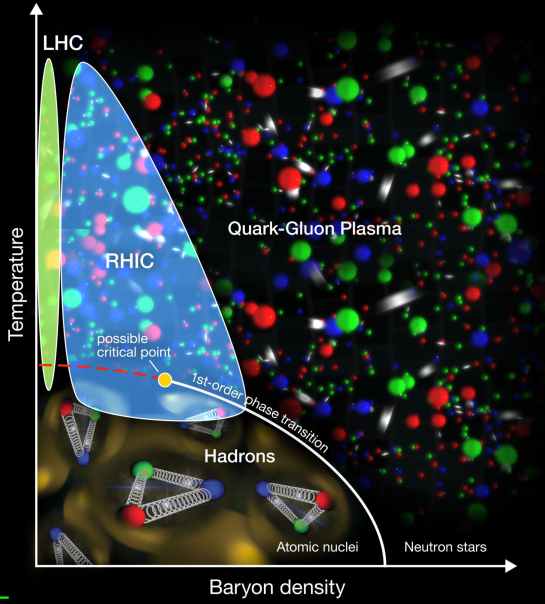
Transport coefficients of transient hydrodynamics for the HRG and thermal-mass quasiparticle models

Gabriel Soares Rocha (*gabriel.soares.rocha@vanderbilt.edu*)

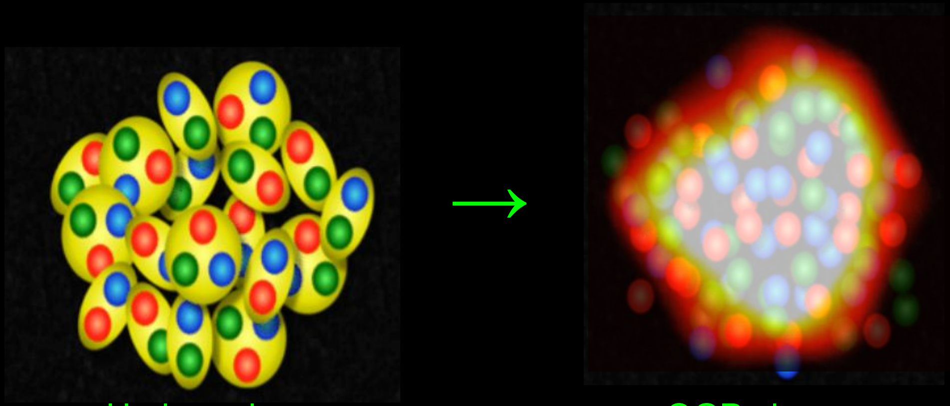
based on *arXiv:2402.06996*
with G. S. Denicol
The 39th Winter Workshop on Nuclear Dynamics – Jackson Hole, WY, USA
February 16th, 2024

Introduction

- With ultra-relativistic Heavy Ion Collisions, nuclear matter in extreme conditions can be studied; Heinz, Snellings Annu. Rev. Nucl. Part. Sci. 63 (2013) 123-151 Gale, Jeon, Schenke Int. J. of Mod. Phys. A, Vol. 28, 1340011 (2013)
- QCD at zero baryon chemical potential: crossover between hadron and QGP phases; Aoki et al. Nature, 443:675–678 (2006) Borsanyi et al, JHEP 11, 077 (2010) 50 yrs of QCD paper EPJ C 83 (2023) 1125 Leutwyler The strong interactions 2212.14791



Credit: BNL



Hadron phase → QGP phase

Images: Sahoo & Nayak Current Science 121, 1403 (2021)

Introduction

- Current understanding of the dynamics implemented in hybrid codes;
- Crucial element: relativistic hydrodynamic models (Solvers: MUSIC, CLVisc, VISHNU);

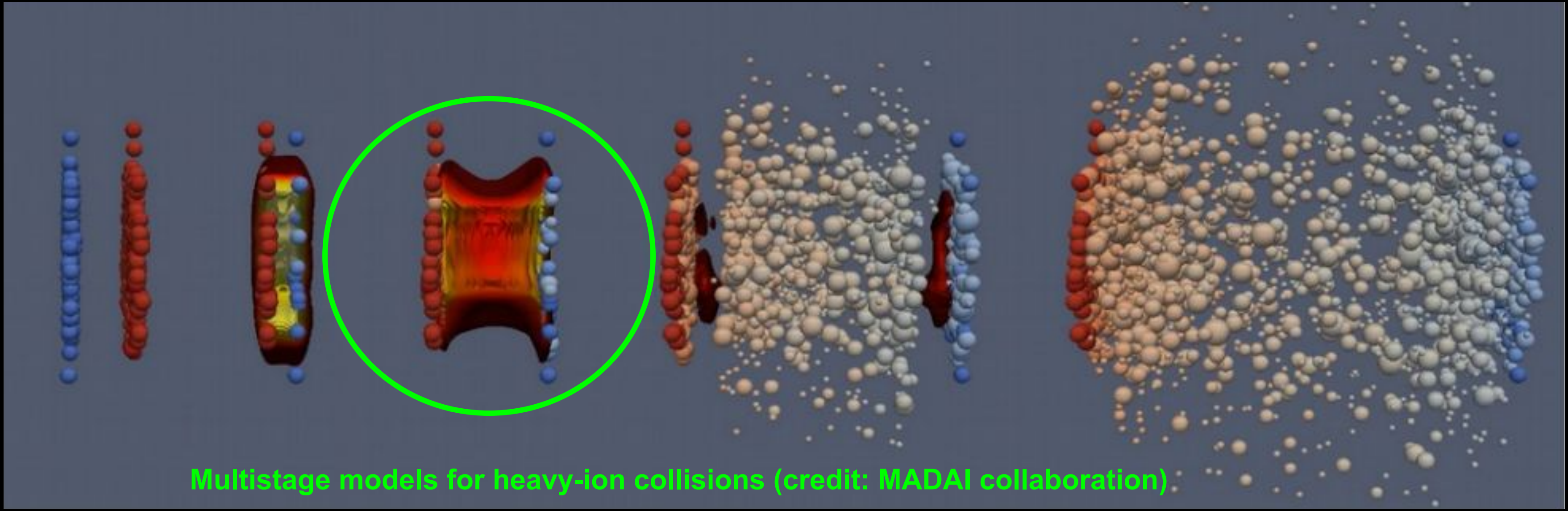
Pand, Petersen & Wang PRC 97, 064918 (2018)
Shen et al arXiv: 1409.8164 (2014)

Schenke Jeon, & Gale, PRC 82, 014903 (2010)
Paquet et al PRC 93, 044906 (2016).

- Kinetic theory as a guide;

Romatschke & Romatschke Cambridge Monographs on Mathematical Physics 1712.05815

Denicol Niemi Molnar Rischke PRD 85, 114047 (2012);
Denicol, Jeon, Gale PRC 90 024912 (2014)



Multistage models for heavy-ion collisions (credit: MADAI collaboration)

Hydrodynamic variables and equations of motion

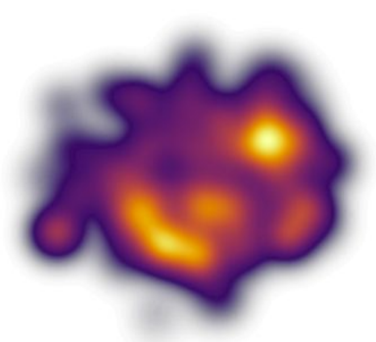
- Hydrodynamics: dynamics of coarse grained macroscopic variables (temperature/energy density, velocity);
- Basic equations of motion: local conservation of energy and momentum;

$$\partial_\mu T^{\mu\nu} = 0$$

continuity equations

$$\partial_t \left(\begin{matrix} \text{energy/} \\ \text{momentum} \end{matrix} \right) + \vec{\nabla} \cdot \left(\text{ fluxes } \right) = 0$$

(Credit: https://www.waves.com.br/expedicao/arraial-do-cabo-de-gala/)



http://jetscape.org/sims/

Hydrodynamic variables and equations of motion

- Dissipative fluids: $T^{\mu\nu} = T^{\mu\nu}_{\mathrm{eq}} + T^{\mu\nu}_{\mathrm{diss}}$

Equilibrium part/
Thermodynamic relations

Non-equilibrium components

$$\varepsilon_0 u^\mu u^\nu - P_0(\varepsilon_0)\Delta^{\mu\nu} \qquad - \Pi\Delta^{\mu\nu} + \pi^{\mu\nu}$$

energy density

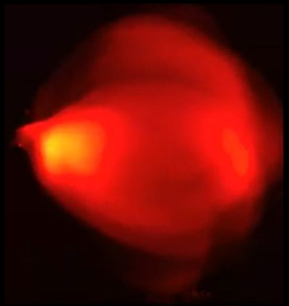
pressure

bulk viscous pressure

anisotropic pressure

$$\Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu$$

space-like projector



Credit: Chun Shen
https://youtu.be/G-Fbon0YQak

Hydrodynamic variables and equations of motion

- Dissipative fluids: $T^{\mu\nu} = T^{\mu\nu}_{\text{eq}} + T^{\mu\nu}_{\text{diss}}$

Equilibrium part/
Thermodynamic relations

Non-equilibrium components

$$\varepsilon_0 u^\mu u^\nu - P_0(\varepsilon_0)\Delta^{\mu\nu} \qquad - \Pi\Delta^{\mu\nu} + \pi^{\mu\nu}$$

energy density

pressure

bulk viscous pressure

anisotropic pressure

$$\partial_\mu T^{\mu\nu} = 0$$

Not enough for eq + diss

(4 equations, 10 variables)

Constitutive relations/**independent dynamic equations are needed.**

One way: Kinetic theory

Credit: Chun Shen
https://youtu.be/G-Fbon0YQak

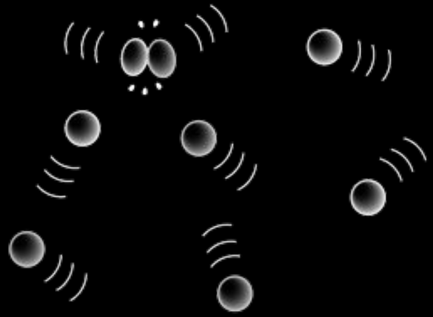
Microscopic derivation from Kinetic Theory

de Groot et al, *Relativistic Kinetic Theory: Principles and Applications* (North-Holland,1980)
Denicol, Rischke Microscopic Foundations of Relativistic Fluid Dynamics. Springer, 2021

- Non-equilibrium dynamics: Boltzmann equation

$$p^\mu \partial_\mu f_{\mathbf{p}} = C[f_{\mathbf{p}}]$$

spacetime dependence   collision term: interaction information



Adapted from: https://qph.cf2.quoracdn.net/main-qimg-ce13aa5d6f2394d9c2dccf6c912e79e4

- Compatibility with conservation laws

$$T^{\mu\nu} = \int dP \; p^\mu p^\nu f_p, \quad \overset{\text{Boltzmann eqn}}{\Longrightarrow} \quad \partial_\mu T^{\mu\nu} = 0$$

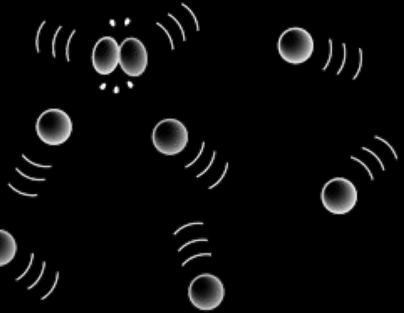
- Boltzmann contains more dynamics than hydro → truncation procedure

*GSR, Wagner, Denicol, Noronha, Rischke [arXiv: 2311.15063]
Denicol Niemi Molnar Rischke PRD 85, 114047 (2012);
Fotakis et al PRD 106, 036009 (2022)
Wagner & Gavassino PRD 109, 016019 (2024)

Microscopic derivation from Kinetic Theory

de Groot et al, *Relativistic Kinetic Theory: Principles and Applications* (North-Holland,1980)
Denicol, Rischke Microscopic Foundations of Relativistic Fluid Dynamics. Springer, 2021

- Non-equilibrium dynamics: Boltzmann equation



$$\underbrace{p^\mu \partial_\mu f_{\mathbf{p}}}_{\text{spacetime dependence}} = \underbrace{C[f_{\mathbf{p}}]}_{\text{collision term: interaction information}}$$

- Boltzmann contains more dynamics than hydro → truncation procedure

→ Exact equations of motion for non-equilibrium fields

Denicol Niemi Molnar Rischke PRD 85, 114047 (2012);
*GSR, Wagner, Denicol, Noronha, Rischke [arXiv: 2311.15063]
de Brito, Denicol [arXiV: 2401.10098]

→ "Small gradients, near equilibrium": relation between fields and $\Pi, \pi^{\mu\nu}$

Struchtrup, Physics of Fluids 16, 3921 (2004)
Fotakis et al PRD 106, 036009 (2022)
Wagner et a PRD 016013 (2022)

→ Closed equations of motion for $\Pi, \pi^{\mu\nu}$

Denicol & Rischke. *Microscopic Foundations of Relativistic Fluid Dynamics*. Springer, 2021
Wagner & Gavassino PRD 109, 016019 (2024)

Equations of motion and heavy-ion collisions

- At the end of this procedure, we have

Israel, W., & Stewart, J. M.. PLA, 58(4), 213-215.(1976)
Denicol Niemi Molnar Rischke PRD 85, 114047 (2012);

$$\tau_\Pi D\Pi + \Pi = -\zeta\theta - \delta_{\Pi\Pi}\Pi\theta + \lambda_{\Pi\pi}\pi^{\mu\nu}\sigma_{\mu\nu},$$

$$\tau_\pi D\pi^{\langle\alpha\beta\rangle} + \pi^{\alpha\beta} = 2\eta\sigma^{\alpha\beta} - \delta_{\pi\pi}\pi^{\alpha\beta}\theta - 2\tau_\pi\omega_\mu{}^{\langle\alpha}\pi^{\beta\rangle\mu} - \tau_{\pi\pi}\sigma_\mu{}^{\langle\alpha}\pi^{\beta\rangle\mu} + \lambda_{\pi\Pi}\Pi\sigma^{\alpha\beta},$$

Equations of motion and heavy-ion collisions

- At the end of this procedure, we have

Israel, W., & Stewart, J. M.. PLA, 58(4), 213-215.(1976)
Denicol Niemi Molnar Rischke PRD 85, 114047 (2012);

⏱ $$D\Pi + \underline{\Pi = -\zeta\theta} - \underline{\delta_{\Pi\Pi}\Pi\theta + \lambda_{\Pi\pi}\pi^{\mu\nu}\sigma_{\mu\nu}},$$

⏱ $$D\pi^{\langle\alpha\beta\rangle} + \underline{\pi^{\alpha\beta} = 2\eta\sigma^{\alpha\beta}} - \underline{\delta_{\pi\pi}\pi^{\alpha\beta}\theta - 2\tau_\pi\omega_\mu{}^{\langle\alpha}\pi^{\beta\rangle\mu} - \tau_{\pi\pi}\sigma_\mu{}^{\langle\alpha}\pi^{\beta\rangle\mu} + \lambda_{\pi\Pi}\Pi\sigma^{\alpha\beta}},$$

Navier-Stokes (first-order terms)

Second-order terms

$$D = u \cdot \partial$$

$$\theta \equiv \nabla_\mu u^\mu$$

Expansion rate

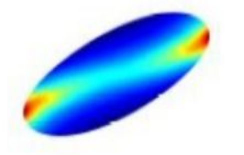
$$\sigma^{\mu\nu} \equiv \nabla^{\langle\mu}u^{\nu\rangle}$$

symmetric-traceless projection

Shear tensor

$$\omega_{\mu\nu} = \frac{1}{2}(\nabla_\mu u_\nu - \nabla_\nu u_\mu)$$

Vorticity tensor



Equations of motion and heavy-ion collisions

- At the end of this procedure, we have

Israel, W., & Stewart, J. M.. PLA, 58(4), 213-215.(1976)
Denicol Niemi Molnar Rischke PRD 85, 114047 (2012);

$$\tau_{\Pi} D\Pi + \Pi = -\zeta\theta - \delta_{\Pi\Pi}\Pi\theta + \lambda_{\Pi\pi}\pi^{\mu\nu}\sigma_{\mu\nu},$$

$$\tau_{\pi} D\pi^{\langle\alpha\beta\rangle} + \pi^{\alpha\beta} = 2\eta\sigma^{\alpha\beta} - \delta_{\pi\pi}\pi^{\alpha\beta}\theta - 2\tau_{\pi}\omega_{\mu}{}^{\langle\alpha}\pi^{\beta\rangle\mu} - \tau_{\pi\pi}\sigma_{\mu}{}^{\langle\alpha}\pi^{\beta\rangle\mu} + \lambda_{\pi\Pi}\Pi\sigma^{\alpha\beta},$$

- Second order transport (MUSIC)
  - Single-particle content;
  - High temperature limit (m/T <<1);
  - Approximate collision term;

$$C[f_{\mathbf{p}}] \simeq -\frac{E_{\mathbf{p}}}{\tau_R}(f_{\mathbf{p}} - f_{0\mathbf{p}})$$

https://webhome.phy.duke.edu/~jp401/music_manual/hydro.html#viscous-hydrodynamics
Denicol, Jeon, Gale PRC 90 024912 (2014)

https://github.com/MUSIC-fluid/MUSIC

Current implementation (MUSIC)

$$\tau_\Pi D\Pi + \Pi = -\zeta\theta - \delta_{\Pi\Pi}\Pi\theta + \lambda_{\Pi\pi}\pi^{\mu\nu}\sigma_{\mu\nu},$$

$$\tau_\pi D\pi^{\langle\alpha\beta\rangle} + \pi^{\alpha\beta} = 2\eta\sigma^{\alpha\beta} - \delta_{\pi\pi}\pi^{\alpha\beta}\theta - 2\tau_\pi\omega_\mu{}^{\langle\alpha}\pi^{\beta\rangle\mu} - \tau_{\pi\pi}\sigma_\mu{}^{\langle\alpha}\pi^{\beta\rangle\mu} + \lambda_{\pi\Pi}\Pi\sigma^{\alpha\beta},$$

- High temperature: second-order ↔ first order and speed of sound

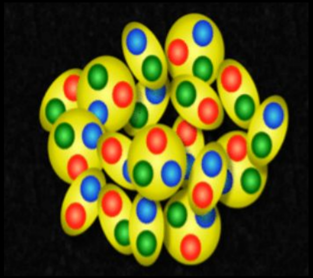
Denicol, Jeon, Gale PRC 90 024912 (2014)

$$\boxed{\tau_\Pi = \frac{\zeta}{15(\varepsilon_0 + P_0)(1/3 - c_s^2)^2}} \qquad \delta_{\Pi\Pi} = \frac{2}{3}\tau_\Pi \qquad \lambda_{\Pi\pi} = \frac{8}{5}\left(\frac{1}{3} - c_s^2\right)\tau_\Pi$$

$$\boxed{\tau_\pi = \frac{5\eta}{\varepsilon_0 + P_0}} \qquad \delta_{\pi\pi} = \frac{4}{3}\tau_\pi \qquad \tau_{\pi\pi} = \frac{10}{7}\tau_\pi \qquad \lambda_{\pi\Pi} = \frac{6}{5}\tau_\pi$$

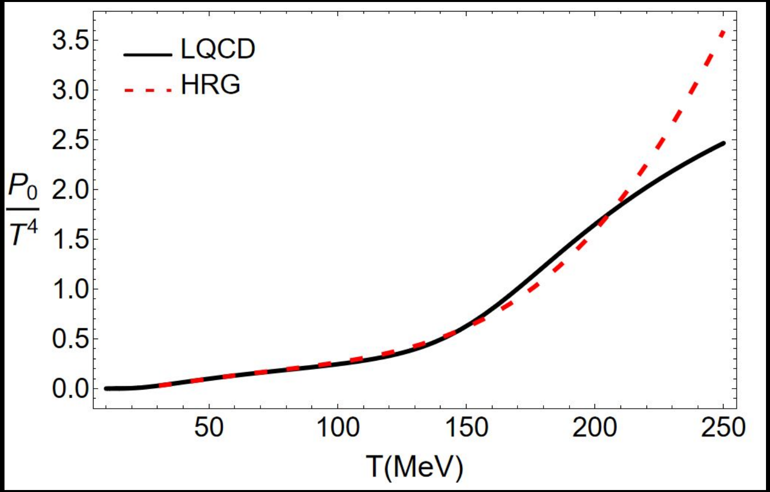
- Goal: update Denicol, Jeon, Gale PRC 90 024912 (2014) with toy models containing more realistic degrees of freedom

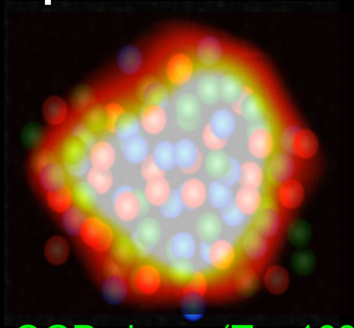
The Hadron Resonance Gas and the Quasiparticle Model



Hadron phase (T < 145 MeV)

crossover region
145 MeV < T < 163 MeV





QGP phase (T > 163 MeV)

Borsanyi et al (Wuppertal-Budapest), JHEP 11, 077 (2010) (adapted);

- Using kinetic theory toy models as guides, how do transport coefficients behave?

The hadron-resonance gas

- Hadron-resonance gas toy model

$$p^\mu \partial_\mu f_{\mathbf{p},i} = C_i[f_\mathbf{p}] \simeq -\frac{E_{\mathbf{p},i}}{\tau_R}(f_{\mathbf{p},i} - f_{0\mathbf{p},i}),$$

$$i = \{\pi^0, \pi^\pm, K^0, K^\pm, \eta, f_0(500), \cdots\}$$

$$f_{0\mathbf{p},i} = g_i e^{-\beta u_\mu p_i^\mu}$$

Relaxation time approximation

J. L. Anderson and H. Witting, Physica 74, 466 (1974)

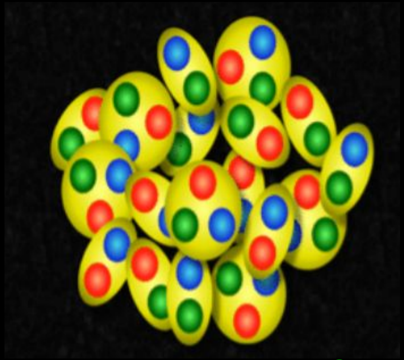
$$T^{\mu\nu} = \sum_{i=1}^{N_{\text{spec}}} \int dP_i p_i^\mu p_i^\nu f_{\mathbf{p},i}.$$

Landau prescription:

$$T^\mu_\nu u^\nu = \varepsilon_0 u^\mu$$

Procedure:

Boltzmann → Hydro truncation + Relaxation time approximation → Hydro equations of motion



Hadron Resonance Gas

Thermal-mass quasiparticle model

- Quasiparticle kinetic theory toy model

P. Romatschke *PRD*, *85*(6), 065012 (2012) .
Calzetta, E., & Hu, B. L.. *PRD 37*(10), 2878 (1988).
Alqahtani et al PRC 92, 054910 (2015)

Jeon & Yaffe PRD, *53*(10), 5799 (1996);
Jeon PRD 52 3591-3642 (1995)

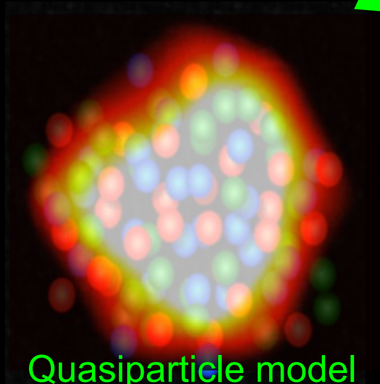
$$p^\mu \partial_\mu f_{\mathbf{p}} + \frac{1}{2}\partial_\mu M^2(T)\partial^\mu_{(p)} f_{\mathbf{p}} = C[f_{\mathbf{p}}]$$

$$s_0(T) = \frac{gM(T)^3}{2\pi^2} K_3(M(T)/T).$$

- ■ The quasiparticles are effective degrees of freedom (neither quarks nor gluons)

Known from lattice data

Borsanyi et al (Wuppertal-Budapest), JHEP 11, 077 (2010);



Quasiparticle model

Thermal-mass quasiparticle model

- Quasiparticle kinetic theory toy model

P. Romatschke *PRD*, *85*(6), 065012 (2012) .
Calzetta, E., & Hu, B. L.. *PRD 37*(10), 2878 (1988).
Alqahtani et al PRC 92, 054910 (2015)

Jeon & Yaffe PRD, *53*(10), 5799 (1996);
Jeon PRD 52 3591-3642 (1995)

$$p^\mu \partial_\mu f_{\mathbf{p}} + \frac{1}{2}\partial_\mu M^2(T)\partial^\mu_{(p)} f_{\mathbf{p}} = C[f_{\mathbf{p}}]$$

- Traditional $T^{\mu\nu}$ gets modified:

$$T^{\mu\nu} \equiv \int dP p^\mu p^\nu f_{\mathbf{p}} + g^{\mu\nu} B$$

Generic non-equilibrium relation

$$\partial_\mu B = -\frac{1}{2}\partial_\mu M^2 \int dP f_{\mathbf{p}}$$

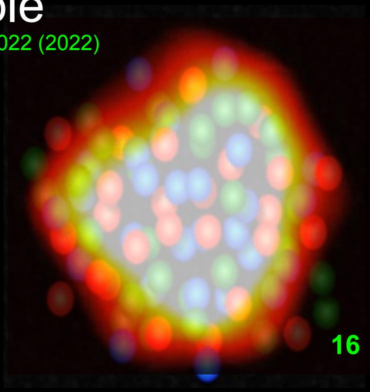
- Judicious definition of temperature: *B* = *B(T)* an equilibrium variable

GSR, Ferreira, Denicol, Noronha, PRD 106, 036022 (2022)

$$\boxed{\int dP f_{\mathbf{p}} \equiv \int dP f_{0\mathbf{p}}}$$

$$T^\mu_\nu u^\nu \equiv \varepsilon u^\mu$$

energy density with dissipative corrections



Thermal-mass quasiparticle model

- Alternative definition of temperature: must use modified version of RTA

GSR, Denicol, Noronha PRL 127, 042301 (2021)
GSR, Ferreira, Denicol, Noronha, PRD 106, 036022 (2022)

$$C[f_{\mathbf{p}}] \underset{\sim}{\propto} -\mathbb{1} + |p^\mu\rangle\langle p^\mu|,$$

Traditional RTA

Counter terms to ensure local conservation laws

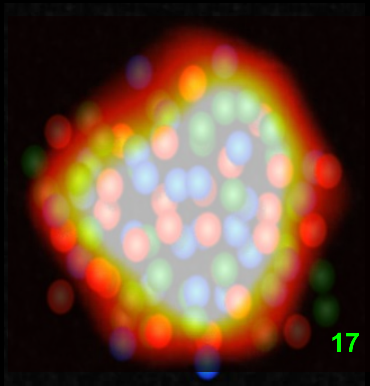
Procedure:

Boltzmann → Hydro truncation + Relaxation time approximation* → Transient equations → change to Landau matching

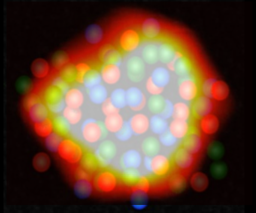
$$\boxed{T^{\mu\nu}_Q = T^{\mu\nu}_L}$$

$$u^\mu_Q = u^\mu_L,$$

$$\begin{aligned}\varepsilon_0(T_Q) + \delta\varepsilon_Q &= \varepsilon_0(T_L)\\ P_0(T_Q) + \Pi_Q &= P_0(T_L) + \Pi_L,\\ \pi^{\mu\nu}_Q &= \pi^{\mu\nu}_L,\end{aligned}$$



Relaxation times, high-temperature limit

	$\tau_\pi = \frac{\#\eta}{\varepsilon_0 + P_0}$	$\tau_\Pi = \frac{\zeta}{\#(\varepsilon_0 + P_0)(1/3 - c_s^2)^2}$
MUSIC Denicol, Jeon, Gale PRC 90 024912 (2014)	5	15
Hadron-resonance toy-model GSR & Denicol [arXiv:2402.06996] 	5	$18\frac{\left(\sum_j g_j\right)\left(\sum_j g_j m_j^4\right)}{\left(\sum_j g_j m_j^2\right)^2} - 3$
Quasiparticle toy-model GSR & Denicol [arXiv:2402.06996] 	5	$\tau_\Pi \simeq \frac{\zeta}{5\left(\frac{1}{3} - c_s^2\right)(\varepsilon_0 + P_0)}$

Relaxation times, high-temperature limit

$$\tau_\pi = \frac{\#\eta}{\varepsilon_0 + P_0}$$

$$\tau_\Pi = \frac{\zeta}{\#(\varepsilon_0 + P_0)(1/3 - c_s^2)^2}$$

MUSIC

Denicol, Jeon, Gale PRC 90 024912 (2014)

5

15

Hadron-resonance toy-model

GSR & Denicol [arXiv:2402.06996]

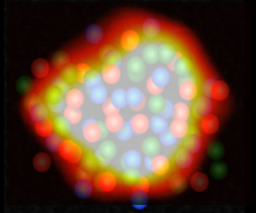
5

16.91 (UrQMD) 293 particles

19.36 (SMASH) 411 particles

Quasiparticle toy-model

GSR & Denicol [arXiv:2402.06996]

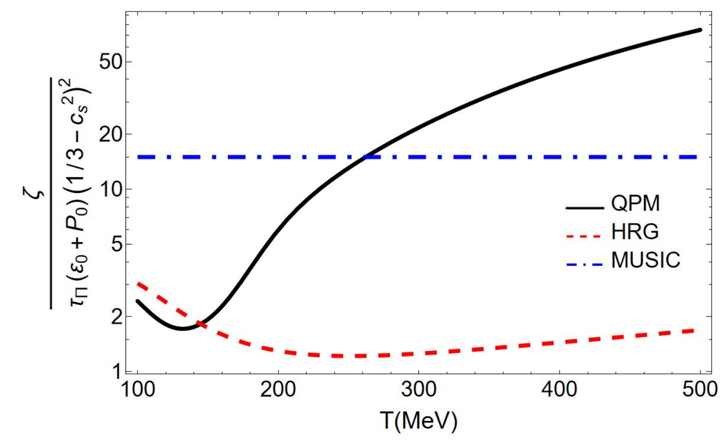


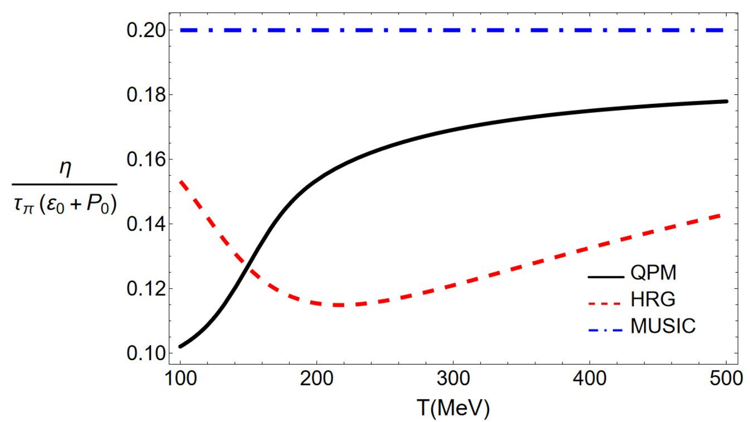
5

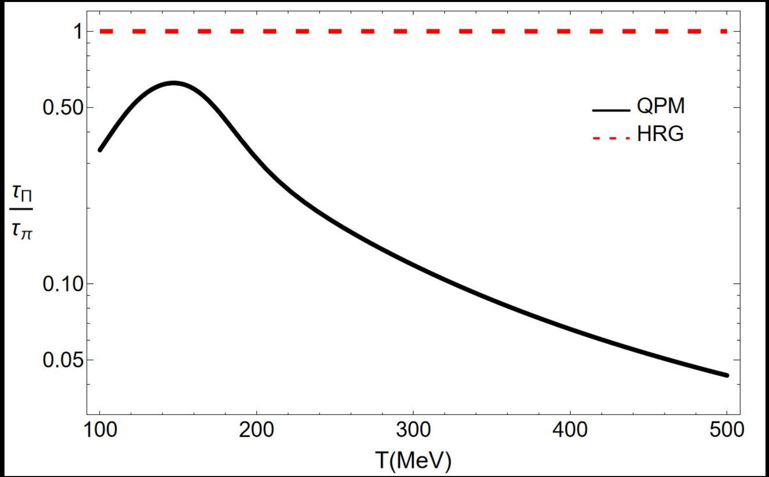
↓Qualitative behavior akin to holography

$$\tau_\Pi \simeq \frac{\zeta}{5\left(\frac{1}{3} - c_s^2\right)(\varepsilon_0 + P_0)}$$

Relaxation times – smaller temperatures



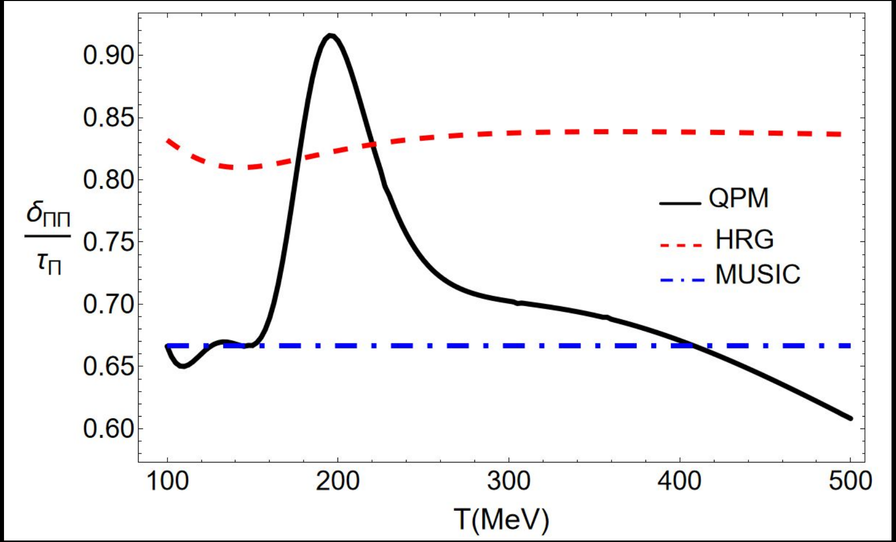


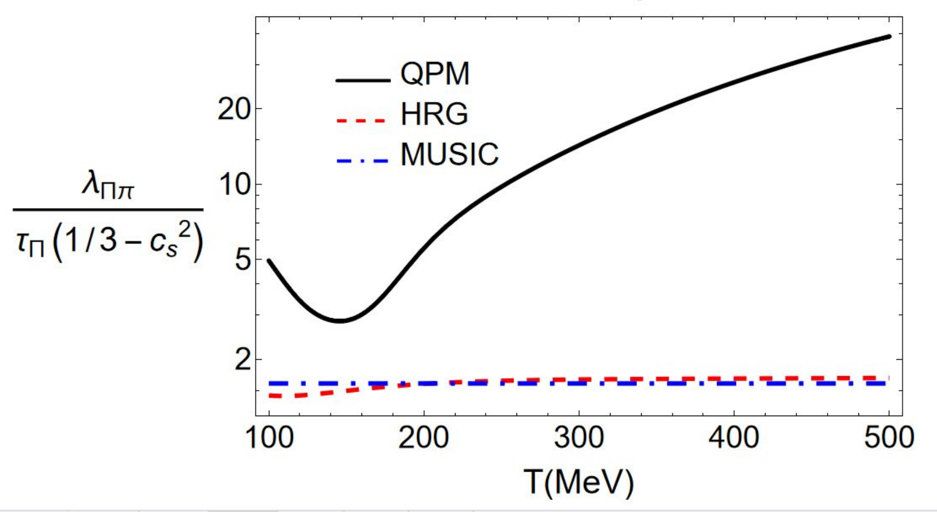


GSR & Denicol [arXiv:2402.06996]

Bulk second-order transport coefficients

$$\tau_\Pi D\Pi + \Pi = -\zeta\theta - \delta_{\Pi\Pi}\Pi\theta + \lambda_{\Pi\pi}\pi^{\mu\nu}\sigma_{\mu\nu},$$

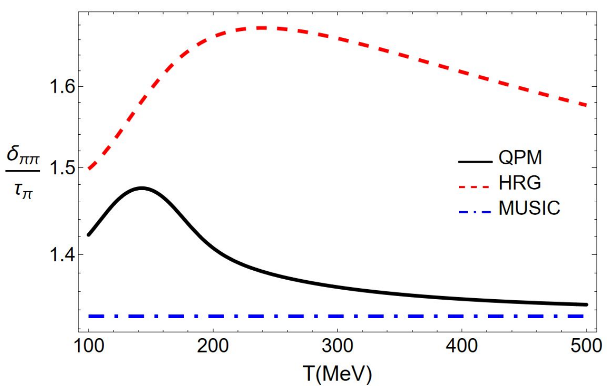


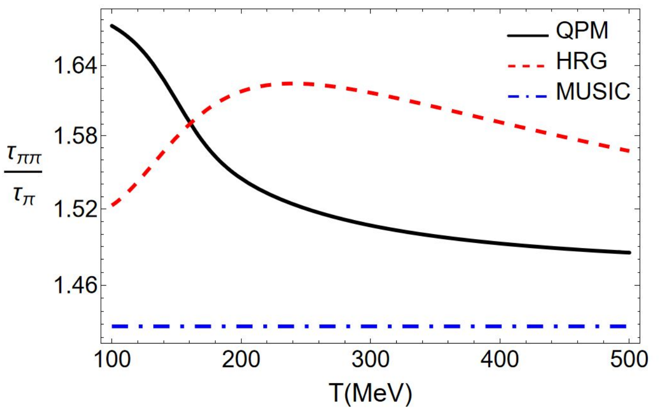


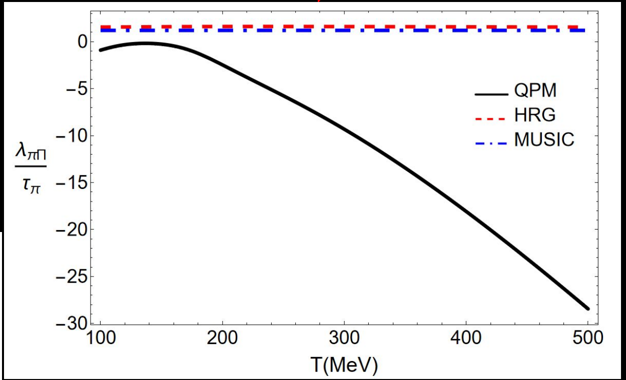
GSR & Denicol [arXiv:2402.06996]

Shear second-order transport coefficients

$$\tau_\pi D\pi^{\langle\alpha\beta\rangle} + \pi^{\alpha\beta} = 2\eta\sigma^{\alpha\beta} - \delta_{\pi\pi}\pi^{\alpha\beta}\theta - 2\tau_\pi\omega_\mu{}^{\langle\alpha}\pi^{\beta\rangle\mu} - \tau_{\pi\pi}\sigma_\mu{}^{\langle\alpha}\pi^{\beta\rangle\mu} + \lambda_{\pi\Pi}\Pi\sigma^{\alpha\beta},$$



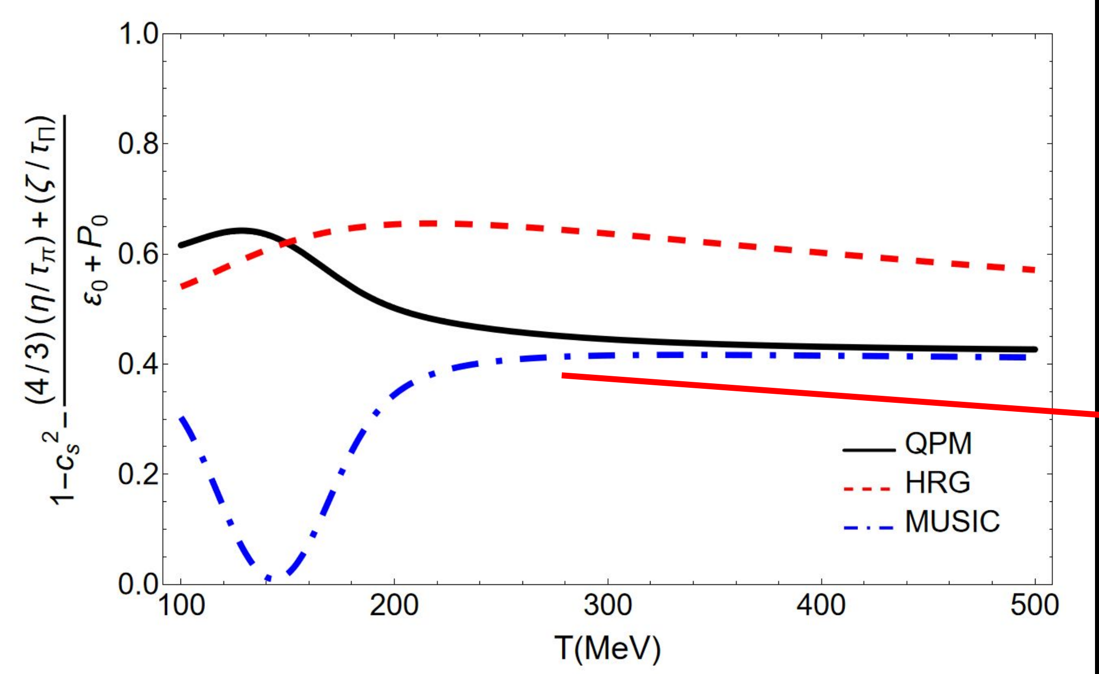




GSR & Denicol [arXiv:2402.06996]

Linear causality

- Constraint on relaxation times $$1 - c_s^2 - \frac{1}{\varepsilon_0 + P_0}\left(\frac{4}{3}\frac{\eta}{\tau_\pi} + \frac{\zeta}{\tau_\Pi}\right) \geq 0$$



Pu, Koide, & Rischke, Phys. Rev. D 81, 114039 (2010)
Olson, Ann. Phys. 199, 18 (1990).
Hiscock & Lindblom, Ann. Phys. 151, 466 (1983).

Can be the source of non-linear causality violations

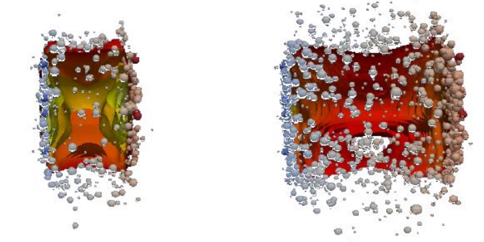
Plumberg et al, PRC 105, L061901 (2022)
Krupczak et al, (2023), arXiv:2311.02210 [nucl-th]

GSR & Denicol [arXiv:2402.06996]

Conclusions

- We have provided updated expressions for various transient hydro transport coefficients using the HRG and QPM;
- We find that:
    - the normalized bulk viscosity has different expressions in the high temperature limit for both models;
    - transport coefficients related to bulk are usually sensitive to temperature;
- Future: finite chemical potential; non-linear causality; momentum-dependent relaxation time.

GSR personal archive



THAT'S ALL FOR TODAY!

BACKUP SLIDES

Transient hydro and Boltzmann moments

Denicol and Rischke. *Microscopic Foundations of Relativistic Fluid Dynamics*. Springer, 2021
Denicol Niemi Molnar Rischke PRD 85, 114047 (2012);

Boltzmann eqn. dynamics→ Moments dynamics

$$\rho_r^{\mu_1\cdots\mu_\ell} = \int dP E_{\mathbf{p}}^r p^{\langle\mu_1} \cdots p^{\mu_\ell\rangle} \delta f_{\mathbf{p}}, \longrightarrow$$

deviation from local equilibrium $\delta f_{\mathbf{p}} \equiv f_{\mathbf{p}} - f_{0\mathbf{p}}$

In particular, $\Pi = \frac{1}{3}(\rho_2 - m^2 \rho_0) \quad \pi^{\mu\nu} = \rho_0^{\mu\nu}$

Transient hydro and Boltzmann moments

Denicol and Rischke. *Microscopic Foundations of Relativistic Fluid Dynamics*. Springer, 2021
Denicol Niemi Molnar Rischke PRD 85, 114047 (2012);

Boltzmann eqn. dynamics→ Moments dynamics

de Brito, Denicol 2401.10098 Full EoMs

$$\rho_r^{\mu_1\cdots\mu_\ell} = \int dPE_{\mathbf{p}}^r p^{\langle\mu_1}\cdots p^{\mu_\ell\rangle}\delta f_{\mathbf{p}},$$

For example:

$$D\rho_r^{\langle\mu\nu\rangle} + \cdots + \alpha_r^{(2)}\sigma^{\mu\nu} = -\sum_n \mathcal{L}_{rn}^{(2)}\rho_n^{\mu\nu}$$

Coupling terms **Navier-Stokes** **Relaxation times**

Hydrodynamics:
reduction of d.o.f's

$$\rho_r^{\mu_1\cdots\mu_\ell} \longrightarrow \{\Pi, \pi^{\mu\nu}\}$$

(Landau matching)

Transient hydro and Boltzmann moments

Denicol and Rischke. *Microscopic Foundations of Relativistic Fluid Dynamics*. Springer, 2021
Denicol Niemi Molnar Rischke PRD 85, 114047 (2012);

Boltzmann eqn. dynamics→ Moments dynamics

$$\rho_r^{\mu_1\cdots\mu_\ell} = \int dPE_{\mathbf{p}}^r p^{\langle\mu_1}\cdots p^{\mu_\ell\rangle}\delta f_{\mathbf{p}},$$

$$\rho_{r,i}^{\mu\nu} = 2\eta_{r,i}\sigma^{\mu\nu} + \mathcal{O}(2),$$

$$\rho_r^{\mu\nu} = \frac{\eta_r}{\eta}\pi^{\mu\nu} \equiv \mathcal{C}_r\pi^{\mu\nu} + \mathcal{O}(2).$$

Struchtrup, Physics of Fluids 16, 3921 (2004)
Fotakis et al PRD 106, 036009 (2022)
Wagner et a PRD 016013 (2022)

Hydrodynamics: reduction of d.o.f's

$$\rho_r^{\mu_1\cdots\mu_\ell} \longrightarrow \{\Pi, \pi^{\mu\nu}\}$$

(Landau matching)

Temperature matching

- We impose that $T_Q^{\mu\nu} = T_L^{\mu\nu}$, and since $u_Q^{\mu} = u_L^{\mu}$,

$$\begin{aligned} \varepsilon_0(T_Q) + \delta\varepsilon_Q &= \varepsilon_0(T_L) \\ P_0(T_Q) + \Pi_Q &= P_0(T_L) + \Pi_L, \\ \pi_Q^{\mu\nu} &= \pi_L^{\mu\nu}, \end{aligned}$$

$$T_Q = T_L - \frac{3\Pi_L}{(\partial\varepsilon_0/\partial T_L)(1-3c_s^2)} + \mathcal{O}(2),$$

$$\Pi_Q = \frac{\Pi_L}{1-3c_s^2} - \frac{9}{2}\frac{\partial c_s^2}{\partial\varepsilon_0}\frac{\Pi_L^2}{(1-3c_s^2)^3} + \mathcal{O}(3).$$

The relaxation time approximation

- All hydro models require inversion of the lin. collision matrix (highly non-trivial);
- Relaxation time approximation (RTA)

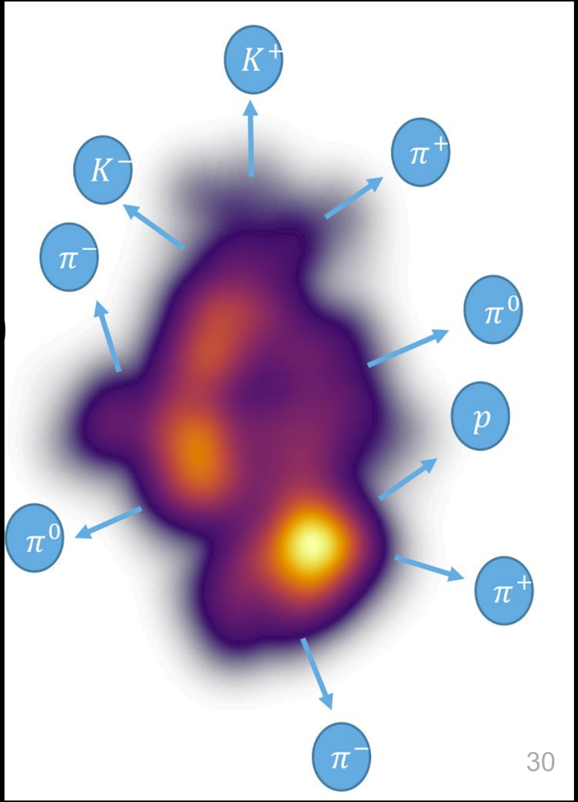
J. L. Anderson and H. Witting, Physica 74, 466 (1974)

$$f_{0\mathbf{p}}\hat{L}\phi_{\mathbf{p}} \simeq -\frac{u_\mu p^\mu}{\tau_R}\left(f_{\mathbf{p}} - f_{0\mathbf{p}}\right)$$

☐widely used in HIC phenomenology: e.g. particlization

☐limited scope
(constant τ_R and Landau matching conditions)

Credit: http://jetscape.org/sims/



Thermal-mass quasiparticle model

- Alternative matching obliges the use of a modified RTA

GSR, Denicol, Noronha PRL 127, 042301 (2021)
GSR, Ferreira, Denicol, Noronha, PRD 106, 036022 (2022)

$$C[f_\mathbf{p}] \underset{\sim}{\propto} -\mathbb{1} + |p^\mu\rangle\langle p^\mu|,$$

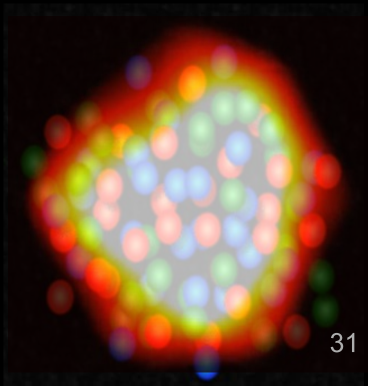
Traditional RTA

Projector in the subspace of conserved quantities in an ***orthogonal basis***

$$C_Q[f_\mathbf{p}] \simeq -\frac{E_\mathbf{p}}{\tau_R} f_{0\mathbf{p}} \left[\phi_\mathbf{p} - \frac{\langle \phi_\mathbf{p} E_\mathbf{p}^2 \rangle_0}{I_{3,0}} E_\mathbf{p} + \frac{\langle \phi_\mathbf{p} E_\mathbf{p} p^{\langle\mu\rangle} \rangle_0}{I_{3,1}} p_{\langle\mu\rangle} \right]$$

$$I_{3,0} = \langle E_\mathbf{p}^3 \rangle_0$$

$$f_{0\mathbf{p}}\phi_\mathbf{p} = f_\mathbf{p} - f_{0\mathbf{p}} \quad \langle \cdots \rangle_0 = \int dP \cdots f_{0\mathbf{p}} \qquad I_{3,1} = (1/3)\langle E_\mathbf{p} \mathbf{p}^2 \rangle_0$$



Thermal-mass quasiparticle model

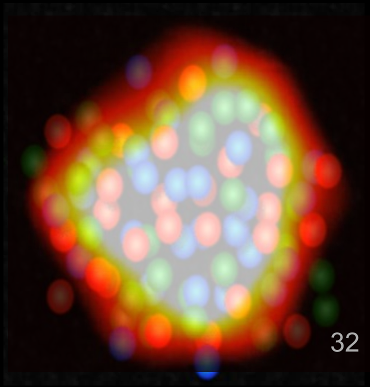
- Alternative matching obliges the use of a modified RTA GSR, Denicol, Noronha PRL 127, 042301 (2021) GSR, Ferreira, Denicol, Noronha, PRD 106, 036022 (2022)

$$C[f_{\mathbf{p}}] \overset{\propto}{\sim} -\mathbb{1} + |p^\mu\rangle\langle p^\mu|,$$

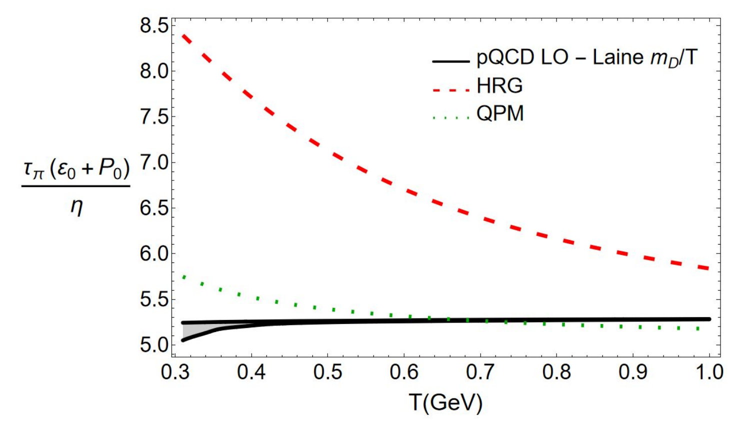
Traditional RTA

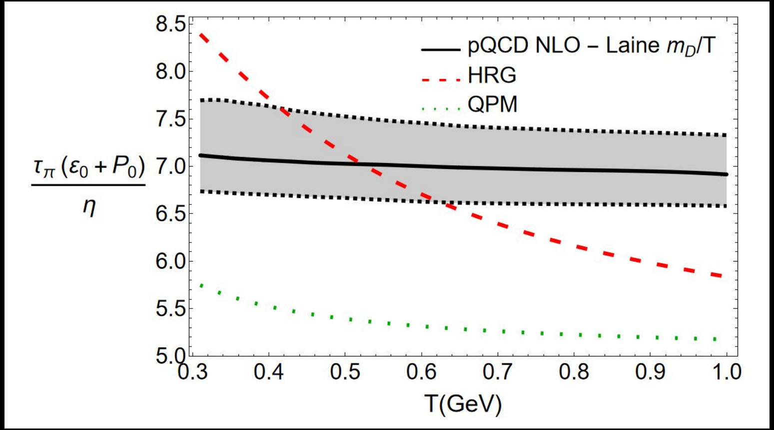
Projector in the subspace of conserved quantities in an ***orthogonal basis***

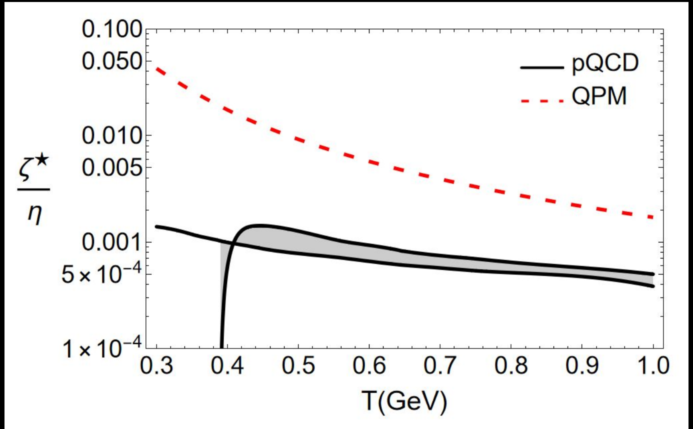
□recovery of fundamental properties of the collision term $\hat{L}1 = 0, \;\; \hat{L}p^\mu = 0.$



Comparison with pQCD

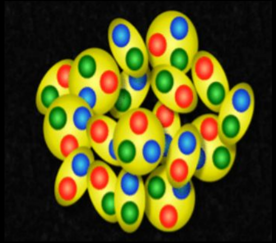
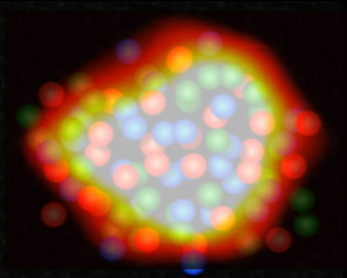






pQCD data - Ghiglieri, Moore, Teaney Phys. Rev. Lett. 121, 052302 (2018)
Debye mass - Laine, Schicho and Schroeder PRD 101 023532
Lattice data - Bazavov et al (HotQCD) PRD 90 094503 (2014), arXiv 1407.6387 [hep-lat];

New results – high-temperature limit, other coefficients

	$\frac{\delta_{\Pi\Pi}}{\tau_\Pi}$	$\frac{\lambda_{\Pi\pi}}{\tau_\Pi(1/3-c_s^2)}$	$\frac{\delta_{\pi\pi}}{\tau_\pi}$	$\frac{\tau_{\pi\pi}}{\tau_\pi}$	$\frac{\lambda_{\pi\Pi}}{\tau_\pi}$
	$\frac{2}{3}$	$\frac{8}{5}$	$\frac{4}{3}$	$\frac{10}{7}$	$\frac{6}{5}$
	$\frac{2}{3}$	2	$\frac{4}{3}$	$\frac{10}{7}$	$\frac{6}{5}$
		$\frac{2}{3}$	$\frac{4}{3}$	$\frac{10}{7}$	$-\frac{3456}{5}\left(\frac{1}{3}-c_s^2\right)^{-1}$