Influence of spatial curvature in early cosmological particle production

Álvaro Parra-López · alvaparr@ucm.es
Complutense University of Madrid & IPARCOS

Article in preparation

In collaboration with
José A. R. Cembranos and Luis J. Garay

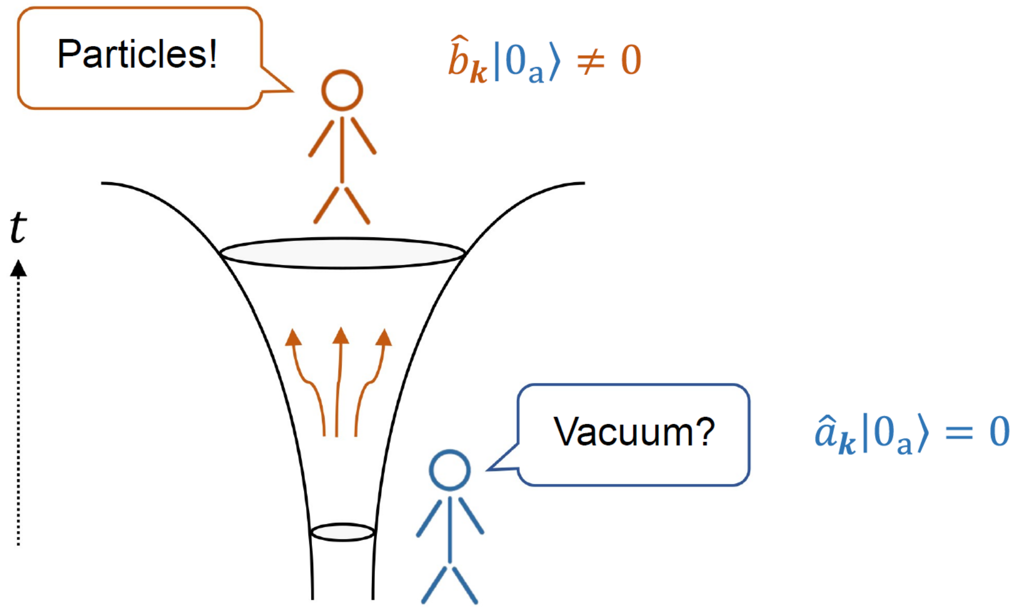
EREP2024 – July 2024

GOBIERNO DE ESPAÑA
MINISTERIO DE CIENCIA, INNOVACIÓN Y UNIVERSIDADES

IPARCOS

Introduction

- QFT in the presence of an external, time-dependent agent
  - Particle production (even from vacuum)
  - Vacuum/particle notion is ambiguous

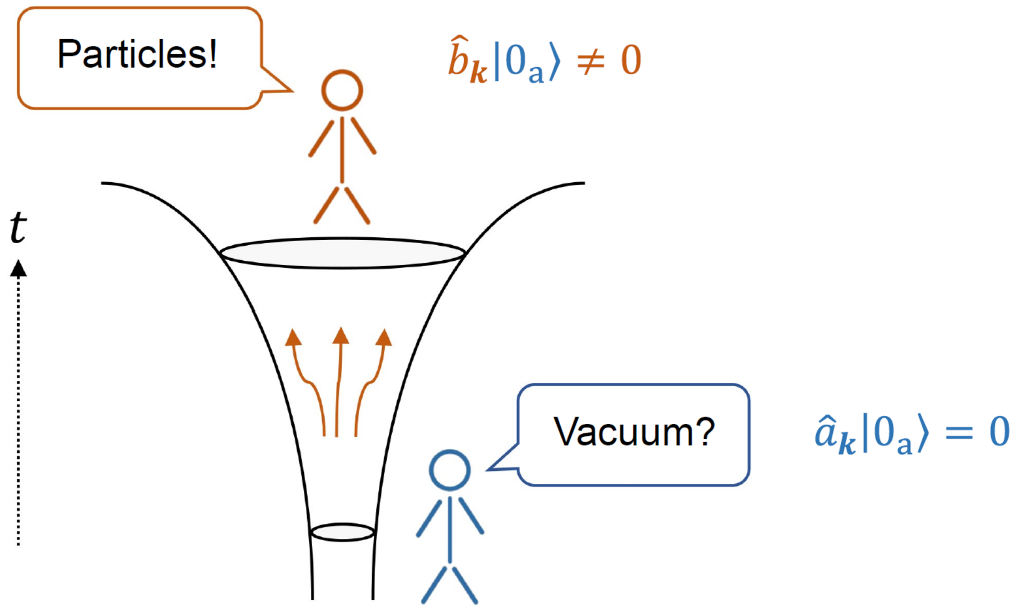


Introduction

- QFT in the presence of an external, time-dependent agent
  - Particle production (even from vacuum)
  - Vacuum/particle notion is ambiguous
- Spatial curvature may affect the Primordial Power Spectrum (PPS)

Bonga '16, '17, Hergt '22

- Does this affect gravitational production (of DM) too?



Scalar field in flat FLRW

- Non-interacting scalar field with action

$$S = -\frac{1}{2}\int d^4x\sqrt{-g}[\partial_\mu\varphi\partial^\mu\varphi + (m^2 + \xi R)\varphi^2]$$

- Non-minimal coupling to the curvature

- We expand the auxiliary field $\chi = a\varphi$ as

$$\chi(\eta, \boldsymbol{x}) = \int_{\boldsymbol{k}} [a_{\boldsymbol{k}} v_k(\eta) + a^*_{-\boldsymbol{k}} v^*_k(\eta)] e^{i\boldsymbol{k}\boldsymbol{x}}$$

(η ↓ Conformal time)

Scalar field in flat FLRW

- Non-interacting scalar field with action

$$S = -\frac{1}{2}\int d^4x\sqrt{-g}[\partial_\mu\varphi\partial^\mu\varphi + (m^2 + \xi R)\varphi^2]$$

- Non-minimal coupling to the curvature

- We expand the auxiliary field $\chi = a\varphi$ as

$$\chi(\eta, \boldsymbol{x}) = \int_{\boldsymbol{k}}[a_{\boldsymbol{k}}v_k(\eta) + a^*_{-\boldsymbol{k}}v^*_k(\eta)]e^{i\boldsymbol{k}\boldsymbol{x}}$$

Conformal time

- The EOM of $\chi(\eta, \boldsymbol{x})$ in k-space is

$$v''_k(\eta) + \omega^2_k(\eta)v_k(\eta) = 0$$

with frequency

$$\omega^2_k(\eta) = k^2 + a^2(\eta)[m^2 + \left(\xi - \frac{1}{6}\right)R(\eta)]$$

- Background determines $a^2(\eta)$ and $R(\eta)$

- Quantization: $a_{\boldsymbol{k}}, a^*_{\boldsymbol{k}} \to \hat{a}_{\boldsymbol{k}}, \hat{a}^\dagger_{\boldsymbol{k}}$

Scalar field in curved FLRW

- Non-interacting scalar field with action

$$S = -\frac{1}{2}\int d^4x\sqrt{-g}[\partial_\mu\varphi\partial^\mu\varphi + (m^2 + \xi R)\varphi^2]$$

- Non-minimal coupling to the curvature

- We expand the auxiliary field $\chi = a\varphi$ as

$$\chi(\eta, \boldsymbol{x}) = \int_{\boldsymbol{k}} [a_{\boldsymbol{k}} v_k(\eta) + a^*_{-\boldsymbol{k}} v^*_k(\eta)] f_{\boldsymbol{k}}(\boldsymbol{x})$$

η → Conformal time

- The EOM of $\chi(\eta, \boldsymbol{x})$ in k-space is

$$v''_k(\eta) + \omega^2_k(\eta) v_k(\eta) = 0$$

with frequency

$$\omega^2_k(\eta) = -h(k) + a^2(\eta)[m^2 + \left(\xi - \frac{1}{6}\right) R(\eta)]$$

- Background determines $a^2(\eta)$ and $R(\eta)$

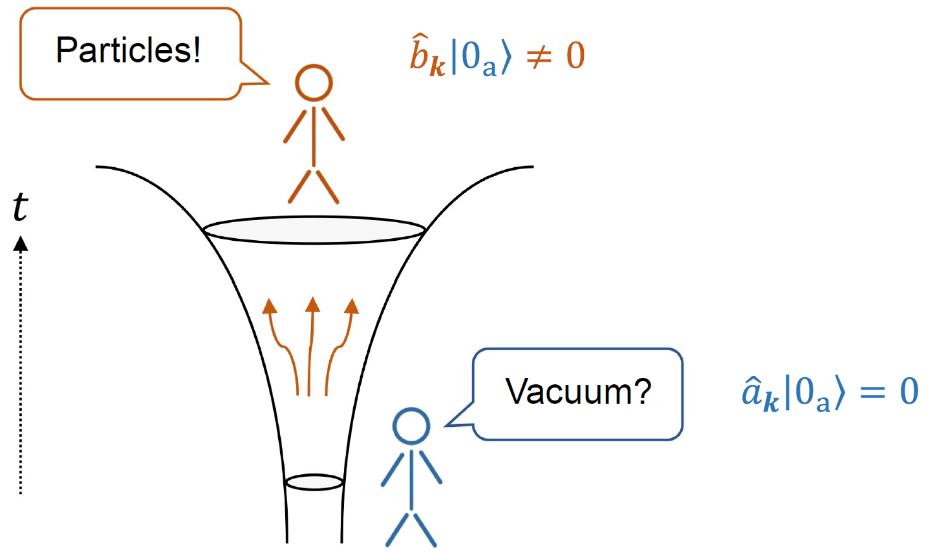
- Quantization: $a_{\boldsymbol{k}}, a^*_{\boldsymbol{k}} \to \hat{a}_{\boldsymbol{k}}, \hat{a}^\dagger_{\boldsymbol{k}}$

Particle production

- Two particular solutions v_k and u_k expand χ with different operators (and vacua),

$$u_k = \alpha_k v_k + \beta_k v_k^*, \quad \hat{b}_k = \alpha_k^* \hat{a}_k - \beta_k^* \hat{a}_k^+$$

$$|\alpha_k|^2 - |\beta_k|^2 = 1$$



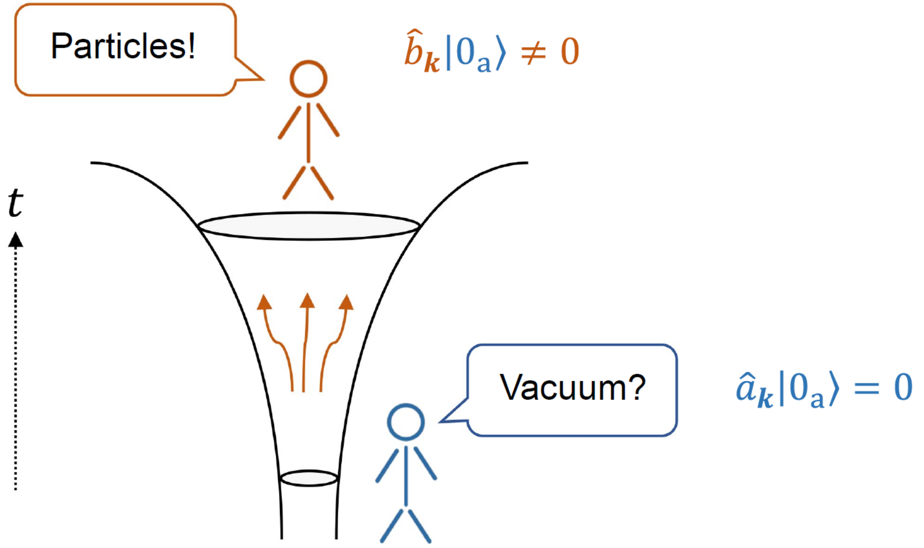
Particle production

- Two particular solutions v_k and u_k expand χ with different operators (and vacua),

$$u_k = \alpha_k v_k + \beta_k v_k^*, \quad \hat{b}_k = \alpha_k^* \hat{a}_k - \beta_k^* \hat{a}_k^+$$

$$|\alpha_k|^2 - |\beta_k|^2 = 1$$

- Can not impose Poincaré symmetry, $\omega_k^2 = \omega_k^2(\eta)$



Particle production

- Two particular solutions v_k and u_k expand χ with different operators (and vacua),

$$u_k = \alpha_k v_k + \beta_k v_k^*, \quad \hat{b}_k = \alpha_k^* \hat{a}_k - \beta_k^* \hat{a}_k^+$$

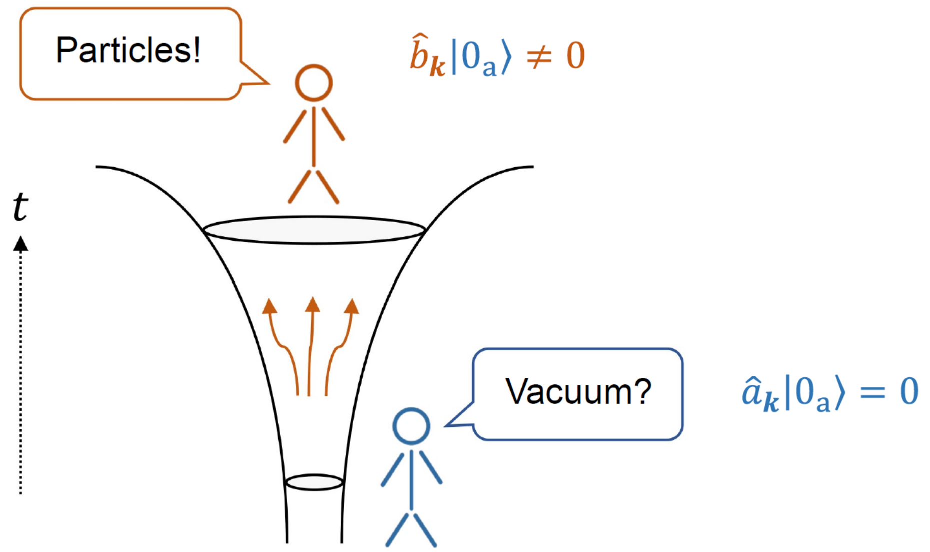
$$|\alpha_k|^2 - |\beta_k|^2 = 1$$

- Can not impose Poincaré symmetry, $\omega_k^2 = \omega_k^2(\eta)$

- If the systems is in the state $|0_a\rangle$ at η_i,

$$N_k(\eta_f) = \langle 0_a | \hat{b}_k^+ \hat{b}_k | 0_a \rangle = |\beta_k|^2$$

- Need to know v_k and u_k at the same time (η_f)



Background dynamics

- Toy model: de Sitter universe + sudden Minkowski

$$u_k(\eta_f) = 1/\sqrt{\omega_k(\eta_f)},\ u_k'(\eta_f) = -i\omega_k(\eta_f)/\sqrt{\omega_k(\eta_f)}$$

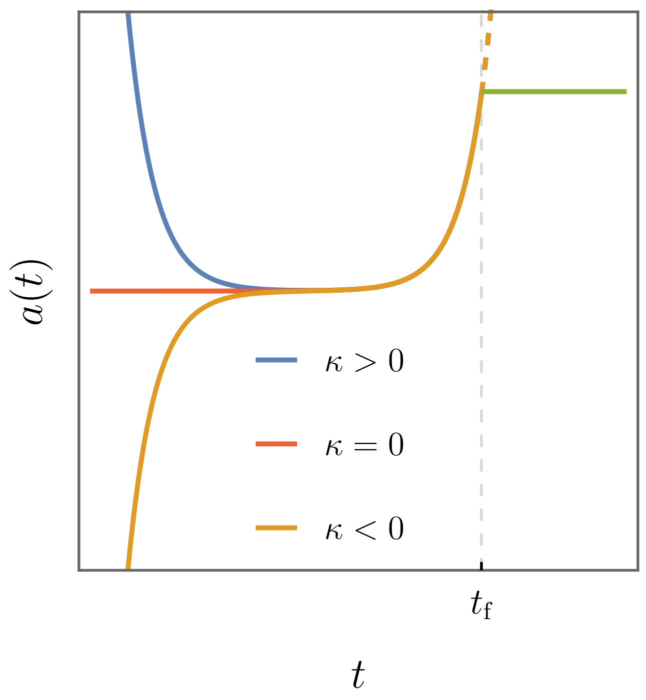
Álvarez-Domínguez *et al.* '23

- Alternative: adiabatic vacuum (smooth transition)

$$u_k'(\eta_f) = -\left(i\omega_k(\eta_f) + \frac{1}{2}\frac{\omega_k'(\eta_f)}{\omega_k(\eta_f)}\right)/\sqrt{\omega_k(\eta_f)}$$

- Choose Bunch-Davies as initial vacuum

- Take $\xi = 1/6$

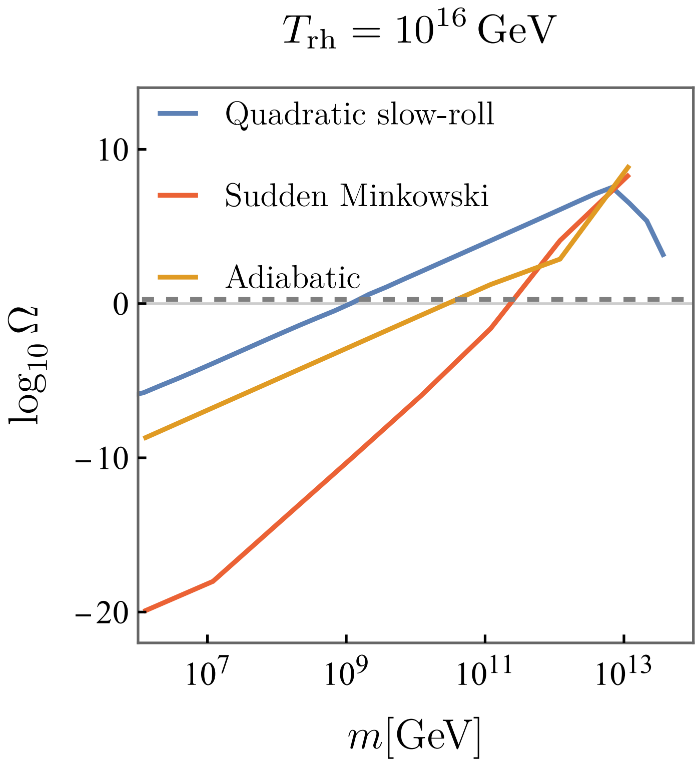


Density and abundance

- The total comoving density of particles is $n(m,\xi) = \int dk\, k^2 |\beta_k|^2$
- The physical density at the end of reheating is $n_{rh}(m,\xi) = n(m,\xi)/a_{rh}^3$
- If the field is non-interacting, the abundance of dark matter today can be expressed in terms of the reheating temperature as

$$\Omega(m,\xi) = \frac{8\pi}{3M_P^2 H_0^2} \frac{g_{*S}^0}{g_{*S}^{rh}} \left(\frac{T_0}{T_{rh}}\right)^3 m n_{rh}(m,\xi)$$

Density and abundance (flat)



- The adiabatic coefficient (the difference between the two inflation exits) is

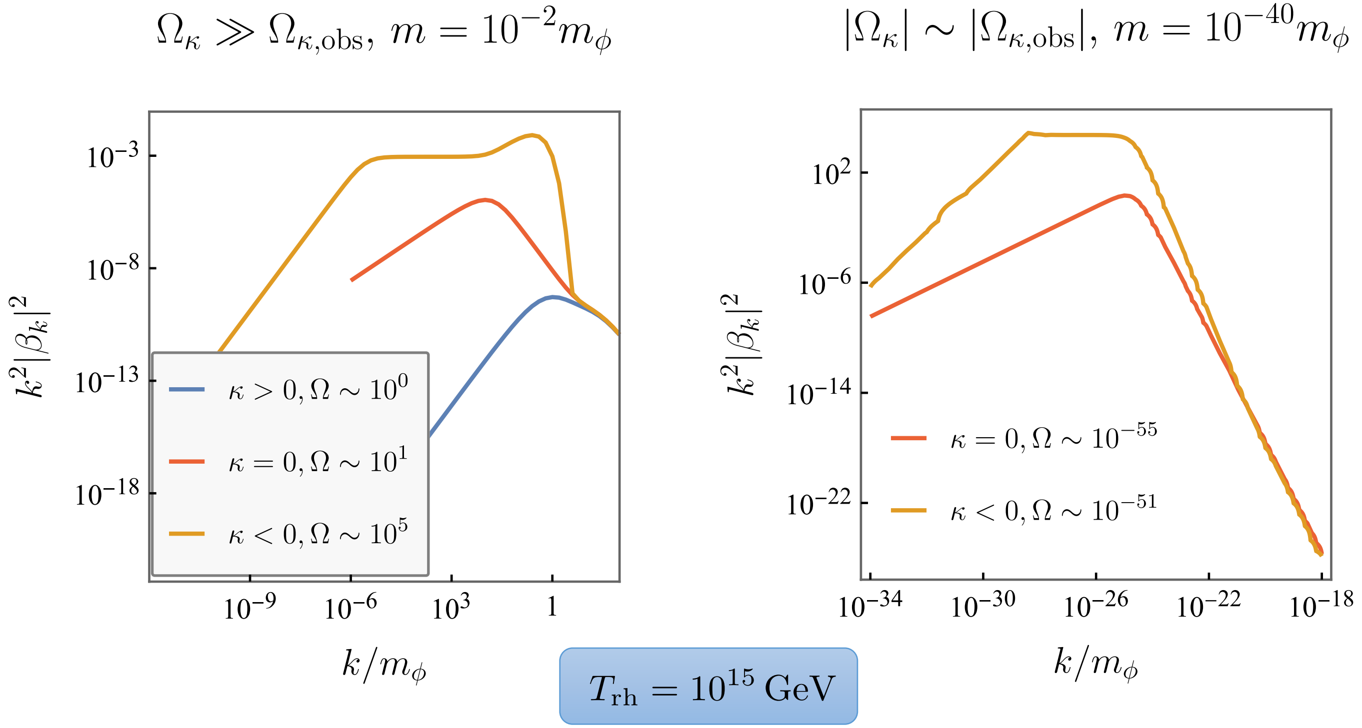
$$\frac{\omega_k'}{\omega_k^2} = \frac{m^2}{H^2\left(k^2\eta^2 + \frac{m^2}{H^2}\right)^{3/2}}$$

- For $m \gg H$, the two toy models coincide

- For $m < H$, the mass dependence seems due to the vacuum choice, not the evolution

Density and abundance (curved)

- There can be differences depending on curvature
- Small wavelengths do not see curvature effects
- Negative curvature increases production
- Positive curvature decreases production



Observation-compatible curvature leads to differences only for very small masses (negligible abundance)

Summary

- QFT in CS leads to gravitational particle production
- Initial spatial curvature can influence particle production for a spectator field
- However, differences are found for too large curvature, or for very small abundances
- Preliminary results suggest that cosmological dark matter production is not affected

Vacuum choice in the flat case

- Spacetime is always expanding → No preferred notion of vacuum a priori

- We take $|0_{\mathrm{a}}\rangle$ to be the Bunch-Davies vacuum, with corresponding v_k fuliflling

$$v_k(\eta \to -\infty) \sim e^{-ik\eta}$$

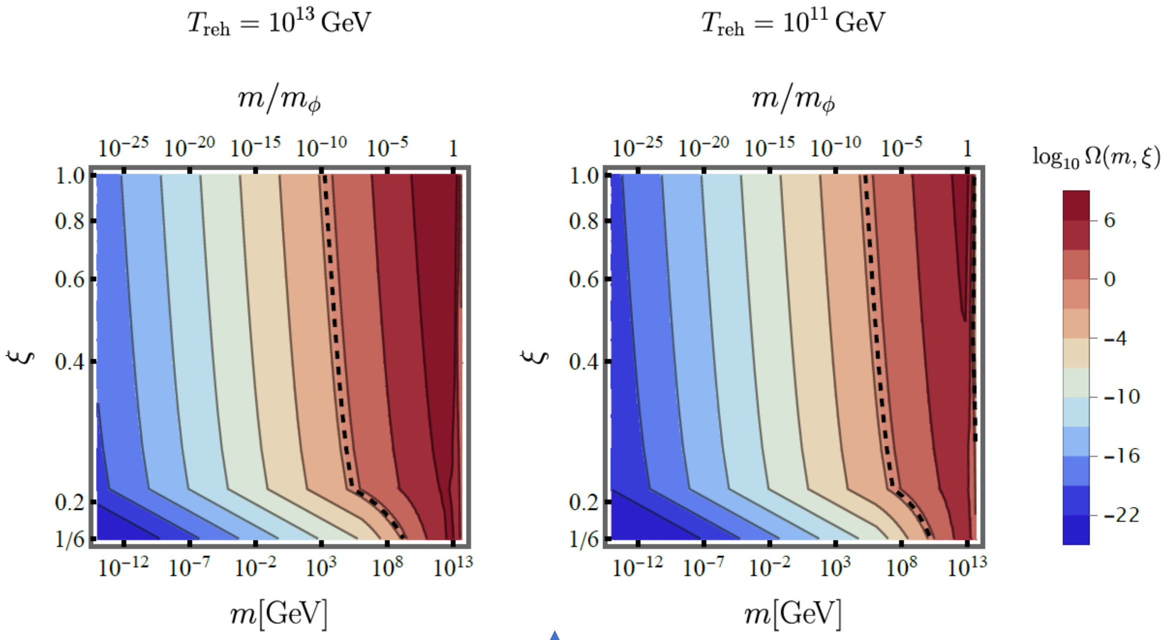
(Assume we recover de Sitter at the beginning of inflation)

- For $|0_b\rangle$ we take the adiabatic vacuum, defined when expansion is very slow, with

$$u_k(\eta_f) = 1/\sqrt{\omega_k(\eta_f)},\ u_k'(\eta_f) = -\left(i\omega_k(\eta_f) + \frac{1}{2}\frac{\omega_k'(\eta_f)}{\omega_k(\eta_f)}\right)/\sqrt{\omega_k(\eta_f)}$$

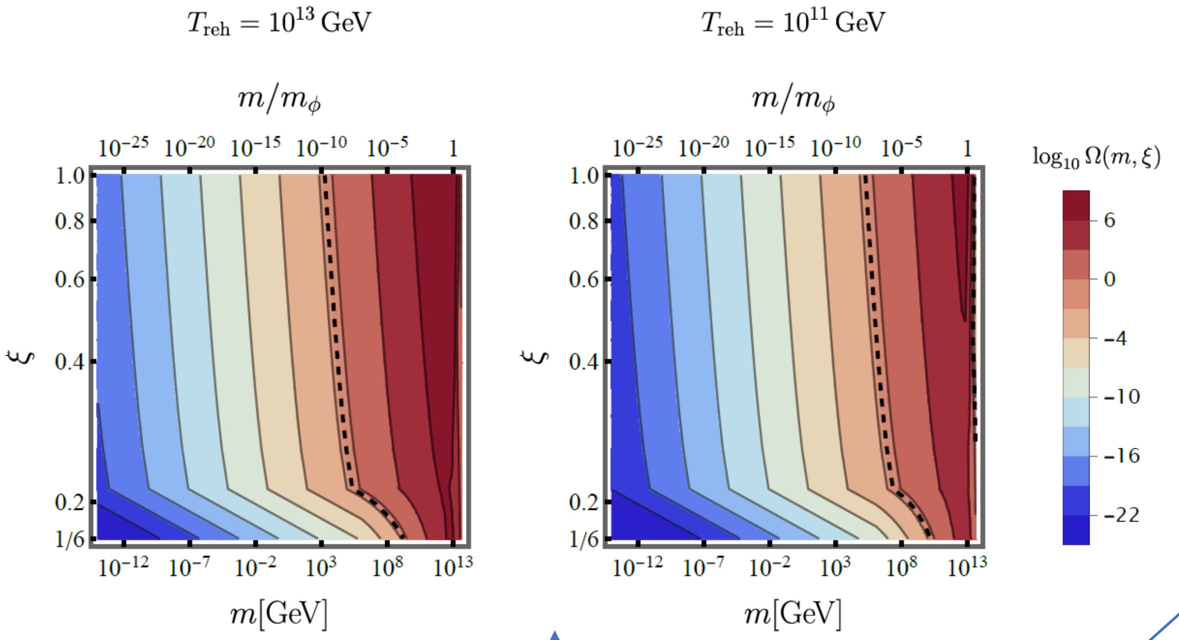
↓ Time at which expansion is adiabatic

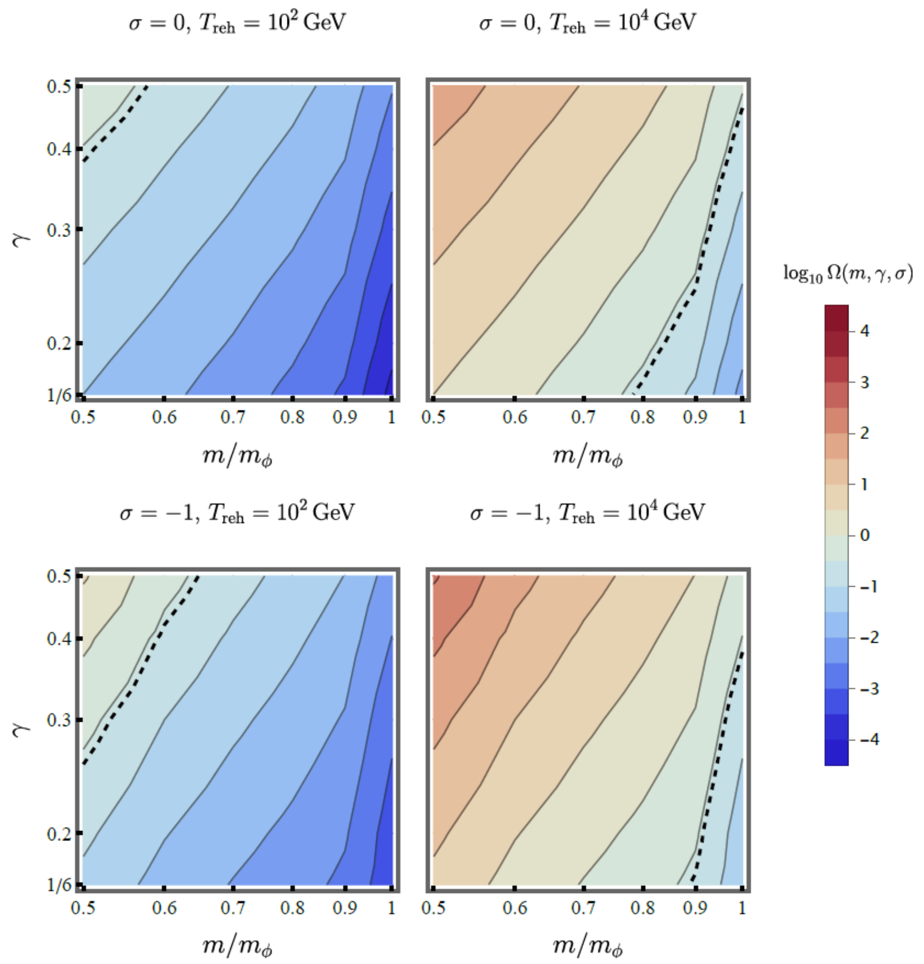
Density and abundance (slow-roll)



Scalar field (flat)

Density and abundance (slow-roll)





Scalar field (flat)

Vector field (flat)