

# Multi-Channel Scattering Calculation for Ultracold Ion-Atom Collisions

Competing excitation quenching and charge exchange in ultracold Li-Ba<sup>+</sup> collisions

HUN  
REN



UNIVERSITY of  
DEBRECEN

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collaboration with University of Freiburg



université  
PARIS-SACLAY

# Introduction

- Ultracold quantum gases (e.g. atoms, molecules, ions, ...)



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- atom – ion mixed system in hybrid traps
- dilute gases  $(10^8 - 10^{13} \text{ atom/cm}^3)$   $\longrightarrow$  long-range interactions relevant: atom-ion  $\sim \frac{1}{R^4}$
- application: precision measurements,  
quantum controlled chemistry,  
quantum technology, ...

# Introduction

- The project focuses on the **theoretical modelling** of the dynamics of **ion-neutral systems at ultracold temperatures** in order to design ways for their full quantum control
- Our aims are connected to **ongoing** cutting-edge experimental investigations with hybrid traps (Uni. Freiburg, Weizmann Inst., Uni. Basel, Uni. Ulm, Uni. Osaka, Uni. Hannover)
- Understanding ion-neutral ultracold collisions

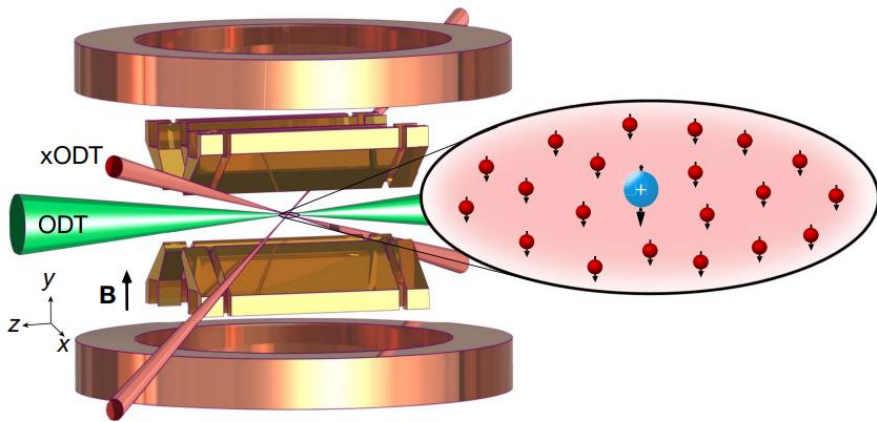
# Introduction

- Understanding such ultracold collisions relies on the combination of various expertise:
  - perform detailed atomic and molecular structure calculations
  - determine electronically excited molecular states generated by lasers in experiments
  - perform dynamical calculations based on the quantum coupled-channel description of the collision including both long-range (electrostatic) and short-range (chemical) interactions
- These methodologies will be applied to two species of atom-ion pairs of relevance for ongoing experiments:

${}^6\text{Li} - {}^{138}\text{Ba}^+$ , in collab. with Uni. Freiburg  
 ${}^{87}\text{Rb} - {}^{88}\text{Sr}^+$ , in collab. with Weizmann Inst.

# Preparation

Experiment: Uni. Freiburg

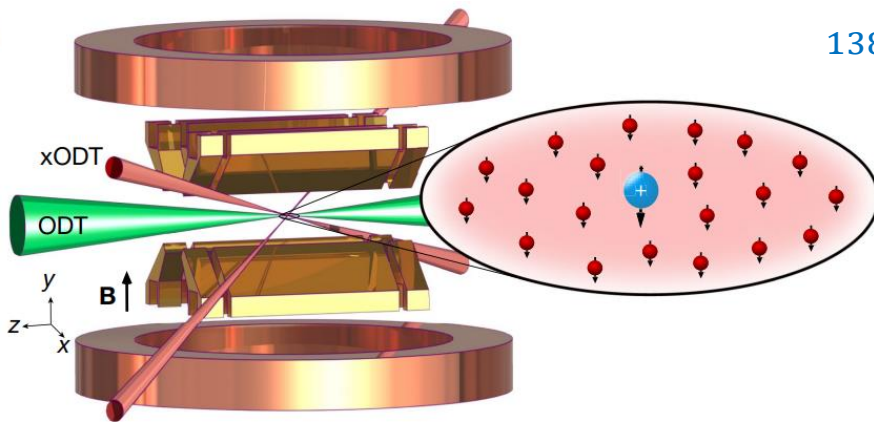


P. Weckesser et. al, Nature, 600:429, 2021



# Preparation

Experiment: Uni. Freiburg



$^{138}\text{Ba}^+$ : single ion

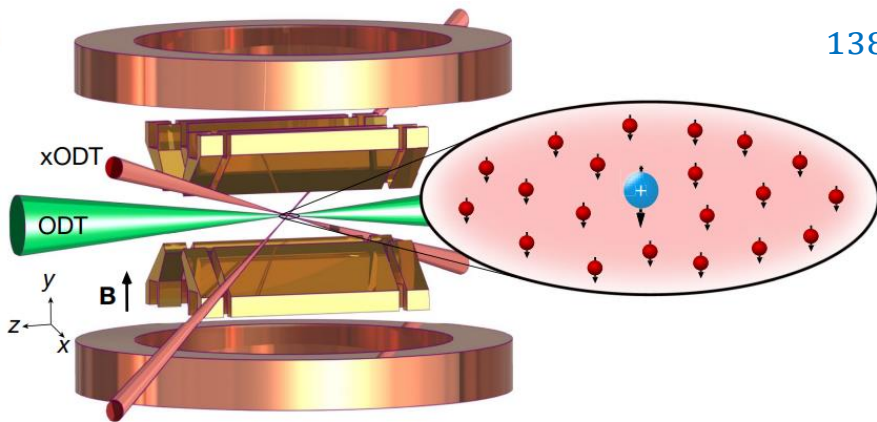
$\sim 365 \mu\text{K}$  in the lab

metastable  $5D_{5/2,3/2}$  excited state  
(lifetime 32 s, 80 s)

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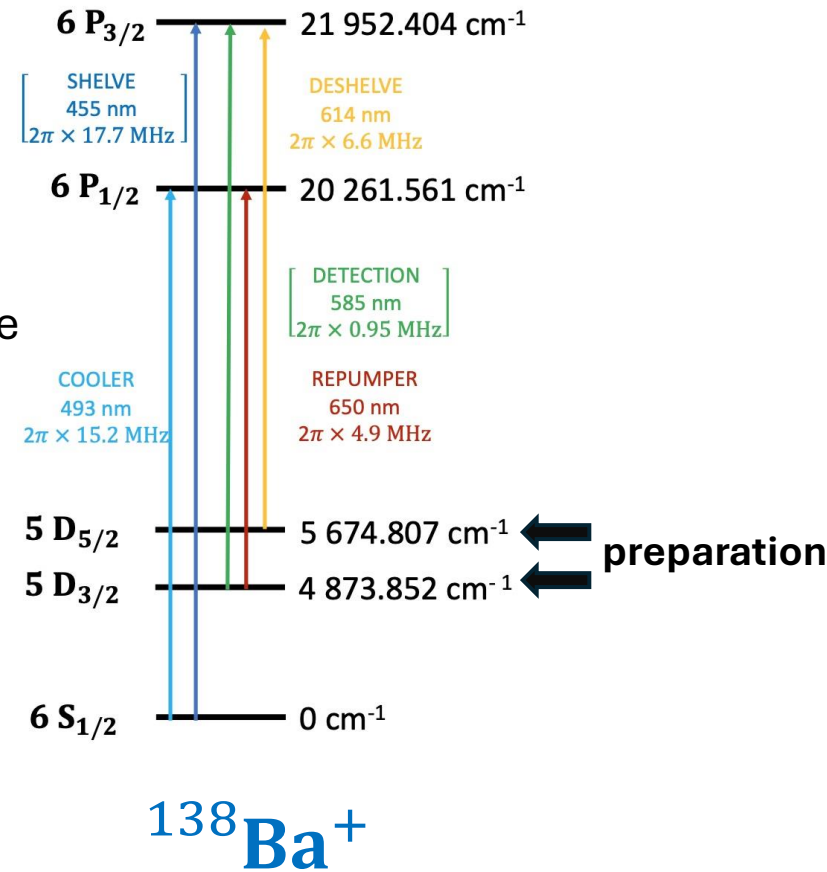
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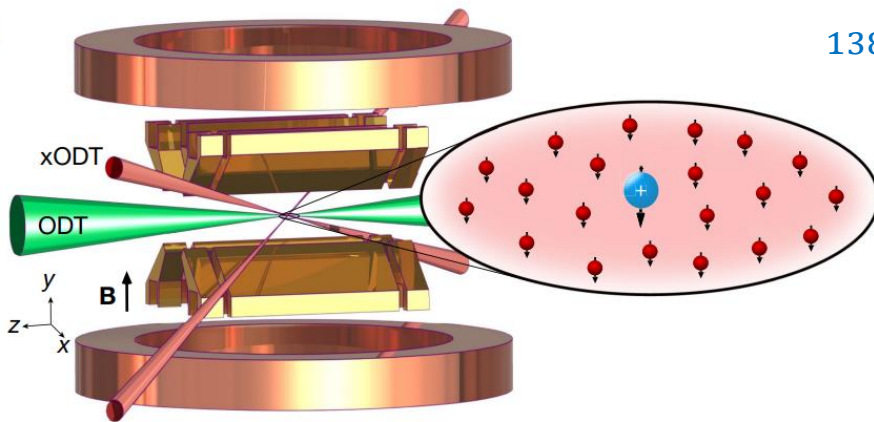
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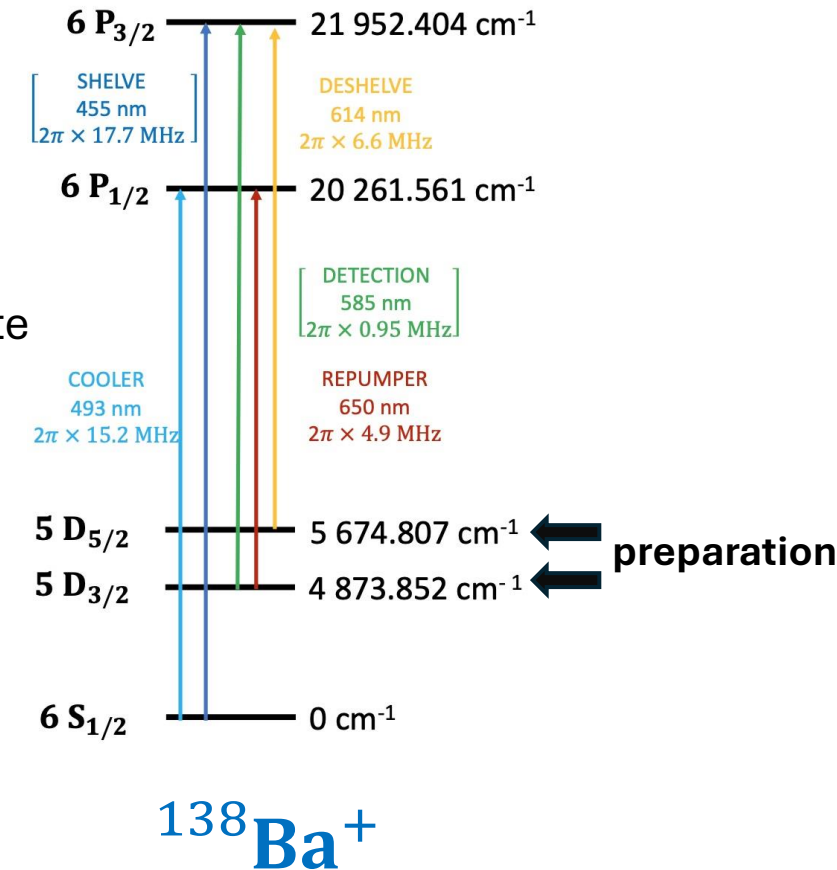
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$^6\text{Li}$ : 1 – 3  $\mu\text{K}$  in the lab

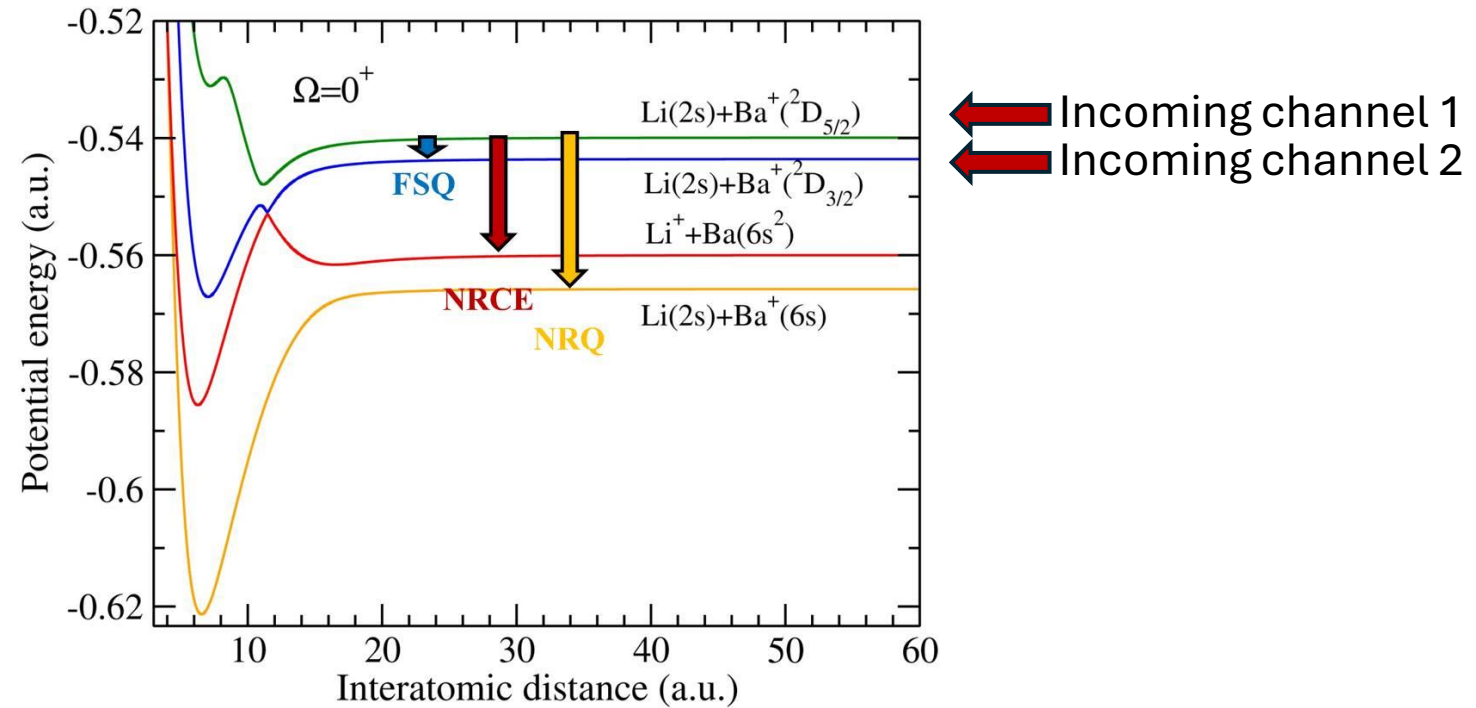
$2S_{1/2}$  ground state

few thousand atom  
close to quantum degeneracy



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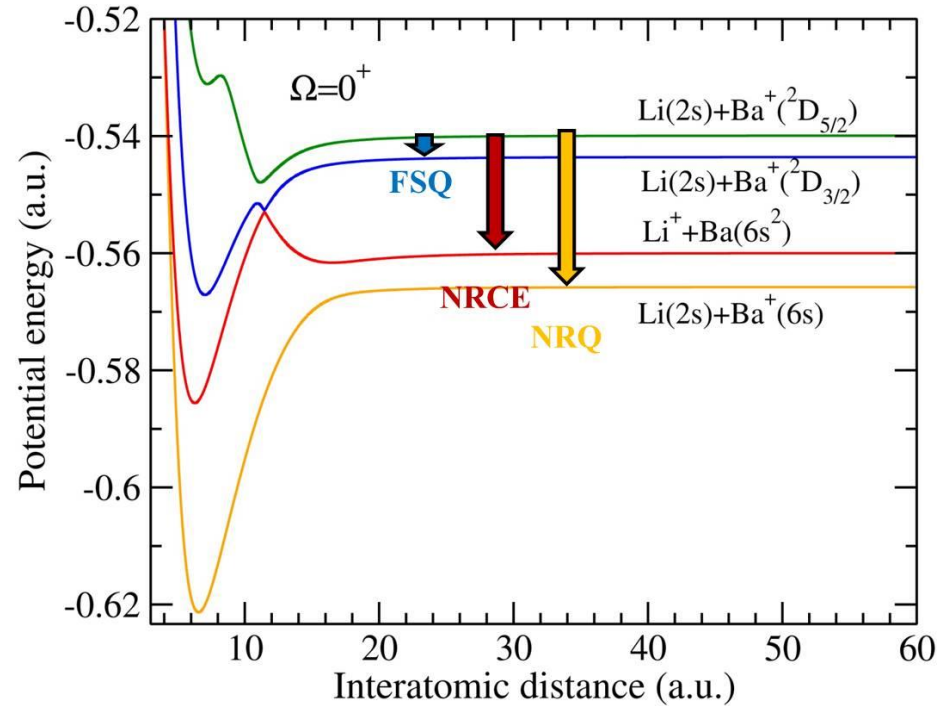
# Observed processes



**Purpose:** theoretical description of the observed processes

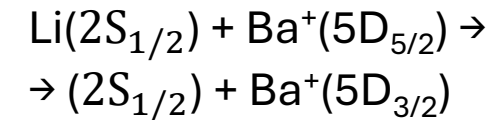
Molecular structure calculated by THEOMOL group

# Observed processes

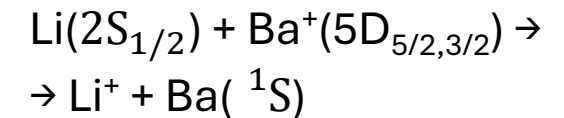


← Incoming channel 1  
← Incoming channel 2

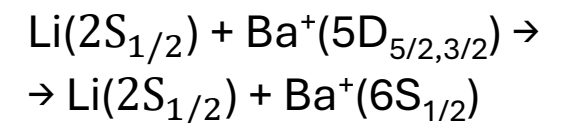
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- NRCE (Non-Radiative Charge Exchange)



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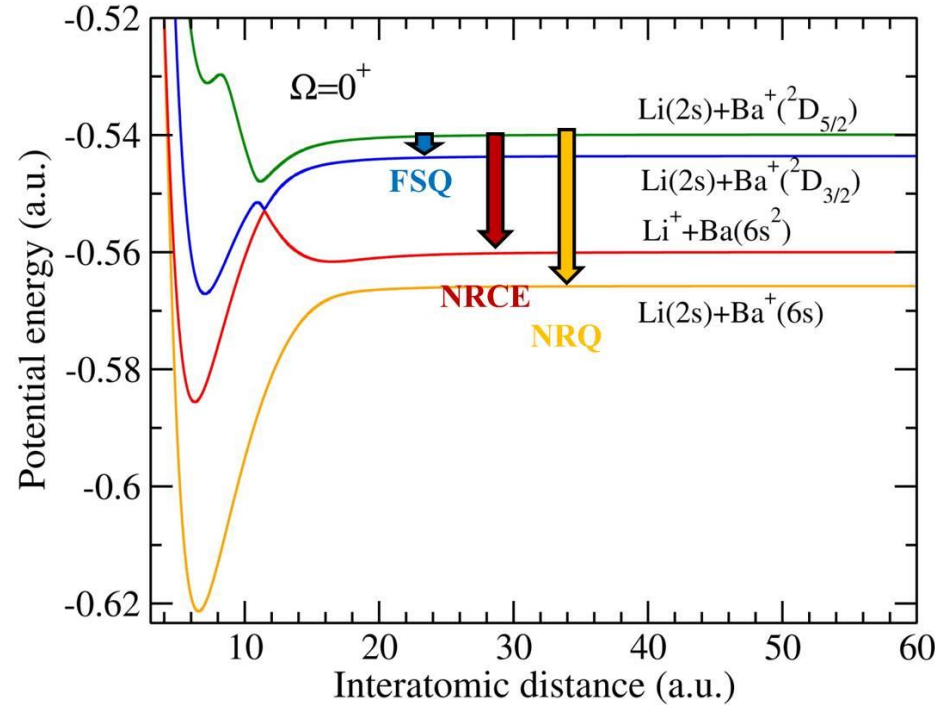
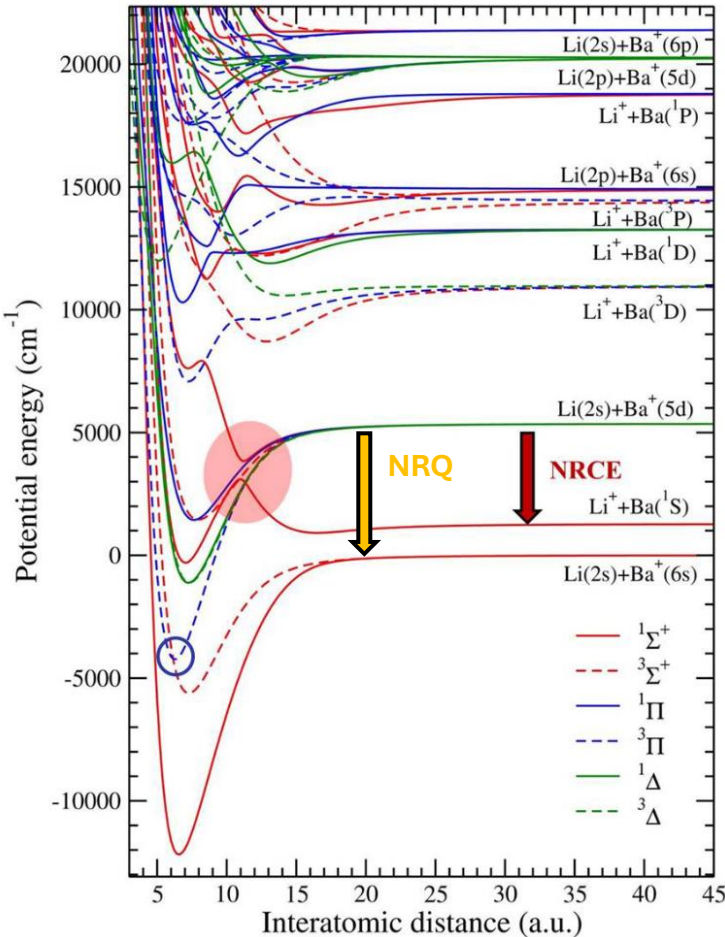


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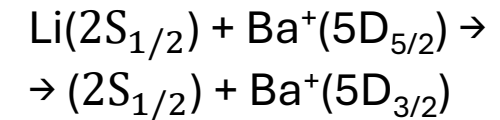
# Observed processes

## Hund's case a)

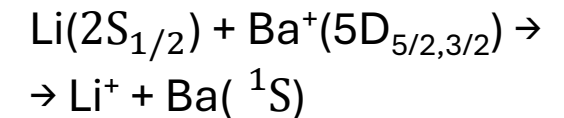


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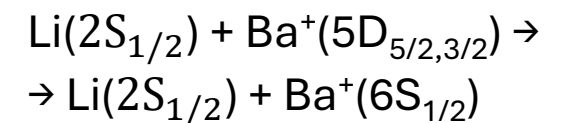
- FSQ (Fine Structure Quenching)



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# Potential energy in the molecular frame

$$H(R) = -\frac{\hbar^2}{2\mu} \frac{d^2}{dR^2} + \underbrace{V(R) + V_{SOC}(R)}$$

	$\Omega=0^+$				$\Omega=0^-$			$\Omega=1$			$\Omega=2$			$\Omega=3$		
	S+S	Ion+S	S+D		S+S	S+D		S+S	S+D		S+D			S+D		
	$1^1\Sigma^+$	$2^1\Sigma^+$	$3^1\Sigma^+$	$1^3\Pi$	$1^3\Sigma^+$	$2^3\Sigma^+$	$1^3\Pi$	$1^3\Sigma^+$	$2^3\Sigma^+$	$1^1\Pi$	$1^3\Pi$	$1^3\Delta$	$1^3\Pi$	$1^1\Delta$	$1^3\Delta$	$1^3\Delta$
$1^1\Sigma^+$	$V(1^1\Sigma^+)$	0	0	$A_{1,4}$												
$2^1\Sigma^+$	0	$V(2^1\Sigma^+)$	$G_{2,3}$	$A_{2,4}$												
$3^1\Sigma^+$	0	$G_{3,2}$	$V(3^1\Sigma^+)$	$A_{3,4}$												
$1^3\Pi$	$A_{4,1}$	$A_{4,2}$	$A_{4,3}$	$V(1^3\Pi) + A_{4,4}$												
$1^3\Sigma^+$					$V(1^3\Sigma^+)$	0	$A_{5,7}$									
$2^3\Sigma^+$					0	$V(2^3\Sigma^+)$	$A_{6,7}$									
$1^3\Pi$					$A_{7,5}$	$A_{7,6}$	$V(1^3\Pi) + A_{7,7}$									
$1^3\Sigma^+$								$V(1^3\Sigma^+)$	0	$A_{8,10}$	$A_{8,11}$	0				
$2^3\Sigma^+$								0	$V(2^3\Sigma^+)$	$A_{9,10}$	$A_{9,11}$	0				
$1^1\Pi$								$A_{10,8}$	$A_{10,9}$	$V(1^1\Pi)$	$A_{10,11}$	$A_{10,12}$				
$1^3\Pi$								$A_{11,8}$	$A_{11,9}$	$A_{11,10}$	$V(1^3\Pi)$	$A_{11,12}$				
$1^3\Delta$								0	0	$A_{12,10}$	$A_{12,11}$	$V(1^3\Delta) + A_{12,12}$				
$1^3\Pi$													$V(1^3\Pi) + A_{13,13}$	$A_{13,14}$	$A_{13,15}$	
$1^1\Delta$													$A_{14,13}$	$V(1^1\Delta)$	$A_{14,15}$	
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16 × 16

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$V(\dots)$  – Hund's case a) potential energy curves

$A_{i,j}$  – spin – orbit coupling matrix element

$G_{2,3}$  – non-adiabatic coupling at the avoided crossing

$A_{5,7}$  – indirect spin – orbit coupling at the crossing



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$1^1\Pi$								$A_{10,8}$	$A_{10,9}$	$V(1^1\Pi)$	$A_{10,11}$	$A_{10,12}$				
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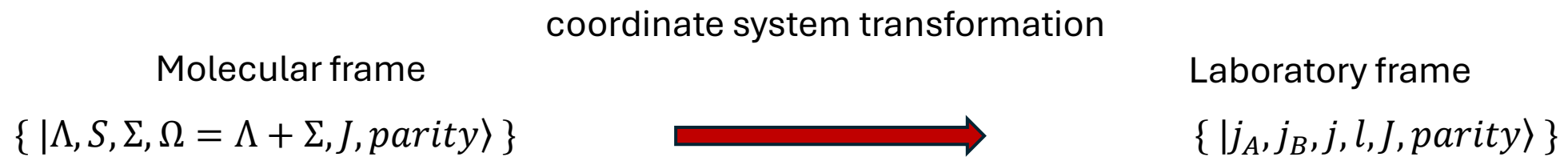
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Diagonalization

Hund's case c) potential energy curves

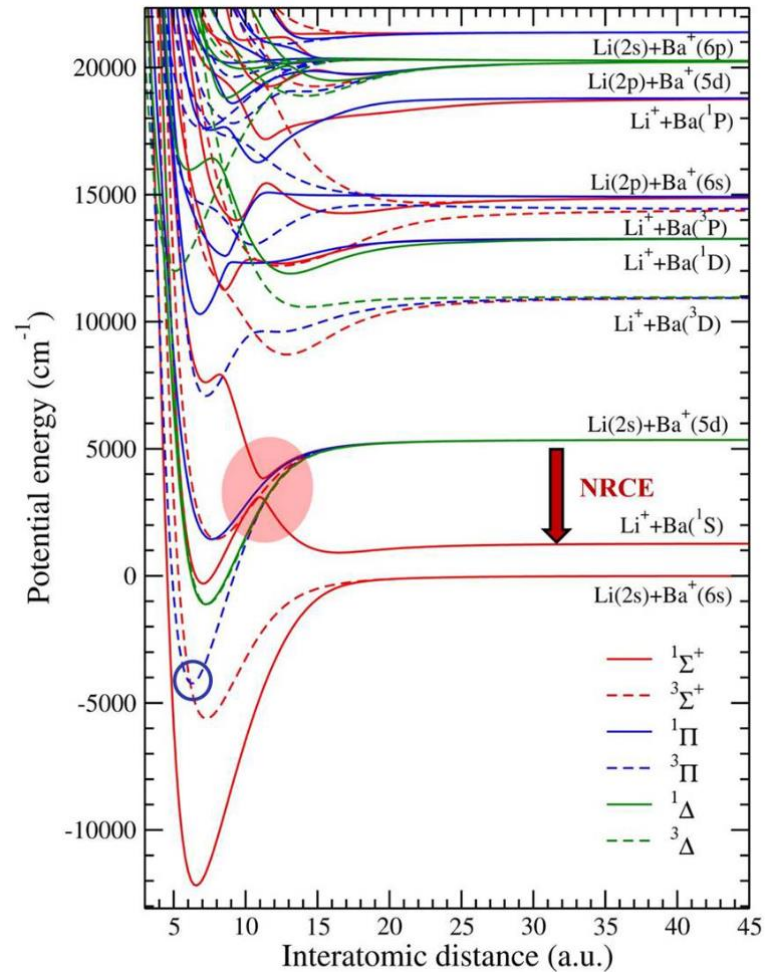
# Frame transformation



# Molecular structure

Molecular frame

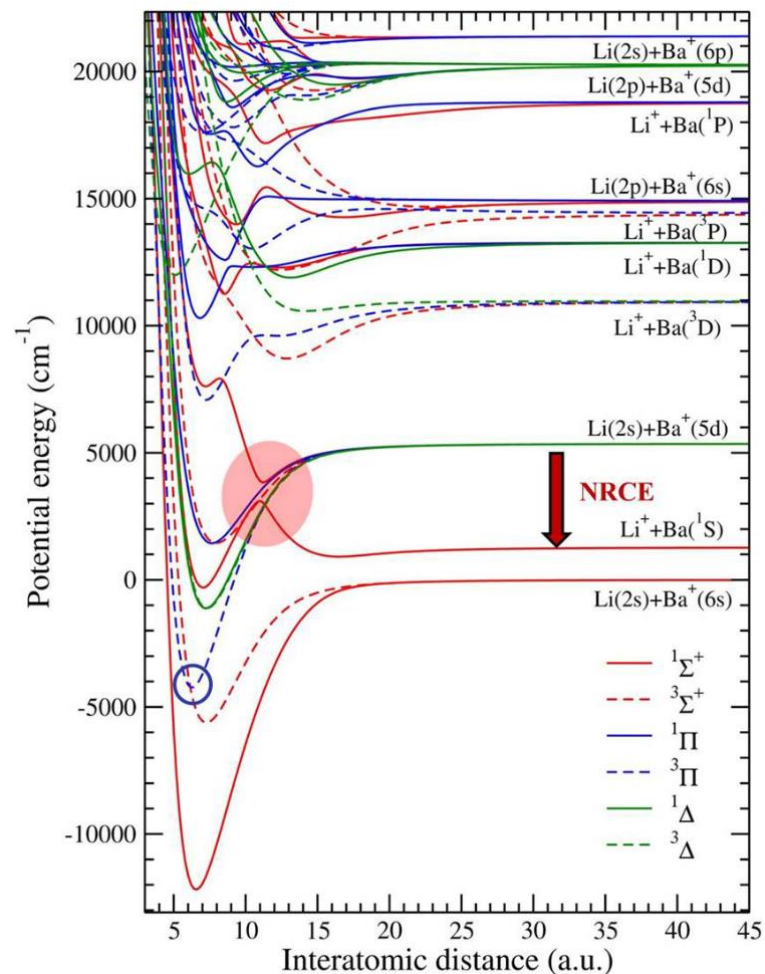
Hund's case a)



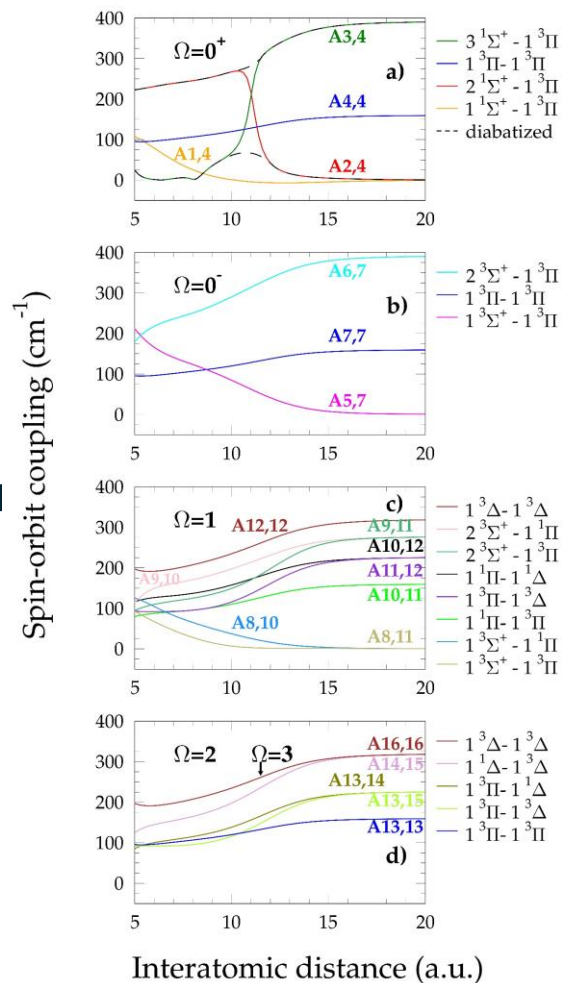
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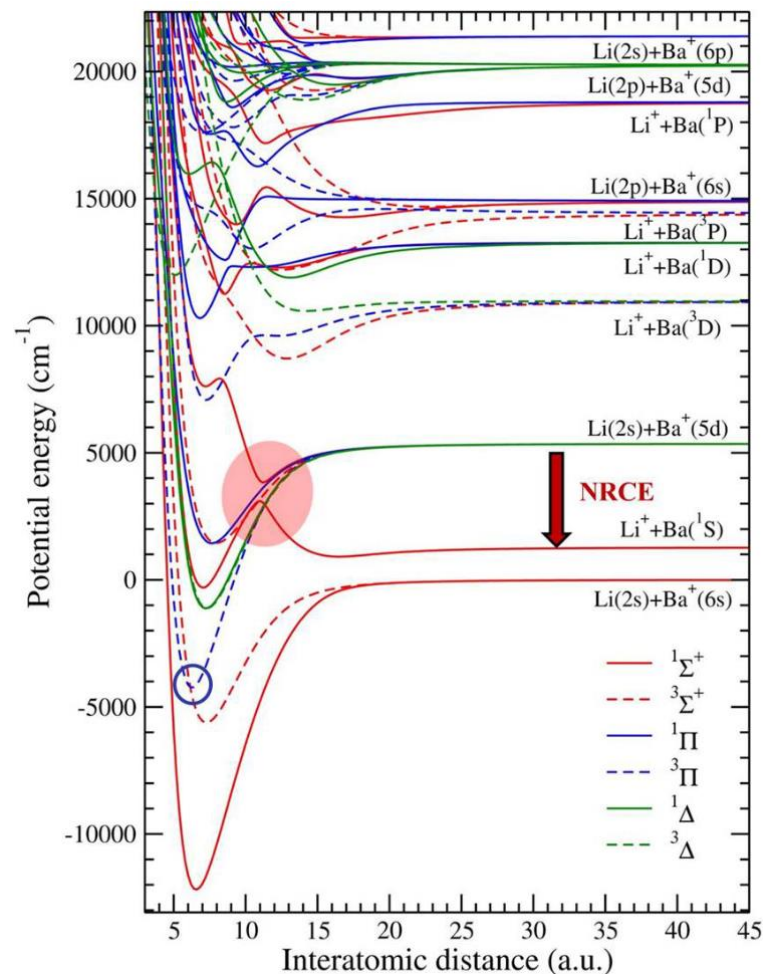
Spin - orbit coupling



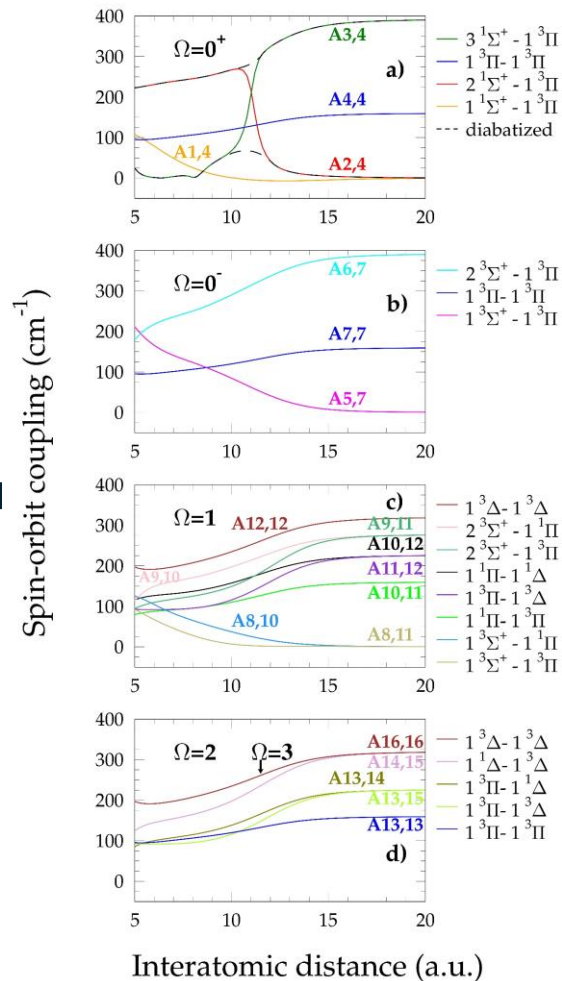
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Molecular frame

Hund's case a)



Spin - orbit coupling



Transformation

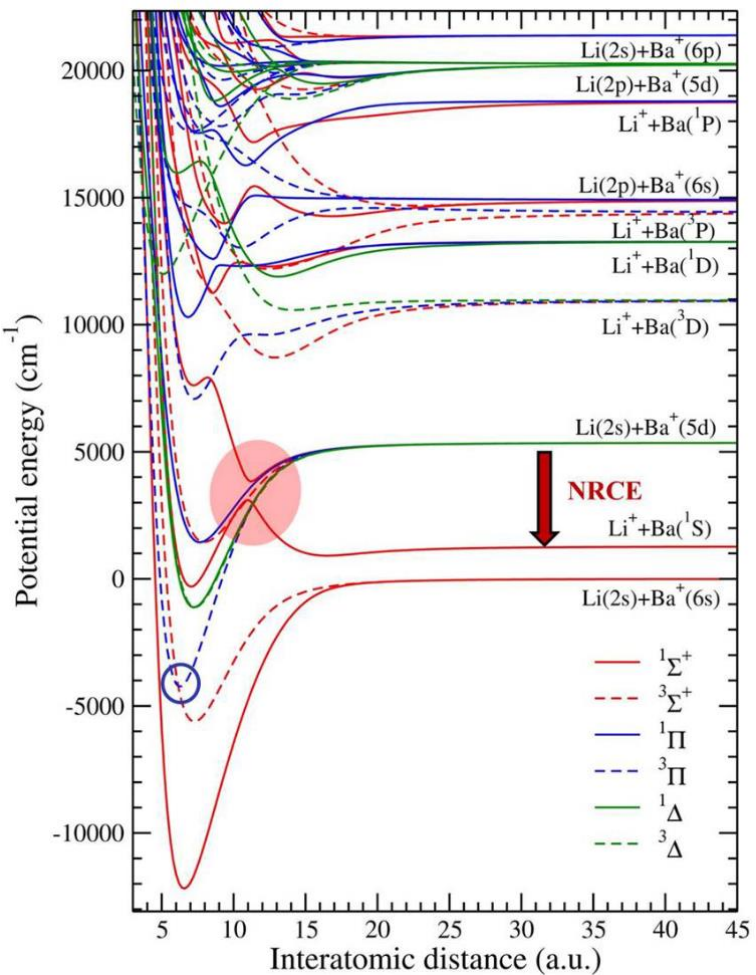
# Molecular structure

Molecular frame

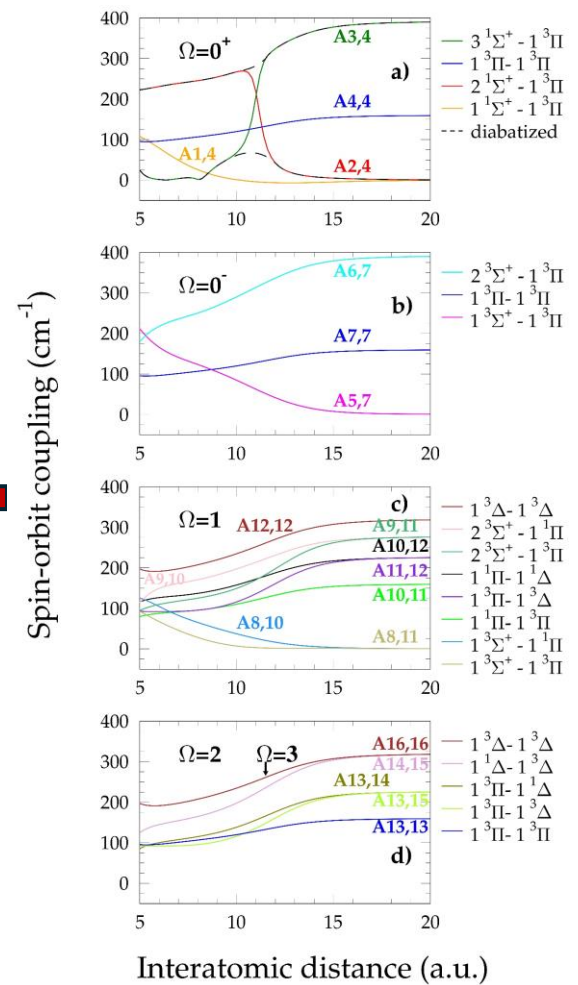
Laboratory frame



Hund's case a)

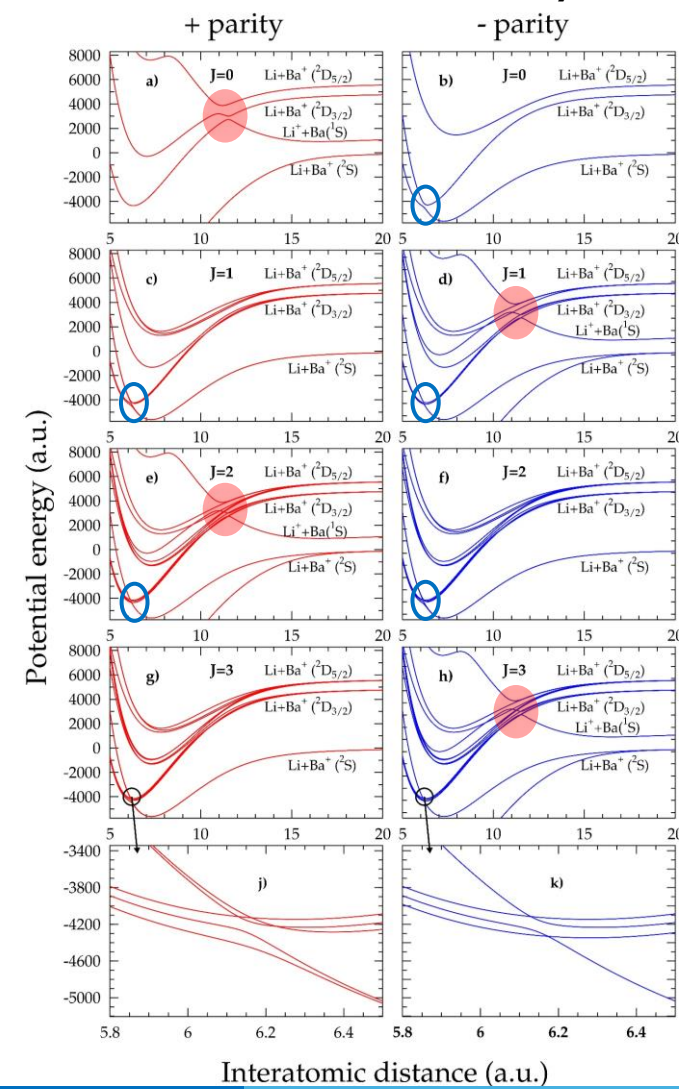


Spin - orbit coupling



Transformation

Hund's case e)



# Multi-Channel Quantum Scattering (MCQS)

Cross section

$$\sigma_{i \rightarrow f}(E) = \frac{\pi}{k_0^2} \sum_J (2J + 1) \sum_{\substack{l_i, j_i, m_i \\ l_f, j_f, m_f}} |S(J, l_f, j_f, m_f, p \leftarrow J, l_i, j_i, m_i, p)|^2$$

where,  $S$  – S – matrix  
 $J$  – total angular momentum quantum number  
 $l$  – partial wave  
 $j$  – electronic angular momentum quantum number  
 $m$  – projectile of the electronic angular momentum  
 $p$  – parity

Rate  $K(E) = \left(\frac{2E}{\mu}\right)^{\frac{1}{2}} \sigma_{i \rightarrow f}(E)$

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$$\sigma_{i \rightarrow f}(E) = \frac{\pi}{k_0^2} \sum_J (2J + 1) \sum_{\substack{l_i, j_i, m_i \\ l_f, j_f, m_f}} |\mathcal{S}(J, l_f, j_f, m_f, p \leftarrow J, l_i, j_i, m_i, p)|^2$$

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 J – total angular momentum quantum number  
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**Rate**  $K(E) = \left(\frac{2E}{\mu}\right)^{\frac{1}{2}} \sigma_{i \rightarrow f}(E)$

## Hamiltonian

$$\mathbf{H}_l(R) = -\frac{\hbar^2}{2\mu} \frac{d^2}{dR^2} \mathbf{I} + \frac{\hbar^2(l(l+1))}{2\mu R^2} \mathbf{I} + \mathbf{V}(R) + \mathbf{V}_{SOC}(R)$$

where,  $\mu$  reduced mass

$\mathbf{I}$  – identity matrix

$\mathbf{V}(R)$  – electronic potential energy matrix

$\mathbf{V}_{SOC}(R)$  – spin – orbit matrix

l – partial wave

## Scattered wave function for the $\alpha$ - channel

$$\psi_\alpha(R) = R^{-1} \sum_i \phi_i(\tau) F_{i\alpha}(R)$$

where,  $\phi_i(\tau)$  – orthonormal basis

$\tau$  – atom/ion all quantum number configuration

$F_{i\alpha}(R)$  – radial channel function

## Coupled equations for $F_{j\alpha}(R)$

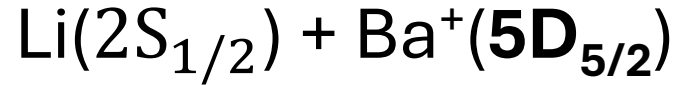
$$\frac{d^2 F_{j\alpha}(R)}{dr^2} = \sum_i [W_{ij} - E \delta_{ij}] F_{i\alpha}(R)$$

where,  $W_{ij} = \frac{2\mu}{\hbar^2} \int \phi_j^*(\tau) \left[ V_{SOC} + V(R) + \frac{\hbar^2(l(l+1))}{2\mu R^2} \right] \phi_i(\tau) d\tau$

# Competing processes

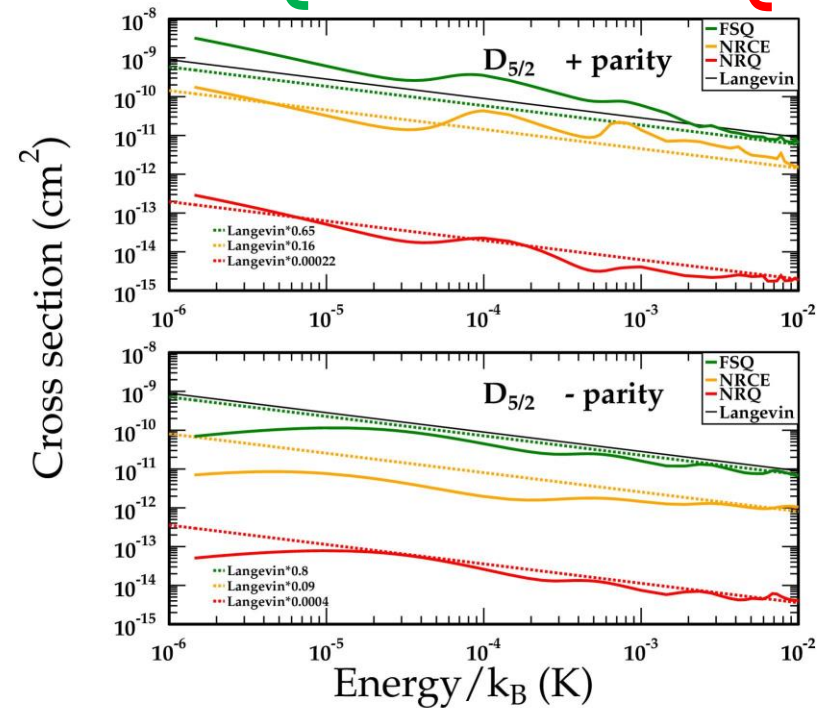
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## Incoming channel



## Cross section

**FSQ** **NRCE** **NRQ**

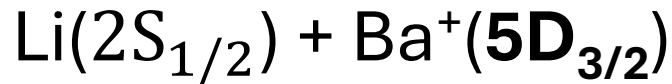
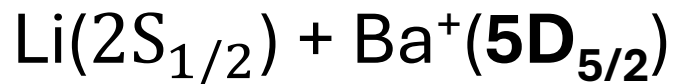


<https://arxiv.org/abs/2406.16017>

# Competing processes

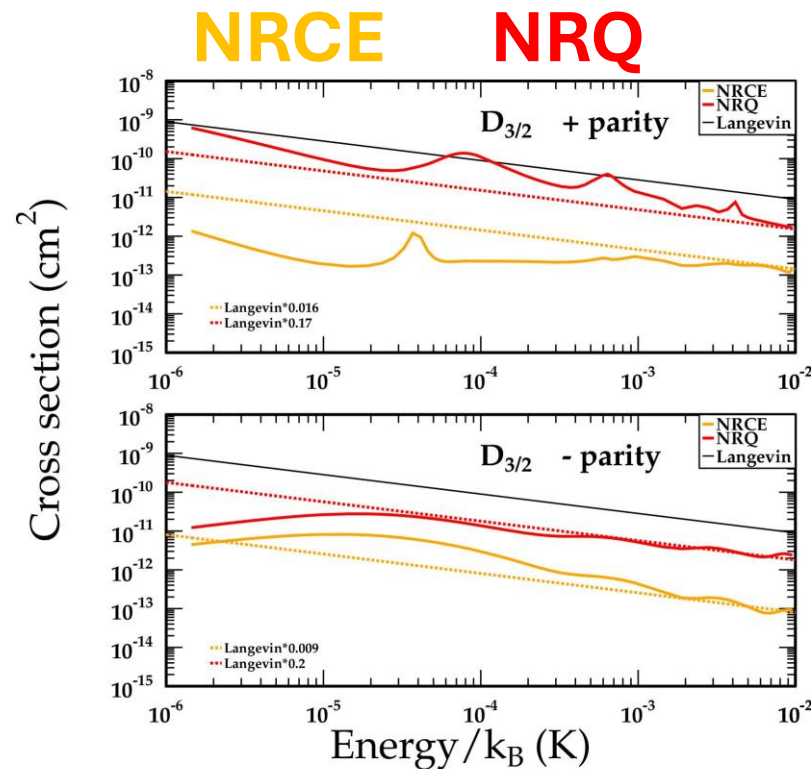
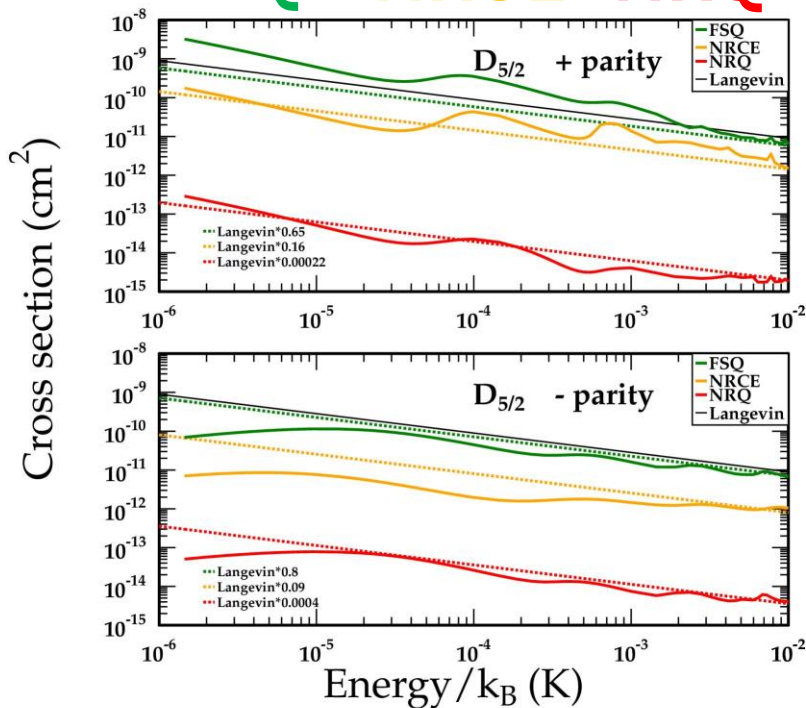
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## Incoming channel



## Cross section

**FSQ** **NRCE** **NRQ**

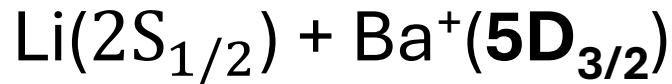
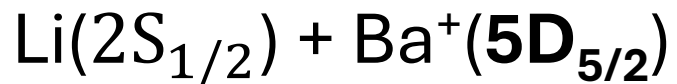


<https://arxiv.org/abs/2406.16017>

# Competing processes

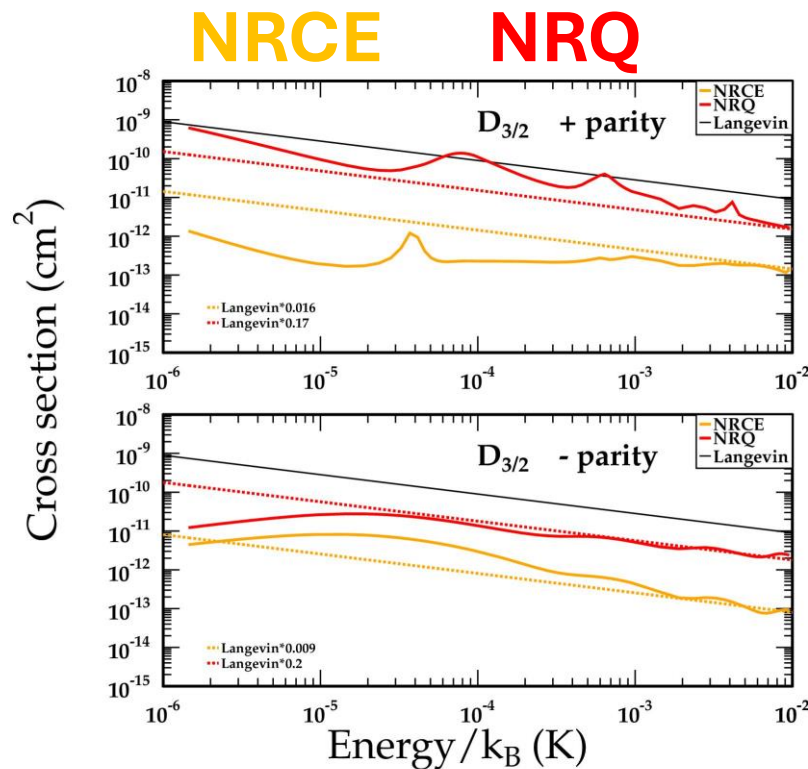
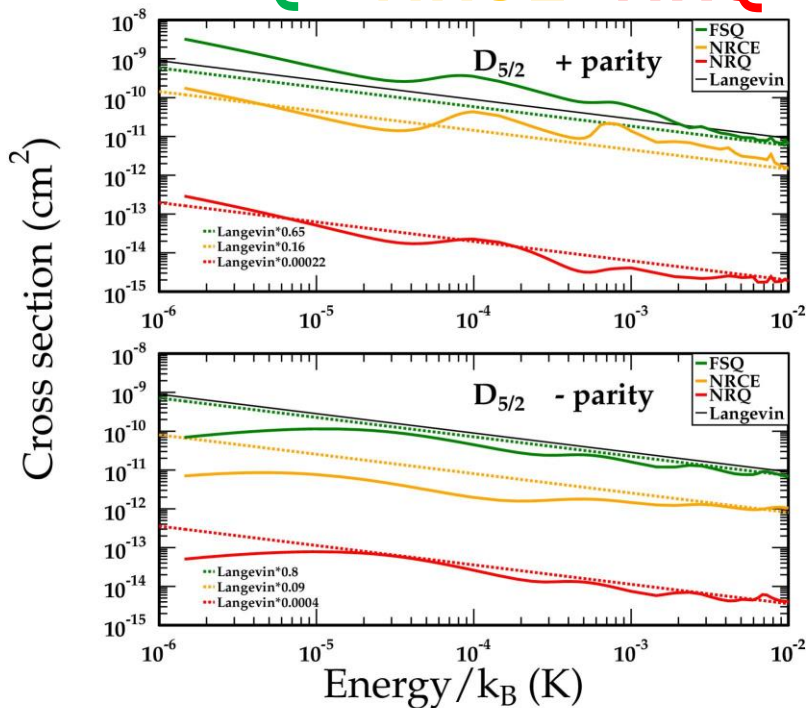
- FSQ (Fine Structure Quenching)
- NRCE (Non-Radiative Charge Exchange)
- NRQ (Non-Radiative Quenching)

## Incoming channel



## Cross section

**FSQ** **NRCE** **NRQ**



Langevin cross section:

$$\sigma_L = 2\pi \sqrt{\frac{\alpha}{2E}}, \quad \alpha - \text{polarizability of the atom}$$

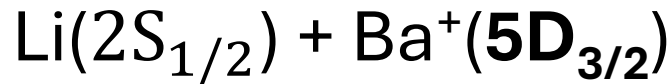
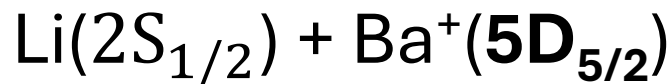
$$E - \text{collisional energy}$$

<https://arxiv.org/abs/2406.16017>

# Competing processes

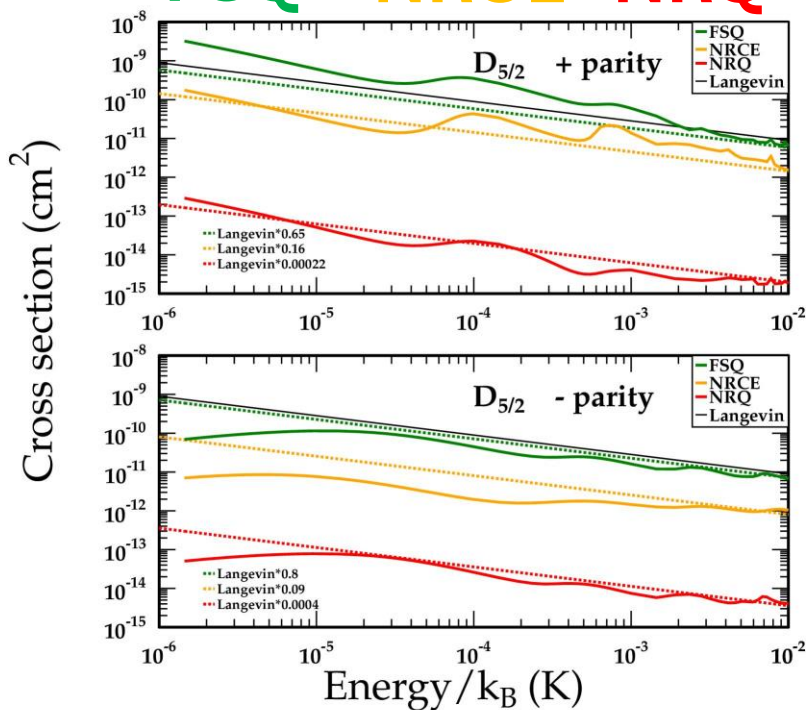
- FSQ (Fine Structure Quenching)
- NRCE (Non-Radiative Charge Exchange)
- NRQ (Non-Radiative Quenching)

## Incoming channel

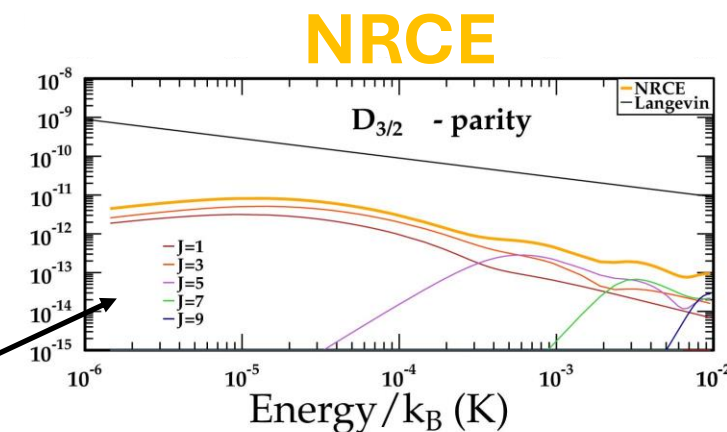
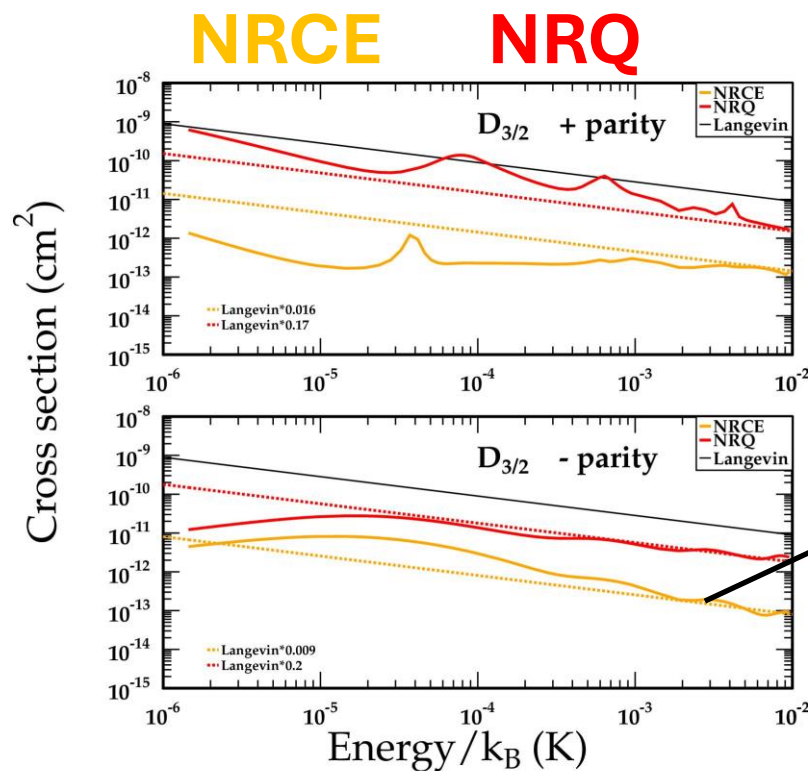


## Cross section

**FSQ** **NRCE** **NRQ**



## Partial cross section



Langevin cross section:

$$\sigma_L = 2\pi \sqrt{\frac{\alpha}{2E}}, \quad \alpha - \text{polarizability of the atom}$$

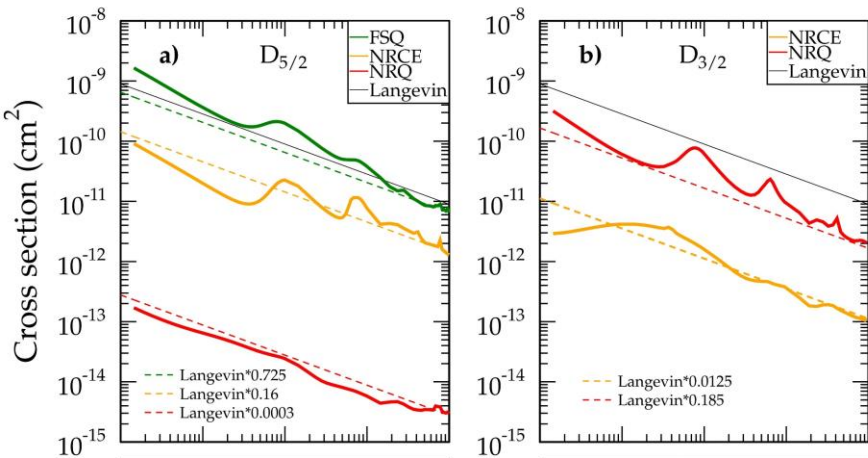
$$E - \text{collisional energy}$$

<https://arxiv.org/abs/2406.16017>

# Cross sections and reaction rates

- FSQ (Fine Structure Quenching)
- NRCE (Non-Radiative Charge Exchange)
- NRQ (Non-Radiative Quenching)

Cross section

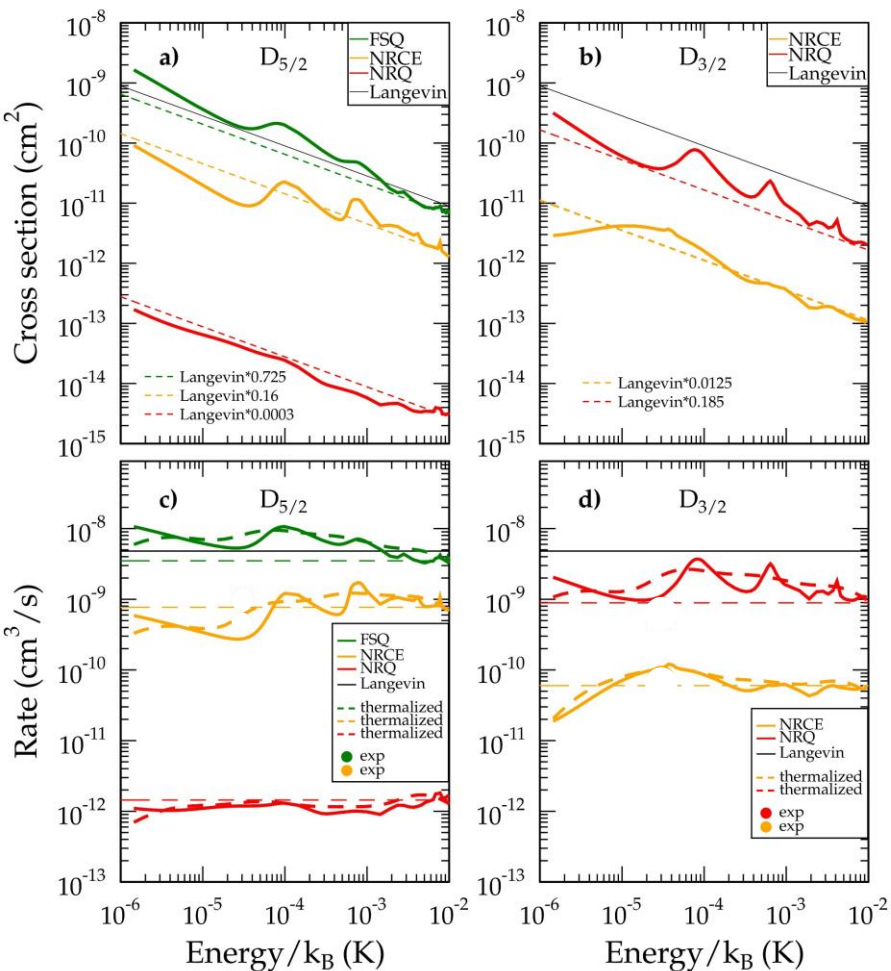


<https://arxiv.org/abs/2406.16017>

# Cross sections and reaction rates

- FSQ (Fine Structure Quenching)
- NRCE (Non-Radiative Charge Exchange)
- NRQ (Non-Radiative Quenching)

## Cross section



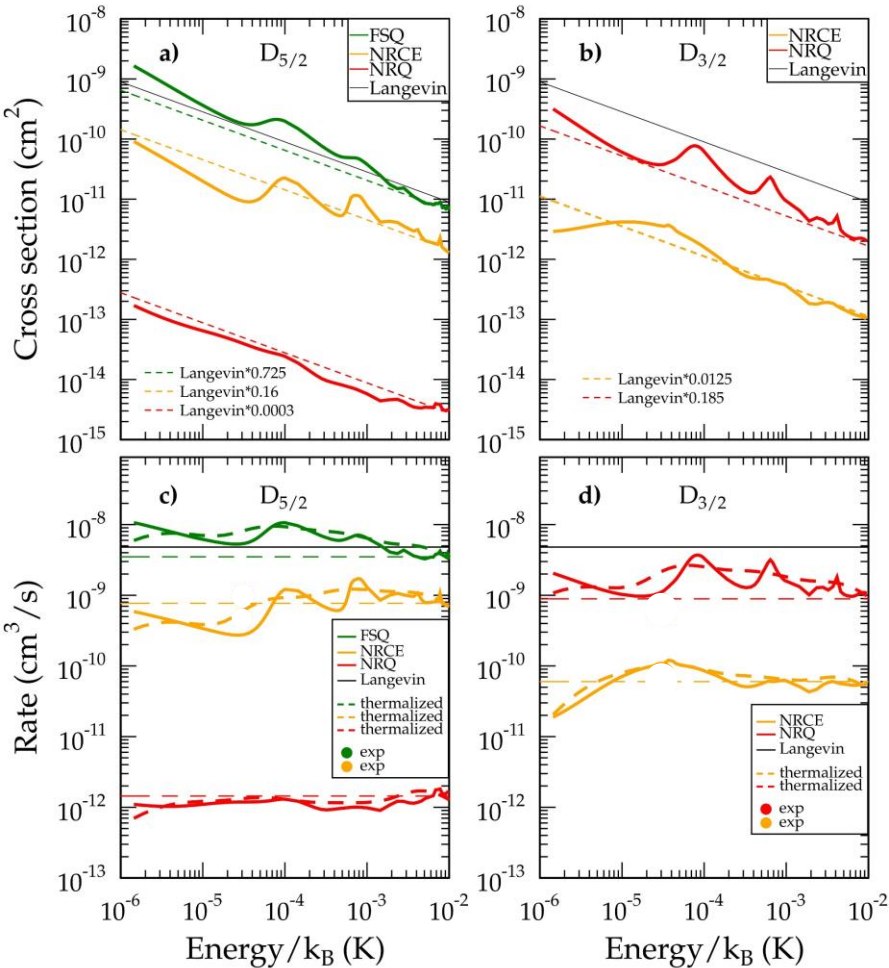
## Rate

<https://arxiv.org/abs/2406.16017>

# Cross sections and reaction rates

- FSQ (Fine Structure Quenching)
- NRCE (Non-Radiative Charge Exchange)
- NRQ (Non-Radiative Quenching)

## Cross section



THEO.

$D_{3/2}$ :  
 $\text{Rate}_{\text{NRCE}} < \text{Rate}_{\text{NRQ}}$

$D_{5/2}$ :  
 $\text{Rate}_{\text{NRCE}} > \text{Rate}_{\text{NRQ}}$

Rate

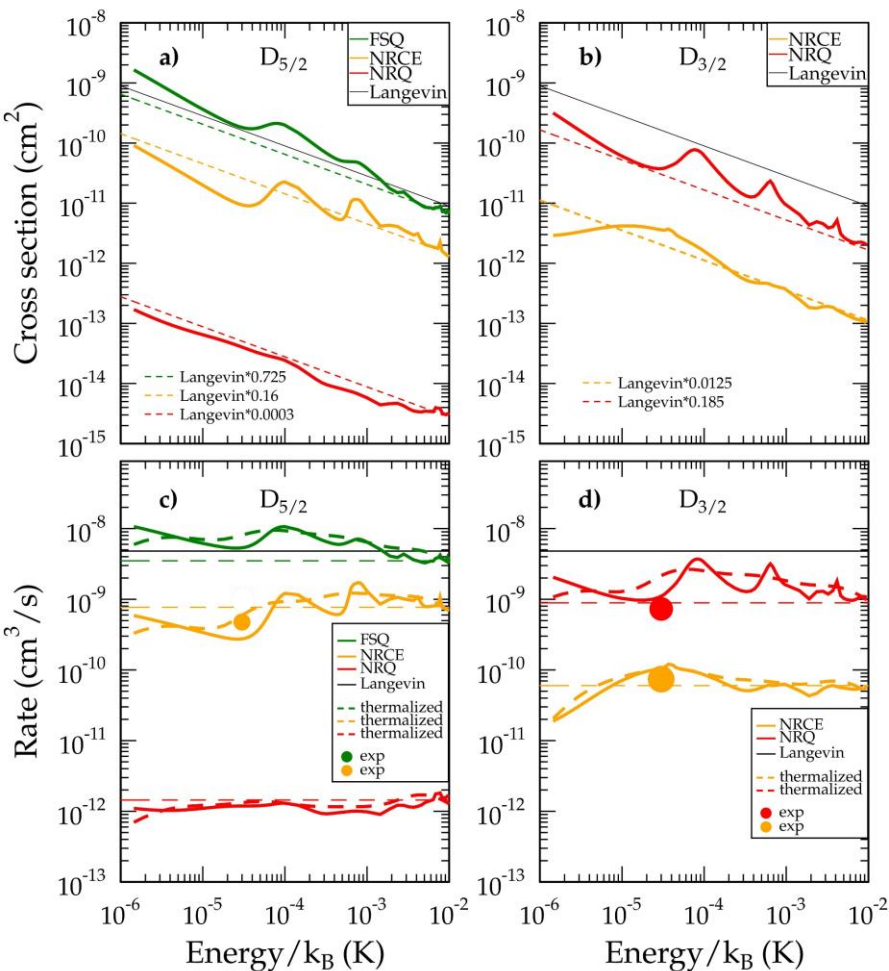
<https://arxiv.org/abs/2406.16017>



# Cross sections and reaction rates + experiment

- FSQ (Fine Structure Quenching)
- NRCE (Non-Radiative Charge Exchange)
- NRQ (Non-Radiative Quenching)

## Cross section



THEO.      EXP.

$D_{3/2}$ :  
 Rate<sub>NRCE</sub> < Rate<sub>NRQ</sub> ✓

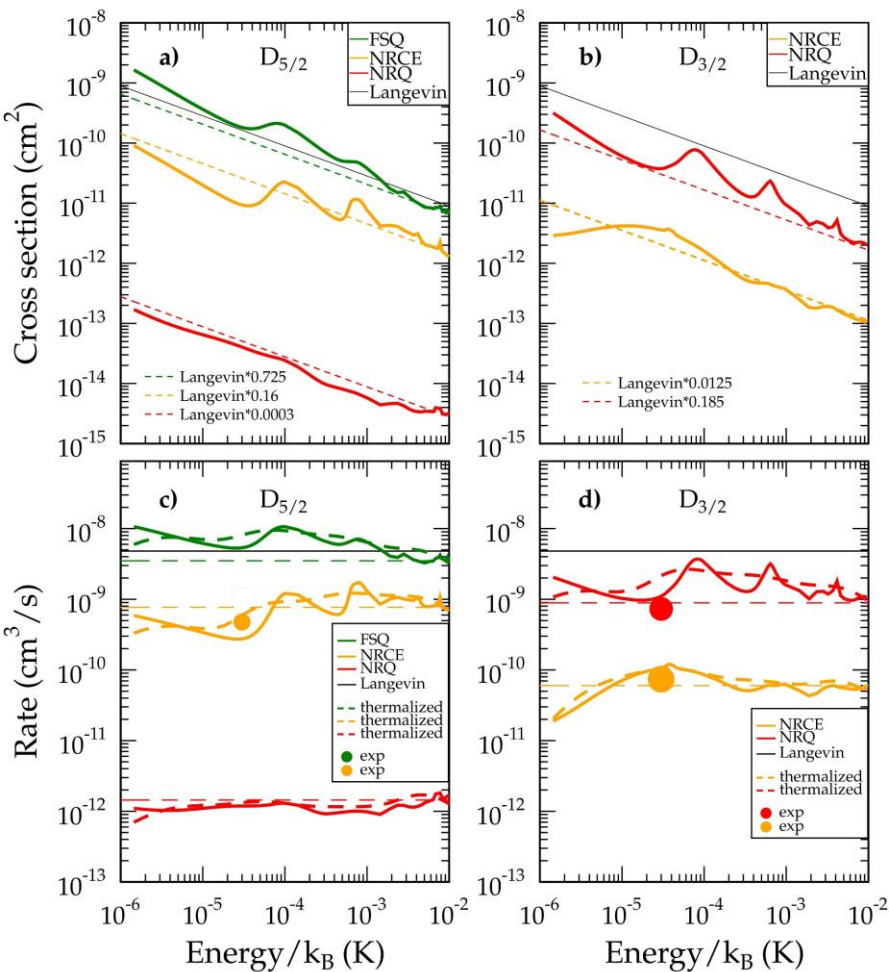
$D_{5/2}$ :  
 Rate<sub>NRCE</sub> > Rate<sub>NRQ</sub> ✓

<https://arxiv.org/abs/2406.16017>

# Cross sections and reaction rates + experiment

- FSQ (Fine Structure Quenching)
- NRCE (Non-Radiative Charge Exchange)
- NRQ (Non-Radiative Quenching)

## Cross section



## Rate

THEO.      EXP.

$D_{3/2}$ :  
 Rate<sub>NRCE</sub> < Rate<sub>NRQ</sub> ✓

$D_{5/2}$ :  
 Rate<sub>NRCE</sub> > Rate<sub>NRQ</sub> ✓

THEO.

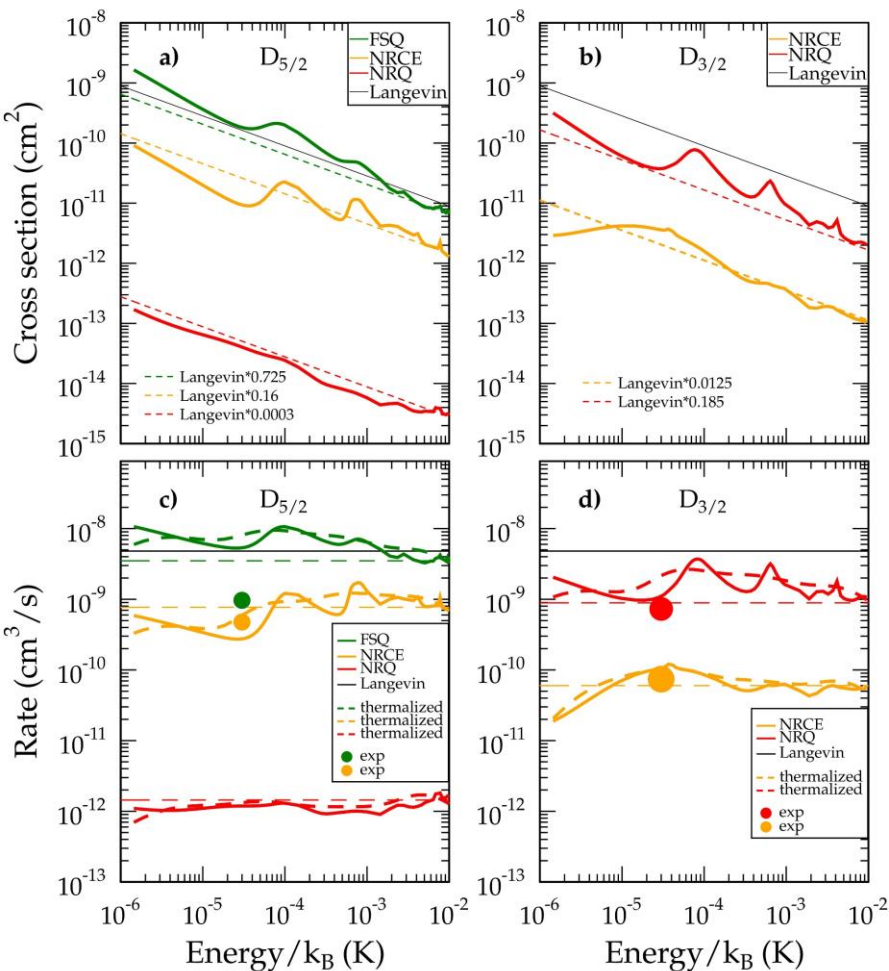
Rate<sub>FSQ</sub> > Rate<sub>NRCE</sub>

<https://arxiv.org/abs/2406.16017>

# Cross sections and reaction rates + experiment

- FSQ (Fine Structure Quenching)
- NRCE (Non-Radiative Charge Exchange)
- NRQ (Non-Radiative Quenching)

## Cross section



## Rate

THEO.

EXP.

$D_{3/2}$ :

Rate<sub>NRCE</sub> < Rate<sub>NRQ</sub> ✓

$D_{5/2}$ :

Rate<sub>NRCE</sub> > Rate<sub>NRQ</sub> ✓

THEO.

EXP.

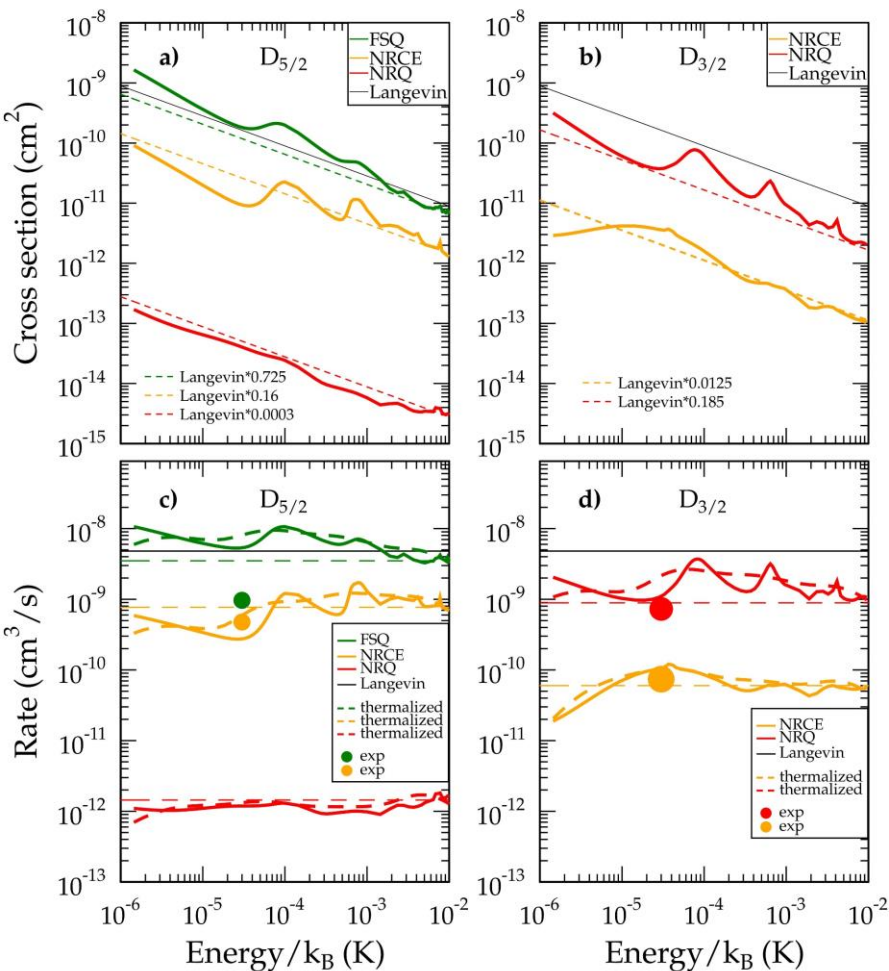
Rate<sub>FSQ</sub> > Rate<sub>NRCE</sub> ✓

<https://arxiv.org/abs/2406.16017>

# Cross sections and reaction rates + experiment

- FSQ (Fine Structure Quenching)
- NRCE (Non-Radiative Charge Exchange)
- NRQ (Non-Radiative Quenching)

## Cross section



## Rate

	THEO.	EXP.	THEO.	EXP.
$D_{3/2}$ :				
Rate <sub>NRCE</sub> < Rate <sub>NRQ</sub>		✓		
$D_{5/2}$ :				
Rate <sub>NRCE</sub> > Rate <sub>NRQ</sub>		✓	Rate <sub>FSQ</sub> > Rate <sub>NRCE</sub>	✓

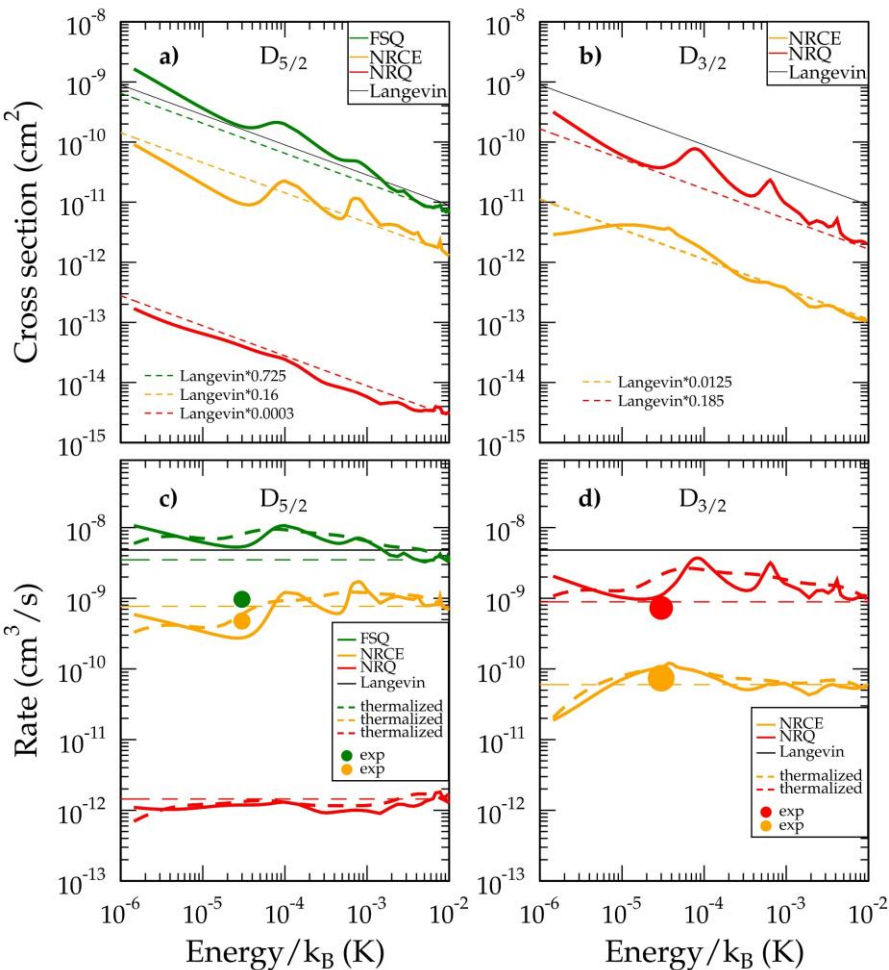
Good agreement with the experiment!

<https://arxiv.org/abs/2406.16017>

# Cross sections and reaction rates + experiment

- FSQ (Fine Structure Quenching)
- NRCE (Non-Radiative Charge Exchange)
- NRQ (Non-Radiative Quenching)

## Cross section

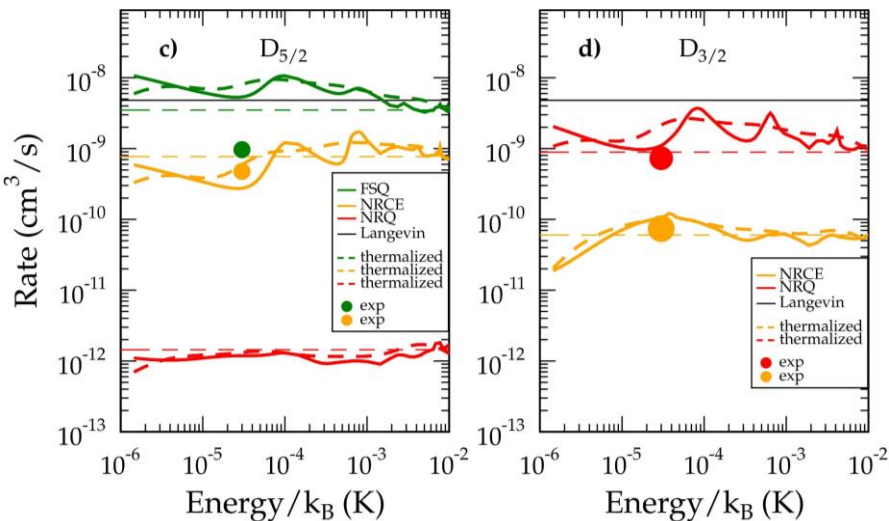


## Rate

	THEO.	EXP.	THEO.	EXP.
$D_{3/2}$ :				
	$\text{Rate}_{\text{NRCE}} < \text{Rate}_{\text{NRQ}}$	✓		
$D_{5/2}$ :				
	$\text{Rate}_{\text{NRCE}} > \text{Rate}_{\text{NRQ}}$	✓	$\text{Rate}_{\text{FSQ}} > \text{Rate}_{\text{NRCE}}$	✓
$D_{3/2}$ :				
	$\frac{\text{Rate}_{\text{NRQ}}}{\text{Rate}_{\text{NRCE}}} \approx 10$			
$D_{5/2}$ :				
	$\frac{\text{Rate}_{\text{NRQ}}}{\text{Rate}_{\text{NRCE}}} \approx 0.001$		$\frac{\text{Rate}_{\text{FSQ}}}{\text{Rate}_{\text{NRCE}}} \approx 10$	$\frac{\text{Rate}_{\text{FSQ}}}{\text{Rate}_{\text{NRQ}}} \approx 10^4$

<https://arxiv.org/abs/2406.16017>

# Summary



- Good agreement between the theory and experiment for the **NRCE** process
- For  $D_{3/2}$  incoming channel the theory predicts that the **NRQ** process has higher probability than the **NRCE**, which is consistent with the experiment
- For  $D_{5/2}$  incoming channel they could not detect **NRQ** in the experiment, our theory can confirm that, it predicts such a low probability
- For  $D_{5/2}$  incoming channel the theory overestimates the **FSQ** process, which could be explained by unaccurate spin – orbit coupling determination → clarification needed



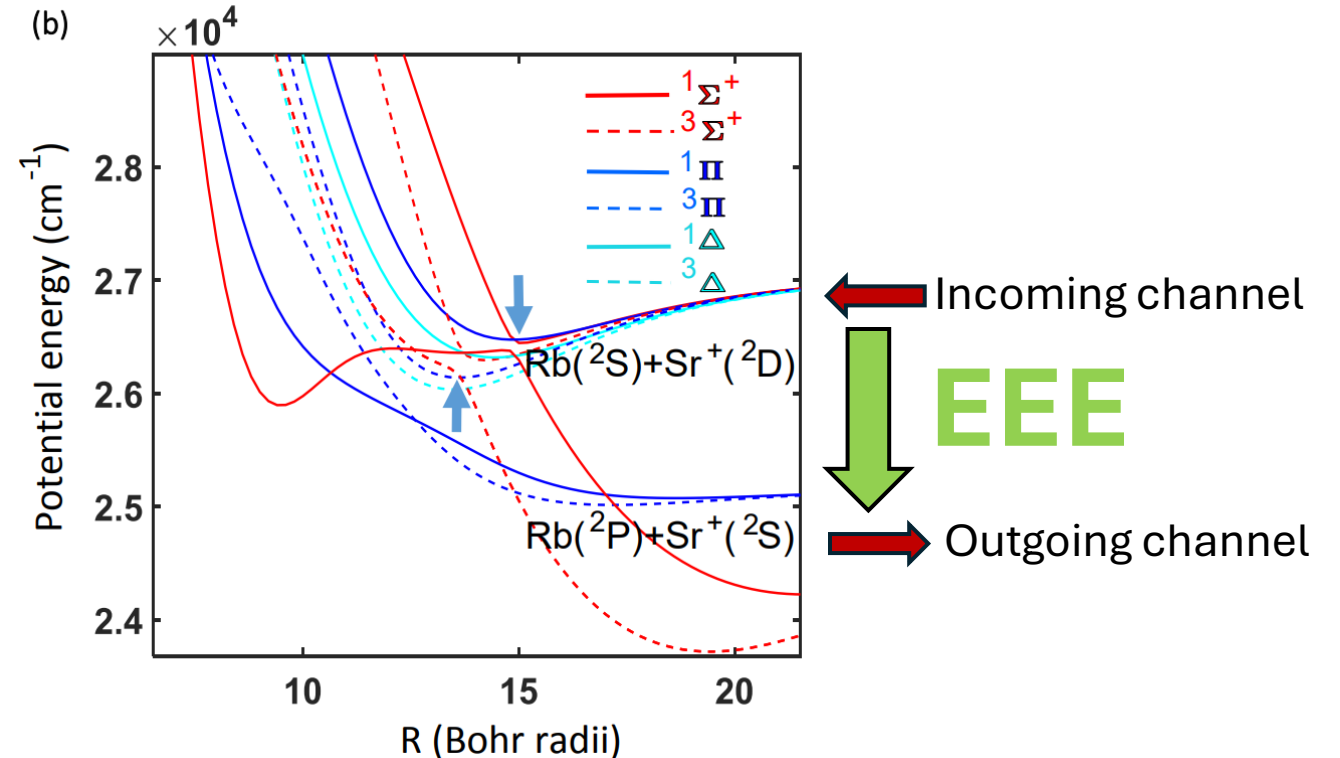
<https://arxiv.org/abs/2406.16017>

# Outlook and perspectives

- Similar investigations for the same class of systems (eg. Rb - Sr<sup>+</sup>)
  - different observed process (**electronic excitation exchange, EEE**)

# Outlook and perspectives

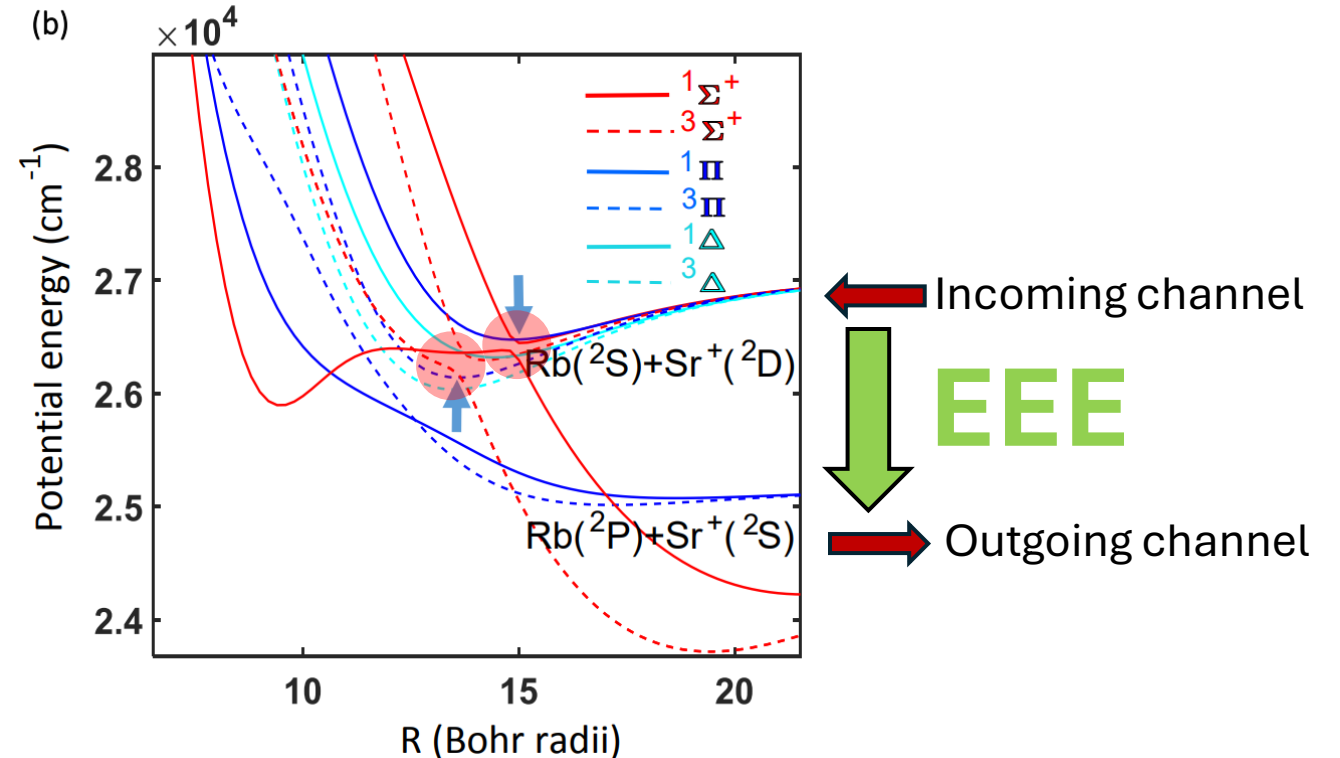
- Similar investigations for the same class of systems (eg. Rb - Sr<sup>+</sup>)
  - different observed process (**electronic excitation exchange, EEE**)





# Outlook and perspectives

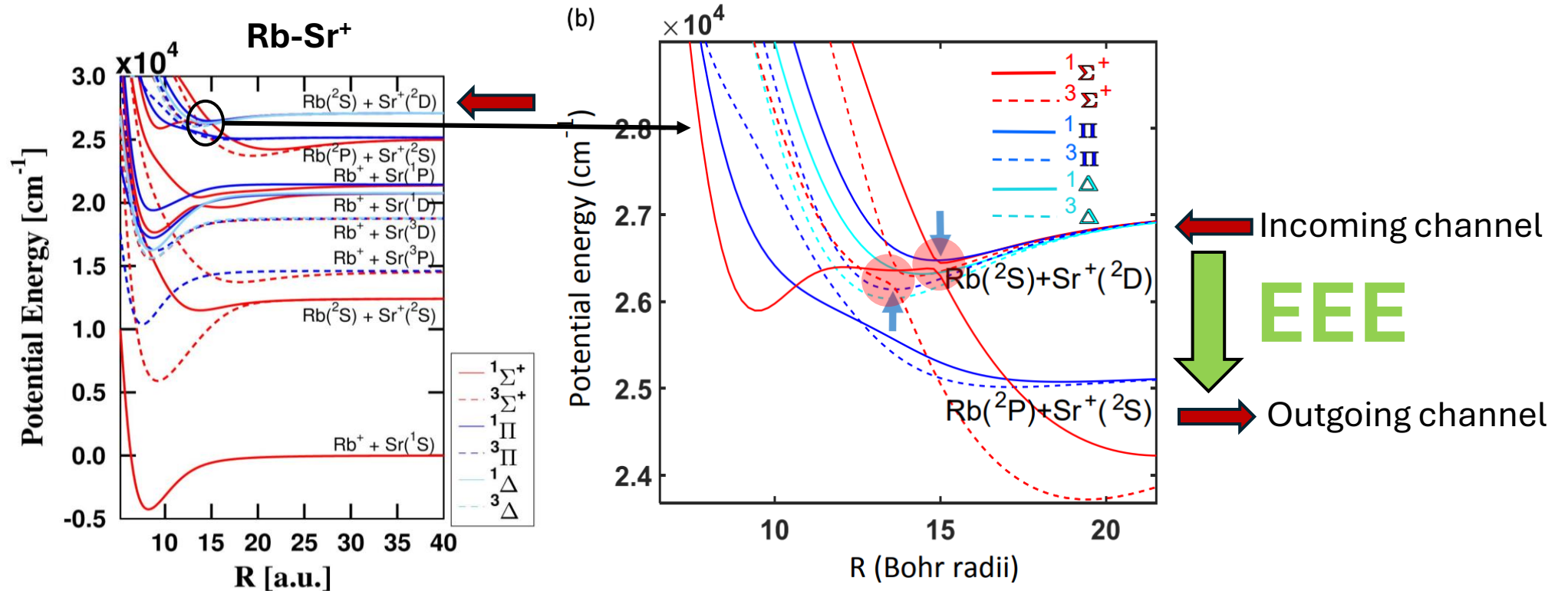
- Similar investigations for the same class of systems (eg. Rb - Sr<sup>+</sup>)
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# Outlook and perspectives

- Similar investigations for the same class of systems (eg. Rb - Sr<sup>+</sup>)
  - different observed process (**electronic excitation exchange, EEE**)
  - more open channel
  - denser asymptotic limits

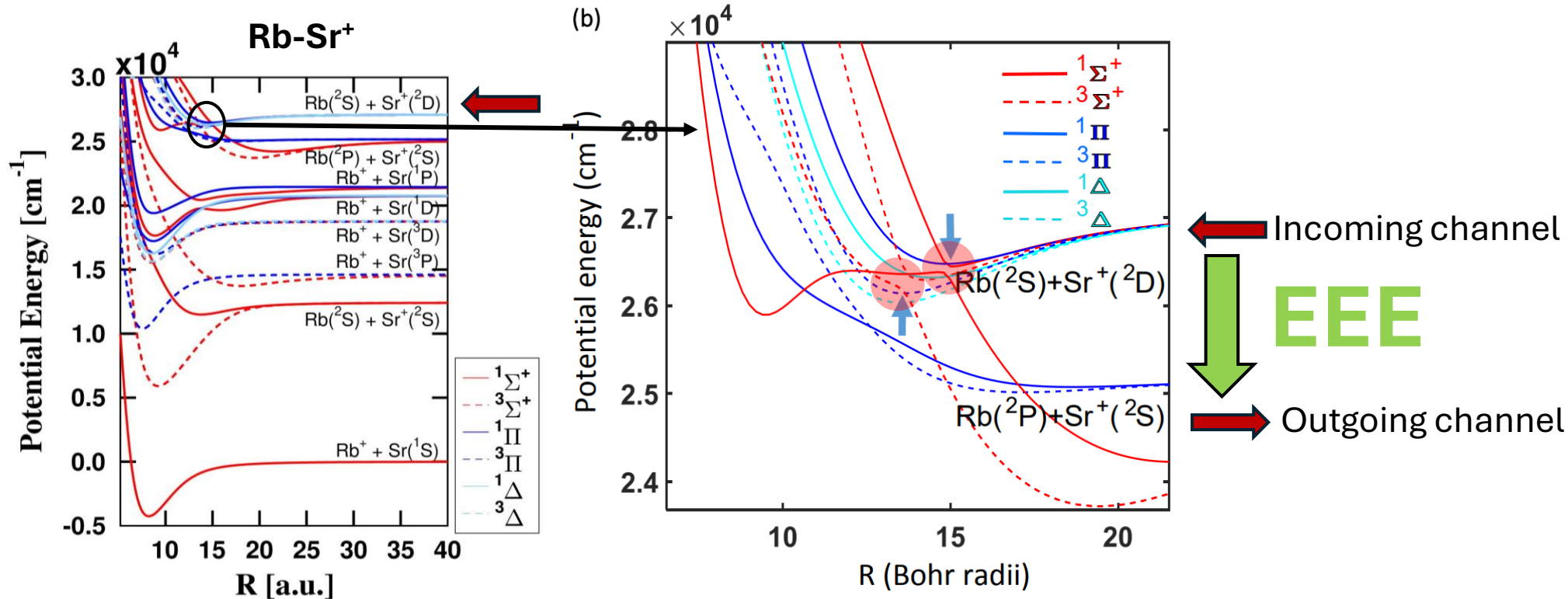
23 × 23



# Outlook and perspectives

- Similar investigations for the same class of systems (eg. Rb - Sr<sup>+</sup>)
  - different observed process (**electronic excitation exchange, EEE**)
  - more open channel
  - denser asymptotic limits
- Considering spin – polarization of colliding partners in the model

23 × 23



# Collaboration

- **THEOMOL group, Aimé Cotton Lab., Orsay, France** & **HUN-REN ATOMKI, Debrecen, Hungary**  
Olivier Dulieu, Nadia Bouloufa-Maafa, Eliane Luc, Romain Vexiau, Xiadong Xing, Ting Xie  
Andrea Orbán
- **Quantum and Atomic Physics group, Uni. Freiburg** & **Uni. British Columbia, Vancouver, Canada**  
Pascal Weckesser, Fabian Thielemann, Tobias Schaetz  
Kirk W. Madison

# Fundings

CNRS International Emerging Action (IEA) – ELKH, 2023-2024



Program Hubert Curien „BALATON” (CampusFrance-GrantNO.49848TC)



NKFIH TÉT-FR, 2023-2024 (2021-1.2.4- TÉT-2022-00069)



French Governmental Scholarship



University of Debrecen



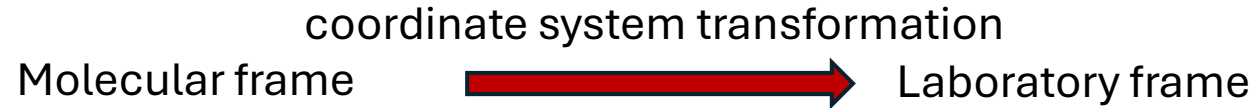
**Thank you for your attention!**

# Rate ratios

Event	Process	Counts	Ratio(%)	$K_{exp.}/K_L^{exp}$	$K_{MCQS}/K_L^{th} ; K_{MCQS-L}/K_L^{th}$
$5D_{3/2}: K_L^{exp} = 4.69 \times 10^{-9} \text{ cm}^3/s; K_L^{th} = 4.81 \times 10^{-9} \text{ cm}^3/s$					
	EC	177	-	-	-
Hot	NRQ	302	90.6(16)	0.154(45) <sub>stat</sub> (27) <sub>sys</sub>	0.21;0.18
Loss	NRCE	31	9.4(16)	0.016(5) <sub>stat</sub> (3) <sub>sys</sub>	0.021;0.012
Total		510			
$5D_{5/2}: K_L^{exp} = 4.81 \times 10^{-9} \text{ cm}^3/s; K_L^{th} = 4.81 \times 10^{-9} \text{ cm}^3/s$					
	EC	41	-	-	-
Cold+Hot	FSQ	42+8	66(6)	0.198(26) <sub>stat</sub> (40) <sub>sys</sub>	1.06;0.725
Loss	NRCE	25	34(6)	0.102(14) <sub>stat</sub> (20) <sub>sys</sub>	0.052;0.16
Total		116			

<https://arxiv.org/abs/2406.16017>

# Frame transformation



Transformation

$$\begin{pmatrix} \Psi_1^{lab} \\ \Psi_2^{lab} \\ \cdot \\ \cdot \\ \cdot \\ \Psi_N^{lab} \end{pmatrix} = \begin{pmatrix} t_{11} & \dots & t_{1N} \\ t_{21} & \dots & t_{2N} \\ \cdot & & \cdot \\ \cdot & & \cdot \\ \cdot & & \cdot \\ t_{N1} & \dots & t_{NN} \end{pmatrix} (\Psi_1^{mol} \quad \Psi_2^{mol} \quad \dots \quad \Psi_N^{mol})$$

$$t_{ij} = \langle j_a j_b j_l J M | \Lambda S \Sigma J M p \rangle = (-1)^{l-\Omega-J} (2 - \delta_{\Lambda,0} \delta_{\Sigma,0})^{-1/2} \\ \times [1 + (-1)^{L_a+L_b+l+p} (1 - \delta_{\Lambda,0} \delta_{\Sigma,0})] \\ \times \sqrt{(2S+1)(2j_a+1)(2j_b+1)} \\ \times \langle l 0 | j - \Omega, J \Omega \rangle \langle L \Lambda | L_a \Lambda_a, L_b \Lambda_b \rangle \\ \times \begin{Bmatrix} L_a & S_a & j_a \\ L_b & S_b & j_b \\ L & S & j \end{Bmatrix} \langle j \Omega | L \Lambda, S \Sigma \rangle$$

- where,  $j_{a/b}$  – A/B atom el. angular momentum quantum number
- $j$  – total el. angular momentum quantum number
- $l$  – partial wave quantum number
- $J$  – total angular momentum quantum number
- $M$  – projectile of total angular momentum quantum number
- $\Lambda$  – projectile of orbital momentum quantum number
- $S$  – spin
- $\Sigma$  – projectile of spin
- $p$  – parity
- $\Omega = \Lambda + \Sigma$
- $\langle \dots | \dots \rangle$  – Clebsch – Gordan coefficient
- $\{ \dots \}$  – 9-j symbol