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# Cosmological study of a symmetric teleparallel gravity model

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Tiago B. Gonçalves (IA-U.Lisboa)  
Luís Atayde (IA-U.Lisboa)  
Noemi Frusciante (U.Napoli)

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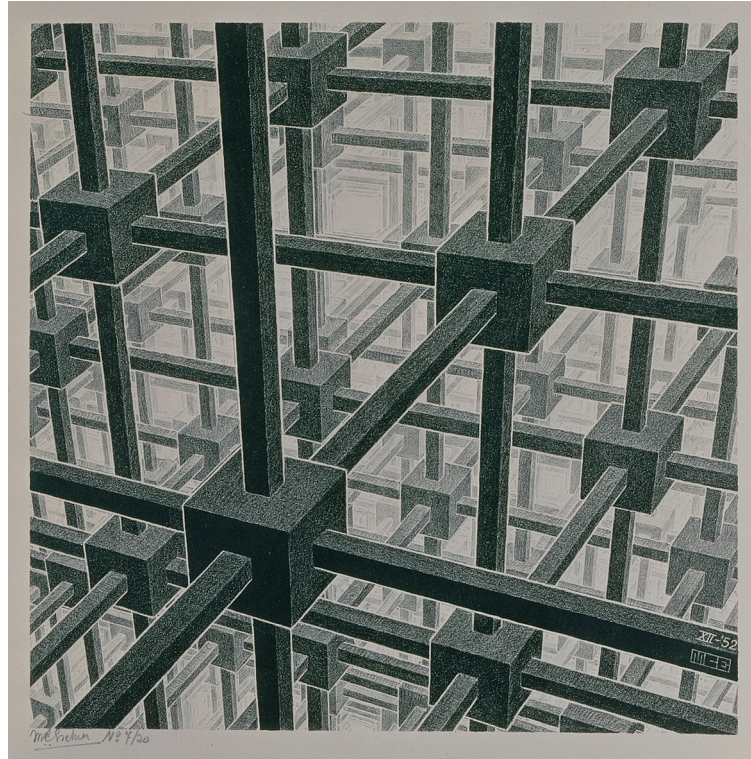


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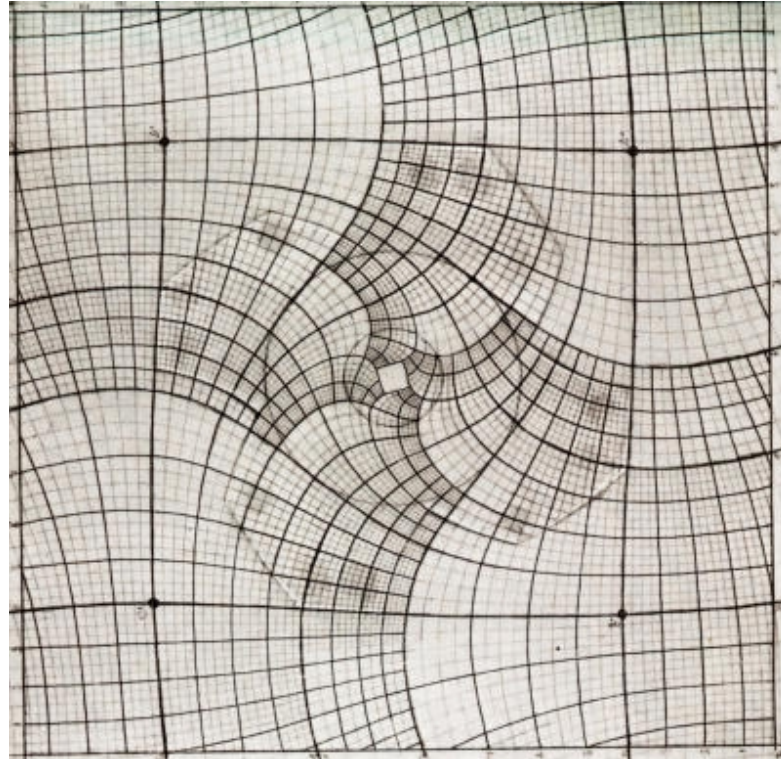




M. C. Escher



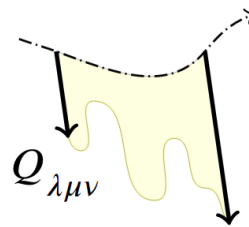
M. C. Escher



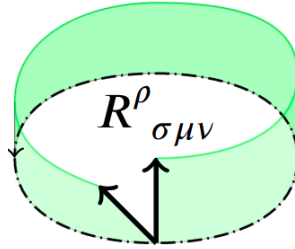
**M. C. Escher**

$$Q_{\alpha\mu\nu} \equiv \nabla_{\alpha}g_{\mu\nu}$$

Non-metricity

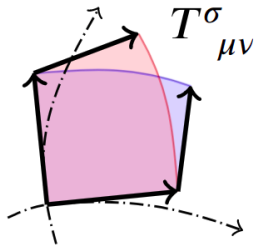


## Curvature

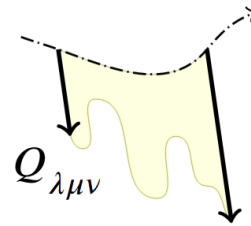


$$Q_{\alpha\mu\nu} \equiv \nabla_{\alpha}g_{\mu\nu}$$

## Torsion



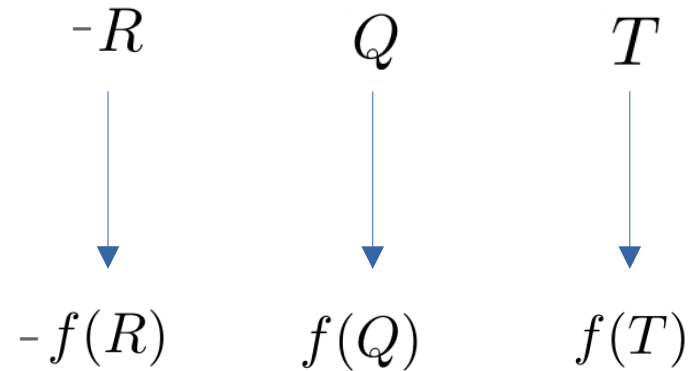
## Non-metricity



$$\mathcal{S} = \int \sqrt{-g} d^4x \left[ -\frac{1}{2\kappa^2} \bigcirc + \mathcal{L}_m(g_{\mu\nu}, \chi_i) \right]$$

The diagram illustrates the decomposition of the Einstein-Hilbert action term. A circle containing the term  $-\frac{1}{2\kappa^2}$  is connected by three arrows to the labels  $-R$ ,  $Q$ , and  $T$ .

$$\mathcal{S} = \int \sqrt{-g} d^4x \left[ -\frac{1}{2\kappa^2} \bigcirc + \mathcal{L}_m(g_{\mu\nu}, \chi_i) \right]$$





# Symmetric teleparallel gravity

f(Q) gravity

Coincident gauge

Flat FLRW

$$Q = 6H^2$$

[Gomes+ 2311.04201]  
[Heisenberg+ 2311.05495]  
[Guzman+ 2406.11621]

# Symmetric teleparallel gravity

f(Q) gravity

Coincident gauge

Flat FLRW

$$Q = 6H^2$$

Pathological f(Q):

strongly coupled

or propagating ghost d.o.f.

$$f_Q = \frac{df}{dQ} > 0$$

[Gomes+ 2311.04201]  
[Heisenberg+ 2311.05495]  
[Guzman+ 2406.11621]

# Logarithmic f(Q)

Logarithmic family of models

$$f(Q) = \frac{\alpha}{2} \sqrt{Q} \ln(\gamma Q) + \beta Q$$

Friedman eq.

$$3\beta H^2 + \frac{3\alpha H_0 H}{\phantom{H}} = \kappa^2 \rho$$



self-accelerating

[Ayuso+ 2111.05061]

DGP = Dvali, Gabadadze, Porrati (5D gravity)

# Logarithmic f(Q)

Logarithmic family of models

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Friedman eq.

$$3\beta H^2 + \underbrace{3\alpha H_0 H}_{\downarrow} = \kappa^2 \rho$$

self-accelerating

3 branches

sLog

$$\alpha = \Omega_{m0} + \Omega_{r0} - \beta$$

$$\alpha < 0$$

sDGP

$$\beta = 1 \text{ and } \alpha < 0$$

$\Lambda$ -nDGP

$$\beta = 1 \text{ and } \alpha > 0$$

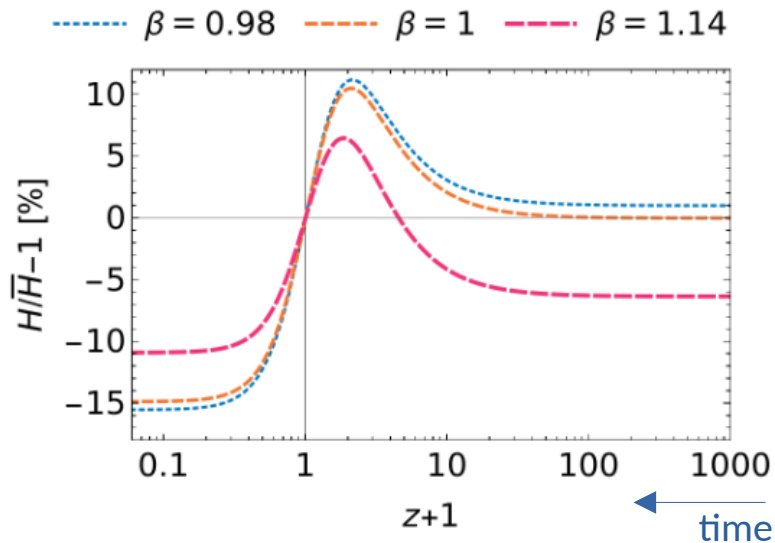
[Ayuso+ 2111.05061]

DGP = Dvali, Gabadadze, Porrati (5D gravity)

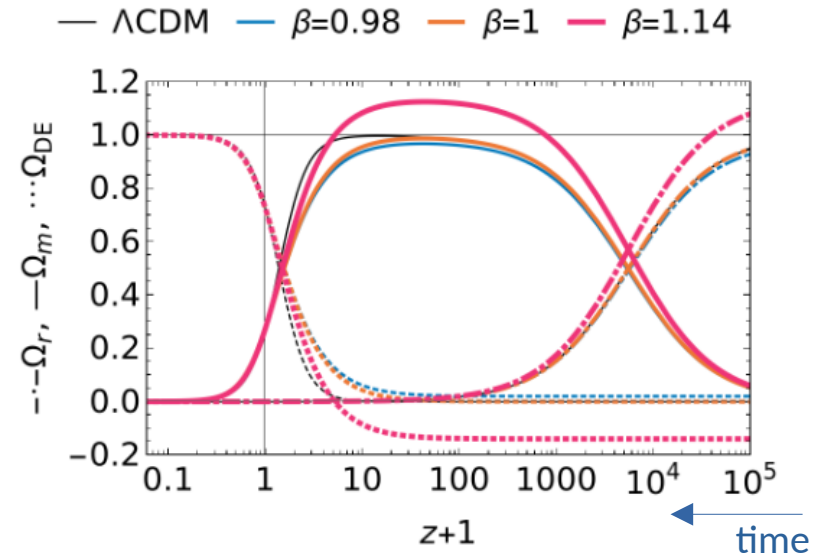
# Background

$$H = \frac{H_0}{2\beta} \left[ \sqrt{4\beta \left( \frac{\Omega_{r0}}{a^4} + \frac{\Omega_{m0}}{a^3} \right) + \alpha^2} - \alpha \right]$$

sLog model: varying  $\beta$



Hubble rate



Density parameters

# Linear perturbations

Newtonian gauge

$$ds^2 = -(1 + 2\Psi)dt^2 + a^2(1 - 2\Phi)\delta_i^j dx^i dx_j$$

Parametrization

effective gravitational coupling	$\mu(a, k)$
effective gravitational slip	$\Sigma(a, k)$

# Linear perturbations

Newtonian gauge

$$ds^2 = -(1 + 2\Psi)dt^2 + a^2(1 - 2\Phi)\delta_i^j dx^i dx_j$$

Parametrization

effective gravitational coupling	$\mu(a, k)$
effective gravitational slip	$\Sigma(a, k)$

quasi-static approximation

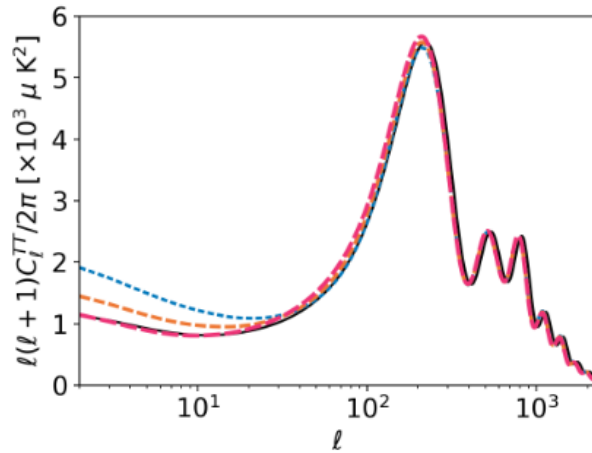
$$\mu(a) = \frac{1}{f_Q}$$

$$\Sigma = \mu$$

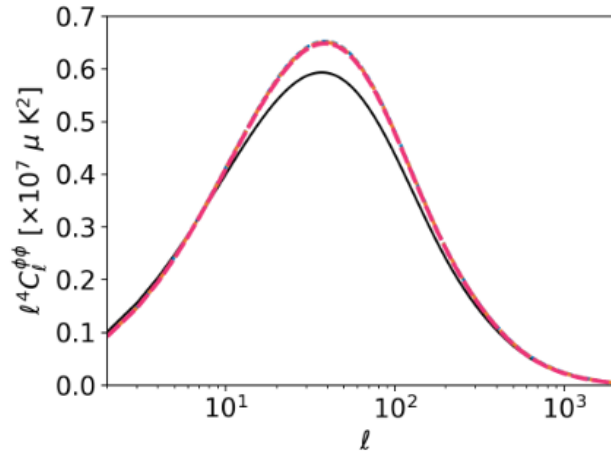
# Linear perturbations

sLog model: varying  $\beta$ , fixing  $\gamma = 0.1$ .

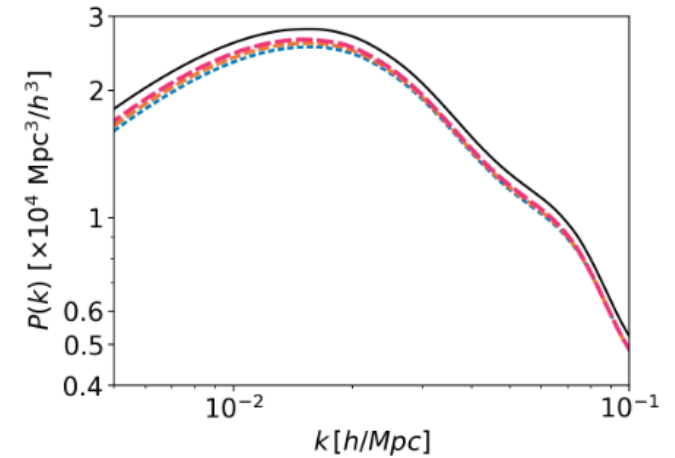
—  $\Lambda$ CDM    -.-  $\beta=0.98$     - -  $\beta=1.0$     - - -  $\beta=1.02$



Temperature-Temperature



Lensing



Matter

[Gonçalves, Atayde, Frusciante 2404.01742]

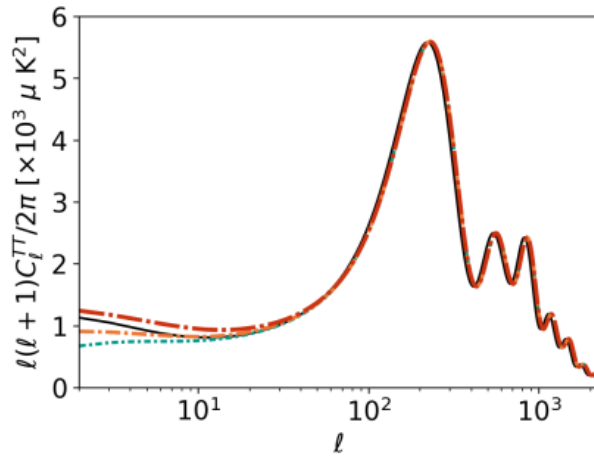
MGCAMB = Modified Growth with Code for Anisotropies in the Microwave Background



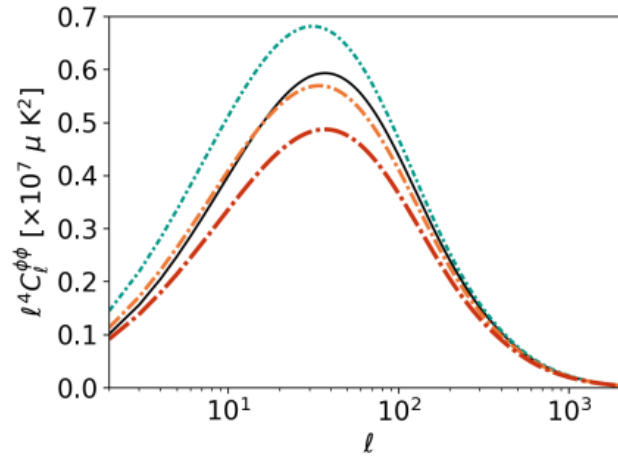
# Linear perturbations

$\Lambda$ -nDGP model: varying  $\gamma$ , fixing  $\alpha = +0.7$  and  $\beta = 1$ .

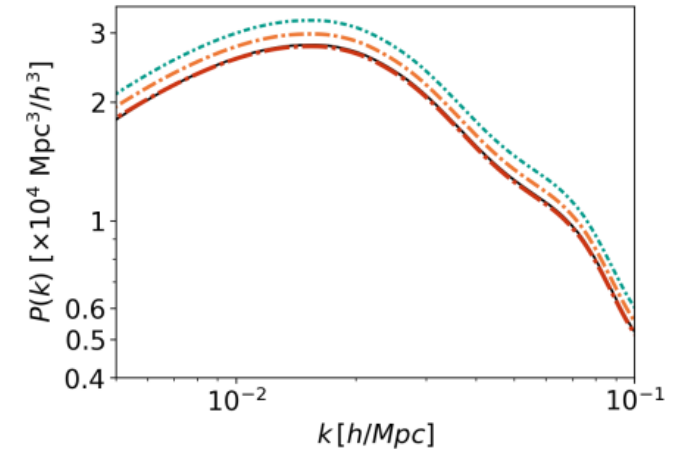
—  $\Lambda$ CDM    -.-  $\gamma=0.05$     -.-  $\gamma=0.1$     -.-  $\gamma=0.2$



Temperature-Temperature



Lensing



Matter

[Gonçalves, Atayde, Frusciante 2404.01742]

MGCAMB = Modified Growth with Code for Anisotropies in the Microwave Background

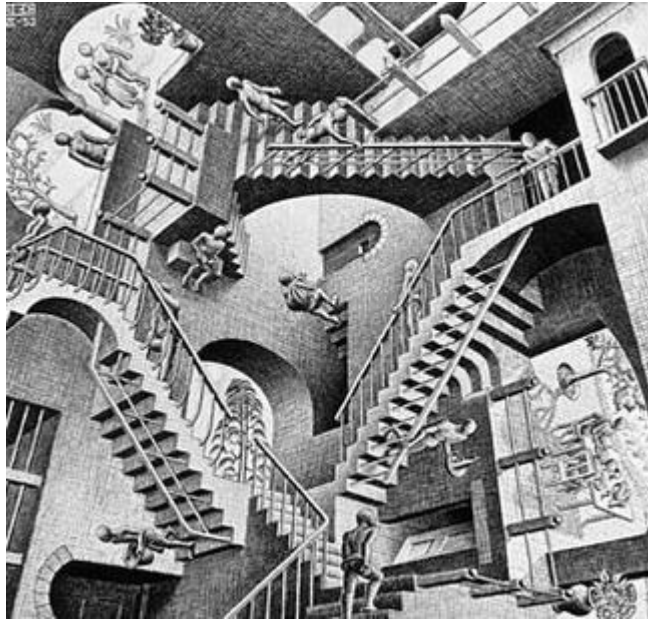
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Self-accelerating branch

Interesting features for S8 tension

No branch with all favourable characteristics



**M. C. Escher**

Thank  
you!

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Fundo Social Europeu



Action

$$\mathcal{S} = \int \sqrt{-g} d^4x \left[ -\frac{1}{2\kappa^2} f(Q) + \mathcal{L}_m(g_{\mu\nu}, \chi_i) \right]$$

Non-metricity and conjugate tensors

$$Q_{\alpha\mu\nu} \equiv \nabla_\alpha g_{\mu\nu} \quad P^\alpha{}_{\mu\nu} = -\frac{1}{2} L^\alpha{}_{\mu\nu} + \frac{1}{4} \left( Q^\alpha - \tilde{Q}^\alpha \right) g_{\mu\nu} - \frac{1}{4} \delta^\alpha_{(\mu} Q_{\nu)}$$

Disformation tensor

$$L^\alpha{}_{\mu\nu} = \frac{1}{2} (Q^\alpha{}_{\mu\nu} - Q_{(\mu\nu)}{}^\alpha)$$

2 independent contractions of non-metricity tensor

$$Q_\alpha = g^{\mu\nu} Q_{\alpha\mu\nu} \quad \text{and} \quad \tilde{Q}_\alpha = g^{\mu\nu} Q_{\mu\alpha\nu}$$

Non-metricity scalar

$$Q = -Q_{\alpha\mu\nu} P^{\alpha\mu\nu}$$

Metric eq.

$$\frac{2}{\sqrt{-g}} \nabla_{\alpha} (\sqrt{-g} f_Q P^{\alpha\mu}{}_{\nu}) + \frac{1}{2} \delta^{\mu}{}_{\nu} f + f_Q P^{\mu\alpha\beta} Q_{\nu\alpha\beta} = T^{\mu}{}_{\nu}$$

Connection eq. (not independent),

when matter Lagrangian is independent of the connection (no hypermomentum)

$$\nabla_{\mu} \nabla_{\nu} (\sqrt{-g} f_Q P^{\mu\nu}{}_{\alpha}) = 0$$

Coincident gauge (coordinate choice)

$$\Gamma^{\alpha}{}_{\mu\nu} = 0$$

Flat FLRW

$$ds^2 = -dt^2 + a(t)^2 \delta_{ij} dx^i dx^j$$

Non-metricity scalar

$$Q = 6H^2$$

Newtonian gauge

$$ds^2 = -(1 + 2\Psi)dt^2 + a^2(1 - 2\Phi)\delta_i^j dx^i dx_j$$

Linear perturbation eqs.

$$-\frac{k^2}{a^2}\Psi = 4\pi G_N \mu(a, k) [\rho\Delta + 3(\rho + p)\sigma]$$

$$\frac{k^2}{a^2}(\Phi + \Psi) = -4\pi G_N \Sigma(a, k) [2\rho\Delta + 3(\rho + p)\sigma]$$

$$\Sigma = \frac{\mu}{2}(1 + \eta)$$

Gauge-invariant density contrast

$$\rho\Delta \equiv \rho\delta + 3\frac{aH}{k^2}(\rho + p)v$$

Density contrast eq.

$$\ddot{\delta} + 2H\dot{\delta} + 4\pi G_N a^2 \rho \mu(a, k) \delta = 0$$

- Strong coupling problem
  - interactions so strong that perturbative methods fail
  - e.g. Vainshtein mechanism screening extra d.o.f.s at large distances; still problematic at small scales
  
- Ghost instability
  - unphysical d.o.f.s
  - wrong sign of kinetic term
  - negative energy states
  - unbounded energy from below
  - unstable vacuum
  - runaway solutions, field grows indefinitely



*[Submitted on 7 Nov 2023]*

## **On the pathological character of modifications of Coincident General Relativity: Cosmological strong coupling and ghosts in $f(\mathbb{Q})$ theories**

Débora Aguiar Gomes, Jose Beltrán Jiménez, Alejandro Jiménez Cano, Tomi S. Koivisto

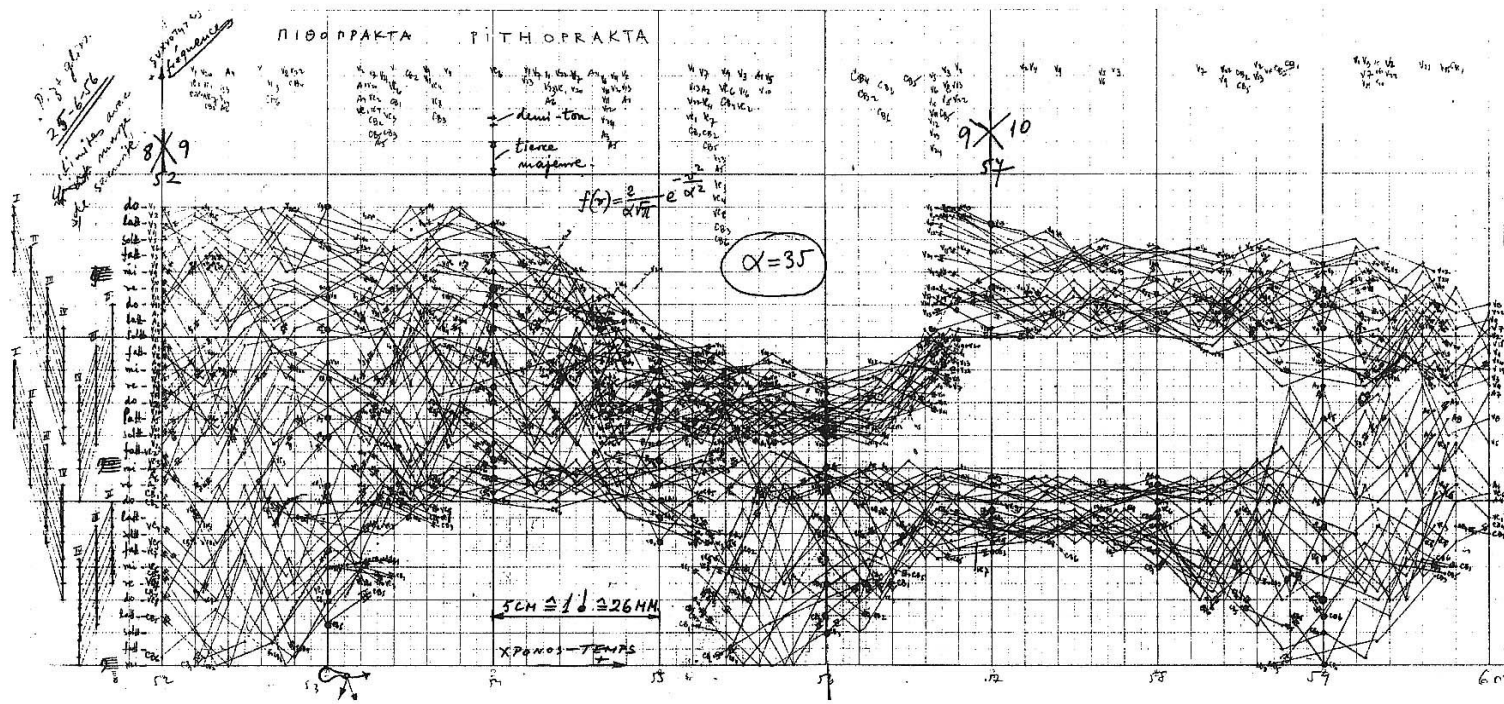
[2311.04201]

*[Submitted on 9 Nov 2023]*

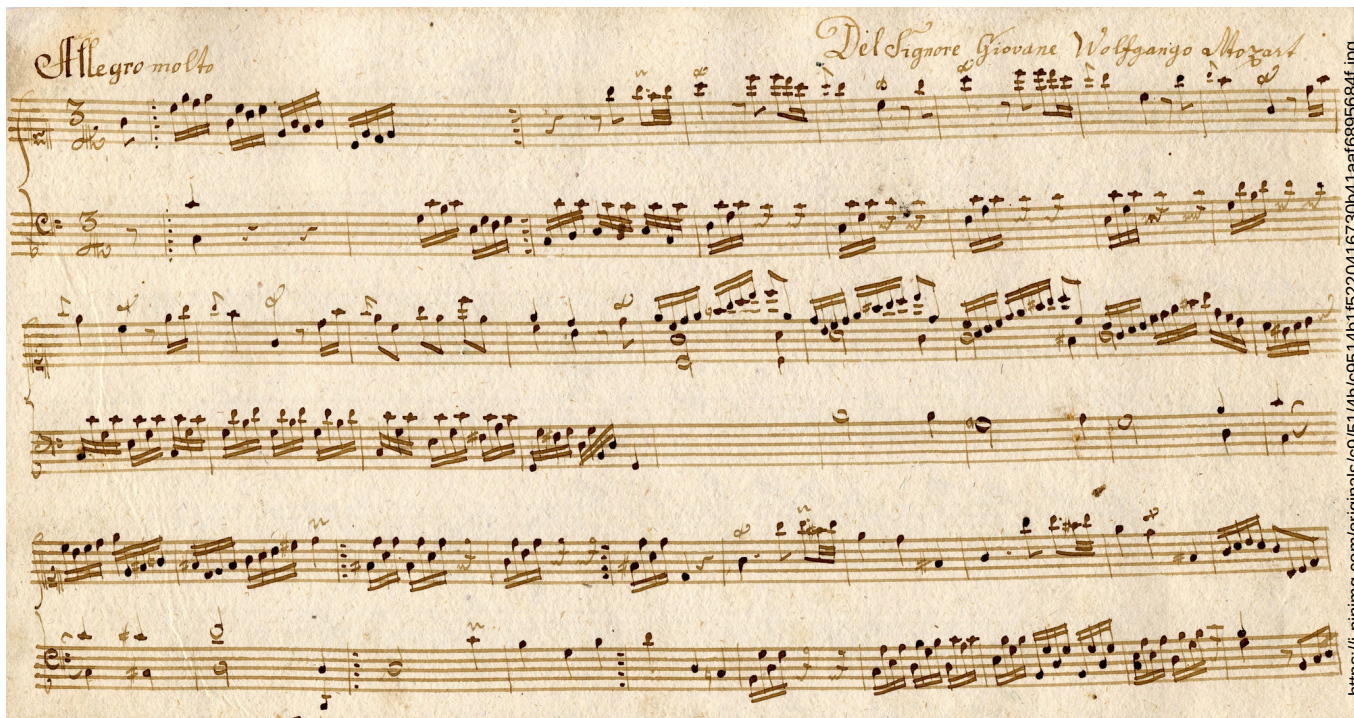
## **Cosmological teleparallel perturbations**

Lavinia Heisenberg, Manuel Hohmann, Simon Kuhn

[2311.05495]



<https://i.pinimg.com/originals/1d/eb/28/1deb28a85ce7c8f1803186a0e9e12af5.jpg>

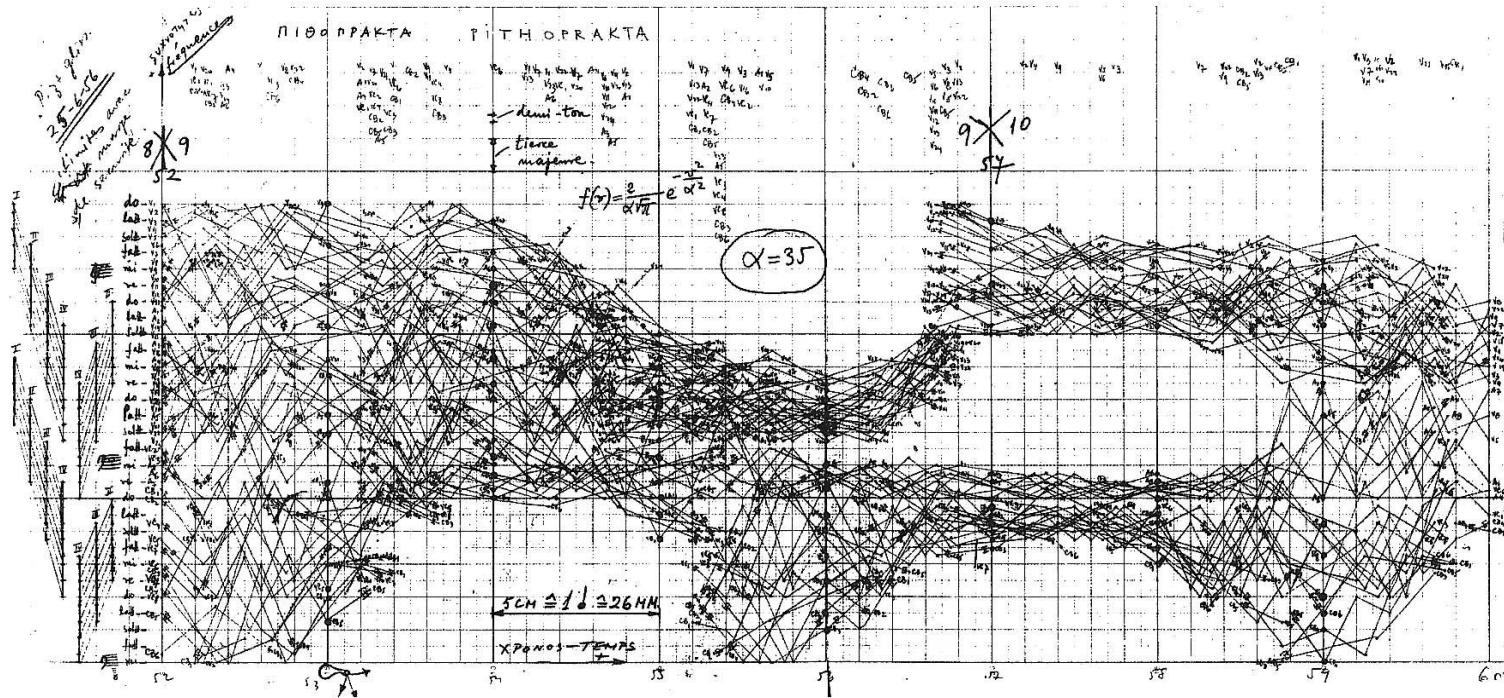


<https://i.pinimg.com/originals/c9/51/4b/c9514b1f5220416730b41aaf6895684f.jpg>

[Mozart]

*Pithoprakta* (1955-56), mesures 52-59 : graphique de Xenakis

Source : Iannis Xenakis, *Musique. Architecture*, Tournai, Casterman, 1976, p. 167



<https://i.pinimg.com/originals/1d/eb/28/1deb28a85ce7c8f1803186a0e9e12af5.jpg>

[Xenakis]

