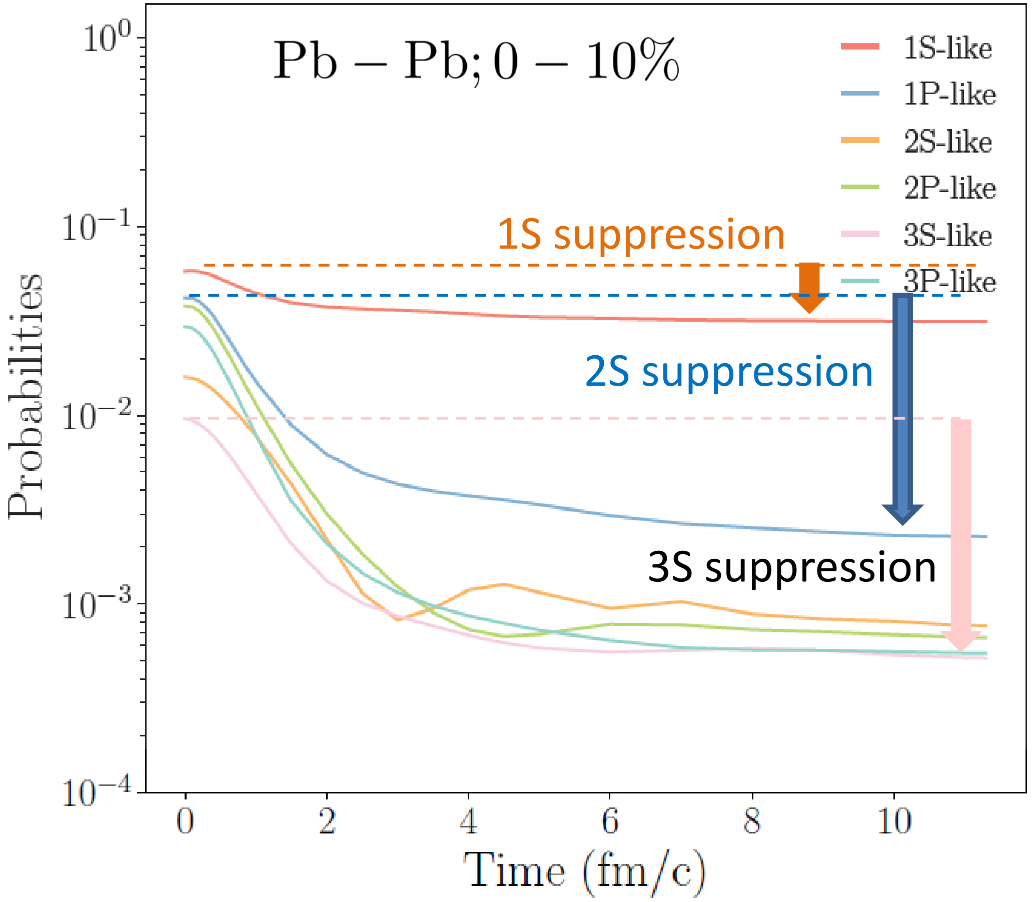
$$T_0 = 600\,\mathrm{MeV} \quad \tau_0 = 1\,\mathrm{fm}$$



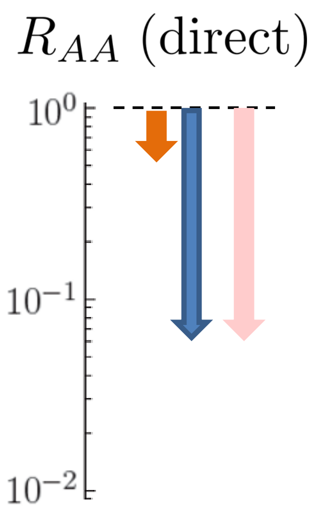
- Initial Singlet: the high-T stage does not completely destroy the correlation at small r.
- Initial octet: o->s transitions act fast to create bound states that survive the QGP cooling
- Lot of uncertainties, but pleads in favor of considering both singlet and octet initial configuration in realistic modelling.

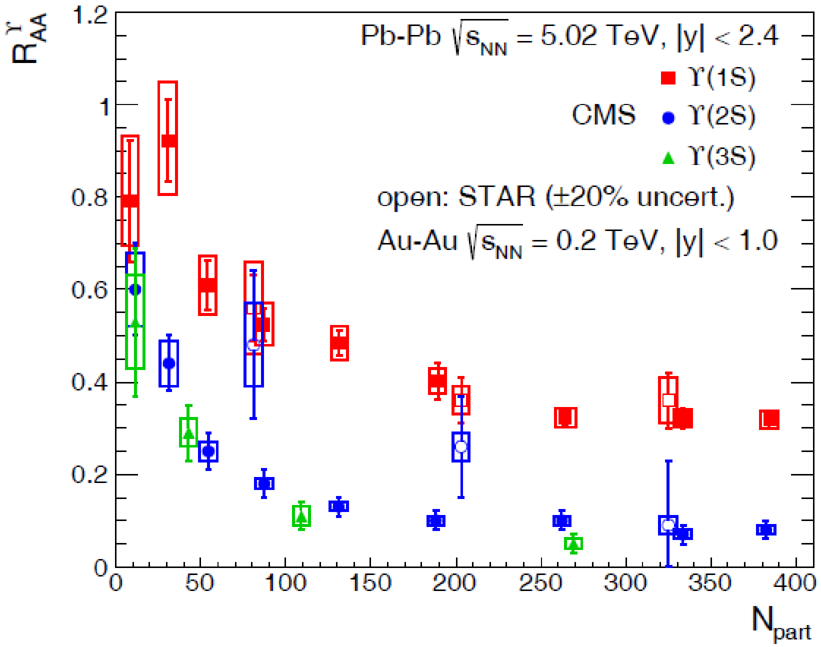
First contact with experimental data ($b\bar{b}$)

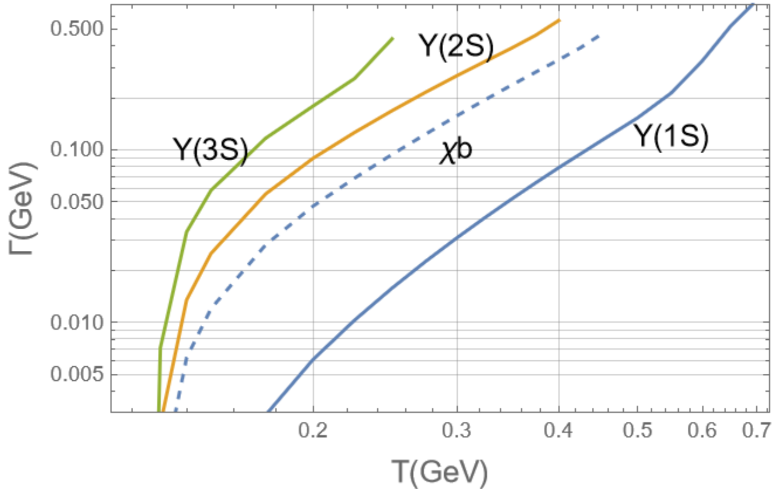
- Calculation of bottomonia yield using the QME with EPOS4 (T,v) profiles and starting from a compact $b\bar{b}$ state.



⇒

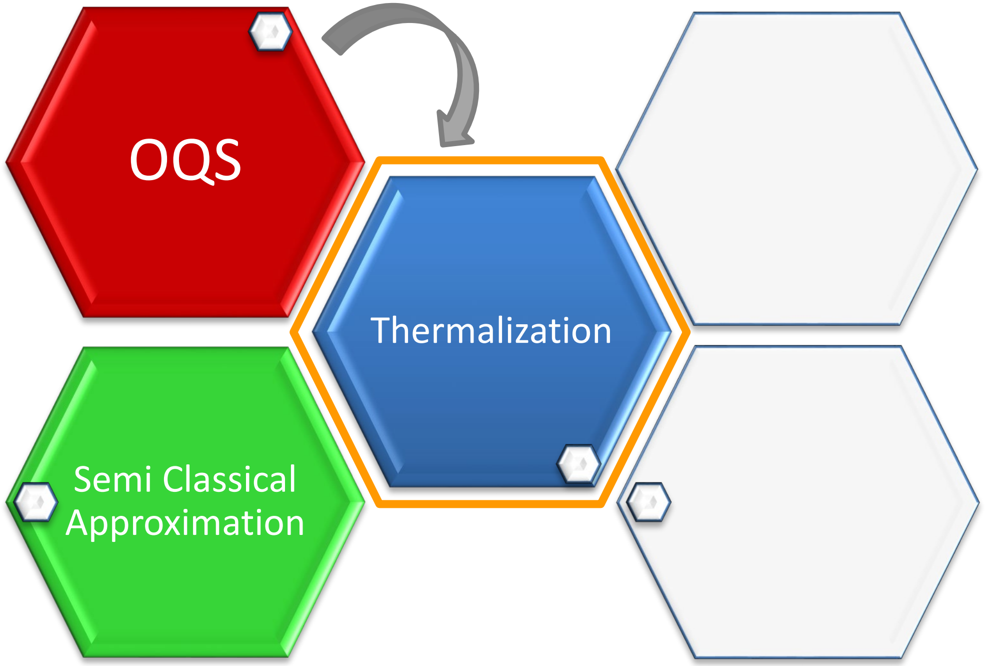






- Similar R_{AA} for Y(3S) and Y(2S) although dissoc. $\Gamma_{3S} >> \Gamma_{2S}$
- **See Stephane Delorme's talk at Hard Probe 2023 for more details.**

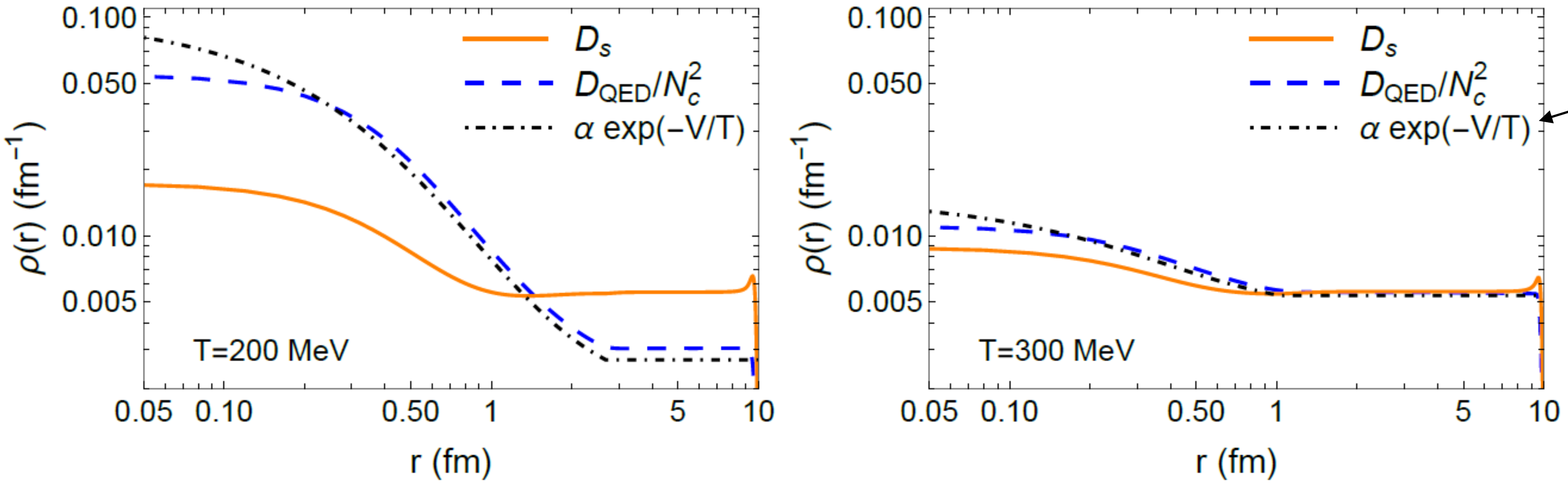
Two topics closely related to OQS



Asymptotic distributions / quarkonium states

- In the simulation, some peak survives in the density, at small relative $c\bar{c}$ distance
- This peak is in direct correlation with the charmonium weight
- Asymptotic states may not be reached in realistic URHIC, but controlling/understanding them is important :
  - Privileged link with lQCD spectral distribution (evaluated in this limit)
  - Better understanding the role of color in QCD

=> Solve $\frac{d}{dt}\mathcal{D}_s = \mathcal{L}[\mathcal{D}_s] \Rightarrow \mathcal{L}[\mathcal{D}_s^{\mathrm{asymp}}] = 0$ (also for the Abelian / QED-like case)

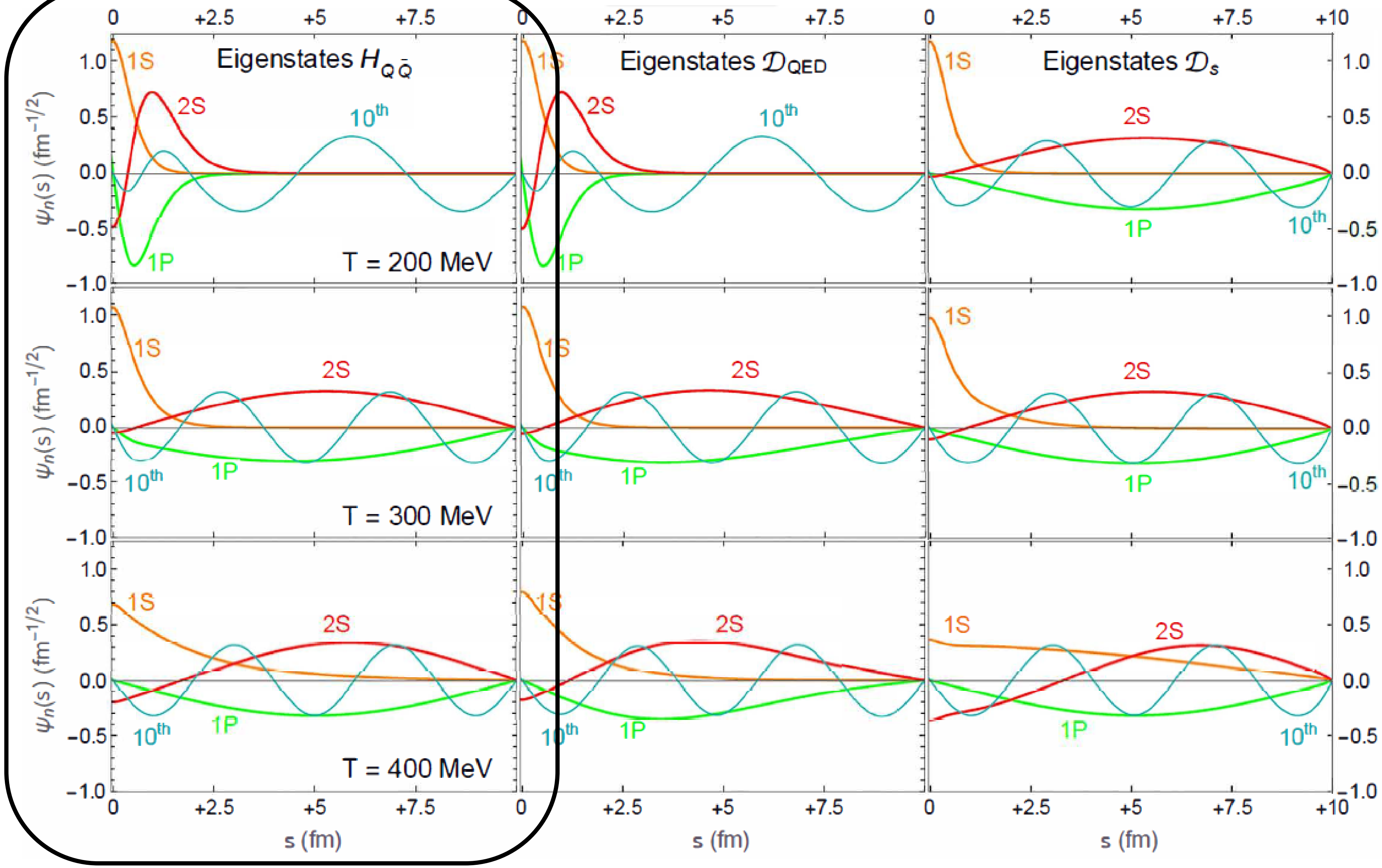


Trivial solution in the semi-classical approximation (Gibbs-Boltzmann)

QCD: Strong reduction of the peak at short relative distance as compared to the Abelian case, probably due to the absence of binding force in the octet configuration

Asymptotic distributions / quarkonium states

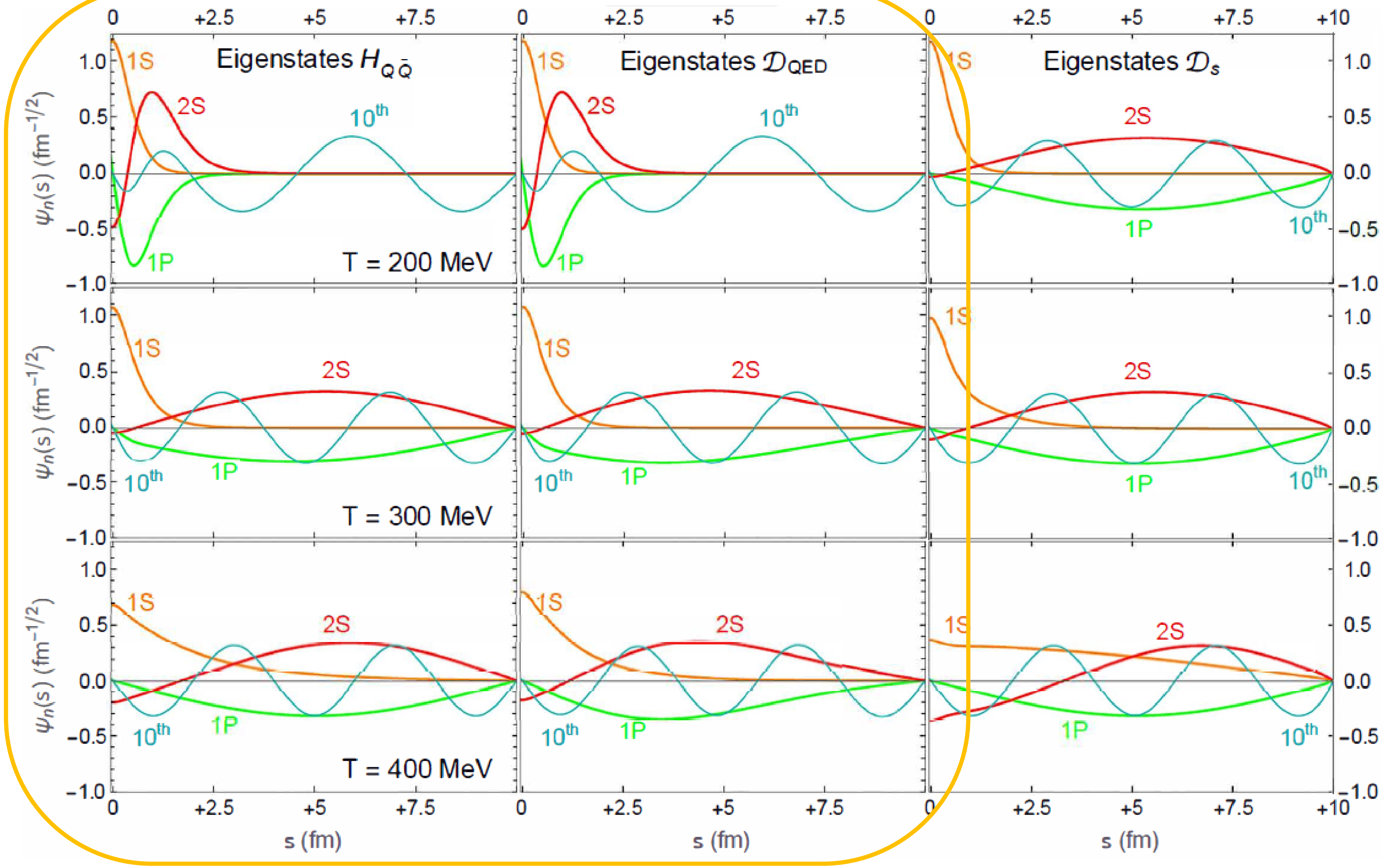
➢ What is an in-medium quarkonium ? Eigenstates of $\mathcal{D}_s^{\mathrm{asymp}}$ compared to those the screened Hamiltonian



➢ Progressive delocalization with increasing temperature (sequential suppression)

Asymptotic distributions / quarkonium states

➢ What is an in-medium quarkonium ? Eigenstates of $\mathcal{D}_s^{\mathrm{asymp}}$ compared to those the screened Hamiltonian

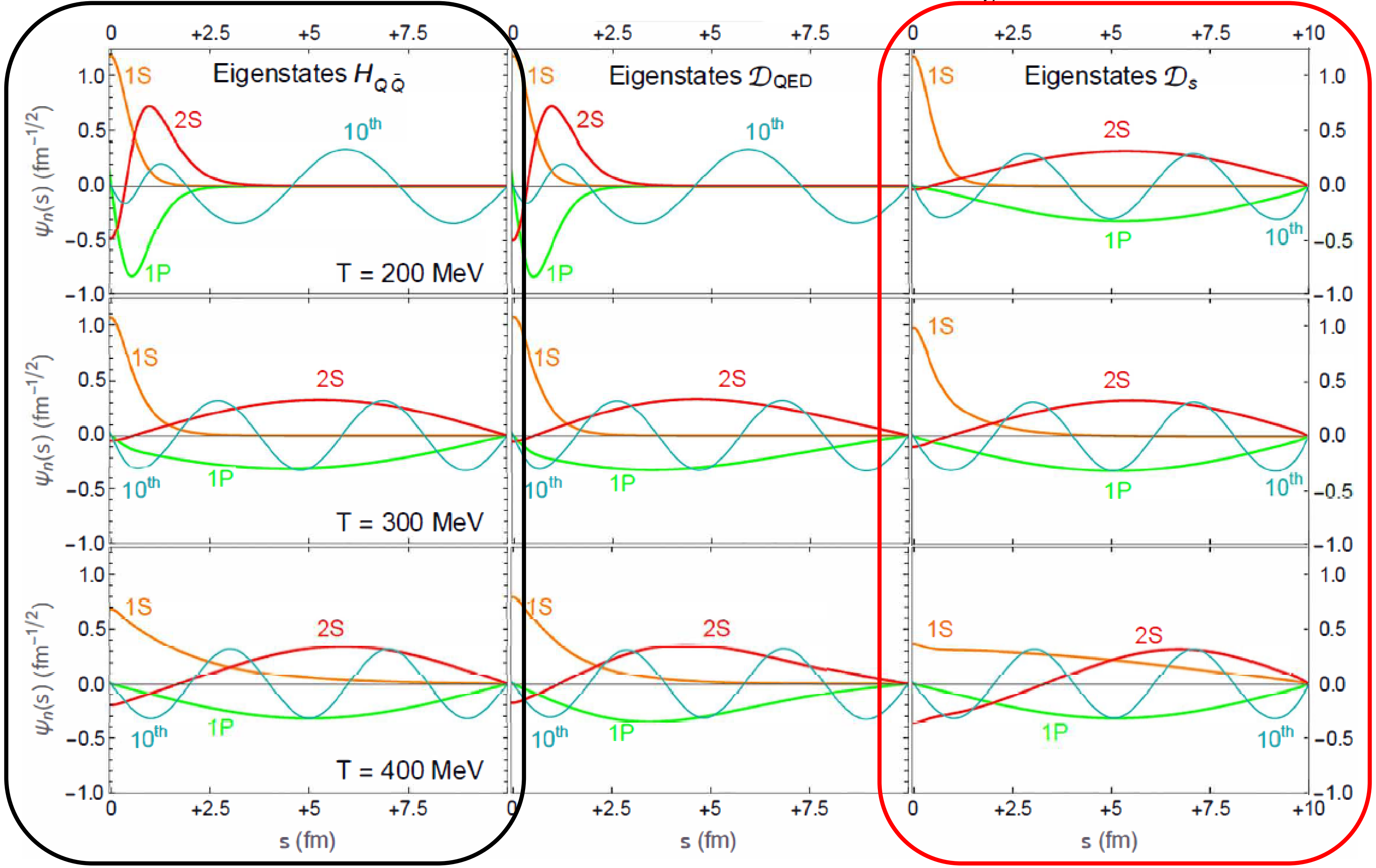


➢ Progressive delocalization with increasing temperature (sequential suppression)

➢ Eigenstates of D_{QED} correspond to those of the screened Hamiltonian

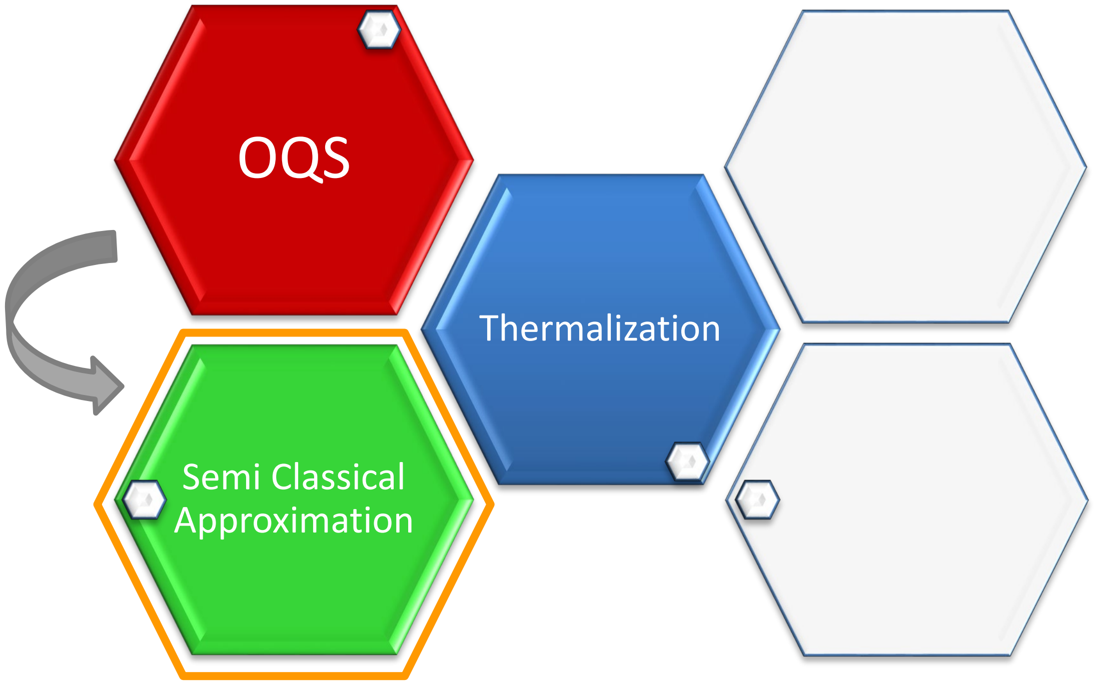
Asymptotic distributions / quarkonium states

- What is an in-medium quarkonium ? Eigenstates of $\mathcal{D}_s^{\text{asymp}}$ compared to those the screened Hamiltonian



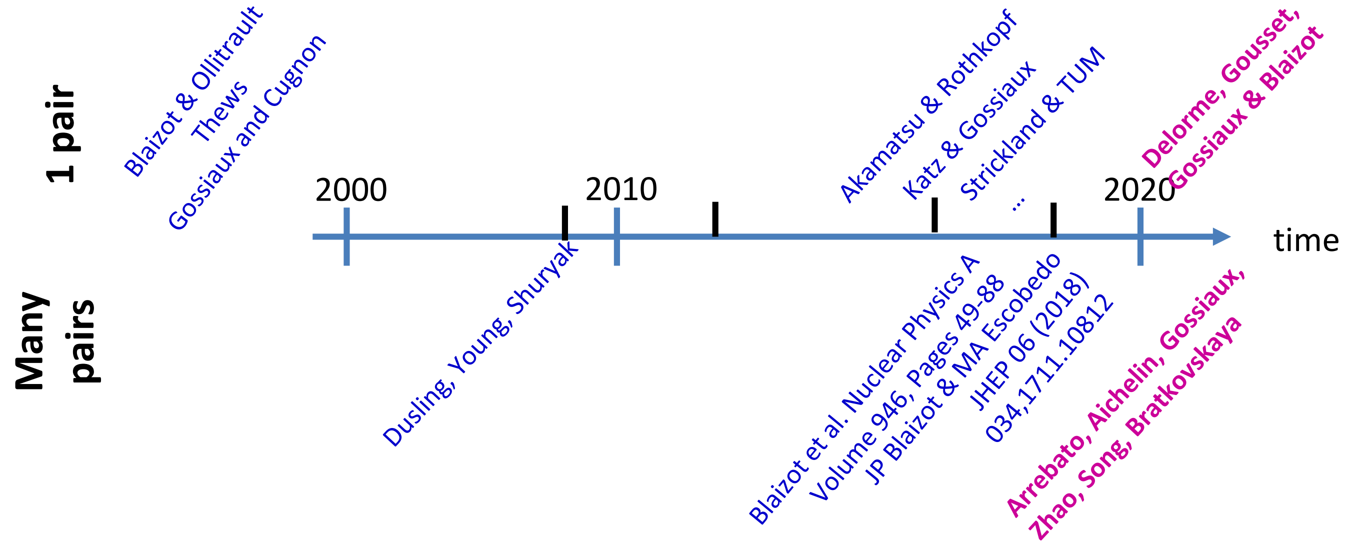
- Progressive delocalization with increasing temperature (sequential suppression)
- Eigenstates of D_{QED} correspond to those of the screened Hamiltonian
- Eigenstates of appear to be more delocalized than those of the screened Hamiltonian…
- …The coupling between QGP and the $Q\bar{Q}$ pair encoded in H_{int} goes beyond the mere screening and this is encompassed in the QME.

Two topics closely related to OQS



Semi-classical approximation

- Dealing with the many ccbar pairs produced in AA collision in a full-quantum way : untractable.
- All state of the art schemes on the market are « semi-classical », in a broad acceptance : Either kinetic rate equations or Langevin/Boltzmann – like, possibly including microscopic degrees of freedom… and some quantum features.

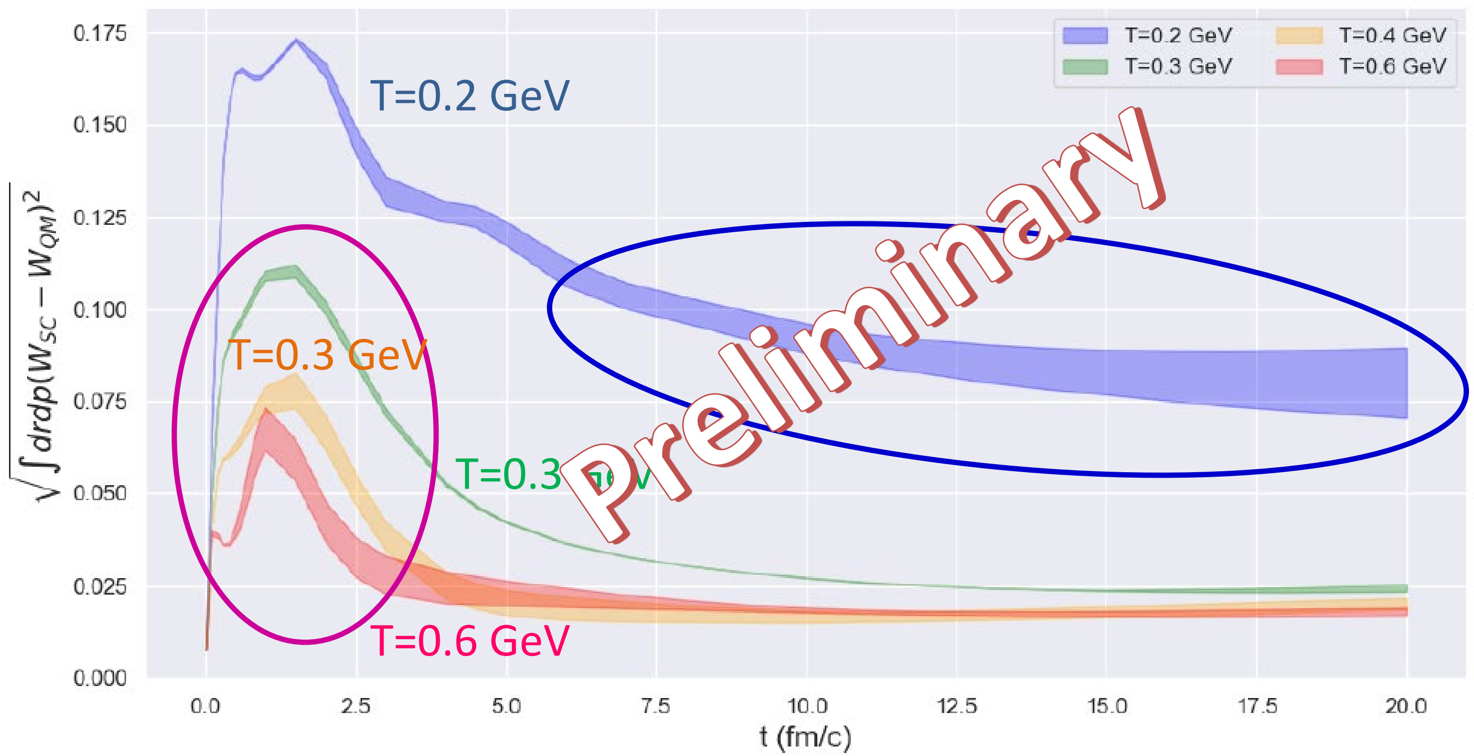


- However, to my knowledge, none of these schemes includes a proper treatment of the color dof, if at all ! Even for the Abelian case, the SC approximation in not benchmarked…

Quantum vs semiclassical dynamics (QED-like)

- Ongoing work (A. Daddi's PhD thesis) to compare SC approximation with benchmark solutions from the QME

Initial state : 1S (vacuum like)



- Band : 2 implementations of the OQS (with and without L_4 term)
- Rise and fall of the deviation for all temperatures ≥ 300 MeV
- As expected, deviations larger for smaller T, where quantum corrections should be more pronounced
- For T=200 MeV, important long lasting deviation, mostly due to differences in the asymptotic ρ_r.

Conclusions and future

- Illustration of a **QME solved exactly**, with some **interesting distinctive features** and a first (not so bad) contact towards experiment using EPOS4 profiles…

- **Novel feature** : discussion of the asymptotic limit of this equation, both for the QED-like and QCD cases… raising some questions to be addressed in a near feature…

- Future: Exact solution of the QME compared with semi-classical solution (adopted in some microscopic models) for the simpler QED-like case and then later for QCD

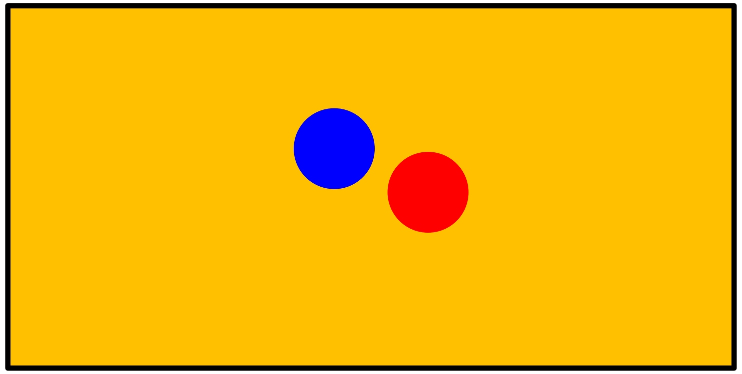
Own conclusion and future

Need more Genuine WWND to improve my skiing !!!

Back up

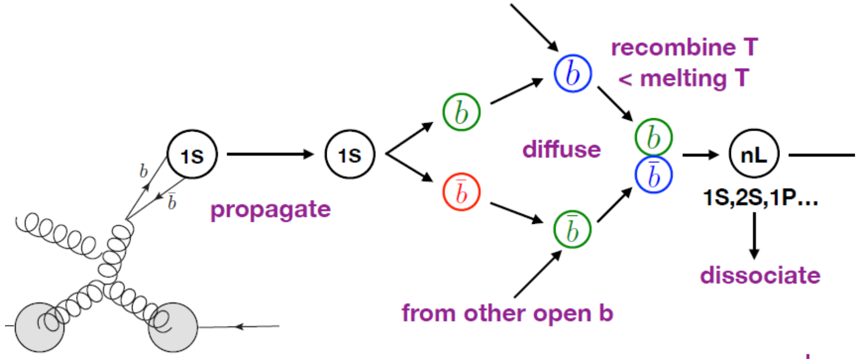
Regeneration: Dilute vs Dense

Bottomia



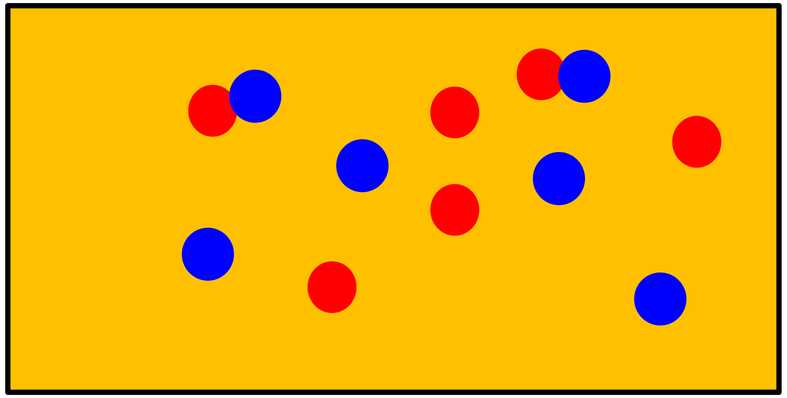
No exogenous recombination : only the b-bbar pairs which are initially close together will emerge as bottomia states

In some SC formalisms : intermediate regeneration



Yao, Mehen, Müller

Charmonia



Exogenous recombination : c & cbar initially far from each other may recombine and emerge as charmonia states

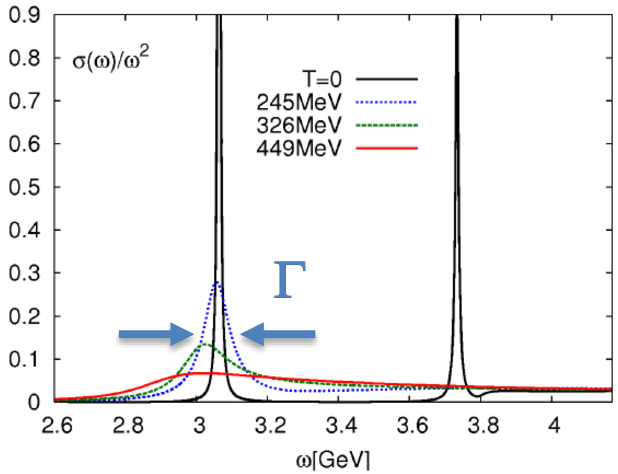
No full quantum treatment possible => need semi-classical approximation(s)

Key question : when does the recombination (dominantly) happen ? Crucial role of the binding force.

One extreme viewpoint : regeneration happens at the end of the QGP (Statistical Hadronization Model)

Quarkonia at finite T

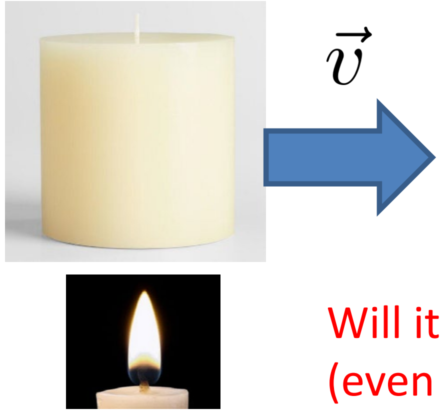
- Pheno: Yet, these pictures might still be compatible with the notion of sequential « suppression »…
- However, this notion has to be made more precise : (LQCD) spectral function lQCD



$$\rho(\omega,p,T)=\frac{1}{2\pi}\mathrm{Im}\int_{-\infty}^{\infty}dte^{i\omega t}\int d^3xe^{ipx}\langle[J(x,t),J(0,0)]\rangle_T$$

At T=245 MeV, ψ' has disappeared but J/ψ still surviving for $\approx 1/\Gamma \approx$ a couple of fm/c … which needs to be compared with the local QGP cooling time τ_{cool} : Γ x τ_{cool} > 1 ⇔ suppressed

- N.B.: The opposite phenomenom might also be relevant: some state above the « melting » temperature can survive (for a short while < $1/\Gamma$) before getting lost definitively.



Will it melt (even party) ?

- Key question : do the quarkonia states (chemically) equilibrate with the QGP ?

Modern era

A special QME: The Lindblad Equation

There are many different QME… a special one :

$$\frac{\mathrm{d}}{\mathrm{d}t}\rho_{Q\bar{Q}}(t) = -i\left[H_{Q\bar{Q}}, \rho_{Q\bar{Q}}(t)\right] + \sum_i \boxed{\gamma_i}\left[L_i \rho_{Q\bar{Q}}(t) L_i^\dagger - \frac{1}{2}\left\{L_i L_i^\dagger, \rho_{Q\bar{Q}}(t)\right\}\right]$$

γ_i Characterize the coupling of the system (Q-Qbar) with the environment

$H_{Q\bar{Q}}$: $\{Q, \bar{Q}\}$ $\underbrace{\text{kinetics + Vacuum potential V}}_{\hat{H}^{(0)}_{Q\bar{Q}}}$ + Lamb shift / screening (every unitary term that is generated by tracing out the environment)

L_i : Collapse (or Lindblad) operators, depend on the properties of the medium

3 important conservation properties :

$\rho^\dagger_{Q\bar{Q}} = \rho_{Q\bar{Q}}$	$\mathrm{Tr}[\rho_{Q\bar{Q}}] = 1$	$\langle\varphi\|\rho_{Q\bar{Q}}\|\varphi\rangle > 0, \forall\|\varphi\rangle$
(Hermiticity)	(Norm)	(Positivity)

… but in general, non unitary !!! (relaxation)

Nice feature : Can be brought to the form of a stochastic Schroedinger equation (quantum jump method : QTRAJ)

A special QME: The Lindblad Equation

Non unitary / dissipative evolution ≡ decoherence

$$\frac{\mathrm{d}}{\mathrm{d}t}\rho_{Q\bar{Q}}(t) = -i\left[H_{Q\bar{Q}}, \rho_{Q\bar{Q}}(t)\right] + \sum_i \gamma_i \left[L_i \rho_{Q\bar{Q}}(t) L_i^\dagger - \frac{1}{2}\left\{L_i L_i^\dagger, \rho_{Q\bar{Q}}(t)\right\}\right]$$

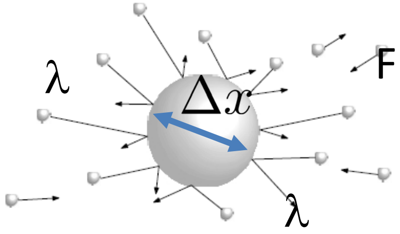
Genuine transitions :
- ✓ Singlet <-> octet
- ✓ Octet <-> octet

Can be reshuffled into non Hermitic effective hamiltonian

$$\hat{H}_{Q\bar{Q},\mathrm{eff}} = \hat{H}_{Q\bar{Q}} - i\sum_j \gamma_j \frac{L_j L_j^\dagger}{2}$$

≡ Dissociation width

For **infinitely massive single Q** and environment wave length λ >> wave packet size Δx:



Fluctuations from env. ⇔

$$\frac{\partial \rho_Q(x_Q, x'_Q)}{\partial t} = -F(x_Q - x'_Q)\rho_Q(x_Q, x'_Q)$$

Decoherence factor: $F \approx \kappa\,(x_Q - x'_Q)^2$

HQ momentum diffusion coefficient (adjoint)

In Q world: smaller objects live longer !

At 1rst order in $1/m_Q$: recoil corrections ⇔ friction / dissipation

Pictorial summary

Inspired from Yao Int. J. of Mod. Phys. A, Vol. 36, No. 20, 2130010 (2021)

τ_E: environment autocorrelation time **τ_S: system intrinsic time scale** **τ_R: system relaxation time**

Subsystem + environment: von Neumann equation

↓ Trace out environment

Subsystem: non-unitary, time-irreversible evolution

↓ Weak syst-environment coupling + Markovian limit $\tau_E \ll \tau_R$

Redfield equation

$$\frac{\partial}{\partial t}\rho_{\mathrm{I}}(t) = -\frac{1}{\hbar^2}\sum_{m,n}\int_0^\infty d\tau \Big(C_{mn}(\tau)\big[S_{m,\mathrm{I}}(t), S_{n,\mathrm{I}}(t-\tau)\rho_{\mathrm{I}}(t)\big] - C^*_{mn}(\tau)\big[S_{m,\mathrm{I}}(t), \rho_{\mathrm{I}}(t)S_{n,\mathrm{I}}(t-\tau)\big]\Big)$$

Similar structure to the Linblad equation but with time delay effects

Two types of dynamical modelling

$m_D \ll E_{\text{bind}}$ $m_D \sim E_{\text{bind}}$ $m_D \gg E_{\text{bind}}$

Quantum Optical Regime

- **Well identified resonances**
- Time long enough wrt quantum decoherence time (once we reach this regime)

Good description with transport models (TAMU, Tsinghua, Duke)

Central quantities :
2->2 and 2->3 Cross sections, decay rates

Equilibrium : exp(-E_n/T) (theorem)

SC Approx: rate equations

?

Quantum Brownian Motion

- Correlations growing with cooling QGP
- **Best described in position-momentum space**
- Time short wrt quantum decoherence time ?

Quantum Master Equations for **microscopic dof (QS and Qbars)**

Equilibrium / asympt* : some limiting cases

SC Approx: Fokker-Planck equations in position-momentum space

* Since one is facing both dissociation and recombination, obtaining a correct equilibrium limit of these models is an important prerequisite !!!

QCD time scales

τ_E: environment autocorrelation time

$$\tau_E \approx \frac{1}{m_D} \approx \frac{1}{CT} \approx \frac{1}{T}$$ (C taken as close to unity)

τ_S: system intrinsic time scale

$$\tau_S \approx \underbrace{\frac{1}{\Delta E}} \approx \frac{1}{m_Q v^2}$$ with $v \approx \alpha_S$ … at the beginning of the evolution

Difference btwn energy levels

τ_R: system relaxation time

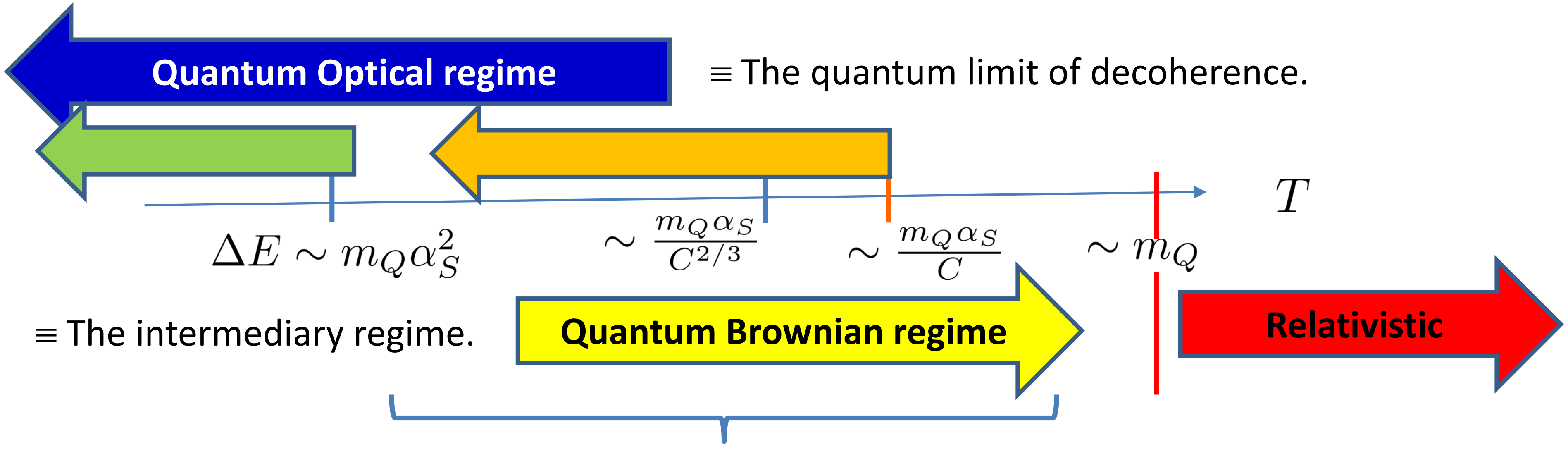
$$\Gamma = \tau_R^{-1} \sim 2\langle\psi|W\psi\rangle \approx \alpha_s T \times \Phi(m_D r) \approx \alpha_s T \times \Phi(\frac{CT}{m_Q \alpha_s})$$

At "small" T $\left(T \lesssim \frac{m_Q \alpha_S}{C}\right)$: dipole approximation : $\Gamma = \tau_R^{-1} \approx \frac{C^2 T^3}{\alpha_s m_Q^2}$

$$\frac{\tau_R}{\tau_E} = \frac{\alpha_s m_Q^2}{CT^2} \gg 1$$ And $$\frac{\tau_R}{\tau_S} = \frac{\alpha_s^3 m_Q^3}{C^2 T^3} \gg 1$$ for $T \lesssim m_Q \frac{\alpha_S}{C^{2/3}}$

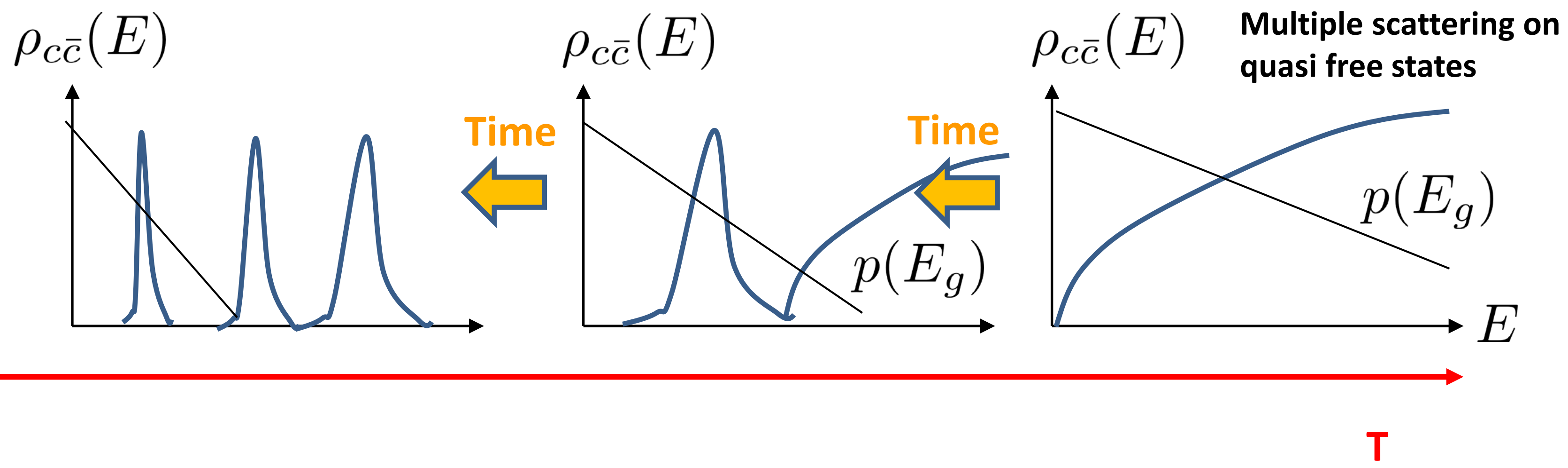
Fine with the Markovian assumption

QCD Temperature scales



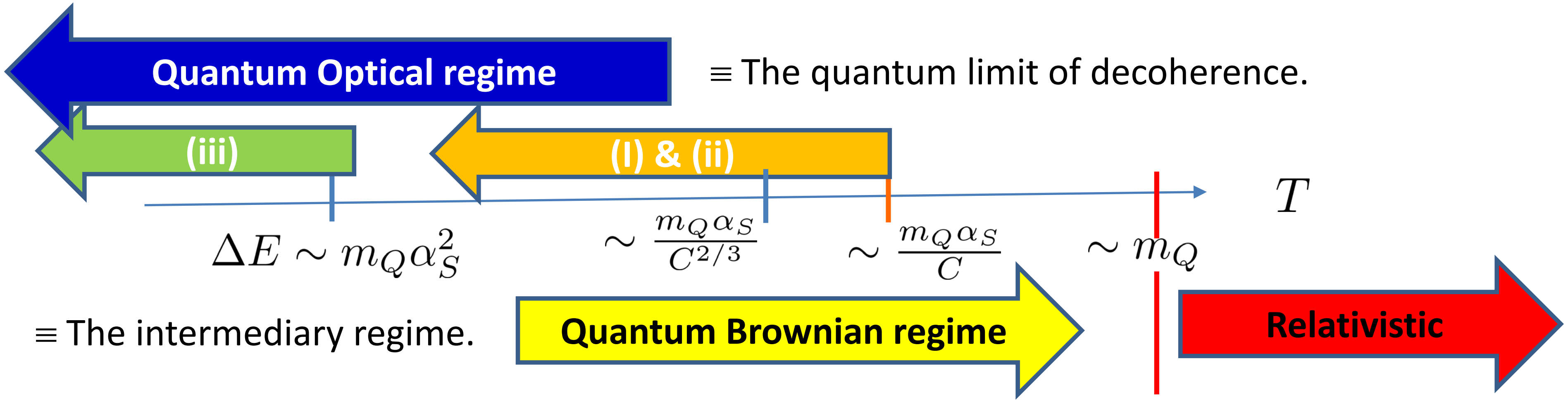
For these « large » temperatures, the Q-Qbar gain enough energy to overwhelm the real binding potential => larger distance => larger decoherence ….

dissociation of well identified levels by scarce "high-energy" modes (dilute medium => cross section ok)



In // : continuous evolution of the $Q\bar{Q}$ spectral function

QCD Temperature scales



Refined subregimes when playing with the scales of NRQCD / pNRQCD (series of recent papers by N. Brambilla, M.A. Escobdo, A. Vairo, M Strickland et al, Yao, Müller and Mehen,…)

NRQCD: $Mv, \Lambda_{\rm QCD}, T \ll \mu_{\rm NR} \ll M$: most general scheme for markovian OQS !

pNRQCD:

(Singlet and octet quarkonium fields)

- (i) $1/r \gg T \sim m_D \gg E$: « strongly coupled » QME same as small dipole limit of NRQCD (applies for small time evolution)
- (ii) $1/r \gg T \gg E \gg m_D$: « weakly coupled » : g T << T : essential contribution is gluo – dissociation from hard mode T : does not apply in QCD
- (iii) $1/r \gg T \sim E \gg m_D$: Quantum optical regime

Recent OQS implementations (single $Q\bar{Q}$ pair)

regime	SU3 ?	Dissipation ?	3D / 1D	Num method	year	remark	ref
NRQCD ⇔ QBM	No	No	1D	Stoch potential	2018		Kajimotoet al. , Phys. Rev. D 97, 014003 (2018), 1705.03365
	Yes	No	3D	Stoch potential	2020	Small dipole	R. Sharma et al Phys. Rev. D 101, 074004 (2020), 1912.07036
	Yes	No	3D	Stoch potential	2021		Y. Akamatsu, M. Asakawa, S. Kajimoto (2021), 2108.06921
	No	Yes	1D	Quantum state diffusion	2020		T. Miura, Y. Akamatsu et al, Phys. Rev. D 101, 034011 (2020), 1908.06293
	Yes ✓	Yes ✓	1D	Quantum state diffusion	**2021**		Akamatsu & Miura, EPJ Web Conf. 258 (2022) 01006, 2111.15402
	No	Yes	1D	Direct resolution	**2021**		O. Ålund, Y. Akamatsu et al, Comput. Phys. 425, 109917 (2021), 2004.04406
	Yes ✓	Yes ✓	1D	Direct resolution	**2022**		S Delorme et al, https://inspirehep.net /literature/ 2026925
pNRQCD (i)	Yes	No	1D+	Direct resolution	2017	S and P waves	N. Brambilla et al, Phys. Rev. D96, 034021 (2017), 1612.07248
(i) Et (ii)	Yes	No	1D+	Direct resolution	2017	S and P waves	N. Brambilla et al, Phys. Rev. D 97, 074009 (2018), 1711.04515
(i)	Yes	No	Yes	Quantum jump	**2021**	See SQM 2021	N. Brambilla et al. , JHEP 05, 136 (2021), 2012.01240 & *Phys.Rev.D* 104 (2021) 9, 094049, 2107.06222
(i)	Yes ✓	Yes ✓	Yes ✓	Quantum jump	**2022**		N. Brambilla et al. 2205.10289
(iii)	Yes ✓	Yes ✓	Yes ✓	Boltzmann (?)	2019		Yao & Mehen, Phys.Rev.D 99 (2019) 9, 096028, 1811.07027
NRQCD & « pNRQCD »	Yes	Yes	1D	Quantum state diffusion	**2022**		Miura et al. http://arxiv.org/abs/2205.15551v1
Other	No	Yes	1D	Stochastic Langevin Eq.	2016	Quadratic W	Katz and Gossiaux

(Year > 2015)

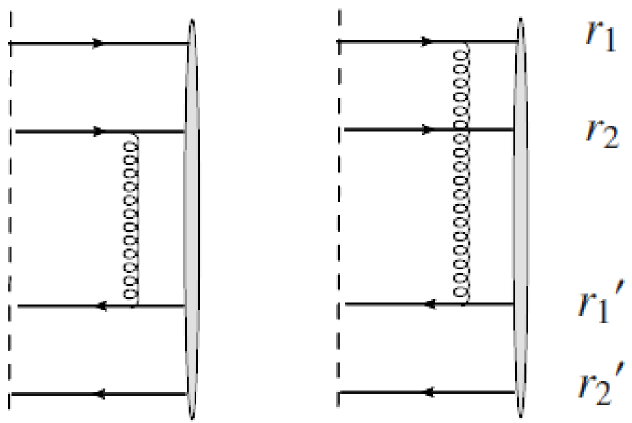
Not exhaustive

See as well table in 2111.15402v1

…

QED-like vs genuine QCD case

Genuine QCD

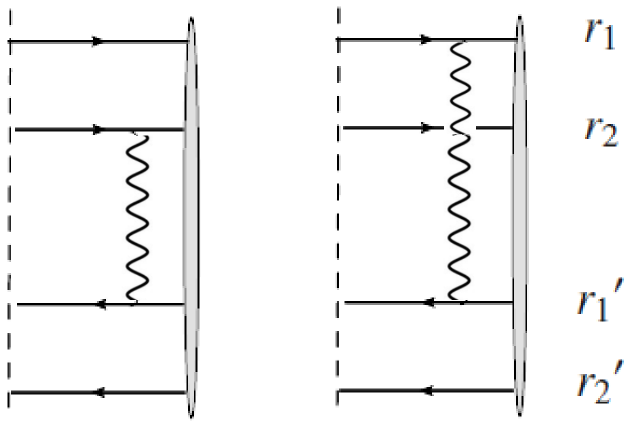


- Scattering from gluons change the color respresentation : o <-> s

$$\mathcal{D}_Q = \begin{pmatrix} \mathcal{D}_s \\ \mathcal{D}_o \end{pmatrix}$$

- No binding potential in the octet chanel => « large » energy gap

QED-like



- Scattering from photons do not change the Casimir : s <-> s

$$\mathcal{D}_Q = \left(\mathcal{D}_s \right)$$

- Usual 1S <-> 1P transitions between bound states.

B-E Quantum Master Equation: QED-like case

- For the relative motion (2 body):

$$\left.\begin{aligned}\vec{s} &= \vec{x}_1 - \vec{x}_2 \\ \vec{s}\,' &= \vec{x}_1' - \vec{x}_2'\end{aligned}\right\} \qquad \vec{r} = \frac{\vec{s}+\vec{s}\,'}{2} \quad \text{and} \quad \vec{y} = \vec{s} - \vec{s}\,'$$

- Near thermal equilibrium, Density operator is nearly diagonal => **semi-classical expansion** (power series in *y* up to 2^{nd} order)

... However, we know from open heavy flavor analysis that it takes some finite relaxation time to reach this state

$$\frac{d}{dt}\mathcal{D}(r,y) = \mathcal{L}\mathcal{D}(r,y)$$

$$\begin{cases} \mathcal{L}_0 = \frac{2i\nabla_y\cdot\nabla_r}{M} \\ \mathcal{L}_1 = i\vec{y}\cdot\nabla V(r) \\ \mathcal{L}_2 = -\frac{1}{4}\vec{y}\cdot(\mathcal{H}(\vec{r})+\mathcal{H}(0))\cdot\vec{y} \\ \mathcal{L}_3 = -\frac{1}{2MT}\vec{y}\cdot(\mathcal{H}(\vec{r})+\mathcal{H}(0))\cdot\nabla_{\vec{y}} \end{cases}$$

$\mathcal{H}(\vec{r})$: Hessian matrix of im. pot. W

$$W(\vec{y}) = W(\vec{0}) + \frac{1}{2}\vec{y}\cdot\mathcal{H}(0)\cdot\vec{y}$$

- Wigner transform -> $\mathcal{D}(\vec{r},\vec{p})$ => $\{\vec{y},\nabla_y\} \rightarrow \{\nabla_p,\vec{p}\}$ Usual Fokker Planck eq.
- Easy MC implementation + generalization for N body system (c-cbar @ LHC)

B-E Quantum Master Equation: QCD case

singlet density matrix

$$\frac{d}{dt}\begin{pmatrix}\mathcal{D}_s\\ \mathcal{D}_o\end{pmatrix} = \mathcal{L}\begin{pmatrix}\mathcal{D}_s(\mathbf{r}_{rel}, \mathbf{r}'_{rel}, t)\\ \mathcal{D}_o(\mathbf{r}_{rel}, \mathbf{r}'_{rel}, t)\end{pmatrix}$$

$$\mathcal{L} = \begin{pmatrix}\mathcal{L}_{ss} & \mathcal{L}_{so}\\ \mathcal{L}_{os} & \mathcal{L}_{oo}\end{pmatrix}$$

octet density matrix

singlet-octet transitions

2 coupled color representations (singlet octet)

Alternate choice : $\begin{pmatrix}\mathcal{D}_0\\ \mathcal{D}_8\end{pmatrix}$

Off color-equilibrium component

With (infinite mass limit)

$$\mathcal{D}_8(r,t) \sim \mathcal{D}_8(r,0)e^{-N_c\Gamma(r)t} \to 0$$

Color equilibration

Example of the $\mathcal{D}_s$ evolution (after semi-classical expansion, i.e power series in y=s-s’) :

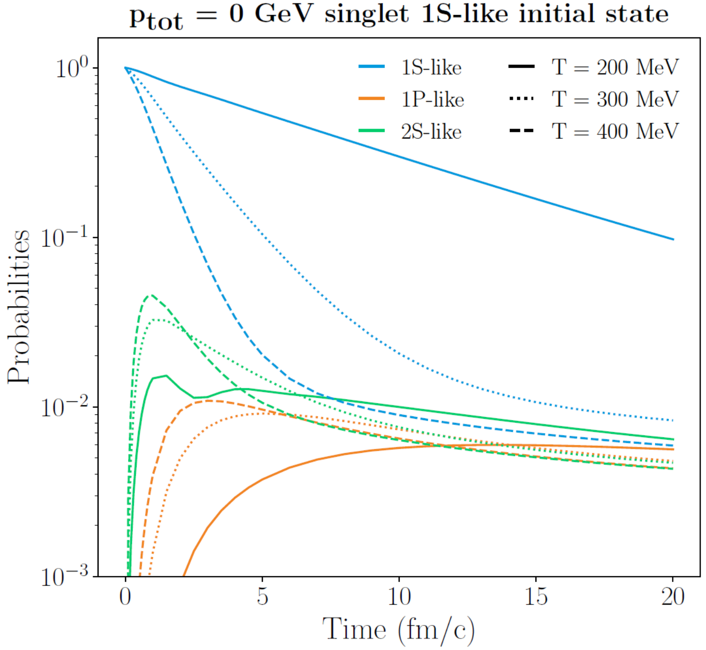
$$\begin{aligned}
(D_s|\mathcal{L}|\mathcal{D}) = & \left(2i\frac{\nabla_{\boldsymbol{r}}\cdot\nabla_{\boldsymbol{y}}}{M} + i\frac{\nabla_{\mathcal{R}}\cdot\nabla_{\boldsymbol{Y}}}{2M} + iC_F\boldsymbol{y}\cdot\boldsymbol{\nabla}V(\boldsymbol{r})\right)D_s\\
& -2C_F\Gamma(\boldsymbol{r})(D_s - D_o)\\
& -\frac{C_F}{4}(\boldsymbol{y}\cdot\mathcal{H}(\boldsymbol{r})\cdot\boldsymbol{y}\,D_s + \boldsymbol{y}\cdot\mathcal{H}(0)\cdot\boldsymbol{y}\,D_o)\\
& -C_F\boldsymbol{Y}\cdot[\mathcal{H}(0)-\mathcal{H}(\boldsymbol{r})]\cdot\boldsymbol{Y}\,D_o\\
& +\frac{C_F}{2MT}\left[\nabla^2W(0) - \nabla^2W(\boldsymbol{r}) - \boldsymbol{\nabla}W(\boldsymbol{r})\cdot\boldsymbol{\nabla}_{\boldsymbol{r}}\right](D_s - D_o)\\
& -\frac{C_F}{2MT}(\boldsymbol{y}\cdot\mathcal{H}(\boldsymbol{r})\cdot\boldsymbol{\nabla}_{\boldsymbol{y}}D_s + \boldsymbol{y}\cdot\mathcal{H}(0)\cdot\boldsymbol{\nabla}_{\boldsymbol{y}}D_o)\\
& -\frac{C_F}{2MT}\boldsymbol{Y}\cdot[\mathcal{H}(0)-\mathcal{H}(\boldsymbol{r})]\cdot\boldsymbol{\nabla}_{\boldsymbol{Y}}D_o.
\end{aligned}$$

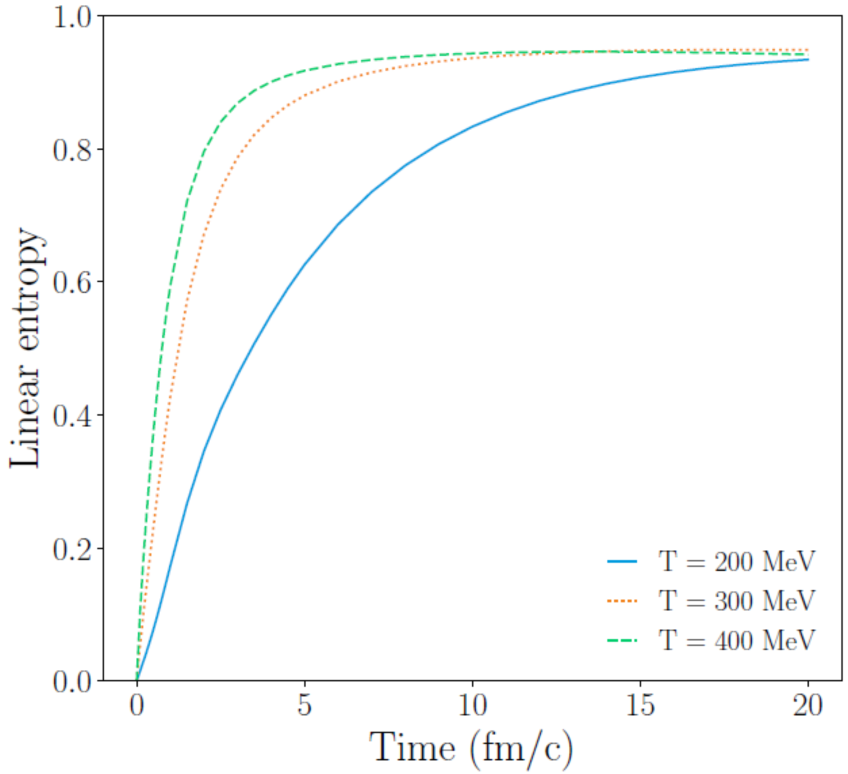
Results for the Linear quantum entropy

$$S_L = \mathrm{Tr}\hat{\rho} - \mathrm{Tr}\hat{\rho}^2 = 1 - \mathrm{Tr}\hat{\rho}^2$$

De Boni, J. High Energ. Phys. (2017) 2017: 64

(results for QED like evolution)





- Suppression and decoherence appear to happen on the same time scale…
- … does not seem in favour of applying classical rate equations (to be investigated further)

B-E Quantum Master Equation: QED-like case

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… However, we know from open heavy flavor analysis that it takes some finite relaxation time to reach this state

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- Wigner transform -> $\mathcal{D}(\vec{r},\vec{p})$ => $\{\vec{y},\nabla_y\}\rightarrow\{\nabla_p,\vec{p}\}$ Usual Fokker Planck eq.
- Easy MC implementation + generalization for N body system (c-cbar @ LHC)

Asymptotic distribution: QED-like

- First, looking at the QED-like case.

$$\frac{d}{dt}\mathcal{D}_s = \mathcal{L}[\mathcal{D}_s] \Rightarrow \mathcal{L}[\mathcal{D}_s^{\mathrm{asymp}}] = 0$$

- Even simpler case : Semi classical approximation : $W(r,p) \propto e^{-\frac{p^2}{m_Q T} - \frac{V(r)}{T}}$

(Wigner representation)

2 lines calculation :

$$\mathcal{L}_2 + \mathcal{L}_3 \to \frac{\mathcal{H}(r)+\mathcal{H}(0)}{2}\partial_p\left[\frac{\partial_p}{2} + \frac{p}{m_Q T}\right] W(r,p) \Rightarrow W(r,p) \propto e^{-\frac{p^2}{m_Q T}}$$

$$\mathcal{L}_0 + \mathcal{L}_1 \to \left(-\frac{p\,\partial_r}{m_Q} + \partial_r V(r)\partial_p\right) \Rightarrow W(r,p) \propto e^{-\frac{p^2}{m_Q T} - \frac{V(r)}{T}}$$

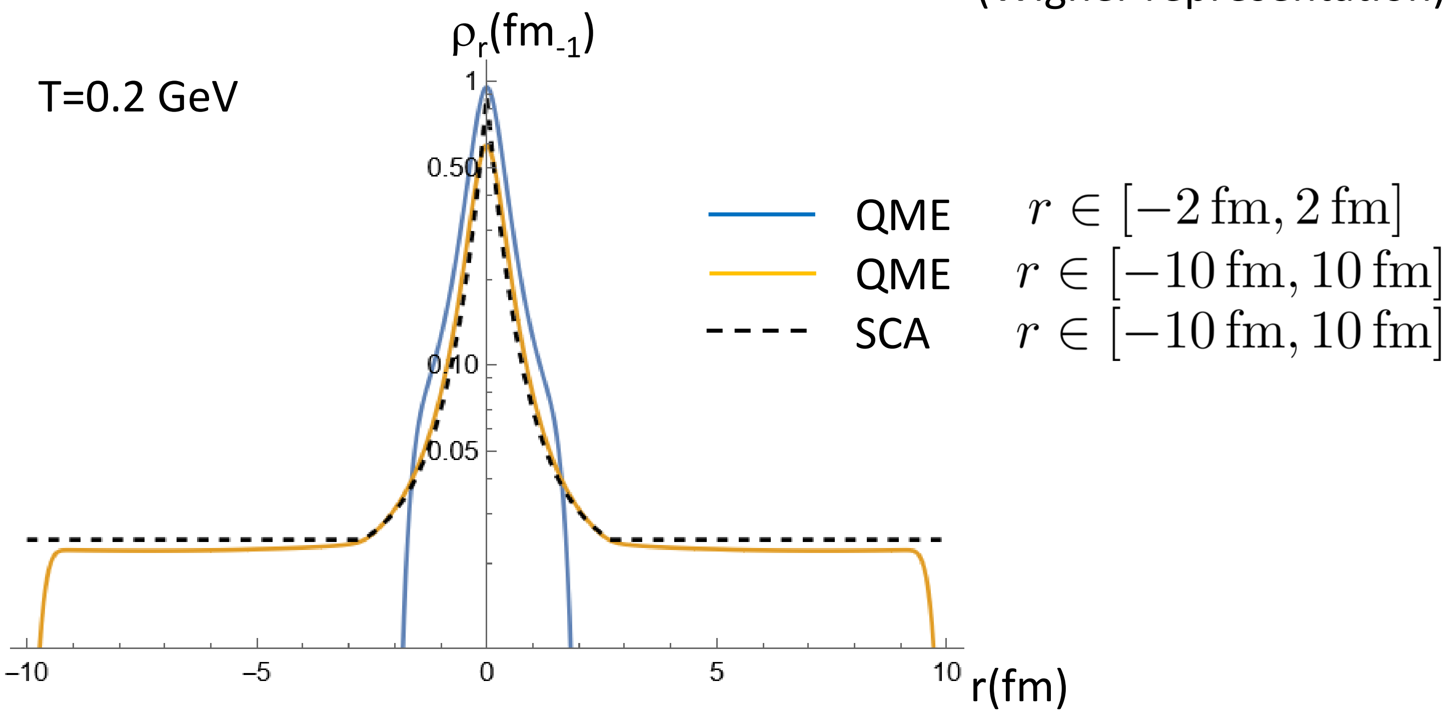
Asymptotic distribution: QED-like

- First, looking at the QED-like case.

$$\frac{d}{dt}\mathcal{D}_s = \mathcal{L}[\mathcal{D}_s] \Rightarrow \mathcal{L}[\mathcal{D}_s^{\rm asymp}] = 0$$

- Even simpler case : Semi classical approximation : $W(r,p) \propto e^{-\frac{p^2}{m_Q T} - \frac{V(r)}{T}}$

(Wigner representation)

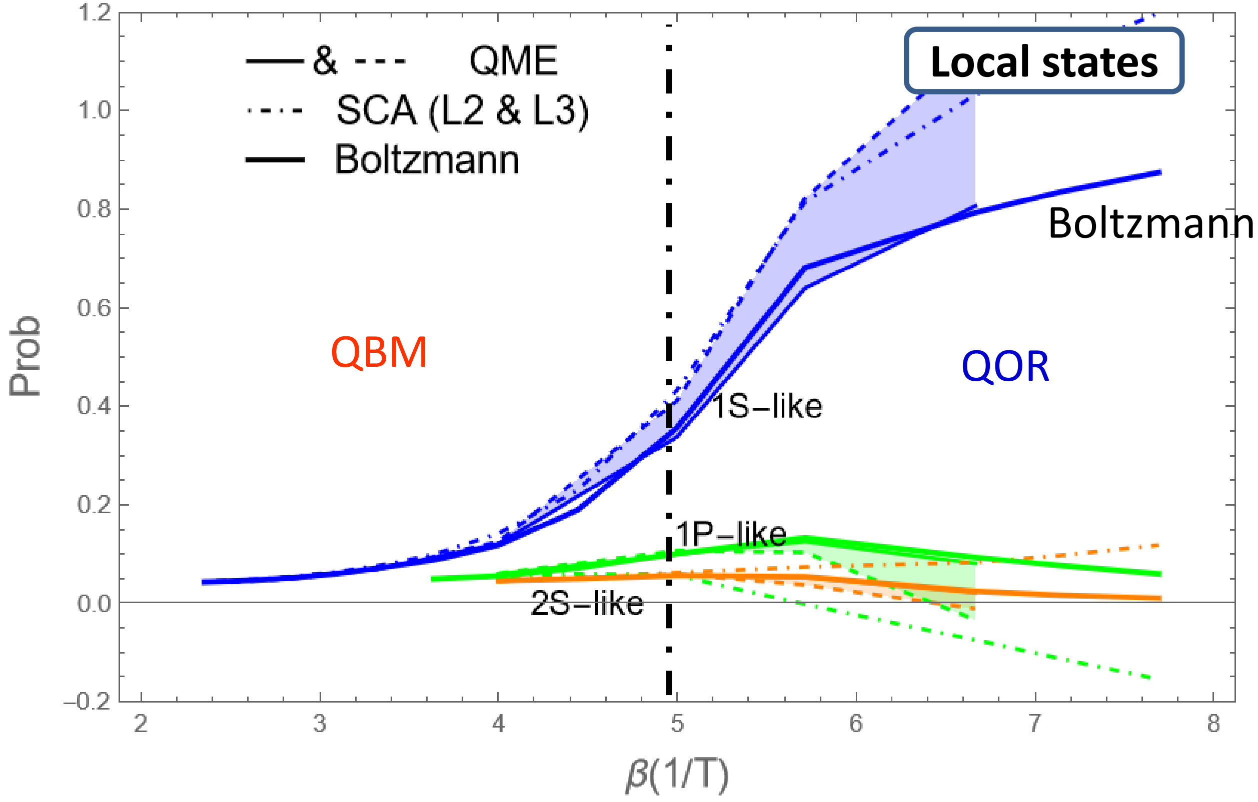


- Peak, independently of box size
- Pretty well described by SC relation (apart around the origin):

Quarkonium-like weights

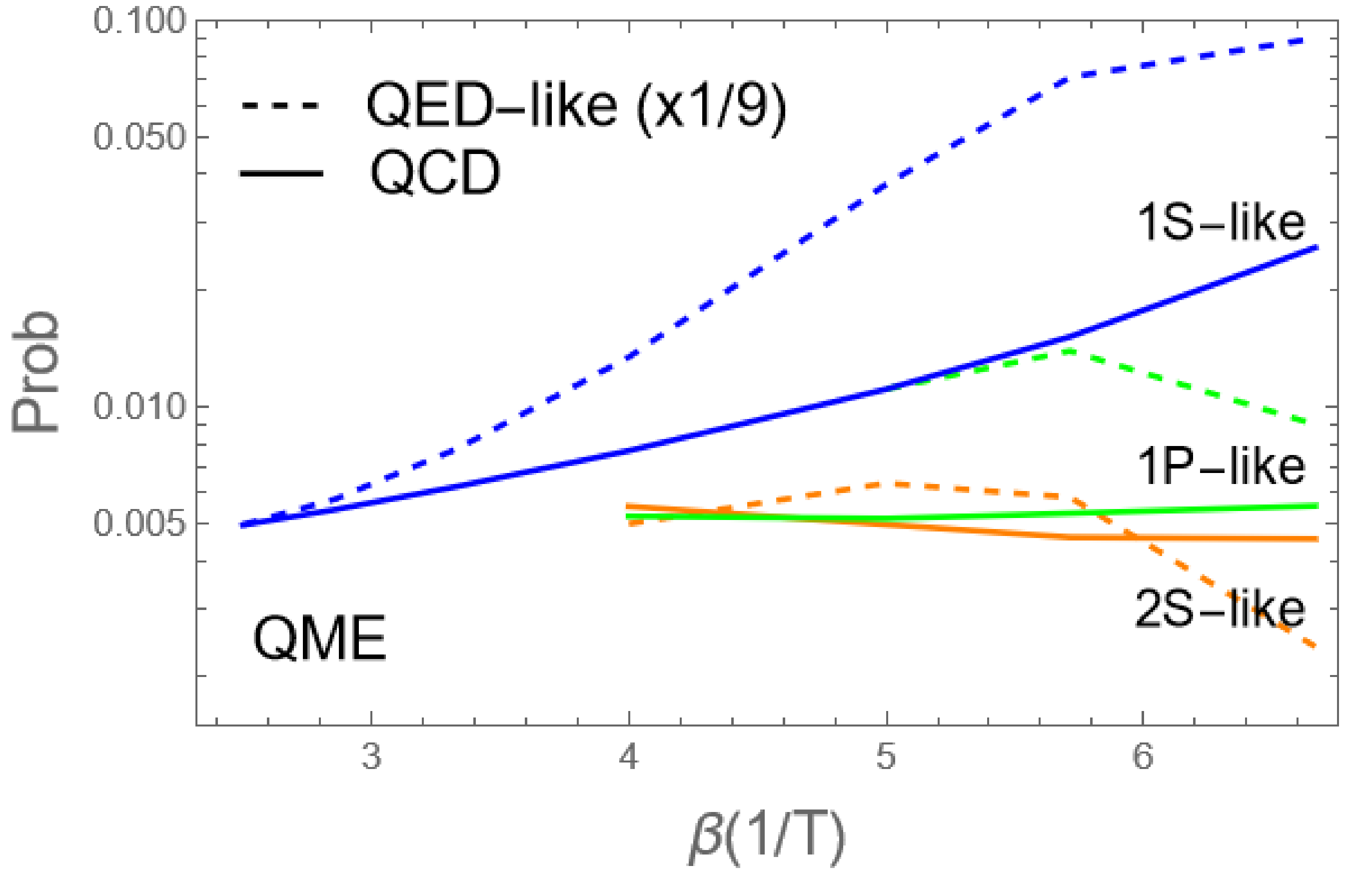
- First, looking at the QED-like case.

SCA : $\text{prob}_n = \langle n | ^{-\frac{V(\frac{x+x'}{2})}{T} - \frac{MT(x-x')^2}{(2\hbar c)^2}} | n \rangle$ Boltzmann : $\text{prob}_n = \frac{e^{-E_n/T}}{Z}$



- Good agreement between QME & SC in the deep QBM regime (expected)
- Good agreement between QME & Boltzmann Ansatz, even in the QOR !!! Not expected at all.
- SCA : not succesfull for 1P-like state

Recent OQS implementations (single $Q\bar{Q}$ pair)



ts

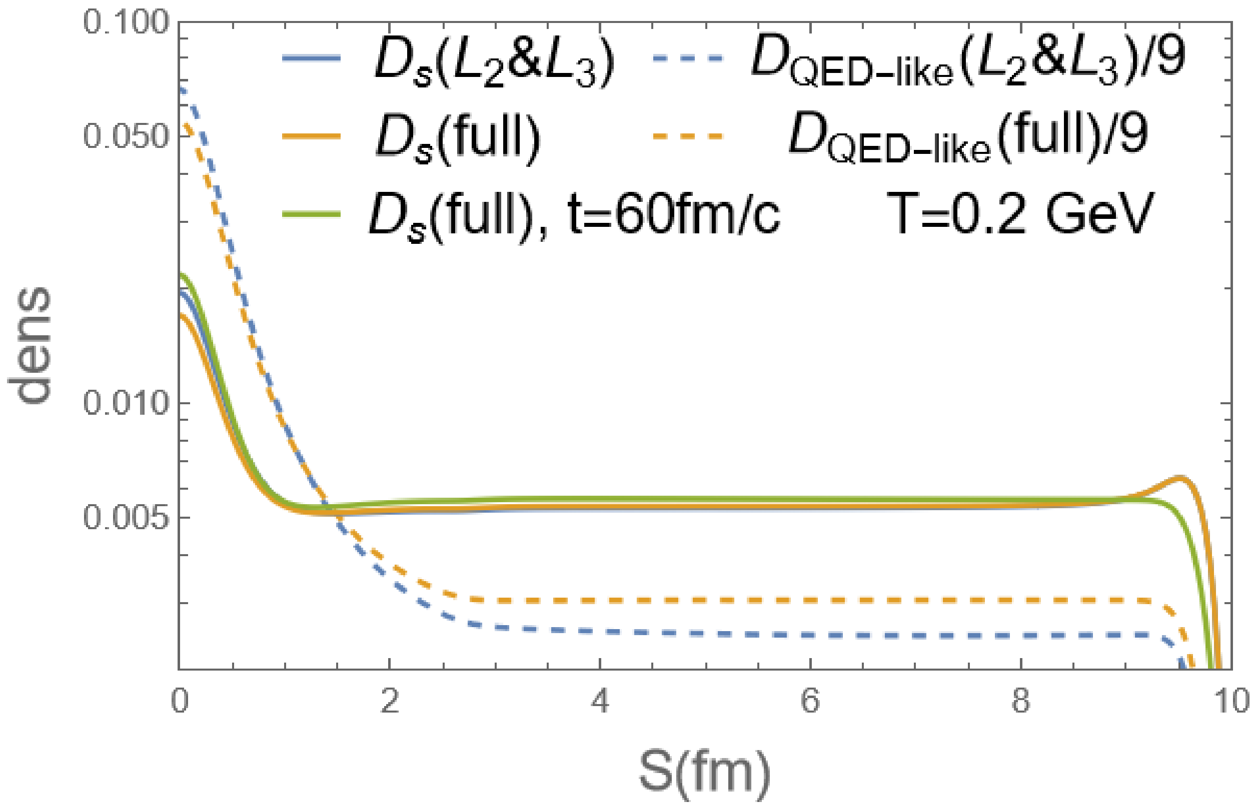
And of course, a lot of questions…

- Is this result correct or the sign of some illness in the QME ? (or the author's mind)
- Can this result be understood by rephrasing things with usual rate equation ?
- Can we put some mathematical modelling on it ?

Asymptotic distribution: QCD

- Now the genuine QCD case
- Solving $\mathcal{L}_{ss} \cdot \mathcal{D}_s = -\mathcal{L}_{so} \cdot \mathcal{D}_o$

With thermalized Ansatz for D_o

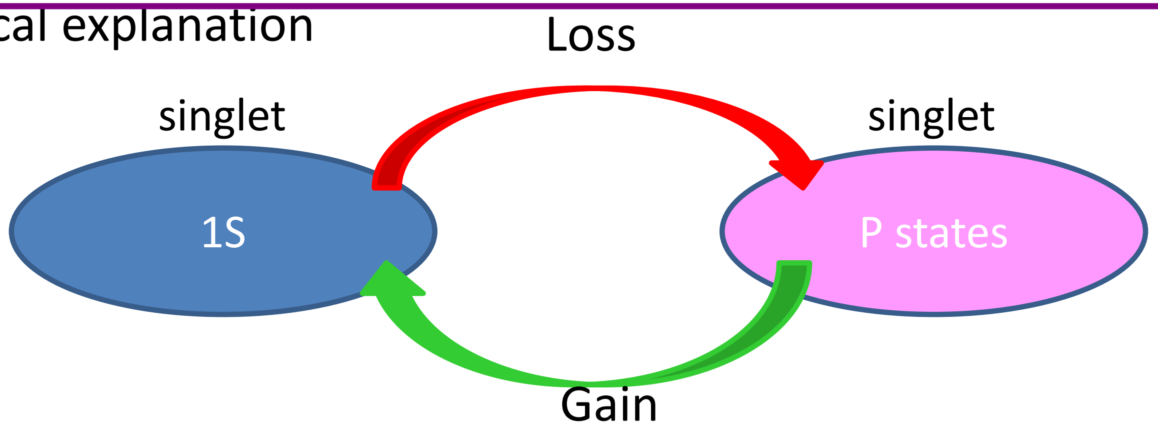


- The peak around origin is reduced in the case of QCD !
- Quarks more deconfined than in QED-like case.
- Reminder : discussion specific to the QBM regime (not the QOR)

Asymptotic distribution: QCD

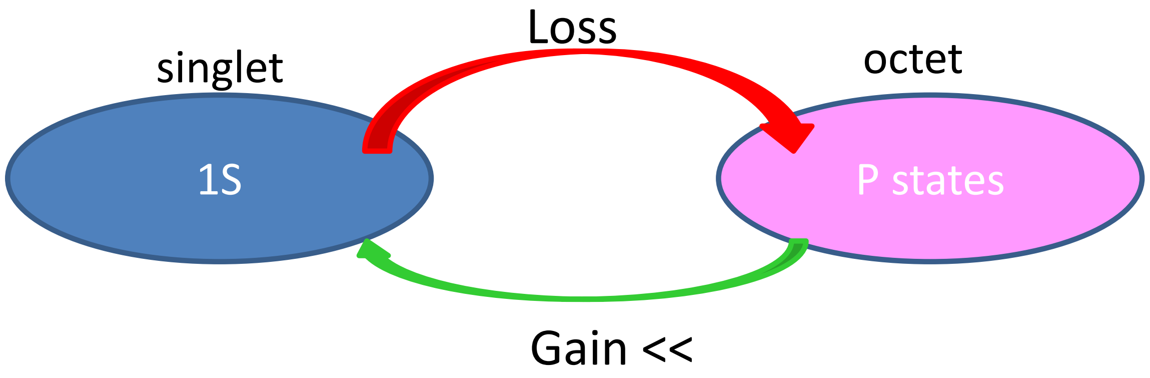
- A possible physical explanation

QED – like :



As the binding potential also acts on P-states (singlet), both densities can increase together when a real potential is applied (gain and loss terms keep α)

QCD :



As there is no potential in the octet chanel, the Q and Qbar quarks have a tendancy to fly apart fast (the asymtotic octet density is found indeed flat).
Hence, the gain term does not increase α to the loss term and the equilibrium limit is displaced wrt QED-like